Copenhagen Business School

Master thesis

cand.merc.(mat.)

Systematic Factor Investing with ESG

Authors:

Christopher Winkel Gorrissen

Erik Harald Bøgh

Philip Maldia Madsen

Supervisor:

Michala Hjelm-Hansen

May 15th 2020

Number of pages: 158

Abstract

Factor investing has a well-established and increasingly important role in an investors' portfolio. Much research has been done on the premiums these factors can provide. This paper uses data, free of survivorship bias, to show further evidence of a value and momentum premium on the equally weighed S&P 500 Index. Furthermore, it extends the research by incorporating the highly popular element, ESG, in several combinations of the joint portfolio of the value factor and the momentum factor. Previous research has indicated that combining these two factors may improve the risk-adjusted returns. This paper will make use of two slightly different portfolio implementation methods, the mixed approach and the integrated approach. These approaches will be applied in order to seek an excess return compared to the benchmark, the equally weighed S&P 500 Index. The findings lead to the conclusion that integrating ESG is in way a no hindrance, on the contrary, it is a particularly useful tool that leads to risk mitigation and powerful enhancement of the value and momentum strategies in our sample period. In addition, the research indicates to the finding that the mixed portfolio approach attains a higher excess return than that of the integrated approach. Moreover, it is concluded that portfolio turnover and transaction costs are an important consideration to take into account and has, dependent on cost assumption, a significant impact on the excess return. For a study such as the one provided in this paper, it usually raises more questions than answers even though the results may seem quite compelling. Since the results in this thesis are based on backtests, it can be argued that these strategies may not hold up in the future. Another question raised, could be whether these strategies could be implemented on other, less liquid, markets than the American stock market. Finally a much more thorough deep-dive could be done into ESG and to discuss whether it actually is a correct measure for companies' sustainability efforts.

Table of Contents

1 Introduction

In this section the motivation for the entire thesis will be explained in subsection 1.1 which will then be followed by the problem statement in section 1.2. Some thoughts of the methodology are given in 1.3 that is then followed up by the assumptions and limitations in 1.4.

1.1 Motivation

Where to place your capital in order to get the best possible return, is a question that has been pondered upon since the dawn of investing, and is still a highly researched area today. Many considerations have to be made before allocating capital towards an investment. What is the objective of investing? Is it to maximise return? If so, how much risk should an investor be willing to accept? Should the investment purpose be slightly more sophisticated and have other equally important goals besides a high return? Would these added purposes function as a restriction or could they actually serve as a driver to seek out investments with the potential to outperform the broader market and lead to excess return? Several approaches can be used to determine when and where an investor should place their capital. All these approaches have certain characteristics that explain the return and risk of the securities. Dividing these investment characteristics into relatable groups is broadly known as factor investing as made popular by Fama and French in their Nobel prize winning three-factor-model. The purpose of this thesis is to contribute with further research on factor investing, namely the value factor and the momentum factor. This thesis will do a deep-dive into these two factors in order to understand how an excess return can be achieved compared to the benchmark of the equally weighted S&P 500 Index over the used backtest period. It will do so by initially looking at the factors alone and combining a vast amount of the factor's underlying metrics in order to find which combination achieves the best results. Following this, the paper will then seek to combine these factors in order to examine if this provides any further benefits to the portfolio. Here, the two very common approaches will be implemented, respectively the mixed approach and the integrated approach. Once this has been implemented and discussed, an integration of the particularly popular and relevant element ESG will be implemented. It will be implemented as a screening tool in order to adapt to an investor's specific preferences, and also as a factor in itself, using the overall ESG score as a metric as well as the scores of the underlying pillars, Environmental, Social and Governance. It will be examined, if this implementation of ESG with the previously examined portfolio's which use the value and momentum factor in a mixed and integrated approach, leads to better or worse results.

1.2 Problem Statement

The main purpose of this paper is to document and investigate the performance of the traditional value and momentum factors as well as their joint performance. Equally important, the objective of this paper is to implement ESG and its various measures as factors and document their standalone performance. The paper will then proceed by incorporating the ESG into a value and momentum framework employing various portfolio implementation techniques. Furthermore, this paper aims at comparing various strategies and implementation approaches by evaluating their performance. The evaluation will in particular focus on the effect of incorporation ESG. In regards to performance, this paper applies the equally weighted S&P 500 Index as its benchmark. In evaluating these strategies the paper will use modern portfolio theory, as well as, various risk and performance measurements, such as the annualized compounded return, standard deviation, information ratio, drawdown, the third and fourth moment, etc.

1.3 Methodology

The applied empirical data used in this paper originates from two financial databases. Since this data complements each other, it has not been necessary to correct for differences in it. Since this has not been necessary the authors of this paper consider that the applied empirical data analysis combined with the founding assumptions and limitations can create a foundation for reliable and credible analysis and conclusion. This thesis is based on several classic and systematic investment strategies on which various analysis' of empirical data have been backtested with inspiration from previous research. Applying this, the paper makes use of a deductive approach where the paper's main questions are answered by the application of general theory and backtest.

1.4 Assumptions and Limitations

To measure value, momentum, and ESG this paper will limit itself to use the S&P 500 Index as its investment universe and use a wide variety of simple portfolio constructions. The objective of this paper is to reveal existing patterns and dynamics governing each strategy and various combinations of these strategies as well as various portfolio approaches for combining the strategies. In achieving this goal, rather than employing complicated approaches to determine optimal strategy combinations, asset allocations, the goal is to maintain a fairly uniform approach across all strategies portfolio constructions. As such, the backtesting of the portfolios will be simple and rule based. Consistent with (Asness, C. (2013)), the data used in the backtest will be done in a rather conservative manner by lagging the financial statements figures by 6 months as well as using the last years sustainability data to ensure data availability for an investor at the time of revising the portfolio composition. A more elaborate data discussion about the investment universe, momentum data, value data, and the ESG data takes place in subsection 2.1 2.2, 2.3, and 2.4, respectively. The authors intention here is, that this methodology minimizes data snooping and biases as much as possible so that the findings do not understate the true gross performances of the documented portfolios.

The financial statement figures are provided by Bloomberg and the sustainability data is supplied by Thomson Reuters, two leading data providers that a lot of investors and asset managers have great confidence in. Furthermore, the authors have deliberately chosen to conduct long-only strategies, rather than employing a long-short approach. This choice is again based on practicability in the eyes of a retail investor. Furthermore, a lot of papers employing long-short strategies imply that the trading strategy is free of transactions, a statement that is not necessarily true. As noted this paper documents gross performances net of transaction costs and frees as well as taxes. However, subsection subsection 8.1 will elaborate on the implications of transaction costs in the end by investigating the performance including various transaction costs.

Having noted this, the authors of this paper are fully aware that the conclusions of this paper are not viable in all situations, however, they provide a solid foundation for reliable results. The authors would also note that past performance is not an indicator of future performance and it is not recommended implementing the strategies presented in this paper without doing further research.

1.5 Thesis Organizer

The paper proceeds as follows. Section 2 outlines the data used for portfolio constructions. Section 3 covers the computations of various portfolio evaluation statistics and measures. Section 4 offers a brief review of value literature and has it focus on documenting and investigating the performance of value strategies. Section 5 also offers a short literature review of the momentum factor before evaluating and further analyzing the momentum trading strategies. Section 6 documents and examines value and momentum jointly across different implementation approaches. These implementation approaches includes mixed and integrated two-factor strategies. Section 7 documents the results of implementing sustainability measures as a factor, furthermore, this section reports and investigates the performances of incorporating ESG into a value and momentum strategy using various implementation approaches. These implementation approaches include mixed and integrated three-factor strategies and two-factor screen strategies. Section 8 briefly examines the implication of incurring transaction costs on the various implementation approaches for the value, momentum, and ESG measure. Section 9 concludes the findings. Lastly, section 10 briefly presents future related research that may be of particular interest to investigate.

2 Data

This subsection proceeds as follow. Subsection 2.1 introduces the employed investment universe. Subsection 2.2 outlines the data used to construct the momentum portfolios. Subsection 2.3 outlines the data used to construct momentum portfolios. Lastly, subsection 2.4 outlines the data utilized to construct sustainability portfolios.

The data used for this thesis comes from Bloomberg and Thomson Reuters. Due to this it has not been necessary to differentiate definitions, statements and key figures other than adding and removing companies that may briefly have been part of our data during which our data covers.

2.1 Investment Universe

The normal S&P 500 Index is market-cap-weighted, which means that every constituent are weighted according to their total market value of outstanding shares. As of March 2020, this has the implication, that the ten largest constituents account for approximately 25% of the performance of the S&P 500 Index. That is, movements of stock prices with higher market capitalizations will have a greater impact on the value of the index than that of companies with a smaller market capitalizations.

This paper uses the S&P 500 equally weighted total return Index, as its investment universe. In other words, this means that the eligible securities for investment at time t will be equivalent to the securities that comprise the S&P 500 Index at time t.

The S&P 500 is widely regarded as the best single gauge of the large cap U.S. equities. There is over USD 9.9 trillion indexed or benchmarked to the index. The S&P 500 is a stock market index that masseurs the stock performance of the 500 leading companies listed on Stock Exchanges in the United States and covers approximately 80% of available market capitalization [1]. Since this paper investigated well-established large American companies that are part of the S&P 500 Index, survival bias is unlikely to be the culprit. Neither will this investigation be culpable of look-ahead bias. All of the investment strategies described in this paper are fully implementable. The investigated S&P 500 Index in this paper is constructed manually over the period 1994:12:30 to 2019:12:31.

[1]The S&P 500 Index covers securities from the NYSE, NASDAQ, and and Cboe BZX Exchange.

The S&P 500 Index is quarterly rebalanced for in and out flows caused by company acquisitions, mergers, defaults, or in the case a company naturally grows (shrinks) in market value so that it becomes (no longer is) a member of the S&P 500 Index. Unfortunately, data providers such as Bloomberg or Thomsom Reuters do not provide data files that solely consists of pricing and accounting data for the S&P 500 constituents and the authors of this paper have not been able to find a complete dataset.

This paper starts of by extracting all daily pricing and accounting available data for all companies that have constituted the S&P 500 Index throughout the sample time. To amend the complication of in and outflow, the paper turns to the ”Leavers and Joiners” file by Thomson Reuters. The list displays 1900 records of companies that have been in and/or out of the S&P 500 Index in the time frame 1994:12:30 to 2019:12:31. Examining the ”Leavers and Joiners” list more carefully, its apparent that some companies like Aetna Inc. show up multiple times. The reason here being that it changed ticker name. To get around this problem for securities like Aetna Inc. this paper differentiates stocks by ISIN code instead. In the end, this resulted in 1174 unique records where 27 of the companies have actually been in and out of the S&P 500 Index more than once. An example of this is Advanced Micro Devices inc. (AMD) that was a constituent throughout 1994:12:30 - 2013:09:23 and again from 2017:03:20 - present. Its worth noting, that the Thomson Reuters list starts off with 482 unique companies albeit the S&P 500 Index should consist of 500 companies at all time. After a couple of years the Thomson Reuters list grows into 500, not being able to find a more accurate and stable list this paper chooses to ignore the missing 18 companies. Meaning that the index will start with some survival biases. Despite theses assumptions and minor flaws in data the manually constructed equally weighted S&P 500 Index with daily adjusted price for example has a relatively good fit to the real equally weighted total return S&P 500 Index with a correlation of 99.85 %, a monthly difference of 0.058 % and the final sum of log returns from the two series deviate just by 6 percent. Everything else equal, this gives a lot of confidence in the manually constructed S&P 500 Index constituent list, thereby also the data used to construct portfolios throughout the entire paper.

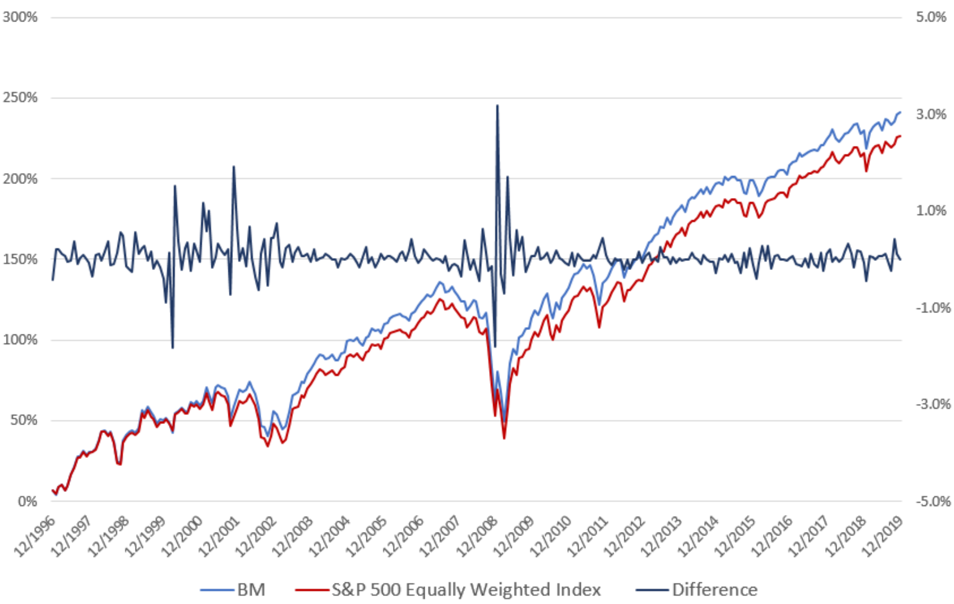


Figure 1: S&P 500 Equally Weighted

2.2 Momentum data

As mentioned in the subsection above, this paper uses the S&P 500 Index as its investment universe meaning that momentum results are fully implementable by investing in the S&P 500 Index. The empirical backtest and analysis conducted on the momentum portfolios in Section 5 uses the S&P 500 Index constituent list created according to the methodology in the previous subsection and matches the constituents at time t with the security pricing data file from Bloomberg. The pricing data file consists of daily closing prices for each of the securities in the time span from 1994:12:30 to 2019:12:31 and are adjusted for dividends. There were a handful of securities exhibiting insufficient pricing data, these securities were neglected throughout the whole paper.

In more detail, "in as of" and "out as of" variables are created to ensure that the implemented momentum portfolios at time portfolio formation time t the portfolio only buys and inspects securities that actually comprises the S&P 500 Index at time t. Note, the momentum portfolios select securities based on the past J- months and holds the best performing securities for the following K- months. This has a important implications. The momentum portfolio cannot buy a security in the upcoming period it eventually joins the S&P 500 Index, however, the day it joins the S&P 500 Index the portfolio can buy this security based on its performance prior to its joining of the S&P 500 Index. Effectively, this dynamical filtering these rules ensure that the portfolios are not exposed to front running and survival biases is keep to a minimum. Further it is worth noting, that the momentum portfolios do not discard securities exiting the S&P 500 Index it will hold these securities till the holding period terminates. The only exception being in case of merger, default, or any other corporate action that causes a security to no longer cease to exist.

In section 5 all the constructed portfolios takes the view as an investor that starts investing 1996:01:01 based on past return data back to 1994:12:30.

2.3 Value Data

Like the momentum portfolios, the value portfolios are fully implementable by investing in the S&P 500 Index. The empirical backtest and analysis conducted on the value portfolios in section 4 uses the S&P 500 Index constituent list created accordingly to the methodology in the previous subsection and matches the constituents at time t the with accounting data file from Bloomberg. The accounting data covers the following daily common value signals *Price-to-Book* (P/B), *Price-to-Earnings* (P/E), *Price-to-Sales* (P/S), *Price-to-Free Cash flow* (P/CF), and the *Dividend yields* (DIV) for each company. These values are lagged 6 months to ensure data availability and stability to investors at the time of revising the portfolio composition.

In the case of data unavailability the implementation discards the respective company. Consider the P/B value portfolio, here the implementation only sort companies that have adequate P/B data at the portfolio formation. This is also the case for value strategies including multiple dimensions. For example in the case of the value strategy sorting companies with respect to P/B and P/E, this portfolio will only sort companies that have adequate P/B and P/E data at the portfolio formation. Companies are indeed by the government required to file their 10-K reports with the SEC within 60 days of their fiscal yearends, but it is not always the case that companies actually comply. Its also worth noting, that using lagged values rather than contemporaneous values will have an impact. For example, if there is a general fall in stock prices during this six months than ratios measures 6 months prior will tend to be lower than ratios measured later.

Consistent with the methodology described in the previous subsection, the value strategy also only buys/inspects portfolios that are a member of the S&P 500 Index at time t to avoid survival bias and front running.

2.4 ESG data

The ESG data used in this thesis is provided by Thomson Reuters. The ESG score is an overall company score based on self-reported information on the Environmental, Social and Governance pillars of the companies. Thomson Reuters uses the rating called the ASSET4 ESG rating which are equally weighted assessments of company performance based on more than 250 key performance indicators that comprise their ASSET4 database. These ratings are then z-scored and normalized to position the scored between 0 and 100%. The score is calculated by using a simple mathematical formula that Thomson Reuters does not disclose. Since it is a formula and not an analyst giving the score they claim the ASSET4 scores a objective.

Environmental Pillar Score

The Environmental score measures a company's impact on living and non-living natural systems in the air, land and water and also complete ecosystems. It aims to reflect the degree of how well a company makes use of best management practices to avoid environmental risks and capitalize on environmental opportunities to be able to generate long term shareholder value.

Social Pillar Score

The Social score measures a company's capacity to generate trust and loyalty with its workforce, costumers, and society as a whole trough use of its best management practices. The aim is to give a reflection of the company's reputation and the health of its license to operate, which are key factors in determining its ability to generate long term shareholder value.

Governance Pillar Score

The Governance score gives a measure of a company's systems and processes which ensures that its board members and executives act in the best interest of the long term shareholders. It reflects a company's capacity through its use of best management practices, to direct and control its rights and responsibilities through the creation of incentives, as well as checks and balances in order to generate long term shareholder value.

3 Methodology

This subsection is organized as follows. Subsection 3.1 outlines the computations applied to evaluate portfolio performances. Subsection 3.2 briefly discuses implications of a backtest and timing. Subsection 3.3 presents portfolio considerations in regards to size and diversification.

3.1 Portfolio Evaluation Computations

This subsection will serve as a brief examination of the measurements used to assess portfolio performance and methodologies to quantify risk. Rather than rigours derivations, this section will be rather cavalier about the underlying mathematics and focus on its application and interpretation.

3.1.1 Return

The Oxford dictionary defines return as a profit on an investment over a given time horizon, expressed as a proportion of the original investment. If P_t defines the terminal price at time t and P_{t-1} denotes the initial price, thus, resulting in a $[0;T]$ time horizon, the time-t single period simple return is defined as:

$$r_t^S = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1 \tag{1}$$

When expanding the single-period return to a multi-period simple return of a portfolio, it is important that the return reflects a cumulative return (implying reinvestment of returns). The cumulative/compounded n-multi-period time-t simple return for a return is computed as a product of a sequence of single-period returns. I.e. the n-period compounded simple time-t return is given as:

$$R_t^S = (\prod_{t=1}^{n} \frac{P_t}{P_t})^{\frac{1}{n}} = (\prod_{t=1}^{n} (1 + r_t^S)^{\frac{1}{n}} \tag{2}$$

Expanding the simple return to a portfolio in a manner such that $R_{t,i}^S$ denotes the simple return at time t for security i, the portfolio simple return is simply the sum of the weighted simple return of constituents of the considered portfolio. Thus, the simple return of a portfolio at time t is expressed as:

$$R_t^S = \sum_{i=1}^{n} w_{t-1,i} R_{t,i}^S \tag{3}$$

I.e. simple returns are security-additive, which is a very desireable property because of obvious computation reasons. The challenge with the simple returns, is however, that multiplying returns close to zero may lead to excessive arithmetic overflow (Miskolczi (2017)). Additionally, these compounding computations are not desireable. The logarithmic returns, on the other hand, are very desireable because of their time-additive property. I.e. obtaining the continuously compounded k-period log return simply by summing each of the log returns in the K-period.

The single period time-t logarithmic return is defined as:

$$r_t^L = \log\left(1 + R_t\right) = \log\left(\frac{P_t}{P_{t-1}}\right) \tag{4}$$

The continuously compounded logarithmic return is simply computed by summing the K-period logarithmic returns i.e. the K-period compounded return is computed as:

$$r_t^L = \sum_{t=1}^{K} \log\left(\frac{P_t}{P_{t-1}}\right) \tag{5}$$

Given the definitions for simple and logarithmic returns it is easily seen that the relation between the simple and the log return is:

$$r_t^S = \exp(r_t^L) - 1 \tag{6}$$

To ease all return computations, avoid unstable approximations and arithmetic overflow this paper will use the time-additive property associated with logarithmic returns combined with the security-additive property of simple returns.

All of the examined portfolios in this paper are equally weighted. Constructing an equally weighted portfolio return is done in the following manner. First, each security return constituting the portfolio is continuously compounded with the time-additive logarithmic property in eq. (5). These returns are then transformed into simple returns with eq. (6) and utilizing the security-additive property these returns are equally weighted using eq. (3), implying that $w = 1/n$ where n is the number of securities. Since the portfolios are rolled over a full 23-year sample period meaning that they are reinvested, revised, and rebalanced throughout the 23-year sample; these returns are kept as logarithmic returns to continue the continuously compounding. Effectively, the formula for the equally weighted portfolio expressed in logarithmic returns collapses to:

$$R_t^L = \log(\sum_{i=1}^{n} w_{t-1,i} \exp(R_{t,i}^L)) \tag{7}$$

Successfully, the case of the portfolio that rebalances its books every third month, results in 95 equally weighted logarithmic portfolio returns that can be summed to a total 25-year full sample compounded return. With the compounded logarithmic returns at disposal the returns are annualized and transformed to simple returns using the relation in (6) for interpretational purposes. The annualization factor is the number of monthly return observations in a year divided by the total months in the sample. So, in the case of 285 months return data, the annualization factor is 12/285.

3.1.2 Standard Deviation

To quantify the risk involved with the portfolios' realized returns, this paper will use standard deviation as risk meassure. The standard deviation measures the amount of variation or dispersion of a set of values relative to its mean. Mathematically speaking the standard deviation is the square root of the variance. Financially speaking, the standard deviation referrers to what is called volatility. A high standard deviation (volatility) indicates that the realized returns are far spread relative to its mean, while a low volatility indicates that the returns are close to its mean. If r_i denotes a set of n returns for a security then the standard deviation is computed as:

$$\sigma(r) = \sqrt{\frac{\sum_{i=1}^{n}(r_i - \bar{r})}{n-1}}$$

where the standard deviation/volatility, $\sigma(r)$, is expressed in percentage. The standard deviation can easily be expressed in annualized terms by multiplying it with the square root of the number of observations as a fraction of a year. So in the case of monthly data, the standard deviation annualization factor is simply, $\sqrt{12}$.

3.1.3 T-test

With the annualized returns and their standard deviation at disposal this paper will conduct statistical tests to determine whether the realized returns are reliable or may be due to chance. The T-test applied in this paper is a statistical hypothesis test in which the test statistic follows a Gaussian distribution under the null hypothesis. This paper is concerned about rejecting two null hypotheses. First, rejecting that the portfolio's return is not different than zero. Secondly, rejecting that a portfolio's excess return (alpha return) is not different than that of the relevant benchmark. Here, the alpha return is computed as the excess return of the portfolio relative to the benchmark and the standard deviation is replaced with the standard deviation of the excess return.

The one sample t-test statistic, applied in this paper, is defined as:

$$t = \frac{\bar{r} - \mu_0}{\sigma/\sqrt{n}}$$

Where $\bar{r}$ is the sample annualized mean from a sample of returns, n denotes the sample size in years, and σ is the estimate of the annualized standard deviation of the population. Lastly, μ_0 denotes the hypothesized population annualized mean. This paper will distinguish between statistical significant and insignificant at the 95% level corresponding to a t-statistic being significant if its $\geq 1.96 \vee \leq -1.96$.

I.e. if a return in this paper is reported statistical significant it implies that it is at the 95% level statistically different from zero.

3.1.4 Skewness

The skewness measures the degree of asymmetry of the return's distribution with a value of zero indicating perfect symmetry around the distribution mean as is the case of a Gaussian distribution. Negative skewness commonly suggests that the left tail is relatively longer than the right and vice versa for positive skewness.

The skewness measurement defines perfect symmetry in terms of probability mass located on each side of the mean and nothing about the actual tail size and length. I.e. a distribution with a thin and long left tail and a equally fatter and shorter right tail has a skewness of zero - since the probability masses balance out.

Nevertheless, a positive skewness suggests a relatively long right tail; the probability mass is concentrated on the right side of the mean. While, a negative skewness, which is most common for financial time series data, suggests a relatively long left tail; the probability mass is concentrated on the left side of the mean.

In this paper, the skewness will be computed using monthly returns. $\widehat{\mathrm{Sk(m)}}$ denotes the estimated monthly sample skewness and is computed as:

$$\widehat{\mathrm{Sk(m)}} = \frac{1}{n}\sum_{t=1}^{n}\left(\frac{r_i - \bar{r}}{\sigma}\right)^3$$

Where σ denotes the standard deviation, r_t denotes the time t return, $\bar{r}$ is the average return, and n is the number of returns.

In regards to the examined portfolios this paper will use the following practical conventions to categorize the magnitude of skewness:

- No skewness if $SK \in [-0.5 : 0.5]$
- Moderate skewness if $SK \in [-1 : -0.5) \vee SK \in [1 : 0.5)$
- High skewness if $SK \in (\infty : -1) \vee SK \in (\infty : 1)$

Everything else equal if a portfolio's return distribution is highly negatively skewed, it implies that the portfolio has a larger likelihood of realizing extreme negative returns than extreme positive returns. However, as already stated, it also suggests that the majority of realized returns will be (slightly) larger than the mean. The reverse is true for highly positive skewed return distribution.

3.1.5 Kurtosis

Traditionally speaking, kurtosis has been explained in terms of the magnitude of the distribution's central peak. This paper will follow the academically acknowledged and more accurate description of Kurtosis outlined by Peter H. Westfall (2014) in his empirical research paper ”Kurtosis as peakedness, 1905-2014 R.I.P”. Peter Westfall states, that the only unambiguous interpretation of kurtosis is in terms of tail extremity; i.e., either existing outliers (for the sample kurtosis) or propensity to produce outliers (for the kurtosis of a probability distribution).

The reference standard is a normal distribution, which has a kurtosis of 3. In this aspect kurtosis is usually denoted in terms of excess kurtosis of 3: Excess kurtosis is simply the kurtosis subtracted by 3. In regards to the examined portfolios this paper will use the practical conventions to categorize the magnitude of kurtosis outlined by Westfall:

- An excess Kurtosis of approximately zero is called Mesokurtic. Compared to the normal distribution, its tails are equally long and proportioned.
- A positive excess kurtosis is called leptokurtic. Compared to the normal distribution, its tails are longer and fatter, and often its central peak is higher and sharper
- A negative excess kurtosis is called platykurtic. Compared to the normal distribution, its tails are shorter and thinner, and often its central peak is lower and broader.

Note the precise wording of ”often” since it is not a definite consequence of excess kurtosis. To sum up, kurtosis only measures the heaviness of tails; or equivalently, the propensity to yield extreme outcomes, and no obvious interpretation of the central peakness. A higher kurtosis means more of the variance is the result of infrequent extreme deviations, rather than to frequent modestly sized deviations.

Taking this to consideration, a portfolio exhibiting a leptokurtic return distribution entails that the propensity of realizing extreme outcomes are much more probable than that of a platykurtic distribution.

3.1.6 Risk-adjusted performance

The extreme global macroeconomic turbulence caused by the financial crisis in 2007-08 prompted a search for a new way of quantifying good performance and risk. Investors shifted their focus from generating raw returns to generating high risk-adjusted returns. The information ratio (IR) addresses this problem by considering the return in terms of risk taken:

$$\mathrm{IR} = \frac{\mathrm{E}\left(R - R^b\right)}{\sigma\left(R - R^b\right)}$$

The idea here, is that IR meassures the amount of excess return, return minus the benchmark return, in terms of excess risk taken, which is the difference between risk of investment and benchmark risk (Lasse (2015)). This paper considers physical cash as its benchmark (i.e. $R^b = 0$), so the IR collapses to:

$$\mathrm{IR} = \frac{E(R)}{\sigma(R)} \tag{8}$$

This number, will of course, be higher than the usual reported IR, since it gives us credit of earning the risk-free rate. However, it is still a good measurement of quantifying how much a strategy earns in terms of return per unit risk taken.

3.1.7 Drawdown

To assess the actual risk of a plunging market the paper will introduce the drawdown measure. The drawdown function is simply the cumulative loss since losses started. So the percentage drawdown is given by:

$$\mathrm{DD}_t = \left(\mathrm{HWM}_t - P_t\right)/\mathrm{HWM}_t$$

Where HWM_t is an abbreviation for high water mark (i.e. highest cumulative return at time t) and P_t is the cumulative return at time t. The DD meassure is zero when the cumulative return is at its peak and positive otherwise. Since the drawdown measurement describes something negative, this is accommodated by putting a minus in front of the value

In this paper we will consider the maximum drawdown (MDD) for the MSs and the SPX. The MDD is naturally given as:

$$\mathrm{MDD}_T = \max_{t \leq T} \mathrm{DD}_t$$

Furthermore, the total length of the drawdown and the time it takes to recovery, are included. The total length denotes the time it takes for the cumulative return to reach its peak again and the recovery denotes the time it takes from the valley to the same peak. (Lasse (2015)).

The drawdown concept will help describe the degree of exposure in a downwards moving economy and at which speed the MS can recover in comparison to the underlying S&P.

3.1.8 Value-at-Risk and Expected Shortfall

Value-at-Risk is a very common risk measure in the financial industry. It can give a holistic view of a portfolio's risk in one number. It is defined as the maximal loss , $\mathcal{L}$, over a given time horizon, T, on a given confidence-level, α.

$$P(\mathcal{L} \geq \mathrm{VaR}_\alpha(\mathcal{L})) = 1 - \alpha \tag{9}$$

$$VaR_\alpha(\mathcal{L}) = inf\{c : P(\mathcal{L} > c) \leq 1 - \alpha\} \tag{10}$$

In the case where $\mathcal{L}$ has a continuous distribution with distribution function $F_\mathcal{L}$, VaR is simplified to:

$$VaR_\alpha(\mathcal{L}) = F^{-1}(\alpha) \tag{11}$$

However, Value at Risk has it limitations. The greatest disadvantage is the uncertainty of what could happen if the VaR estimate is breached. That is, VaR does not explain how much of a drawdown the portfolio might get. Expected Shortfall solves some of the shortcomings of Value at Risk. It is defined as the average of all losses which are greater or equal than the Value at Risk estimate. In other words, the average loss in the worst (1-α) cases. It thus gives the expected value of an investment in the worst cases, however it is important to note that this is not the worst case scenario as that would be a 100% loss assuming no use of leveraged products, that could further make the loss exceed 100% of the initial investment. It is defined as the expected loss on condition that the VaR estimate is breached:

$$ES(\alpha) = E\{\mathcal{L}|\mathcal{L} \geq VaR_\alpha(\mathcal{L})\} \tag{12}$$

$$ES(\alpha) = \frac{\int_\alpha^1 VaR_\gamma(\mathcal{L})d\gamma}{1-\alpha} \tag{13}$$

Figure 2 illustrates a scenario where both portfolio's have the same VaR estimate but ES^2 has a higher expected shortfall than ES^1. This visualization underlines the importance of making use of both risk metrics as Value-at-Risk alone does not provide a thorough insight to the risks of the portfolio.

3.2 Backtest Considerations

The conclusions of this paper are established by employing a backtest. The authors acknowledge, that backtesting is not an exact science and that the choice of portfolio inception heavily impacts the performance of the strategies.

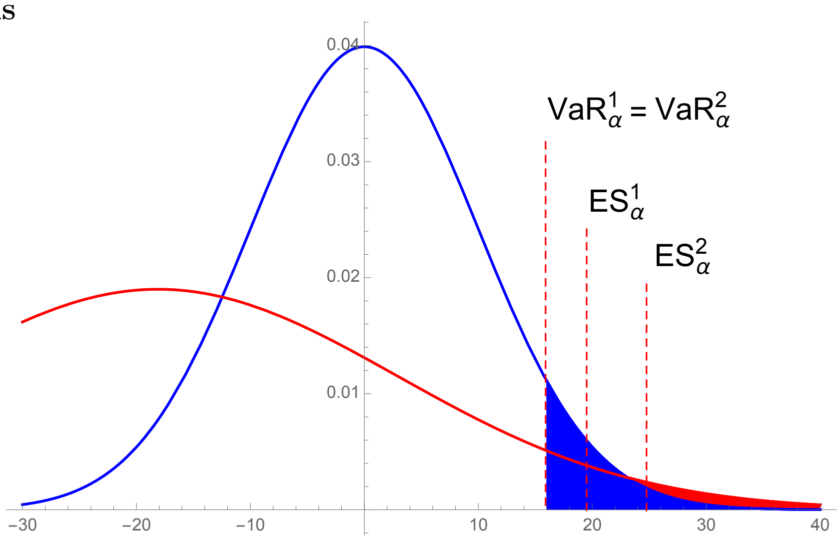


Figure 2: VaR and ES

The starting point of 96/97 for the standalone value and momentum portfolio is somewhat arbitrary. However, given a 23 year sample period there is a high probability that the sample period is neutral since it consists several economic cycles. This includes the Dot-com bubble in the late 90's and early 00's as well as large bull run leading up to the global financial crisis in 08. Figure 3 presents the risk-adjusted return (highlighted by the left a-xis) and return (highlighted by the right-axis) realized at different inception dates (highlighted by the x-axis) for the equally weighted S&P 500 Index. The figure, gives a good idea of how altering the inception date can result in very different performances. Figure 3 highlights importance of altering inception dates is sensitive to large economic cycles. The equally weighted S&P 500 Index has a risk-adjusted return of 0.46 when initiated in 1997 while it has

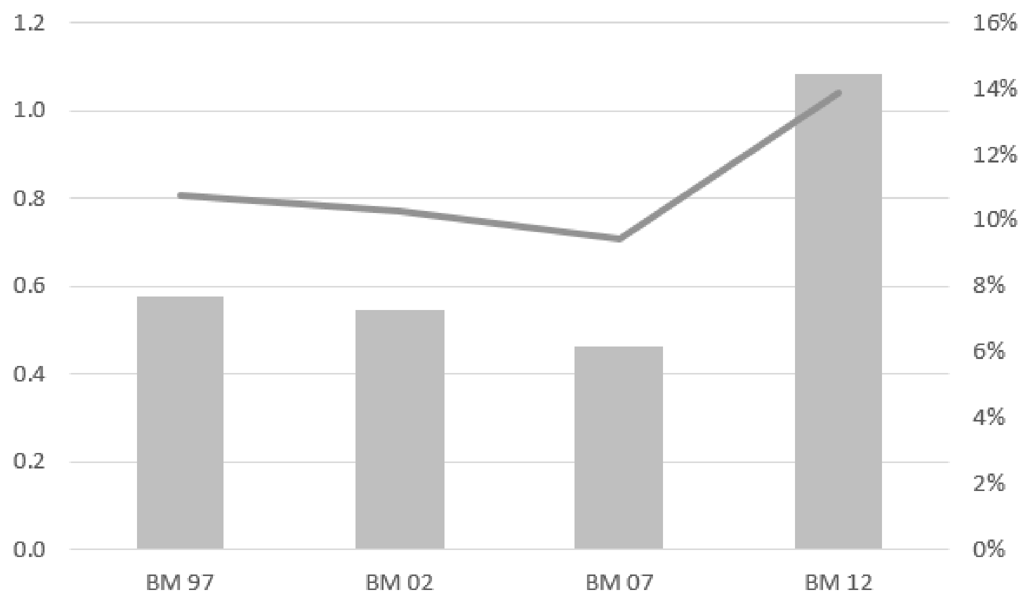


Figure 3: Caption

a risk-adjusted return of 1.08 initiated 2012 after financial crisis and its aftermath.

All the portfolios in this paper are equally weighted series of portfolios set one month apart. Which may minimize the effect of inception date. However, this is most likely to minimize underlying daily, weekly, and monthly trends and fluctuations that might be manifested in the market and not larger economic cycles. Nonetheless, it is a good methodology to amplify the significance and confidence of the conducted tests

In regards, to the portfolios including ESG measures the portfolios will run from 2006:01:01 to 2019:12:31 because of scarce data availability earlier than this. This means the documented portfolios skip the Dot-com bubble and the large market rising and initiated two years prior to the financial crisis in 2007-08. As documented in 7.2 this slightly impairs the performance of the value and momentum strategy initiated in 1997.

Another way approach of backtesting is to have a "in-sample" period and "out-of-sample" period. By using a in-sample period to find the best performing strategies and then testing them out-of-sample can be a great robustness test of the best strategy. The reason for not using a in- and out-of-sample backtest in this paper is that the authors felt the break up of pricing data would not have been optimal. If the data was broken into 50-50 so the in-sample period would be running from 95-07 and the out-of-sample period would be running from 07-19. Then the out-of-sample period would just be one long bull run and therefore created biases against strategies that do better in bear markets. One could have change the in-sample and out-of-sample period to be a more 30-70, so in-sample was running from 95-04 and out-of-sample would then be running from 04-19. However, after adjusting for timing risk the in-sample would start in 97, making the in-sample period only 7 years and mostly viewing the Tech bubble.

Therefore, to get the most "neutral" results, a full sample backtest has been determined as the best option. Since a full sample backtest has been used the main focus of the paper has not been to find one α generating strategy but more describing underlying trends in the marked. This trends will be analysed throughout the paper and used to generate portfolios with an above marked return, smaller drawdowns and lower volatility. However, past performance is not an indicator of future performance and that backtesting is not an exact science. So these results may not be the same going forward. The reason why using a backtest looking for patterns in marked still has its relevance might best be summed up by Mark Twain *"History doesn't repeat itself but it often rhymes"*.

3.3 Portfolio Considerations

As a rule throughout all portfolio construction in this paper, all portfolio are constructed with the 10-quantiles selection methodology. For example for the momentum strategies, monthly returns are ranked in ascending order and divided into 10 equally sized quantiles, which we call deciles. Using the S&P 500 Index as the investment universe, this means, that the first decile in a given month consists of the approximately 50 worst performing securities in terms of returns. While the 10th decile will consist of the 50 securities delivering the highest returns. It is approximately 50, because as mentioned earlier, the S&P500 Index constructed for this paper will roughly at any given time consist of 500 companies. More importantly, this selection methodology ensures a controlled and constant selection process throughout the whole strategy, which is a necessity when backtesting strategies.

In addition to this selection methodology, all portfolios are constructed in a manner that attempts to eliminate (most of) the idiosyncratic risk. This is achieved by constructing portfolios that are sufficiently large to ensure that the portfolios are considered widely diversified. Recall, that the portfolios are all (daily) equally weighted portfolios and that the variance of an equally weighted portfolio security can be expressed as:

$$\sigma_P^2 = \frac{1}{N}\bar{\sigma}_j^2 + \frac{N-1}{N}\bar{\sigma}_{jk} \tag{14}$$

Where $\bar{\sigma}_j^2$ denotes the average variance of the j securities and $\bar{\sigma}_j^2$ is the average covariances between the j and k securities. There are N values of J, so clearly, term 1 is an average of the variances. There are N values of J and *(N-1)* values of K. There are *(N-1)* of K because K cannot be equal to J - there is one less value of K than J. Resulting in *N(N-1)* covariance terms. Ultimately, this means that the second value is indeed also an average of the covariances. Equation 14 gives a intuitive understanding of what occurs to the variance of a portfolio, when including additional securities. The contribution to the portfolio variance of the individual securities goes to zero as N gets very large. however, the contribution of the covariance terms does not go to zero, rather, it approaches the average covariance as N increases. In other words, as the number of securities grows equation 14 reaches its asymptotic limit, $\lim_{n\to\infty} \sigma_P^2 = \bar{\sigma}_{ij}$. This is because the idiosyncratic risk (the individual security variance) can be diversified away, this however, is not the case for the variance caused by the covariance terms - which is called systematic risk.

Table 1 illustrates the relationship when dealing with S&P 500 Index equities. The table is a 10-year snapshot of 500 companies in the period of 2000:01:03 - 2009:12:01. Since the S&P 500 Index changed a lot throughout this 10 year window, the companies examined will obviously not all comprise the S&P 500 Index throughout this time window. However, these are all companies which all have been S&P 500 Index constituents in the full sample period 1994:12 - 2019:12. This assumption obviously deviates from the real diversification effect throughout this inspected time window, nevertheless, it still provides a good estimation of the diversification effect. The average variance and covariance of returns were computed using monthly log returns for all of the securities in the S&P 500 Index during this interval. The average variance was 0.01610; the average covariance was 0.00430. I.e. the average variance constitutes approximately 73% of the portfolio's variance, this is idiosyncratic risk that can be eliminated by holding a widely diversified portfolio.

Throughout this time period the equally weighted S&P 500 index has an annual standard deviation of 22.19 %. The expected variance found in table 1 for large portfolio, i.e. the average covariance of

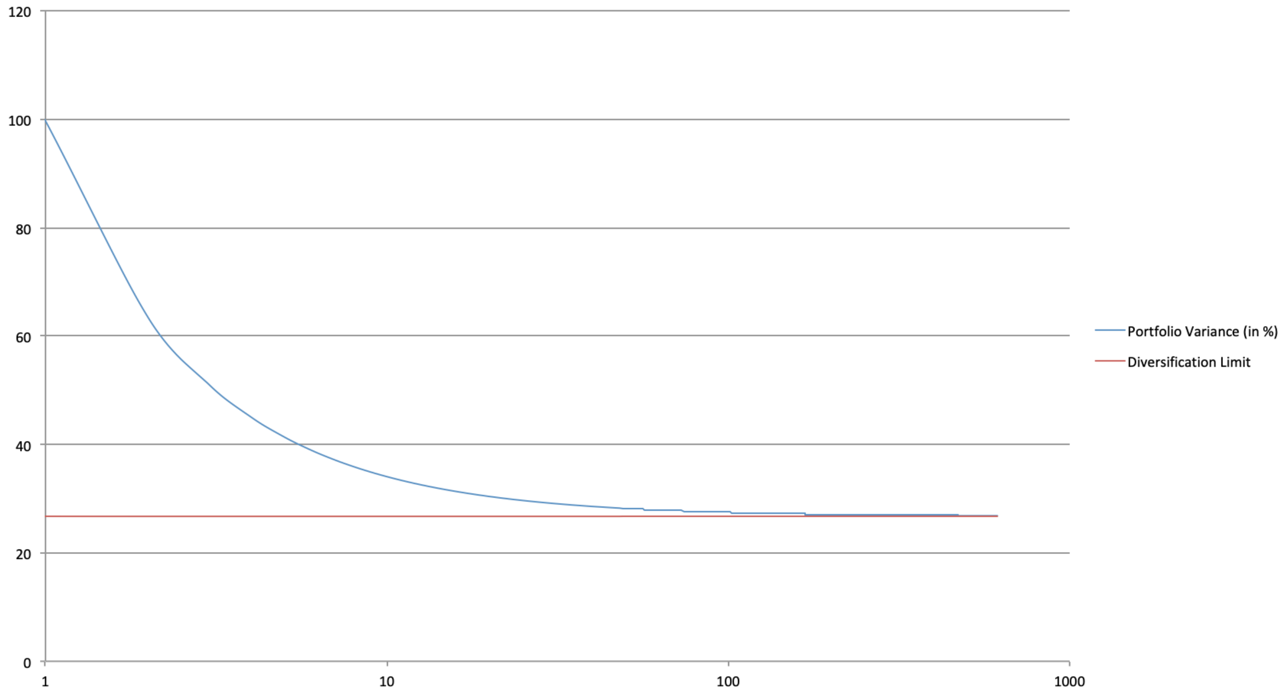
0.00430 translates into a annual standard deviation of 22.73%. This quick quality check speaks to the confidence of these examples.

Table 1: Diversification effect in the equally weighted S&P 500 Index over the sample period of 2000:01:03 - 2009:12:01.
The table depicts the average variance, average covariance, and the sum of these two terms which is the expected portfolio variance as a function of the number of securities added to a portfolio. The average variance and covariance are calculated using monthly return data for each of the 500 examined companies throughout the sample period of 2000:01:03 to 2009:12:01. For simplicity, these values are multiplied by 100. The 500 inspected companies do not comprise the S&P 500 Index throughout this period, but are companies that have been a member of the Index throughout the full sample period from 1994:02 - 2019:12.

Diversification Effect			
Number of securities	Average Variance	Average Covariance	Expected Portfolio Variance
1	1.6099	0	1.6099
2	0.8049	0.2152	1.0201
3	0.5366	0.2868	0.8236
4	0.4025	0.2869	0.7253
5	0.3220	0.3228	0.6663
6	0.2683	0.3587	0.6270
7	0.2300	0.3689	0.5989
8	0.2012	0.3766	0.5778
9	0.1789	0.3826	0.5683
10	0.1610	0.3873	0.5416
15	0.1073	0.4017	0.5090
20	0.0805	0.4089	0.4894
30	0.0537	0.4160	0.4697
40	0.0402	0.4196	0.4599
50	0.0322	0.4218	0.4540
100	0.0161	0.4261	0.4422
200	0.0080	0.4282	0.4363
300	0.0054	0.4289	0.4343
400	0.0040	0.4293	0.4333
500	0.0032	0.4295	0.4327
Infinity	0	0.4303	0.4303

As more securities are added to the portfolio, the average expected variance of the portfolio declines until it approaches the average covariance. Figure 4 illustrates the same phenomenon in terms of percentage. The vertical axis is the risk as a percentage of the risk of a single individual security in the S&P 500. The horizontal axis is the number of securities in the portfolio. This figure clearly shows how the variance of the portfolio declines dramatically within the first 10 assets included in the portfolio.

Figure 4: The diversification effect of adding securities for the equally weighted S&P 500 Index over the sample period of 2000:01:03 - 2009:12:01.
The figure illustrates how the total risk for a portfolio declines as a function of number of securities added to a portfolio in percentage. The blue graph represents the portfolio increasing its number of securities, while the red graph shows the asymptotic lower boundary of risk. The average variance and covariance are calculated using monthly return data for each of the 500 examined companies throughout the sample period of 2000:01:03 to 2009:12:01. The 500 inspected companies do not comprise the S&P 500 Index throughout this period, but are companies that have been a member of the Index throughout the full sample period from 1994:02 - 2019:12.



Intuitively, construction portfolios with a framework of choosing the top decile returns, worst book to market ratio, or any other trait will almost surely lead to a portfolio of securities exhibiting higher correlations than a randomly/average selected portfolio. Since the computations in this paragraph assumes that a security contributes with average variance and covariance, the results here may very well be slightly too optimistic. To avoid as much idiosyncratic risk as possible while considering the incremental transaction costs associated with adding securities to a portfolio, this paper will attempt to keep portfolio sizes to a minimum of 50 securities and a maximum of 100 securities.

4 The Value Factor

This section proceeds as follows. Subsection 4.1 covers some of the predominant existing work done on value factor investing, while subsection 4.2 reveals some of the existing behavioral explanations underlying value investing. Subsection 4.3 outlines how the value portfolios are constructed. Subsection 4.4 and 4.5 examine the performances of one-dimensional value trading results and multiple-dimensional value trading results, respectively. Subsection 4.6 investigates key crash characteristics for the value portfolios. Subsection 4.7 considers the turnover rates associated with value investing. Lastly, Subsection 4.8 concludes.

4.1 Literature Review

The value factor utilizes the idea of investing in stocks that appear to be trading at a value that is less than their intrinsic or book value. The goal in this trading strategy is to find stocks that are being underestimated by the market. For this to be possible the market must be imperfect as in the case where the market overreacts to positive or negative news thus resulting in stock price movements that do not correspond to the long-term fundamentals of a company. These overreactions present an opportunity to earn a profit by buying the corresponding shares of the companies at discounted prices or selling at overvalued prices. Value investing originally has its roots back from Benjamin Graham and David Dodds and their book publishing Security Analysis in the after-time of the great depression. This long-term strategy is used by successful contrarian investors such as Warren Buffett and Bill Ackman. Various metrics can be used to find the intrinsic value of a stock. Typically, it is a combination of studying financial statements and analyzing more intangible assets such as a company's financial aspects such as the brand, business model, target market and competitive advantage.

Much academic empirical research has been done on the existence of the value-premium. It had a major breakthrough in the academic world following the publishing of two papers by Fama & French. Davis, French and Fama (2000) researched the value-premium for all industrial companies traded on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX),and the National Association of Securities Dealers Automated Quotation (NASDAQ) from the Great depression in 1929 until 1997. This research, excluded financial companies and companies within transport and supply. Where they divided the companies into three groups based on their B/M-ratio. A group of growth stocks with the 30% lowest B/M ratio, a group with the 40% stocks that were neither growth nor value stocks and a group of value stocks consisting of the companies with the 30% highest B/M ratio. To control for an eventual size factor the stocks were further allocated into two groups; large cap and small cap, depending on whether the company was above or below the median for stocks on the NYSE. The portfolio returns were calculated by market weighting the stock returns against the market value in relation to the relevant portfolios market value. They managed to show that across all the stocks on average there had been a highly significant value-premium of 0.46 percent per month throughout the data period running from 1927 to 1997 which was equivalent to a yearly premium of ca. 5.5%. They also found that during the same time period small cap companies had a value premium of 0.56 percent on average per month which is noticeably larger than for large cap companies which in the same period had an average value-premium of 0.29 percent per month.

In another paper by Lakonishok, Schleifer and Vishny (1994)[2] they find that the search for deep-value stocks is worth-while. By dividing the stocks into ten deciles instead of just three portfolios they show that the ten percent stocks with the highest B/M ratio have a significantly higher return than the top 30 percent. The same result was achieved when using P/CF as sorting criteria which gives a yearly value-premium of 11 percent during the period running from 1963 to 1990. Furthermore, their research demonstrates that the slightly more sophisticated two-dimensional strategies where stocks are sorted using two criteria instead of merely one, does pays off. For a portfolio consisting of the ten percent stocks with the lowest P/C and low sales growth it was possible to attain an average return of 22.1 percent a year while a growth portfolio containing the 10 percent stocks with highest P/C and largest sales growth only gave a return of 11.4 percent per year. Financial companies have not been included in the data of the of the aforementioned research. However, research on this has been made. Barber and Lyon (1997) examined whether a value premium for financial companies exists compared to non-financial companies on the American stock market by dividing the companies into deciles based the stocks B/M ratio. During the period 1973 to 1994 a value-premium for financial companies was found to be 1.1 percent per month whereas for the number for non-financial was slightly higher at 1.44 percent with the difference not being statistically significant. Furthermore, it is worth noting that financial companies on average have the same value as non-financial companies where the difference in return is not explained by a potential size factor.

Another research of the value-premium was done by Risager (2013) who examined how value stocks

[2] From hereon denoted as LSV(1994)

fared during the IT-bubble and the financial crisis where it was found that value-premium had deteriorated in the period 1998 to 2010, however value stocks have still fared better than growth stocks. It was also shown that in 1998 and 1999 a negative value-premium was observed which corresponds well with the hype surrounding growth and particular IT stocks. It was not until 2007 just prior to the financial crisis that growth stocks ones again outperform value stocks and again in 2009 and 2010 after the peak of the crisis. As stated above there has been researched into the premium of the value factor showing that over the long term value investing has outperformed investing in growth stocks. Since the return on the total stock market is a market weighted average return of the two portfolios it can further be deducted that value stocks also have outperformed the market. However, one should not assume that value investing is risk-free which certainly has been underlined by the financial crisis.

Even though most academic papers agree that value stocks historically have performed better than growth stocks the reason for this is not exactly clear. As previously mentioned, it was Fama & French (1992) and LSV (1994) who put focus on the presence of the value premium. Even though they both agree on the empirical evidence of it, they also propose different interpretations of the value premium in the form of rational and irrational forces. Traditional rational financial theory believes in efficient markets, where return and risk go hand in hand which therefore gives a higher return on value stocks compared to growth stocks due to the higher risk associated with value stocks. Behavioral theory does not deny that risk may play an important role with regards to the pricing of stocks. According to Risager (2013) the most important reason for why the value factor outperforms the growth factor is due to investors being over optimistic of growth stocks future performance and forgetting that even the greatest growth stocks can be overpriced which eventually will lead to a correction when the market figures out that the expected earnings were excessive. The efficient market hypothesis states that higher risk must be compensated by an accordingly higher expected return. Therefore, the evidence of a value premium could lead an investor to believe that risk must thus be higher since value stocks on average have outperformed growth stocks and growth stocks therefore should have a lower risk. However, this theory (Ilmanen 2011) has had a hard time justifying that there should be a higher risk attached to investing in value stocks compared to growth stocks due to the lack of empirical data. Cai(1997) researched into whether value portfolios have higher volatility than growth portfolios which was done by comparing some risk targets for both value and growth stocks measures using Book to Market B/M ratio on the Japanese stock market. He found that there was only a minor difference in the risk of the value and growth stocks which therefore only explained a small part of the difference in return. Risager (2013) also researched into the volatility of value and growth stocks on several markets and was able to report similar findings as Cai(1997) with this larger test.

Cai(1997) also found identical market exposure but also opens for the discussion about how well value and growth stocks fare during a recession. He found that value stocks in no way were outperformed by growth stocks, as for example during the recession that occurred in Japan from 1989 to mid-1990. LSV (1994) likewise found a positive value premium during periods with recession which also has been seen on other markets that value stocks should perform worse during recessions than growth stocks.

4.2 Behavioral Explanation

The previous subsection covers literature work attempting to explain the value premium using the efficient market hypothesis. In the next section a review of some selected literature on behavioral finance will be examined. It will examine the impact of psychological factors on the financial markets that lead investors to often acting irrational. It will also cover the limits to arbitrage trading that restrict the possibilities to exploit these imperfections in the market.

Using extrapolation of the historical earnings growth far into the future Ilmanen (2011) says that the main behavioral expectations for future growth have been over optimistic. He shows that the expectations for growth on American stocks have been too optimistic and have consequently disappointed when released. Even though growth companies have grown faster than value companies and continue to do this for some years, over time they will however reverse towards the mean. Since the market underestimates this mean reversion there has been a tendency for companies to perform well despite low prior sales growth or low forecasts for future growth. According to La Porte et Al. (1997) they find higher statistic significance for falls in the prices of growth stocks when the quarterly reports are announced while the price of value stocks on the contrary increase and that this difference explains up to a third of the value premium on the American stock market from 1971 to 1993. This disappointment factor is most likely even larger since companies give ongoing indirect and direct information, for instance as analysts publish market analysis about companies and the market as a whole.

Another explanation of the value premium could be found in the systematic optimism of analysts and their overweight of buy recommendations. This could be due to their own personal gain from a bull market and ordinary investors' preference for popular growth stocks. Asset managers tendency to likewise outweigh these kind of stocks could also be a cause since an eventual loss would be easier to explain to clients. Furthermore, herd behavior and other social influences such as information cascades can also cause and contribute to speculation. Barberis and Thaler (2002) present seven different forms of psychological bias in relation to investor behavior which form a basis for how the financial market agents make judgements:

- **Overconfidence:** Research shows that people believe too much in the own capabilities. People tend to indicate too small outcomes for their estimates along with inaccurate probabilities for these outcomes.

- **Optimism and wishful thinking:** People tend to have over optimistic interpretations and expectations of future outlooks. In a survey (Buehler, Griffin and Rose, 1994) over 90% believed that they were above average when self-rating activities such as driving skills or personality aspects like humour and how they perceived how agreeable they were.

- **Representativeness:** Irrational investors put too much weight on factors that do not have major influence on what is being weighed. Furthermore, there is a tendency to put too much weight on little data, such as the hot-hand phenomenon. If a basketball player has made three shots in a row fans will believe the player is on a hot streak and will score again even though no such evidence may exist in the data.

- **Conservatism:** If one as an investor takes a sample that is not representative of the entire population one weighs too much weight on former results. That is, if a sample is representative of an underlying model, people will tend to overweight it whereas if there is no significant underlying model people will over-emphasize the base rates relative to the sample size.

- **Belief Perseverance:** People stick to their decisions for longer than is rational and they are unwilling to search and accept new information that is contrary to their original stance. If such information should be established, some studies have found that a confirmation bias could lead people to misinterpret the evidence and understand it as being in favor of their hypothesis.

- **Anchoring:** Investors put too much weight on their initial view and anchor themselves to this existing view. Experiments usually show that when adjustments are made to the initial view they are typically not sufficient.

- **Availability bias:** Events that have happened recently weigh too much in peoples' minds. While this is a sensible approach, not all memories are equally retrievable. This may lead to biased estimates with more salient and recent events weighed heavier and mislead the estimate.

Apart from the psychological bias Thaler and Barberis also mention several reasons why it is not possible for investors to utilize inefficient markets perfectly. In an efficient market prices are always trading at the fundamental value of the stock thus meaning that there is "no free lunch". However "No free lunch" does not necessarily mean that prices are right. The following bullets cover some of the traditional mechanics that prevent arbitrage trading:

- **Fundamental Risk:** Arbitrage is defined as a self-financing and risk-free investment where you buy an undervalued asset and simultaneously sell an overvalued asset. To be able to hedge the position perfectly, the two assets must be perfect substitutes with perfect negative correlation. However this is not the case and the investor is still be exposed to fundamental risk.

- **Noise Trader Risk:** Even if one achieved a perfect correlation, it is by no means sure that the other investors and the market in general behave rationally, which could push the price further away from the fundamental and true value. In addition to this, a so-called agent problem may occur which the authors call "the separation of brains and capital" since the portfolio managers do not invest their own money.

- **Implementation Cost:** This is the most obvious since certain transaction costs such as the bid-ask spread, brokerage fee and market impact are connected to the practical execution of arbitrage trading. Also great restrictions can be made on shorting assets.

4.3 Value Trading Strategies

From the existing literature it can be concluded that a value premium has been present and it has been possible to harvest it using strategies that makes use of the value metrics to decide which stocks to buy. This section uses value metrics such as Price/Book, Price/Earning, Price/Sales, Price/free-cash flow and dividend yield will be used in the proposed strategies. Notably, non-positive value metrics will be excluded. When this is covered, the section will dive deeper by combining these metrics in order to examine if the combined information these value metrics provide can deliver a greater value premium. For simplicity this section will solely be pursuing long-only strategies and will use these metrics to decide which stocks to buy. The value metrics lay the foundations for our value strategies and are grounded in the following reasoning:

Price to Book (P/B) also known as book value which is a measure of a company's assets and compares them to the stock price. In the case where the price is lower than the value of assets the stock is undervalued under the condition that the company is not in any financial hardship, however this varies by industry. It is calculated by dividing a company's stock price per share by its book value per share:

$$\text{P/B Ratio} = \frac{\text{Market price per Share}}{\text{Book value per Share}}$$

This metric thus uses the stock markets value of the company as a forward-looking metric in order to give an indication of what the market believes the future cash flows will be. It can provide an important reality check for investors who want growth at a reasonable price. A classic sign of over-valued growth stocks is typically seen in the case where a company has both a high P/B ratio and a low return on equity (ROE). Using this as the sole value metric for a trading strategy an investor must believe that there is value to be found by buying stocks that trade in the market with a discount compared to the net asset value of the company.

Price to Earning (P/E) Ratio is a commonly used metric as it is easy to compare between different companies. It relates a company's share price to its earnings. This is also known as the price multiple or earnings multiple. It is used to determine the relative value of a company's shares in a point-by-point comparison. A high P/E ratio could indicate that the company is over-valued or else that investors are expecting higher growth rates in the future.

$$\text{P/E Ratio} = \frac{\text{Market value per Share}}{\text{Earnings per share}}$$

The P/E ratio can differ a lot between sectors as each sector normally has its own range of P/E ratio. Typically the P/E of technology companies tend to be higher than that of industrial companies. That could be due to the willingness of investors to pay for the larger upside that many technology and pharmaceutical companies can offer. Using the current P/E of a sector and comparing it to the historical average can help indicate if the sector is undervalued. Likewise, if the P/E is higher than average this could deem the sector overvalued.

Free cash flow (FCF) is the cash generated from companies' revenue or operations after the cost of expenditures have been deducted. Free cash flow is the cash remaining after expenses have been paid here including operating expenses and large purchases called capital expenditures which is the purchase of assets like equipment or upgrading a manufacturing plant. If the company is generating free cash flow, it will have money left over to invest in the future of the business, pay off debt, pay dividends or reward shareholders by issuing a share buyback. In order to use it as a value metric, this paper will combine it with the price of the stock thus getting the price to free cash flow ratio (P/FC):

$$\text{P/FC Ratio} = \frac{\text{Market Capitalization}}{\text{Free Cash Flow}}$$

A lower value indicates that the stock is relatively cheap whereas the high indicates it being overvalued. Like other equity value metrics, it is best compared against other similar companies in the same sector.

Price-to-Sales (P/S) Ratio compares a company's stock price to its revenue thus giving an indication of the value placed on each dollar of a company's sales or revenue. The P/S ratio is calculated by dividing the market capitalization of the company by its total sales over a designated period, typically a year. Another approach is on a per-share basis by dividing the stock price by sales per share. It shows how much investors are willing to pay per dollar of sales which is best used when compared to other companies in the same sector. A low value could indicate that the stock is undervalued and likewise a high value could suggest that it is overvalued. It is important to note that the P/S Ratio does not take into account if the company has any earnings or subsequently will make any. Furthermore the ratio does not account for debt loads or the status of the given company's debt load.

$$\text{P/S Ratio} = \frac{\text{Market value per Share}}{\text{Sales per share}}$$

It is very useful as a valuation tool since it indicates what investors are willing to pay per currency of sales. Like the other ratios, the P/S ratio is best used when comparing companies in the same sector. Intuitively, a high ratio indicates an overvalued stock whereas an low ratio indicates a undervalued stock.

Dividend yields are the reward shareholders earn for being a holder of equity in a company. These can be issued as cash payments, as shares of stock or as cash dividends which are the most common. Since dividends mean money leaving the company's books for good this has an impact on the share price. It rises on the announcement by approximately the amount of dividend declared and declines by a similar amount at the opening of the session after dividend pay-out. The dividend strategy is the odd one out as this could be a factor in itself. As it can take many years for a value stock to trade at the value that investors know to be true, receiving income along the way makes that wait slightly more appealing. Dividends are in some cases also included in the quality factor, since many investors insist on companies having a long history of paying dividends before investing in them.

The following subsection will cover all the unique value trading combinations possible using the five value metrics. The number of unique feasible value trading strategies is computed using:

$$\text{nCr} = \begin{pmatrix} n \\ r \end{pmatrix} = \frac{n!}{k!(n-k)!} \tag{15}$$

Remembering that $\frac{n!}{(n-k)!}$ gives all the permutations and the k! in the denominator disregards the duplicates, thus the case of five measures yields:

$$\begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ 5 \end{pmatrix} = 2^5 - 1 = 31$$

31 strategies as the maximum amount of unique value trading combinations. In regards to the data used for the various value metrics this paper will be using data lagged by six months. The reason for this is that even after companies release their financial statements, errors and correction may occur after their release. Therefore, to ensure that the value trading strategies are fully implementable for an investor at the time the paper will lag all financial statements by 6 months.

The following section will emphasize on the five single value metrics as unique strategies all initiated at the same time thus holding some timing risk. The subsequent section will then take this timing risk into account and initiate the trading strategies a month apart throughout the initial year in order to take underlying risk and market structures into considerations. This is more thoroughly elaborated in the value data subsection 2.3.

4.4 One-Dimensional Value Trading Results

Table 2 reports the performance characteristics for value decile portfolios that are each based on the five one-dimensional value measures addressed in the previous subsection, respectively. The portfolios all start June 1996 and terminate December 2019. The performance characteristics include annualized compounded returns, the corresponding standard deviation, *t*-statistic, and the information ratio. For example Panel A displays the performances characteristics for the portfolio that divides the universe of stocks annually into 10 (B/P) deciles at each portfolio formations using six months lagged price-to-book data. In respect to the annualized compounded returns for the portfolios over the postformation, the low P/B (values) securities exhibit an annual compounded return of 12.16 percent and the low P/B (glamour) securities have an annualized compounded return of 12.38 percent, for a difference of -0.22 percent.

The natural question is: what does the P/B ratio capture? Unfortunately, the P/B reflects several factors. The price-to-book reflects the value investors attach to a company's equity relative to the book value of its equity. A high P/B may describe a company with a lot of intangible assets, such as research and development (R&D) capital, that are not included in the accounting book values because R&D is expensed. It could be a company with an attractive growth component that does not enter the book value but is included in the market price. It could also be a low risk company whose future cash flows are discounted at low rates. Finally, a high P/B may describe an overvalued glamour stock. This however, is not was the evidence indicates. The point here being that the equivocal results are due to the nature of variable. The P/B is not as well-defined a factor when it comes to economical interpretation as the others examined in this subsection, and this is reflected by the glamour portfolio outperforming the value portfolio. The findings here being in contrast to previous research, for example the work by Fama & French (1992). One explanation may be the fact, that the economy is moving towards a more service-based economy. This type of economy is heavily impacted by intangibles' ability to generate cash flows. As the current accounting procedure for P/B does not measure intangible assets, this may cause P/B to misclassify value stocks as growth stocks because companies show smaller assets on their balance sheet than they truly have. Its important to mention, that this narrative does not mean that the value premium is structurally impaired, however, it does indicate that perhaps other value multiples that capture intangible assets may realize a higher value premium.

Table 2: Performance Characteristics for Decile Portfolios Based on One-Dimensional Sortings by various Measures of Value.
The table presents performance characteristics for 10 decile portfolios that are formed in descending order based on P/B, P/S, P/FC, P/E, and DIV value signals in the period of 1996:06:30 - 2019:12:31. P/B is the ratio of stock price to book value; P/S is the ratio of stock price to sales; P/FC is the ratio of stock price to cash flow; P/E is the ratio of stock price to earnings, and DIV is the dividend yield measure. r denotes annualized compounded return, σ is the standard deviation denoted in percentage, t(r) is the t-statistic, and IR is the information ratio. All of the computations are determined using realized monthly return data for each of the decile portfolios.

	Glamour								Value	
	1	2	3	4	5	6	7	8	9	10
					Panel A: P/B					
r	12.38	9.82	9.20	10.92	12.01	11.19	10.81	9.92	10.01	12.16
σ	20.26	18.58	17.92	18.31	18.19	19.18	19.16	19.50	20.85	22.11
t(r)	2.93	2.53	2.46	2.86	3.17	2.80	2.71	2.44	2.30	2.64
IR	0.61	0.53	0.51	0.60	0.66	0.58	0.56	0.51	0.48	0.55
	Glamour								Value	
	1	2	3	4	5	6	7	8	9	10
					Panel B: P/FC					
r	7.35	9.84	11.38	12.30	13.90	12.86	12.13	11.91	10.96	11.69
σ	22.35	19.44	17.22	17.25	16.28	16.53	17.18	17.91	18.95	22.35
t(r)	1.58	2.43	3.17	3.42	4.09	3.73	3.39	3.19	2.77	2.51
IR	0.33	0.51	0.66	0.71	0.85	0.78	0.71	0.67	0.58	0.52

Table 2, Panel B presents the results for P/FC. Low P/FC securities are identified with value stocks because their growth rate of cash flow is expected to be low. In other words, their prices are low per dollar of cash flow. Conversely, high P/FC stocks are glamour securities. The annualized compounded return for the tenth decile (value) P/FC stocks have a return of 11.69 percentage points, whereas the first decile (glamour) stocks realize a return of 7.35 percent, for a difference of 4.34 percentage points. Sorting on P/FC thus appears to produce a large positive spread in contrast to the P/B ratio. T

Table 2 – *Continued*

	Glamour 1	2	3	4	5	6	7	8	Value 9	10
Panel C: P/E										
r	6.83	10.00	11.88	11.84	12.22	11.51	10.05	10.76	11.14	11.47
σ	24.81	19.40	18.02	17.43	17.07	17.42	18.05	18.31	19.14	20.31
t(r)	1.32	2.47	3.16	3.26	3.43	3.17	2.67	2.82	2.79	2.71
IR	0.28	0.52	0.66	0.68	0.72	0.66	0.56	0.59	0.58	0.57
Panel D: P/S										
r	8.46	10.86	10.94	11.54	10.32	11.12	10.63	10.55	11.67	12.17
σ	23.49	18.79	18.00	18.18	18.04	17.94	18.73	19.39	19.62	22.01
t(r)	1.73	2.77	2.91	3.04	2.74	2.97	2.72	2.61	2.85	2.65
IR	0.36	0.58	0.61	0.63	0.57	0.62	0.57	0.54	0.60	0.55
Panel E: DIV										
r	9.11	9.57	10.61	10.73	12.48	12.35	11.17	11.08	10.31	10.35
σ	21.30	19.75	18.73	19.27	18.52	17.62	17.83	17.07	16.87	18.30
t(r)	2.05	2.33	2.72	2.67	3.23	3.36	3.01	3.11	2.93	2.97
IR	0.43	0.48	0.57	0.56	0.67	0.70	0.63	0.65	0.61	0.62

Panel C of Table 2 presents the results of portfolio deciles sorted on the ratio of P/E. Over the post-formation years, the annualized compounded return for the tenth decile is 11.47 percent and 6.83 percent for the first decile, thus this value measure produces a fairly large return and exhibits the widest margin between value and glamour of 4.46 percent. Albeit the P/E exhibit the largest spread between the glamour and value stocks, its worth emphasizing, that it does not produce the largest return.

Examining the P/S portfolio deciles, in Panel D, the value portfolio produces a return of 12.17 percent while the glamour realizes 8.47 percent, for a difference of 3.17. While this spread is lower than the spread obtained for the P/E measure, the P/S measure produces the largest returns of all value multiples. However, the large returns are accompanied by a larger standard deviation, therefore, lower risk-adjusted returns.

The large margin between value and glamour for P/FC, P/E, and P/S, may indicate that these value multiples capitalize the investments generated by intangible assets and almost surely indicate that they a more powerful value multiple with current account standards. Nevertheless, the P/B measure produces 0.47 percentage points more annualized compounded return.

Finally, Panel E reports the results for the portfolios sorting with respect to the DIV measure. Recall, the dividend yield portfolios are based on dividend yield excluding companies that do not payout dividend. Very interestingly, the value deciles of the dividend yield measure realize decent returns with less volatility than any of the other value measures. This may no be very surprising, since dividend payouts consist of regular dividend payments i.e. a consistent cash flow and the fact that dividend paying companies tend to continue to deliver dividends (a change in the dividend payout structure is usually perceived negatively by the stock market). Moreover, the prices are adjusted for dividends, which naturally will have a positive impact on the standard deviations - because the volatility of the dividend payout is smoothed out over the entire sample period.

Table 3 reports the performance characteristics of the equally weighted value series as well as both the equally and market weighted S&P 500 Index. The reported equally weighted series of value portfolios equally weights 12 separately constructed value portfolios measuring that use the same value multiple but are initiated one month apart. The implication with this methodology is that uses 12 months more data than simply examining one value portfolio. However, implementing portfolios in this manner leaves a lot up to chance, potential data snooping, and risk. To avoid this, the portfolios in the rest of the value section will take timing risk into consideration. Table 3 reports the performance characteristics of the value deciles from 2 as well as the equally weighted S&P 500 Index.

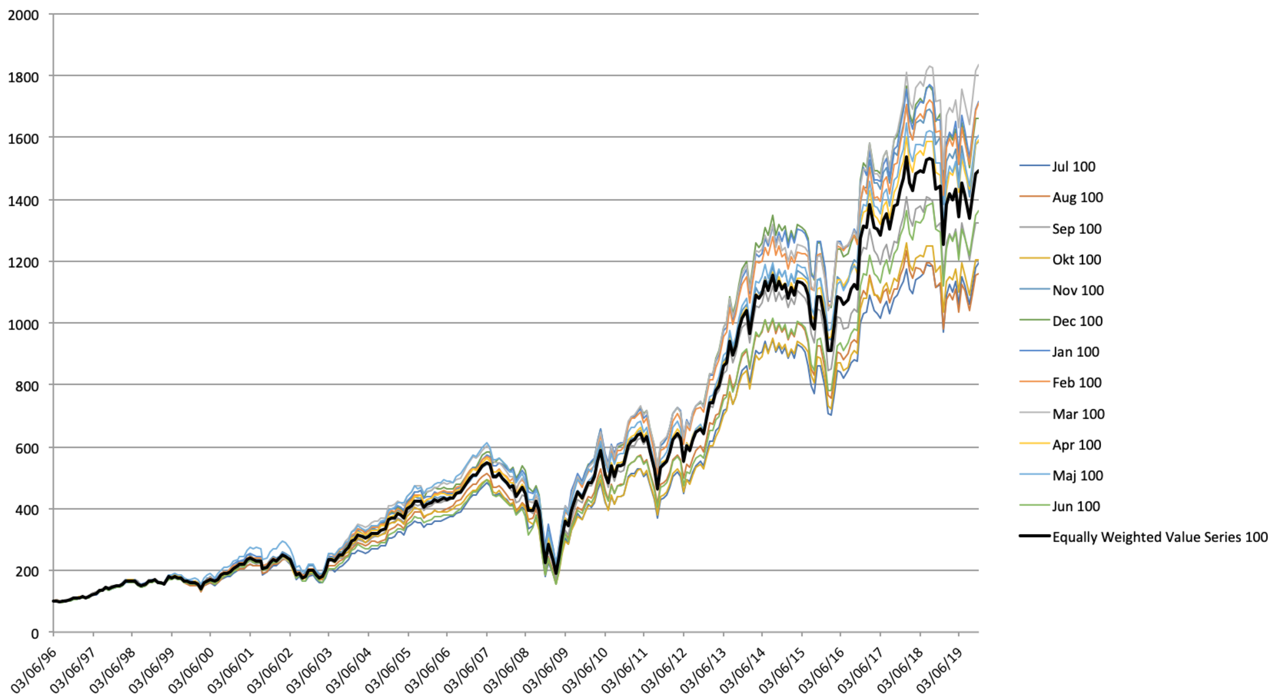
Table 3: Performance Characteristics the equally weighted series of value portfolios based on One-Dimensional sortings by various measures of Value.
The table presents performance characteristics for equally weighted series of value Portfolios that are formed in descending order based on P/B, P/S, P/FC, P/E, and DIV value signals in the period of 1996:06:30 - 2019:12:31.P/B is the ratio of stock price to book value; P/S is the ratio of stock price to sales; P/FC is the ratio of stock price to cash flow; P/E is the ratio of stock price to earnings, and DIV is the dividend yield measure. r denotes annualized compounded return, σ is the standard deviation denoted in percentage, IR is the information ratio, t(r) denotes the t-statistic, and α is the return subtracted by the equally weighted S&P 500 Index's return. All of the computations are determined using realized monthly return data for each of the decile portfolios.

Strategy	r	σ	IR	t(r)	α
P/B	12.16	22.11	0.55	2.64	1.18
P/E	11.47	20.31	0.57	2.71	0.50
P/S	12.17	22.01	0.55	2.65	1.20
P/FC	11.69	20.35	0.52	2.51	0.72
DIV	11.35	18.30	0.62	2.97	0.37
Average	11.77	21.02	0.56	2.75	0.79
Equally Weighted S&P 500	10.97	18.50	0.59	2.91	0.00

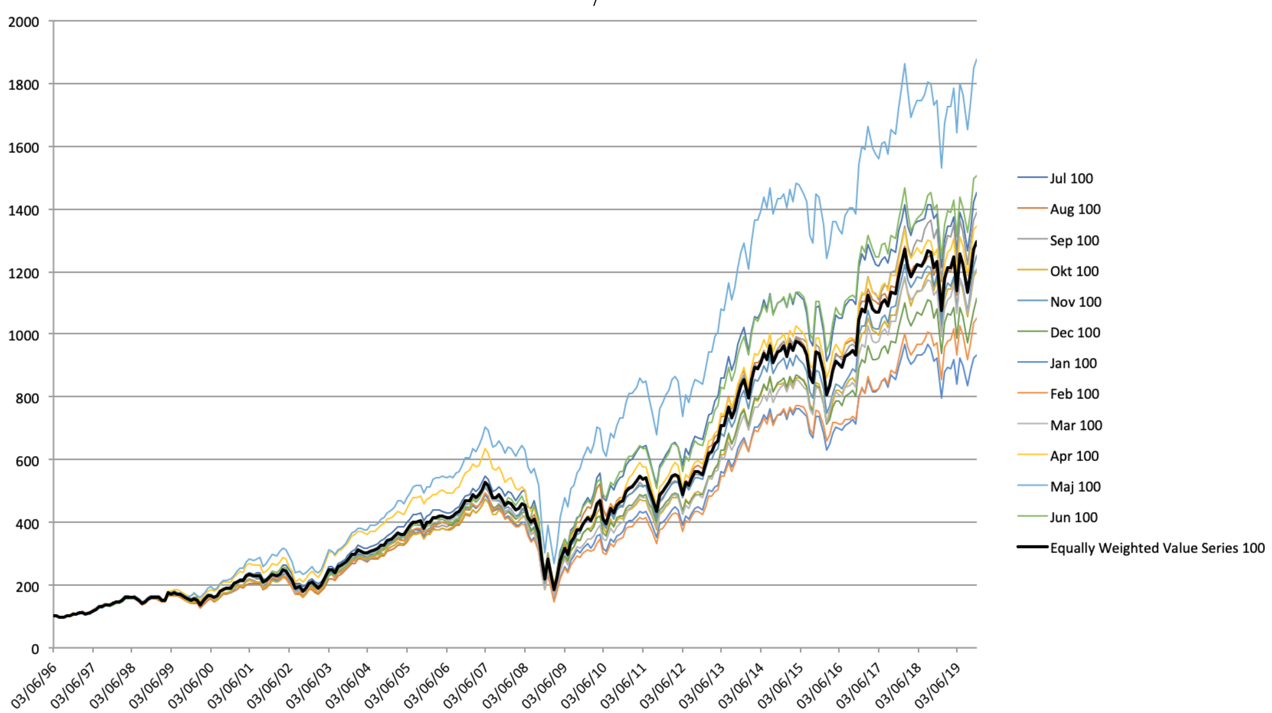
Zeroing in on the difference Figure 5 Panel A, B, C, D, and E plot the realized compounded return for the various measures of value and its constituents. The portfolios start with a $100 notional 1996:06:30 and the portfolios conclude 2019:12:31. The constituent in the P/B that realizes the largest compounded return is approximately 1.58 times larger than the worst portfolio; this number is 1.36 for P/E, 2.01 for P/S; 1.30 for P/FC; and 1.54 for DIV portfolios. These are quite significant and alarming margins and are likely to be caused by underlying market structural dynamics and noise. This paper will not go into the governing dynamics causing the spreads, nor will it exploit these huge margins. Rather, the goal is to maintain a simple and fairly uniform and consistent approach to minimize unwanted effects of data snooping.

Figure 5: Realized compounded returns in the period of 1996:06:30 to 2019:12:31 for the equally weighted series of value portfolios and its constituents.
The January portfolios reflects the value portfolios initiating in January, the same is the case for the February, March, April etc. portfolios, respectively. The Equally weighted portfolio is highlighted in black and is a equally weighted series of all of the value portfolios. The realized compounded return is computed using monthly return data throughout the full 23-year sample period.

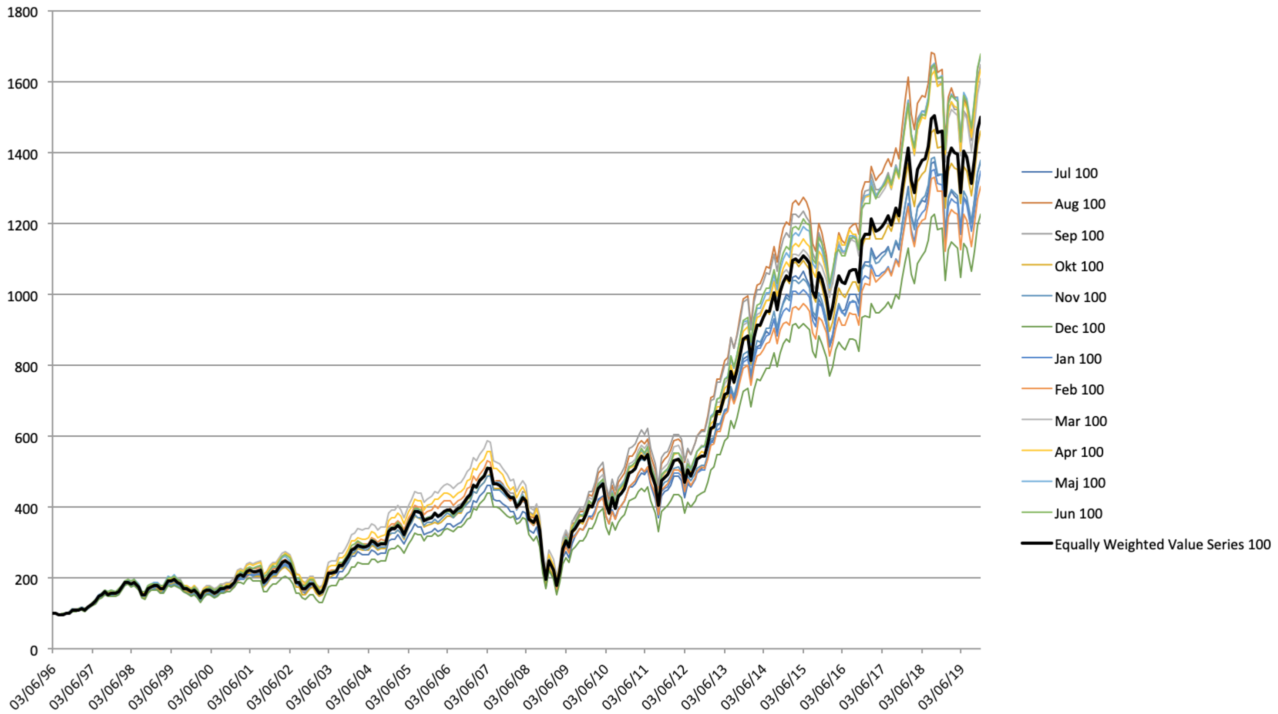
Panel A: P/B Value Portfolios



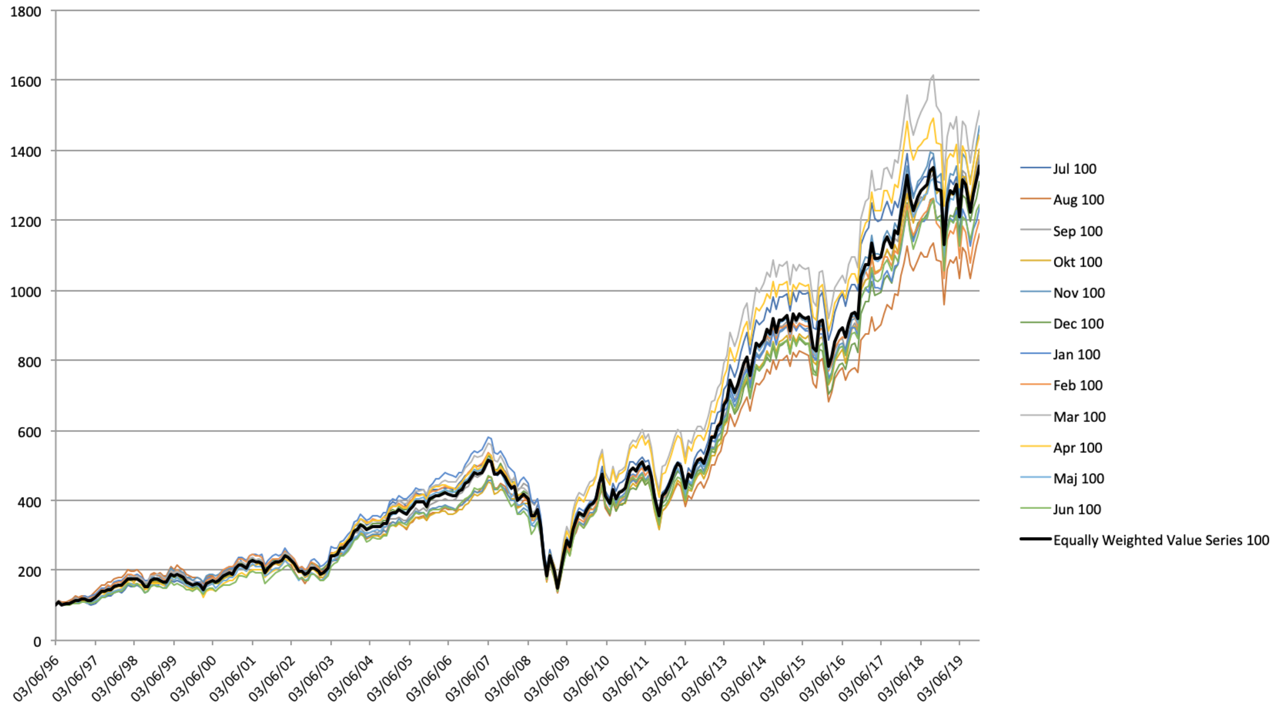
Panel B: P/E Value Portfolios



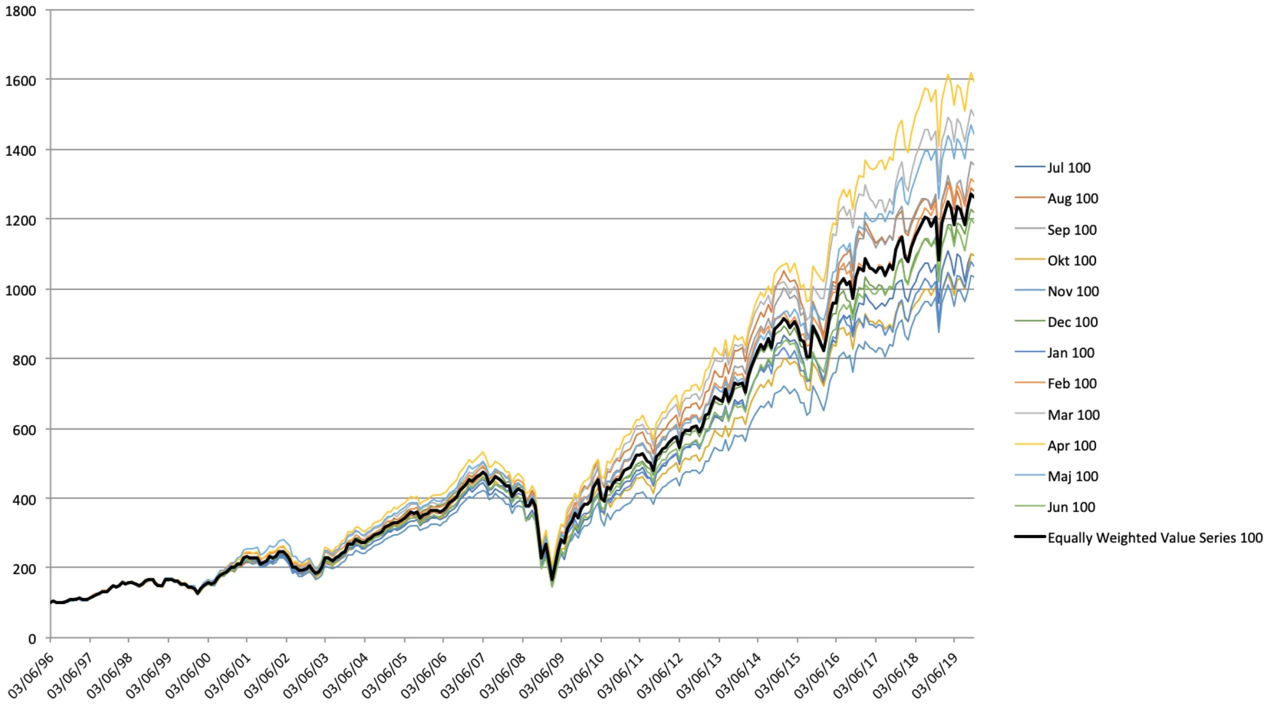
Panel C: P/S Value Portfolios



Panel D: P/FC Value Portfolios



Panel E: DIV Value Portfolios



In summary, this subsection confirms and extends results from existing literature conducted on other markets and time frames. The evidence indicates that the value strategies based on classification of various measures of value produce large returns over the 23-year sample period from 1996:06:30 to 2019:12:31. In contrast to previous work, the evidence suggests a relatively smaller premium emerges in the S&P 500 Index over the sample time. Especially, the P/B value measure underperforms compared to research by Fama & French (1992), the result here implying that the glamour portfolios actually perform superior. This paper finds, that the P/S value measure realizes the best return and that the dividend yield signal produces the best risk-adjusted return. When it comes to the strong risk-adjusted return by the dividend yield signal, it is in particular the low volatility that is the driving force. Furthermore, this paper finds evidence that the performance of the value portfolios to a large degree depend on the inception month. To avoid underlying structural market dynamics, seasonal trends, and data snooping this paper will continue examining equally weighted value series. Taking timing risk into account, the P/S measure still produces the largest return while the DIV signal still realizes the largest risk-adjusted return by far.

The next subsection will continue the investigation of value driving measurements by expanding the research to include more sophisticated portfolios that are constructed using multiple-dimensional classifications.

4.5 Multiple-Dimensional Value Trading Results

With inspiration from LSV (1994) this subsection will explore potential opportunities by classifying portfolios using respectively two, three, four, and five measures of value simultaneously. Consisting with Fitzgibbon et al. (2017) this paper considers two implementation approaches, namely, portfolios constructed by mixing or integrating value measures. The mixed portfolio implementation entails equally weighting multiple factors i.e. equally weighting the union of multiple factors. The integrated implementation approach equally weights the average highest scoring stocks on the sorted measures of value. Consequently, a mixed portfolio sorting based on P/B and P/E will be a portfolio equally weighting approximately 50 P/B stocks and 50 P/E stocks. A total of approximately 100 stocks that may very well score very low on one factor and high on the other. In contrast, an integrated portfolio sorting based on P/B and P/E will consist of approximately 50 stocks exhibiting a combined high score examining both factors. This subsection will primarily examine the integrated portfolios, since a vast majority of the integrated portfolios performed better and require less transactions. Less transactions, because it selects the top decile in a ranking scheme that finds a total average score for the number of used value measures in the respective strategy. Nevertheless, the end of the section will include results for the mixed portfolios.

As mentioned in the value trading strategies subsection 4.3, sorting based on multiple dimensions gives rise to additional 26 portfolios. Table 4 reports the performance characteristics of the ten possible combinations of sorting value portfolios based on two measures as well as the equally weighted S&P 500 Index. In addition to this, Benefits(r) and Benefits(IR) categorizes whether the two dimensional classification results in worse (W) performance, in-between (I) performance, or strictly better (B) performance.

First of all, it is evident from Table 4 that all the two-dimensional value portfolios produce returns in two digits and above with all return statistics being significantly different from zero at the 0.05 alpha level. In addition comparing, all of the portfolios produces positive alpha relative to the equally weighted total return S&P 500 Index. The evidence is somewhat consistent with the deep value work by LSV (1994), 3/10 of the portfolios produce better returns, 3/10 produce returns in-between the two measures of value. However, 4/10 of the portfolios actually end up performing worse. The story is different when taking risk into consideration. Here 6/10 of the portfolios perform better and 4/10 perform in-between of what they did separately. Very interestingly, the portfolios that include the dividend yields measure actually perform better in terms of risk-adjusted return while performing worse in terms of absolute returns. This is most likely due to the nature of the dividend yields portfolio, that will deliver a (relatively low) constant cashflow with little volatility. This result shows that the dividend measure has great explanatory power for volatility.

Table 4: Performance Characteristics for the equally weighted series of value portfolios based on two-Dimensional sortings by various measures of Value.
The table presents performance characteristics for equally weighted series of value portfolios that are formed by equally weighting two measures of value simultaneously in the period of 1996:06:30 - 2019:12:31. r denotes annualized compounded return, σ is the standard deviation denoted in percentage, IR is the information ratio, t(r) denotes the t-statistic, and α is the excess return relative to the equally weighted S&P 500 Index. All of the computations are determined using realized monthly return data for each of the portfolios Lastly, Benefits(r) and Benefits(IR) denotes effect of sorting based on the respective two value measures W denotes a worse performance, I is a performance in-between the two measures, B denotes a strictly better performance in comparison to both value measures.

Strategy	r	σ	IR	t(r)	α	Benefits(r)	Benefits(IR)
P/B & P/E	11.83	21.97	0.57	2.80	0.85	W	B
P/B & P/S	12.01	21.97	0.55	2.68	1.03	I	I
P/B & P/FC	11.77	22.42	0.52	2.57	0.80	I	I
P/B & DIV	11.01	18.80	0.59	2.89	0.10	W	I
P/E & P/S	12.32	20.75	0.59	2.91	1.35	B	B
P/E & P/FC	11.39	20.50	0.56	2.72	0.41	W	B
P/E & DIV	11.29	17.21	0.66	3.21	0.32	W	B
P/S & P/FC	12.55	21.65	0.58	2.84	1.58	B	B
P/S & DIV	11.71	18.66	0.63	3.07	0.74	B	B
P/FC & DIV	10.76	19.00	0.57	2.77	-0.21	I	I
Market weighted S&P 500 index	6.16	15.22	0.41	1.94	-4.18		
Equally Weighted S&P 500 index	10.97	18.50	0.59	2.91	0.00		

Table 4 also includes the performance of the market weighted S&P 500 Index to put the portfolios in additional perspective. Comparing the value portfolios to the market weighted S&P 500 Index, all of the portfolios produce significantly larger returns and risk-adjusted returns. They do however, come bearing significantly larger risk, everything else equal. Much of this risk can be attributed to the fact that the value portfolios are equally weighted portfolios. This means that lower market valued companies which usually exhibit more volatile stock prices have the same portfolio weight as large companies who's stock prices tend to be less volatile. Yet, they do also bear additional risk in comparison to the equally weighted S&P 500 Index (Research by S&P Dow Jones Indices).

Table 5 expands the research to portfolio sorting on three-dimensions. Here the Benefits(·) notation refers to any feasible portfolio construction using any combination and dimension of the three value measures. For example sorting on P/B & P/E & P/S performs superior with respect to both absolute return and risk-adjusted return across all feasible portfolio constructions using these three measures of value. Specifically, the strategy is superior to P/B & P/E, P/B & P/S, P/E & P/S, P/B, P/E, and P/S in terms of performance. The evidence indicates that adding an extra dimension in the sorting criteria does not impair any of the portfolio constructions. By contrast it improves 4/10 of the portfolio performances. Again all of the portfolios including the dividend yield measure exhibit lower volatilities than those without. This supports the claim that the dividend yield measures select stocks with low volatility. Another example of this is the P/B and P/S portfolio that produces 12.01 percent, adding P/FC increases the return to 12.62 percent with a standard deviation of 22.42, however, adding the dividend yield to the portfolio instead yields a return of 12.61 percent with a standard deviation of 19.28.

Table 5: Performance Characteristics for the equally weighted series of value portfolios based on three-Dimensional sortings by various measures of Value.
The table presents performance characteristics for equally weighted series of value portfolios that are formed by equally weighting three measures of value simultaneously in the period of 1996:06:30 - 2019:12:31. r denotes annualized compounded return, σ is the standard deviation denoted in percentage, IR is the information ratio, t(r) denotes the t-statistic, and α is the excess return relative to the equally weighted S&P 500 Index. All of the computations are determined using realized monthly return data for each of the portfolios Lastly, Benefits(r) and Benefits(IR) denotes effect of sorting based on the respective three value measures W denotes a worse performance, I is a performance in-between the two measures, B denotes a strictly better performance in comparison to to any feasible portfolio construction using any of three value measures.

Strategy	r	σ	IR	t(r)	α	Benefits(r)	benefits(IR)
P/B & P/E & P/S	12.78	20.63	0.62	3.04	1.81	B	B
P/B & P/E & P/FC	11.75	21.21	0.55	2.71	0.77	I	I
P/B & P/E & DIV	11.17	18.07	0.62	3.03	0.20	I	I
P/B & P/S & P/FC	12.62	22.42	0.56	2.76	1.65	B	I
P/B & P/S & DIV	12.61	19.28	0.65	3.21	1.64	B	B
P/B & P/FC & DIV	11.30	20.36	0.55	2.71	0.32	I	I
P/E & P/S & P/FC	12.26	21.04	0.58	2.85	1.28	B	I
P/E & P/S & DIV	11.71	17.85	0.66	3.21	0.73	I	I
P/E & P/FC & DIV	11.33	18.16	0.62	3.06	0.36	I	I
P/S & P/FC & DIV	11.81	19.29	0.61	3.00	0.84	I	I
Market weighted S&P 500 index	6.16	15.22	0.41	1.94	-4.18		
Equally Weighted S&P 500 index	10.97	18.50	0.59	2.91	0.00		

Finally, Table 6 reports the performances of the portfolios sorting based on four and five dimensions. While all portfolios produce performance statistics that are highly significant, all of the statistics are inferior to portfolio constructions using same respective measures of value in lower dimensions.

Table 6: Performance Characteristics for the equally weighted series of value portfolios based on four and five Dimensional sortings by various measures of Value.
The table presents performance characteristics for equally weighted series of value portfolios that are formed by equally weighting four and five measures of value simultaneously in the period of 1996:06:30 - 2019:12:31. r denotes annualized compounded return, σ is the standard deviation denoted in percentage, IR is the information ratio, t(r) denotes the t-statistic, and α is the excess return relative to the equally weighted S&P 500 Index. All of the computations are determined using realized monthly return data for each of the portfolios Lastly, Benefits(r) and Benefits(IR) denotes effect of sorting based on the respective four and five value measures W denotes a worse performance, I is a performance in-between the two measures, B denotes a strictly better performance in comparison to both value measures.

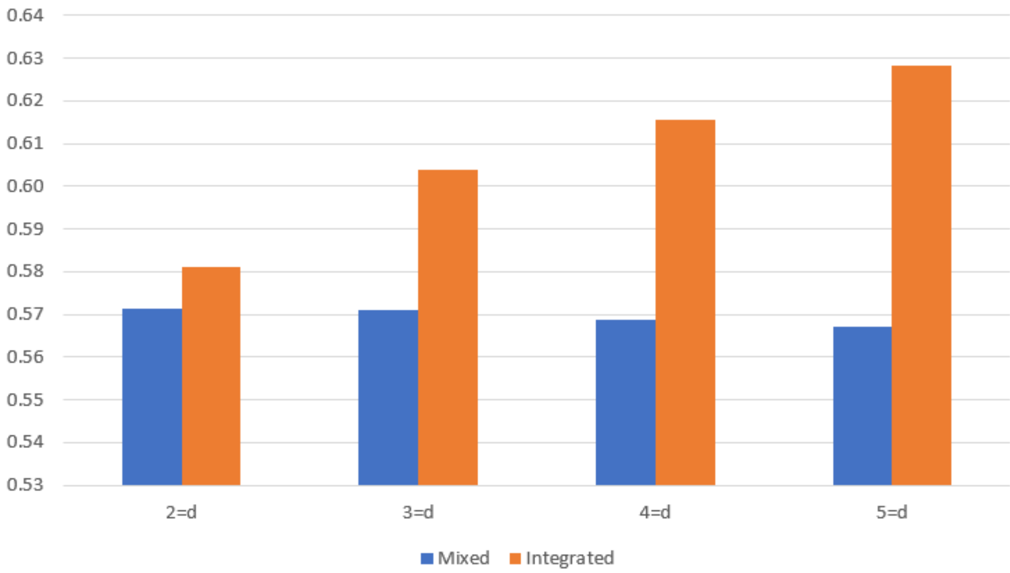
Strategy	r	σ	IR	t(r)	α	Benefits(r)	Benefits(IR)
P/B & P/E & P/S & P/FC	12.43	21.28	0.58	2.86	1.45	I	I
P/B & P/E & P/S & DIV	11.73	18.54	0.63	3.10	0.75	I	I
P/B & P/E & P/FC & DIV	11.68	19.61	0.60	2.92	0.70	I	I
P/B & P/S & P/FC & DIV	12.43	20.09	0.62	3.03	1.45	I	I
P/E & P/S & P/FC & DIV	12.01	18.69	0.65	3.17	1.12	I	I
P/B & P/E & P/S & P/FC & DIV	12.47	19.84	0.63	3.08	1.49	I	I
Market weighted S&P 500 index	6.16	15.22	0.41	1.94	-4.18		
Equally Weighted S&P 500 index	10.97	18.50	0.59	2.91	0.00		

Table 7 displays the returns and risk-adjusted returns of all the 26 value portfolio combinations using the integrated and a mixed portfolio implementation approaches. The applied coloring scheme highlights the best return and risk-adjusted return for each strategy. The evidence highly suggests that the integrated implementation approach produces better returns and low risk portfolios. This is emphasized by the fact, that the 22 of the 26 integrated portfolios realize the best risk-adjusted return coupled with the fact that 9 of the integrated highest scoring risk-adjusted returns are portfolios realizing larger returns for the mixed portfolios. Table 6 presents average materialized information ratio for the mixed and integrated portfolio approach across multiple dimensions. Note, that the y-axis denoting the information ratio starts at 0.53. Nonetheless, the Table 7 depicts a monotonically increasing information ratio with respect to number of dimensions used for classification for the integrated portfolio. While the mixed portfolio actually realizes a stagnant information ratio across the number of applied dimensions. On average, the evidence here highly indicates, that the integrated portfolio approach produces higher risk-adjusted returns than the mixed portfolio approach. Furthermore, adding a dimension to the sorting criteria on average increases the obtained information ratio.

Table 7: Comparison of mixed and integrated implementation approach using various measures of value in the sample period of 1996:06:30 - 2019:12:31.
Annualized compounded returns and information ratios for the integrated and mixed implementation approach for various measures of value. The statistics are computed using monthly data throughout the entire sample from 1996:06:30 to 2019:12:31. The applied coloring scheme highlights the best return and risk-adjusted return for each strategy across different holding periods

Strategy	Return (Int)	IR	Return (Mix)	IR
P/B & P/E	11.83	0.57	11.84	0.57
P/B & P/S	12.01	0.55	11.95	0.56
P/B & P/FC	11.77	0.52	11.62	0.53
P/B & DIV	11.07	0.59	12.02	0.59
P/E & P/S	12.32	0.59	12.02	0.58
P/E & P/FC	11.39	0.56	11.46	0.55
P/E & DIV	11.29	0.66	11.58	0.60
P/S & P/FC	12.55	0.58	11.74	0.54
P/S & DIV	11.71	0.63	12.04	0.61
P/FC & DIV	10.76	0.57	11.53	0.58
P/B & P/E & P/S	12.78	0.62	11.97	0.58
P/B & P/E & P/FC	11.75	0.55	11.52	0.55
P/B & P/E & DIV	11.17	0.62	11.70	0.58
P/B & P/S & P/FC	12.62	0.56	11.68	0.55
P/B & P/S & DIV	12.61	0.65	11.89	0.59
P/B & P/FC & DIV	11.29	0.55	11.61	0.56
P/E & P/S & P/FC	12.26	0.58	11.69	0.56
P/E & P/S & DIV	11.71	0.66	11.87	0.60
P/E & P/FC & DIV	11.33	0.62	11.46	0.57
P/S & P/FC & DIV	11.81	0.61	11.72	0.58
P/B & P/E & P/S & P/FC	12.43	0.58	11.67	0.56
P/B & P/E & P/S & DIV	11.73	0.63	11.79	0.58
P/B & P/E & P/FC & DIV	11.68	0.60	11.40	0.56
P/B & P/S & P/FC & DIV	12.42	0.62	11.60	0.57
P/E & P/S & P/FC & DIV	12.09	0.65	11.58	0.57
P/B & P/E & P/S & P/FC & DIV	12.47	0.63	11.52	0.57
Average	11.88	0.60	11.71	0.57

Figure 6: Integration benefits when increasing number of dimensions.
The figure presents the average realized information ratios for each respective number of dimensions. The average is simply computed by averaging all IRs within each dimension for all of the equally weighted series of value portfolios using a integrated and a mixed portfolio implementation approach.



The results in this subsection point towards several propositions. First, implementing value strategies that sort based on multiple dimensions produce positive returns that are statistically significant at the 95% confidence level. Secondly, with the exception of the two-dimensional value strategies adding another dimension results in strictly better performance or performance better than the worst denominator. The two-dimensional value sorting may result in better, worse, or in-between performance with respect to return, however, taking risk into account, it only results in performance in-between and better. Third, three-dimensional value sorting results only results in strictly better and in-between performance across dimensions. Adding four and five dimensions to the value classification does not yield better performance than those obtained using three dimensions. However, averaging the portfolios information ratios show that the information ratio is monotonically increasing in number of dimensions, while the mixed portfolio does not capitalize any benefits of multiple dimensions. Finally, the integrated implementation approach yields the overall best performances, in particular, the integrated portfolios results in less volatile portfolios.

Now, this subsection will shed light on the portfolio deciles of the highest return strategy. Table 8 reports the performance characteristics of the value strategy that sorts on P/B, P/S, and P/E over the full sample period 1996:06:30 - 2019:12:31. The 10-decile integrated portfolios are formed in ascending order based on rankings from the three measures of value. The value portfolios is equivalent to first decile and the glamour is equal to the tenth decile. The first decile produces a return of 12.78 percent and the tenth decile produces a return of 9.65 percent, for a difference of 3.13 percent. The results here show close to monotonically increasing returns as you shift from glamour deciles to value deciles. This indicates, that the multiple dimensional sorting exhibit even more well defined return results than the one dimensional sorting.

Table 8: Performance Characteristics for Decile Portfolios Based on Three-Dimensional Sortings by various Measures of Value.
Value portfolio characteristics for the P/B & P/S & P/E in the full period from 1996:06:30 - 2019:12:31. The 10-decile integrated portfolios are formed in ascending order based on a joint rank using all three measures of value. The first decile is the bottom 10 percent and the tenth portfolio is the top 10 percent. r is the compounded return, σ denotes the standard deviation, t(r) denotes the t-statistic, the α denotes the excess return from the portfolio in relation to the equally weighted S&P 500 Index, the IR denotes the annualized information ratio, SK(m), and K(m) represents the full period realized skewness and excess kurtosis of the monthly returns.

Return Statistic			Value decile portfolio											
	Best 1 %	Best 5 %	1	2	3	4	5	6	7	8	9	10	S&P 500	EW S&P 500
r	11.58	12.38	12.78	10.95	10.13	10.37	10.18	11.26	11.18	10.13	10.59	9.65	6.16	10.97
σ	24.58	21.79	20.63	19.14	18.79	19.14	18.43	19.00	17.51	17.68	18.47	23.14	18.41	18.50
t(r)	2.26	2.73	2.97	2.74	2.58	2.60	2.65	2.84	3.06	2.75	2.75	2.00	1.94	2.84
α	0.61	1.41	1.81	-0.03	-0.85	-0.60	-0.80	0.29	0.21	-0.84	-0.38	-1.33	-4.73	0,00
IR	0.47	0.57	0.62	0.57	0.54	0.54	0.55	0.59	0.64	0.57	0.57	0.42	0.40	0.59
SK(m)	-0.44	-0.69	-0.64	-0.66	-0.64	-0.68	-0.69	-0.52	-0.54	-0.64	-0.53	-0.47	-0.83	-0.61
K(m)	6.50	5.68	5.70	4.84	3.38	3.88	3.70	5.06	3.01	3.07	2.17	3.41	2.30	3.55

Table 8 also shows all decile portfolios monthly skewness and kurtosis. First of all, All portfolios including the equally weighted S&P 500 Index exhibit moderate negative skewnesses. In respect to kurtosis all of the 10-decile portfolios exhibit relatively large kurtosises compared to the equally weighted S&P 500 index. Examining the skewnesses across the 10-decile portfolios, it is evident that going from glamour to value, the portfolios become slightly more negatively skewed. In addition, the excess kurtosis is almost monotonically increasing, going from the glamour to the value portfolios including the more extreme value portfolios.

The moderate negative skewness suggests that the return distributions have long left tails. The increasing kurtosis across the 10-deciles implies that the return distributions exhibit an increasing amount of probability mass in the tails. Practically such a cocktail, implies that the value and glamour portfolios both realise extreme losses. Nonetheless, the value portfolios realize relatively larger extreme outcomes at a relatively larger likelihood. In other words, while the value portfolios still outperform the glamour and equally weighted S&P 500 index over time, the Sharpe ratio and alpha seem to radically understate the significance of its crash potential. This is a conjecture that this paper will expand on in the following subsection.

Lastly, Table 8 portrays the performance of portfolios sorting based on the best 5 and 1 percent are less than the first decile. These portfolios will naturally consist of less stocks, approximately 25 and 5 stocks, which is most likely the culprit of these two portfolios. First, decreasing the number of stocks will almost surely increase the idiosyncratic risk embedded in the portfolios (as documented in subsection 3.5). Secondly, fewer stocks will intuitively increase the weights of each single stock in the portfolio, effectively requiring each single stock always to continually perform. The results are not unanimous for all multiple-dimensional value portfolios. Some of the portfolios do deliver higher returns but at the cost of more risk as emphasized by the two points above. This paper will conclude, that the portfolios sorting based on the bottom decile consisting of approximately 50 securities deliver a satisfying and more stable return almost certainly due to diversification effects. The next subsection will investigate the conjecture that the value portfolios do indeed come bearing significant crash risks.

Prior to this, this subsection will first investigate the implications of changing the holding periods of the value portfolio from the well documented 12 months to respectively 6 and 18 months, respectively. Table 9 presents the compounded annualized returns and information ratios for each of the 31 value sortings that rebalances its portfolio every 6, 12, and 18 months. The applied coloring scheme highlights the best return and risk-adjusted return for each strategy. Consistent with existing work Table 9 shows that the value premium requires rather long horizons to be favourable. The value portfolios sorting every six months perform inferior on all various measures of value with the exception of the dividend yield measure. Increasing the holding period from 6 to 12 months, strictly increases the return and risk-adjusted return with the exception of the dividend yield measure. Expanding the holding periods to 18 months does in some cases result in better returns, however, the real benefit is reflected in the risk-adjusted returns. Here the 18 months portfolios actually realize better risk-adjusted returns albeit the best return is realized for the 12 months portfolios. This indicates that the 18 month strategies perhaps come bearing less risk, nonetheless, the overall average picture puts the 12 month strategies in front.

Table 9: Comparison various holding lengths of the integrated value portfolios throughout the entire sample period of 1996:06:30 - 2019:12:31.
The table presents annualized compounded returns and information ratios for each strategy across different holding lengths. The statistics are computed using monthly data throughout the entire sample from 1996:06:30 to 2019:12:31. The applied coloring scheme highlights the best return and risk-adjusted return for each strategy.

Strategy	Return (6m)	IR (6m)	Return (12m)	IR (12m)	Return (18m)	IR (18m)
P/B	11.66	0.52	12.16	0.55	11.61	0.55
P/E	11.86	0.54	11.47	0.57	11.94	0.55
P/S	10.81	0.53	12.17	0.55	11.64	0.58
P/FC	11.10	0.49	11.69	0.52	11.30	0.52
DIV	11.71	0.63	11.35	0.62	11.25	0.63
P/B & P/E	11.42	0.54	11.83	0.57	12.12	0.60
P/B & P/S	11.97	0.54	12.01	0.55	12.12	0.55
P/B & P/FC	11.08	0.48	11.77	0.52	11.52	0.53
P/B & DIV	10.86	0.57	11.07	0.59	11.03	0.61
P/E & P/S	11.78	0.56	12.32	0.59	12.33	0.61
P/E & P/FC	10.72	0.51	11.39	0.56	10.88	0.54
P/E & DIV	10.96	0.64	11.30	0.66	11.43	0.67
P/S & P/FC	11.78	0.54	12.55	0.58	11.86	0.56
P/S & DIV	11.63	0.62	11.71	0.63	11.74	0.64
P/FC & DIV	10.75	0.56	10.76	0.57	10.27	0.55
P/B & P/E & P/S	11.93	0.57	12.78	0.62	12.88	0.64
P/B & P/E & P/FC	10.72	0.50	11.75	0.55	11.58	0.56
P/B & P/E & DIV	10.65	0.58	11.17	0.62	11.61	0.66
P/B & P/S & P/FC	11.56	0.50	12.62	0.56	12.11	0.56
P/B & P/S & DIV	12.05	0.61	12.61	0.65	12.70	0.68
P/B & P/FC & DIV	11.07	0.53	11.29	0.55	10.75	0.55
P/E & P/S & P/FC	11.41	0.53	12.26	0.58	11.57	0.56
P/E & P/S & DIV	11.26	0.63	11.71	0.66	11.92	0.68
P/E & P/FC & DIV	11.06	0.60	11.33	0.62	10.99	0.61
P/S & P/FC & DIV	11.59	0.59	11.81	0.61	11.35	0.58
P/B & P/E & P/S & P/FC	10.86	0.50	12.43	0.58	11.95	0.57
P/B & P/E & P/S & DIV	10.88	0.58	11.68	0.63	11.87	0.67
P/B & P/E & P/FC & DIV	10.88	0.54	11.58	0.60	11.24	0.59
P/B & P/S & P/FC & DIV	11.78	0.57	12.43	0.62	11.93	0.60
P/E & P/S & P/FC & DIV	11.42	0.60	12.01	0.65	11.62	0.62
P/B & P/E & P/S & P/FC & DIV	11.36	0.56	12.47	0.63	12.00	0.62
Average	11.31	0.56	11.88	0.60	11.65	0.59

4.6 Value Crashes

As documented in the subsections above, all value portfolios produce highly statistical significant positive return, but since the inception date, there has been a number of long periods over which value under-performed significantly. Figure 7 presents the cumulative monthly returns for all of the series of value portfolios and the equally weighted S&P 500 Index and highlights the largest sustained crash.

The realized value drawdowns highly coincide with the contemporaneous market weighted S&P 500 Index realizing its largest drawdown during the financial crisis 2007-08.

Table 10 presents examines the worst realized drawdowns associated with the value portfolios as well equally weighted S&P 500 Index. In addition, Table 10 includes the length of the crash and the time it took to recover from the worst point. A few keypoints emerge form Table 10 and Figure 7:

1. While all the value portfolios have outperformed the capitalization and equally weighted S&P 500 Index in terms of return and IRs, there are period over which value portfolios realize substantially larger losses and crashes, which are not directly reflected in the returns and IRs.

2. Figure 7 shows that value portfolios exhibit strong co-movement with the contemporaneous equally weighted S&P 500 Index. Investigation of the maximum drawdowns confirms this, all of the value portfolios realize their largest loss 2009:03 the same month the the contemporaneous market has its greatest decline.

3. Furthermore Table 10 depicts the relationship between sustained drawdown lengths and recoveries for multiple-dimensional value portfolios with the recoveries being almost half the magnitude of the drawdown lengths. Coupling this with the general V-shaped recoveries for a vast majority of the value portfolios exhibited in Figure 7 leads to the conclusion that the value portfolios recovery very swiftly during crises. In short, value stocks are very attractive when a market "bottoms".

4. Another important point that emerges from Table 10 is that the relatively longer recovery associated with the P/FC measure diminishes drastically as other measures of value are included to form a integrated portfolio. This is an additional attribute that strengthens the choice of multiple-dimensional value sorting.

Figure 7: Realized compounded returns in the period of 1996:06:30 to 2019:12:31 for all equally weighted series of value portfolios and the equally weighted S&P 500 Index .
The figure presents the cumulative monthly returns from 1996:06 to 2019:12 for all equally weighted series of value portfolios and the equally weighted S&P 500 Index with an initial notional of $100.

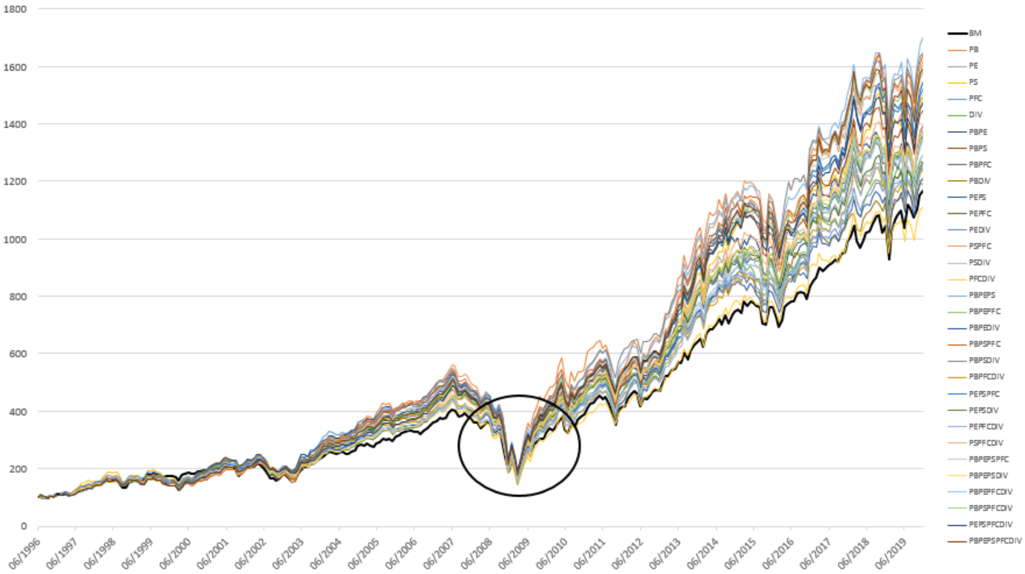


Table 10: Value portfolio drawdown characteristics in the full period from 1996:06:30 - 2019:12:31. This table presents the worst realized maximum drawdown (MDD) for all of the value portfolios over the full 23-year sample period using monthly return data. Additionally, the table presents the corresponding length and recovery time for the portfolios. MDD(%) denotes the worst realized maximum drawdown in percentage. Length(m) denotes the time it took from peak to valley to initial peak value, and the recovery(m) time is the time it takes from valley to new initial peak value. Both length(m) and recovery(m) are specified in number of months.

Strategy	MDD (%)	Length(m)	Recovery(m)
P/B	65.69	33	12
P/E	65.05	44	24
P/S	65.10	45	25
P/FC	60.29	64	43
DIV	64.54	42	21
P/B & P/E	67.36	45	24
P/B & P/S	66.12	43	22
P/B & P/FC	70.44	46	25
P/B & DIV	66.67	43	22
P/E & P/S	64.60	45	24
P/E & P/FC	64.84	45	24
P/E & DIV	60.10	43	22
P/S & P/FC	66.49	46	25
P/S & DIV	62.55	42	21
P/FC & DIV	66.30	56	35
P/B & P/E & P/S	65.21	42	22
P/B & P/E & P/FC	68.18	45	24
P/B & P/E & DIV	62.60	42	21
P/B & P/S & P/FC	68.29	34	13
P/B & P/S & DIV	62.75	33	12
P/B & P/FC & DIV	68.68	45	24
P/E & P/S & P/FC	65.69	43	22
P/E & P/S & DIV	58.70	34	13
P/E & P/FC & DIV	61.87	45	24
P/S & P/FC & DIV	62.69	34	13
P/B & P/E & P/S & P/FC	67.33	43	22
P/B & P/E & P/S & DIV	60.97	41	20
P/B & P/E & P/FC & DIV	65.70	42	21
P/B & P/S & P/FC & DIV	65.24	34	13
P/E & P/S & P/FC & DIV	60.41	34	13
P/B & P/E & P/S & P/FC & DIV	65.19	42	21
Equally weighted S&P 500 Index	57.39	42	21
Market weighted S&P 500 Index	54.70	80	48

zeroing in on the relationship between sustained drawdown lengths and recovering, this paper will look deeper into the one dimensional value strategies - the actual building-blocks of the papers value portfolios. In particular the P/B, P/E, P/S, and DIV measures recover rapidly, whereas the P/FC has a recovery lengths of 43 months in contrast to 12, 23, 25, and 24, respectively.

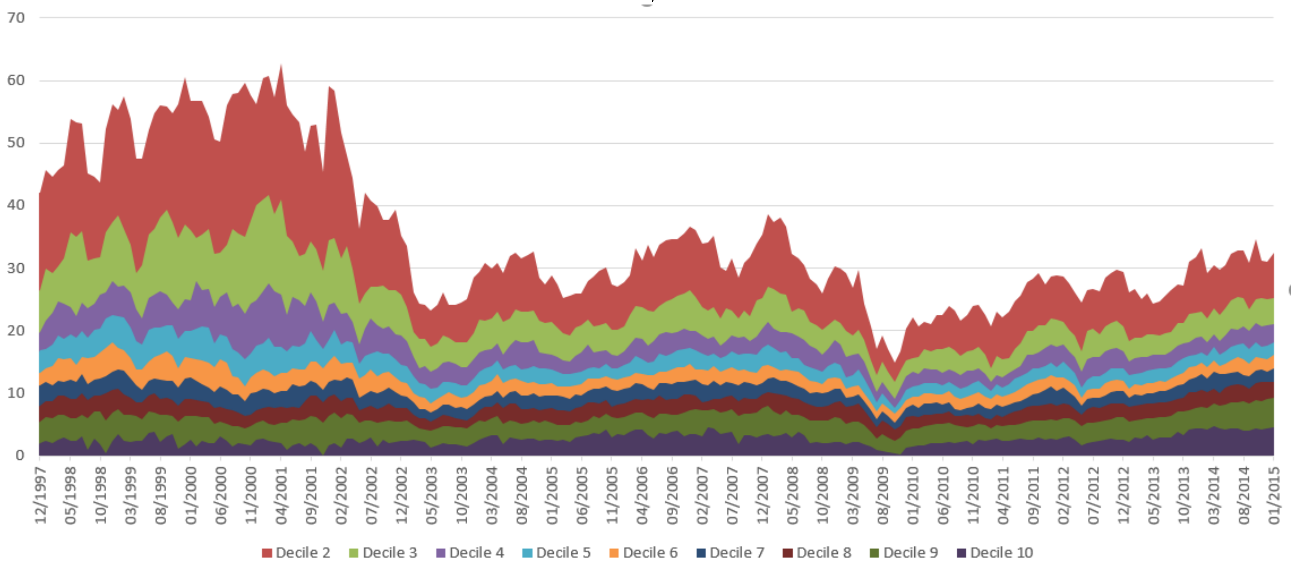
Figure 8, Panel A presents the median P/FC for each of the last 9 deciles from 1996:06:30 to 2019:12:31. The first decile is excluded because simply it marginalizes the rest of the deciles in the figure. Nonetheless, the highest P/FC values for the 2nd decile represented the lowest values for the 1st decile. Note that the P/FC values are realized P/FC values for the 9 deciles and are therefore 6 months lagged values. Panel B presents the median of the S&P 500 Index (highlighted by the blue graph belonging to the left axis) as well as the the ratio between the median P/FC of the 10th decile and the S&P 500 Index (highlighted by the red graph belonging to the right axis). Intuitively, the spread gives an indication of how cheap the 10th decile is compared to the rest of the market. A low spread indicating that the 10th portfolio is relatively cheap and a high spread indicating it is relatively expensive.

Figure 8, Panel A shows that when the stock market declines during the financial crises all P/FC deciles are temporarily depressed. Panel B, shows how the 10th decile gets relatively more and more expensive in the years from 2001 to 2007, suggesting that the P/FC portfolio's stock prices are rising but also that the portfolio is getting relatively more expensive relative to the market. The spread peaks fall 2007 and experiences a huge decline as the contemporaneous S&P 500 market declines. Highlighted in Figure 8, Panel A the 9 deciles converge as the market plummets. Examining Figure 8, Panel B its apparent that the spread declines, implying that the 10th decile median declines relatively more. The very low spread in end 2009 indicates, that the 10th decile is very cheap relative to the market. Evidently, as the market stabilizes and prices increase, the spread increase as a consequence. The 10th decile median does rise rapidly (the spread increase), however, as indicated by the spread it takes several years before the it reaches the same levels as it had before the crisis. One explanation may be, that buying companies that are relatively cheap in respect to free cash flows is very sensitive to market conjunctures. The aftermath of the financial crisis is characterized by tightened money lending policies and companies in general writing off and restructuring their existing debt. These are all factors that heavily impact the ability to generate high free cash flows. This may be especially true for companies prices at low dollar per free cash flow (value stocks). Whether it is true or not the evidence suggests, that the market prices the value stocks relatively lower to the market than the prior to the crisis. In other words, the evidence implies that investors do not have same confidence in the low price-to-free-cash-flow as they did prior the crisis. As a consequence, the value stocks recovery very slowly.

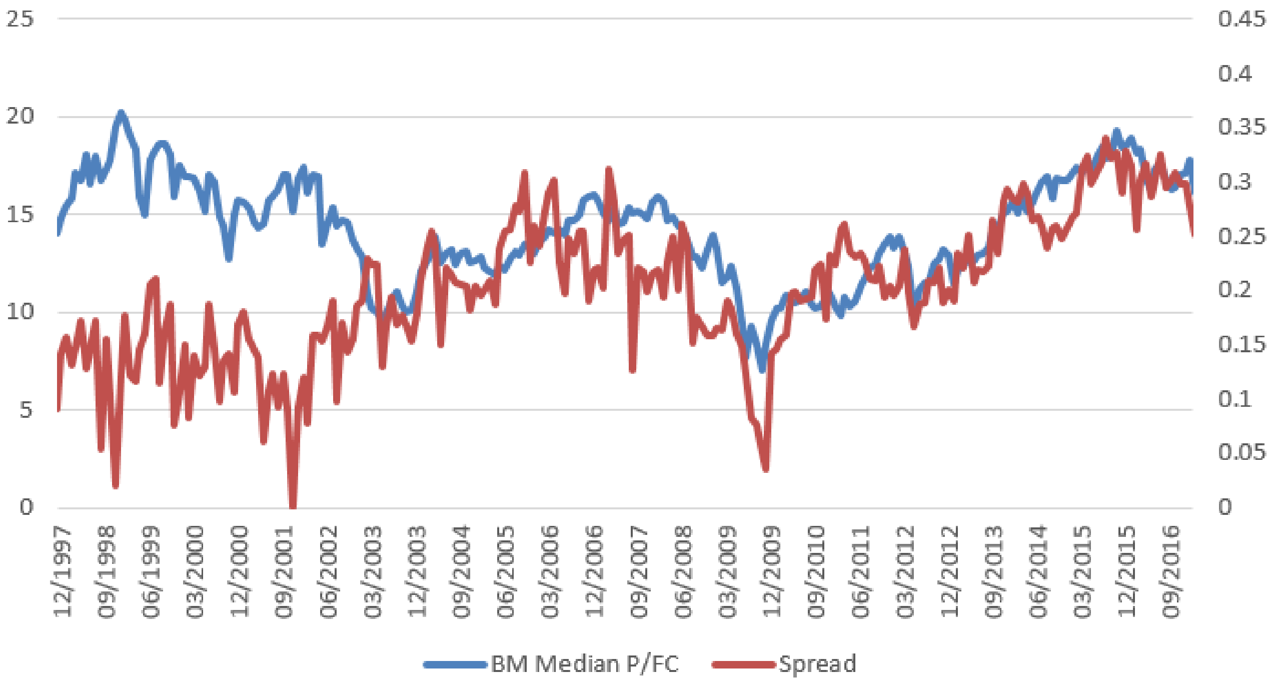
Another explanation could be that investors especially tend to turn to stocks that are solid and stable value investments. An excellent example of this, is the rapid recovery by P/B multiple. The evidence suggests, that in market turmoil investors turn to company's that are fairly priced with respect to their assets. Moreover, this suggests, that investors turn to tangible assets with actual dollar value instead of intangible assets during market meltdowns. This argument is in line with swift also fairly swift recovery by the dividend yield measure, suggesting investors have confidence in the stability of dividend-paying stocks.

Figure 8: P/FC deciles through time for the equally weighted series of value from 1997:12:31 - 2015:01:01. Panel A presents the deciles for the P/FC value measure throughout the sample period from 1997:12:31 - 2015:01:01. The deciles are formed by ranking all stocks in the S&P 500 Index by their P/FC value in a ascending order each month. Panel B presents the equally weighted S&P 500 Index median P/FC (primary axis). The red graph is the spread (secondary axis) between the equally weighted S&P 500 Index and the ten decile.

Panel A: P/FC Deciles



Panel B: P/FC Decile Spread



4.6.1 Value Drawdown Deep-Dive

The next section will examine the drawdown over the sample period for a select few of the value strategies. The strategies chosen here are some of the best performing value strategies. Looking at

Figure 9 below the four best performing value strategies are visualized where the two major crises that impacted the financial markets during the sample period are clearly visible.

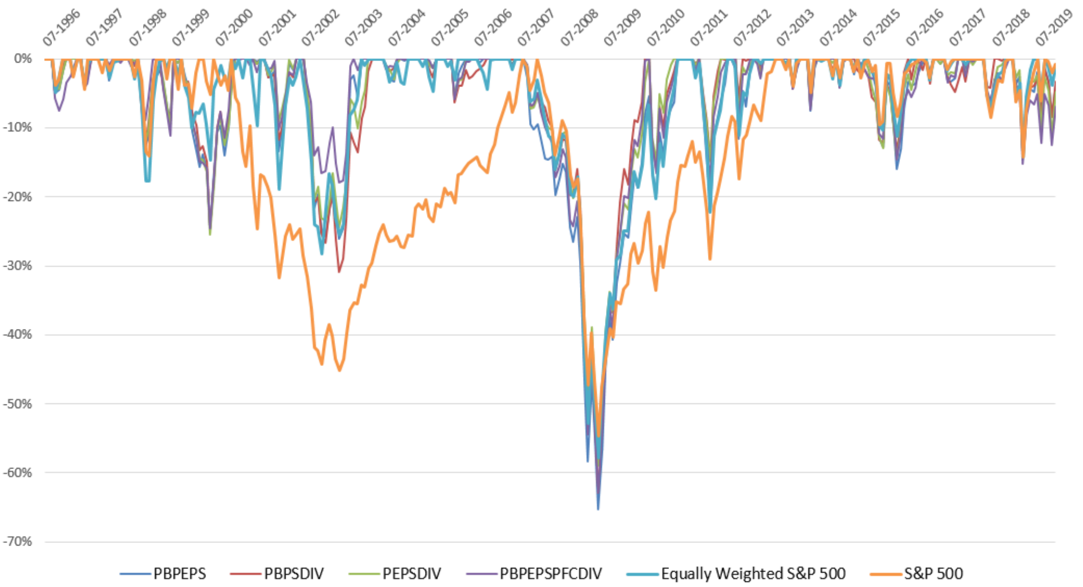


Figure 9: Value Drawdown

The first major drawdown is the Dot-com bubble which extended from 2000 until mid 2003. Here it can be observed that the drawdown for the equally weighted S&P 500 Index reaches -28% whereas the market weighted S&P 500 Index has a drawdown of -45%. The value strategies appear to follow the equally S&P 500 Index closely both under- and outperforming it slightly. The value strategy that handles the dot-com bubble best is the P/B, P/E, P/S, P/FC, DIV only getting a drawdown of 18% before it makes what appears like a V-shaped recovery thus recovering much more rapidly than the market-weighted S&P 500 Index and again slightly more than the equally weighted S&P 500 Index. With regards to the other value strategies these appear to follow the path of the equally weighted S&P 500 Index apart from the P/B, P/S, DIV strategy that gets a worse drawdown of 31% and has a slightly slower recovery compared to our equally weighted S&P 500 Index. A reason for the slightly lower drawdown in the value strategies could be explained by the previously findings of Risager (2013) who found that the value premium had decreased in the period between 1998 and 2010 and that a negative value premium was in effect in 1998 and 1999. The reason for this was the abundant hype around growth stocks thus pushing these kind of stocks valuations to the extreme. As a Danish saying goes: "High to fly, deep to fall" albeit meaning that growth stocks might have surged pre-crisis but would also see a steep fall when the crisis hits. Having implemented a value strategy approach to the portfolio an investor could have dodged this drawdown led by growth stocks in the broader market, albeit not received the return these delivered during the hype.

Looking at the next major drawdown of the sample period, the financial crisis of 2007-08 where a much deeper drawdown can be observed. Here the broader market in the form of both the market weighted S&P 500 Index and equally weighted S&P 500 Index as well as the value strategies take a massive hit in the form of a drawdown of 60-65%. The equally weighted value strategies and the equally weighted S&P 500 attain a slightly lower drawdown of 55% thus not indicating any difference between growth or value stocks. However, when observing the recovery from the all-time-low, our value strategies repeats the behaviour from the dot-com bubble and once again recovers faster than the overall market, albeit not in a V-shape but in more of a U-shape.

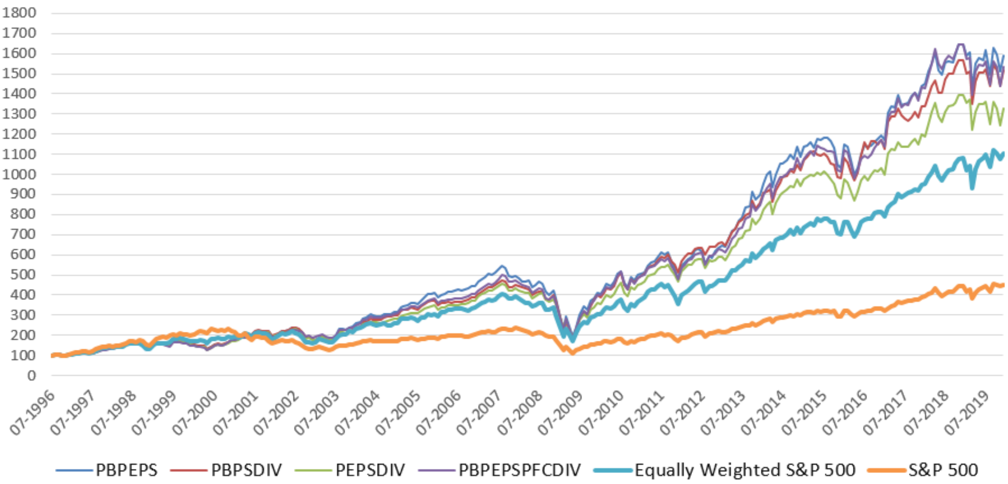


Figure 10: Value return on 100 dollars

In Figure 10 above the four best performing value strategies have been selected and their performance has been visualized. All strategies are initiated at index 100, thus equivalent to investing a $100 in each of the four value strategies and both equally weighted S&P 500 Index index and market-weighted S&P 500 Index index. Running this throughout the sample period, a tremendous difference in return becomes clearly visible. The market-weighted S&P 500 Index attains a return of around 365% over the sample period of 23 years which may be considered a very decent return. However, observing the equally weighted S&P 500 Index over the same period, the return is an astonishing 1100% where much of the return is attributable to the size factor. This comes into play when equal-weighing the index thus letting the companies with smaller market-cap constitute a larger part whereas the heavy-weights in the index conversely represent a smaller part. Looking at the value strategies proposed in this paper, although based on the equally weighted S&P 500 Index, we observe that they clearly tweak the portfolios and attain even higher returns. The strategies P/B & P/E & P/S, P/B & P/S &

DIV and P/B P/S DIV and P/B & P/E & P/S & P/FC & DIV all attain returns around 1550% by the end of our sample period thus implicating that a value premium is definitely existent and possible to attain.

4.7 Turnover Considerations

Figure 11 presents the total amount of stock purchases for the various value portfolios over various holding periods from 1998:12:31 to 2019:12:31. The start date is assured that the portfolios are stable and fully functioning simultaneously so that the number of stock purchases for the various strategies are commensurable. First and foremost, it is important to emphasize the fact that one of the desireable attributes, the value strategy posses is its historically low turnover. Especially, the turnover for the value strategies are significantly lower compared to the momentum strategies which are investigated in the next section. Assessing the turnover for the one-dimensional value sorting strategies the P/S and dividend measure require 369 and 375 fewer transactions compared to the P/FC measure, respectively. Further Examination of

Figure 11: Total number of stock purchases plot for various Value portfolios over various holding periods throughout the entire sample period from 1998:12:31 - 2019:12:30.
The figure presents the total number of new stocks purchased throughout the entire sample period for each value strategy holding 6, 12, and 18 months, respectively. All portfolios start 1998:12:31 and concludes in 2019:12:31.

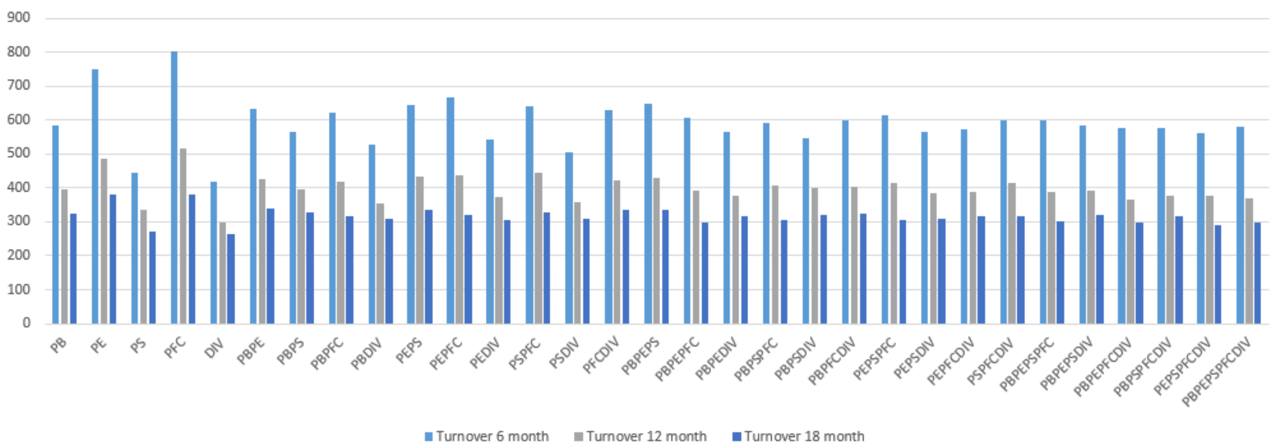
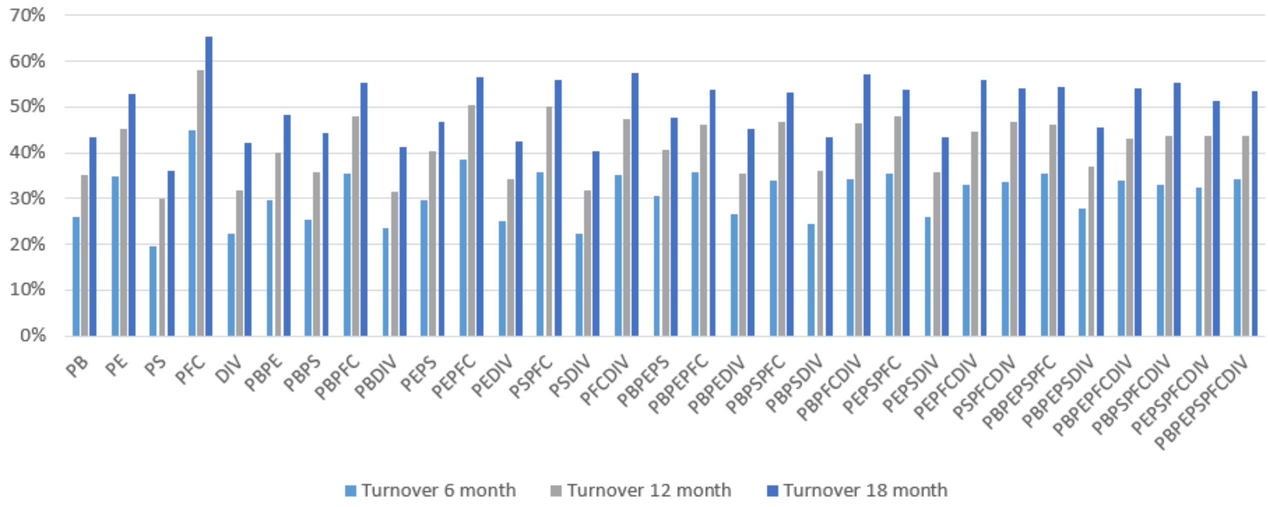


Figure 12 presents the relationship between each strategy and various holding periods and their turnover rate throughout the entire sample period. Naturally, increasing the holding period increases the amount of data required to implement the portfolio, therefore, the 6 months portfolios requires less data than the longer holding periods and therefore starts 1996:06:30, the 12 months portfolios start 1996:12:31, and lastly the 18 months portfolios start 1997:06:30 and all portfolios conclude 2019:12:30.

Thus, the portfolios examined here do posses timing risk which is in contrast to the equally weighted series of value portfolios examined elsewhere. The turnover rate is calculated as the ratio between the sum of new stock names in the portfolio and the sum of stocks for a given measure of value throughout the entire sample.

Figure 12: Turnover plot for various Value portfolios and various holding periods throughout the entire sample period from 1996:06:30 - 2019:12:31.
The figure presents turnover rate for each value strategy holding 6, 12, and 18 months, respectively throughout the entire sample period. The 6 months portfolios requires less data than the longer holding periods and therefore starts 1996:06:30, the 12 months portfolios start 1996:12:31, and lastly the 18 months portfolios start 1997:06:30 and all portfolios conclude 2019:12:31. The turnover rate is calculated as the ratio between new stocks purchased and the total amount of stocks in the portfolio throughout the entire sample period.



Intuitively, increasing the holding period increases the turnover rate because the likelihood that the first decile consists of the same companies as the previous book decreases as the time since the last formation period increases. In other words, the higher turnover rate implies that the portfolio has a larger stock rotation from formation period to formation period. Nonetheless, as evidenced by Figure 11 the higher turnover rate does not outweigh the fact that the portfolios are less frequently rebalanced. The overall number of new stocks associated with the longer holding periods is significantly lower than portfolios that rebalance more frequently. Coupling these results together with the evidence from Table 9 there is no argument in favor of selecting the transaction heavy 6 month portfolio when it performs worse than the 18 months portfolios. Neither would it be beneficial to sacrifice an average of 0.22 percent annually compounded interest over 23-years ($\approx 4\%$ over 23 years) for approximately 200 stock transactions.

4.8 Value Conclusion

The results in this section point towards a number of propositions. First, trading strategies that buy stocks which have low prices relative to book value, earnings, sales, free cashflow, and dividend yield highly outperform strategies buying stocks with relatively high prices (glamour) as well as the equally weighted market portfolio over the 1996:06:30 to 2019:12:31 period. Thus the evidence suggests that market participants have consistently overestimated future growth rate of glamour relative to value stocks. Secondly, constructing value portfolios that sort based on multiple dimensions (measures of value) result in strictly better performance or performance better than its worst constituent (with the exception of two-dimensional sortings). Furthermore, evidence suggests that when constructing these deep value portfolios, the integrated portfolio approach is superior to mixed implementation. Thirdly, while the value portfolios realize significant returns they do come bearing high crash risks with varying swift recoveries. These crashes have severe impact on the temporary performance of the value meassures. For example, evidence implies that the P/FC is affected by a lack in confidence form investors, where more solid and stable value measures such as the dividend yield measure persists to perform well. Examining various holding lengths, the results suggests that value premium is best captured for 12 months with 6 months performing far worse and 18 months returns diminishing. Lastly, the examined value strategies exhibit overall desireable turnover rates for various portfolio sortings and holding periods, that are not likely to deter the attractive returns generated by the value strategies.

5 The Momentum Factor

This section proceeds as follows. Subsection 5.1 covers some of the groundbreaking empirical work done on the momentum factor. Subsection 5.2 covers the methodology for constructing the momentum portfolios. Subsection 5.3 presents the performances of various momentum portfolios. Subsection 5.4 outlines key points for the momentum crashes. Subsection 5.5 takes a deep dive into some of the strongest momentum performances and 5.6 investigates the time-varying beta of some of these strategies. Subsection 5.7 reveals the turnover rates associated with momentum investing. Lastly, 5.8 concludes.

5.1 Literature Review

The momentum factor reflects future excess returns to equity securities with stronger past performances. In other words, the momentum factor refers to the fact, that security prices tend to exhibit a short-term pattern; in which strong performing securities will continue to perform strongly and poorly performing securities will continue to perform poorly.

There exists an extensive body of financial literature documenting the predictability of security returns based on past pricing history. Jegadeesh and Titman (1993) were the pioneers to document the existence of the MF phenomena on the US security market. Studying data from the CRSP daily return file [3] Jegadeesh and Titman considered strategies that selected securities based on their return over the past 1, 2, 3, and 4 quarters of a year, while also considering holding periods that varied from 1 to 4 quarters. This resulted in a total of 16 various strategies. They also examined a second set of 16 portfolios that skipped a week between the formation of the portfolio and the selection period in order to avoid market interference [4]. The imposed momentum strategies documented that buying the past top decile winners and selling the past bottom decile losers, yielded a significant abnormal return in the period 1965-1989. They found that the most successful trading strategy - in terms of raw return - selected securities based on their return over the past previous 12 months and then holds this portfolio for the following 3 months. This particular momentum strategy generated a statistical significant return of about 1.31% per month in the sample period of 1965 to 1989.

Moreover, additional evidence by Jegadeesh and Titman indicates that the abnormal return cannot

[3] At the time, this data file covered return data of securities in the NYSE and AMEX from 1965 - 1989.
[4] This includes bid-ask spread, price pressure, and lagged reaction effects.

be attributed by their systematic risk, implying that the profits may be due to market inefficiencies. Their results document that the momentum strategies' positive returns are short- rather than long-term; beyond the 12 month mark, the portfolios realize mostly negative returns for the subsequent 24 months. This indicates that the momentum strategies do not select securities that have unconditionally high expected returns, but rather it selects securities with a short and temporarily high return. They attribute the momentum factor to be caused by investor behavior; investors who buy past winners and sell past losers temporarily move the security prices away from their long-term prices. This is in contrast to common interpretations that the momentum strategies' profits are due to overreaction and return persistence. It is however, consistent with existing literature done by DeLong, Shleifer, Summers, and Waldman (1990) who study the implication of "positive feedback traders".

In a study of mutual fund performances, Carhart (1997) expanded upon the Fama-French Model to a Four-Factor Model, which includes momentum as an additional explanatory variable. Rouwenhorst's research (1998) found momentum in Europe using a sample of 2.000 securities and a year later (1999) he also found momentum in emerging equity markets. Fama and French (2012) acknowledged the vast increasing focus on momentum and found strong significant evidence of positive momentum returns in North America, Europe, and Asia Pacific. Based on their empirical evidence they also confirmed the robustness of the Four-Factor model proposed by Carhart.

The findings of Jegadeesh and Titman were later on extended upon by Cliff Asness (1995, 1997). Not only did he confirm the earlier momentum findings, he also expanded on the findings by indicating the optimal time horizon. In his empirical research, he found that the deciles tend to exhibit mean reversion over the long-term, i.e. the top decile would outperform in the short-term (1-12 months) and underperform in the long-term (3-5 years out). The long-term outperformance, was however, already proposed by De Bondt and Thaler (1985, 1987) who demonstrated long-term price reversals. The evidence of negative serial correlation in the long-term was later supported and documented by Fama and French (1998). Moreover, the reversal from positive to negative serial correlation underline a very important fact namely that the success of the MSs are very reliant on a relatively high security turnover.

In summary, empirical research has found evidence that the momentum strategy exhibits positive serial correlations in the short-term and a long-term reversal i.e. a negative serial correlation in the long-run. Ultimately, the results are evidence of security return predictability and in this context, momentum is at the very least, a challenging factor to put into the framework of the random walk hypothesis which is required for efficient markets.

The theory underlying this risk premium is still under heavy discussion. The two prevailing prepositions are both based on behavioral science. First, investors either overreact (Hong, Lim, and Stein (1999)) or underreact (Chan, Jegadeesh, and Lakonishok (1996)) to news, both of which may be the cause of the momentum factor. Second, investors tend to herd - a study by Grinblatt, Titman, and Wermers (1995) - shows that mutual funds base a lot of their investment strategies on past performances. More recent academic research, proposed by Vayanos and Woolley (2011), suggests that a majority of institutional funds weigh their value and momentum portfolio based on recent past performances. For example, if their value portfolio declines they will increase their momentum portfolio and vice versa, which makes sense since these two portfolios are negatively correlated. Both interpretations that supports Jegadeesh and Titman's initial proposal, that *"(...) transactions by investors who buy past winners and sell past losers move prices away from their long-run values temporarily and thereby cause prices to overreact"* (Jegadeesh and Titman (1993), pp. 90).

5.2 Momentum Trading Strategy

If stock prices exhibit positive or negative serial correlation in the short run or long run, respectively, then profitable trading strategies, that selects stock based on their past returns, will exist. The momentum strategies proposed in this paper will select stocks based on returns over the past J months and it will hold this portfolio for the following K months. For simplicity this paper will only focus on a long-only (buy-only) style investment, exploiting the fact that empirical evidences suggests that:

$$\mathrm{E}\left(r_{it} - \bar{r}_t | r_{it-1} - \bar{r}_{t-1} > 0\right) > 0 \tag{16}$$

where r_{it} is the return on security i in period t and $\bar{r}_t$ denotes the cross-sectional average of the sample. As mentioned in subsection subsection 2.2 existing research studies suggest, that the momentum factor is most prominent in a short-term period (3-12 months). With this knowledge at our disposal, this paper will examine the set of holding periods with $K = \{1, 3, 6, 9, 12\}$ months and a set of selection periods with $J = \{1, 3, 6, 9, 12\}$ months. Moreover, in line Jegadeesh and Titman (1993), this paper will include a second set of selection periods, namely, selection periods that exclude the most recent month in the selection period. This methodology results in 25 portfolios with regular selection periods and 20 portfolios, in which the selection period excludes the last month, hereby, yielding a total of 45 different MS portfolios. Specifically, in a given month t, the securities are ranked in ascending order on the basis of their returns in the past K months. Based on these rankings, a portfolio is constructed that equally weighs each security in the top decile (top 10%). Consider the case of the $J = K = 12$ months portfolio, which is constructed in January 1996 (1996:01:02), this portfolio will consist of the top decile from the past 12-months (data ranging from 1994:12:30 to 1995:12:30) and holds this portfolio for the following 12 months. When, the holding period concludes, the portfolio rebalances its book to now consists of the new top decile.

To amplify the significance of the tests conducted in this paper, the tests will include portfolios with overlapping holding periods. Therefore, in any given month t, each portfolio will hold an equally weighted series of portfolios that are constructed in the previous J months. As an example, the $J = K = 12$ portfolio, is an equally weighted (*1/12*) series of $J = K = 12$ momentum strategies constructed in January, February, March, April, May, June, July, August, September, October, November, and December during 1996. This methodology requires 24 months of data and with data from 1994:12:30 this means that the portfolio is first successfully constructed 1997:01:01. Going forward all of the 45 examined portfolios are equally weighted series $1/X$ momentum portfolios set apart with 1 month. It effectively, ensures that the portfolios' performances are not too dependent on their starting point and underlying structural market dynamics.

The momentum strategies imposed in this paper are conducted on the sample data from 1994:12:30 to 2019:12:31 and are all constructed in the exact logic as described above.

5.3 Momentum Strategy Results

The annualized compounded returns of the 45 different momentum portfolios are reported in Table 11, as well, as their corresponding t-statistics, which are listed in the parentheses[5]. The returns of the momentum portfolios are all positive and exhibit statistically significant t-statistics at the 95%

[5]The annualized returns and standard deviations are obtained by compounding the full sample daily returns into monthly and then converting them to annualized returns and standard deviations. These values allow computations of the corresponding t-statistics given a full sample period of $n = 23$ years.

confidence level. This does however not hold for the momentum portfolios with $K = 1$ and $K = 3$ months holding periods that select stocks based on a selection period of $J = 1$ month. In respect to absolute returns the momentum portfolios in general produce higher returns when comparing it to the capitalization and equally weighted S&P 500 index, which both realize an annualized compounded return of 6.65 and 11.05 percentage, respectively. For example, the momentum portfolio that selects stocks based on their last $J = 6$ months return and holds these stocks for the subsequent $K = 6$ months produces a return of 12.41 percent.

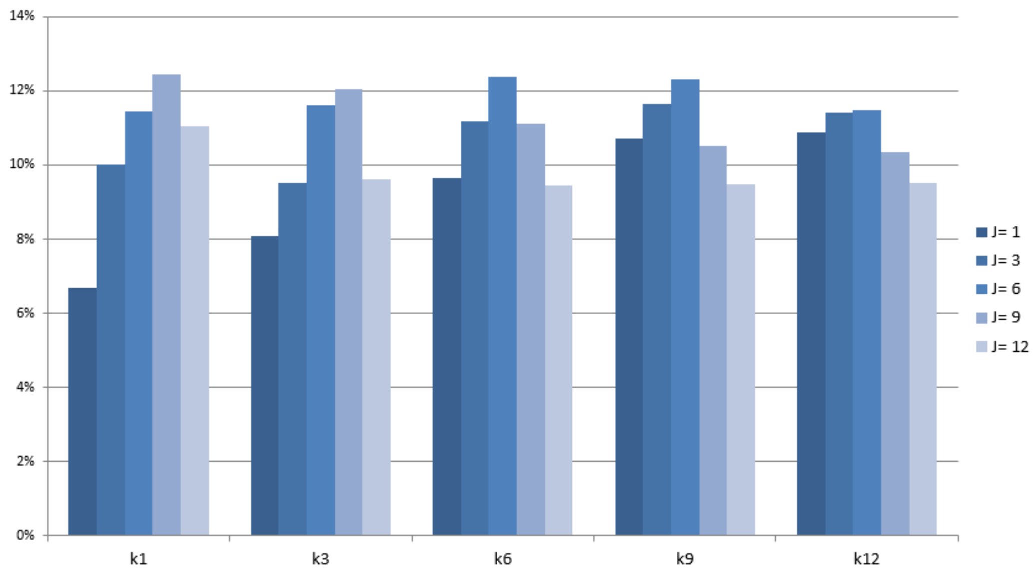
Table 11: Annualized compounded returns for Momentum strategy portfolio in the sample period of 1997:01:01 - 2019:12:31.
The momentum strategies are formed based on a J-month selection period and are held for K-months. The values for K and J indicating the different strategies formations are found in the first column and row, respectively. The securities are ranked in ascending order with respect to each security's return on a basis of the J-months selection. The equally weighted momentum portfolios consists of securities in the top decile and are long portfolios. This Table reports the annualized returns for each long portfolio. The $J = \{1, 3, 6, 9, 12\}$ selection periods are without excluding the most recent month in the selection period. While, the $J = \{3-1, 6-1, 9-1, 12-1\}$ selection periods disregards the most recent month in its J-month selection. The corresponding t-statistics are reported in the parentheses.

K $J =$	1	3	3-1	6	6-1	9	9-1	12	12-1
1	6.67	10.00	9.62	11.44	12.04	12.43	13.37	11.02	11.76
	(1.53)	(2.55)	(2.25)	(3.02)	(2.96)	(3.15)	(3.26)	(2.76)	(2.85)
3	8.07	9.52	9.40	11.63	12.05	12.14	12.23	9.60	9.83
	(1.94)	(2.38)	(2.24)	(2.89)	(2.86)	(2.96)	(2.91)	(2.31)	(2.29)
6	9.65	11.17	10.95	12.37	12.41	11.11	11.39	9.44	9.61
	(2.30)	(2.72)	(2.57)	(3.07)	(3.00)	(2.68)	(2.70)	(2.18)	(2.18)
9	10.71	11.68	11.29	12.30	12.11	10.52	10.76	9.48	9.49
	(2.49)	(2.84)	(2.64)	(3.04)	(2.93)	(2.48)	(2.49)	(2.13)	(2.10)
12	10.87	11.42	11.00	11.48	11.07	10.33	10.47	9.50	9.43
	(2.53)	(2.78)	(2.59)	(2.78)	(2.64)	(2.41)	(2.42)	(2.13)	(2.08)

In regards to the hypothesis of using the selection criteria of excluding the most recent month in the selection period, Table 11 displays inconclusive signals. None of the strategies selecting based on the past $J = 3-1$ months perform better than those selecting based on the past $J = 3$ months. However, strategies with longer selection periods such as the $J = 9-1$ months all perform superior to those that select based on the past $J = 9$ months. Comparing the $J = 6-1$ and the $J = 12-1$ to the $J = 6$ and $J = 12$, respectively, the results are again mixed. In summary, the idea of excluding the most recent month in the selection period does not produce stable concise results. Furthermore, claiming the hypothesis holds, will require the portfolios excluding the most recent month to produce statistically larger returns than the portfolios that do not exclude the most recent month, which is not the case.

Figure 13 presents a visualization of the annual compounded returns in Table 11 for different holding periods and formation periods (selection periods). The findings are consistent with existing research. The Table and Figure indicate, that the momentum strategies are strongest in the intermediate term ($K = 6$ and $K = 9$ months) and tends to be weak in very short formation periods ($K = 1$ month) and slightly weaker in very long formation periods (K=12).

Figure 13: Realized compounded returns in the period of 1997:01:01 to 2019:12:02 for various equally weighted series of momentum portfolios.
The figure visualizes the annualized compounded returns from the various portfolio combinations of $J = \{1, 3, 6, 9, 12\}$ and $K = \{1, 3, 6, 9, 12\}$. The risk-adjusted returns are obtained by using monthly return data for each strategy.



Furthermore, the evidence suggests that there is a dependency between formation length and holding length. From Table 11 it is very easy to see that the $K = 1$ month holding period produce higher returns for longer formation periods (highlighted by the light columns). The same is true for the $K = 3$ holding period. While, longer holding periods as $K = 9$ and $K = 12$ months produces higher returns for lower formation periods (highlighted by the dark columns). Generally speaking, this means that as you move from left to right shifting to longer holding periods, the shorter formation periods (highlighted by the dark columns) realizes stronger returns while the long formation period (highlighted by the light columns) weaken.

Notably, as an extension to current literature, the findings suggest that the momentum portfolio that rebalances its books every single month performs very well. Consistent with the argumentation above, for longer formation periods of the $k = 1$ months holding period yields higher returns than the equally weighted S&P 500 index.

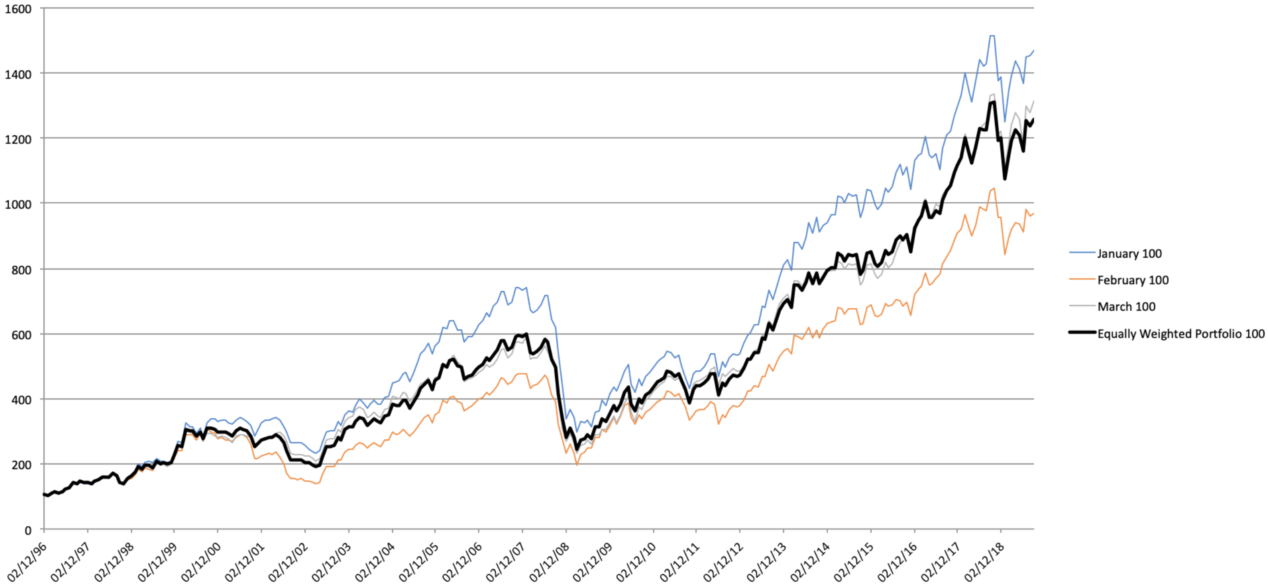
In summary, the evidence highly suggests, that the optimal holding period depends on the formation period. Short formation periods perform well with longer holding periods, while long formation periods perform well with shorter holding periods. This means an investor that aims at levering short-term momentum may benefit from longer holding lengths and vice versa. A casual interpretation of the findings is, that the momentum factor has a price appreciation cycle of approximately 9-15 months from the start of its "momentum" (uptrend). This finding has little economic intuition, but rather it may be explained by implementation implications. If momentum tends to exist for around 12-14 month, then formation periods of $J = 12$ months will have a high likelihood of buying these stocks. Conversely, lower formations periods such as the $J = 1$ month periods will have a relatively lower likelihood of capturing the momentum stocks. However, when the lower portfolio buys momentum stocks it will hold them for a longer time, ultimately materializing more of the momentum appreciation Selecting a formation and holding period that sums to around 12-14 may just be the perfect trade-off between the likelihood of selecting a momentum stock and maximizing the realized return.

As mentioned in the subsection 5.2 all of the listed momentum portfolios are constructed as an equally weighted series of K momentum portfolios. These K momentum strategies are all constructed with one month apart so that each of the equally weighted momentum strategies constituents have overlapping holding periods. For example, the $J = K = 3$ months momentum portfolio, is a portfolio that equally weighs a series of 3 different $J = K = 3$ momentum portfolios starting in January, February, and March 1997. Including a forth portfolio starting in April is futile, as the first monthly return is simply going to be equal to the forth monthly return for the portfolio starting in January. The $J = K = 3$ months momentum strategy starting in January concludes in March. Starting up in April it will select the stocks performing best the last $J = 3$ months. In other words, the second formation period for the January portfolio is identical to starting an additional $J = K = 3$ months momentum strategy in April. Effectively, this overlapping structure ensures that the momentum portfolios are less effected by underlying market structural dynamics.

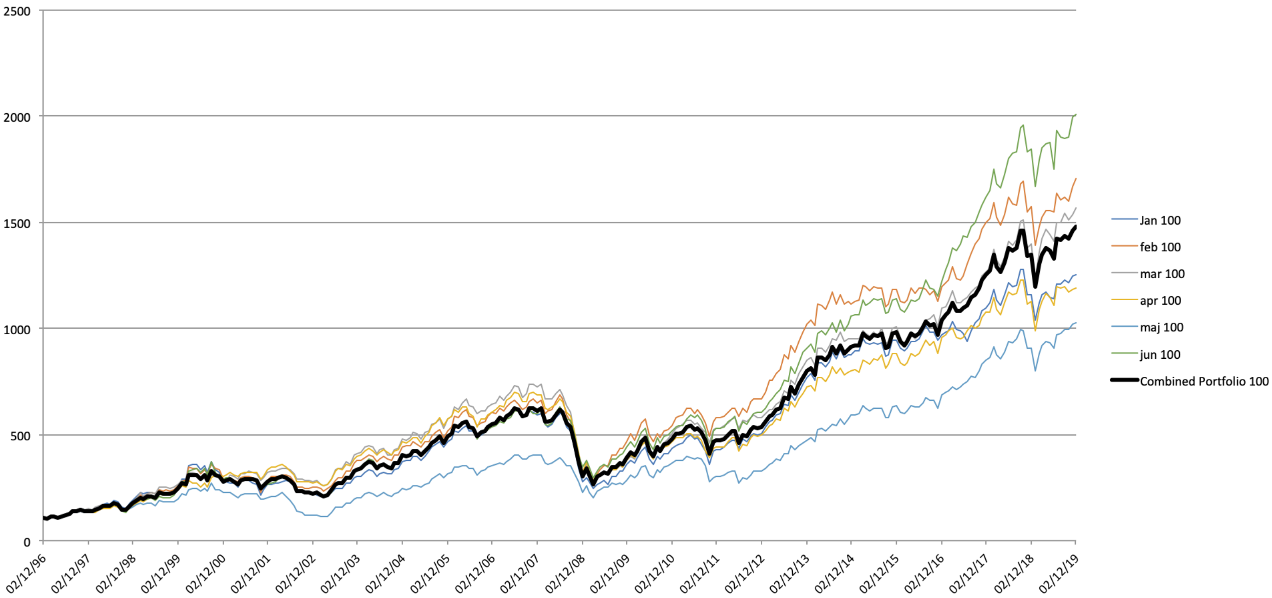
The figures below depict the compounded log returns for the equally weighted momentum portfolio and its constituents, which have a selection period of *J=6* months and holds these securities for the following 3, 6, 9, and 12 months, respectively. In each of the figures the black labelled graph is the equally weighted series of momentum portfolios, and are equivalent to the momentum portfolios listed in Table 11. The evidence indicates that the variation of realized compounded log returns is most stable for portfolios with lower holding periods, the instability increases as the holding period increases with the $J = 9$ months portfolios exhibiting most variation.

Figure 14: Realized compounded returns in the period of 1997:01:01 to 2019:12:31 for various equally weighted series of momentum portfolios and its constituents.
The portfolios have $K = \{3, 6, 9, 12\}$ months holding periods respectively for Panel A, B, C, and D and select securities based on returns from the previous $J = 6$ months. The January portfolios reflects the momentum portfolios initiating in January, the same is the case for the February, March, April etc. portfolios, respectively. The equally weighted portfolio is highlighted in black and is a equally weighted series of all of the portfolios. The realized compounded return is computed using monthly return data throughout the full 23-year sample period.

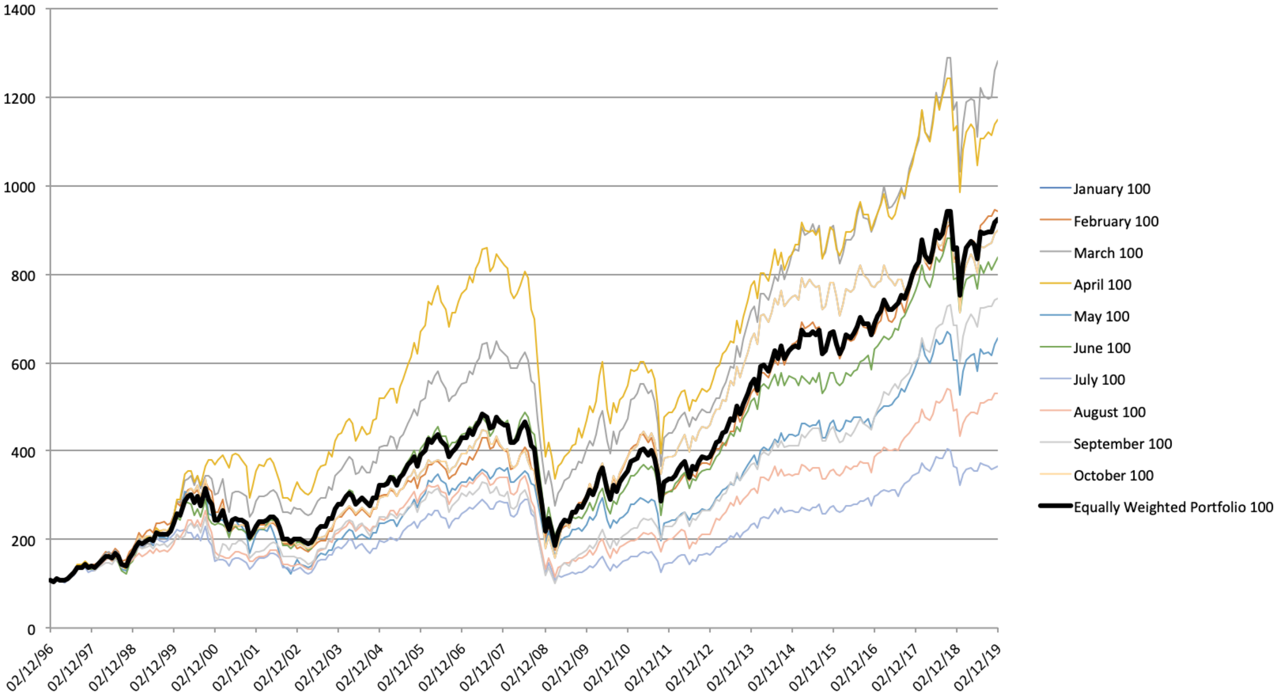
Panel A: $K = 3 \wedge J = 6$ Momentum Portfolios



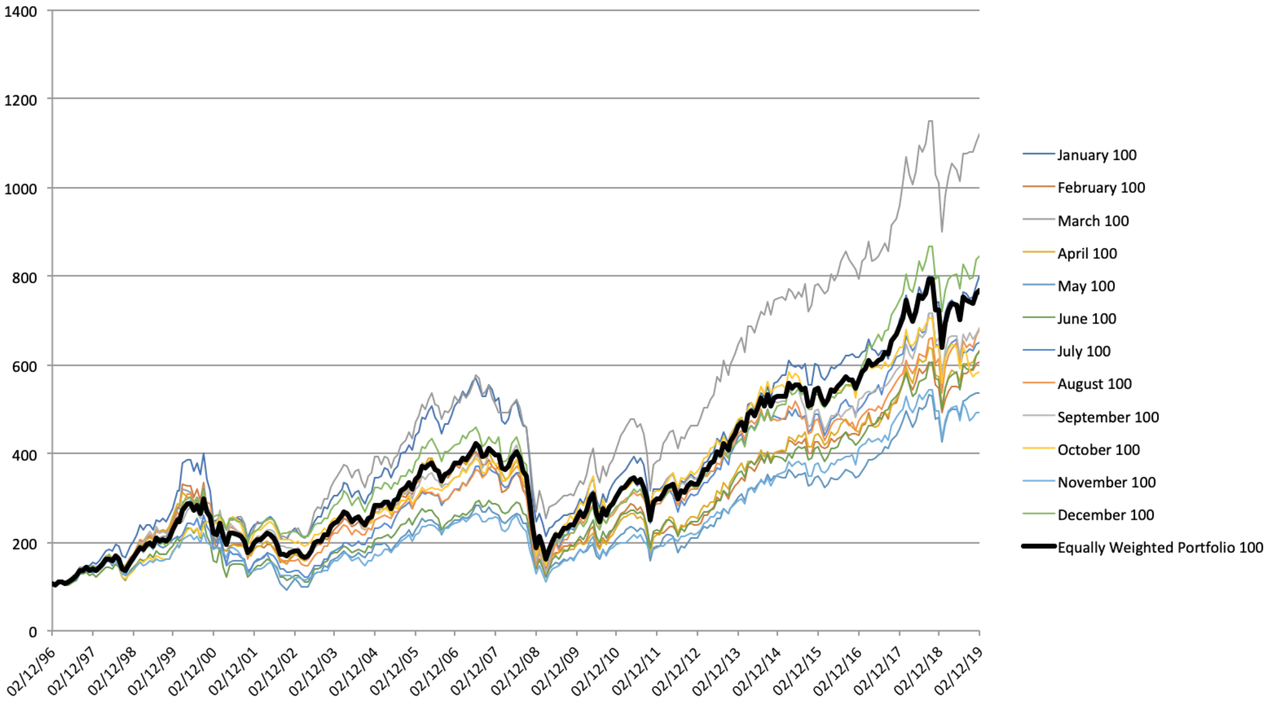
Panel B: $K = 6 \wedge J = 6$ Momentum Portfolios



Panel C: $K = 9 \wedge J = 6$ Momentum Portfolios



Panel D: $K = 12 \wedge J = 6$ Momentum Portfolios



As evidenced in the figures above, the stability of the returns decrease as the holding period increases with the portfolios holding *J=9* months exhibiting the largest variance among its constituents. The largest compounded return for the *J=3* months portfolio is approximately 1.5 times larger than the corresponding smallest realized compounded return; the same number is 1.9 for the *J=6* months portfolio; 3.5 times for the *J=9* months portfolio; and 2.2 times for the *J=12* months portfolio.

The huge variation among the momentum portfolios' constituents looking at the full period realized compounded returns are the exact reason that this paper investigates the equally weighted series of momentum portfolios instead of its constituents. As stated earlier, this methodology, reduces the likelihood that the listed performance characteristics for the portfolios are not due to seasonal/monthly underlying structural market dynamics manifested in the markets. In other words, the equally weighted portfolio allows investigation of the momentum premium without being overly exposed to potential seasonal/monthly risk premiums.

As mentioned in subsection 3.1.6 portfolio managers and investors are in general far more interested in the risk-adjusted returns than the absolute returns. One can always increase the return by increasing the risk profile! To depict an overview of the risk involved with the returns, the risk-adjusted performances of the 45 strategies the IRs are listed in Table 12. To simplify the results in Table 12 a colouring scheme has been applied. The colouring scheme is applied with respect to the IR of the equally weighted S&P 500 Index, which has a IR of 0.60. A rich green colour implies that the IR is 0.65 or higher; a lighter green implies the IR is 0.60 or higher; a yellow colour implies an IR below 0.60 and lastly, orange indicates a IR lower than 0.50.

As indicated by the colouring scheme, the risk is tightly related to the return. Examining the risk-adjusted returns it is apparent, that the portfolios that select stocks based on the past set of $J = \{1, 3, 12\}$ months do not come with less risk. Not very surprisingly, in line with the return results, the portfolios with shorter formation periods and longer holding periods produce stronger risk-adjusted returns. This is also true for the portfolios with longer formation periods and short holding periods. Nonetheless, comparing these strategies to the equally weighted S&P 500 Index they all bear significant risk relative to their inferior returns. At this point, they are all deemed less attractive.

Furthermore, Table 12, reinforces the observation, that the momentum strategies, that have a selection period that excludes the most recent month, in general does not yield indisputable superior performance. In terms of risk-adjusted returns, the $J = 3 - 1$ months portfolios are all strictly worse across different holding periods; the same is true for the $J = 6 - 1$ months portfolios with the exception of the $K = 1 \wedge J = 6 - 1$ months portfolio and the $J = 9 - 1$ months portfolios are no longer strictly superior to those that select based on the last $J = 9$ months; lastly, the $J = 12 - 1$ months portfolios yield somewhat the same performance as those of $J = 12$ months.

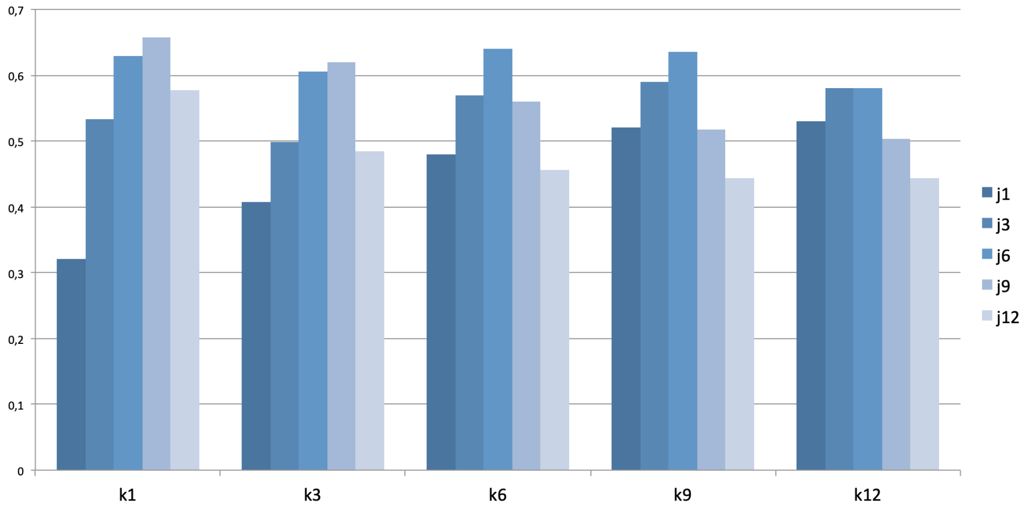
It is worth noting that the equally weighted S&P 500 Index with an IR of 0.60 is a highly competitive benchmark. The regular market capitalization weighted S&P 500 Index yields an IR of 0.40, which results in almost all of the portfolios beating its benchmark in a risk-adjusted return framework. However, since the momentum portfolios are all equally weighted, it is only fair to at least compare it to both the equally- and market capitalization weighted indices.

Table 12: Momentum strategy portfolio risk-adjusted return in the sample period 1997:01:01 - 2019:12:31.
This table presents the information ratios for each of the 45 differently constructed momentum portfolios over the 23-year full sample period form 1997:01:01 to 2019:12:31. The information ratio (IR) meassures the return after adjusting for the risk involved. The IR here is computed as the ratio between the annualized return and standard deviation for the given portfolio construction. The colouring scheme is applied with respect to the IR of the equally weighted S&P 500 Index, which has a IR of 0.60. A rich green colour implies implies that the IR is 0.65 and higher; a lighter green implies the IR is 0.60 or higher; a yellow colour implies an IR below 0.60 and lastly, orange indicates a IR lower than 0.50.

K $J =$	1	3	3-1	6	6-1	9	9-1	12	12-1
1	0.32	0.53	0.47	0.63	0.62	0.66	0.68	0.58	0.59
3	0.41	0.50	0.47	0.60	0.60	0.62	0.61	0.48	0.48
6	0.48	0.57	0.54	0.64	0.63	0.56	0.56	0.46	0.45
9	0.52	0.59	0.55	0.63	0.61	0.49	0.49	0.42	0.42
12	0.53	0.58	0.54	0.58	0.55	0.50	0.50	0.44	0.43

Figure 15 visualizes the risk-adjusted returns in Table 12 for the different holding periods and formation periods. The findings the very similar to the return results. There is a well defined dependency between formation period and holding period. As you move from left to the the right shifting to longer holding periods, the shorter formation periods (highlighted by the dark columns) produces unparalleled high risk-adjusted returns.

Figure 15: Realized risk-adjusted returns in the period of 1997:01:01 to 2019:12:31 for various equally weighted series of momentum portfolios.
The figure visualizes the annualized risk-adjusted returns from the various portfolio combinations of $J = \{1, 3, 6, 9, 12\}$ and $K = \{1, 3, 6, 9, 12\}$. The risk-adjusted returns are obtained by using monthly return data for each strategy.



In summary, the risk-adjusted return results are very consistent with the return results, suggesting a full momentum cycle of 9-15 months. Examining the risk-adjusted returns, the results arrive at the same conclusion, that excluding one month in the formation period has little evidence to support its claims.

Having established that the momentum portfolios are in fact very profitable. Table 13 presents performance characteristics and return moments for all of the $J = K = 6$ -months momentum portfolio deciles. The portfolios examined so far have all been momentum strategies with a selection criteria based on the 10th return decile. Consistent with the existing litterateur, a strong momentum premium emerges over the 23-year sample data: The 10th winner decile strongly outperforms the loser decile. The winner decile has an annualized compounded return of 12.31 % while the loser decile realizes a return of 7.14%. In contrast, the winner decile has an excess annualized return of 1.41% and 6.69% in comparison to the equally weighted S&P 500 Index and the market capitalization weighted S&P 500 Index, respectively. Consistent with this alpha, the portfolio has IR which is 8% and 58% higher than these two indices, respectively.

Table 13 also shows that the winner portfolios are considerably more negatively skewed than the loser portfolios. While the winners still outperform the losers overtime, the IR and alpha understates the significance of crashes. Examining the skewness of the portfolios, the skewness is monotonically decreasing with the 9th decile being most negatively skewed, moving to the 10th decile and more extreme deciles the skewness increases slightly. The 10th decile has a negative skewness of -0.72, while the 1st decile has a positive skewness of 0.10. The opposite is true for the kurtosis, here the loser portfolios appear to exhibit larger kurtosis than the winner portfolios. This indicates that the loser portfolios have a larger likelihood of realizing relatively negative extreme outcomes, however, these relatively extreme outcomes are of a smaller magnitude, indicated by the lower skewness, than those of the winner portfolios. Vice versa, the winner portfolios exhibit a large negative skewness coupled with a smaller kurtosis, suggesting they realize extremely negative outcomes of a larger magnitude than those of the loser portfolios, but that the likelihood of these extreme outcomes is smaller.

Table 13: Momentum portfolio characteristics in the full sample period from 1997:01:01 - 2019:12:31.

This table presents characteristics of the monthly momentum decile portfolio return over the 23-year full sample period from 1997:01:01 to 2019:12:31. r is the compounded return, σ denotes the standard deviation, t(r) denotes the t-statistic, the α denotes the excess return from the portfolio in relation to the equally weighted S&P 500 Index, the IR denotes the annualized information ratio, SK(m), and K(m) represents the full period realized skewness and excess kurtosis of the monthly returns.

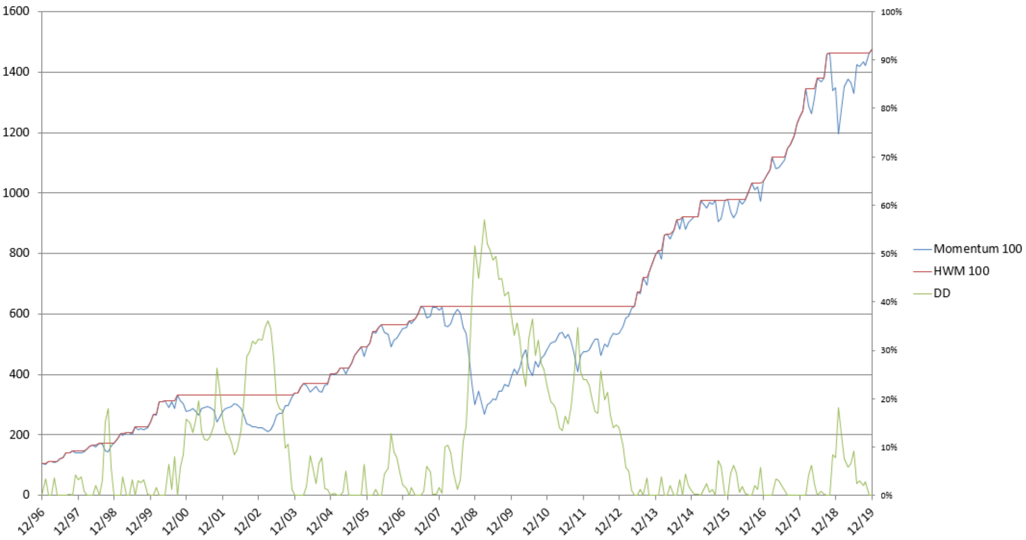
Return Statistic	Momentum decile portfolio													
	1	2	3	4	5	6	7	8	9	10	Top 5%	Top 1%	S&P500	EW S&P 500
r	7.14	10.09	10.36	11.79	11.48	10.74	10.49	10.59	11.40	12.37	12.41	13.19	6.16	10.96
σ	31.80	23.03	20.39	18.28	16.91	16.45	15.98	16.04	15.90	19.20	21.30	26.78	15.22	18.41
t(r)	(1.08)	(2.10)	(2.44)	(3.09)	(3.26)	(3.13)	(3.15)	(3.17)	(3.44)	(3.09)	(2.79)	(2.36)	(1.94)	(2.86)
α	-3.82	-0.87	-0.60	0.83	0.52	-0.22	-0.47	-0.37	0.44	1.41	1.45	2.23	-4.73	0
IR	0.22	0.44	0.51	0.65	0.68	0.65	0.66	0.66	0.72	0.64	0.55	0.49	0.40	0.60
SK(m)	0.10	-0.29	-0.44	-0.44	-0.50	-0.65	-0.84	-0.90	-0.97	-0.72	-0.56	-0.27	-0.83	-0.63
K(m)	2.83	4.31	4.38	3.72	3.53	3.42	3.34	3.57	2.76	1.91	1.49	1.18	2.30	3.48

In regards of choosing the optimal decile for harvesting this strong present momentum premium, it is worth noting that selecting securities constituting the top 5% or 1% to gain additional alpha (albeit the lower relative return for risk) will decrease the portfolio size dramatically. Choosing either of these two will decrease the portfolio size from the region of 50 securities to the region of approximately 25 securities and 5 securities, respectively. As shown in Section 3.5 decreasing the number of securities in the portfolio size will inevitably lead to additional idiosyncratic risk. This paper will proceed in the following manner, concluding that the 10th decile consisting of approximately 50 securities delivers strong and pervasive momentum alpha returns and IRs while being a well diversified portfolio.

5.4 Momentum Crashes

The momentum strategies' compounded annual returns are in general large and highly statistically significant, but since 1997:01:01 there have been a number of long periods over which momentum underperformed dramatically. Figure 16 highlights the largest sustained drawdown (highlighted in green) realized all momentum strategies (the discretionary choice here is $J = K = 6$ months momentum portfolio), namely the one starting March 2009 and lasting to about march 2013. The starting date is not coincidentally March 2009, the "market bottoms" following the stock market decline associated with the financial crisis.

Figure 16: Realized compounded returns in the period of 1997:01:01 to 2019:12:31 for the equally weighted momentum portfolio that has K=6 months holding period and select securities based on returns from the previous J=6 months.
The figure presents the cumulative monthly returns from 1997:01:01 to 2019:12:31 for investing in the $J = K = 6$ months momentum portfolio. The initial notional is \$100 and the final cumulative return is \$ 1477.30



Zeroing in on the financial crisis crash, looking exclusively at the portfolios that do not exclude the most recent month in its selection criteria (the results, however, are very similar), Table 14 examines the realized tail risk associated with the momentum portfolios as well as the capitalization and equally weighted S&P 500 Index. The Table displays the maximum drawdowns realized for each of the portfolios, as well as the corresponding length and recovery time denoted in months [6]. Several key points emerges from Table 14 as well as from figure 16:

1. While the momentum portfolios have generally outperformed the capitalization and equally weighted S&P 500 Indices in terms of returns and IRs, there are relatively long periods over which momentum portfolios experience severe losses and crashes. Said differently, the superior performance in terms of alpha and IRs, clearly understates the significance of these crashes.

2. All of momentum portfolios experience their maximum drawdown 2009:03:02 when the contemporaneous equally weighted S&P 500 Index (market weighted S&P 500 Index) experienced its largest monthly decline of 16% (19%), indicating that all of the the portfolios are highly exposed to the market. The clustering evident in Table 14 and Figure 16, also indicate that the crashes do not occur in days or weeks; it is not a jump poisson process. Rather, they take it slow and follow the market in some degree, over multiple months.

3. Further examination reveals that the large crash performances are highly correlated with volatility spikes in the contemporaneous market. Figure 17 illustrates how the large negative monthly returns of the $J = K = 6$ -months portfolio occur at the same time as the six months rolling volatility (standard deviation) of the equally weighted S&P 500 Index spikes. While it might not be surprising, that extreme returns occur when the market is highly volatile, note that the effect is asymmetric. The extreme momentum gains are not as large or concentrated in time as the extreme losses.[7]

[6] The drawdown characteristics are computed using monthly return data for each of the evaluated portfolios in the period of 1996:12:31 to 2019:12:31.

[7] Figure 17 plots the equally weighted S&P 500 Index market volatility and the monthly returns of the momentum portfolio that selects its securities based on the last 6 months and rebalances its books every 6 months. The volatility estimation is computed using a 6 months rolling market model regression with monthly data. Consequently, the time varying volatility starts 6 months later than usual, meaning that the plot spans the sample period starting 1997:11:31 and concludes 2019:12:31.

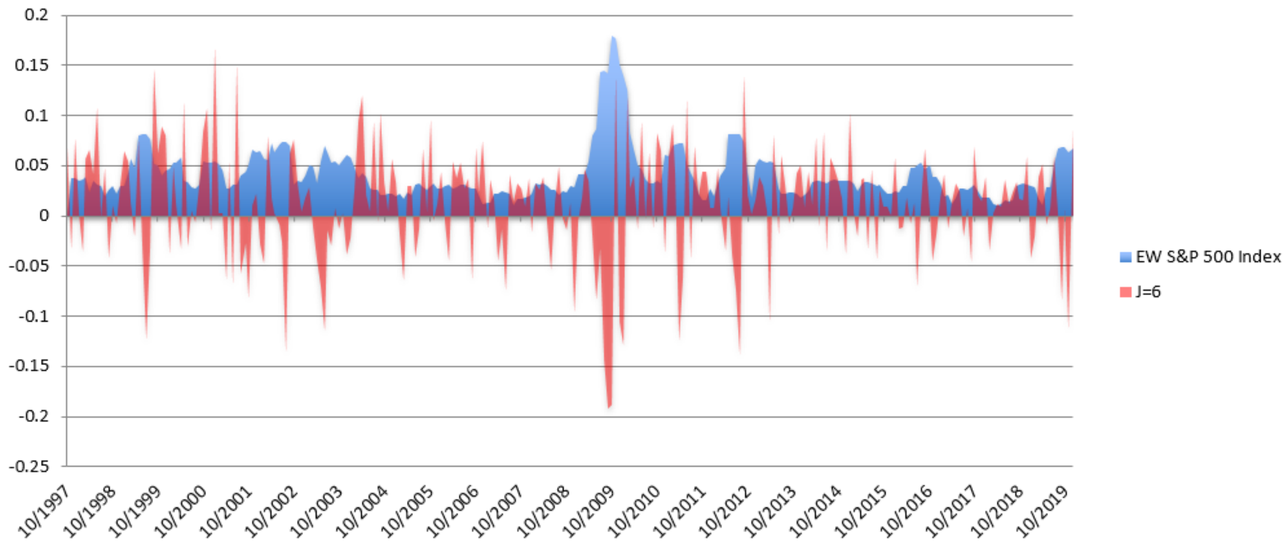
Table 14: Momentum portfolio drawdown characteristics in the full period from 1997:01:01 - 2019:12:31.

This table presents the worst realized maximum drawdown (MDD) for all of the momentum portfolios over the 23-year full sample period from 1997:01 to 2019:01 using monthly return data. Additionally, the table presents the corresponding length and recovery time for the portfolios. MDD denotes the worst realized maximum drawdown in percentages. Length(m) denotes the time it took from peak to valley to initial peak value, and the recovery(m) time is the time it takes from valley to new initial peak value. Both length(m) and recovery(m) are specified in number of months.

K	$J = 1$	3	6	9	12
$K = 1$ MDD	63.86	58.68	54.11	55.82	56.73
Length(m)	68	56	64	66	71
Recovery(m)	48	36	50	52	54
$K = 3$ MDD	65.72	60.45	59.32	58.69	59.15
Length(m)	65	63	66	64	68
Recovery(m)	48	46	52	50	59
$K = 6$ MDD	63.99	60.28	57.08	58.20	60.05
Length(m)	69	65	70	73	77
Recovery(m)	48	48	49	52	56
$K = 9$ MDD	62.18	59.08	56.98	59.16	61.40
Length(m)	46	45	49	52	56
Recovery(m)	25	24	45	50	55
$K = 12$ MDD	61.69	58.68	57.53	59.47	61.51
Length(m)	44	46	66	69	71
Recovery(m)	23	25	45	48	50
Benchmarks	MDD	Length(m)	Recovery(m)	Start	End
equally weighted S&P 500 Index	57.39	42	21	2007:06:01	2011:01:03
S&P 500 Index	54.70	80	48	2007:10:01	2013:04:01

Figure 17: Realized simple returns and six months rolling volatilities in the period of 1997:11:31 to 2019:12:31 for the $J = K = 6$ months portfolio and the equally weighted S&P 500 Index, respectively.

The volatility estimation of the equally weighted S&P 500 Index is computed using a 6 months rolling market model regression with monthly data. The return data for the $J = K = 6$ months momentum portfolio is computed using monthly data. Since the rolling volatility estimation requires six months of data the plot first starts running 1997:11:31 and concludes 2019:12:31.



Lastly, Table 14 outlines three well defined patterns highlighted by the light green colouring scheme. Firstly, The maximum realized drawdowns are somewhat concavely shaped with respect to the selection criteria, the lowest MDDs are realized for portfolios with a selection criteria between 3 and 9 months and the largest are found in the lower extreme, given their respective holding periods. Secondly, with respect to the corresponding length and the recovery time associated the results are equivalent; the the portfolios with shorter look back windows perform superior to those with larger look back windows. Finally, there appears to be a relationship between the holding period and the sustained drawdown. Holding periods of $K = 9 \wedge K = 12$ months exhibit significantly smaller drawdown lengths and much more swift recoveries.

Going forward, the rest of the paper will shift its focus towards the holding period that on average provided superior returns, IRs, and maximum drawdowns characteristics, namely, the portfolios that have a selection period of $J = 6$ and holding periods of $K = \{3, 6, 9, 12\}$ months, respectively.

5.5 Momentum Portfolio Performance

Extending risk beyond return and their corresponding standard deviation Table 15 introduces the last two moments, namely, the skewness and excess kurtosis of the monthly full period realized returns for the $J = \{3, 6, 9, 12\}$ MSs that rebalances its books every $K = 6$-months, as well, as the relevant benchmarks. Additionally, Table 15 reports the alpha return with respect to the EW S&P 500 Index.

First of all, all of the portfolios generate a positive alpha return in comparison to the relevant benchmark. They all also exhibit a moderate negative skewness and leptokurtic return distributions. The equally weighted S&P 500 Index has a moderate skewness of -0.63 and a leptokurtic excess kurtosis of 3.48. The momentum strategies, also display varying degrees of moderate skewness and leptokurtic return distributions. However, these kurtosises are in general of much smaller magnitude than the relevant benchmark. Statistically speaking, the equally weighted S&P 500 Index exhibiting higher excess kurtosis suggests that the benchmark has a lot more probability mass in its tails in comparison to the momentum strategy portfolios. This fact, coupled with the moderate negative skewness, implies that the benchmark has a much larger likelihood of realizing large negative returns in comparison to the MSs. (Ivanovski, Stojanovski, and Narasanov (2015)).

Table 15: Momentum portfolio characteristics for the momentum portfolios that select stocks based on the last J=6 and rebalances its portfolios K={1,3,6,9,12} throughout the full sample period from 1997:01 - 2019:12.
r is the compounded return, σ denotes the standard deviation, the α denotes the excess return from the portfolio in relation to the equally weighted S&P 500 Index, the IR denotes the annualized information ratio, SK(m), and K(m) represents the full period realized skewness and excess kurtosis of the monthly returns.

	J=6						
K	1	3	6	9	12	S&P500	EW S&P500
r	11.44	11.63	12.37	12.30	11.48	6.16	10.96
σ	18.16	19.27	19.34	19.77	19.77	15.22	18.41
α	0.59	0.67	1.41	1.34	0.52	-4.8	0
IR	0.63	0.60	0.64	0.63	0.58	0.41	0.60
SK(m)	-0.29	-0.58	-0.72	-0.74	-0.70	-0.913	-0.63
K(m)	0.95	1.74	1.87	1.63	1.68	1.21	3.48

The statistical moments are a great indication of general exposure during the whole life span of the investment strategies. To assess the actual risk exposure of the worst losing event, this paper will reinspect the maximum drawdown, the corresponding length, and recovery period in Table 16. The majority of the evidence is consistent with that of the statistical moments, however, the moments do understate the potential tail risk associated with all of momentum strategies, except for the portfolio that rebalances its books every single month. Impressively, the $K = 1$ month MS realizes a maximum drawdown of 54% in comparison to the rest of the momentum strategies that realize maximum drawdowns in the region of 57% equivalent to that of the equally weighted S&P 500 Index.

Table 16: Drawdown report for the J=6 selected momentum portfolios in the full sample period from 1997:01:01 - 2019:12:30.
This table presents the maximum drawdowns of the $J = 6$ momentum portfolios and S&P 500 Index benchmark over the full sample period from 1997:01:01 - 2019:12:30. The maximum drawdown denotes the percentage loss from the highest accumulated return at time t to its valley. The length denotes the number of months from the peak to valley to initial peak, while the recovery period is the number of months from valley to initial peak value.

	J=6				
K	Maximum drawdown	Length	Recovery	Start date	End date
1	-54.11	64	50	2008:01:02	2013:06:03
3	-59.32	66	52	2008:01:02	2013:08:01
6	-57.08	70	49	2007:06:01	2013:05:01
9	-56.98	66	45	2007:06:01	2013:08:01
12	-57.53	66	45	2007:06:01	2013:01:02
EW S&P 500 Index	-57.39	42	21	2007:06:01	2011:01:03
S&P 500 Index	-54.70	80	49	2007:06:01	2013:04:01

In other words, the evidence presented in Table 16, show that the significant alpha returns realized by the momentum strategies do not come at the expense of significantly larger drawdowns relative to the equally weighted S&P 500 Index. In addition, the momentum strategies exhibit far more attractive statistical return moment. However, in regards to the length and recovery time of the realized drawdown they all behave inferior to the equally weighted S&P 500 Index.

Assuming these portfolios are widely diversified, these substantially varying drawdown characteristics suggests that the momentum portfolios potentially exhibit large differences in its market betas which can explain their behavior - a conjecture that will be more formally investigated in the following section.

5.6 Time-varying Beta

The Capital Asset Pricing Model (CAPM) states that the return of a security i is given as:

$$r_i = r_f + \beta_i(r_m - r_f) + \epsilon_i \tag{17}$$

Where r_i is the expected return of security i, r_f is the risk free rate, and r_m denotes the expected market return. Lastly, we have β_i, which can be interpreted as expressing the amount of market risk captured by security i. β_i is computed as the covariance between the security and the market over the variance of the market, that is $\beta_i = \frac{\text{Cov}(r_i, r_m)}{\text{Var}(r_m)}$. Without further assumption, it should be noted that (17) is a generic expression, it has been expanded upon. Nevertheless, (17) will always hold since one can define $\epsilon_i \equiv r_i - \beta_i(r_m - r_f)$.

The CAPM states that the return of a security is solely determined by the market risk exposure - not the idiosyncratic risk constituted by the security alone. The market risk exposure in the CAPM is governed by β_i, which has the following dynamics:

- $\beta_i < 0$ implies that security i is negatively correlated with the market.
- $\beta_i > 0$ implies that security i is positively correlated with the market.
- $\beta_i = 0$ implies that security i is uncorrelated with the market.

Suppose that all securities in the S&P 500 Index economy conform to this one-factor model and have permanent market betas and zero alphas (abnormal returns) on that factor. Then no trading strategies should be able produce a true abnormal profit relative to the market. A momentum strategy in this world, however, may be able to generate positive alpha. Given a positive (negative) realization of contemporaneous market returns, the winner portfolios constructed by selecting previous past winner returns are more like to select high beta (low beta) securities. This means that the long-only momentum portfolio should exhibit high beta (low beta) securities if the contemporaneous market is rising (falling). If the market return is slightly auto correlated, as suggested by the high momentum returns, meaning that strong returns in the past will yield strong returns going forward, then the momentum portfolio will generate a positive alpha in the CAPM framework.

To investigate the dynamics governing the momentum portfolio, this paper inspects the time-varying beta of the two portfolios $J = 6$ months momentum portfolios exhibiting the largest maximum drawdown difference. Figure 18 plots the market betas for the $J = 6$ and the $K = 1$ portfolio, which has a maximum drawdown of 47.81% and Figure Figure 19 plots the market betas for the $J = 6$ and $K = 9$ portfolio, which has a maximum drawdown of 61.10%. The market betas are estimated using 12 months rolling market model regression with monthly return data.

Consistent with the selection criteria and this argumentation, the momentum portfolios in general exhibits large market betas in upwards moving markets and lower market betas when the market moves downward. Another intuitive pattern apparent in the two figures above, is that increasing the holding/rebalancing period decreases the stability of the betas. The portfolio that rebalances its books every month has a maximum beta of 2.25 and a minimum beta of -0.05 in comparison to the portfolio that rebalances every 9 months which has a maximum beta of 1.89 and minimum beta of 0.51. The decreasing beta stability when increasing the rebalancing frequency is apparent for all of the $J = \{1, 3, 6, 9, 12\}$ month holding portfolios. Ignoring the nature of the equally weighted portfolio construction, naturally, increasing the rebalancing frequency will increase the amount of securities changed as time goes and thus almost surely the likelihood that the portfolio selects high (low) beta securities when the market rises (and falls). This intuition, seems to be more true in the case of a falling market than a rising market. The $K = 9$ month portfolio in average exhibits larger betas when the market is rising compared to the $K = 1$ month portfolio.

Figure 18: Market beta for the $J = 6$ and $K = 1$ momentum portfolio in the period of 1997:12:31 to 2019:12:31. The betas are estimated by running a set of 12-months rolling regression of the momentum portfolio returns on the contemporaneous S&P 500 Index market return.

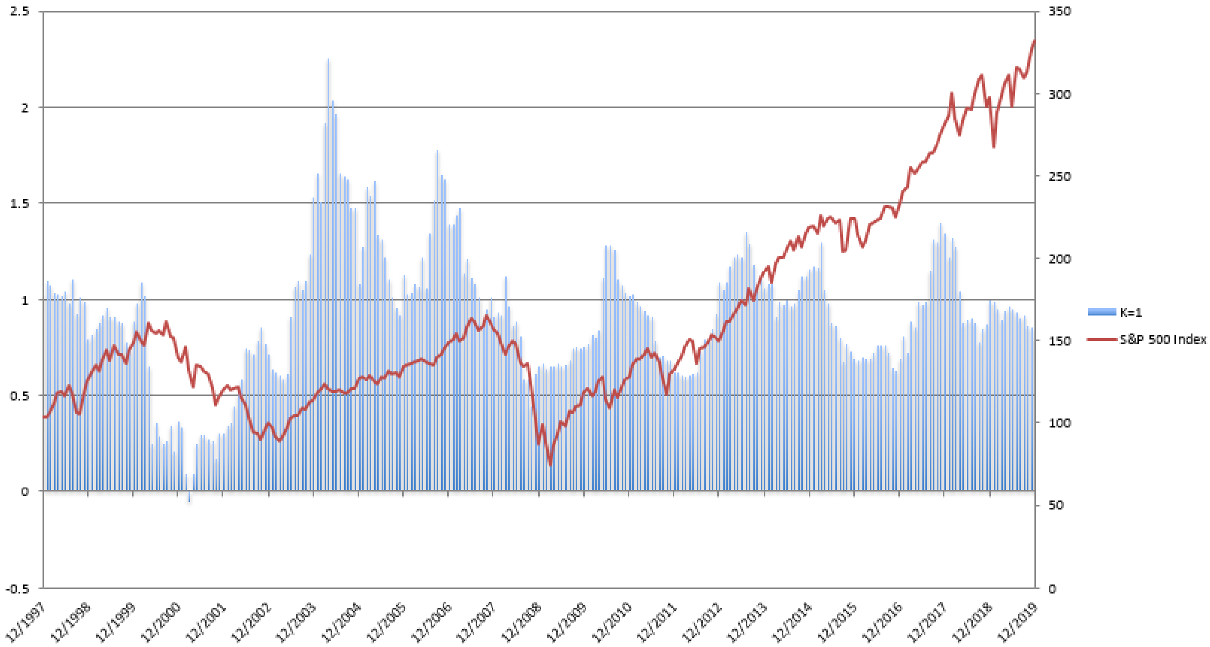
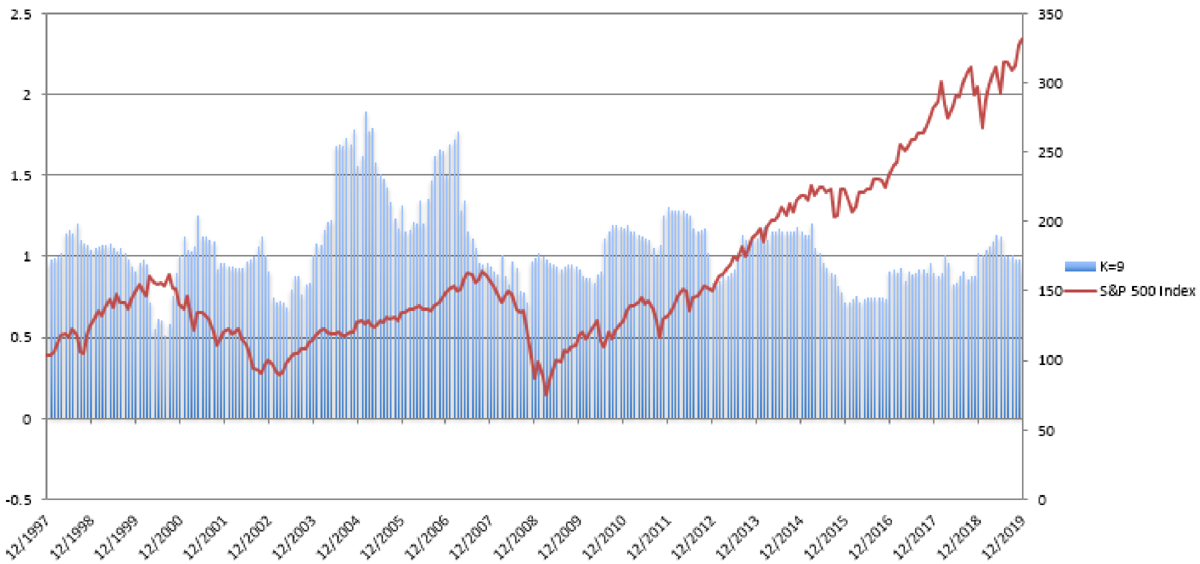


Figure 19: Market beta for $J = 6 \wedge K = 9$ momentum portfolio in the period of 1997:12:31 to 2019:12:31. The betas are estimated by running a set of 12-months rolling regression of the momentum portfolio returns on the contemporaneous S&P 500 Index market return.

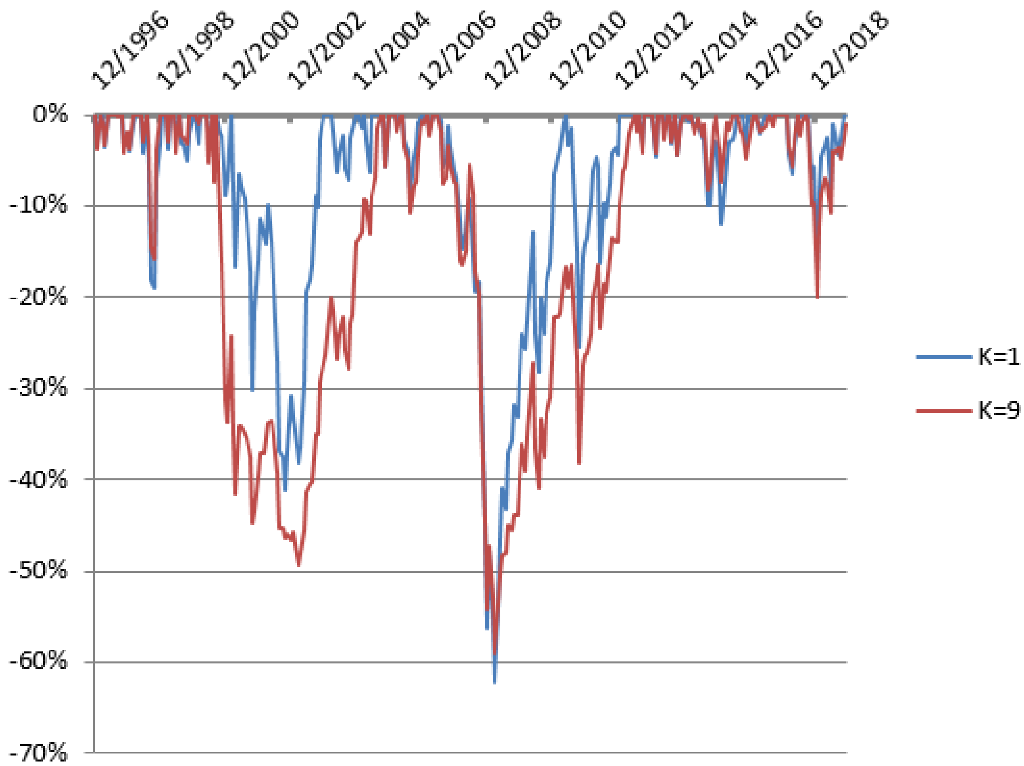


In the period of 2001 - 2007, while the contemporaneous S&P 500 Index surges upwards, the $J = 6$ and $K = 1$ momentum strategy exhibits high betas in the range of 1.5 and 2. Effectively, the portfolio is picking winners from the last 6 months, holding these the subsequent month, and the fact that these securities delivered the highest (lowest) return in the past month results in high (low) beta portfolios. As the contemporaneous S&P 500 Index plunges, the momentum strategy will be exposed to its relatively high beta portfolio for the whole $K = 1$ month duration. One month into the crash, the $J = 6$ and $K = 1$ months portfolio will select the best performing securities from the previous 6 months i.e. the best performing securities five months before the crash and one month into the crash. The portfolio will proceed in the same manner every single month, six months into the crash, the portfolio will be selecting the best performing securities during the crash. These securities will in general exhibit a lower beta (if they fell less than the market) or in rare cases, a negative beta (if they rose while the market falls). This is exactly what the figures depict, the $K = 1$ month portfolio exhibits beta values in the range of zero after the dot com bubble and 0.5 in the aftermath of the financial crisis.

Now Consider the $J = 6$ and $K = 9$ months portfolio, this portfolio will rebalance its books every 9 months. Logically, this portfolio will have a propensity to stick to securities that had appropriate beta values at its inception but these beta values will almost surely not be appropriate when the contemporaneous market is volatile (and plunging). Compared to the $K = 1$ month portfolio, this portfolio exhibits beta values in range of 0.55 and 0.70 in the aftermath of the dot com bubble and the financial crisis, respectively. In other words, decreasing the rebalancing frequency will almost surely lead to less appropriate security selections in a volatile market. The same argumentation is true when the market starts to rally upwards again.

Note, that the reported momentum portfolios are all equal series of K identical portfolio constructions initiated one month apart. The implication of this is that the beta sensitivity with respect to the rebalancing frequency is going to be slightly reduced, nevertheless, the overall relationship between the time-varying beta and the rebalancing frequency will still be evident. Consider, the $K = 9$ portfolio which 1/9 equally weighs the $J = 6$ and $K = 9$ portfolio with 9 different inceptions set a month apart. Four months into a hypothetical crash, this portfolio will have 4 of its portfolios constructed including crash data, while 5 of its portfolios will still consist of securities that performed well before the crash. Hypothetical or not, the intuition above is consistent with the findings in drawdown report in Table 14 and for the third and fourth return moment in Table Table 15. The momentum portfolio rebalancing its books every month exhibits the smallest maximum drawdown, skewness, and kurtosis among the set of $J = 6$ momentum strategies. The findings are perhaps even more apparent in Figure 20 below.

Figure 20: Drawdown plot for the momentum that rebalances its books every K={1,9} months and selects based on the previous J={1, 6, 9, 12} months.
The figure presents the realized drawdowns for the portfolio that rebalances its books every $J = \{1, 9\}$ and selects stocks based on the previous J=6 months. The drawdowns are computed using monthly return data for each portfolio series.



The relatively lower time-varying beta value for the $K = 1$ portfolio decreases the initial drawdowns in the dot-com bubble in the early 00s and results in a recovery in 2003:12:03 compared to the $K = 9$ portfolio which recovers in 2004:12:01. Additionally, during the financial crisis the more frequent rebalancing reduces the maximum drawdown by 7%.

In summary, its worth emphasizing that the portfolios that selected securities based on the last 6 months performed superior in terms of maximum drawdowns sustained. The results are somewhat counterintuitive, decreasing the selection period to 1 month and increasing the rebalancing frequency will result in worse drawdowns as is evident in Table 14. However, within each given set of $J = \{3, 6, 9, 12\}$ selection period, increasing the rebalancing frequency to $K = 1$ month does result in smaller maximum drawdowns. The argument here being, that a higher rebalancing frequency will increase the likelihood of consisting of more appropriate beta values in a volatile market. Lastly, the benefits stemming from frequent rebalancing are asymmetrical, the more sensitive time-varying beta gains from decreasing the drawdown are out outweighed by the on average less sensitive and lower beta values when the contemporaneous market is moving upwards. As a result, the $K = 9$ months portfolio has an annualized compounded return of 12.37% while the $K = 1$ month portfolio "only" realizes a return of 10.01%

Its important for the reader to note, that this explanation implies that the entire return source can solely be explained by β and CAPM. In other words, that there is no violation of the one-factor CAPM and that the markets are efficient and behave as the CAPM dictates. These surprisingly high risk-adjusted returns might indicate that the markets are in fact not efficiently priced. The topic of efficient and inefficient market pricings are out of this paper's scope and we will refer the interested reader to: *Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency (1993)* by Narasimhan Jegadeesh and Sheridan Titman.

5.7 Turnover Considerations

From the perspective of real-life implementation turnover is an important feature. Figure 21 depicts the relationship between the average yearly security purchases for each portfolio construction throughout the entire sample period from 1997:01:01 - 2019:12:31. The inspected portfolios are not equally weighted series of momentum portfolios as the rest of the paper - they are single momentum portfolios with one inception date, the set of $J = \{1, 3, 6, 9, 12\}$ portfolios are initiated in 1996:01, 1996:03, 1996:06, 1996:09, and 1996:12, respectively. The different inception dates reflect the fact that the higher formation periods require additional data. Nonetheless, the figures start measuring the incurred turnovers from 1997:01:01 to ensure that the portfolios are commensurable.

Three key points emerges from Figure 21 panel A:

1. The average yearly security purchase is decreasing in its holding period - rebalancing the portfolio less frequently will decrease the turnover rate.

2. The average yearly security purchase is decreasing in its selection period - increasing the look back window will increase the likelihood of portfolios being more similar.

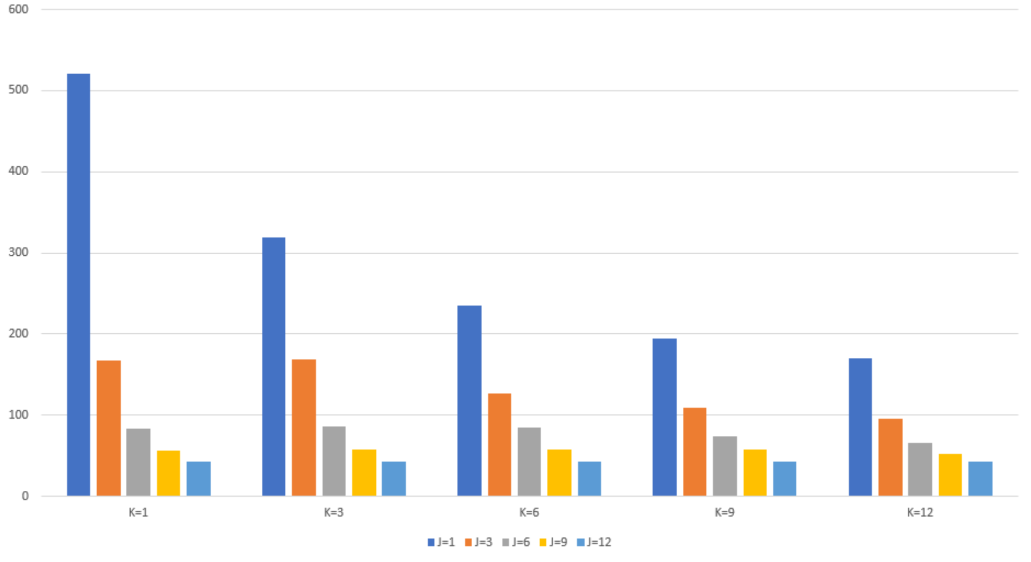
3. Increasing both the holding and selection period simultaneously also (slightly) decreases the turnover rate. This effect is most dominant when the holding period exceeds the selection period, because this means that the next rebalancing period will look at an entirely different look back window.

Its worth stressing the fact that replicating the turnovers for the equally weighted series of K momentum portfolios for any of the momentum portfolios will indisputably increase the turnover rate dramatically to the point that the holding periods have no real impact on the impact. The selection period will however still play an impact as long as the selection period is shorter than the respective holding period.

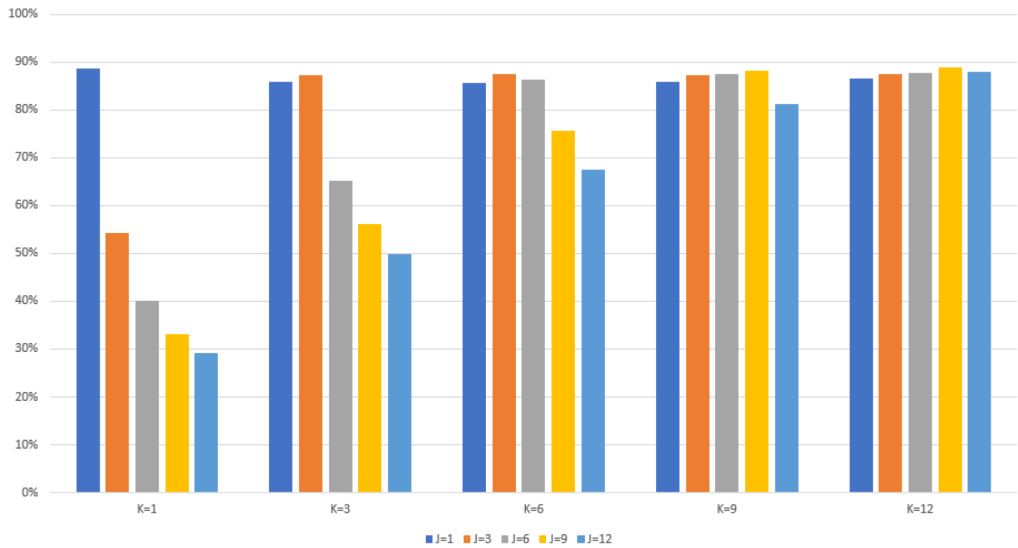
Figure 21, Panel B presents the turnover rates for the 25 various momentum portfolios. The evidence shows, that the majority of the momentum portfolios have very high turnover rates with most of the portfolios exhibiting turnover rates around 88 percent. This is in line with the proposition, that the momentum factor has an appreciation of 9-15 months. Examining the formation period $J = 12$ months for the increasing holding periods shifting from left to right the "probability" that the stock remains in the portfolio decreases drastically. Assuming the momentum cycle holds, the stocks that manage to be included in the $K = J = 12$ months momentum strategy twice in row may be stock performing very well in each formation period and not the time portfolio actually holds the stock. Again, these figure assumes that the investor takes timing risk when implementing the strategy, regardless, it still sheds light to underlying dynamic that frequent rebalancing and shorter look back windows will undeniably increase the money required to manage this type of investment.

Figure 21: Turnover plot for the momentum portfolios that rebalances its books every K={1, 3, 6, 9, 12} months and selects based on the previous J={1, 3, 6, 9, 12} months throughout the entire sample period from 1997:01:01 - 2019:12:31.
Panel A presents the average yearly newly purchased stocks for momentum portfolios selecting based on the previous $J = \{1, 3, 6, 9, 12\}$ months and rebalances its portfolios every $K = \{1, 3, 6, 9, 12\}$ months, respectively. The average yearly new stock purchases are calculated as the ratio between new stock names in the portfolio and the total amount of stocks throughout the entire sample period. Panel B presents the realized turnover rates incurred at each formation period for the various momentum portfolios with a formation periods of $J = \{1, 3, 6, 9, 12\}$ months and holding length of $K = \{1, 3, 6, 9, 12\}$ months. Both Panel A and Panel B use the full 23-year sample period 1997:01:01 - 2019:12:31.

Panel A



Panel B



5.8 Momentum Conclusion

This section provides several comprehensive results. First, trading strategies that buy past winners produce statistically significant positive returns with holding periods of 1 to 6 months and selection periods in the range of 6 and 9 months performing superior. The evidence here suggesting, that the momentum has an appreciation cycle of 9-15 months. These results are consistent with existing work and indicate that the momentum is most dominating in the intermediate term. Second, further investigation shows that that buying past winners highly outperforms past losers. Third, evidence shows that these high returns come bearing severe risk and crash potentials realizing long periods of severe losses. Further analysis, finds that the momentum crashes are highly correlated with contemporaneous market volatility. Forth, investigation of the momentum strategies time-varying market betas suggests that the momentum portfolios are very pro cyclical. Lastly, applying turnover rates as a proxy for potential transaction costs do imply that momentum portfolios are cost heavy especially for short holding periods.

6 Mixed versus Integrated Portfolio Approach

The previous sections found consistent and pervasive momentum and value premiums across the S&P 500 Index. Whilst, in depth analysis determined the various effects of the variables governing each investment factor. This section will investigate their returns jointly by implementing a portfolio using both factors. Additionally, this section will have a particular interest in investigating how the combined returns depend on their implementation approach.

The section proceeds as follows. Subsection 6.1 motivates the use of value and momentum in one portfolio. This is followed by subsection 6.2 where the mixed and integrated portfolio approaches are introduced. Subsection 6.3 covers the the implementation of the two portfolio approaches which is followed by results for the mixed approach in subsection 6.4 and for the integrated approach in subsection 6.5. In subsection 6.6 different holdings lengths of the portfolios are compared which is then followed up by turnover considerations in subsection 6.7. Finally a conclusion of results found in this section are concluded upon in subsection 6.8.

6.1 Value and Momentum Incentive

This subsection will look into the main incentives for constructing a long-only value and momentum portfolio compared to the traditional value and momentum trading strategies involving long and short positions. For the value trading strategy this involves buying value stocks and selling glamour stocks. In regards to the momentum portfolio, this means buying past winners and selling past losers. Best documented in Value and Momentum Everywhere (Asness, Moskowitz, Pedersen (2013)), these two investment styles exhibit negative correlations in the range of -0.4 to -0.6 across various equity, currency, fixed income, commodity, and other asset classes. The negative correlation is most prominent during market crashes, roughly speaking this is the result of the short positions of momentum producing returns that tend to outweigh the losses of its long positions, while the value momentum still realizes severe losses. However, as documented in both the value and momentum sections in this paper, the long only value and momentum will not be negatively correlated. Nonetheless, the long-only value and momentum separately exhibit daunting crash potentials that would scare of any portfolio manager. In this context it is definitely worth exploring the hedging potential of combing these otherwise very strong premiums. Table 17 presents the long-term correlation between the five measures of value and the momentum strategy that selects its stocks based on the last $J = \{1, 3, 6, 9, 12\}$ months and holds these for the subsequent $K = 6$ months. The important finding here is that all of the value and momentum strategies are less correlated to each other, than they are respectively, to the equally weighted S&P 500 Index. Table 18 presents the full correlation table for all dimensions of value measure and the momentum strategy that selects its stocks based on the last $J = \{1, 3, 6, 9, 12\}$ months and holds these for the subsequent $K = 6$ months. From Table 18 a clear pattern emerges; the correlations increase as the selection period decreases with the high selection periods offering correlations of 0.51-0.63 and the short selection periods offering high correlations in the 0.80's. Further research shows, that these results are also consistent across different common holding periods. In other words, the vast amount of correlation data suggests that the two strategies offer appealing diversification opportunities compared to using the S&P 500 Index, especially using the momentum strategies with higher selection periods.

Table 17: Long-term correlations for various measures of value and momentum strategies with K = 6 and various selection periods.
The table presents the full sample period correlation between various value strategies and momentum strategies that hold their portfolios for $K = 6$ months and have various selection periods. Furthermore, the table presents the long-term correlation between each strategy and the equally weighted S&P 500 Index (BM). The correlations are computed using full sample monthly returns for each strategy respectively from 1997:01:01 - 2019:12:31.

	BM	P/B	P/S	P/E	P/FC	DIV
BM	*1.00*	0.93	0.94	0.95	0.95	0.85
J=1	0.95	0.86	0.88	0.89	0.89	0.77
J=3	0.89	0.79	0.81	0.83	0.82	0.70
J=6	0.82	0.71	0.74	0.76	0.74	0.62
J=9	0.78	0.65	0.68	0.71	0.69	0.56
J=12	0.76	0.61	0.64	0.68	0.66	0.51

Table 18: Long-term correlations for various measures of value and momentum strategies with K = 6 and various *J* selection periods.
The table presents the full sample period correlation between various value strategies and momentum strategies that hold their portfolios for $K = 6$ months and have various selection periods. The correlations are computed using full sample monthly returns for each strategy respectively from 1997:01:01 - 2019:12:31.

Value/Momentum	*J=12*	*J= 9*	*J= 6*	*J= 3*	*J= 1*
PB	0.61	0.65	0.71	0.79	0.87
PS	0.63	0.67	0.73	0.80	0.87
PE	0.67	0.71	0.76	0.83	0.89
PFC	0.65	0.69	0.74	0.82	0.89
DIV	0.51	0.55	0.61	0.70	0.76
PBPE	0.58	0.63	0.69	0.77	0.84
PBPS	0.64	0.68	0.74	0.81	0.88
PBPFC	0.60	0.64	0.70	0.78	0.84
PBDIV	0.53	0.58	0.64	0.72	0.79
PEPS	0.65	0.69	0.75	0.81	0.88
PEPFC	0.61	0.65	0.71	0.78	0.85
PEDIV	0.54	0.59	0.65	0.73	0.80
PSPFC	0.66	0.69	0.75	0.82	0.88
PSDIV	0.60	0.64	0.70	0.78	0.84
PFCDIV	0.55	0.60	0.66	0.74	0.81
PBPEPS	0.62	0.66	0.72	0.79	0.86
PBPEPFC	0.59	0.64	0.70	0.77	0.84
PBPEDIV	0.54	0.59	0.65	0.73	0.80
PBPSPFC	0.63	0.67	0.73	0.80	0.86
PBPSDIV	0.58	0.62	0.68	0.76	0.84
PBPFCDIV	0.55	0.59	0.65	0.73	0.81
PEPSPFC	0.63	0.67	0.73	0.80	0.86
PEPSDIV	0.59	0.64	0.70	0.77	0.84
PEPFCDIV	0.56	0.61	0.67	0.75	0.82
PSPFCDIV	0.60	0.64	0.70	0.77	0.84
PBPEPSPFC	0.62	0.66	0.72	0.79	0.85
PBPEPSDIV	0.58	0.63	0.68	0.76	0.83
PBPEPFCDIV	0.56	0.60	0.66	0.74	0.81
PBPSPFCDIV	0.58	0.63	0.69	0.76	0.83
PEPSPFCDIV	0.60	0.64	0.70	0.77	0.84
PBPEPSPFCDIV	0.59	0.63	0.69	0.77	0.84

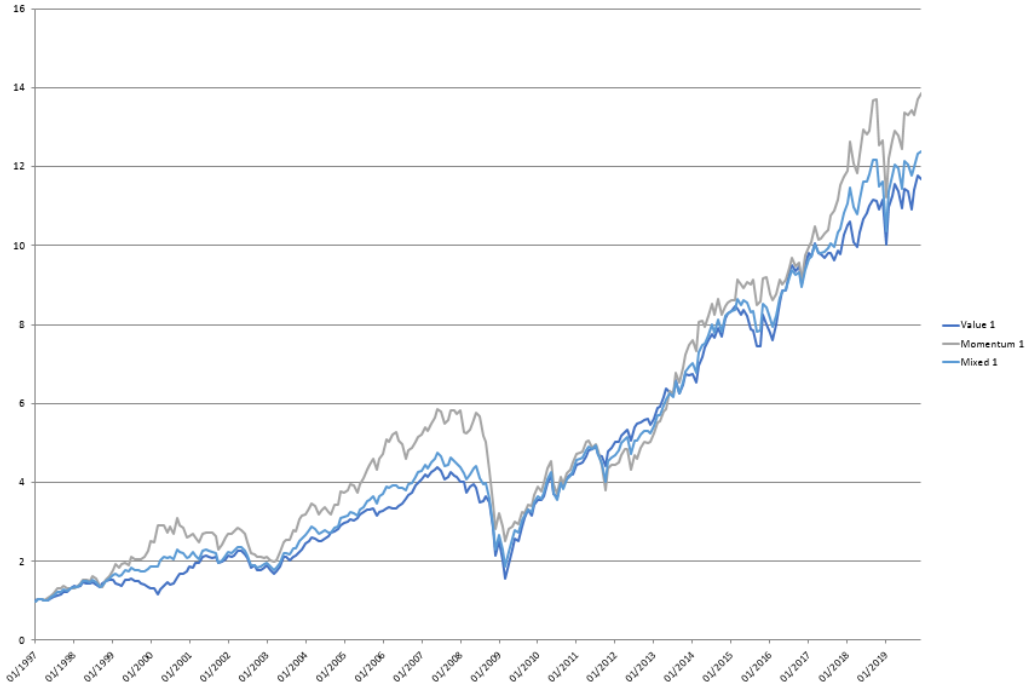
Figure 22, Panel A presents cumulative returns of the value (highlighted in blue), momentum (highlighted in red), and the equally weighted mixed portfolio (highlighted in green). Despite the fact that all three portfolios produce impressive cumulative returns, the figure highlights the diversification hedging effect. Figure 22, Panel B truly highlights the most desirable situation, namely, the situation where the momentum portfolio increase and the value portfolio decreases, while the mixed portfolio steadily increases in a less volatile manner. The result shows how the mixed portfolio actually hedges itself against the contemporaneous market dot-com bubble decline in the 00's.

Furthermore, the mixed portfolio realizes an annual standard deviation of 18.54 percent, while the value and momentum strategies have standard deviations of 19.83 and 19.77 percent, respectively. These results are consistent across different portfolio combinations and confirms the initial conjecture; that mixing strategies with low correlations leads to overall less volatile price movements and joint returns, that smooth out the price movements.

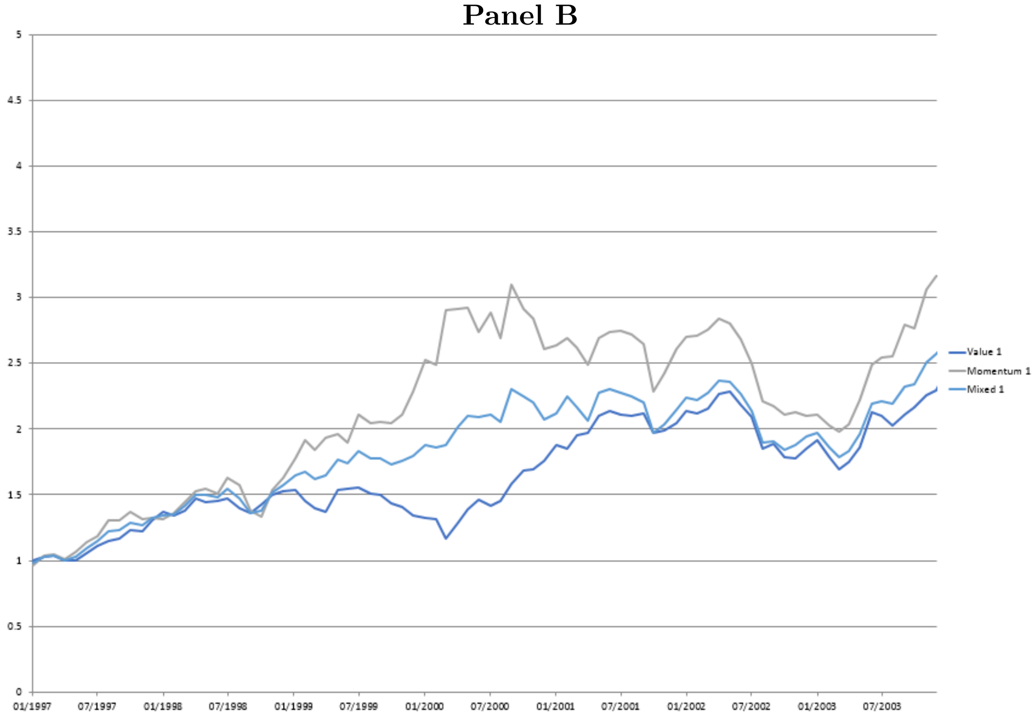
Figure 22: Realized compounded returns in the period of 1997:01:01 to 2019:12:31 for the value, momentum and mixed portfolios.

Panel A presents the realized compounded returns in the period of 1997:01:01 to 2019:12:31 starting with a notional of $1 for the value strategy sorting on all five measures of value, the momentum strategy selecting stocks based on the last $J = 6$ months. It then holds these stocks for the following $K = 12$ months, and the mixed portfolio that equally weights both strategies. Panel B zooms in on the *prime example* of why an investor would combine both portfolios. The realized compounded returns are computed using monthly return data throughout the 22-year sample period.

Panel A



Panel B



On the basis of potential diversification and risk hedging effects, this paper will construct portfolio combinations based on the lowest obtained correlation between the respective value and the momentum strategies, its returns, and the information ratio. This score is computed using the following formula:

$$Score = (\text{Inverse Correlation} \cdot Return \cdot IR) \cdot 100 \tag{18}$$

Table 19 and Table 20 reports the 15 highest ranking value classifications and the 3 highest ranking momentum strategies, respectively. Coupling the results from Table 17 and those from the value section, it might not come as a surprise that the dividend yield measure scores excellently using this scoring scheme. The dividend yield offers very attractive correlations down to 0.51, while it also produces high returns at very attractive risk levels (low standard deviation). On the other hand, for the momentum rankings the $J = 6$ to $J = 12$ months portfolios are selected rather than the $J = \{1, 3\}$. Somewhat surprisingly, despite the $J = \{1, 3\}$ selection periods producing strong returns and risk-adjusted returns, they are discarded because of their very high correlations to the various value strategies. The $J = 12$ months momentum strategy provides the most attractive correlations overall, but it does not produce returns and risk-adjusted returns in the magnitude of the $J = \{6, 9\}$ months portfolios, ultimately, leaving it in the bottom of the top 3 momentum scores.

Its worth noting, that in the case of the integrated portfolio approach, which is elaborated upon in the next subsection, the above correlation argument is not as valid. The integrated approach selects stocks that score the on average highest value and momentum scores. These stocks may not have same low correlation. However, assuming that the low long-term correlation also holds for the stocks exhibiting both high value and momentum scores this may be a very strong portfolio. If you can capitalize the correlation benefit holding stocks strongly exposed to both factors this may likely result in a stronger portfolio than holding stocks that are "only" exposed to only factor, respectively. In line with this, the relatively lower correlations between the factors compared to the correlations to the equally weighted S&P 500 Index is also a compelling argument. Another more qualitative argument is that stocks exposed to one factor may have offsetting effects to stocks with high exposure to another factor. A offsetting effect, which may outweigh the potential positive diversification effects. The actual results for the various portfolio approaches are documented later in this section.

Table 19: List of the 3 scoring momentum strategies
This table presents the 15 top scoring value strategies in descending order using equation 18. The inverse correlation, return, and information ratio are computed using monthly data throughout the entire sample period 1997:01:01 - 2019:12:31.

Top 15 Value	
Value strategy	Score*
DIV	6.715
P/E DIV	6.104
P/B P/S DIV	6.055
P/E P/FC DIV	5.569
P/B P/E DIV	5.536
P/B DIV	5.534
P/E P/S DIV	5.510
P/E P/S P/FC DIV	5.446
P/S DIV	5.445
P/E DIV	5.430
P/B P/E P/S P/FC DIV	5.354
P/B P/E P/S DIV	5.305
P/B P/E P/FC DIV	5.247
P/S P/FC DIV	5.176
P/B P/E P/S	5.101

*Score = $(InverseCorrelation \cdot Return \cdot IR) \cdot 100$

Table 20: List of the top scoring value strategies
This table presents the three best scoring momentum strategies in descending order using equation 18. The Inverse correlation, return, and information ratio are computed using monthly data throughout the entire sample period 1997:01:01 - 2019:12:31.

Top 3 Momentum	
Momentum strategy	Score*
6 month	5.447
9 month	4.920
12 month	3.980

Score = $(InverseCorrelation \cdot Return \cdot IR) \cdot 100$

6.2 Introduction to Mixed and Integrated Portfolio Approach

In order for an investment succeed, the approach of portfolio implementation is an important element to consider. According to recent research by Fitzgibbons et al. (2017), seemingly minor differences in the portfolio construction can lead to significant differences in performance outcomes for long-only portfolio style investing. These "styles" refer to the investment approach of the portfolio and are also known as "smart beta". They are based on well-known and generally accepted factors such as the previously examined value factor and momentum factor, which can give an edge to the implementation of the portfolio. Particularly for investors who are interested in portfolios where more than one factor is used, so called multi-style investors, should be aware that these kind of portfolios require thorough implementation in order to reap the benefits of combining factors. The most obvious multi-style portfolio approach would be the portfolio mix. This approach consists of building a portfolio by combining separate long-only portfolios for each individual style. This style is very appealing because it is simple, flexible and lets investors gain control over allocations across styles.

The other very common approach is the integrated approach, which likewise, is quite simple and intuitive. This approach is constructed by making an aggregated ranking of stocks that includes all the styles the investor may be interested in. These styles are then, in combination with the investors preferred allocation across styles, ranked accordingly to create a multi-style long-only portfolio in one step. Both the mixed and integrated portfolio approaches are very common in all sorts of investment products ranging from Index products and exchange-traded funds to active portfolios managed by investment managers.

Much investigation and research has gone into how to approach portfolio implementation. In Long Only Style Investing: Don't Just Mix, Integrate (Fitzgibbons et al. 2017) it describes an in-depth coverage and investigation of the same two approaches that will also be implemented in this paper, namely the mixed approach and the integrated approach. They find that using the approach of integrating the investment styles in the long-only portfolio construction yields a first-order effect on the performance, thus generating benefits such as including stocks that have a balanced positive style exposure and avoiding stocks with offsetting style exposures. Over their sample period running from February 1993 to December 2015 on the MSCI World Index they find empirically, that the integration of styles improves the excess return by 1% and increases the information ratio by 40% relative to the mixed portfolio approach. However, they conclude that this does not necessarily mean that the mixed approach will always lag that of the integrated approach, as there will be periods where it will outperform. These are though, ironically, during periods where the underlying styles of the tracking portfolio would have poor performance. Assuming that both the value factor and momentum factor needs to be considered for prospective returns, they thus come to the overall conclusion that the integrated approach is best since it blends each stock's value and momentum score, thereby not leaving out any information that can only be attained when combining the scores of the factors.

The measurable criterias are not the only aspects to take into consideration. Taking a more practical real-world approach, aspects that are harder to quantify such as governance, transparency and ease of performance attribution also need to be considered. In an article by Research Affiliates they find that the qualitative criteria strongly support a mixed multi-factor strategy. They also argue that the investment industry is made for complexity and is perhaps driven by the conviction that more complexity, should be able to earn a greater fee. However, this increased complexity can be unnecessary and even problematic, since it can make it more difficult to fathom. They appropriately quote Einstein: "Everything should be made as simple as possible, but not simpler." A more straightforward strategy not only makes it easier for investment teams to explain the strategy to stakeholders, it would also be able to reduce the labour costs associated with executing the investment strategy and the due diligence process. Thus, using multi-factor index strategy that has simple, transparent and rules-based structures makes it much easier for the investor's understanding of the main return premiums, since these will have the same focus five years in the future as they have today. Furthermore, having clear, transparent and simplified strategies can work as a surrogate for trust, which is an essential part for helping investors to stay the course and reap the benefits of a multi-factor approach.

In the following section we will summarize the difference between the two most common portfolio approaches, mixed and integrated. Initially we will examine our previously discussed value factor and momentum factor to demonstrate how these play out in the mixed portfolio approach which will then be followed by a similar example in the case of the integrated portfolio approach to demonstrate any differences. Finally we will address the highly popular element to investing, ESG, and what options exists to incorporate this into a successful multi-style portfolio.

6.3 Mixed versus Integrated Implementation

The first thing to consider when combining two separate trading strategies into one single portfolio is the strategy-specific parameters governing each portfolio. One of the conclusions for value investing, documented in section section 4 is that value has previously performed strongest with a resorting frequency of 12 months. This is not the case for the momentum strategy. The conclusions documented in section 5, shows that the momentum factor performs superior within 6 and 9 months with the returns diminishing as the holding periods approach 12 months. One must keep in mind that these are results established based on a backtest and should not be considered as the truth going forward but rather suggestions based on historical data. As the classic wording goes: "Past performance is not indicative of future performance".

One implementation style would be to treat each of the factors as separate legs in a joint portfolio. The implementation challenge here is found in the integrated model. Treating each factor as separate legs in a integrated model would imply updating the *Momentum Rank* every 6 months and the *Value Rank* every 12 months. This may however create an asymmetrical ranking influence in favor of the factor with the highest rebalancing frequency. Consider the case of a steep market incline, allowing momentum to update its *Momentum Ranks* will almost surely result in very different *Total Rank(Value Rank, Momentum Rank)* system compared to that at the previous formation period. Specifically this may result in a bunch of ("better") high ranking momentum stocks, nonetheless, it may also affect the value stocks which were originally selected because of their high value ranks and mediocre momentum ranks. Conversely, allowing value to re-rank more frequently than momentum may result in the opposite phenomena.

In addition, one could argue that the asymmetric influence gained by more frequent re-ranks is stronger for the momentum strategy. Recall, the backtested momentum strategy builds on "real time" data, while the backtested value trading strategy, as discussed in the value data subsection 2.3, builds on conservative 6 months lagged data. Intuitively speaking, this means that the backtested momentum strategy is more likely to reflect a real time momentum investment, than the backtested value strategy mimics a real time value investment. In a real life setting, a value investor would also build his/hers value ranks on real time data, consequently making the value portfolio more sensitive to contemporaneous market fluctuations. This is not necessarily an issue when examining the performances separately, but when integrating the strategies it does indisputably give more ranking influence to the momentum strategy. This is particularly emphasized in market turbulent times, because it is ranking is more sensitive to the contemporaneous market. Thus, allowing momentum to re-rank more frequently will presumably further extend momentum's influence. This conjecture is supported by the fact that the momentum turnover rate is immensely larger than that realized by the value strategy. Throughout the full sample period, the momentum strategy realizes an average turnover rate of 80.5 percent for the $K = 6$ months holding periods across different J look backs, 86.1 percent for $K = 9$ months, and 88.0 percent for $K = 12$ months. Conversely, the value strategy has an average turnover rate of 30.9, 41.7, and 49.9 percent for the value strategies that hold their portfolios for 6, 12, and 18 months. In summary, the re-ranking of stocks with respect to measures of value is much less sensitive when compared to the momentum strategy.

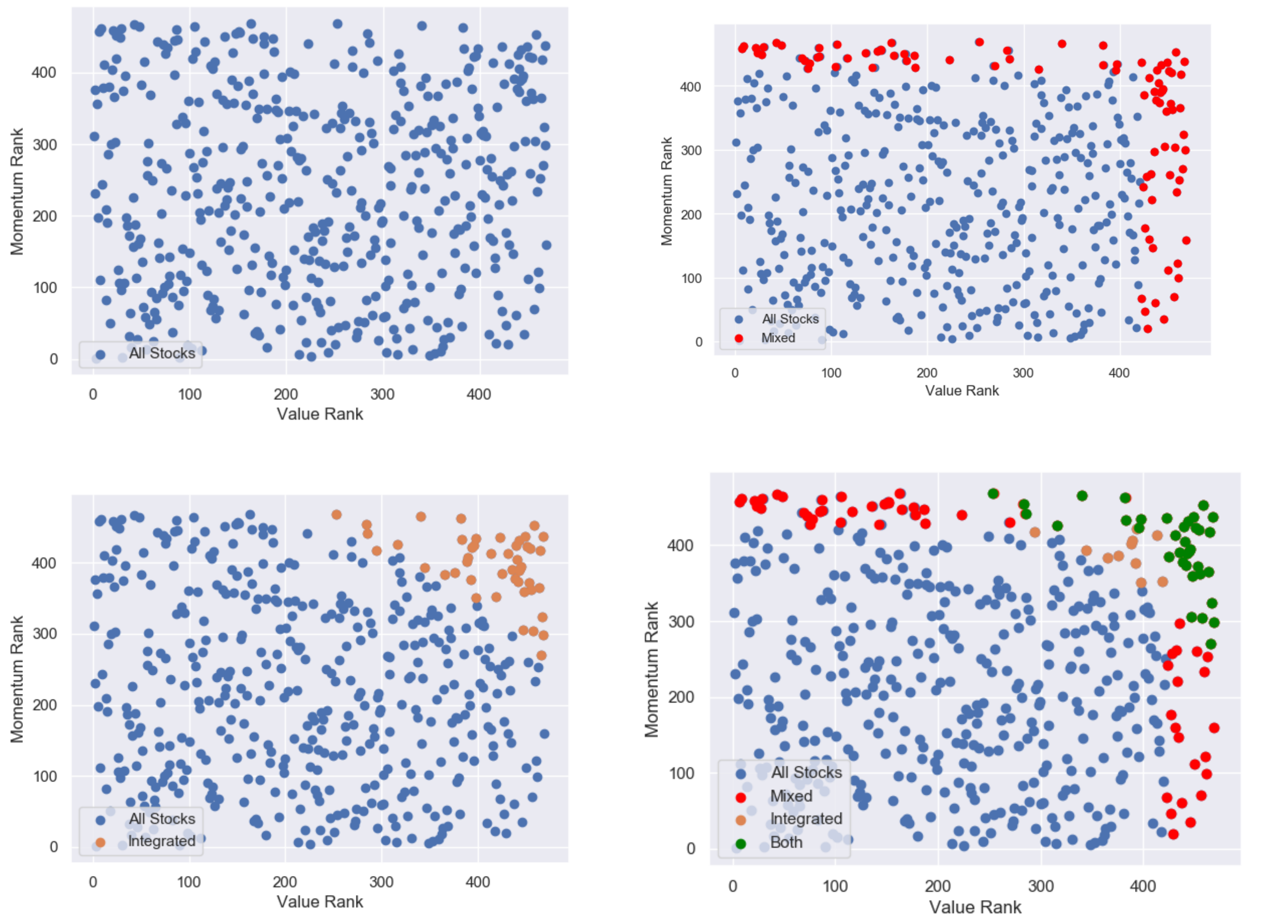
Instead of developing more sophisticated/complicated rankings regimes to correct some of the outlined complications with the two legged integrated implementation approach, this paper will for simplicity implement one common for both strategies. This rule will also apply for the mixed portfolio approach to avoid effects deeming the two portfolios incommensurable, albeit that the mentioned challenges do not apply for the mixed implementation. This means that the value strategy and momentum strategy will rank the stock universe simultaneously throughout the entire backtest and hold these stocks for an equal amount of time. Furthermore, to avoid completely ignoring the results drawn from back-testing each strategy separately, this section will backtest holding periods of $K = \{6, 9, 12\}$ months for various value measures and momentum strategies. To narrow down the field of strategy combinations, on going backtests will only include the momentum strategy that selects stocks based on the past $J = 6$ months returns, since these formations perform superior on the selected K holding periods.

An overview of the two portfolio implementation approaches are listed in the table below. As aforementioned, the mixed approach ranks the value stocks and momentum stocks separately and combines the top 10 percent value stocks and top 10 percent momentum stocks to form an equally weighted portfolio. The integrated approach ranks the stocks using an integrated *Total Rank(Value Rank, Momentum Rank)* system that selects stocks with high *total rank*. Since the *total rank* is a function of both the *value rank* and *momentum rank*, the integrated approach will select stocks that possess high value and momentum attributes. This portfolio is then constructed by selecting the top 10 percent stocks of the *total rank*. The table below gives a simplified overview of the selection process:

Comparing the "Mix" and "Integrate" Approaches	
Mixed Portfolio Construction Approach	Integrated Portfolio Construction Approach
1. Identify top value stocks and momentum stocks, separately.	1. Blend each stock's value and momentum score into one average composite (or "integrated") measure
2. Identify top momentum stocks	2. Choose the stocks with the highest "integrated" score
3. "Mix" the top value stocks with the top momentum stocks to create the portfolio	

Figure 23 presents a (2010 - 2011) snapshot of the value strategy sorting based on P/B, P/S, DIV and the momentum strategy that selects based on the past $J = 6$ months returns both strategies with a holding period of $K = 12$ months for the two implementation approaches. The y-axis represents the *momentum ranks* and the x-axis represents the *value ranks* across all approximately 500 companies. The top left panel shows the stocks that the mixed portfolio will select: high ranking momentum stocks and high ranking value stocks. The bottom left panel depicts the integrated approach, and in contrast to the mixed approach the integrated approach selects stocks with high *total ranks*. In general this means selecting stocks with either high individual value or momentum ranks or stocks with semi high value and momentum ranks, especially not stocks exhibiting high value (momentum) ranks and low momentum (value ranks). The bottom right corner gives an accurate visualization of the different stock compositions for each portfolio approach.

Figure 23: Visualization of the mixed and integrated portfolio composition taken from 2010-2011. The figures present a visualization of how the mixed and integrated portfolio approaches select stocks in 2010-2011. The value strategy is based on the P/B, P/S, and dividend yield measure and rebalances every 12 months, while the momentum strategy selects stocks based on the last $J = 6$ months and holds these securities the subsequent $K = 12$ months. The y-axis represents the momentum ranks and the x-axis represents the value ranks. The top left figure depicts the total examined stock universe in this snapshot. The top right figure presents how the mixed portfolio approach will select its top decile. The bottom left figure presents how the integrated portfolio approach will select its top decile. The bottom right figure depicts the difference in stock composition for the mixed and the integrated portfolio approach.



Both portfolio consist of stocks along the extremes of our data, which makes good sense since the aim is to select the best performing stocks. As expected the mixed portfolio approach covers all the stocks along the top and right corner whereas the integrated approach only selects the stocks that achieve high ranking in both factors.

In order to better illustrate how the two implementation techniques are different and appreciate how the end portfolios are materially different, Table 21 illustrates how the ranking of momentum, value, and the total rank are evaluated. The table presents the same strategy as Figure 23, namely, the value strategy that is formed based on the P/B, P/S, and DIV measures and the momentum strategy that selects stocks based on the last $J = 6$ months and rebalances every $K = 12$ months. The table illustrates how a investment manager would choose the stocks with the most attractive prospective returns. A manager using the mixed approach would initially rank the stocks separately based on their value metric with 1 being the best in the sample. In this case this would be done by ranking the stocks by using the P/B, P/S and DIV and combining a score for this. For example, Pepco Holdings LLC attains the highest value rank in this particular snapshot applying the following approach to ranking:

$$Value_{Rank} = \frac{\sum_{i=1}^{N} V_i(\cdot)}{\text{N}} \tag{19}$$

Where $V_i(\cdot)$ denotes the value rank of measure i and N is the number of applied value measures. The momentum approach is slightly more straightforward. It simply ranks the stocks in ascending order based on their previous 6 month return. Thus, for an investment manager preferring the mixed portfolio approach and a equal weighted factor allocation of four stocks, they will select the two best performing value stocks and momentum stocks and weigh them equally. This amounts to an equally weighted portfolio of the following two value stocks Centurylink inc. and Pepco Holdings LLc (highlighted in blue) and the following two momentum stocks Goodrich Corp. and Motorola Mobility Holdings LLc (highlighted in purple). The investment manager preferring the integrated portfolio approach would first compute the momentum and value ranks and then the average rank of the momentum and value factor. He/she will then select the four best performing value and momentum stocks based on the *Total Rank*. This amounts to an equally weighted portfolio of FirstEnergy Corp., PPL corp., Motorola Mobility Holdings LLc, and Pepco Holdings LLC (highlighted in light blue).

Table 21: Evaluation of value and momentum ranking characteristics.
The table reports the returns for each value measure and the momentum strategy for 15 discretionary stocks selected in 2010. In addition, it shows the separate ranks for value, momentum, and the combined total rank score.

Name	PB	PS	DIV	Momentum	Value Rank	Momentum Rank	Total Rank	Comment
Walmart Inc	2.3676	0.3723	2.88	3.52	8	7	6	
FirstEnergy Corp	0.8715	0.61	6.22	15.24	4	4	1	Unique to "Integrated"
Expedia Group Inc	1.1008	0.9249	2.57	8.13	11	5	7	
ConocoPhillips	0.8873	0.3428	5.26	-10.39	3	13	7	
CenturyLink Inc	0.6343	0.8707	14.48	-12.62	2	14	7	Unique to "Mix"
Darden Restaurants Inc	2.2105	0.584	4.11	2.15	8	9	11	
Dell Inc	3.3569	0.4635	0.00	-1.96	13	11	15	
Abbott Laboratories	1.3018	0.8776	8.71	2.16	6	8	5	
Goodrich Corp	3.0258	1.5156	1.29	33.79	14	2	7	Unique to "Mix"
Leggett & Platt Inc	1.8959	0.835	5.60	-19.48	7	15	13	
McDonald s Corp	4.3622	2.6146	3.82	17.01	14	3	11	
PPL Corp	0.9495	0.8666	8.38	7.99	5	6	2	Unique to "Integrated"
Ventas Inc	2.0262	4.9648	6.64	-3.32	11	12	14	
Motorola Mobility Holdings LLC	1.5136	0.6198	0.00	42.98	10	1	2	Common to both
Pepco Holdings LLC	0.8174	0.5041	7.00	0.86	1	10	2	Common to both

By observing the rankings for the integrated stocks, it can be seen that three of the four stocks achieve the rank of two, which simply means that they share this rank. Applying these two approaches can lead to two vastly different end portfolios. Much like Fitzgibbons et al. (2017) we also attain an overlap in 50 of the stocks (Pepco Holdings LLC and Motorola Mobility Holdings LLC) that are both included in the Mixed and integrated portfolio. Why the relatively low overlap? Stocks that are relatively cheap (expensive) with respect to some measure of value have a propensity to have poor (great) recent returns, in other words, as documented in subsection 6.1 these two factors are far from perfectly correlated. The mix portfolio approach processes information in a sequential manner. It identifies the best value and momentum stocks in a silo manner. While, the integrated portfolio processes all relevant information simultaneously, thus, correctly incorporating the offsetting nature of value and momentum. Likewise stated by Fitzgibbons et al. (2017) a high ranked value stock with a terrible momentum rank would only have a mediocre integrated ranking which would not allow it an allocation in the portfolio. In the mix this stocks would clearly make it into the portfolio since it scores top grades in the value silo.

6.4 Mixed Portfolio Results

This subsection will present the results for the various selected value and momentum strategies based on the selection rationale in subsection 6.1. Table 22 presents the annualized compounded returns (r), standard deviation (σ),t-statistics, information ratios (IRs), and alphas (α) for the mixed value and momentum portfolios that holds its stocks for $K = 12$ months with the momentum strategy selecting based on the past $J = \{6, 9, 12\}$ months.

Table 22, Panel A presents results for the various value portfolios mixed with the momentum strategy that selects based on the past $J = 6$ months return. In the value section the table also includes two categorical columns to determine whether the mixed portfolios realize worse (W) performance, in-between (I), or better performance (B) than it could achieve by its constituents. The overall picture suggests, that 9/15 of the portfolios realize better returns than they could have with a standalone value or momentum strategy, and perhaps even more impressively 13/15 of the portfolios realize improved risk-adjusted return performances. Notably, its worth emphasizing that the mixed portfolio approach produces much less volatile performances, as a lot of the standalone value strategies have standard deviations in the range of 20-22 percent with a the momentum strategy being 19 percent. The drastic decrease in volatility here can most likely be attributed to the correlation benefits discussed in subsection 6.1.

Table 22: Performances for mixed value and momentum portfolios that holds for K = {12} months with the momentum selecting based on the past J = {6, 9, 12} months returns.
This table presents the performance characteristics for the mixed portfolio combining value and momentum equally weighted. Panel A presents the results for the various value measures that hold 12 months and the momentum strategy that holds $K = 12$ months and selects stocks based on the past $J = 6$ months. Panel B presents the same strategies but now with momentum selecting based on the last $J = 9$ months, and lastly Panel C has $J = 12$ months for its momentum selection. r denotes the annualized compounded returns, σ is the annual standard deviation, IR is the information ratio, t(r) is the t-statistic for the returns, and α is the strategies returned subtracted by the equally weighted S&P 500 Index. All metrics are computed using monthly data throughout the sample from 1997:01:01 - 2019:12:31.

Panel A							
Strategy mix J=6 month	r	σ	IR	t(r)	α	Benefits(r)	Benefits(IR)
DIV	11.56	17.42	0.66	3.25	0.81	B	B
P/E & DIV	11.49	17.09	0.67	3.29	0.74	B	B
P/B & P/S & DIV	12.54	18.20	0.67	3.30	1.50	I	B
P/E & P/FC & DIV	11.65	17.77	0.66	3.21	0.90	B	B
P/B & P/E & DIV	11.51	17.51	0.66	3.22	0.76	B	B
P/B & DIV	11.50	17.73	0.65	3.18	0.75	B	B
P/E & P/S & DIV	11.75	17.62	0.67	3.27	1.00	B	B
P/E & P/S & P/FC & DIV	11.60	17.75	0.65	3.20	0.84	I	I
P/B & P/S & P/FC & DIV	12.18	18.64	0.65	3.20	1.42	I	I
P/S & DIV	11.75	17.83	0.66	3.23	0.99	I	B
P/B & P/E & P/S & P/FC & DIV	12.16	18.54	0.66	3.21	1.41	I	B
P/B & P/E & P/S & DIV	11.81	17.86	0.66	3.24	1.06	B	B
P/B & P/E & P/FC & DIV	11.87	18.31	0.65	3.18	1.12	B	B
P/S & P/FC & DIV	11.86	18.39	0.64	3.16	1.10	B	B
P/B & P/E & P/S	12.27	19.00	0.65	3.16	1.52	I	B
Average	11.80	17.97	0.66	3.22	1.04		
Equally weighted S&P 500 Index	10.75	18.54	0.58	2.83	0		

Table 22, Panel B presents results for the various value portfolios mixed with the momentum strategy that selects based on the past $J = 9$ months return. Recall from the momentum results Table 12 and Table 11, that increasing the selection period to $J = 9$ months and $J = 12$ months in Panel C will results in inferior standalone momentum results. Nonetheless, all of the mixed strategies produce statistically significant returns in the 11 percent range resulting in all strategies realizing positive alpha against the equally weighted S&P 500 Index. All of the strategies realize returns in-between the standalone constituents almost surely due to the low return realized by the momentum strategy. Despite the fact that the standalone value strategies can produce higher returns, 4/15 of the strategies manage to produce better risk-adjusted returns. As mentioned, the momentum strategy is most likely the constraining factor, conversely this means that from a standalone momentum portfolio perspective the mixed portfolio actually results in a strict improvement in terms of risk-adjusted returns. The evidence here indicates that the diversification effect of combining value and moment is very strong, especially for the standalone momentum perspective.

Table 22 – *Continued*
Panel B

Strategy mix J=9 month	r	σ	IR	t(r)	α	Benefits(r)	Benefits(IR)
DIV	11.04	17.52	0.63	3.09	0.29	I	B
P/E & DIV	11.05	17.19	0.64	3.15	0.29	I	I
P/B & P/S & DIV	11.78	18.29	0.64	3.16	1.03	I	I
P/E & P/FC & DIV	11.11	17.91	0.62	3.04	0.36	I	I
P/B & P/E & DIV	11.05	17.60	0.63	3.08	0.30	I	B
P/B & DIV	11.04	17.81	0.62	3.04	0.28	I	B
P/E & P/S & DIV	11.27	17.73	0.64	3.12	0.52	I	I
P/E & P/S & P/FC & DIV	11.15	17.85	0.62	3.06	0.40	I	I
P/B & P/S & P/FC & DIV	11.66	18.79	0.62	3.04	0.90	I	I
P/S & DIV	11.31	18.07	0.63	3.07	0.56	I	I
P/B & P/E & P/S & P/FC & DIV	11.61	18.69	0.62	3.04	0.85	I	I
P/B & P/E & P/S & DIV	11.34	17.96	0.63	3.09	0.59	I	I
P/B & P/E & P/FC & DIV	11.32	18.46	0.61	3.00	0.56	I	B
P/S & P/FC & DIV	11.34	18.55	0.61	2.99	0.58	I	I
P/B & P/E & P/S	11.86	19.09	0.62	3.04	1.11	I	I
Average	11.33	18.10	0.63	3.07	0.57		
Equally weighted S&P 500 Index	10.75	18.54	0.58	2.83	0.00		

Finally, Table 22, Panel C presents results for the various value portfolios mixed with the momentum strategy that selects based on the past $J = 12$ months return. As previously mentioned, this standalone momentum strategy is the worst performing of the three momentum strategies, which naturally impacts the mixed portfolio performances negatively. However, one must not forget the beneficial correlations of this strategy as evidenced in Table 17.

The average returns fall by 0.47 percent causing some of the portfolios to deliver negative alphas. Albeit the larger correlation benefits, the standard deviation is affected with a slight increase. This leaves these mixed portfolios all performing in-between the standalone constituents, a result almost surely dominated by the inferior momentum standalone performance. Again, with respect to the standalone momentum portfolios, the mixed portfolios result in portfolios which are far superior in terms of return and risk-adjusted return. In other words, the mixed strategies do not produce performances that are worse than its standalone constituents.

Table 22 – *Continued*

Panel C

Strategy mix J=12 month	r	σ	IR	t(r)	α	Benefits(r)	Benefits(IR)
DIV	10.59	17.78	0.60	2.92	-0.17	I	I
P/E & DIV	10.60	17.41	0.61	2.98	-0.15	I	I
P/B & P/S & DIV	11.33	18.52	0.61	3.00	0.58	I	I
P/E & P/FC & DIV	10.61	18.18	0.58	2.86	-0.14	I	I
P/B & P/E & DIV	10.62	17.81	0.60	2.92	-0.14	I	I
P/B & DIV	10.63	18.02	0.59	2.89	-0.13	I	I
P/E & P/S & DIV	10.84	17.96	0.60	2.96	0.09	I	I
P/E & P/S & P/FC & DIV	10.67	18.07	0.59	2.89	-0.08	I	I
P/B & P/S & P/FC & DIV	11.18	19.08	0.59	2.87	0.42	I	I
P/S & DIV	10.82	18.31	0.59	2.90	0.07	I	I
P/B & P/E & P/S & P/FC & DIV	11.15	18.96	0.59	2.88	0.39	I	I
P/B & P/E & P/S & DIV	10.90	18.18	0.60	2.94	0.15	I	I
P/B & P/E & P/FC & DIV	10.84	18.74	0.58	2.83	0.08	I	I
P/S & P/FC & DIV	10.80	18.83	0.57	2.81	0.04	I	I
P/B & P/E & P/S	11.41	19.32	0.59	2.89	0.66	I	I
Average	10.87	18.35	0.59	2.90	0.11		
Equally weighted S&P 500 Index	10.75	18.54	0.58	2.83	0.00		

In summary, an equal allocation of value and momentum exposure produces high and significant returns with the vast majority of the portfolios yielding better returns and information ratios than could be obtained by the standalone styles. For example the realizes portfolio sorting of the dividend yield measure with momentum selecting on $J = 6$ months obtains 11.54 percent annualized compounded return, 17.42 percent standard deviation, and 0.66 IR, which is a 0.06 percent return increase and a 0.04 IR increase. Furthermore, the evidence of lower volatile portfolios across all mixed portfolios compared to their standalone constituents suggests that the diversification effect offers a more "smooth ride" through the economic cycles and meltdowns for the multi-factor strategy than the single factor strategy. Unfortunately, as J increases, the correlation between the two single-factor strategies slightly decrease, however the average volatility slightly increase. In other words, there is little evidence to support that the slightly less correlated value and momentum portfolios will lead to significant diversification effects. This may however, be caused by the fact that the momentum portfolio's volatility increases as J increases.

6.5 Integrated Portfolio Results

Like the previous subsection, this subsection will present the results for the various selected value and momentum strategies based on the selection rationale in subsection 6.1. Table 23 presents the annualized compounded returns (r), standard deviation (σ), t-statistics, information ratios (IRs), and alphas (α) for the integrated value and momentum portfolios that holds its stocks for $K = 12$ months with the momentum strategy selection based on the past $J = \{6, 9, 12\}$ months.

Table 23, Panel A presents the results for the various value portfolios integrated with the momentum strategy that selects based on the past $J = 6$ months return. First, all of the reported portfolios produce statistically significant returns in the 10 and 11 percent range resulting in a vast majority of portfolios generating positive alpha and 3/15 realizing negative alpha. Compared to Panel A for the mixed portfolios, the evidence suggests that selecting stocks scoring both high value and momentum rankings do not result in as impressive returns as the mixed portfolio approach. Conversely, the evidence suggests that the true power of the integrated portfolio approach manifests itself as low volatile portfolios. The integrated portfolios realize an average volatility of 16.25 percent, 2.29 percent below the equally weighted S&P 500 Index and 1.69 percent below the mixed portfolio approach for the exact same standalone constituents. In terms of risk-adjusted returns, the lower volatility more than makes up for the lack of high returns, resulting in the integrated portfolio realizing larger risk-adjusted returns, than the mixed portfolio exclusively because of the lower volatility.

Table 23: Performances for integrated value and momentum portfolios that holds for K = {6, 9, 12} months with the momentum selecting based on the past J = {6, 9, 12} months returns.
This table presents the performance characteristics for the integrated portfolio combining value and momentum equally weighted. Panel A presents the results for the various value measures that hold 12 months and the momentum strategy that holds *K=12* months and selects stocks based on the past *J=6* months. Panel B presents the same strategies but now with momentum selecting based on the last *J=9* months, and lastly Panel C has *J=12* months for its momentum selection. r denotes the annualized compounded returns, σ is the annual standard deviation, IR is the information ratio, t(r) is the t-statistic for the returns, and α is the strategies returned subtracted by the equally weighted S&P 500 Index. All metrics are computed using monthly data throughout the sample from 1997:01:01 - 2019:12:31.

Panel A

Strategy int J=6 month	r	σ	IR	t(r)	α	Benefits(r)	Benefits(IR)
DIV	10.97	15.10	0.73	3.56	0.22	I	B
P/E & DIV	10.93	15.46	0.71	3.46	0.18	I	B
P/B & P/S & DIV	10.87	16.65	0.65	3.20	0.12	I	I
P/E & P/FC & DIV	11.00	15.88	0.69	3.39	0.25	I	I
P/B & P/E & DIV	11.00	16.04	0.69	3.36	0.24	I	B
P/B & DIV	10.71	16.27	0.66	3.22	-0.47	I	B
P/E & P/S & DIV	11.09	16.05	0.69	3.39	0.34	I	B
P/E & P/S & P/FC & DIV	11.02	16.33	0.67	3.31	0.27	I	B
P/B & P/S & P/FC & DIV	10.73	16.92	0.63	3.11	-0.03	I	B
P/S & DIV	11.04	16.22	0.68	3.33	0.28	I	B
P/B & P/E & P/S & P/FC & DIV	10.81	16.52	0.65	3.21	0.06	I	B
P/B & P/E & P/S & DIV	10.93	16.37	0.67	3.27	0.17	I	B
P/B & P/E & P/FC & DIV	10.75	16.27	0.66	3.24	-0.00	I	B
P/S & P/FC & DIV	11.23	16.52	0.68	3.33	0.48	I	B
P/B & P/E & P/S	10.91	17.11	0.64	3.12	0.16	I	B
Average	10.93	16.25	0.67	3.30	0.18		
Equally weighted S&P 500 Index	10.75	18.54	0.58	2.83	0.00		

Table 23, Panel B presents the results for the various value portfolios integrated with the momentum strategy that selects based on the past $J = 9$ months return. Interestingly, 8/15 of the integrated portfolios manage to perform better than the standalone portfolio albeit only 1/15 of the portfolios can be categorized as producing better returns. In addition, these portfolios on average realize 0.17 percent less volatility than the portfolios in Panel A. This evidence reinforces the proposition, that the integrated approach of selecting stocks that are relatively cheap for some measures of value while also exhibiting momentum signals, truly shine in picking low volatile stocks that produce acceptable returns.

Table 23 – *Continued*

Panel B

Strategy int J=9 month	r	σ	IR	t(r)	α	Benefits(r)	Benefits(IR)
DIV	10.48	14.62	0.72	3.51	-0.28	B	B
P/E & DIV	10.41	15.35	0.68	3.32	-0.35	I	B
P/B & P/S & DIV	10.18	16.44	0.62	3.03	-0.58	W	I
P/E & P/FC & DIV	10.43	15.67	0.67	3.26	-0.33	I	B
P/B & P/E & DIV	10.23	15.92	0.64	3.15	-0.52	W	B
P/B & DIV	10.10	15.87	0.64	3.12	-0.66	I	I
P/E & P/S & DIV	10.38	15.97	0.65	3.18	-0.38	I	I
P/E & P/S & P/FC & DIV	10.30	16.28	0.63	3.10	-0.45	W	I
P/B & P/S & P/FC & DIV	10.13	16.59	0.61	2.99	-0.62	I	B
P/S & DIV	10.46	16.13	0.65	3.18	-0.30	I	B
P/B & P/E & P/S & P/FC & DIV	10.11	16.43	0.62	3.01	-0.64	I	I
P/B & P/E & P/S & DIV	10.07	16.22	0.62	3.04	-0.69	W	I
P/B & P/E & P/FC & DIV	10.23	16.07	0.64	3.12	-0.53	W	B
P/S & P/FC & DIV	10.57	16.48	0.64	3.14	-0.18	I	B
P/B & P/E & P/S	10.18	17.10	0.59	2.90	-0.63	W	W
Average	10.28	16.08	0.64	3.02	-0.48		
Equally weighted S&P 500 Index	10.75	18.54	0.58	2.83	0.00		

Table 23, Panel C presents the results for the various value portfolios integrated with the momentum strategy that selects based on the past $J = 12$ months return. These portfolios exhibit the lowest returns documented so far in this subsection, but also the lowest standard deviation. With respect to risk-adjusted returns this leaves them with an average IR of 0.61 which is 0.3 larger than the equally weighted S&P 500 Index, but lower than the portfolios is Panel B and remarkably lower than those in Panel A.

Table 23 – *Continued*

Panel C

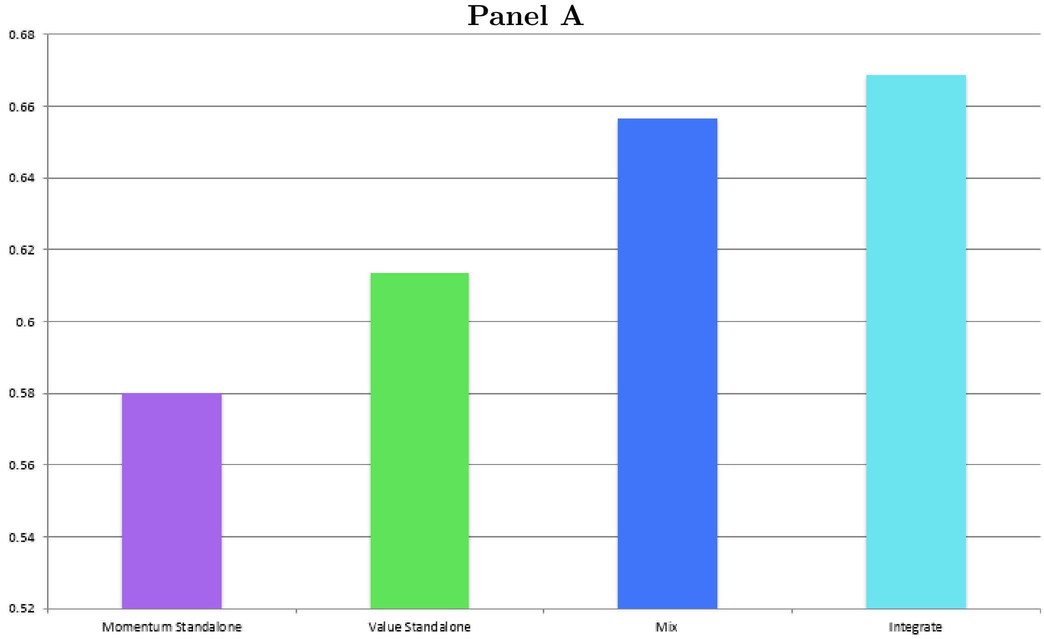
Strategy int J=12 month	r	σ	IR	t(r)	α	Benefits(r)	Benefits(IR)
DIV	10.32	14.39	0.72	3.51	-0.44	B	B
P/E & DIV	10.03	15.27	0.66	3.22	-0.73	I	I
P/B & P/S & DIV	9.66	16.39	0.59	2.89	-1.09	I	W
P/E & P/FC & DIV	9.88	15.70	0.63	3.08	-0.87	I	B
P/B & P/E & DIV	9.69	15.77	0.61	3.01	-1.06	I	I
P/B & DIV	9.58	15.56	0.62	3.02	-1.17	I	I
P/E & P/S & DIV	9.74	16.00	0.61	2.98	-1.01	I	I
P/E & P/S & P/FC & DIV	9.94	16.224	0.61	3.00	-0.82	I	I
P/B & P/S & P/FC & DIV	9.55	16.60	0.58	2.82	-1.21	I	I
P/S & DIV	9.88	16.16	0.61	3.00	-0.88	I	I
P/B & P/E & P/S & P/FC & DIV	9.70	16.41	0.59	2.89	-1.06	I	I
P/B & P/E & P/S & DIV	9.40	16.39	0.57	2.81	-1.35	I	I
P/B & P/E & P/FC & DIV	9.85	16.07	0.61	3.00	-0.90	I	I
P/S & P/FC & DIV	9.96	16.38	0.61	2.98	-0.79	I	I
P/B & P/E & P/S	9.50	17.13	0.55	2.72	-1.26	I	I
Average	9.78	16.03	0.61	3.00	-0.98		
Equally weighted S&P 500 Index	10.75	18.54	0.58	2.83	0.00		

In summary, this subsection documents that the portfolios, selecting stocks with the on average strongest exposure to both single-factor strategies, obtain very impressive standard deviations and risk-adjusted returns. For example the portfolio sorting on the dividend yield measure with momentum selecting on $J = 6$ months obtains an annualized compounded return of 10.97 percent, standard deviation of 15.10 percent, and an IR of 0.73. Comparing this to its standalone constituents suggests, that the integrated approach offers a strong combined exposure to both single-factors with a return in-between the single-factors strategies, but at a much lower risk and importantly at a relatively higher risk-adjusted return. Furthermore, as the selection period J increases, the integrated approach experiences a slight decrease in volatility but a more than offsetting decrease in realized returns. A casual interpretation of the evidence, is that, by aggregating factor information at the security level, the integrated approach manages to reduce volatility by avoiding stocks that exhibit high exposure to just one signal factor.

Figure 24 presents the average materialized information ratios for the average single-factor value and momentum strategies IRs, as well as the average mix and integrated IRs from Panel A in Table 22 and Table 23. Panel B, plots the realized returns and standard deviations for the strategies in Panel A. In line with the established results above, mixing the portfolios realize better risk-adjusted returns, than the single-factor strategies with the dominant source being high returns and lower standard deviations. Selecting stocks based with high composite rankings turns out to deliver even stronger risk-adjusted returns, primarily caused by the low risk stocks constituting the integrated portfolios.

Figure 24: Average risk-adjusted returns across different strategies for the full sample period
Panel A presents the average risk-adjusted returns denoted by the information ratios for each strategy. The average risk-adjusted returns are computed for all strategies are computed using monthly return data. The average mixed portfolio is computed as the average IRs of each of the 15 examined strategies for $K = 12$ with the momentum selecting based on $J = 6$, the same goes for the integrated portfolio. The momentum strategy's average risk-adjusted return is simply 0.58 equivalent to that of the momentum $K = 12$ and $J = 6$ portfolio, while the value strategy averages 15 stand-alone value measures. Panel B shows average σ on the primary axis and the average simple return on the secondary axis.

Panel A



Panel B

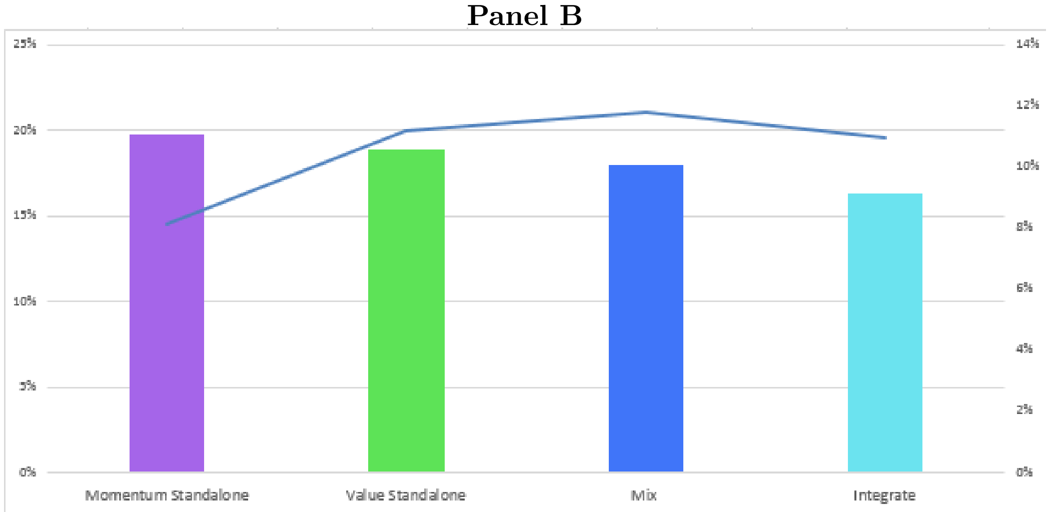
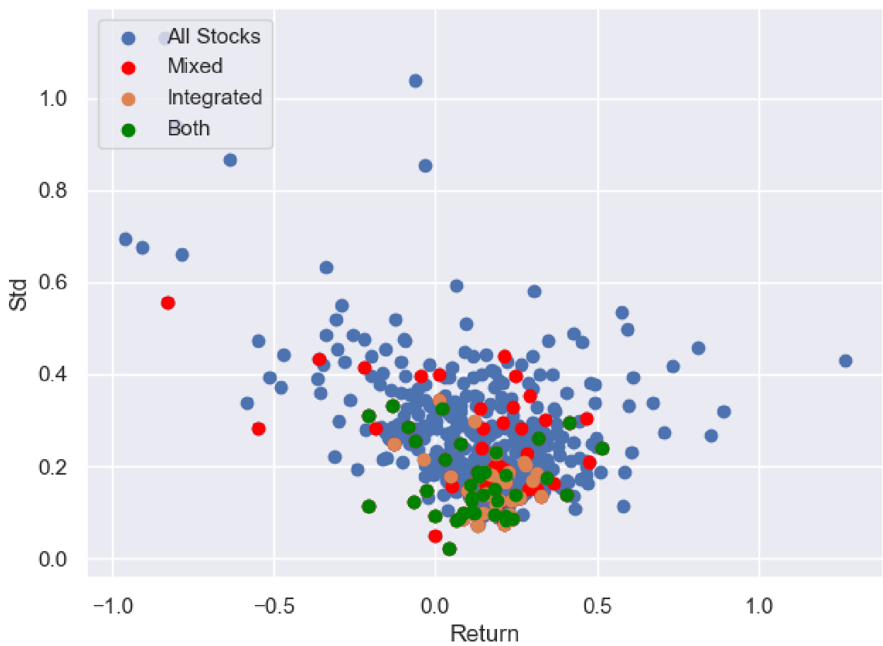


Figure 25 presents the realized annualized compounded log returns and standard deviations of all S&P 500 stocks in the period of 2010-2011. The mixed portfolio consists of the stocks highlighted in red and the integrated portfolio stocks are highlighted in orange. The visualization is computed using monthly return data for the mixed and integrated strategies sorted on the dividend yield measure and the momentum strategy that selects based upon the last $J = 6$ and holds this entire portfolio for the subsequent $K = 12$ months. The figure is consistent with the propositions stated in the previous paragraph; selecting stocks with high composite ranks, yields low volatile portfolios with relatively smaller positive returns. While selecting stock that score high value and momentum ranks separately, results in a more dispersed risk/reward profile. Generally, the mixed portfolio consists of stocks that materialize a wider return margin in both extremes and also exhibit relatively more risk than the integrated portfolio approach. It may be noted that this pattern from this one-year snapshot of this particular strategy is very consistent throughout the sample time and across different strategies.

Figure 25: Visualization of log returns and standard deviations of selected stocks using mixed and integrated portfolio approaches.
The figure presents 12 months realized log returns and standard deviations of the S&P 500 stocks in the period of 2010-2011. Both the mixed and integrated portfolios ranks on the Dividend yield measure and the momentum $J = 6$ months portfolio and rebalances the portfolio every $K = 12$ months. The mixed portfolio is highlighted red, the integrated portfolio is highlighted in orange, the stocks that overlap in the mixed and integrated portfolios are highlighted in green.



6.6 Comparison of various holding lengths

As discussed in subsection 6.3 the mixed and integrated portfolios documented in this paper will be implemented as one leg with a common holding length K. The multi-factor strategies documented so far have been on a $K = 12$ months rebalancing frequency. As discussed earlier, the momentum strategies excel on a $K = 6$ months rebalancing frequency while the value strategies historically perform superior on a $K = 12$ months holding period. To extend the provided research of the previous subsections, this subsection will increase the scope of investigation to examine the multi-factor strategies with the set of $K = \{6, 9, 12, 18\}$ months holding periods. In terms of previous documentation, the set of $K = \{9, 18\}$ holding periods is new for the standalone momentum and value strategies, respectively. That is, there is no documentation so far for these exact holding period for their standalone performances, however, this solution is a deliberate solution to the common K holding period rule.

Table 24, Panel A reveals several interesting results for the mixed strategy. First and most importantly, the evidence suggests that the $K = 9$ common holding period on average produces the highest annually compounded return of 12.26 percent, for a difference of 0.46 percent and 0.06 percent compared to the average returns of the $K = 12$ and $K = 6$ months holding periods, respectively. Taking risk-adjusted returns into consideration, the $K = 9$ months holding period also produces the best performances together with the $K = 6$ months holding portfolios, which both are roughly 5 percent and 12 percent better than the $K = 12$ and $K = 18$ months holding periods, respectively. Second, the $K = 6$ months holding periods performs very well, in terms of return and risk-adjusted return it is actually superior to the set of $K = \{12, 18\}$ month holding periods. This may not be very surprising, since the standalone momentum $K = J = 6$ months portfolio produces a 0.49 percent higher return than the average $K = 12$ month portfolio with an additional 0.04 of IR. In this perspective it may not be very astounding that the $K = 9$ month portfolio offers the best compromise when the objective is to equally weight these two single-factors into a multi-factor portfolio.

Table 24: Performances for mixed and integrated value and momentum portfolios that holds for $\mathbf{K = \{6, 9, 12, 18\}}$ months with the momentum selecting based on the past $\mathbf{J = \{6\}}$ months returns. This table presents the performance characteristics for the integrated portfolio combining value and momentum equally weighted. Panel A presents the results for the mixed portfolio approach with various value measures that holds for $K = \{6, 9, 12, 18\}$ months combined with the momentum strategy that holds for $K = \{6, 9, 12, 18\}$ and selects stocks based on the past $J = 6$ months. Whereas, Panel B presents the results for the integrated portfolio approach with various value measures that holds for $K = \{6, 9, 12, 18\}$ months combined with the momentum strategy that holds for $K = \{6, 9, 12, 18\}$ and selects stocks based on the past $J = 6$ months. $r(\cdot)$ denotes return denotes annually compounded returns for a given length K months in percentage, while $IR(\cdot)$ denotes the annual information ratio for a given length K months. Both metrics are computed using monthly data throughout the sample from 1997:01:01 - 2019:12:31.

Panel A

Strategy mix J=6 month	r(6)	IR(6)	r(9)	IR(9)	r(12)	IR(12)	r(18)	IR(18)
DIV	12.08	0.70	12.06	0.70	11.56	0.66	10.97	0.61
P/E & DIV	11.79	0.70	11.90	0.71	11.49	0.67	11.07	0.62
P/B & P/S & DIV	12.53	0.70	12.67	0.70	12.54	0.67	11.82	0.63
P/E & P/FC & DIV	12.24	0.70	12.18	0.69	11.54	0.66	11.12	0.60
P/B & P/E & DIV	11.83	0.68	11.90	0.69	11.51	0.66	11.25	0.62
P/B & DIV	11.86	0.68	11.94	0.68	11.50	0.65	10.99	0.61
P/E & P/S & DIV	12.11	0.70	12.18	0.70	11.75	0.67	11.36	0.62
P/E & P/S & P/FC & DIV	11.97	0.69	12.04	0.69	11.60	0.65	11.24	0.61
P/B & P/S & P/FC & DIV	12.67	0.69	12.64	0.69	12.18	0.65	11.55	0.60
P/S & DIV	12.27	0.69	12.29	0.69	11.75	0.66	11.25	0.60
P/B & P/E & P/S & P/FC & DIV	12.45	0.68	12.54	0.68	12.16	0.66	11.62	0.61
P/B & P/E & P/S & DIV	12.01	0.68	12.19	0.69	11.81	0.66	11.39	0.62
P/B & P/E & P/FC & DIV	12.20	0.67	12.27	0.68	11.87	0.65	11.31	0.60
P/S & P/FC & DIV	12.50	0.69	12.43	0.68	11.86	0.64	11.28	0.58
P/B & P/E & P/S	12.47	0.66	12.67	0.67	12.27	0.65	11.82	0.60
Average	12.20	0.69	12.26	0.69	11.80	0.66	11.34	0.61

Examining Table 24 Panel B, the integrated portfolio results emerge very similar results to the mixed portfolios. On average the $K = 9$ months holding period comes out on top in terms of returns and risk-adjusted returns. The $K = 6$ months is slightly behind, followed by the $K = 12$ and then $K = 18$ month strategies, the same sequence as for the mixed portfolios. This is somewhat unexpected, as documented in the previous subsection, these two multi-factor approaches select quite different stocks that materialize different performances.

Table 24 – *Continued*
Panel B

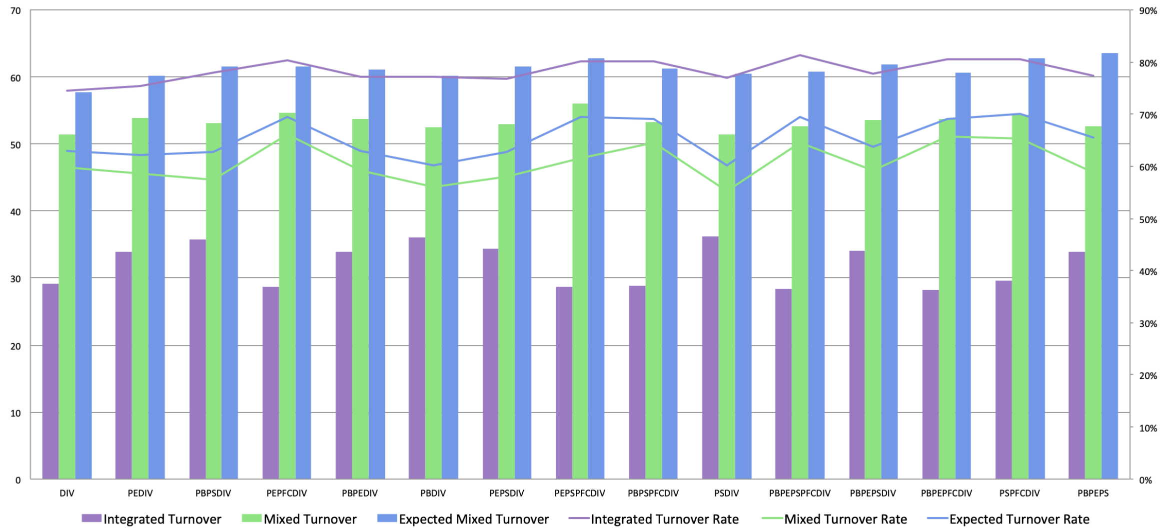
Strategy int J= 6 month	r(6)	IR(6)	r(9)	IR(9)	r(12)	IR(12)	r(18)	IR(18)
DIV	10.75	0.69	11.26	0.73	10.97	0.73	10.82	0.71
P/E & DIV	10.58	0.68	11.08	0.72	10.93	0.71	11.20	0.70
P/B & P/S & DIV	11.14	0.66	11.27	0.68	10.87	0.65	10.69	0.62
P/E & P/FC & DIV	10.88	0.68	11.32	0.72	11.00	0.69	10.55	0.64
P/B & P/E & DIV	10.75	0.66	11.17	0.69	11.00	0.69	11.00	0.67
P/B & DIV	10.62	0.64	10.89	0.66	10.71	0.66	10.31	0.62
P/E & P/S & DIV	11.16	0.70	11.48	0.72	11.09	0.69	11.34	0.67
P/E & P/S & P/FC & DIV	10.97	0.67	11.39	0.70	11.09	0.69	10.63	0.63
P/B & P/S & P/FC & DIV	10.94	0.65	11.13	0.66	10.73	0.63	10.50	0.61
P/S & DIV	11.41	0.70	11.62	0.72	11.04	0.68	10.80	0.63
P/B & P/E & P/S & P/FC & DIV	10.87	0.66	11.12	0.67	10.81	0.65	10.53	0.62
P/B & P/E & P/S & DIV	11.01	0.67	11.32	0.69	10.93	0.67	11.01	0.64
P/B & P/E & P/FC & DIV	10.54	0.64	11.10	0.68	10.75	0.66	10.49	0.63
P/S & P/FC & DIV	11.56	0.70	11.71	0.71	11.23	0.68	10.72	0.62
P/B & P/E & P/S	10.80	0.63	11.26	0.66	10.91	0.64	10.96	0.61
Average	10.93	0.67	11.27	0.70	10.93	0.67	10.77	0.64

In summary, this rather brief subsection documents the annualized compounded returns and the IRs of various mixed and integrated portfolios. The evidence is strongly in favour of the $K = 9$ month holding periods. The actual excess performance documented for the $K = 9$ months holding period, is not statistically significantly different then the rest; however, on a more intuitive level the argument that $K = 9$ is the best compromise for the value and momentum to work optimally together is rather compelling. With the $K = 9$ time horizon the mixed portfolios on average outperform the integrated portfolios, however, risk-averse investors selecting the integrated portfolios sorting on the dividend yield measure will outperform the mixed portfolios in terms of risk-adjusted return.

6.7 Turnover Considerations

Figure 26 presents the turnovers realized for the various mixed and integrated portfolios examined in the previous subsections. The x-axis denotes the value sortings and these strategies are mixed and integrated with the momentum strategy that selects stocks based on the last $J = 6$ months performance and the multi-factor portfolio which holds these stocks the subsequent $K = 12$ months. The primary axis denotes the average yearly purchased stocks for the respective multi-factor strategy (these belong to the columns). The secondary axis denotes the turnover rate in percentage (these belong to the curved graphs).

Figure 26: Turnover plot for the mixed and integrated portfolios that rebalances every K = 12 months.
This figure presents the integrated and mixed portfolio turnovers for various value and momentum strategies that rebalance every $K = 12$ months with the momentum strategy selecting stocks based on the past $J = 6$ months return. The x-axis denotes the value sortings, the primary axis is the average yearly new stock purchases, and the secondary axis denotes the turnover rate in percentage. The strategies run over the sample period from 1997:01:01 - 2019:12:31



Several key points emerge from this figure. First, in addition to the turnover implications discussed earlier for the standalone value and momentum strategies, the evidence suggests that the mixed multi-factor improves trading efficiency. The blue columns denote the expected mixed turnover for each of the various strategies. The expected turnover is calculated as the sum of stock purchases for the standalone value and momentum strategies, divided by 23 (number of years in the sample). As depicted in the figure, the implemented mixed portfolio realizes a turnover of approximately 50 instead of 60 stocks, for a difference of roughly 16-17 percent on average. There are two explanations for these trading efficiencies. The first possible explanation can be found in the implementation of the mixed portfolios. The mixed multi-factor model can technically be decomposed into two single-factor strategies which are equally weighted. The authors of this paper have deliberately chosen that the multi-factor strategies only consists of unique stocks. Unique, in the sense that in the case that the standalone momentum and value strategies both select stock XYZ, then the multi-factor strategy will only hold one XYZ stock. This is to avoid over exposure to one single stock, despite the fact, that it has exposure to both single-factors. This will naturally, lower the number of stocks that the multi-factor purchases, linearly with the number of stocks that the standalone strategies both would want. This happens more often than one would expect, Figure 23 gives a good quantification of the occurrence. Recall, that the top right figure depicts the stocks that the mixed portfolio selects (highlighted in red) and how the stocks rank on value (the x-axis) and momentum (the y-axis). The top quadrant of this figure, accounting for roughly 8-10 percent of the total mixed portfolio, are the stocks that will only be bought once albeit their high ranks and presence in both standalone value and momentum portfolios. The second possible explanation, is more of an intuitive implication of the multi-factor implementation. Consider the case of rebalancing the mixed multi-factor strategies, this could potentially involve selling a stock from the value portfolio and buying the exact same stock in the momentum portfolio. For example, in the case that a stock price drastically increases deeming it too expensive for the value portfolio, however, this price appreciation might be the precise reason that it could be included in the momentum portfolio. Third, in regards to the integrated multi-factor strategy it does not run into the same implications as the mixed multi-factor strategy. From a implementation technical standpoint, the integrated approach explicitly takes all value and momentum ranking information in at same time, thus avoiding the situation of double exposure to stock XYZ. This approach will only select stocks in the top quadrant in the top right figure in Figure 23, a consequence of this is that the integrated portfolios will hold less stocks at any time t than the mixed portfolio. In this regard, the integrated portfolio manages to exhibit a balanced exposure to both factors with a much smaller portfolio. However, indicated by the curved graphs and the secondary axis, the average turnover rates for the mixed integrated portfolio is 78 percent while the mixed portfolio is 60 percent, a difference of 18 percentage points. This is almost surely attributed to the fact that the mixed portfolios generally holds twice as many stocks meaning that the turnover netting effect, described in the previous paragraph, will be much larger as well, but also because the selection criteria is much less sensitive. Whereas, the high concentration criteria for the integrated approach intuitively will result in a smaller likelihood that a stock XYZ will meet the criteria once the portfolio rebalances.

6.8 Mix Versus Integrated Conclusion

This section establishes several interesting propositions. First, an equal allocation of value and momentum exposures produces high and significant returns and risk-adjusted returns with a vast majority of the portfolios outperforming those standalone value and momentum portfolios. In addition, the evidence indicates that the portfolio realize lower volatilities, which may be attributed to the potential diversification effect. Second, integrating value and momentum into one portfolio produces similar returns compared to the standalone portfolio, however, these returns come at a much lower volatility leading to overall better risk-adjusted returns. This evidence indicates, that stocks with high exposure to two factors rather than one bear much less risk and produces strong risk-adjusted returns. Third, on average the integrated portfolio approach outperform the mixed approach in terms of risk-adjusted return. Nonetheless, both the mixed and integrated portfolios produce remarkably larger risk-adjusted returns compared to the standalone value and momentum portfolio. Third, comparison of different holding periods reveal that the common holding length $K = 9$ months performs superior for both the mixed and integrated portfolio approach. This can be attributed to the fact that the value and momentum factors have different appreciation cycles, a holding length of $K = 9$ month offers the best compromise. Lastly, investigating the turnover rates associated with the mixed and integrated portfolio approaches. The mixed and integrated approaches have turnover rates of approximately 60 percent and 80 percent, respectively. Thereby, a reduction of the turnover rates in comparison to the standalone momentum portfolio. The evidence here suggesting a potential trading efficiencies when that may be because of stocks migrating from one factor to another.

7 ESG Implementation

This section proceeds as follows. Subsection 7.1 outlines the increasing importance and incentive of using ESG, its limitations, and its potential applications. Subsection 7.2 documents the attractive performance of a standalone sustainability (ESG) approach, when deploying it as a factors. Subsection 7.3 presents the realized tail risk associated with the sustainability portfolios. Subsection 7.4 summaries the results for the standalone ESG portfolios. Subsection 7.5 documents the performance of incorporating the various sustainability measures for various portfolio approaches. These approaches include mixed and integrated two-factor strategies and three-factor strategies. Subsection 7.6 presents an in depth analysis of all of the various portfolio approaches sorting on the ESG measure exclusively. In particular the subsection documents the tail-risks associated with the strategies. Subsection 7.7 concludes.

7.1 The ESG approaches

Over time, investors are not only interested in the financial outcomes of their investments but also the impact of their investment and the role of their assets in promoting global issues such as climate action has gained tremendous interest throughout the last couple of decades. According to research done by Bank of America Merill Lynch (2019), Europe is a region that in particular has embraced ESG investing. They state that this is due to the increased influence of eco-political parties together with an increase in the amount of "green" investors aiming at making an impact. A demographic that also has embraced this rather new investment approach are millennial's. In a study from 2006 called Cone Millennial Cause Study (2006) it was found that more than 80% of millennial's are more likely to trust a company or purchase a company's products, when the company has a reputation of being a socially or environmentally responsible. Moreover, the lion share of those surveyed were more likely to turn down a product or service from a company that was perceived to be socially or environmentally irresponsible.

When using ESG as a factor, investors can evaluate companies using ESG criteria as a framework to screen investments or to assess risks in its investment decision-making. It can take several forms. According to Morgan Stanley (2016) leading ESG companies do not necessarily hav to focus on having a positive impact or do any good for the planet or individuals in need. They see it as an investment strategy in itself that as a framework should be applied to all investment diligence processes in order to identify and mitigate economic, social and governance risks, that could negatively impact an investment. An example of this could be a company with inadequate environmental controls, which could lead to damages on the company's revenues, profitability valuation or reputation. The BP's Deepwater Horizon oil spill is a prominent example of this. An investor does not necessarily need to be a "do-gooder" to be aware of how a weak ESG profile can materially affect investment returns.

Within Environmental factors typical data includes waste management, water management, resource use, environmental disclosure, environmental impact and reduction of pollution and emissions. Environmental risks that are created by business activities can have actual or potential negative impact on air, land, water, ecosystems or human health. Environmental positive outcomes include avoiding or minimizing Environmental liabilities, lowering costs and increasing profitability through energy and other efficiencies and thus reducing regulatory, litigation and reputational risks.

The Social aspect refers to the impact that companies can have on society. They can be addressed by a company's social activities such as encouraging labor-management relation, promoting health and safety and focusing on product integrity and protecting human rights. Giving focus to these elements and achieving social positive outcomes, can lead to increasing productivity and morale, increasing brand loyalty and reducing turnover and absenteeism of employees. More specifically, these typically includes stakeholder analysis, workplace mentality, human rights, diversity community relationships, corporate citizenships and philanthropy.

Looking at issues with regards to Governance risks these concern the ways companies are run. More specifically policies concerning diversity, corporate risk management, excessive executive compensation, corporate, brand independence using company governance activities such as increasing diversity and accountability of the board, protecting shareholders and their rights and reporting and disclosure of information. Governance positive outcomes include aligning interests of shareowners and management and thus avoiding unpleasant financial surprises.

According to a survey carried out by UBS Asset Management (2019), it was found that 78% of asset owners are already to some degree implementing ESG into their investment processes. The survey also found that 17% are interested in the trend and are considering measures to take ESG into account wheres merely 5% were not interested in it. Thus, ESG has indeed become a megetrend and will most likely have defining impact in the long-term. Several approaches to use ESG as an investing strategy exists. The most common approaches are:

- **Best In Class/Positive Screening:** The approach seeks to invest in stocks that achieve a higher ESG performance rating than their industry peers. Investors use this to concentrate their capital in industries, stocks or projects that are considered "best-in-class" in each of ESG metrics when comparing to their peers. Different approaches to this exist, some investors look for companies that are actively improving their ESG score while others look for companies that already have established a high ESG score.

- **Corporate Engagement and Shareholder Action:** The approach makes use of company shareholder power in order to influence corporate behaviour either through direct corporate communication such as direct communication with the senior management and board or through filing shareholder proposals and proxy votings that follow ESG guidelines.

- **ESG Integration:** The approach seeks to take a quantitative and systematic view on ESG and uses the Environmental, Social and Governance pillars in a financial analysis framework. This should be interpreted as a holistic approach to investment analysis, where the material factors, being the ESG factors and traditional financial factors, are identified and assessed as the foundation for an investment decision. This integration means that the ESG criteria influences the investment analysis at all levels, from stock selection to asset allocation.

- **Impact Investing:** This approach targets investments that have a direct impact on solving environmental or social issues. This could be to reduce CO2 emissions, save water, improve access of education and housing to deprived communities, while still seeking to generate compelling returns. However, active ownership requires both time and resources, but according to Principles of Responsible Investing these efforts are worthwhile in mitigating risk and enhancement of investment returns.

- **Negative/Exclusionary Screening:** This approach specifically excludes companies from their

portfolio that have a score that does not meet the pre-determined ESG score criteria required for the portfolio. As noted by Clifford Asness, negative screening does not in it itself necessarily generate alpha. The profit goal is not the only motive, as asset owners can have goals that extend beyond the bottom line which means that many would trade a few extra basis points in order to leave a smaller carbon footprint or other ESG-associated benefit. Since the investment managers are accountable to their communities they aim to reflect the values of these communities and thus will continue to exclude companies and sectors that do not align with their evolving standard. Many investors tend to use a mix of both the negative
screening and positive screening in order to screen out stocks which they are morally opposed to and proactively choosing stocks that are making a difference in areas they care about.

- **Norms-based Screening:** This approach makes use of the international standards set by OECD, UN, ILO and UNICEF. The purpose is to screen investments based on compliance with international norms and either exclude or underweight investments that are not in line with these.

- **Sustainability Themed Investing:** This approach aims to invest in themes that have a particular focus on sustainability issues. Examples of this could be investments in renewable energy, clean technology, water, food and positive-impact climate change investments.

Hence, there are several ways of making use and implementing ESG into an investment process. The authors of this paper are enticed by the concept of integrating ESG in a value and momentum framework for several reasons. With respect to value investing, ESG will capture non-financial issues as governance, corporate culture and employee satisfaction which almost surely plays a detrimental role in a companies valuation and expected growth. From the perspective of a value investor these underlying issues are very well known, however, the ESG framework may help in boiling down the addressed issues to single metrics. In this perspective value and sustainable investing concepts are very closely aligned. Even the definition of Sustainable - meaning to be able to maintain over a long horizon - is at the heart of value investing with respect to businesses' long-term growth. As a consequence of this close conceptual alignment, both factors will also exhibit commonalities in their implementation with respect to their stock selection procedure and holding lengths.

As for the case of applying the ESG framework in a momentum style driven investment, this paper finds inspiration from a paper conducted by Morgan Stanley (2016). According to Morgan Stanley, the amount of capital going into U.S. investments utilizing sustainable investment criteria is approximately USD 8.72 trillion almost threefold the amount in 2012 which was USD 3.74 trillion. Their analysis suggests, that the huge buy pressure on sustainable investments are driven by client demands. This paper argues, that integrating high ESG ranking stocks into the momentum factor driven investment may aid the momentum style in capturing this rapid demand which almost surely will lead to large price appreciations. Furthermore, research on integrating ESG into a value, growth and momentum approach (Lars Kaiser (2018)) in both the U.S. and European equity space reveals potential risk mitigation and return enhancement. His results were most prominent in the European equity space. From the perspective of the S&P 500 Index this may indicate, that there may be a long-term positive outlook applying the sustainability measures.

The next subsection will document the standalone ESG performance characteristics, which will lay the foundation for its integrating in the following subsection.

7.2 ESG Standalone Results

Table 25 reports the results for the sustainability decile portfolios that sort on ESG, Environment, Social, and Governance measures of ESG throughout the 2005:01:01 - 2019:12:31 sample period. The procedure is similar to the value trading strategy. At formation, the four ESG implementations will evaluate each stocks previous year-ends score (Refinitive ESG data provided by Thomson Reuters) and rank them in a descending order. Each measure of ESG will than place the stock universe into ten equally sized deciles with the first decile consisting of the approximately 50 highest scoring stocks and the tenth decile consisting of the approximately 50 worst scoring stocks given their respective measure of ESG. The reported ESG strategies in this subsection equally holds the stocks in each decile for the following 12 months, thus rebalancing its portfolios every year in this exact procedure. The ESG portfolios are first up and running 2005:01:01 because of limited ESG data availability. Going further back in time before 2004 will involve data scarcity and thereby universe shrinkage (since the procedure for lacking data throughout the paper has been to neglect the stock). Furthermore, the reported ESG strategies represent equally weighted overlapping series of ESG strategies set to start one month apart to avoid timing risk. On the technical level, this involves running 12 ESG strategies using the same year-end 2004 data in 2005 with the first portfolio starting in January, the next starting in February and so forth. This means, that the reported ESG strategies actually first start 2006:01:01, since it takes 12 months for the equally weighted series of various sustainable measures to get up and running simultaneously. It is worth emphasizing, that whether ESG can be crowned as a traditional systematic risk factor or not is not the focus of this paper. This paper simply implements ESG, at the technical level, to the value and momentum factors and evaluates its performance as a potential new factor.

Table 25, Panel A reports the decile portfolio performances for the portfolios sorting on the overall ESG ranking. Comparing the two decile extremes, the highest scoring ESG portfolios produce a highly statistical significant annualized compounded return of 11.91 percent, while the lowest scoring portfolio realizes a return of 9.60 percent, for a relatively large annually return difference of 2.31 percent. In addition, the highest scoring portfolio exhibits lower volatility than the lowest scoring. Thus, resulting in a overall risk-adjusted return of 0.60 for the highest ESG scoring portfolio in contrast to an IR of 0.48 for the lowest ESG scoring portfolio. With this different time horizon the equally weighted S&P 500 index realizes an annual compounded return of 9.83 percent (with a t-statistic of 1.86), standard deviation of 19.75 percent, and a risk-adjusted return of 0.50. Comparing the highest scoring ESG portfolios to this benchmark, will actually result in a convincing outperformance across all three measurements.

Table 25: Performance Characteristics for the equally weighted series of decile portfolios based on the various measures of ESG.
The table presents performance characteristics for the 10 equally weighted series of decile portfolios that are formed in descending order based on their ESG score from high score portfolio (1) to lowest (10). r denotes annualized compounded return, σ is the standard deviation denoted in percentage, t(r) denotes the t-statistic, IR is the information ratio, and α denotes alpha. All of the computations are determined using realized monthly return data for each of the decile portfolios. Panel A denotes the performances characteristics for the decile portfolios sorting on the overall ESG score. Panel B reports the same characteristics for the sustainable decile portfolios sorting on the environment measure. Panel C reports for the Social measure. lastly, Panel D reports for the Governance measure.

	High score								Low score	
	1	2	3	4	5	6	7	8	9	10
	Panel A: ESG									
r	11.19	11.67	9.29	10.22	9.14	7.55	9.18	10.68	9.24	9.59
σ	18.77	18.86	19.56	20.52	19.74	21.36	20.72	20.82	19.99	20.05
t(r)	2.23	2.32	1.78	1.86	1.73	1.32	1.66	1.92	1.73	1.79
IR	0.60	0.62	0.48	0.50	0.46	0.35	0.44	0.51	0.46	0.48
α	1.36	1.84	-0.54	0.39	-0.69	-2.28	-0.67	0.85	-0.59	-0.24

Table 25, Panel B presents the decile portfolio performances sorting on the Environmental score. The highest scoring portfolio has a slightly larger return with a return of 11.25 percent, but a considerably larger standard deviation of 19.96, resulting in a IR of 0.56. While the lowest scoring portfolio has a return of 9.98 percent and a standard deviation of 20.10, for a return difference of 1.27 percent. High Environmental stocks are identified with stocks that displays best management practices to avoid environmental risk and capitalize potential environmental opportunities and vice versa for low Environmental stocks. This is somewhat consistent with the higher returns of the high Environmental stocks, but the findings do not suggests that Environmental score is a good proxy for potential risk.

Table 25– ***Continued***

	High score							Low score		
	1	2	3	4	5	6	7	8	9	10
	Panel B: Environment									
r	11.25	10.49	11.28	10.59	10.50	7.72	8.61	7.72	8.53	9.98
σ	19.96	18.16	18.69	19.47	18.66	22.08	22.85	22.14	20.46	20.10
t(r)	2.11	2.16	2.26	2.04	2.11	1.31	1.41	1.30	1.56	1.86
IR	0.56	0.58	0.60	0.54	0.56	0.35	0.38	0.35	0.42	0.50
α	1.42	0.65	1.44	0.76	0.67	-2.11	-1.23	-2.11	-1.31	0.15

Examining Table 25, Panel C the decile portfolios that select their stocks based on highest and lowest Social scores yield a compounded annual return of 9.73 and 9.47 percent respectively, for a difference of 0.27 percent. High Social scoring stocks are identified by companies with high employment quality, human rights, and product responsibility and vice versa for the low Social scoring companies. The low marginal spread between the highest and lowest scoring decile coupled with relatively low returns may suggest company and employee relations are not a necessarily indicative of high return. Nonetheless, the evidence of slightly increasing standard deviation going from the highest scoring to lowest scoring decile, indicate that companies with healthy company employee relations and high product responsibility are (slightly) less volatile. In support of this argument, Lee and Faff (2019) demonstrate that higher Social performance is accompanied by smaller idiosyncratic risks for equities.

Lastly, Table 25, Panel D presents the performance of the decile portfolios that select their stocks based on Governance scores. High Governance scoring stocks are characterized by strong shareholder rights, vision and strategy, and board structure. The highest Governance portfolio yields the largest return of all four variables with a compounded annual return of 11.87 percent, for a difference of 2.31 percent comparing it to the lowest Governance portfolio. This impressive return is accompanied by a equally impressive standard deviation of 18.15, resulting in a risk-adjusted return of 0.65. Recall, that these numbers are actually very high in the new time sample, the return produces an alpha of 2.03 percent. Consistent with the high alpha and relatively lower volatility the IR is 30 percent higher than the benchmark. This evidence suggests, that the Governance measure may have excellent explanatory return and volatility power. A conjecture that is supported by Gombers et al. (2003) who find strong correlation between shareholders rights and return.

Table 25 – *Continued*

	High score								Low score	
	1	2	3	4	5	6	7	8	9	10
Panel C: Social										
r	9.73	10.22	10.97	9.53	9.20	9.76	10.41	9.54	8.96	9.47
σ	18.07	19.67	20.61	20.47	20.85	21.52	18.91	19.99	20.14	19.95
t(r)	2.02	1.94	1.99	1.74	1.65	1.70	2.06	1.79	1.67	1.77
IR	0.54	0.52	0.53	0.47	0.44	0.45	0.55	0.48	0.45	0.47
α	-0.10	0.39	1.14	-0.30	-0.64	-0.07	0.58	-0.29	-0.87	-0.38
Panel D: Governance										
r	11.87	9.11	8.49	9.17	10.24	10.00	10.08	10.37	8.85	9.56
σ	18.15	20.03	20.64	19.55	19.25	21.18	20.49	20.17	20.43	20.66
t(r)	2.45	1.70	1.54	1.76	1.99	1.77	1.84	1.92	1.62	1.73
IR	0.65	0.46	0.41	0.47	0.53	0.47	0.49	0.51	0.43	0.46
α	2.03	-0.71	-1.35	-0.66	0.40	0.17	0.25	0.54	-0.98	-0.27

To attain a broader overview, the results of the highest scoring decile portfolios with various measures of sustainability are shown in Table 26 below. The table compares the various realized sustainability portfolios with the equally weighted S&P 500 Index and makes a very compelling case with respect to the listed 3 performances metrics. The next subsection, will take a closer look into the tail risk associated with the sustainability portfolios - to test whether their low standard deviation understates the potential crash risk and time with underperformance.

Table 26: Performance Characteristics for the equally weighted series of decile portfolios based on the various measures of ESG.
The table presents performance characteristics for the equally weighted series of the highest portfolios deciles in the period of 2006:01:01 - 2019:12:31. r denotes the annualized compounded returns, σ is the annual standard deviation, IR is the information ratio, t(r) is the t-statistic for the returns, and α is the strategies returned subtracted by the equally weighted S&P 500 Index. All metrics are computed using monthly data throughout the sample from 2006:01:01 - 2019:12:31.

	r	σ	IR	t(r)	α
ESG	11.19	18.77	0.60	2.23	1.36
Environment	11.25	19.96	0.56	2.11	1.42
Social	9.73	18.07	0.54	2.02	-0.10
Governance	11.87	18.15	0.65	2.45	2.03
Equally Weighted S&P 500 Index	9.83	19.75	0.50	1.86	0.00

7.3 ESG Standalone Drawdown

Further analysis of the ESG pillars are shown in the form of drawdowns in Figure 27 and Table 27 below. The figure visualises the drawdown of the benchmark equally weighted S&P 500 Index and the highest scoring decile of the various sustainability portfolios. Like the previously documented value and momentum portfolios, the worst realized maximum drawdown occurs in March 2009. The figure and table reveal several interesting insights. First, the equally weighted S&P 500 Index realizes a drawdown of 57.39 percent, which is 11.09 percentage points larger than the lowest sustainability drawdown and 3.02 percentage points larger than the highest sustainability drawdown. In addition, the sustainability portfolios on average experience drawdown lengths of 31.5 months approximately 10.5 months lower than the equally weighted S&P 500 Index, while also on average, enduring lower length recovery ratios.

In relation to the value and momentum factor, the sustainability portfolios actually provide a much more compelling annualized compounded returns than the average standalone value $K = 12$ months portfolio (10.15 percent) and the momentum $J = K = 6$ strategy (8.17 percent) accompanied with lower risk and risk-adjusted return. Remember, starting portfolios at the market peaks during 2006 will have a tremendous impact on the following years because of the financial crisis in 2008. For example, starting the value and momentum portfolio in start 2010 will yield an average annualized cumulative return of 13.91 percent and 14.20 percent, respectively for the value and momentum strategy. This goes to show, that the inception date is very impactful which will be elaborated on in subsection 8.2, and perhaps more importantly that the sustainability portfolios are very powerful.

Zeroing in on the different sustainability strategies, one very insightful result is that the Governance measure producing the highest return and risk-adjusted return comes bearing remarkably less crash risk compared to the rest of the sustainability portfolios as well as the equally weighted S&P 500 Index. This evidence supports the argument that the companies with strong client relations, board structure, and vision and strategy are attributes that can explain company returns and risk. A supporting explanation is that strong Governance intuitively plays a huge role in times with high market turmoil.

Figure 27: Drawdown plot for the various sustainability portfolios that rebalances every $K = 12$ months ESG.
The figure presents the realized drawdowns for the various sustainability portfolios including the equally weighted S&P 500 Index in the period of 2006:01:01 - 2019:12:31. The drawdowns are computed using monthly return data for each portfolio series.

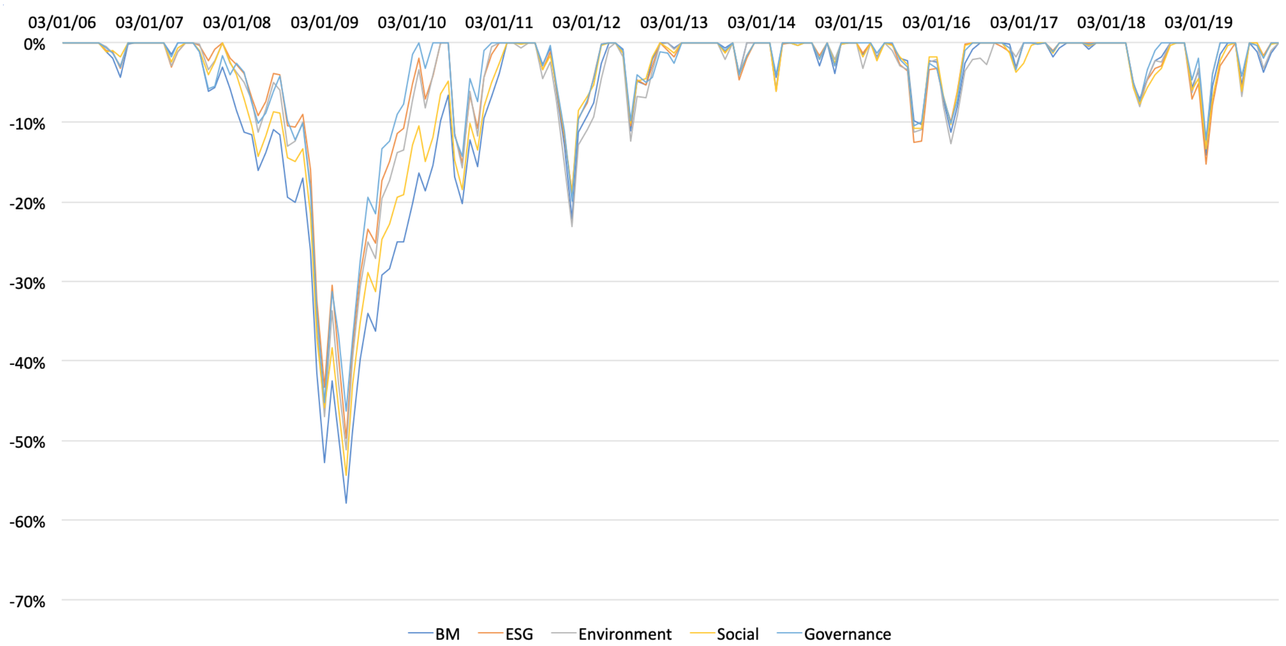


Table 27: ESG measure drawdown characteristics in the full period from 2006:01:01 - 2019:12:31. This table presents the worst realized maximum drawdown (MDD) for all of the ESG portfolios over the full 14-year sample period using monthly return data. Additionally, the table presents the corresponding length and recovery time for the portfolios. MDD(%) denotes the worst realized maximum drawdown in percentage. Length(m) denotes the time it took from peak to valley to initial peak value, and the recovery(m) time is the time it takes from valley to new initial peak value. Both length(m) and recovery(m) are specified in number of months.

Strategy	MDD (%)	Length(m)	Recovery(m)
ESG	49.70	29	12
Environmental	51.18	29	12
Social	54.37	38	29
Governance	46.30	30	9
Equally weighted S&P 500 Index	57.39	42	21

7.4 ESG Standalone Summary and Incentives

This section establishes several insightful and compelling findings. First, trading strategies that buy stocks which have high overall ESG, Environmental, or Governance scores, outperforms strategies buying stocks with low scores. The Social score has little to no return spread, however, buying stocks with high Social scores does come with low standard deviations. The evidence here indicating that companies with high employment quality, product responsibility etc. bear lower risk. Its worth reemphasizing that taking the new time horizon into account, all of the sustainability portfolios highly outperform the average momentum and value strategies across all performance metrics. Second, the sustainability measures come bearing relatively less crash risk, while 3/4 of the portfolios also produce relatively higher returns than the equally weighted S&P 500 Index. Third, the evidence suggests that companies with strong shareholder rights, vision and strategy, and board structure produce very impressive returns, risk-adjusted returns and drawdown measures. Last, the overall ESG sustainability that captures all three sustainability measures and additional sustainability information also produces very appealing performance characteristics.

Based on these findings and the complementing arguments outlined in the last paragraph of subsection 7.1 the authors of this paper find the idea of integrating ESG into the value and momentum multi-factor strategy very compelling.

7.5 Don't just mix your ESG, integrate it

Table 28 reports the annualized compounded returns, standard deviation, and IRs for the five strongest performing integrated and mixed multi-factor value and momentum strategies with a overlap of 3 strategies. The respective five strongest mixed and integrated portfolios here being selected throughout the same sample period ESG runs, that is 2006:01:01-2019:12:31. Consistent with the previous findings, the reported strategies have a common holding length of $K = 9$ months and uses the momentum strategy that selects stocks based on the past $J = 6$ months.

In contrast to the previous much longer sample time period (1997:01:01-2019:12:31), the multi-factor strategies do not all outperform the equally weighted S&P 500 Index in respect of alpha return. This is primarily caused by the momentum strategy. As documented in subsection 5.4, selecting stocks based on past strong performance comes bearing huge crash risk and long periods (several years) of severe losses. Initiating momentum strategies two years prior to a financial crisis lead to long lasting and detrimental impacts, which will impair results for the remaining investment horizon. The evidence for value crashes in subsection 4.6 finds, that buying stocks that are priced relatively cheap in respect to some value metric yields a very steep drawdown in a crash but equally rapid in recovery. Consequently, the majority of the selected multi-factor strategies that equally weighs momentum and value stocks underperforms the S&P 500 Index in terms of return, nonetheless, all of the integrated strategies and two of the mixed portfolios still outperform in terms of risk-adjusted return. Like in the previous single-factor subsections, Table 28 also includes two categorical columns to determine whether the multi-factor portfolios realize worse (W) performance, in-between (I), or better performance (B) than it could achieve by its constituents. The constituents here being the standalone various mixed value and momentum portfolios. Consistent with the findings in subsection 6.5, buying stocks with the highest on average standalone momentum and value characteristics offer strictly better risk-adjusted returns. Particularly, the integrated portfolios exhibit very low volatilities. 2/5 of integrated portfolio may not outperform the equally weighted S&P 500 Index in this time sample with respect to returns, but all portfolios still buys stocks with acceptable returns at a lower volatility, which leads to better information ratios that are 15-20 % larger than the equally weighted S&P 500 Index. In this context, the integrated multi-factor style shows strong robustness in its performance attributes, even with an inception two years prior to the financial crisis.

The mixed multi-factor portfolios, are very affected by the new inception date. The standalone $K = 9$ and $J = 6$ momentum portfolio realizes a return of 7.13 percent with a volatility of 20.50 percent.[8] Whereas, the standalone value portfolios in Table 28 with the new time horizon produce an average return of 9.94 percent coupled with a volatility of 18.54 percent. In the perspective of a momentum investor, mixing the momentum with value results in strictly better performance metrics. Naturally, the poor momentum portfolio performing impairs the mixed multi-factor strategy, especially in terms of return. Nonetheless, consistent with subsection 6.4 the diversification effect of equally weighing both single-factors results in lower volatile portfolios for all 5/5 portfolios and 3/5 of the mixed strategies produce stronger risk-adjusted returns.

Table 28: Performance characteristics for best performing mixed and integrated momentum and value portfolios from 2006:01:01 - 2019:12:31.
The table presents the top five best performing mixed and integrated portfolios in the same time horizon that the ESG data runs from 2006:01:01 to 2019:12:31. r denotes the annualized compounded returns, σ is the annual standard deviation and IR is the information ratio. The performance metrics are computed using monthly return data. Lastly, B(r), B(σ), and B(IR) denotes whether the portfolios result in performance worse denoted by W, in-between denoted by I, or better performance denoted by B, than could be achieved by its constituents.

Top 5 Integrated w. momentum J=6						
Strategy	r	σ	IR	B(r)	B(σ)	B(IR)
DIV	9.72	16.73	0.58	B	B	B
P/S DIV	9.91	17.32	0.57	W	B	B
P/E P/S DIV	9.98	17.06	0.59	W	B	B
P/E P/FC DIV	10.00	17.45	0.57	W	B	B
P/E DIV	9.66	16.70	0.58	W	B	B
Top 5 Mix w. momentum J=6						
P/E DIV	9.44	18.42	0.51	W	B	B
P/B P/S DIV	10.10	19.69	0.51	W	B	W
P/E P/S DIV	9.02	19.42	0.46	W	B	W
DIV	9.34	18.96	0.49	W	B	B
P/B P/S P/FC DIV	9.96	20.32	0.50	W	B	B
Equally Weighted S&P 500 index	9.83	19.70	0.50			

Having documented the results of the mixed and integrated value and momentum multi-factor in this new time sample as well as the sustainability standalone portfolios, this subsection will technically treat the sustainability portfolios as four separate factors and offer insights to their joint returns. In

[8] The performance statistics are computed using monthly return data in the sample period of 2006:01:31 - 2019:12:31

line with the discussion in subsection 6.3 the multi-factor portfolios will be treated as one single leg, hence all single-factors will have a common holding length $K = 9$. Consistent with previous implementations, this subsection will employ data in a conservative manner. The accounting data for the value strategies is lagged by a minimum of 6 months and the sustainability data employed is from year *t-1*. This methodology increases the credibility of the end results, because it ensures that the data employed in the backtest is equivalent to that of an investor at the time revising the portfolios. This is particularly conservative choice with respect to the sustainability data, which is released and available as of January each year. In addition, the sustainability data is not subject to revision/calibration like the financial statement figures employed in the value strategy. Lastly, the momentum strategy will continue to use daily contemporaneous stock prices to compute monthly compounded returns.

Table 29 presents the various value and momentum mixed multi-factor strategies for an investor that incorporate various sustainability measures into their mixed multi-factor framework. Here the Benefits($\cdot$) notation, denotes whether the portfolio has performed better, in-between in this case being equal, or worse than the respective various integrated value and momentum multi-factor strategies. All of the value and momentum mixed multi-factor portfolios, with four exceptions for one metric each, exhibit stronger performance metrics across the whole line, when including the the four measure to its multi-factor strategy, respectively. The evidence highly indicates, that a value and momentum mixed multi-factor investor can greatly benefit from including sustainability into his/hers investment portfolio. Since the examined portfolios value and momentum multi-factor portfolios are identical but subject to different measures of sustainability the main difference will reflect the unique characteristics of each separate sustainability measure. Consistence with the standalone ESG subsection 7.2, sorting on ESG, Environmental, and Governance measures, separately, produces multi-factor strategies that yield high returns and relatively low volatilities. Whereas, sorting on the Social measure will produce relatively lower returns but relatively more attractive low volatilities.

Table 29: Performance characteristics for the mixed value, momentum, and sustainability portfolios that run from 2006:01:01 - 2019:12:31.
The table presents performance characteristics for the five strongest value and momentum mixed multi-factor strategies that includes various sustainability measures, respectively. The reported performance characteristics are computed using monthly return data. r denotes the annualized compounded returns, σ is the annual standard deviation and IR is the information ratio. Lastly, B(r), B(σ), and B(IR) denotes the effect of sorting based on the respective four sustainability measures, W denotes a worse performance, I is an identical performance, B denotes a strictly better performance in comparison to the mixed value and momentum multi-factor strategy.

	r	σ	IR	B(r)	B(σ)	B(IR)
Panel A: ESG						
P/E DIV	9.95	18.45	0.54	B	W	B
P/B P/S DIV	10.36	19.10	0.54	B	B	B
P/E P/S DIV	10.12	18.77	0.54	B	B	B
DIV	9.91	18.73	0.53	B	B	B
P/B P/S P/FC DIV	10.17	19.60	0.52	B	B	B
Average	10.10	18.93	0.53			
Panel B: Environment						
P/E DIV	10.13	18.81	0.54	B	W	B
P/B P/S DIV	10.57	19.46	0.54	B	B	B
P/E P/S DIV	10.34	19.12	0.54	B	B	B
DIV	10.15	19.13	0.53	B	B	B
P/B P/S P/FC DIV	10.49	20.01	0.52	B	B	B
Average	10.34	19.31	0.54			
Panel C: Social						
P/E DIV	9.51	18.30	0.52	B	B	B
P/B P/S DIV	9.98	18.84	0.53	W	B	B
P/E P/S DIV	9.80	18.52	0.53	B	B	B
DIV	9.46	18.43	0.51	B	B	B
P/B P/S P/FC DIV	9.83	19.35	0.51	W	B	B
Average	9.64	18.69	0.52			
Panel D: Governance						
P/E DIV	10.22	18.43	0.55	B	W	B
P/B P/S DIV	10.51	19.13	0.55	B	B	B
P/E P/S DIV	10.37	18.71	0.55	B	B	B
DIV	10.24	18.67	0.55	B	B	B
P/B P/S P/FC DIV	10.57	19.44	0.54	B	B	B
Average	10.40	18.88	0.55			
Equally Weighted S&P 500 index	9.83	19.70	0.50			

Table 30 presents the performance characteristics of an investor applying the integrated portfolio approach and sorts on value, momentum, and the various sustainability measures. Here the Benefits($\cdot$) notation, denotes whether the portfolio has performed better, in-between in this case being equal, or worse than the respective various integrated value and momentum multi-factor strategies. For example the very first strategy in Panel A, reflects a portfolio that buys stocks that scores the highest on average dividends, momentum, and ESG scores. The evidence highly implies, that buying stocks with the highest average value, momentum, and sustainability measure produces much more powerful performances than sorting exclusively on value and momentum. As documented in Table 30, 15/20 of integrated value and momentum portfolios experience increased returns when adding the respective sustainability measures to the overall integrated ranking scheme. In terms of worsening returns, the Social measure has 3/5 of the portfolio that would produce higher returns excluding the sustainability from the value and momentum integration. Using the Social measure with the momentum and value strategy sorting on the P/E, P/FC, and dividend yield measure actually underperforms both the value and momentum integrated multi-factor, as well as the standalone Social pillar. The fact that the portfolio is outperformed by the value and momentum integrated multi-factor is most likely due to the fact that the Social pillar has poor return explanatory power, as evidenced in the ESG standalone subsection 7.2. The fact that the portfolio is outperformed by both its constituents suggests, that high ranking value and momentum stocks may have offsetting return effects on high ranking Social stocks and the other way around.

Examining the standard deviation and the IR performance metrics, all of the portfolio exhibit very low volatilities. Especially the very first dividend sorting strategy in Panel A has an annual volatility of 15.01 percent, while the equally weighted S&P 500 Index has a volatility of 19.70, for a difference of 4.69 percentage points. The average volatilities for all of the portfolios is 16.19 percent, whereas the value and momentum integrated portfolio has an average standard deviation of 17.05 percent and the standalone ESG portfolios have an average standard deviation of 18.73 percent.[9] The high returns coupled with the low volatilities results in 19/20 of the integrated multi-factor strategies producing better IRs than the value and momentum multi-factor strategy could achieve alone. In addition, the vast majority (17/20) of the integrated three-factor strategy outperform the standalone ESG portfolios in respect to risk-adjusted returns.[10] The large volatility benefits, strongly advocates the risk and risk-adjusted return obtained from buying stocks that exhibit on average high value, momentum, and ESG scores.

[9]The performance statistics are computed using monthly return data in the sample period of 2006:01:31 - 2019:12:31

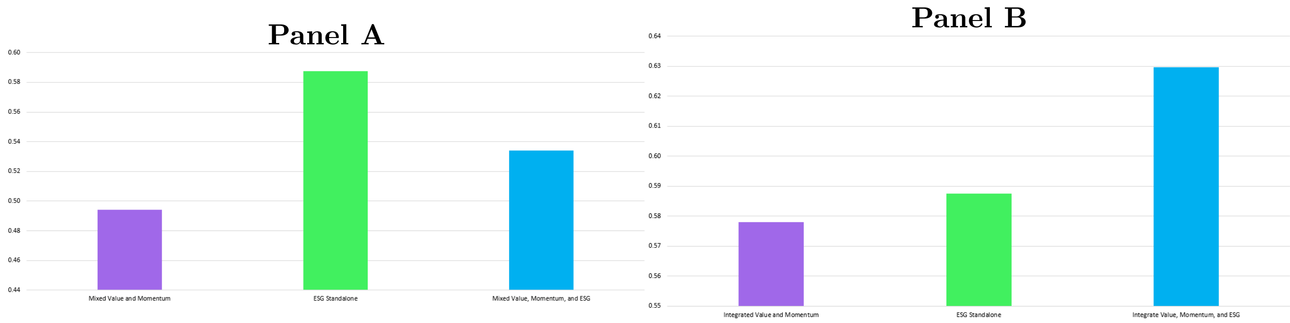
[10]The comparison is done by comparing the three-factor strategies with the annualized compounded returns in Table 26 for each sustainability measure, respectively.

Table 30: Performance characteristics for the integrated value, momentum, and sustainability portfolios that run from 2006:01:01 - 2019:12:31.
The table presents performance characteristics for the five strongest value and momentum integrated multi-factor strategies that integrated various sustainability measures. The reported performance characteristics are computed using monthly return data. r denotes the annualized compounded returns, σ is the annual standard deviation and IR is the information ratio. Lastly, B(r), B(σ), and B(IR) denotes effect of sorting based on the respective four sustainability measures, W denotes a worse performance, I is an identical performance, B denotes a strictly better performance in comparison to the integrated value and momentum multi-factor strategy.

	r	σ	IR	B(r)	B(σ)	B(IR)
Panel A: ESG						
DIV	10.09	15.01	0.67	B	B	B
P/S DIV	10.51	16.23	0.65	B	B	B
P/E P/S DIV	10.49	16.13	0.65	B	B	B
P/E P/FC DIV	9.95	16.60	0.60	W	B	B
P/E DIV	10.46	15.55	0.67	B	B	B
Average	10.30	16.10	0.64			
Panel B: Environmental						
DIV	10.19	15.65	0.65	B	B	B
P/S DIV	10.42	16.81	0.62	B	B	B
P/E P/S DIV	10.57	16.52	0.64	B	B	B
P/E P/FC DIV	9.87	17.12	0.58	W	B	B
P/E DIV	10.54	16.21	0.65	B	B	B
Average	10.32	16.46	0.63			
Panel C: Social						
DIV	10.00	15.45	0.65	B	B	B
P/S DIV	9.80	16.62	0.59	W	B	B
P/E P/S DIV	9.91	16.24	0.61	W	B	B
P/E P/FC DIV	9.56	16.70	0.57	W	B	I
P/E DIV	10.13	15.82	0.64	B	B	B
Average	9.88	16.17	0.61			
Panel D: Governance						
DIV	10.41	15.12	0.69	B	B	B
P/S DIV	10.00	16.48	0.61	B	B	B
P/E P/S DIV	10.13	16.31	0.62	B	B	B
P/E P/FC DIV	10.17	16.58	0.61	B	B	B
P/E DIV	10.51	15.81	0.66	B	B	B
Average	10.24	16.06	0.64			
Equally Weighted S&P 500 index	9.83	19.70	0.50			

Figure 28, Panel A plots the average risk-adjusted return obtained when mixing the value and momentum multi-factor with the ESG measure. In line, with the results in Table 29 an investor applying the mixed portfolio approach aiming for exposure to both the value and momentum factors can strongly improve hers risk-adjusted return by implementing the sustainability measure as a factor. Furthermore, the evidence in Table 29 indicates, that she will also improve the realized returns and the volatility that comes with these returns. The improvement is mainly attributed to the fact, that the ESG standalone portfolio performs superior to the value and momentum multi-factor in this time horizon. The story is different examining the integrated portfolio. The integrated value and momentum portfolios are not as tainted by the financial crisis as the mixed portfolio approach. They produce returns in the proximity of the equally weighted S&P 500 Index and persist to construct relatively lower volatile portfolios with better IRs. Figure 28, Panel B visualizes the materialized risk-adjusted returns for the various integrated portfolios. The average acquired risk-adjusted return by the integrated value and momentum portfolios is slightly less than the average ESG standalone portfolios. Consistent with the results in Table 30, combining value, momentum and ESG in a integrated multi-factor strategy fashion, produces on average larger risk-adjusted returns than could have been obtained using either constituent. Moreover, as mentioned earlier, the larger risk-adjusted returns can be attributed to the high returns that are accompanied by very low volatility. The evidence highly suggesting, that an investor applying the integrated portfolio approach aiming for value and momentum exposure can strongly increase his/hers return and risk profile by buying stocks with on average high value, momentum, and sustainability measures.

Figure 28: Average risk-adjusted returns across different strategies for the full sample period
This figure presents the average risk-adjusted returns denoted by the information ratios for each strategy. Panel A shows the average risk adjusted return for the mixed value and momentum strategy, the ESG standalone portfolios, and the mixed value, momentum, and ESG portfolios. The average mixed and integrated value and ategies in Table 28. The average average ESG standalone IR is computed by averaging the four sustainability measures. Lastly, the mixed and integrated value, momentum, and ESG portfolios' IR is computed by averaging the portfolios in Table 29 and Table 30.



The subsection has established several key proposition. First, the annualized compounded return by the selected standalone momentum strategy and various value strategies are slightly impaired when initiated two years prior to the financial crisis. The integrated and mixed value and momentum strategies realize lower returns relative to the equally weighted S&P 500 Index, nonetheless acceptable returns accompanied by strictly lower volatilities. Second, including the various sustainability measures to the mixed value and momentum portfolio results in 19/20 of the portfolios producing better returns, 17/20 of the portfolios realizing lower volatility, and all of the portfolios producing stronger risk-adjusted returns. Third, the integrated portfolio results show, that integrating various ESG measures resulted in 15/20 of the portfolios producing larger returns. All 20/20 portfolios realizes lower volatilities and 19/20 produce better returns. Perhaps more interestingly, the integrated portfolio manages to outperform 17/20 of the standalone ESG portfolios in terms of risk-adjusted returns, in contrast to the mixed portfolio, which all underperformed the standalone ESG portfolios. Fourth, its worth adding these results indicate, that the integrated portfolio implementation approach exhibit strong robustness.

Next, this subsection will investigate the performances of positive screening. As noted in subsection 7.1, there exists several approaches for an investor reach his/hers sustainability targets. Technically applying the various sustainability measures as factors was one approach. Another approach, is to simply exclude stocks that do not meet some pre-determined ESG threshold from the investment universe. Most arguments against are centered around lost opportunity. The process of positive screening will shrink the universe and thus the opportunity set. While investors relies on diversification, an increasing amount of investors want their investment to reflect their values and beliefs also. In this context, some investor may apply exclusionary screening, which could includes excluding sin stocks such as alcohol, tobacco, gambling, sex-related industries and weapons manufacturers, but they can also be defined by regional and societal expectations that vary widely across the globe. Ironically, a lot of the above-mentioned sins stocks actually score high sustainability scores. For example ConocoPhilips (Oil company) exhibits Environmental and Social pillar scores in the 80's and 90's; Philip Morris International Inc. (Tobacco company) has Social and Environmental scores in the 80's and 90's, and Boeing (aerospace and weapons company) has scores for all four sustainability measures in the 90's. This subsection will only subject the positive screening processes to exclude stocks that scores below the mean sustainability measure. Recall, the sustainability scores are normalized to percentages between 0 - 100. This will alter the investment universe to only include companies that demonstrate above average sustainability practices, thus shrinking the universe to half its size. The investment universe reduction is applicable to both the mixed and integrated portfolios.

Table 31 documents the performance of the mixed value and momentum multi-factor strategy on an investment universe that has been positively screened applying the various sustainability measures, respectively.

The results in Table 31 show, that the mixed value and momentum two-factor positively screening for various above average sustainability measures produce higher returns for 19/20 of the portfolio. 13/20 of the strategies realize lower volatilities, leading to 16/20 of the portfolios producing better risk-adjusted returns than they would without the sustainability screenings. The fact that 13/20 of the portfolios realize lower volatilities is somewhat unexpected, as mentioned the screened portfolios are half the size. Thus, the screened portfolios of approximately 25 stocks compared to the value and momentum portfolio of 50 stocks is supposedly exposed to relatively more idiosyncratic risk. The evidence is however consistent with the previous evidence and the narrative, that sustainable companies' stock prices are less volatile.

Comparing the mixed value and momentum screened approach to the mixed value, momentum, and sustainability approach, 13/20 of the mixed value and momentum and sustainability portfolios in Table 29 produce lower standard deviations, respectively. This indicates, that applying the sustainability measures as factors in a mixed portfolio setup, rather than applying the screening approach, lowers the risk relatively more. There may be several attributes contributing to this result. First, the mixed approach may take advantage of positive diversification effects from the additional sustainability companies added to the portfolio. This explanation is emphasized by the evidence indicating, that the sustainability measures select less risky stocks. Second, another important aspect to address is the vast difference in the two portfolio approaches. The mixed portfolios do entail a larger portfolio book, thus almost surely a lot more transactions. The mixed portfolio has three factors, namely, the value sorting factor, the momentum factor, and the sustainability sorting factor. Each factor which will select their top decile consisting of approximately 50 stocks for a total of roughly 150 stocks. While the screened value and momentum portfolio has two separate factors, selecting their top decile in the above average sustainability investment universe consisting of approximately 250 stocks, resulting in a portfolio of roughly 50 stocks.

Table 31: Performance characteristics for the mixed value and momentum portfolio applying various measures of sustainability screenings.
The table presents performance characteristics for the mixed value and momentum portfolios using various measures of sustainability screenings throughout 2006:01:01 - 2019:12:31. r denotes the annualized compounded returns, σ is the annual standard deviation and IR is the information ratio. The reported performance characteristics are computed using monthly return data. Lastly, B(r), B(σ), and B(IR) denotes the effect of sorting based on the respective four sustainability measures, W denotes a worse performance, I is an identical, B denotes a strictly better performance in comparison to the mixed value and momentum multi-factor strategy.

Panel A: ESG Screen

	r	σ	IR	B(σ)	B(IR)	
P/E DIV	9.77	18.22	0.54	B	B	B
P/B P/S DIV	10.66	19.96	0.53	B	W	B
P/E P/S DIV	10.22	18.83	0.54	B	B	B
DIV	10.16	17.86	0.57	B	B	B
P/B P/S P/FC DIV	10.45	20.71	0.50	W	B	I
Average	10.25	19.11	0.54			

Panel B: Environment Screen

	r	σ	IR	B(σ)	B(IR)	
P/E DIV	9.84	18.37	0.54	B	B	B
P/B P/S DIV	10.72	20.23	0.53	B	W	B
P/E P/S DIV	10.29	19.04	0.54	B	B	B
DIV	9.71	17.55	0.55	B	B	B
P/B P/S P/FC DIV	10.54	21.02	0.50	B	W	I
Average	10.22	19.24	0.53			

Panel C: Social Screen

	r	σ	IR	B(σ)	B(IR)	
P/E DIV	9.65	18.31	0.53	B	B	B
P/B P/S DIV	10.48	20.04	0.52	B	W	B
P/E P/S DIV	10.07	18.92	0.53	B	B	B
DIV	9.60	18.25	0.53	B	B	B
P/B P/S P/FC DIV	10.18	20.89	0.49	B	W	W
Average	10	19.28	0.52			

Panel D: Governance Screen

	r	σ	IR	B(σ)	B(IR)	
P/E DIV	9.70	18.31	0.53	B	B	B
P/B P/S DIV	10.59	20.06	0.53	B	W	B
P/E P/S DIV	10.14	18.97	0.53	B	B	B
DIV	9.43	18.48	0.51	B	B	B
P/B P/S P/FC DIV	10.44	20.79	0.50	B	W	I
Average	10.06	19.32	0.52			
Equally Weighted S&P 500 index	9.83	19.70	0.50			

Gaining perspective, Table 32 reports the average performance characteristics for the two-factor mixed value and momentum strategy, three-factor mixed value, momentum, and sustainability strategy, and lastly the the two-factor value and mixed which screens for various sustainability measures. The evidence, highly suggests the benefits for including the sustainability measure into the standalone value and momentum framework. Selecting the implementation approach the results reveal that on average the screening approach produces similar performance to the three-factor approach with approximately 100 fewer stocks.

Table 32: Average performance characteristics for the mixed value and momentum portfolios and different portfolio approaches for including sustainability.
The table presents the average performance characteristics for the mixed value and momentum portfolio denoted by the Two-factor mix, the mixed value, momentum, and sustainability portfolio is called the Three-factor mix, and the value and momentum portfolio applying sustainability as a positive screening is the Two-factor screen. The portfolios are initiated 2006:01:01 and terminate 2019:12:31. r denotes the annualized compounded returns, σ is the annual standard deviation, IR is the information ratio, lastly Stocks denotes the approximate number of stocks the strategy holds at any given time. The performance metrics are computed using monthly return data.

	r	σ	IR	Stocks
Two-factor mix	9.57	19.36	0.49	100
Three-factor mix	10.12	18.95	0.53	150
Two-factor screen	10.13	19.11	0.53	50

Table 34 reports the performance of the integrated value and momentum multi-factor strategy on an investment universe that has been positively screened applying the various sustainability measures, respectively.

The results show, that the integrated value and momentum two-factor positively screening for various above average sustainability measures produce higher returns for 13/20 of the portfolio. All 20/20 of the strategies realize lower volatilities, thus 16/20 of the portfolios result in larger risk-adjusted returns than they would have obtained without the sustainability screenings. This result, is similar to the sustainability screening approach for the mixed value and momentum portfolios. Also in line with the previous results, the integrated screening approach lowers transaction costs because the reduced investment universe results in approximate portfolio stockholding of 25.

The positive effect from the sustainability screen is much larger for the integrated model. The mixed two-factor strategy gained an average of 0.25 percent lower volatility across the 20 strategies. As highlighted by Table 34 the integrated two-factor screen strategy gains an average of 0.85 percent lower volatility across the 20 strategies. Again, quite remarkably considering the smaller stockholding and supposedly larger idiosyncratic risk and smaller diversification effects. Furthermore, the average integrated screen approach has 0.11 percentage points higher return, resulting in a 0.03 points higher risk-adjusted return. The overall evidence, highly suggests that investing in an investment universe consisting of above average sustainability companies gives rise to remarkably large performance statistics and transaction costs enhancements. Despite, the fact that the integrated screened approach

Comparing the integrated value and momentum screened approach to the integrated value, momentum, and sustainability approach, 7/20 of two-factor screen portfolios realize higher returns, 6/20 of the portfolios come bearing less risk, and 5/20 of the portfolios produce better risk-adjusted returns. Notably, the integrated three-factor strategy strongly outperforms the two-factor integrated screening approach on all performance metrics. The three-factor integrated strategy performs exceptionally strong on the Governance pillar. For example, the integrated three-factor strategy sorting on the DIV measure has a return of 10.41 percent with a standard deviation of 15.12 percent, producing the highest scoring risk-adjusted return of 0.69. This is not the case for the two-factor screening strategy. The integrated two-factor Governance screen strategy produce the poorest average risk-adjusted returns, while the ESG screen produces the highest average risk-adjusted returns.

Diving deeper into the two portfolio approaches. The integrated three-factor selects the average top 10 % scoring value, momentum, and sustainability stocks, while the integrated two-factor screen approach selects the average to 10% scoring value and momentum stocks conditioned that they are above the average sustainability measure. The three-factor strategy implicitly has the same conditioning as the two-factor screening approach; selecting the top 10% average scoring value, momentum, and Governance stocks will almost surely not result in stocks scoring below average Governance scores. Ultimately, in terms of return the evidence suggests, that the value and momentum strategy synergies well from selecting high Governance scoring stocks and not just above average. Nevertheless, taking all sustainability measures into perspective, the overall average return enhancement is 0.23 percentage points and it comes with twice the portfolio size.

Table 33: Performance characteristics for the integrated value and momentum portfolio applying various measures of sustainability screenings.
The table presents performance characteristics for the integrated value and momentum portfolios using various measures of sustainability screenings throughout 2006:01:01 - 2019:12:31. r denotes the annualized compounded returns, σ is the annual standard deviation and IR is the information ratio. The reported performance characteristics are computed using monthly return data. Lastly, B(r), B(σ), and B(IR) denotes the effect of sorting based on the respective four sustainability measures, W denotes a worse performance, I is an identical, B denotes a strictly better performance in comparison to the integrated value and momentum multi-factor strategy.

Panel A: ESG Screen						
	r	σ	IR	B(r)	B(σ)	B(IR)
DIV	10.28	15.40	0.67	B	B	B
P/S DIV	10.26	16.29	0.63	B	B	B
P/E P/S DIV	10.78	15.90	0.68	B	B	B
P/E P/FC DIV	9.91	16.73	0.59	W	B	B
P/E DIV	10.06	15.64	0.64	B	B	B
Average	10.26	15.99	0.64			
Panel B: Environmental Screen						
	r	σ	IR			
DIV	9.70	15.77	0.61	B(r)	B(σ)	B(IR)
P/S DIV	10.89	16.51	0.66	B	B	B
P/E P/S DIV	10.83	16.44	0.66	B	B	B
P/E P/FC DIV	9.79	17.07	0.57	B	B	B
P/E DIV	9.98	16.06	0.62	W	B	I
Average	10.24	16.37	0.63			
Panel C: Social Screen						
	r	σ	IR	B(r)	B(σ)	B(IR)
DIV	9.94	15.76	0.63	B	B	B
P/S DIV	9.94	16.81	0.59	B	B	B
P/E P/S DIV	10.16	16.29	0.62	B	B	B
P/E P/FC DIV	9.57	16.60	0.58	W	B	B
P/E DIV	10.01	15.70	0.64	B	B	B
Average	9.92	16.23	0.61			
Panel D: Governance Screen						
	r	σ	IR	B(r)	B(σ)	B(IR)
DIV	9.31	15.76	0.59	W	B	B
P/S DIV	9.46	16.62	0.57	W	B	I
P/E P/S DIV	10.00	16.41	0.61	B	B	B
P/E P/FC DIV	9.00	17.02	0.53	W	B	W
P/E DIV	9.31	16.12	0.58	W	B	I
Average	9.42	16.39	0.58			
Equally Weighted S&P 500 index	9.83	19.70	0.50			

Table 34: Average performance characteristics for the integrated value and momentum portfolios and different portfolio approaches for including sustainability.
The table presents the average performance characteristics for the integrated value and momentum portfolio denoted by the Two-factor int, the integrated value, momentum, and sustainability portfolio is called the Three-factor int, and the value and momentum portfolio applying sustainability as a positive screening is the Two-factor screen. The portfolios are initiated 2006:01:01 and terminate 2019:12:31. r denotes the annualized compounded returns, σ is the annual standard deviation, IR is the information ratio, lastly Stocks denotes the approximate number of stocks the strategy holds at any given time. The performance metrics are computed using monthly return data.

	r	σ	IR	Stocks
Two-factor int	9.85	17.05	0.58	50
Three-factor int	10.19	16.20	0.63	50
Two-factor screen	9.96	16.20	0.61	25

Five significant results emerge throughout this subsection. First, initiating value and momentum portfolios two years prior to the financial crisis slightly impairs the return by integrated approach and especially the mixed approach, but both realize a risk-adjusted return of 0.58 and 0.49, respectively, compared to the equally weighted S&P 500 Index that obtains an IR of 0.50. Second, including the various sustainability measure to the integrated and mixed portfolio on average yields an improved performance across all metrics with an IR of 0.63 and 0.53, for a 8 percent IR and 9 percent increase, respectively. Third, on average the mixed two-factor screen approach produces a return accompanied by volatility, producing an IR of 0.53. Thus, the two-factor mixed approach outperforms the mixed value and momentum portfolio and produces similar performance to the mixed value, momentum, and sustainability portfolios on all performance multiples while lowering the stockholding at time t by approximately 100 and 50 stocks, respectively. Forth, on average the relatively smaller integrated two-factor screen portfolio produces higher return than the value and momentum portfolio along with a large volatility reduction of 0.85 percent, leading to a superior risk-adjusted return and a stockholding reduction of 25. Fifth, on average the integrating value, momentum, and various sustainability measures lead to overall highest risk-adjusted return of 0.63 at a stockholding of 50. Screening the sustainability measure, rather than integrating it, leads to very close performance and a stockholding of 25. As a portfolio manager both cases may be very interesting depending on client demands. For example, albeit the fact that the integrated sustainability approach has an approximate stockholding of 50 stocks it constructs a portfolio of sustainability leading companies, whereas, the screened approach constructs a relatively smaller portfolio of above average sustainability.

7.6 Risks

Table 35 presents the performance and risk characteristics for all of the value and momentum strategies employing the ESG measure for various portfolio approaches. Thereby, excluding the strategies sorting on the Environmental, Social, and Governance measures. The deliberate choice examining the ESG measure exclusively is based on simplicity and the fact that the ESG measure takes all the three measures into account making it a well rounded measure for sustainability. In addition to the previous applied performance multiples, this subsection includes two new risk measures, namely value at risk (VaR(·)) and the expected short fall (ES(·)).

Several key points emerge from Table 35. First, within each respective approach, the two-factor, three-factor, and two-factor screen portfolio approach, on average with respect to risk-adjusted returns, the integrated approaches outperform the mixed portfolio approach by 17, 21, and 19 percent, respectively. As indicated by the returns and information ratios, this outperformance is highly driven by the lower volatilities.

Second, consistent with the narrative of the integrated approach constructing less risky portfolios, the realized excess kurtosises for the same three approaches, in the same order, are 26, 48, and 52 percent higher for the mixed approach. All of the portfolios in Table 35 are moderately negative skewnesses indicating that the probability mass is concentrated in the left tails (negative ends) of the return distributions. coupling the negative skewness with the higher kurtosises by the mixed approaches suggests, that the mixed portfolios exhibit a large probability of realizing relatively larger losses than the integrated approach. This argument is consistent examining the on average realized relatively larger drawdowns and drawdown durations for the mixed portfolio approach across the three different portfolio approaches.

Third, quantifying the loss in left tail, the VaR(95%) measure denotes the percentage amount the portfolios loses with a probability of 0.05 percent any given month. Again, emphasizing that the portfolio initiate two years prior to the financial crisis the vast majority of the mixed approached strategies exhibit VaR(95%) measures similar to the equally weighted S&P 500 Index. In line with the drawdowns and the third and fourth moments, the on average realized VaR(95%) values are relatively smaller for the integrated approach compared to the mixed approach.

Fourth, on average the mixed two-factor screen realizes a VaR(95%) of 9.11 percent which is relatively smaller than the mixed three-factor of 9.52 percent, while the on average expected percentage loss below the 0.05 quantile is 13.38 percent (highlighted by the ES(95%) measure) which is relatively larger than the 13.21 percent realized by the mixed-three factor. The same result is true for the integrated three-factor and the integrated two-factor screen. This evidence suggests, that albeit the two-factor screen approaches may lose less in a 0.05 probability loss situation they can expect to lose relatively more in a below 0.05 probability loss situation compared to the three-factor approaches. In other words, in the very extreme end of the left tail - below the 0.05 quantile - the two-factor screen approaches realize larger losses than the three-factor approaches. This is in line with the on average 3.54 and 9.50 percentage points larger maximum drawdowns for the mixed and integrated approach, respectively. One may add, that the on average 9.50 percentage points larger maximum drawdown for the integrated two-factor is highly understated by the third and fourth moment as well as the VaR(95%) and ES(95%) measures.

Table 35: Performance and risk characteristics for all implementation approaches of the value, momentum, and ESG portfolios in the period from 2006:01:01 - 2019:12:31.
The table presents performance and risk characteristics for all mixed and integrated implemented portfolio approaches of the value, momentum and ESG portfolios starting 2006:01:01 terminating 2019:12:31. r denotes the annualized compounded return, σ is the annual standard deviation, IR is the information ratio, M(%) denotes the worst realized maximum drawdown in percentage. L(m) denotes the time measured in months it took from peak to valley to initial peak value, R(m) is the recovery time which denotes the months it takes from valley to new initial peak value. SK(m) denotes the monthly skewness, K(m) is the monthly excess kurtosis, which are both measured in months. Lastly the VaR(95%) is the 95% monthly historical Value at Risk and ES is the 95% monthly historical Expected Shortfall. All computations are conducted on monthly return data. The average L(m) and R(m) is rounded to the nearest integer.

Strategy	r	σ	IR	M(%)	L(m)	R(m)	SK(m)	K(m)	VaR(95%)	ES(95%)
Mixed two-factor										
P/E DIV	9.44	18.42	0.51	57.86	45	21	-0.60	3.51	10.77	13.40
P/B P/S DIV	10.10	19.69	0.51	60.38	45	24	-0.97	4.36	10.19	13.78
P/E P/S DIV	9.02	19.42	0.46	58.51	45	21	-0.83	3.55	9.49	13.17
DIV	9.34	18.96	0.49	59.80	42	21	-0.81	3.89	11.11	13.97
P/B P/S P/FC DIV	9.96	20.32	0.50	59.20	45	24	-0.77	3.24	10.59	13.59
On Average	9.57	17.05	0.49	59.15	44	22	-0.80	3.71	10.43	13.57
Integrated two-factor										
DIV	9.72	16.73	0.58	53.29	45	24	-0.80	2.92	10.12	13.56
P/S DIV	9.91	17.32	0.57	53.91	45	24	-0.76	2.96	9.85	12.40
P/E P/S DIV	9.98	17.06	0.59	56.13	57	36	-0.80	3.09	10.65	13.00
P/E P/FC DIV	10.00	17.45	0.57	53.75	45	24	-0.78	2.70	10.25	12.26
P/E DIV	9.66	16.70	0.58	55.01	46	25	-0.79	3.04	10.33	12.71
On Average	9.85	17.05	0.58	54.42	48	27	-0.79	2.94	10.24	12.79
Sustainability standalone										
ESG	11.19	18.77	0.65	49.70	29	12	-0.64	4.43	9.25	12.65
Environmental	11.25	19.96	0.56	51.18	29	12	-0.55	4.08	9.26	12.93
Social	9.73	18.07	0.54	54.37	38	29	-0.61	4.24	8.93	11.88
Governance	11.87	18.15	0.65	46.30	30	9	-0.53	4.64	8.88	12.08
On Average	11.01	14.99	0.60	50.39	32	16	-0.58	4.34	9.08	12.38
Mixed three-factor										
P/E DIV	9.95	18.45	0.54	56.32	38	22	-0.87	3.98	8.92	12.88
P/B P/S DIV	10.36	19.10	0.54	55.75	34	14	-0.87	4.40	9.88	13.36
P/E P/S DIV	10.12	18.77	0.54	55.08	42	22	-0.85	3.91	9.50	13.02
DIV	9.91	18.73	0.53	56.23	42	22	-0.99	4.40	9.12	13.26
P/B P/S P/FC DIV	10.17	19.60	0.52	54.66	42	22	-0.79	4.01	10.20	13.52
On Average	10.10	18.93	0.53	55.61	40	20	-0.87	4.14	9.52	13.21
Integrated three-factor										
DIV	10.09	15.01	0.67	43.67	34	14	-1.03	3.14	7.30	10.45
P/S DIV	10.51	16.23	0.65	47.07	34	14	-0.92	2.39	8.14	11.08
P/E P/S DIV	10.49	16.13	0.65	47.04	42	22	-0.97	2.75	8.12	11.14
P/E P/FC DIV	9.95	16.60	0.60	47.91	38	22	-0.96	2.80	8.35	11.58
P/E DIV	10.46	15.55	0.67	44.86	38	22	-0.96	2.87	7.94	10.77
On Average	10.30	16.10	0.65	46.11	37	19	-0.97	2.79	7.97	11.00
Equally weighted S&P 500 Index	9.80	19.70	0.50	57.39	42	21	-0.76	4.28	10.78	13.36

Table 35 – *Continued*

Strategy	r	σ	IR	M(%)	L(m)	R(m)	SK(m)	K(m)	VaR(95%)	ES(95%)
Mixed two-factor screen										
P/E DIV	9.77	18.22	0.54	57.86	45	21	-0.99	4.15	7.54	12.79
P/B P/S DIV	10.66	19.96	0.53	60.38	45	24	-0.94	4.94	9.43	14.02
P/E P/S DIV	10.22	18.83	0.54	58.51	45	21	-0.86	3.61	8.65	13.03
DIV	10.16	17.86	0.57	59.80	42	21	-1.14	4.84	9.10	12.82
P/B P/S P/FC DIV	10.45	20.71	0.50	59.20	45	24	-0.77	3.86	10.86	14.26
On Average	10.23	19.11	0.54	59.15	45	22	-0.94	4.28	9.11	13.38
Integrated two-factor screen										
DIV	10.28	15.40	0.67	56.32	38	22	-1.20	3.86	7.17	10.85
P/S DIV	10.26	16.29	0.63	55.75	34	14	-0.88	2.00	8.92	11.10
P/E P/S DIV	10.78	15.90	0.68	55.08	42	22	-0.88	2.23	7.45	10.66
P/E P/FC DIV	9.91	16.73	0.59	56.23	42	22	-0.96	2.99	8.01	11.49
P/E DIV	10.06	15.64	0.63	54.66	42	22	-1.05	2.99	7.73	10.88
On Average	10.27	15.99	0.64	55.61	40	20	-0.99	2.81	7.85	11.00
Equally weighted S&P 500 Index	9.80	19.70	0.50	57.39	42	21	-0.76	4.28	10.78	13.36

7.7 ESG Conclusions

A spark increasing amount of attention by retail and institutional investors has been directed towards sustainable investments. As highlighted by Subsection 7.4 using sustainability measures as a factors and buying stocks scoring high sustainability scores outperforms buying stocks with low sustainability scores as well as the equally weighted S&P 500 Index (with the exception of the Social measure). Compelled by the performance of these sustainability strategies, subsection 7.5 proceeds by employing mixed, integrated, and screening approaches to the various sustainability measures as a value and momentum investor. The evidence highly indicates, that on average the mixed and integrated value and momentum investor benefits from including various sustainability measures. Especially, the integrated approach displays robust and pervasive performance across all sustainability sortings, outperforming all the integrated value and momentum portfolios and 17/20 of the standalone sustainability strategies.

Employing the sustainability measures as positive screening for above average scoring companies for various sustainability measures produces on average stronger return, standard deviation, and risk-adjusted return for the mixed and integrated value momentum approach compared to neglecting the sustainability measures. Comparing the options of employing the mixed value, momentum, and sustainability approach (three-factor approach) against the mixed value and momentum approach on universe screened by various sustainability measures (two-factor screen approach), the two-factor screen approach on average arguably outperforms.

On average, the mixed two-factor screen produces a return of 10.13 percent 0.01 percentage points above the mixed three-factor, with volatility of 19.11 percent 0.16 percentage points below the mixed three-factor, leading to both strategies producing a risk-adjusted return of 0.53. However, implied by the two vastly different portfolio approaches the two-factor screen requires approximately 50 stocks whereas the mixed three-factor requires roughly 150 stocks. Examining this optionality for the integrated approach, on average the integrated three-factor approach harvests an addition 0.23 percentage points of return at the exact same volatility, leading to a IR of 0.63, which is slightly superior to the two-factor screen approach. In contrast to the large portfolio differences for the mixed approach, this comes at merely 25 additional stocks.

Putting all risk measures into perspective and the end performance, the on average integrated approaches perform superior to the mixed approaches. On average the integrated approaches produce acceptable or higher returns at a much lower volatility while exhibiting lower potential tail risk. Needless to say at this point all the evidence highly suggests, a value and momentum investor using the equally weighted S&P 500 Index as benchmark may materialize strong benefits from including sustainability into hers portfolio despite her portfolio approach preference. These results, may be attributed to the sustainability measures' ability to capture stocks exhibiting strong intangible assets which also plays a vital role in a companies valuation and future growth. Another factor could be that the large increasing interest in high scoring sustainability companies drives stock prices up.

8 Implementation Costs

Many of the previously examined strategies seem quite compelling on paper, however, the results in the real world can often be much less convincing and perhaps even quite disappointing. Transaction costs and other related practical hurdles can have a considerable impact on performance. According to an article by Roboco Asset Management, factor-based strategies tend to lead to higher turnover than passive market-weighted strategies, which is reasoned by the more frequent rebalancing in order to perpetuate the exposures to the different factor premiums. For factor-based strategies it is crucial to take implementation costs into considerations. According to a paper by the EDHEC-Risk Institute, estimating these transaction costs is rather complicated and remains a challenge. They find that when replicating a number of generic factor based indices, the costs can vary significantly depending on simply the size of the investment universe.

When implementing a trading strategy, two types of costs need to be considered. The transaction costs in the form of commissions and other direct costs on each trade. The other is funding costs, which is for strategies that either make use of leverage or borrow shares in order to short them; however, since this is not applicable for the strategies implemented in this paper, it will refrain form delving further into this. Looking into transaction costs these are made up of several direct and indirect costs. The direct costs include commission to broker-dealers and are typically either a fixed fee per share traded or a small percentage of the total value traded, typically noted in basis points. Lasse Heje Pedersen draws out three stylized types of transaction costs. The first is increasing transaction costs as a function of trade size. This is the most common approach for liquid electronic markets where the commissions and bid-ask spreads are small for professional traders. Here, the direct costs are not the major drag on the investment, but rather the indirect costs incurred as a consequence of the small amount of shares that can be traded at the bid or ask price for a large investor such as a hedge fund. Thus, the larger the position traded, the more it might move the prices indirectly increasing the costs. An option to avoid these kinds of transaction costs would be to divide the trade into several smaller sized blocks and patiently trade them over time. The second type of transaction cost is the constant transaction cost also called the proportional transaction cost due to the average cost being constant. This means that the transaction cost neither increases or decreases with the size of the trade. In practice this means that the entire trade can be traded at the bid or ask price and sometimes as a result of negotiation, at the mid-price between the bid and ask price. This can happen if the market has a large tick size, which lets market makers earn a lot from the bid-ask spread also resulting in more market makers offering trades at these prices. An example of this is the market for exchange-traded funds with very liquid underlying securities such as the *EUROSTOXX 50 Index* or S&P 500 Index, which is completely dominated by market makers. The last stylized transaction cost mentioned is the decreasing transaction cost which is mainly applicable in the over-the-counter (OTC) markets. In these markets, small orders tend to be more expensive than large orders, since the manual search for counterpart takes the same time for a small order as a large order. Thus not necessarily making the small order worth the time spent if it were to cost the same as the large. As a result the percentage cost for small orders are typically higher than for large orders. In this market it is best to trade in sizes that are worthwhile for the dealer. Considering the indirect costs, these consists of the bid-ask spread and the market impact, the trade incurs on the market price. The bid-ask spread is the spread between the bid, the price a buyer is willing to purchase the specific stock for and the ask price, the price a seller is willing to sell the same stock at. As the ask price will be higher than the bid price, naturally, a spread will exist between these two. How the trades are executed can have a significant impact on the costs associated with the transaction. As this paper solely makes use of the equities asset class, it is fair to assume that these are mostly traded on liquid electronic markets and not illiquid OTC stocks that require a more manual execution process as mentioned above. Trading algorithms are a highly utilized tool that minimize market impact dramatically if applied smartly. With the use of these, large investors such as hedge funds and other asset managers can move size in and out of equities equivalent of entire day's volume without moving the price. Of course, this requires the right conditions. These kind of transactions do not occur in what would be known as the "lit" market, where the market traders can observe by looking at the order depth on the bid and ask side. These trades cross on what is called dark venues or more commonly named dark pools. These are trading venues only accessible for sophisticated professional traders such as specialized broker-dealers and market-makers. These venues differ from the lit venues in the sense that no information such as prices and sizes are visible. Traders can place an order that is solely active on the dark venues and select certain parameters so that it only crosses with a minimum size and price, such that there is no impact in the market. Further explanation of these kinds of algorithms are beyond the scope of this paper, however, understanding that these tools exist and can play a major role in minimizing the market impact when rebalancing portfolios is worth noting.

In the table below three implementation costs assumptions have been implemented on an ESG integrated portfolio, an ESG mixed portfolio, an ESG screened integrated portfolio and an ESG screened mixed portfolio in order to get an impression of whether a real-world implementation would be possible and profitable. The implementations are based on three cost scenarios each having assumed different costs in order to attain a better idea of what tolerance of costs the portfolios can handle, before the performance is dragged completely by the transaction costs. In order to get a sense of the magnitude of transaction costs for professional traders this paper will make use of the transaction costs estimates provided by Frazzini, Israel, Moskowitz (2012). They suggest three costs estimates each depending on the size of the trade compared to the daily volume. For a sample running from 1998 to 2011 they find a value weighted average transaction cost of 9.5 bps.[11] For large orders, that constitute more than one percent of the daily volume, the average transaction costs is 27 bps. Finally, for very large orders that constitute more than 10% of the daily volume the transaction costs end around 40 bps.

The transaction costs are first incurred at the initiation of the portfolio when all the stocks in the portfolio are purchased for the first time. These are then held until the next rebalance of the portfolio, where both transaction costs on new stocks entering the portfolio are added, as well as, transaction costs for stocks leaving the portfolio. Stocks that neither leave or enter, naturally have no transaction costs. For example, for a portfolio assuming transaction costs of 10 bps, these are incurred for the entire portfolio when it is created. When the portfolio then rebalances, a transaction costs of 10 bps are incurred on all the stocks leaving the portfolio, the cost of selling them, and likewise a 10 bps costs on new stocks entering the portfolio. For stocks remaining in the portfolio these are simply held onto and add no further costs. In the case of the mixed portfolio this holds for stocks migrating from one factor to another.

Table 36 presents the top five selected integrated and mixed three-factor and two-factor screen approaches, receptively. In line with he results in subsection 7.5, these portfolio run from 2006:01:01 - 2019:12:31. Observing Table 36 for the mixed three-factor portfolio all the strategies with a cost of 40 bps underperform slightly apart from the strategy consisting of the P/B, P/S and DIV measures, which manages to slightly overperform. For a cost of 27 bps for the mixed strategies only one of the strategies outperform the benchmark. Assuming a cost of 10 bps for mixed only 3/5 mixed strategies overperform the benchmark.

[11] This gives higher weight to large trades.

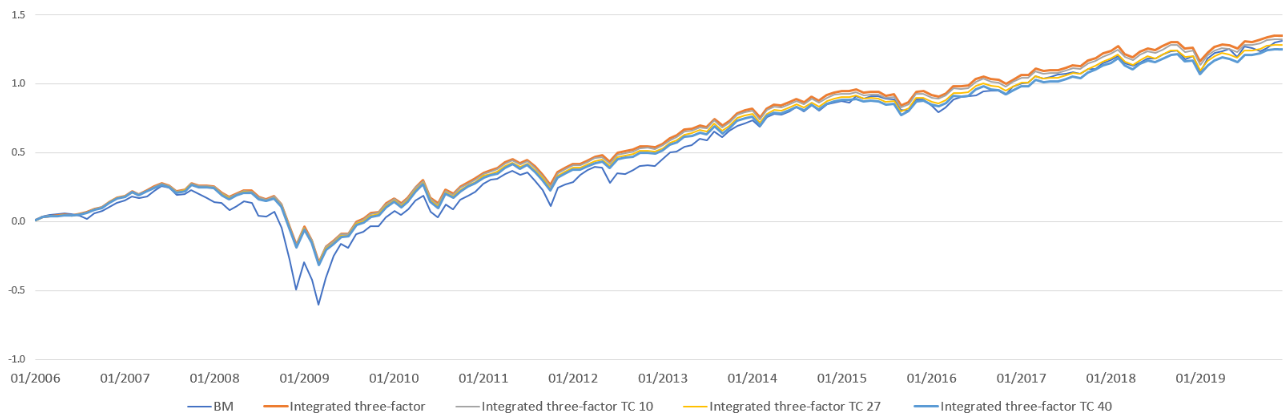
Table 36: Performance characteristics for the integrated value, momentum, and sustainability portfolios that run from 2006:01:01 - 2019:12:31.
The table presents performance characteristics with and without transaction cost, bps is the basis points it costs to buy or sell a stock. The performance is the annualized compounded return.

Panel ESG				
Mixed three-factor	0 bps	10 bps	27 bps	40 bps
P/E DIV	9.95	9.82	9.61	9.44
P/B P/S DIV	10.36	10.23	10.02	9.86
P/E P/S DIV	10.12	9.99	9.78	9.61
DIV	9.91	9.79	9.57	9.41
P/B P/S P/FC DIV	10.17	10.03	9.81	9.64
Integrated three-factor				
DIV	10.09	9.89	9.56	9.31
P/S DIV	10.51	10.31	9.97	9.71
P/E P/S DIV	10.49	10.29	9.95	9.69
P/E P/FC DIV	9.95	9.74	9.34	9.13
P/E DIV	10.46	10.26	9.92	9.67
Mixed two-factor screen				
P/E DIV	9.77	9.63	9.38	9.19
P/B P/S DIV	10.66	10.52	10.27	10.09
P/E P/S DIV	10.22	10.07	9.83	9.64
DIV	10.16	9.98	9.68	9.45
P/B P/S P/FC DIV	10.45	10.29	10.02	9.82
Integrated two-factor screen				
DIV	10.28	10.05	9.66	9.37
P/S DIV	10.26	10.03	9.64	9.34
P/E P/S DIV	10.78	10.54	10.14	9.84
P/E P/FC DIV	9.91	9.67	9.27	8.97
P/E DIV	10.06	9.83	9.44	9.15
Equally Weighted S&P 500 index	9.83			

Moving on to the integrated three-factor portfolios, a somewhat similar pattern can be observed. It becomes clear that the implementation cost of 40 bps has a heavy drag on all the strategies as all of the strategies underperform the benchmark. When implementing a cost of 27 bps the results are slightly different as 3/4 of integrated strategies outperform the benchmark. For a implementation cost of 10 bps 4/5integrated strategies overperform the benchmark with three of them delivering close to 0.50 percentage points more than the benchmark which makes the integrated strategies performance slightly better than that of the mixed strategies. Investigating, the portfolio that utilize ESG screening the mixed two-factor screen strategies are also clearly affected by a cost of 40 bps. All the strategies underperform the benchmark apart from the P/B, P/S, and DIV strategy, that still delivers superior return. For a cost assumption of 27 bps one more strategy joins the ”overperformers”, making 2/5 of strategies producing returns above the benchmark. For a cost of 10 bps 4/5 of the strategies beat the benchmark, which is better than both of the three-factor portfolios. The integrated two-factor screen has zero portfolios outperform the equally weighted S&P 500 Index at 40 bps, 1/5 at 27 bps, and 4/5 at 10 bps.

However, analyzing the relative transaction cost robustness for each portfolio approach the story is different. Recall, that the mixed three-factor strategy holds approximately 150 stocks, while the mixed two-factor screen strategy holds 50 stocks at any given time. A lower stockholding will almost surely lead to a larger turnover rate, which is reflected directly in the incurred transaction costs. The same is true for the integrated approaches, the integrated three-factor has a stockholding of 50 and the integrated two-factor screen has a stockholding of 25 stocks. On average, the percentage decrease for the mixed three-factor at 40 bps as opposed to 0 bps is 5.05 percent, while this number is 6.00 for the mixed two-factor screen. The integrated three-factor with a stockholding of 50 stocks realizes a percentage decrease of 7.75 percent comparing 40 bps to 0 bps, and the integrated two-factor screen with a stockholding of 25 stocks realizes a large decrease of 9.01 percent. In regards to the impact of including transactions costs, the evidence implies that the integrated two-factor screen gross returns overstate real-life implementation, while the mixed three-factor shows the strongest robustness. Lastly, Figure 29 presents a visualization of the transaction cost implementation on a discretionary integrated three-factor strategy for the various cost assumptions. Despite these added transaction costs, the portfolio's manage to deliver excess return compared to the benchmark and the low volatility is persistent.

Figure 29: Integrated three-factor model with TC



9 Conclusion

This paper provides comprehensive evidence on the return premium in value and momentum strategies employed on the S&P 500 Index. Firstly, a variety of value strategies that involve buying relatively cheap (value) stocks have outperformed glamour strategies over the period of 1996:06:30 - 2019:12:31. This indicates, that the market has consistently overestimated future growth rates of glamour stocks relative to value stocks. Moreover, the empirical analysis suggests, that on average, the more sophisticated value strategies sorting on multiple dimensions, produces higher risk-adjusted returns or at worst, results in-between its constituents. The lion share of all the value strategies outperform the equally weighted S&P 500 Index in terms of return, standard deviation, and risk-adjusted return. Applying maximum realized drawdown as a risk measure, the value strategies experience severe losses in the magnitude of 60 to 70 percent. Secondly, trading strategies that buy past winners produce statistically significant returns and high risk-adjusted returns. For example, the strategy which is studied in most detail, selects stocks based on their compounded return over the past 6 months and holds these stocks the following 6 months, realizes a yearly return of 12.37 percent. Additional evidence indicates, that the momentum factor has a price appreciation cycle of 9-15 months. Furthermore, still considering a long-style only universe, this paper finds evidence of a strong correlation structure between value and momentum strategies. Based on the strong correlation structure and the appealing argument of selecting stocks with strong exposure to both the value and momentum factor, this paper proceeds by employing both mixed and integrated portfolio implementation approaches. The findings suggests that employing momentum strategies selecting stocks based on the past 6 months and holding these stocks for the following 9 months, while selecting (mixed approach) and sorting (integrated approach) on various value strategies with the same holding period, produces strong performances. A vast majority of the mixed strategies materialize returns and risk-adjusted returns superior to respective standalone value and momentum strategies. The integrated approach produces acceptable returns, in-between its standalone constituents, and much lower volatilities leading to superior risk-adjusted returns. Further analysis indicates that combining the value and momentum to one portfolio, rather than holding two separate value and momentum portfolios, produces strong potential trading efficiencies reducing the overall turnover rates. This efficiency is strongest for the momentum with only slightly better results for the integrated approach.

Having established the prevalent and strong performance by the standalone value and momentum factors, as well as their joint returns, this paper aims to meet the increasing demand and attention directed towards ESG. This is done by both implementing a screening approach and treating the various sustainability measures as factors on a technical level, which reveals two very interesting results. To start with, trading strategies buying stocks with high sustainability measures on the overall ESG score, Environmental, Social or Governance scores, strongly outperform stocks with low sustainability scores. In addition, selecting stocks with high scores on the four measures of sustainability provide relatively lower volatilities with the Social score exhibiting 2.51 percentage points lower volatility. This evidence suggests that high sustainability measures may possess strong explanatory power for volatility. In line with this, all sustainability measures experience lower maximum drawdowns.

The most prominent finding in this paper is, that on average, incorporating the various sustainability measures in the value and momentum multi-factor strategy, reveals a strong and pervasive outperformance across all portfolio implementation approaches. On average, both the mixed value, momentum, and sustainability strategies as well as the mixed value, momentum and sustainability screen strategies strongly outperform the mixed value and momentum strategies along with the equally weighted S&P 500 Index. This strong relation is also evident for the integrated approaches. One interpretation of these results is that the stronger performance may be attributed to the sustainability measures' ability to capture stocks exhibiting strong intangible assets, which also plays a vital role in a companies valuation and future growth. Another element could be that the increased interest and demand for high scoring sustainability companies, drives the stock price up.

Further investigation suggests that both the mixed and integrated three-factor strategies experience large tail risk reductions when incorporating the sustainability measures. For example, on average the mixed and integrated two-factor strategies realize a maximum drawdown of 59.15 and 54.42 percent, while the mixed and integrated experience relatively lower maximum drawdowns of 55.61 and 46.11 percent.

Finally, the gross returns of the 5 best performing strategies across all portfolio approaches are tested for robustness against transaction costs. Incurring relatively conservative transaction costs of 27 and 10 bps, 7/19 and 15/19 of the strategies that already produce alpha persists to outperform the equally weighted S&P 500 Index.

10 Perspectivation

Even though the results presented in this thesis appear very compelling, a paper like this typically opens up for more questions than it answers.

An extension of the research done in this paper could be to examine if the same portfolio approaches consisting of value, momentum combine with ESG can likewise attain remarkable and attractive returns on other financial markets than that of the equally weighted S&P 500 Index. This could for instance be any of the other American stock indices including or on the European market where ESG investing is a much more utilized approach. Another interesting conjecture to research, is whether the results are prevalent across different asset classes. Furthermore, the research could be well complemented by including the possibility of shorting stocks. Implementing this, funding costs would have to be considered too, which also is out of scope of this paper.

In addition to the ideas above, practitioners would be very interested in a more thorough examination of transaction costs and market impact than what was found sufficient for the needs in this paper. Here, one could take into consideration the steep decline in direct transaction costs such as commission due to much more competition and more efficient trading systems. However, should one implement this on a much less liquid market, this would perhaps be offset by the higher indirect costs such as the bid-ask spread.

11 Bibliography

11.1 References

Asness, C. (1995), *“The Power of Past Stock Returns to Explain Future Stock Returns”*, Working paper, Goldman Sachs Asset Management.

Asness, C. (1997), *“The Interaction of Value and Momentum Strategies”*, Financial Analysts Journal, March/April 1997.

Asness, C., Moskowitz, T., Pedersen, L. (2013), *Value and Momentumer Everywhere.*, The Journal of Finance, Volume 68, Issue 3.

Asness, Liew, Pedersen (2016),*”Deep Value”*, AQR working paper.

Barber, B., Lyon, J. (1997). Firm Size, Book-to-Market ratio, and Security Returns: A Holdout Sample of Financial Firms. Journal of Finance, Vol. 52, pp. 875-883.

Barberis, N., Thaler, R. (2002). *A Survey of Behavioral Finance.* Working paper, National Bureau of Economic Research.

Buehler, R., Griffin, D., Ross, R. (1994). *Exploring the ”Planning Fallacy”: Why People Underestimate Their Task Completion Times.* Journal of Personality and Social Psychology, Vol. 67, No. 3.366-381

Cai, J. (1997). *Glamour and Value Strategies on the Tokyo Stock Exchange.* Journal of Business, Finance and Accounting, Vol. 24, pp. 1291-1310.

Carhart, M. (1997), *“On Persistence in Mutual Fund Performance”*, the Journal of Finance 52(1), 57-82.

Chan, L. K. C., N. Jegadeesh, and J. Lakonishok, (1996), *”Momentum Strategies”*, Journal of finance, Vol. 51.

David Ruppert & David S. Matteson. (2015) *Statistics and Data analysis for Financial Engingeering*, 2nd ed.

Davis, J. L., Fama, E. F., French, K. R. (2000). *Characteristics, Covariances, and Average Returns: 1929 to 1997*, Journal of Finance, pp. 389-406.

De Bondt, Werner F. M. and Richard Thaler, (1985),*”Does the stock market overreact?”*, Journal of finance Vol. 40, pp. 793-805.

De Bondt, Werner F. M. and Richard Thaler, (1985),*”Further Evidence on Investor Overreaction and Stock Market Seasonality”*, Journal of finance Vol. 42, pp. 557-581.

Esakia, M., Goltz F., Sivasubramanian S., Ulahel J., *Smart Beta Replication Costs*, EDHEC-Risk

Institute, 2017.

Fama, E. (1963). *"Mandelbrot and the Stable Paretian Hypothesis"*, Journal of Business, Vol. 36, pp. 394-419.

Fama, Eugene and Kenneth French, (1988),*"Permanent and temporary components of stock prices"*, Journal of Political Economy 96, 246-273.

Fama, E., French, K. (1992). *The Cross-Section of Expected Stock Returns.* The Journal of Finance, vol. 47, pp. 427-465.

Fama, Eugene F., and Kenneth R. French (1993), *"Common Risk Factors in the Returns on Stock and Bonds"*, Journal of Financial Economics 33, 3-56.

Fama, E. French, K., (2012). *"Size, value, and momentum in international stock returns"*, Journal of Financial Economics, Elsevier, vol. 105(3), pp. 457-472.

Fitzgibbons, S. Hect, McQuinn, Serban (2017). *Portfolio Construction Matters: A Simple Example Using Value and Momentum themes.*

Fitzgibbons, S., Friedman, J, . Pomorski, L, . Serban, L, . (2017). *Long-Only Style Investing: Don't Just Mix, Integrate*, Journal of Investing, Volume 26, Number 4.

Frazzini, A., Israel, I., Moskowitz, T., (2012). *Trading Costs of Asset Pricing Anomalies*, Fama-Miller Working Paper; Chicago Booth Research Paper No. 14-05.

Gombers, P. Ishii, J., and Metrick, A. (2003). *Corporate governance and equity prices*

Grinblatt, M., Titman, and R. Wermers (1995),*"Momentum Strategies, Portfolio Performances, and Herding: A Study of Mutual Fund Behavior"*, American Economic Review, vol. 85, 1088-1105.

Hong, Harrison, Terence Lim, and Jeremy C. Stein (2000), *"Bad News Travels Slowly: Size, Analyst Coverage, and the Profitability of Momentum Strategies"*, Journal of Finance, 55(1): 265-295.

Ilmanen, A. (2011). *Expected Returns: An Investor's Guide to Harvesting Market Rewards.* Wiley.

J. Bradford de Long, Andrei Shleifer, Lawrence H. Summers, Robert J. Waldmann (1990), *Positive Feedback Investment Strategies and Destabilizing Ration Speculation*, The Journal of Finance Vol. 45, pp. 379-395.

Jegadeesh, N. (1990). Evidence of Predictable Behavior of Security Returns. Journal of Finance(45), pp. 881-898.

Jegadeesh, N., Titman, S., (1993) *"Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency"*, The Journal of Finance , Vol. 48 No.1. (Mar., 1993), pp. 65-91.

Jennifer Bender, Remy Briand, Dimitris Melas, Raman Aylur Subramnian. (2013) *Foundations of*

Factor Investing, Research insight by MSCI.

Lakonishok, J., Schleifer, A., Vishny, R. (1994). Contrarian Investment, Extrapolation and Risk. Journal of Finance, vol. 49, pp. 1541-1578.

Kaiser, L. (2018) *ESG Integration: Value, Growth and Momentum*

Lee, D. and Faff, R. (2009). *Corporate hypothesis testing with the Sharpe ratio*

Porta, L., Lakonishok, J., Shleifer, A., & Vishny, R., (1997). Good News for Value Stocks: Further Evidence on Market Efficiency. *Journal of Finance, Vol. LV(5), pp. 2017-2069.*

Pedersen, LH., (2015), *Efficiently inefficient*, 1st ed.

Lo, A. W., MacKinaly, A. C., (1990),*"When Are Contrarian Profits Due to Stock Market Overreaction?"*, Review of Financial Studies(3), pp. 175-205.

Miskolczi, P., (2017), *"NOTE ON SIMPLE AND LOGARITHMIC RETURN"*, ABSTRACT Vol. 11. number 1-2, 127-136.

Westfall, P., (2014), *"Kurtosis as peakedness, 1905-2014. R.I.P."*.

Risager, O. (2013). Value Investing in World Equity. København: McGraw-Hill

Rouwenhorst, K.G., (1998), *"International momentum strategies"*, Journal of Finance 53, 267–284.

Rouwenhorst, K.G., (1999), *"Local return factors and turnover in emerging stock markets"*, Journal of Finance 54, 1439–1464.

Subramanian, S., Kabra, M., Chopra, S., Yeo, J,. Strzelinska, P., Huang, L., Bank of America Merill Lynch (2019), *ESG from A to Z: a global primer*

Piani, V. et al, (2018), *A Practical Guide To Active Investing In Listed Equities*, Principles For Responsible Investing, United Nations

Vayanos, Dimitri and Paul Woolley, (2011), “An institutional theory of momentum and reversal”, London School of Economics (LSE), Working paper.

Zoran Ivanovski, Toni Stojanovski, Zoran Narasanov, (2015), *"Volatility and Kurtosis of daily stock returns at MSE"*, UTMS Journal of Economics 6 (2), 209-221

11.2 Links

Morgan Stanley Sustainable Signals:

https://www.morganstanley.com/content/dam/msdotcom/en/assets/pdfs/Morgan_Stanley_ISI_Sustainable_Signals.pdf

Raju, V,. *You Are Not Doing ESG Investing (2020)* Morgan Stanley

https://www.morganstanley.com/im/en-us/individual-investor/insights/articles/you-are-not-doing

html?cid=SM_CORP__LINKEDIN_morgan-stanley-investment-management_20200310&linkId=84049274&

fbclid=IwAR1iWcQRqQHnsL5ksHwAK3Dp_5Vcdujvo3ZaZ1GTBNA_pS8963RSIe2Zi1U

Roboco Asset management: Factor Investing Challenges Limiting Turnover:

https://www.robeco.com/en/insights/2017/05/factor-investing-challenges-limiting-turnover.

html

Conecomm 2006 Millennial Cause Study:

https://www.conecomm.com/research-blog/2006-millennial-cause-study

Research Affiliates: Food For Thought Integrating vs Mixing:

https://www.researchaffiliates.com/en_us/publications/articles/673-food-for-thought-integratin

html?fbclid=IwAR2Tas96pYTvDNULGq0Xm_J-vzrSZdGqSCP2tNj85hv3ymgIbli8TkVB2zA

UBS Asset Management: ESG: a game changer?

https://www.ubs.com/global/en/asset-management/insights/sustainable-and-impact-investing/

2020/esg-a-game-changer.html

Refinitiv: ESG Scores Methodology:

https://www.refinitiv.com/content/dam/marketing/en_us/documents/methodology/esg-scores-methodo

pdf