Choice of Social Security System

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Abstract

This paper examines the impact of unemployment insurance and social assistance on wages, unemployment, education and inequality when educational choice and the decision to pay into an unemployment insurance fund is endogenous and when workers are heterogenous in terms of initial wealth. The higher the worker's wealth, the lower social assistance the worker receives if unemployed. The model can help shed light on the puzzle why only some workers pay into an unemployment insurance fund as well as the relative compressed wage structure in countries with generous social assistance and unemployment insurance. We use this set up to consider whether we should as society increase social assistance or unemployment insurance, if the aim is to reduce inequality.

Keywords: Voluntary unemployment insurance, unemployment, search, education, welfare, inequality.

JEL codes: J6, I24

1 Introduction

The welfare state is, according to Encyclopaedia Britannica, a concept of government in which the state plays a key role in the protection and promotion of the economic and social well-being of citizens. It is based on the principles of equality of opportunity, equitable distribution of wealth, and public responsibility for those unable to avail themselves of the minimal provisions for a good life. A fundamental feature of the welfare state is social insurance.

The aim of this paper is to analyse the choice of social security system in a welfare state with voluntary unemployment insurance (where individuals cover most of the insurance costs) and social assistance.

At the turn of the 20th century, trade unions in many European countries offered voluntary unemployment benefits to unemployed workers. However, the system was unable to cope with mass and prolonged unemployment during the interwar years. The models of unemployment insurance (UI) then adopted consisted of either public subsidies to voluntary trade union run systems (so-called Ghent system) or unified compulsory state schemes. Over time, most countries converted to compulsory systems and voluntary UI administered by trade union linked funds survived only in Denmark, Sweden and Finland.

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At first sight, the Danish unemployment benefit seems to be quite generous, namely 90% of previous wages. However, there is a maximum of €2,470 per month, so the Danish UI is less generous for high-income groups. The maximum duration is 2 years and people are only eligible by working at least 6 months within the last 36 months. After the UI period, employees can apply for social assistance depending on their age, marital status, and wealth as social assistance is means-tested. This is also the case for those who do not participate in the programme either by choice or because they are not eligible.

The choice of social security system has important implications on inequality. Parsons et al (2015) describe the voluntary public UI programme as requiring a premium payment, implying that a significant fraction of the labour force, approximately 20 percent are uninsured. They may have to rely on social assistance if they lose their job. Social assistance is challinging both because its level depends on whether the individual has other means of support (such as a spouse working or assets to sell) and other conditions such as years of residence in the country. Therefore, voluntary UI systems may increase inequality (compared to a compulsory system) as uninsured workers may have little or no income if fired. At the same time, a cap on UI as well as a lower unemployment risk, may imply that high income people have lower incentives to obtain insurance which instead may reduce inequality (compared to a situation with voluntary insurance but no cap), if those workers lose their job.

In this paper we analyse the choice of social security system in a welfare state with voluntary unemployment insurance and social assistance. Individuals valuation of social insurance varies depending on their degree of risk aversion. We analyse the choice of alternative ways to protect workers facing unemployment risks depending on how risk averse individuals are.

The design of the UI system can curb broad participation. Premiums are similar for all workers despite large differences in unemployment risk as the government subsidies the remaining costs. This pricing structure encourages adverse selection as the insurance is more attractive for those with a high risk of unemployment. Also means-tested social assistance is available to uninsured unemployed workers as well as those whose unemployment insurance benefits have exhausted. This implies a risk for charity hazard as some individuals reject to obtain insurance cover because they anticipate governmental aid. The coverage is low among those with generous treatment under the safety net programme. Parsons et al (2015) find that the voluntary UI programme has predictable selection effects, including adverse selection across risk classes and a substantial charity hazard. Also, section 2 shows with graphs and regressions evidence consistent with the existence of adverse selection and charity hazard in the Danish voluntary unemployment benefit programmes. These two risks are considered in the model as they influence the choice of the social security system.

In our model individuals face two important choices: they first decide whether to remain uneducated or to acquire education and then decide whether they want to pay into an UI fund or to rely on social assistance if they become unemployed. Workers will therefore be divided into four different types, uneducated uninsured workers, educated uninsured workers, uneducated insured workers as well as educated insured workers. Individuals make education and insurance choices taking into account that the generosity of the social security system will influence the potential wage level received while employed and the probability of being employed for each type of worker. These effects and the choices made will have consequences for inequality.

The UI system does not target different education levels directly, but education is generally correlated with income, so a more generous system for low-income groups means a more generous system for uneducated workers. A cap on UI is for example one way to make the system more generous for low income (more often uneducated) workers. For simplicity we refer in our analysis to UI benefits for educated and uneducated workers.

The main contributions of our paper are twofold. First, we show the mechanisms by which selection effects inherent to the social security system affect both labour market outcomes and educational choices in an economy with voluntary unemployment insurance. Second, we calibrate the model to Danish data in order to quantify the impact of increasing social assistance and UI on labour market performance, welfare and inequality and to discuss how a policy maker obtains the most bang for the buck for a given degree of risk aversion.

This paper relates to the literature concerned with measuring the effects of potential UI benefits on unemployment and on the quality of post-unemployment jobs. More generous UI benefits insure workers against the earnings losses that accompany job loss, but they also distort the incentives to work. Some benefit recipients may search less intensively for a new job or simply wait longer until they take a job they would have accepted earlier, when benefits were lower. Acemoglu and Shimer (1999) develop a general equilibrium model of search and matching with risk averse agents and incomplete insurance and show that moderate unemployment insurance increases output. Many empirical papers study the distortions induced by UI to labour supply (some examples are Moffitt 1985; Meyer 1990; Krueger and Meyer 2002). Empirical evidence from the Great Recession in the USA confirms the prevailing wisdom by showing that extensions had a positive effect on the unemployment duration of likely benefit recipients (for example Rothstein 2011; Farber and Valletta 2013, Hagedorn et al 2013; Schmieder et al 2016). The UI benefits do not only help recipients to stay out of poverty but also allow them to search longer for a new job that matches their skills. In this way, UI benefits may improve the allocation of unemployed job seekers and vacant jobs. A less developed empirical literature shows generally small impacts on reemployment wages (for example Lalive 2007; Card et al 2007, van Ours and Vodopivec 2008, Lalive et al 2015). Two recent studies find contradictory effects on wages. Schmieder et al (2016) estimate that UI benefits extensions reduce reemployment wages in Germany. This effect combines two key policy parameters: the effect of UI on reservation wages and the effect of non-employment durations on wage offers. The effect of UI on wages they find arises mainly from substantial negative non-employment duration effects. Nekoei och Weber (2017) also take into account that an unemployed agent's job opportunities, skills, and UI benefits decrease the longer she remains without a job when they study how an extension of potential duration of UI benefit affects laid-off agents' search decisions in Austria. They find that an increase in potential benefit duration causes workers to stay unemployed longer and also causes workers to obtain jobs that pay on average 0.5 percent higher wage. Similarly, Arni et al (2013) show that temporary reductions in UI benefits as punishment for noncompliance with eligibility requirements lower the quality of post-unemployment jobs both in terms of job duration as well as in terms of earnings. Chetty (2008) stresses that unemployment insurance enables productive job search by liquidity constrained households. Our paper fits into this literature by describing the mechanisms by which more generous unemployment benefits affect wages, employment and inequality.

Few papers combine unemployment insurance and education choices, which is what we do in the present paper. Recently, Landersø and Heckman (2016) show that intergenerational educational mobility is low in welfare state

economies like Denmark. Our paper may provide a theoretical foundation in which to analyse which factors related to the welfare state are important for educational choices as well for the choice of participating in an UI fund. Liu (2012) analyses the interrelationship between labour market outcome and educational choices and predicts that higher UI benefit encourages individuals to take education. In our model the effect on education depends on which group is most benefited by the more generous UI.

We analyse the effects of three alternative ways to protect workers facing unemployment risks in an economy characterised as a welfare state depending on how risk averse individuals are. The main result of our analysis is that the effect of social insurance is multi-faceted and complex. The choice of the social security system depends on several factors, including how the policy maker prioritizes among the diverse effects which alternative policies have on different variables, as well the degree of risk aversion in society. The model lets us determine what policy is most effective depending on whether the policy maker cares about welfare or prefers variables that are easier to observe such as unemployment. We also show the preferred decisions of a policy maker who cares most about inequality.

The paper is organised in the following way. We first document the relationship between wealth and being member of an unemployment insurance fund. Next, in Section 3, we set up a theoretical model featuring risk aversion and unemployment as well as endogenous job supply, unemployment, wages, and both education and unemployment insurance decisions. In Section 4, the equilibrium with risk neutrality is described. The budget restriction is given in Section 5. Section 6 considers the impact on wages, unemployment, unemployment insurance, education, and inequality for increases in social assistance and the replacement rate. The equilibrium with risk aversion is described in Section 7. The simulation results in Section 8 include counterfactuals for both risk neutrality and risk aversion. Section 9 examines welfare implications and Section 10 attempts to answer the question: 'which policy gives more bang for the buck?'. The last Section concludes.

2 Wealth and Social Security

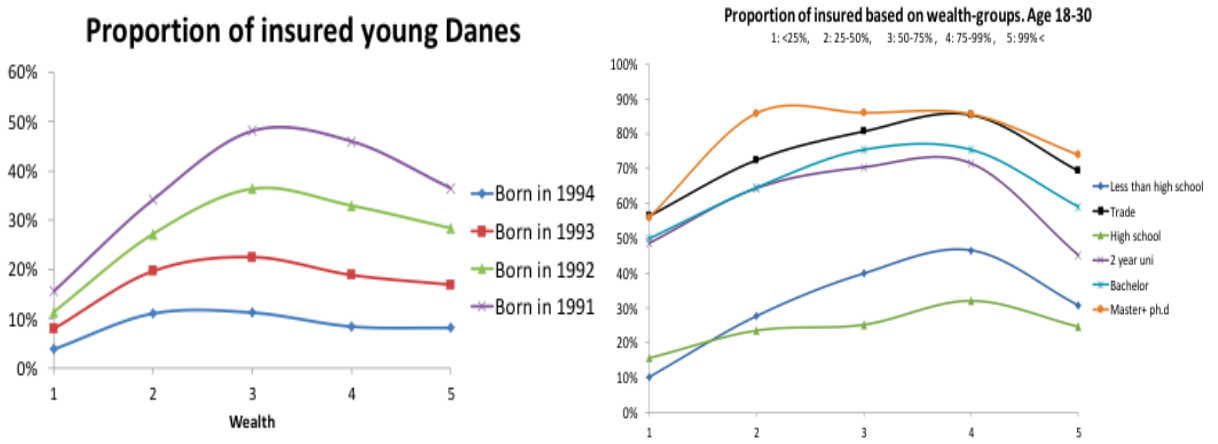
The design of the UI programme in Denmark implies a risk for both adverse selection and charity hazard. We describe in this section statistics of the Danish social security system showing evidence consistent with the existence of both.

The risk for adverse selection is a result of premiums being basically identical for all workers despite large differences in unemployment risk. This means that UI is more attractive for those with a high risk of unemployment. We can show (see online appendix) that there is a positive correlation between the unemployment rate and the fraction of the labour force paying into an UI fund across Danish municipalities. This is an indication of the presence of adverse selection, that is, that individuals with higher unemployment risk find UI more attractive than individuals with lower unemployment risk.

Charity hazard means that the UI coverage is low among those with generous treatment under the safety net programme. The left panel of Figure 1 shows the relationship between the wealth of young individuals (19 to 22 years old) divided into quintiles and the share paying into an unemployment insurance fund. For the lowest

three quintiles higher wealth is associated with a higher proportion of workers paying into an UI fund. The lowest quintiles are more likely to receive social assistance in case of unemployment. Many young individuals are still students and may not have faced the insurance choice yet. In the right panel of Figure 1 we extend the age interval and split the sample into different educational groups to focus more on those individuals who are more likely to have already entered the labour market. The positive relationship between initial wealth and paying into an UI fund is present for most educational groups except when they reach the highest wealth group. Notice that the highest wealth group means in the right panel the wealthiest 1 percent. The relationship is less clear for the master and PhD students whose labour force participation is probably just a minor complement to their studies. Individuals with lower wealth find the UI less attractive than individuals with higher wealth, indicating the presence of charity hazard.

Figure 1: Relationship between wealth and paying into an Unemployment Insurance fund in 2013



Note: Danish register data in 2013. Left figure: The relationship between the wealth of young individuals for different cohorts (19 to 22 years old) divided into income quintiles and the share paying into an unemployment insurance fund. Right figure: The relationship between the wealth of individuals, divided into income quintiles at different educational levels.

The correlation between UI fund membership and wealth is confirmed by a regression for young workers (20 to 25 years old) where we control for education, parents' membership of a UI fund, and industry. We consider that for this young age group we can regard wealth as an exogenous variable and not something which is accumulated as part of each individual's saving decision. Results for the 21 year old are presented Table (1) and the Appendix supplies additional results for 20 and 22-25 year old. The regressions indicate a negative correlation between wealth levels and the decision to become member of a UI fund when entering the labour force.

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Last, we show in Figure 2 that the decision to acquire unemployment insurance is rarely reversed over the working life period. Many workers start paying into a UI fund in the beginning of their 20s and once they start

Table 1: Wealth and membership of an UI fund at age 21

	(1)	(2)	(3)	(4)	(5)
Wealth	0.0317***	0.0261***	0.0282***	0.0131***	0.0139***
	(0.00361)	(0.00352)	(0.00354)	(0.00311)	(0.00313)
Education		0.0369***		0.0156***	
		(0.00312)		(0.00272)	
Educational group			Yes		Yes
Industry				Yes	Yes
Constant	0.965***	0.944***	0.932***	0.976***	0.969***
	(0.00152)	(0.00282)	(0.00384)	(0.00316)	(0.00409
Observations	14157	13892	13892	10835	10835

Note: Regression where the dependent variable is whether or not the individual is paying into an unemployment insurance fund. Only individuals who are 21 years old are taken in consideration. Wealth is a continuous variable that is scaled with 100,000. Wealth ranges from -100,000 to 100,000, as we can see, it is significant in all regressions. Education is a categorical variable that is coded 0 for those who have an education lower than High school, 1 for those who have a regular High school education, 2 for those with an business High school education, and 3 for those who have a college education, or higher. Educational groups is a categorical variable that describes the different directions one can choose within education. This can be, for example, construction, law, finance, etc. Industry describes which industry the individual is working within.

paying into a UI fund, they continue to do so. Few move in and out of a UI fund.

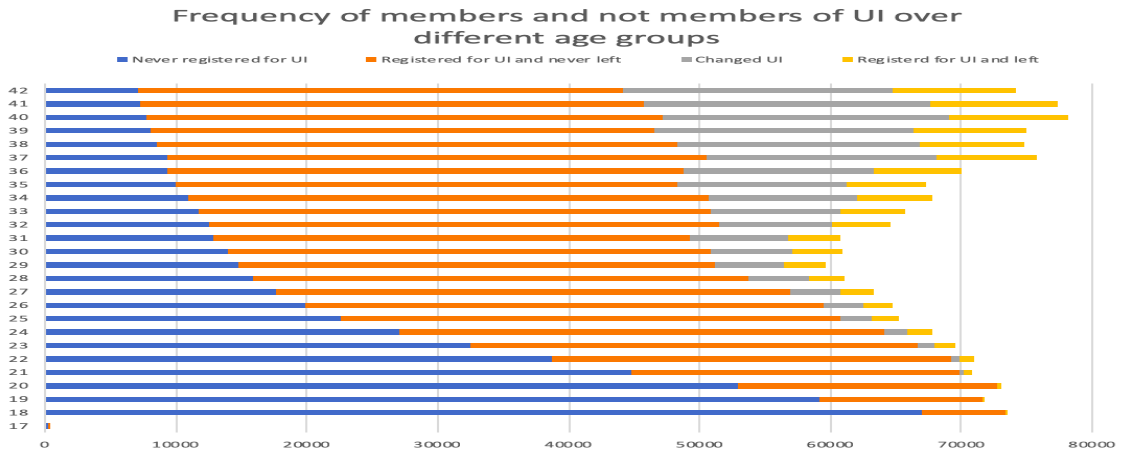
We have shown indications of the presence of adverse selection and charity hazard, which can affect the choice of the social security system. In the next section we set up a model that will be consistent with the presence of these two risks.

3 The Model

We develop an equilibrium model where individuals have two decisions to make. First, they decide if they want to educate themselves or not before entering the labour market. After entering the labour market, they choose if they want to obtain UI. At the point of time when they make the educational choice decision, individuals are not aware of their level of wealth. We justify this simplifying assumption by considering that wealth mainly comes from inheritance, which normally is received after the education decision has been taken. The education decision depends on the ability level of the worker, $e \in [0, 1]$.

The level of wealth is revealed by the time the individuals enter the labour force. We want to analyse how the exogenous wealth level affects the individuals' decision, so we disregard savings and accumulation of wealth while being a member of the labour force (See Fontaine et al 2020). When workers enter the labour force they are endowed with wealth, $F \in [0, 1]$,. When workers decide whether to pay into a UI fund or not, they are aware that

Figure 2: UI membership, membership shifts and age



social assistance (SA) is an alternative only for those with low wealth. Workers with high wealth levels are not eligible for SA.

3.1 Matching

The matching function is given by $X^m = (v^m)^\alpha (u^m)^{1-\alpha}$, $m = h, l$, where X^m is the number of matches, v^m is the number of vacancies in sector m, and u^m is the number of unemployed workers in sector m. Uninsured and insured workers will compete for the same jobs and workers apply for jobs in the sector supplying jobs to their specific eduational level.

The transition rates for a particular worker i searching for a job in sector m is then $(\theta^m)^\alpha$ where $\theta^m = v^m/u^m$ is labour market tightness. The rates at which vacant jobs become filled are $(\theta^m)^{\alpha-1}$, $m = h, l$.

3.2 Workers

Given the education decision they have taken, workers may be educated with high productivity $m = h$ or uneducated with low productivity $m = l$. Workers then choose to pay into an unemployment fund or rely on social assistance, SA. The present discounted value of the expected income of an unemployed worker, U_z^m, $m = h, l$ and an employed workers, E_z^m, not paying into an unemployment insurance fund are determined by

$$rU_z^m = \mathcal{U}(I_{uz}^m) + (\theta^m)^\alpha (E_z^m - U_z^m), \tag{1}$$

$$rE_z^m = \mathcal{U}(I_{ez}^m) - s(E_z^m - U_z^m), \tag{2}$$

where instantaneous utility is given by

$$\mathcal{U}(I_j^m) = \left\{ \begin{array}{ll} \frac{1}{1-q}\left(I_j^m\right)^{1-q} & \text{for } q < 1 \\ \ln(I_j^m) & \text{for } q = 1 \end{array} \right\}$$

and where income is $I_{uz} = z(F) + rF - t$, $I_{ez}^m = w_z^m + rF - t$, $(\theta^m)^\alpha$, $m = h, l$ is the worker's transition rate into a job. The variable $z(F)$ is the income received while unemployed which is decreasing in wealth and at a decreasing rate, $z'(F) < 0$, $z''(F) > 0$ and rF is the return to wealth per period as r is the interest rate, which is equal to the discount rate. The parameter t is a lump sum tax rate used as one way to finance unemployment insurance.[1] We furthermore need that $z(0) = \underline{z}$ is finite. The variable w_z^m is the wage level for uninsured workers. The parameter s represents exogenous job separation, identical for all worker types, and a is the rate by which workers leave the labour force, also identical for all worker types. The parameter q is the degree of relative risk aversion.

If workers have chosen to pay into an unemployment insurance fund, then the values U_b^m (while unemployed) and E_b^m, $m = h, l$ (while employed) are determined by:

$$rU_b^m = \mathcal{U}\left(I_{ub}^m\right) + (\theta^m)^\alpha \left(E_b^m - U_b^m\right) - aU_b^m, \tag{3}$$

$$rE_b^m = \mathcal{U}\left(I_{eb}^m\right) - s\left(E_b^m - U_b^m\right) - aE_b^m. \tag{4}$$

where $I_{ub}^h = \bar{b} + rF - k - t$, and $I_{ub}^l = bw_b^l + rF - k - t$ and $I_{eb}^m = w_b^m + rF - k - t$ and where $\bar{b}$ is the unemployment insurance educated workers receive, which is at a ceiling and therefore independent of wages, b is the fraction of wages an uneducated worker receives and k is what an insured worker pays to be a member of a UI fond. [2] We assume that average unemployment insurance benefits are higher than payments into an unemployment insurance fund and taxes. That is, we assume $\bar{b} > k + t$ and $bw_b > k + t$. Employed insured workers receive the wage level w_b^m and pay k into an unemployment insurance fund. As above, all insured workers pay the lump sum tax t.

3.3 Firms

The present discounted value of a filled job, J_j^m, $m = h, l$, $j = b, z$ is

$$rJ_b^m = y^m - w_b^m - s\left(J_b^m - V^m\right), \; m = h, l, \tag{5}$$

$$rJ_z^m = y^m - w_z^m - s\left(J_z^m - V^m\right), \; m = h, l, \tag{6}$$

[1] We do this as unemployment insurance is partly paid by unemployment insurance premium and partly by taxes. This is done in order to keep unemployment insurance premia equal for different groups of workers even though unemployment rates on average are different.

[2] We assume these payments are independent of wages as all workers usually pay around the same amount.

where y^m is productivity, $w_j^m,\ j = b, z,\ m = h, l$ is the wage level and $V^m,\ m = h, l$, is the value of a vacancy determined by

$$rV^m = (\theta^m)^{\alpha-1}\left(\left(1-\hat{F}^m\right)\left(J_b^m - V^m\right) + \hat{F}^m\left(J_z^m - V^m\right)\right) - c,\ m = h, l, \tag{7}$$

where $(\theta^m)^{\alpha-1},\ m = h, l$ is the rate by which a firm fills a vacancy, c are the costs of supplying a vacancy and $\hat{F}^m$ is the fraction of uninsured workers among the unemployed workers.

3.4 Wages

Wages are determined by Nash Bargaining, with equal bargaining power, giving the maximisation problem

$$\max_{w_j^m}\left(E_j^m - U_j^m\right)^{1/2}\left(J_j^m - V^m\right)^{1/2},\ m = h, l\ j = b, z,$$

We assume that all uneducated workers have wages under the cap and all educated workers have wages over the cap.[3] This means that uneducated workers have unemployment benefits proportional to their wages, while educated workers have unemployment benefits that are independent of their wages. Using equations (1) -(7) as well as free entry, $V_j^m = 0$, and that firms only know the workers' expected wealth (workers are not disclosing their true wealth) and thereby expected social assistance, $\mathbb{E}\mid_{\hat{F}^m} z(F)$ we obtain the wage equations for uninsured and insured workers where $\tilde{r} = r + s$.

$$w_z^m = y^m - \frac{\left(\mathcal{U}\left(I_{ez}^m\right) - \mathbb{E}\mid_{\hat{F}^m}\mathcal{U}\left(I_{uz}^m\right)\right)}{\mathcal{U}'\left(I_{ez}^m\right)}\frac{\tilde{r}}{\tilde{r} + (\theta^m)^{\alpha}}\ m = h, l, \tag{8}$$

$$w_b^h = y^h - \frac{\left(\mathcal{U}\left(I_{eb}^h\right) - \mathcal{U}\left(I_{ub}^h\right)\right)}{\mathcal{U}'\left(I_{eb}^h\right)}\frac{\tilde{r}}{\tilde{r} + (\theta^h)^{\alpha}}, \tag{9}$$

$$w_b^l = y^l - \frac{\left(\mathcal{U}\left(I_{eb}^l\right) - \mathcal{U}\left(I_{ub}^l\right)\right)}{\mathcal{U}'\left(I_{eb}^l\right)}\frac{\tilde{r}}{\tilde{r} + (\theta^l)^{\alpha}}, \tag{10}$$

For linear utility the expressions simplify to become

$$w_z^m = \frac{y^m\left(\tilde{r} + (\theta^m)^{\alpha}\right) + \mathbb{E}\mid_{\hat{F}^m} z(F)\tilde{r}}{2\tilde{r} + (\theta^m)^{\alpha}}\ m = h, l, \tag{11}$$

$$w_b^h = \frac{y^h\left(\tilde{r} + \left(\theta^h\right)^{\alpha}\right) + \bar{b}\tilde{r}}{2\tilde{r} + (\theta^h)^{\alpha}}, \tag{12}$$

[3] These are simplifying assumptions to capture the fact that a much larger share of educated workers have wages over the cap.

$$w_b^l = y^l \frac{\tilde{r} + \left(\theta^l\right)^\alpha}{(2-b)\,\tilde{r} + \left(\theta^l\right)^\alpha} \ . \tag{13}$$

For given labour market tightness we have the following direct effects on wages. Insured workers paying into a UI fund will have wages increasing in b or in $\bar{b}$ and workers not doing so, will see their wages reduced by expected wealth as expected social assistance, $\mathbb{E}\mid_{\hat{F}^m} z(F)$, is falling in wealth, F. A better outside option, a higher b or $\bar{b}$ increases wages while a lower outside option (lower social assistance when expected wealth increases) reduces wages.

3.5 Education

When a worker decides whether to acquire education or remain an uneducated worker, he or she compares the value of unemployment as an educated worker (including the associated costs of education) to the value of unemployment as an uneducated worker. We assume that education is acquired before entering the labour market and therefore also before deciding whether the worker wants to pay into an unemployment insurance fund. Workers that find it optimal to acquire education view this as a once and for all investment in human capital. The cost of higher education depends negatively on individual ability, $e \in [0,1]$, and is given by $o(e)$, where $o'(e) < 0$ and $o''(e) > 0$.The marginal worker has an ability level, $\hat{e}$, which makes him or her just indifferent between acquiring higher education and remaining an uneducated worker. Workers proceed to higher education if the expected income gain of education exceeds their cost of education. By using equations (1) or (3) and (2), we write the condition determining the ability level of the marginal worker acquiring education as:

$$\mathbb{E}rU_z^h - o(\hat{e}) = \mathbb{E}rU_z^l. \tag{14}$$

We can use equations (1) and (14) to obtain

$$\mathbb{E}\frac{\tilde{r}\mathcal{U}\left(I_{uz}^h\right) + \left(\theta^h\right)^\alpha \mathcal{U}\left(I_{ez}^h\right)}{\tilde{r} + \left(\theta^h\right)^\alpha} - \mathbb{E}\frac{\tilde{r}\mathcal{U}\left(I_{uz}^l\right) + \left(\theta^l\right)^\alpha \mathcal{U}\left(I_{ez}^l\right)}{\tilde{r} + \left(\theta^l\right)^\alpha} = o(\hat{e}). \tag{15}$$

Equation (15) gives $\hat{e}$ as a function of the endogenous variables $\theta^m,\ m = h, l$. Workers with $e \leq \hat{e}$, choose not to acquire education, whereas workers with $e > \hat{e}$ acquire education. Hence, $\hat{e}$ and $1 - \hat{e}$ constitute the uneducated and educated labour forces, respectively. The left hand side of equation (15) is the expected income gain associated with attaining education. This gain needs to be positive in order for, at least some workers to acquire higher education. The fact that productivity is higher for highly educated workers, which gives rise to an educational wage premium, provides incentives for higher education. In order to guarantee a nontrivial interior solution where at least some, but not all, individuals choose to acquire education, we assume that the individual with highest ability faces a very low cost of education, more specifically $o(1) = 0$, and the individual with the lowest ability faces very high cost of education, i.e., $lim_{e \to 0} o(e) = \infty$. With risk neutrality equation simplifies (14) to

$$\frac{\hat{r}\mathbb{E}\mid_{\hat{F}^h} z(F)2+\left(\theta^h\right)^\alpha y^h}{2\hat{r}+\left(\theta^h\right)^\alpha}-\frac{\hat{r}\mathbb{E}\mid_{\hat{F}^l} z(F)2+\left(\theta^l\right)^\alpha y^l}{2\hat{r}+\left(\theta^l\right)^\alpha}=o(\hat{e}). \tag{16}$$

3.6 Unemployment

Unemployment rates are derived from the flow equilibrium equations for the 2 worker types, $sn^m=\left(\theta^m\right)^\alpha u^m$, $m=h,l$. We therefore obtain

$$u^m=\frac{s}{s+\left(\theta^m\right)^\alpha},\ \ m=h,l. \tag{17}$$

The total unemployment rate is given by:

$$u_{tot}=\hat{e}u^l+(1-\hat{e})\,u^h. \tag{18}$$

We note that total unemployment depends on the fraction of educated workers. If the unemployment rate is higher for uneducated workers than educated workers, then an increase in the fraction of uneducated workers, $\hat{e}$, will increase unemployment. We need that labour market tightness for educated workers is higher than for uneducated workers, $\theta^h>\theta^l$, so that educated workers have a lower unemployment rate than uneducated workers.

3.7 Labour Market Tightness

Using equation (5), (7) for the firms hiring insured workers and (6) for firms hiring uninsured workers, and assuming free entry, $V_j^m=0$, we obtain equations to determine labour market tightness as a function of wages

$$(r+s)\,c(\theta^m)^{1-\alpha}=y^m-\left(\left(1-\hat{F}^m\right)w_b^m+\hat{F}^m w_z^m\right),\ m=h,l, \tag{19}$$

In the case of no risk aversion we obtain after substiting for wages from equations (11) - (13) for the two sectors

$$c(\theta^h)^{1-\alpha}=\frac{y^h-\left(1-\hat{F}^h\right)\bar{b}-\hat{F}^h\mathbb{E}\mid_{\hat{F}^h} z(F)}{2\widetilde{r}+\left(\theta^h\right)^\alpha}, \tag{20}$$

$$\tilde{r}c(\theta^l)^{1-\alpha}=y^l-\left(\left(1-\hat{F}^l\right)y^l\frac{\tilde{r}+\left(\theta^l\right)^\alpha}{(2-b)\,\tilde{r}+\left(\theta^l\right)^\alpha}+\hat{F}^l\frac{y^l\left(\tilde{r}+\left(\theta^l\right)^\alpha\right)+\mathbb{E}\mid_{\hat{F}^l} z(F)\tilde{r}}{\left(2\tilde{r}+\left(\theta^l\right)^\alpha\right)}\right), \tag{21}$$

Labour market tightness falls in wages and the fraction of workers insured (as they receive higher wages) and increases in productivity.

3.8 Unemployment Insurance

When a worker decides whether or not to pay into an unemployment insurance fund, she or he compares the value of doing so, to the value of not doing so, i.e the marginal worker in terms of wealth, $\hat{F}^m$ solves:

$$rU_z^m = rU_b^m,\ m = h, l.$$

Substituting for the values from equations (1) and (3) we obtain

$$\mathcal{U}\left(I_{uz}^m\right)\tilde{r} + \left(\theta^l\right)^\alpha \mathcal{U}\left(I_{ez}^m\right) = \mathcal{U}\left(I_{ub}^m\right)\tilde{r} + \left(\theta^l\right)^\alpha \mathcal{U}\left(I_{eb}^m\right),\ m = h, l. \tag{22}$$

In the case of linear utility we obtain for the educated and uneducated workers

$$z(\hat{F}^h) = 2\frac{\tilde{r} + \left(\theta^h\right)^\alpha}{2\tilde{r} + \left(\theta^h\right)^\alpha}\bar{b} - k\frac{\tilde{r} + \left(\theta^h\right)^\alpha}{\tilde{r}} - \left(\theta^h\right)^\alpha \frac{E\left|_{\hat{F}^h}\right. z(F)}{2\tilde{r} + \left(\theta^h\right)^\alpha}, \tag{23}$$

$$z(\hat{F}^l) = \frac{2\left(\tilde{r} + \left(\theta^l\right)^\alpha\right)^2 by^l}{\left(2\tilde{r} + \left(\theta^l\right)^\alpha\right)\left(\left(2-b\right)\tilde{r} + \left(\theta^l\right)^\alpha\right)} - k\frac{\left(\tilde{r} + \left(\theta^l\right)^\alpha\right)}{\tilde{r}} - \left(\theta^l\right)^\alpha \frac{\mathbb{E}\left|_{\hat{F}^l}\right. z(F)}{2\tilde{r} + \left(\theta^l\right)^\alpha}. \tag{24}$$

We note that higher unemployment insurance will tend to increase the fraction of workers in each group, the educated and the uneducated, respectively, paying into an unemployment insurance fund. This is the case both directly as the worker will receive a higher payment if unemployed but also indirectly as insured workers will receive higher wages.

4 Equilibrium with risk neutrality

In this section we characterize the equilibrium. We only find closed form solutions for the case of risk neutrality. But insurance is mostly needed when individuals are risk averse, so we also include a calibration to Danish data and simulations for diverse degrees of risk aversion in Section 8.

4.1 Relative wages

When we compare wages for the four different groups of workers we obtain the following results.

Proposition 1. *Educated insured workers receive higher wages than uneducated insured workers, $w_b^h > w_b^l$ if labour market tightness in the sector employing educated workers is higher than in the sector employing uneducated workers, $\theta^h > \theta^l$. Wages received by uninsured educated workers is higher than wages received by uninsured uneducated workers if labour market tightness in the sector employing educated workers is higher than in the sector employing uneducated workers, $\theta^h > \theta^l$ as well as expected social assistance is lower for uneducated uninsured than educated uninsured, $\mathbb{E}\,|_{\hat{F}^l}\, z(F) < \mathbb{E}\,|_{\hat{F}^h}\, z(F)$ (s.c.) which corresponds to there being more insured educated workers than uneducated workers, $\hat{F}^h < \hat{F}^l$. Educated insured workers receive higher wages than educated uninsured workers, $w_b^h > w_z^h$ if and only if unemployment insurance for educated workers is higher than their expected social assistance, $b^h > \mathbb{E}\,|_{\hat{F}^h}\, z(F)$. Finally, insured uneducated workers receive higher wages than uninsured uneducated workers, $w_b^h > w_z^l$ if and only if their replacement rate is higher than their expected social assistance, $b > \mathbb{E}\,|_{\hat{F}^l}\, z(F)$ (s.c.) and productivity is relatively high.*

Proof. Comparing equation (12) and (13) we obtain that $w_b^h > w_b^l$ if and only iff $\theta^h > \theta^l$. Comparing equation (11) for $m = h$ and $m = l$ we obtain that $w_z^h > w_z^l$ if and only if $\theta^h > \theta^l$ and $\mathbb{E}\,|_{\hat{F}^l}\, z(F) < \mathbb{E}\,|_{\hat{F}^h}\, z(F)$. Furthemore, from equation (12) and (11) for $m = h$ we have that $w_b^h > w_z^h$ if and only if $b^h > \mathbb{E}\,|_{\hat{F}^h}\, z(F)$. Finally, comparing equation (13) to equation (11) for $m = l$ we have $w_b^l > w_z^l$ if and only if $b > \frac{\left(2\tilde{r}+\left(\theta^l\right)^{\alpha}\right)\mathbb{E}|_{\hat{F}^m} z(F)}{y^l(\tilde{r}+(\theta^l)^{\alpha})+\mathbb{E}|_{\hat{F}^m} z(F)\tilde{r}}$.

□

4.2 Labour market tightness

As we will see below, it cannot be determined whether educated or uneducated workers become members of a UI fund in higher proportion. Hence, we may have either $\hat{F}^l > \hat{F}^h$ or the reverse. But for unemployment of uneducated workers to be higher than unemployment for educated workers, $u^h > u^l$, we need that labour market tightness in the sector supplying jobs to educated workers is higher than labour market tightness in the sector supplying jobs to uneducated workers. The higher productivity of educated workers will tend to increase labour market tightness for those workers. Higher wages will instead tend to reduce labour market tightness. Educated workers receive higher UI which will tend to increase their wages. If social assistance is also higher for educated workers, this will have a similar effect on wages and therefore also tend to reduce labour market tightness.

4.3 Unemployment insurance

Again, it cannot be determined whether a higher or lower fraction of educated workers than uneducated workers pay into a UI fund. The higher wages received by educated workers will tend to increase the fraction of educated workers paying into a UI fund (lower $\hat{F}^h$), therefore tend to cause the following relationship, $\hat{F}^l > \hat{F}^h$, while the higher unemployment rate facing uneducated workers will tend to increase the fraction of uneducated workers paying into a UI fund (lower $\hat{F}^l$), tending to imply the reverse relationship, $\hat{F}^l < \hat{F}^h$.

5 Budget Restriction

We only consider fully financed changes in unemployment insurance, which means that unemployment insurance changes are always followed by adjustments in the tax rate so as to balance the budget constraint.[4] As the labour force is normalized to one, the budget constraint can expressed as follows:

$$t = (1-\hat{e})\left(\left(1-\hat{F}^h\right)\left(\bar{b}u^h - k\right) + \int_0^{\hat{F}^h} z\left(F\right)u^h dF\right) + \hat{e}\left(\left(1-\hat{F}^l\right)\left(bw_b^l u^l - k\right) + \int_0^{\hat{F}^l} z\left(F\right)u^l dF\right). \quad (25)$$

On the right-hand side, the first term represents the expenditures corresponding to educated unemployed workers, divided into insured and uninsured workers. Uninsured unemployed workers receive different social assistance dependent on their wealth level. Similarly, the second term is the expenses on uneducated workers, which both consists of insured and uninsured unemployed uneducated workers. We assume that education is financed through lump sum taxes.

6 Comparative Statics with risk neutrality

In this section we examine the impact on wages, education, unemployment insurance as well as inequality due to an increase in social assistance and unemployment insurance with closed form solutions assuming risk neutrality as we did in section 4.

6.1 Higher social assistance

When social assistance increases, there is a direct positive impact on uninsured workers' wages and thereby labour market tightness and a direct negative impact on the fraction of uneducated workers paying into a UI fund. The total impact on labour market tightness and unemployment insurance is given by the following proposition.

[4] There is a small variation in the contributions to UI through taxes, specially during booms. During recessions the tax rate varies by at most 45 percent.

Proposition 2. *When social assistance, z increases, then wages for uninsured workers increase, $dw_z^m/dz > 0$, wages for insured workers may increase or fall, dw_b^m/dz, $m = h, l$. Labour market tightness may increase or fall, $d\theta^l/dz < 0$, the fraction of workers being insured falls, $d\hat{F}^m/dz > 0$, $m = h, l$. The impact on education is ambiguous.*

Proof. We differentiate equation (21) and (24) with respect to θ^l, $\hat{F}^l$ and z to obtain

$$\frac{d\theta^m}{dz} = \frac{-z'(\hat{F}^m)\tilde{r}\hat{F}^m - (w_b^m - w_z^m)(\theta^m)^\alpha}{D^m}\frac{\partial w_z^m}{\partial z},$$

where $\frac{\partial w_z^m}{\partial z} > 0$ is the direct impact of higher social assistance and is therefore positive. The sign is indeterminate as the first term of the numerator is positive and the second term is negative. The determinate is negative

$$D^m = \left(\tilde{r}(1-\alpha)c(\theta^m)^{-\alpha} + \left(1-\hat{F}^m\right)\frac{\partial w_b^m}{\partial \theta^m} + \hat{F}^m\frac{\partial w_z^m}{\partial \theta^m}\right)\left(z'(\hat{F}^m)\tilde{r} + (\theta^m)^\alpha\frac{\partial w_z^m}{\partial \hat{F}^m}\right)$$

$$-\left(\alpha(\theta^m)^{\alpha-1}(w_b^m - w_z^m - k) + \left(\frac{\partial w_b^m}{\partial \theta^m}x^m - (\theta^m)^\alpha\frac{\partial w_z^m}{\partial \theta^m}\right)\right)\left(w_b^m - w_z^m - \hat{F}^m\frac{\partial w_z^l}{\partial \hat{F}^l}\right) < 0,$$

where $x^h = \left(\theta^h\right)^\alpha$and $x^l = \left(b\tilde{r} + \left(\theta^l\right)^\alpha\right)$. Also

$$\frac{d\hat{F}^m}{dz} = -\frac{\alpha(\theta^m)^{\alpha-1}(w_b^m - w_z^m - k)\hat{F}^m + \tilde{r}(1-\alpha)c + \frac{\partial w_b^h}{\partial \theta^h}\varsigma^m}{D^m}\frac{\partial w_z^m}{\partial z} > 0,$$

where $\varsigma^h = \left(\theta^h\right)^\alpha$ and $\varsigma^l = \left(b\tilde{r}\hat{F}^l + \left(\theta^l\right)^\alpha\right)$. Wages are derived to be

$$\frac{dw_z^m}{dz} = \frac{\tilde{r}}{(2\tilde{r} + (\theta^m)^\alpha)}\left(\frac{\alpha(\theta^m)^\alpha}{\theta^m}\frac{y^m - \mathbb{E}\mid_{\hat{F}^m} z(F^m)}{(2\tilde{r} + (\theta^m)^\alpha)}\frac{d\theta^m}{dz} + \frac{d\mathbb{E}\mid_{\hat{F}^m} z(F^m)}{dz} + \frac{d\mathbb{E}\mid_{\hat{F}^m} z(F^m)}{d\hat{F}^m}\frac{d\hat{F}^m}{dz}\right),$$

$$\frac{dw_b^l}{dz} = \frac{\alpha\left(\theta^l\right)^\alpha}{\theta^l}y^l\frac{(1-b)\tilde{r}}{\left((2-b)\tilde{r} + (\theta^l)^\alpha\right)^2}\frac{d\theta^l}{dz},$$

$$\frac{dw_b^h}{dz} = \frac{\alpha\left(\theta^h\right)^\alpha}{\theta^h}\tilde{r}\frac{y^h - \bar{b}}{\left(2\tilde{r} + (\theta^l)^\alpha\right)^2}\frac{d\theta^h}{dz},$$

Finally, the impact on education is found by differentiating equation (16) with respect to z and $\hat{e}$ to obtain

$$\frac{d\hat{e}}{dz} = \frac{1}{o'(\hat{e})}\left(\frac{2\hat{r}}{2\hat{r} + (\theta^h)^\alpha}\frac{dw_z^h}{dz} - \frac{2\hat{r}}{2\hat{r} + (\theta^l)^\alpha}\frac{dw_z^l}{dz}\right),$$

where the sign depends on the impact on wages in the educated sector relative to in the uneducated sector and relative labour market tightness.

□

When social assistance increases, there is a direct positive impact on uninsured workers' wages. This will tend to reduce labour market tightness in both sectors and make it less attractive to pay into an unemployment insurance fund. The former effect reduces wages for insured workers and modifies the increase in uninsured workers' wages. The latter effect implies that a lower fraction of the workers will receive the relative higher wages, which insured workers receive. This effect therefore tends to increase labour market tightness. The impact on labour market tightness is therefore ambiguous. The impact on education is also ambiguous as it depends on relative wage increases for uninsured workers as well as the relative impact on labour market tightness and therefore expected employment for educated versus uneducated workers.

Proposition 3. *When social assistance, z increases, unemployment for workers, du^m/dz, $m = h, l$, may increase or fall. As education may increase, and therefore shift workers into the sector with lower unemployment, the impact on total unemployment is ambiguous.*

Proof. We differentiate equation (17) with respect to social assistance z to obtain:

$$\frac{du^m}{dz} = -\frac{\alpha s \left(\theta^l\right)^{\alpha-1}}{\left(s + \left(\theta^l\right)^{\alpha}\right)^2} \frac{d\theta^l}{dz}, \;\; m = h, l,$$

which is negative if $d\theta^m/dz > 0$. As $du_{tot}/dz = \left(d\hat{e}/dz\right)\left(u^l - u^h\right) + \hat{e}\left(du^l/dz\right) + \left(1 - \hat{e}\right)\left(du^h/dz\right)$ and as $\left(d\hat{e}/dz\right)$ has ambiguous sign it follows that du_{tot}/dz has ambiguous sign.

□

We now turn to inequality. When social assistance increases we obtain

Proposition 4. *When social assistance increases, z increases, then wage inequality between insured workers and unsured workers falls, $d\left(w_b^m/w_z^m\right)/dz < 0$. Wage inquality between (un)insured educated and (un)insured uneducated workers, $d\left(w_z^h/w_z^l\right)/dz$, $d\left(w_b^h/w_b^l\right)/dz$, may fall or increase.*

Proof. The first result follows as if $d\theta^m/dz > 0$ then dw_b^m/dz increases less than dw_z^m/dz and if $d\theta^m/dz < 0$ then $dw_b^m/dz < 0$ and there is still a positive impact on dw_z^m/dz. As the relative impact on educated and uneducated workers' wages is ambiguous the impact on $d\left(w_z^l/w_z^l\right)/dz$ and $d\left(w_b^h/w_b^l\right)/dz$, become ambiguous.

□

Proposition 5. *When social assistance increases, z increases, the impact on unemployment inequality, $d\left(u^l/u^h\right)/dz$, is ambiguous.*

Proof. This results follows immediately as the impact on relative labour market tightness is ambiguous.

□

6.2 Higher unemployment insurance for uneducated workers

When the replacement rate b increases, there is a direct positive impact on wages and thereby labour market tightness and the fraction of uneducated workers paying into a UI fund. The total impact on labour market tightness and unemployment insurance is given by the following proposition.

Proposition 6. *When the replacement rate for uneducated workers, b increases, then wages for uneducated insured workers increase, $dw_b^l/db > 0$ and for uneducated uninsured workers the impact is ambiguous, dw_z^l/db. Labour market tightness in the uneducated sector falls, $d\theta^l/db < 0$, the fraction of uneducated workers being insured increases, $d\hat{F}^l/db < 0$. The impact on education is also ambiguous. There is no impact on educated workers.*

Proof. We differentiate equation (21) and (24) with respect to θ^l, $\hat{F}^l$ and b to obtain

$$\frac{d\theta^l}{db} = \frac{-(z'(\hat{F}^l)\tilde{r} + \left(\theta^l\right)^\alpha \frac{\partial w_z^l}{\partial \hat{F}^l})\left(1-\hat{F}^l\right)\frac{\partial w_b^l}{\partial b} + \left(w_b^l - w_z^l - \hat{F}^l\frac{\partial w_b^l}{\partial \hat{F}^l}\right)\left(\frac{\partial w_b^l}{\partial b}\left(b\tilde{r} + \left(\theta^l\right)^\alpha\right) + \tilde{r}w_b^l\right)}{D^l} < 0,$$

$$\frac{d\hat{F}^l}{db} = \frac{\left(\frac{\partial w_b^l}{\partial \theta^l}\left((2-b)\,\tilde{r} + \left(\theta^l\right)^\alpha\right) - \frac{\left(w_b^l - w_z^l - k\right)\alpha}{\left(\theta^l\right)^{1-\alpha}}\right)\left(1-\hat{F}^l\right) + \frac{\partial w_z^l}{\partial \theta^l}\left(\left(\theta^l\right)^\alpha + \left(2\tilde{r} + \left(\theta^l\right)^\alpha\right)\hat{F}^l\right) + \frac{\tilde{r}(1-\alpha)c}{\left(\theta^l\right)^\alpha}2\left(\tilde{r} + \left(\theta^l\right)^\alpha\right)}{D^l\left((2-b)\,\tilde{r} + \left(\theta^l\right)^\alpha\right)}\tilde{r}w_b^l < 0,$$

where

$$D^l = \left(\frac{\tilde{r}\,(1-\alpha)\,c}{(\theta^l)^\alpha} + \left(1-\hat{F}^l\right)\frac{\partial w_b^l}{\partial \theta^l} + \hat{F}^l\frac{\partial w_z^l}{\partial \theta^l}\right)\left(z'(\hat{F}^l)\tilde{r} + \left(\theta^l\right)^\alpha\frac{\partial w_z^l}{\partial \hat{F}^l}\right)$$

$$-\left(\frac{\partial w_b^l}{\partial \theta^l}\left(b\tilde{r} + \left(\theta^l\right)^\alpha\right) - \left(\theta^l\right)^\alpha\frac{\partial w_z^l}{\partial \theta^l} + \left(w_b^l - w_z^l - k\right)\alpha\frac{1-\hat{F}^l}{\left(\theta^l\right)^{1-\alpha}}\right)\left(w_b^l - w_z^l - \hat{F}^l\frac{\partial w_b^l}{\partial \hat{F}^l}\right) < 0,$$

and

$$\frac{dw_b^l}{db} = \frac{w_b^l\tilde{r}}{(2-b)\,\tilde{r} + (\theta^l)^\alpha}\left(1 + \frac{\alpha\left(\theta^l\right)^\alpha}{\tilde{r} + (\theta^l)^\alpha}\frac{(1-b)}{\theta^l}\frac{d\theta^l}{db}\right) > 0,$$

$$\frac{dw_z^l}{db} = \frac{\tilde{r}}{2\tilde{r} + (\theta^l)^\alpha} \left(\frac{d\mathbb{E} \mid_{\hat{F}^l} z(F)}{d\hat{F}^l} \frac{d\hat{F}^l}{db} + \frac{\alpha \left(\theta^l\right)^\alpha}{2\tilde{r} + (\theta^l)^\alpha} \frac{y^l - \mathbb{E} \mid_{\hat{F}^l} z(F)}{\theta^l} \frac{d\theta^l}{db} \right),$$

Finally, the impact on education is found by differentiating equation (16) with respect to b and $\hat{e}$ to obtain

$$\frac{d\hat{e}}{db} = -\frac{1}{o'(\hat{e})} \frac{2\hat{r}}{2\hat{r} + (\theta^l)^\alpha} \left(\frac{d\mathbb{E} \mid_{\hat{F}^l} z(F)}{d\hat{F}^l} \frac{d\hat{F}^l}{db} + \frac{\alpha \left(\theta^l\right)^\alpha}{2\hat{r} + (\theta^l)^\alpha} \frac{y^l - E \mid_{\hat{F}^l} z(F)}{\theta^l} \frac{d\theta^l}{db} \right)$$

where the sign is equal to the sign of dw_z^l/db.

□

When the replacement rate for uneducated workers increases, there is a direct positive impact on uneducated insured workers' wages. This will tend to reduce labour market tightness in the sector hiring uneducated workers and make it more attractive to pay into an unemployment insurance fund. This latter effect tends to raise wages for uneducated uninsured workers, which reduces labour market tightness in the uneducated sector further, both due to the wage effect and due to the shift into more uneducated insured workers, who are paid more. Labour market tightness in the uneducated sector therefore falls. The direct positive impact on uneducated insured workers' wages is stronger than the indirect impact through reduced labour market tightness, and hence w_b^l increases. The impact on uneducated uninsured workers' wages is ambiguous as the positive impact on their wages through the lower expected wealth and therefore higher expected social assistance may be smaller or larger than the lower job supply. When more uneducated workers choose to insure themselves it implies that the ones that remain uninsured are expected to be less wealthy on average. Education may increase or fall as the negative impact on labour market tightness facing uneducated workers may be smaller than the positive impact on expected social assistance.

Proposition 7. *When the replacement rate for uneducated workers, b increases, unemployment for uneducated workers increases, $du^l/db > 0$. There is no impact on educated workers.*

Proof. We differentiate equation (17) with respect to the replacement rate b

$$\frac{du^l}{db} = -\frac{\alpha s \left(\theta^l\right)^{\alpha-1}}{\left(s + (\theta^l)^\alpha\right)^2} \frac{d\theta^l}{db} > 0, \;\; m = h, l,$$

which is positive as $d\theta^l/db < 0$. As $d\theta^h/db = 0$ it follows that $du^h/db = 0$.

□

With respect to inequality we obtain the following. When the replacement rate for uneducated workers increases there is no impact on educated workers. Hence the impact on inequality is given by the following proposition.

Proposition 8. *When the replacement rate for uneducated workers, b increases, then wage inequality between educated insured workers and uneducated insured workers falls, $d\left(w_b^h/w_b^l\right)/db < 0$, wage inequality between educated insured and educated uninsured workers is unaffected, $d\left(w_b^h/w_z^h\right)/db = 0$, wage inquality between uninsured educated and uninsured uneducated workers, $d\left(w_z^h/w_z^l\right)/db$, may fall or increase and wage inequality between uneducated insured and uneducated uninsured workers, $d\left(w_b^l/w_z^l\right)/db$, is ambiguous.*

Proof. As educated workers are not affected, the first two results follow immidiately as $dw_b^l/db > 0$. As the impact on uneducated uninsured workers' wages is ambiguous the impact on $d\left(w_z^h/w_b^l\right)/db$ and $d\left(w_b^l/w_z^l\right)/db$, become ambiguous.

□

Proposition 9. *When the replacement rate for uneducated workers, b increases, then unemployment inequality becomes more unequal, $d\left(u^l/u^h\right)/db > 0$.*

Proof. This results follows immidiately from equation (17) as unemployment falls in labour market tightness and labour market tightness for the educated sector, θ^h is unaffected whereas labour market tightness for the uneducated sector falls: $du^l/db > 0$ and $du^h/db = 0$ then $d\left(u^l/u^h\right)/db > 0$.

□

6.3 Higher unemployment insurance for educated workers

When the replacement rate for educated workers $\bar{b}$ increases, there is a direct positive impact on wages and thereby labour market tightness and the fraction of educated workers paying into a UI fund. The total impact on labour market tightness and unemployment insurance is given by the following proposition.

Proposition 10. *When the replacement rate for educated workers, $\bar{b}$ increases, then wages for educated insured workers increase, $dw_b^h/d\bar{b} > 0$ and for educated uninsured workers, $dw_z^h/d\bar{b}$, the impact is ambiguous. Labour market tightness in the educated sector falls, $d\theta^h/d\bar{b} < 0$, the fraction of educated workers being insured increases, $d\hat{F}^h/d\bar{b} < 0$. The impact on education is ambiguous. There is no impact on uneducated workers.*

Proof. We differentiate equation (21) and (24) with respect to θ^l, $\hat{F}^l$ and b to obtain

$$\frac{d\theta^h}{d\bar{b}} = \frac{-\left(z'(\hat{F}^h)\tilde{r} + \left(\theta^h\right)^\alpha \frac{\partial w_z^h}{\partial \hat{F}^h}\right)\left(1-\hat{F}^h\right) + \left(w_b^h - w_z^h - \hat{F}^h \frac{\partial w_z^h}{\partial \hat{F}^h}\right) 2\left(\tilde{r} + \left(\theta^h\right)^\alpha\right)}{D^h} \frac{\partial w_b^h}{\partial \bar{b}} < 0$$

$$\frac{d\hat{F}^h}{d\bar{b}} =$$

$$\frac{\left(\tilde{r}\left(1-\alpha\right)\frac{c}{(\theta^h)^\alpha}+\frac{\partial w_z^h}{\partial\theta^h}\right)+\frac{2\tilde{r}+\left(\theta^h\right)^\alpha}{2(\tilde{r}+(\theta^h)^\alpha)}\left(1-\hat{F}^h\right)\left(\frac{\partial w_b^h}{\partial\theta^h}-\frac{\partial w_z^h}{\partial\theta^h}-\frac{\left(\theta^h\right)^\alpha\alpha\left(w_b^h-w_z^h\right)}{\theta^h(2\tilde{r}+(\theta^h)^\alpha)}\right)}{D^h/2\left(\tilde{r}+\left(\theta^h\right)^\alpha\right)}\frac{\partial w_b^h}{\partial\bar{b}}<0,$$

where

$$D^h=\left(\frac{\tilde{r}\left(1-\alpha\right)}{(\theta^h)^\alpha}c+\left(1-\hat{F}^h\right)\frac{\partial w_b^h}{\partial\theta^h}+\hat{F}^h\frac{\partial w_z^h}{\partial\theta^l}\right)\left(z'(\hat{F}^h)\tilde{r}+\left(\theta^h\right)^\alpha\frac{\partial w_z^h}{\partial\hat{F}^h}\right)$$

$$-\left(\theta^h\right)^\alpha\left(\frac{\alpha}{\theta^h}\left(w_b^h-w_z^h\right)+\left(\frac{\partial w_b^h}{\partial\theta^h}-\frac{\partial w_z^h}{\partial\theta^h}\right)\right)\left(w_b^h-w_z^h-\hat{F}^h\frac{\partial w_z^h}{\partial\hat{F}^h}\right)<0$$

and

$$\frac{dw_b^h}{d\bar{b}}=\frac{\tilde{r}}{\left(2\tilde{r}+\left(\theta^h\right)^\alpha\right)}\left(1+\frac{y^h-\bar{b}}{2\tilde{r}+\left(\theta^h\right)^\alpha}\frac{\alpha\left(\theta^h\right)^\alpha}{\theta^h}\frac{d\theta^h}{d\bar{b}}\right)>0,$$

$$\frac{dw_z^h}{d\bar{b}}=\frac{\tilde{r}}{2\tilde{r}+\left(\theta^h\right)^\alpha}\left(\frac{\partial\mathbb{E}\mid_{\hat{F}^h}z(F)}{\partial\hat{F}^h}\frac{d\hat{F}^h}{d\bar{b}}+\frac{y^h-\mathbb{E}\mid_{\hat{F}^h}z(F)}{2\tilde{r}+\left(\theta^h\right)^\alpha}\frac{\alpha\left(\theta^h\right)^\alpha}{\theta^h}\frac{d\theta^h}{d\bar{b}}\right).$$

Finally, the impact on education is found by differentiating equation (16) with respect to $\bar{b}$ and $\hat{e}$ to obtain

$$\frac{d\hat{e}}{d\bar{b}}=-\frac{1}{o'(\hat{e})}\frac{2\hat{r}}{2\hat{r}+\left(\theta^h\right)^\alpha}\left(\frac{d\mathbb{E}\mid_{\hat{F}^h}z(F)}{d\hat{F}^h}\frac{d\hat{F}^h}{db}+\frac{\alpha\left(\theta^h\right)^\alpha}{2\hat{r}+\left(\theta^h\right)^\alpha}\frac{y^h-E\mid_{\hat{F}^h}z(F)}{\theta^h}\frac{d\theta^h}{db}\right),$$

where the sign is equal to minus the sign of dw_z^h/db.

□

When an uneducated worker's replacement rate increases, there is a direct positive impact on educated insured workers' wages. This will tend to reduce labour market tightness in the sector hiring educated workers and make it more attractive to obtain unemployment insurance. This latter effect tends to raise wages for educated uninsured workers, which reduces labour market tightness in the educated sector further, both due the wage effect and due to the shift into more educated insured workers, who are paid more. Labour market tightness in the sector hiring educated workers therefore falls. The direct positive impact on educated insured workers' wages is stronger than the indirect impact through reduced labour market tightness. Therefore, uneducated insured workers' wages increase. The impact on educated uninsured workers' wages is ambiguous as the positive impact on their wages through the lower expected wealth and therefore higher expected social assistance may be smaller or larger than the lower transition rate into a job. The impact on education is ambiguous as the negative impact on labour market tightness

facing educated workers may be smaller than the positive impact on expected social assistance.

Proposition 11. *When the replacement rate for uneducated workers, b increases, unemployment for educated workers increase,* $du^h/d\bar{b} > 0$. *There is no impact on uneducated workers.*

Proof. We differentiate equation (17) with respect to the replacement rate $\bar{b}$

$$\frac{du^h}{d\bar{b}} = -\frac{\alpha s\left(\theta^h\right)^{\alpha-1}}{\left(s+\left(\theta^h\right)^{\alpha}\right)^2}\frac{d\theta^h}{d\bar{b}} > 0, \;\; m = h, l,$$

which is positive as $d\theta^h/d\bar{b} < 0$. As $d\theta^l/d\bar{b} = 0$ it follows that $du^l/d\bar{b} = 0$.

□

We finally analyse inequality. When the replacement rate for educated workers increases there is no impact on uneducated workers. Hence the impact on inequality is given by the following proposition.

Proposition 12. *When the replacement rate for uneducated workers, b increases, then wage inequality between educated insured workers and uneducated insured workers increase,* $d\left(w_b^h/w_b^l\right)/d\bar{b} > 0$, *wage inequality between uneducated insured and uneducated uninsured workers is unaffected,* $d\left(w_b^l/w_z^l\right)/d\bar{b} = 0$, *wage inquality between uninsured educated and uninsured uneducated workers,* $d\left(w_z^h/w_z^l\right)/d\bar{b}$, *may increase or fall and wage inequality between uneducated insured and uneducated uninsured workers,* $d\left(w_b^l/w_z^l\right)/d\bar{b}$, *increases for* $y^h > 2\bar{b}$.

Proof. As uneducated workers are not affected, the first two results follow immidiately as $dw_b^h/d\bar{b} > 0$. As the impact on educated uninsured workers' wages is ambiguous the impact on $d\left(w_z^h/w_z^l\right)/d\bar{b}$ become ambiguous whereas we can show that $d\left(w_b^l/w_z^l\right)/d\bar{b} > 0$ for $y^h > 2\bar{b}$.

□

Proposition 13. *When the replacement rate for educated workers,* $\bar{b}$ *increases, then unemployment inequality between educated and uneducated workers becomes less unequal,* $d\left(u^l/u^h\right)/d\bar{b} < 0$.

Proof. This result follows immidiately from equation (17) as unemployment falls in labour market tightness and labour market tightness for the educated sector, θ^h is unaffected whereas labour market tightness for the uneducated sector falls $du^h/d\bar{b} > 0$ and $du^l/d\bar{b} = 0$ then $d\left(u^l/u^h\right)/d\bar{b} < 0$.

□

7 The equilibrium with risk aversion

When we include risk aversion we cannot derive any explicit conditions for relative variables in the model. We therefore assume we are in a realistic scenario where the following claim holds:

Claim 14. With risk aversion, we consider realistic equilibria where the following holds. Labour market tightness facing educated workers is higher than labour market tightness facing uneducated workers, $\theta^h > \theta^l$ as this insures that educated workers are less unemployed than uneducated workers, $u^h < u^l$. Wages for educated workers are higher than wages for uneducated workers, $w_j^h > w_j^l$ $j = b, z$. Educated workes may be more or less insured than uneducated workers, $\hat{F}^h \lesseqgtr \hat{F}^l$.

An equilibriun fulfilling these requirements will therefore be the starting point, when we consider the impact of an increase in unemployment insurance in the simulations in Section 8.

8 Counterfactuals

In the comparative statistics Section 6 we analysed the effects of an increase in the generosity of the insurance system (both unemployment insurance and social assistance). The comparative statistics were limited by the fact that many impacts were ambiguous. Also, the analytical results were derived under the assumption of risk neutrality. In this section we use simulations to complete the analytical results already shown with the inclusion of risk aversion. This allows us to shed light on ambiguous results in the setting of the simulations. Furthermore, we can consider the impact of the insurance system on welfare as well as relative welfare.

We simulate an increase in the generosity of the insurance system in a model calibrated to fit the Danish economy in 2018, the country chosen for the empirical observations in the previous section, where voluntary unemployment insurance is prevalent. Educated workers are defined as those with tertiary education, while uneducated workers have up to secondary education. Table 2 shows the parameters of the Danish economy the model is calibrated to approximate. Educated workers have lower unemployment and higher wages than uneducated workers. Uneducated workers are much less likely to be recipients of unemployment insurance and receive on average higher social assistance, which fits the model, but receive slightly higher unemployment insurance than educated workers. This is because the cap on unemployment insurance is relatively low and therefore some of the low educated workers actually have wages high enough to reach the cap.

The discount rate is set to $\rho = 0.03$, (See Millard and Mortensen 1997) and $\alpha = 0.5$ (Petrongolo and Pissarides 2001). The parameter values for productivity, hiring costs, search efficiency, separation rates and educational costs are chosen to match the data given the level of risk aversion ($q = 0; 0.2$ and 0.5) are given in Table 3.

We consider the following educational costs function: $c(e_i) = g/e_i$ and the social assistance are related to wealth in the following way: $z(F) = z/F$.

Table 2: Wages, unemployment, fraction receiving UI insurance and education in Denmark in 2018

Data			
	Below or secondary education	Tertiary education	Share
Unemployment	$u^l = 0.056$	u^h =0.041	$u^h/u^l = 0.73$
Education			$\hat{e} = 0.63$
Wages/DKR	$w^l = 168127$	$w^h = 446686$	$w^h/w^l = 1.67$
UI payment	$bw^l = 59816$	$\bar{b} = 58571$	$\bar{b}/bw^l = 0.98$
Social assistance	$\mathbb{E}\mid_{\hat{F}^l}\left(z\left(\hat{F}^l\right)\right) = 61829$	$\mathbb{E}\mid_{\hat{F}^h}\left(z\left(\hat{F}^h\right)\right) = 60361$	$\frac{\mathbb{E}\mid_{\hat{F}^l}\left(z\left(\hat{F}^l\right)\right)}{\mathbb{E}\mid_{\hat{F}^h}\left(z\left(\hat{F}^h\right)\right)} = 0.98$
UI recipients	$\hat{F}^l = 0.59$	$\hat{F}^h = 0.16$	$\frac{1-\hat{F}^l}{1-\hat{F}^h} = 0.488$

Note: Data from Statistikbanken.dk. Educational level 'Below or secondary education' includes also vocational training.

Table 3: Parameter values with risk aversion, new model

q	c	r	s^h	s^l	z	y^h	y^l	b	$\bar{b}$	g	d	v^h	k	t
0.0	0.4	0.02	0.0627	0.055	0.22	1.69	1	0.80	0.78	0.3975	0.5475	-0.1095	0.01	0.08
0.2	0.4	0.02	0.0651	0.05655	0.22	1.69	1	0.80	0.78	0.339	0.547	-0.01327	0.01	0.052
0.5	0.4	0.02	0.0689	0.0595	0.22	1.69	1	0.80	0.78	0.3286	0.55	-0.02	0.01	0.0524

In order to be able to match the data, we need two modifications of the model. First, we allow social assistance to be higher for uneducated workers than educated workers, which reflects that social assistance is quite generous for individuals with hardly any wealth. Second, we include parameters, v^m, $m = h, l$ which measures other external factors which may affect the decision about entering an unemployment insurance fund. For example, whether parents have been member of an unemployment unsurance fund or not is a determinant factor for whether their offspring pay into one. This parameter may differ for educated and uneducated workers.

In our simulations the budget constraint is always fulfilled when we assume risk neutrality and we explicitly include it when we assume risk aversion. That is, we also calibrate the lump sum tax needed for the budget constraint to be fulfilled. Notice that the calibrated tax rate is low because it is only the extra resources needed to cover unemployment insurance expenses beyond the ones financed by the fund members. This is consistent with the functioning of a voluntary unemployment insurance system of the kind we base our model on.

When we use the term relative average wages, we refer to average wages of educated workers relative to average wages for uneducated workers. The average wage of educated (uneducated) workers take into account that wages are different for insured and uninsured workers, given their level of education.

We consider the impact of an increase in the generosity of the insurance system on wages, unemployment, unemployment insurance, education and welfare. We examine the impact on these variables when we increase social assistance from $z = 0.22$ to z' $= 0.27$; unemployment insurance for uneducated from $b = 0.8$ to b' $= 0.85$ and unemployment insurance for educated workers from $\bar{b} = 0.78$ to $\bar{b}' = 0.83$.

8.1 Impact on Wages

We examine the impact on wages when we increase social assistance from $z = 0.22$ to $z' = 0.27$, unemployment insurance for uneducated workers from $b = 0.8$ to $b' = 0.85$ and unemployment insurance for educated workers from the benchmark case where $\bar{b} = 0.78$ to $\bar{b'} = 0.83$.

All figures in this section and Section 9.2 present percentage changes in the corresponding variable, for example wages in Figure 3, when the social security become more generous and given a balanced budget. In Figure 3 we observe that higher social assistance raises both wages for uneducated and educated workers, and mainly for the noninsured workers. But as there are more uneducated uninsured workers than educated uninsured workers, uneducated uninsured workers' wages increase the most. The higher the degree of risk aversion, the larger the impact on uninsured workers. Figure 3 panel b shows that relative wages decrease both for insured and uninsured workers as uneducated workers get higher wage increases whether they are insured or not. The percentage decrease of relative wages of workers on average is even larger as more workers become uninsured, where differences in wage increases are larger.

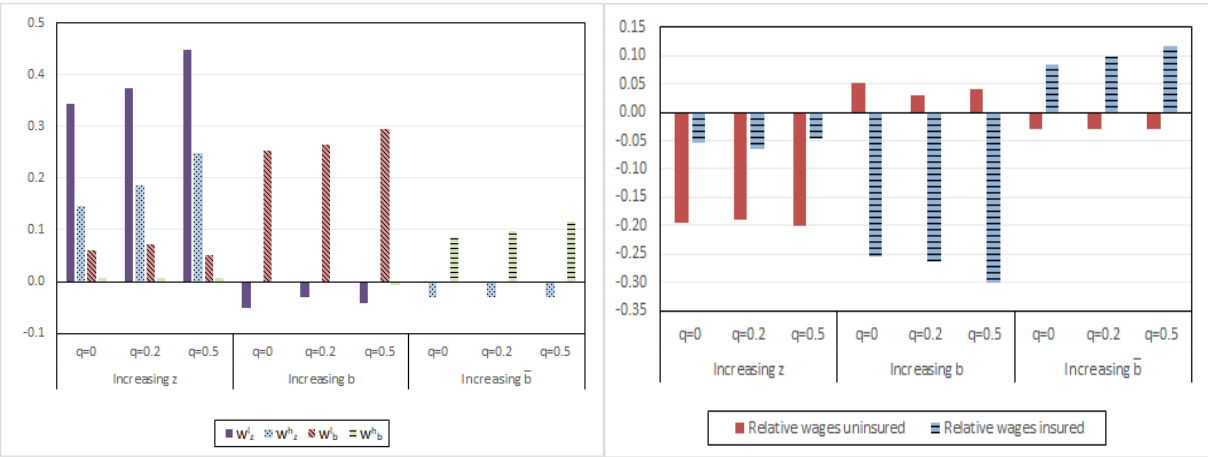
We showed in the analytical results that when unemployment insurance for uneducated workers increases, then wages for uneducated insured workers increase (proposition 8) and that wage inequality between educated insured workers and uneducated insured workers falls. There is no impact on educated workers. In the simulations, (see Figure 3) uneducated insured workers' wages increase whereas the uneducated uninsured workers' wages fall as labour market tightness falls. The higher the degree of risk aversion, the larger the impact on insured workers. Figure 3 shows that relative wages increase for uninsured workers when the wages of uneducated uninsured workers fall, while it decreases for insured workers when the wages of uneducated insured workers increase. Average relative wages still increase because most of the uneducated workers are uninsured.

Similarly, we showed in the analytical results that when unemployment insurance for educated workers increases, then wages for educated insured workers increase (Proposition 10) and wage inequality between educated insured workers and uneducated insured workers increase (Proposition 12). There is no impact on uneducated workers. In the simulation (right panel of Figure 3), educated insured workers' wages increase and educated uninsured workers' wages fall as labour market tightness falls in the sector hiring educated workers. It also shows that relative wages decrease for uninsured workers when the wages of educated uninsured workers fall, while it increases for insured workers when the wages of educated insured workers increase.

8.2 Impact on Unemployment

The effect of a more generous social assistance on unemployment was ambiguous in the analytical results (Proposition 3). The impact on unemployment inequality was ambiguous too (Proposition 5). In our simulations (Left panel of Figure 4), higher social assistance reduces unemployment. This is the case as higher social assistance makes it less attractive to pay into an unemployment insurance. This has two effects. The first is that uninsured workers' wages increase tending to reduce labour market tightness and the second is that fewer workers obtain insurance and as

Figure 3: Wages



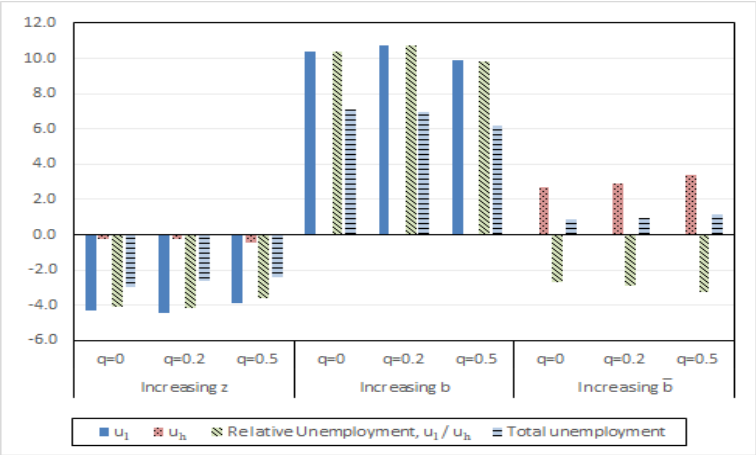
uninsured workers receive lower wages than insured workers, this effect reduces firms' expected wages which tend to increase labour market tightness. The latter effect dominates for both educated and uneducated workers. Thus unemployment falls.

The effect is larger for uneducated workers who are uninsured in a larger proportion. Therefore, the unemployment rate of uneducated workers falls relative to the unemployment rate of educated workers. Educated and uneducated workers become more equal measured in terms of unemployment. The impact is the strongest when risk aversion is low because in that case less workers are insured initially.

The analytical results revealed that when unemployment insurance for uneducated workers increases, unemployment for uneducated workers increases while there is no impact on educated workers (Proposition 7). Unemployment inequality increases (Proposition 9). In our simulations (middle panel of Figure 4), the increase in wages for uneducated workers as well as the shift into there being more of them, will reduce labour market tightness. Therefore, unemployment of low educated workers unambiguously increases. This will increase unemployment inequality.

The analytical results also showed that when unemployment insurance for educated workers increases, unemployment for educated workers increase. There is no impact on uneducated workers (Proposition 11). Consequently, unemployment inequality between educated and uneducated workers becomes less unequal (Proposition 13). In our simulations (right panel of Figure 4), an increase in the replacement rate for educated workers will increase their wages and increase the fraction of them being insured. Both effects reduce labour market tightness for educated workers. Consequently, the unemployment rate of educated workers increases which reduces inequality measured in unemployment. The impact is the strongest the higher the degree of relative risk aversion.

Figure 4: Unemployment



8.3 Impact on the number of insured workers

We showed in the analytical results (Proposition 2) that the fraction of workers being insured falls when social assistance increases. Our simulations (left panel of Figure 5) also show that higher social assistance reduces the number of insured workers. The impact is the strongest for uneducated workers as social assistance is relative more important for these workers.

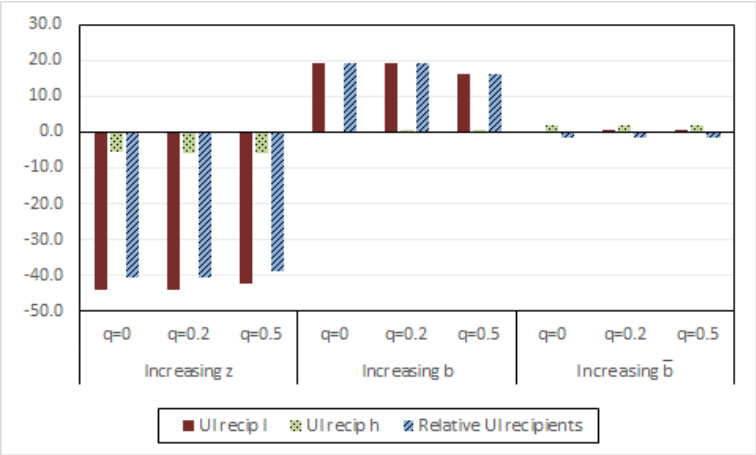
We showed in the analytical results (Proposition 6) that the fraction of uneducated workers being insured increases when unemployment insurance for uneducated workers increases. Our simulations (middle panel of Figure 5) also show that the direct impact on how attractive it is to be become insured as well as the increase in wages for uneducated workers will increase the fraction of insured uneducated workers.

Similarly, we showed in the analytical results (Proposition 10) that the fraction of educated workers being insured increases when unemployment insurance for educated workers increases. Our simulations (right panel of Figure 5) confirms that an increase in wages for educated workers as well as a direct increase the relative benefits of being insured result in a higher fraction of them being insured. The results seems not to depend heavely on the degree of relative risk aversion for low value of risk aversion. However, for $q = 0.5$ the results are a bit smaller.

8.4 Impact on education

The effect of a more generous social assistance on education was ambiguous in the analytical results (Proposition 2). In our simulations (left panel of Figure 6), an increase in social assistance will tend to reduce education as uneducated workers gain both in terms of the largest wage increase and the sharpest reduction of unemployment.

Figure 5: Insurance



The effect of a more generous unemployment insurance for uneducated workers on education was also ambiguous in the analytical results (Proposition 6). In our simulations (middle panel of Figure 6) the reduction of wages received by uninsured uneducated workers and the fact that uneducated workers' unemployment rate will increase imply that it becomes less attractive to be uneducated. This results in more educated workers. The increase in the number of educated workers is increasing in the degree of relative risk aversion. Even the effect of a more generous unemployment insurance for educated workers on education was ambiguous in the analytical results (Proposition 10). In our simulations (right panel of Figure 6) the wage reduction for uninsured educated workers as well as the unemployment rate for educated workers will reduce the incentives to obtain education and the fraction of educated workers fall. The impact is stronger the higher the degree of risk aversion.

9 Welfare

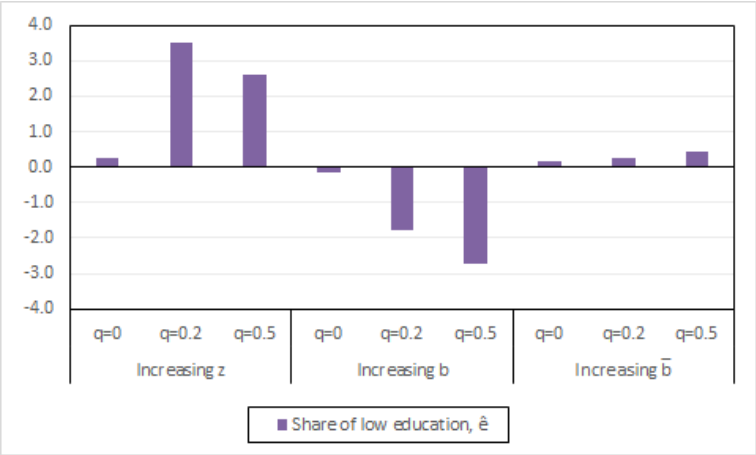
Welfare depends on the asset equations, the flow equilibrium conditions and the government budget restriction in (25). Assuming no discounting, i.e., $r \to 0$, we can write the welfare function as

$$W = \hat{e}W^l + \int_{\hat{e}}^{1} W^h de, \tag{26}$$

where

$$W^l = \left(\left(1-u^l\right)y^l - u^l c\theta^l\right), \tag{27}$$

Figure 6: Fraction of Uneducated



$$W^h = \left(1 - u^h\right) y^h - u^h c\theta^h - o\left(e\right). \tag{28}$$

With the assumption of risk neutral individuals, we ignore distributional issues and hence wages will not feature in the welfare function. Welfare of educated uninsured workers is higher than welfare of uneducated uninsured workers as $\left(1 - u^h\right) y^h - u^h c\theta^h - o\left(e\right) > \left(1 - u^l\right) y^l - u^l c\theta^l$. This is a consequence of the assumption needed in order to have a positive share of educated individuals.

If we include risk aversion, equations (27) and (28) are modified to

$$W^l = \left(1 - \hat{F}^l\right)\left(u^l \mathcal{U}\left(I^l_{ub}\right) + n^l \mathcal{U}\left(I^l_{eb}\right)\right) + \int_0^{\hat{F}^l} \left(u^l \mathcal{U}\left(I^l_{uz}\right) + n^l \mathcal{U}\left(I^l_{ez}\right)\right) dF^l, \tag{29}$$

$$W^h = \left(1 - \hat{F}^h\right)\left(u^h \mathcal{U}\left(I^h_{ub}\right) + n^h \mathcal{U}\left(I^h_{eb}\right)\right) + \int_0^{\hat{F}^h} \left(u^h \mathcal{U}\left(I^h_{uz}\right) + n^h \mathcal{U}\left(I^h_{ez}\right)\right) dF^h - o\left(e\right), \tag{30}$$

and subject to the budget restriction in equation (25). With risk aversion, wages and taxes will figure in the welfare function. We also here obtain, as with risk neutrality, that an interior solution for education implies that welfare of educated workers is higher than welfare of the uneducated workers.

9.1 Efficient outcome with risk neutrality

Risk neutrality usually implies that the economy will be efficient in the absense of unemployment insurance, social assistance and taxes. We first derive the social optimum by maximising the welfare function, equation (26) with respect to θ^m, $m = h, l$ and $\hat{e}$. The socially optimal solution, is where the variables $\theta^{m*}, m = h, l$ and $\hat{e}^*$ solve the following conditions

$$\frac{dW}{d\hat{e}} = -\left(\frac{\left(\theta^{h*}\right)^{\alpha} y^h - sc\theta^{h*}}{s + \left(\theta^{h*}\right)^{\alpha}} - \frac{\left(\theta^{l*}\right)^{\alpha} y^l - sc\theta^{l*}}{s + \left(\theta^{l*}\right)^{\alpha}}\right) + o\left(\hat{e}^*\right) = 0, \tag{31}$$

$$\frac{dW}{d\theta^m} = \frac{\alpha\left(y^m \left(\theta^{m*}\right)^{\alpha-1} - \frac{1-\alpha}{\alpha} c \left(\theta^{m*}\right)^{\alpha}\right) - cs}{s + \left(\theta^{m*}\right)^{\alpha}} = 0,\ m = h, l. \tag{32}$$

We then compare the socially optimal solution to the market outcome. Rewriting equation (16), (20) and (21) when $r \to 0$ we obtain the equations to determine θ^m, $m = h, l$ and $\hat{e}$:

$$\frac{s\mathbb{E}\mid_{\hat{F}^h} z(F) 2 + \left(\theta^h\right)^{\alpha} y^h}{2s + \left(\theta^h\right)^{\alpha}} - \frac{s\mathbb{E}\mid_{\hat{F}^l} z(F) 2 + \left(\theta^l\right)^{\alpha} y^l}{2s + \left(\theta^l\right)^{\alpha}} = o(\hat{e}). \tag{33}$$

$$c(\theta^h)^{1-\alpha} - \frac{y^h - \left(1 - \hat{F}^h\right)\bar{b} - \hat{F}^h \mathbb{E}\mid_{\hat{F}^h} z(F)}{2s + \left(\theta^h\right)^{\alpha}} = 0, \tag{34}$$

$$sc(\theta^l)^{1-\alpha} - y^l + \left(\left(1 - \hat{F}^l\right) y^l \frac{s + \left(\theta^l\right)^{\alpha}}{(2-b)\, s + \left(\theta^l\right)^{\alpha}} + \hat{F}^l \frac{y^l \left(s + \left(\theta^l\right)^{l}\right) + \mathbb{E}\mid_{\hat{F}^l} z(F) s}{\left(2s + \left(\theta^l\right)^{\alpha}\right)}\right) = 0, \tag{35}$$

It follows that with risk neutrality and if the Hosios condition holds, $\alpha = 0.5$ (remember the worker's bargaining power is equal to a half) the market solution coincides with the social optimum in the absence of taxes, social assistance and unemployment insurance as equations (33)-(35) then become

$$\frac{\left(\theta^h\right)^{\alpha} y^h}{2s + \left(\theta^h\right)^{\alpha}} - \frac{\left(\theta^l\right)^{\alpha} y^l}{2s + \left(\theta^l\right)^{\alpha}} = o(\hat{e}). \tag{36}$$

$$0.5(\theta^h)^{\alpha-1}\left(y^h - c\left(\theta^h\right)\right) - cs = 0, \tag{37}$$

$$0.5(\theta^l)^{\alpha-1}\left(y^l - c\left(\theta^l\right)\right) - cs = 0, \tag{38}$$

and using equation (32) in equation (31) gives $\frac{dW}{d\hat{e}} = \frac{\left(\theta^{h*}\right)^{\alpha} y^h}{2s+(\theta^{h*})^{\alpha}} - \frac{\left(\theta^{l*}\right)^{\alpha} y^l}{2s+(\theta^{l*})^{\alpha}} = o(\hat{e}^*)$, $\frac{dW}{d\theta^m} = 0.5\left(y^m\left(\theta^{m*}\right)^{\alpha-1} - c\left(\theta^{m*}\right)^{\alpha}\right) - cs = 0, \;\; m = h, l$ for $\alpha = 0.5$. Including unemployment insurance and social assistance will, on the other hand, increase wages and therefore reduce tightness and also move education away from the efficient outcome.

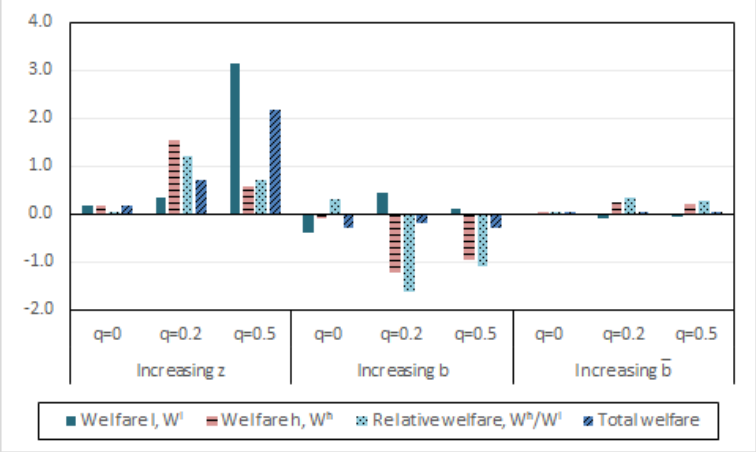
9.2 Simulation results for welfare

Figure 7 shows how the combined changes in wages, unemployment, education decisions, unemployment insurance decisions, etc., originating in the alternative policy changes (including the fact that these policies must be financed through higher taxes) affect welfare.

A more generous social assistance increases the welfare of both educated and uneducated workers, relative and total welfare. Higher unemployment insurance for uneducated workers only increases welfare of the uneducated workers when they are risk averse while it decreases welfare of educated workers, decreases relative welfare (except in the case of risk neutrality) and total welfare.

Finally, higher unemployment insurance for educated workers only increases welfare of the educated workers when they are risk averse while it is welfare reducing for uneducated workers, increases relative welfare and hardly affects total welfare.

Figure 7: Welfare



10 Which policy gives more bang for the buck?

We have showed the effect of the alternative policies on wages, unemployment, the decisions to get education and unemployment insurance and welfare. But we could not directly compare across policies because they have different costs for a policy maker. In order to compare the impact of different policies we need to transform the percentage changes into elasticities that say how much more, for example welfare the policy maker can get with an extra unit of social assistance. The changes in variables in this section are measured in elasticities normalised by the required change in taxes in order to keep the budget restriction balanced.

The percentage increase in taxes needed to support the increase in generosity of the unemployment insurance for the uneducated workers is as large as the percentage increase in taxes needed to support the more generous social assistance when risk aversion is $q = 0.2$. It is a bit larger with risk neutrality and a bit smaller when risk aversion is $q = 0.5$. The similarity of taxes needed means the comparison of the effects of these two policies resembles that in the previous diagrams where just percentage changes were shown. But there are much less unemployed educated workers in need of insurance, so an increase in the generosity of the unemployment insurance for the educated workers requires much lower taxes. This means that when we compare in terms of elasticities the effects in the right panel of all figures become larger in relation to the left and middle panel than what was the case in the last section.

10.1 Welfare elasticities

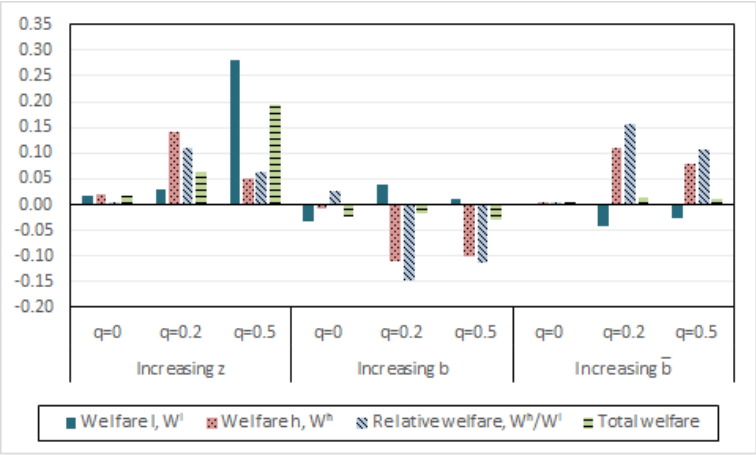
A first answer to the question of which policy gives more bang for the buck, would be to see what policy increases welfare the most for a given increase in taxes. A policy maker who values educated and uneducated workers equally, would choose the policy that offers higher increase in total welfare, that is, an increase in social assistance (see Figure 8). This would be especially the case if the degree of risk aversion is high. If the policy maker cares more about inequality in welfare, then the preferred policy would be a more generous unemployment benefit as the decrease in relative welfare is largest at least for some degree of risk aversion.

Welfare is a challenging measure for a policy maker to focus on as it is not observable. An alternative is to focus on measures that are easier to observe for the workers such as unemployment or the level of education.

10.2 Unemployment elasticities

A policy maker who cares about reducing total unemployment would also choose to increase social assistance. On the other hand, a policy maker focusing on reducing unemployment inequality would instead choose a more generous unemployment insurance for the educated (see Figure 9).

Figure 8: Welfare elasticities



10.3 Education elasticities

And finally, considering Figure 10 we observe that a policy maker having preferences for increasing the fraction of educated workers in the economy, corresponding to reducing the share of low educated, would find more generous unemployment insurance for the uneducated workers preferable. As in the middle panel of Figure 10 we observe the largest drops of uneducated workers when b increases, especially if the degree of risk aversion is high.

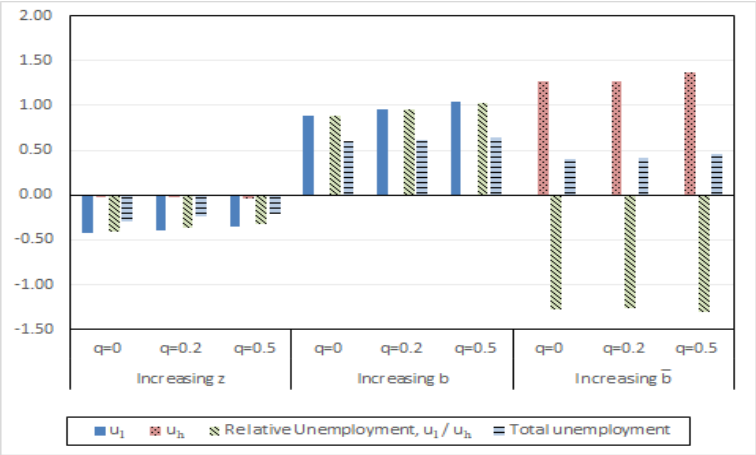
There is no absolute answer to the question of which policy gives more bang for the buck without knowing how the policy maker prioritizes among the diverse effect the alternative policies have on the different variables.

This model and calibrations in this paper helps understand what policy is most effective for each desired outcome, given the degree of risk aversion.

11 Conclusion

We have analysed the impact of three alternative ways to protect workers facing unemployment risks in an economy characterised as a welfare state inhabited by risk averse workers. We show how the results depend on the degree of relative risk aversion. We had in mind the Danish welfare state where individuals bear a part of the cost of their unemployment insurance but are at the same time protected by a generous social assistance system. Workers in the model can choose to insure themselves against the risk of unemployment. The risk of becoming unemployed can be affected not only by acquiring unemployment insurance but also by acquiring education that opens the door to a skilled portion of the labour market. All these features play a role in the determination of a new equilibrium when policy changes. The choice of policy would in our model depend on which outcomes the

Figure 9: Unemployment elasticities



policy maker prioritizes. Different policies (a more generous social assistance or a more generous unemployment insurance for uneducated or educated workers) would affect several variables of importance such as unemployment, wages, and the level of education. In this paper, we therefore find that the policy maker faces several trade-offs when deciding how to design the social security system and the optimal policy depends on the desired outcome.

We show some of these relationships analytically and then complete with simulations adapted to the Danish economy. The results show that the impact of the chosen policy is heavily dependent on the degree of the risk aversion. The results indicate that a policy maker who wishes to increase total welfare should consider a policy that increases social assistance, which also would decrease the total unemployment rate in the economy. However, a policy maker who seeks to reduce inequality should instead consider more generous unemployment benefits because it would lead to less welfare inequality, while an increase in unemployment insurance only for the educated would reduce unemployment inequality. If the policy maker prioritizes an increase in the share of educated workers, it would be prefereable to implement a more generous unemployment insurance for the uneducated. This paper therefore gives the framework in which to analyse the choice of social security system, dependent on the estimated degree of relative risk aversion and which are the economic goals.

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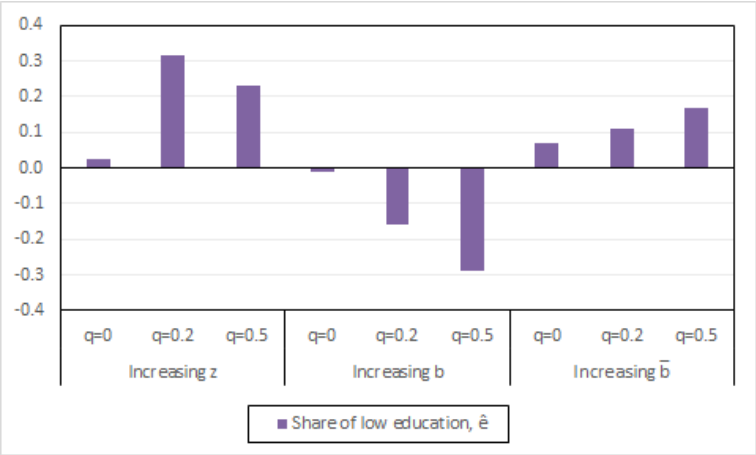
Figure 10: Education elasticities



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A Appendix

Table 4: Wealth and membership of an UI fund at age 20

	(1)	(2)	(3)	(4)	(5)
Wealth	0.0224***	0.0212***	0.0218***	0.0111***	0.0113***
	(0.00323)	(0.00320)	(0.00322)	(0.00262)	(0.00263)
Education		0.0128***		0.00435*	
		(0.00261)		(0.00603)	
Educational group			Yes		Yes
Industry				Yes	Yes
Constant	0.977***	0.973***	0.967***	0.989***	0.987***
	(0.00139)	(0.00184)	(0.00280)	(0.00207)	(0.00274)
Observations	11544	11359	11359	9060	9060

Table 4 is showing the results from a regression where the dependent variable is whether or not the individual is paying into an unemployment insurance fund. Only individuals who are 20 years old are taken in consideration. Wealth is a continuous variable that is scaled with 100 000. Wealth ranges from -100 000 to 100 000. As we can see, it is significant in all regressions. Education is a categorical variable that is coded 0 for those who have an education lower than High school, 1 for those who have a regular High school education, 2 for those with an business High school education, and 3 for those who have a college education, or higher. Educational groups is a categorical variable that describes the different directions one can choose within education. This can be, for example, construction, law, finance, etc. Industry describes which industry the individual is working within.

Table 5: Wealth and membership of an UI fund at age 22

	(1)	(2)	(3)	(4)	(5)
Wealth	0.0290***	0.0212***	0.0239***	0.00912***	0.0998***
	(0.00392)	(0.00389)	(0.00390)	(0.00364)	(0.00363)
Education		0.0482***		0.00200***	
		(0.00373)		(0.00330)	
Educational group			Yes		Yes
Industry				Yes	Yes
Constant	0.951***	0.917***	0.902***	0.969***	0.960***
	(0.00173)	(0.00370)	(0.00474)	(0.00385)	(0.00486)
Observations	15316	14922	14922	11664	11664

Table 5 is showing the results from a regression where the dependent variable is whether or not the individual is paying into an unemployment insurance fund. Only individuals who are 22 years old are taken in consideration. Wealth is a continuous variable that is scaled with 100 000. Wealth ranges from -100 000 to 100 000. Education is a categorical variable that is coded 0 for those who have an education lower than High school, 1 for those who have a regular High school education, 2 for those with an business High school education, and 3 for those who have a college education, or higher. Educational groups is a categorical variable that describes the different directions one can choose within education. This can be, for example, construction, law, finance, etc. Industry describes which industry the individual is working within.

Table 6: Wealth and membership of an UI fund at age 23

	(1)	(2)	(3)	(4)	(5)
Wealth	0.0313***	0.0211***	0.0239***	0.0118***	0.0128***
	(0.00383)	(0.00384)	(0.00383)	(0.00340)	(0.00339)
Education		0.0538***		0.0259***	
		(0.00351)		(0.00310)	
Educational group			Yes		Yes
Industry				Yes	Yes
Constant	0.943***	0.897***	0.885***	0.954***	0.945***
	(0.00181)	(0.00407)	(0.00490)	(0.00434)	(0.00518)
Observations	16278	15776	15776	12474	12474

Table 6 is showing the results from a regression where the dependent variable is whether or not the individual is paying into an unemployment insurance fund. Only individuals who are 23 years old are taken in consideration. Wealth is a continuous variable that is scaled with 100 000. Wealth ranges from -100 000 to 100 000. Education is a categorical variable that is coded 0 for those who have an education lower than High school, 1 for those who have a regular High school education, 2 for those with an business High school education, and 3 for those who have a college education, or higher. Educational groups is a categorical variable that describes the different directions one can choose within education. This can be, for example, construction, law, finance, etc. Industry describes which industry the individual is working within.

Table 7: Wealth and membership of an UI fund at age 24

	(1)	(2)	(3)	(4)	(5)
Wealth	0.0338***	0.0217***	0.0243***	0.0114***	0.0126***
	(0.00377)	(0.00376)	(0.00377)	(0.00349)	(0.00350)
Education		0.0549***		0.0321***	
		(0.00314)		(0.00300)	
Educational group			Yes		Yes
Industry				Yes	Yes
Constant	0.939***	0.885***	0.874***	0.930***	0.920***
	(0.00183)	(0.00422)	(0.00491)	(0.00497)	(0.00575)
Observations	16827	16209	16209	12636	12636

Table 7 is showing the results from a regression where the dependent variable is whether or not the individual is paying into an unemployment insurance fund. Only individuals who are 24 years old are taken in consideration. Wealth is a continuous variable that is scaled with 100 000. Wealth ranges from -100 000 to 100 000. Education is a categorical variable that is coded 0 for those who have an education lower than High school, 1 for those who have a regular High school education, 2 for those with an business High school education, and 3 for those who have a college education, or higher. Educational groups is a categorical variable that describes the different directions one can choose within education. This can be, for example, construction, law, finance, etc. Industry describes which industry the individual is working within.

Table 8: Wealth and membership of an UI fund at age 25

	(1)	(2)	(3)	(4)	(5)
Wealth	0.0321***	0.0192***	0.0204***	0.00834**	0.00855**
	(0.00381)	(0.00379)	(0.00379)	(0.00348)	(0.00347)
Education		0.0584***		0.0326***	
		(0.00300)		(0.00284)	
Educational group			Yes		Yes
Industry				Yes	Yes
Constant	0.934***	0.871***	0.853***	0.932***	0.918***
	(0.00191)	(0.00449)	(0.00529)	(0.00466)	(0.00531)
Observations	16676	15951	15951	12495	12495

Table 8 is showing the results from a regression where the dependent variable is whether or not the individual is paying into an unemployment insurance fund. Only individuals who are 25 years old are taken in consideration. Wealth is a continuous variable that is scaled with 100 000. Wealth ranges from -100 000 to 100 000. Education is a categorical variable that is coded 0 for those who have an education lower than High school, 1 for those who have a regular High school education, 2 for those with an business High school education, and 3 for those who have a college education, or higher. Educational groups is a categorical variable that describes the different directions one can choose within education. This can be, for example, construction, law, finance, etc. Industry describes which industry the individual is working within.