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Identifying production units with outstanding performance

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Abstract. In many applications of data envelopment analysis, there are situations in which a central body manages a large set of similar units delivering some services. In such multi-unit organizations, the central management desires a mechanism by which the local management of each unit is incentivized to perform towards the improvement of the performance of the organization as a whole. In a recent paper, Afsharian et al. (2017) have proposed a system of incentives under these circumstances. In their approach, – which relies on the original concept of super-efficiency – units with outstanding performance are identified and incentivized by some reward compatible with the level of their impact on the overall performance of the organization. We discuss why the conventional super-efficiency approach may not be optimal in such situations. We revisit the definition of the collective impact and propose a new method, which can identify – in a controlled manner – a subset of *k* outstanding DMUs among *n* existing units in the system. The proposed approach is illustrated using data from a German retail bank.

Keywords: Data envelopment analysis; Units with outstanding performance; Centralized management; Mixed-integer linear program.

1. Introduction

In applications of Data Envelopment Analysis (DEA), there are situations in which a central body manages a large set of similar decision making units (DMUs) delivering some services. In such multi-unit organizations, the central management desires a mechanism by which the local management of each unit is incentivized to perform towards the improvement of the performance of the organization as a whole. In a recent paper, Afsharian et al. (2017) have proposed a DEA-based system of incentives under these circumstances.

In their approach, units with outstanding performance are identified and incentivized by some reward compatible with the level of their impact on the overall performance of the organization. As discussed by the authors, this process is done to ensure that these already exceptional units still act in the best interest of the whole system of units, cf. also the idea of incentivizing innovations in Agrell et al. (2002).

Afsharian et al. (2017) relied on the concept of super-efficiency for the purpose of rewarding the outstanding DMUs. In this short communication, we discuss why their approach may not be optimal in such situations, and we propose a new method, which can identify – in a controlled manner – a subset of *k* outstanding DMUs among *n* existing units in the system. In a broader context, the proposed approach can also be used in cases where there exist natural monopolies instead of usual competitive markets and where a regulator is acting as the contracting party, cf. e.g. Agrell and Bogetoft (2017) or Bogetoft and Otto (2010, Ch.10).

2. Preliminaries and fundamentals

Suppose that there exist a set of $N=\{1,...,n\}$ DMUs, a set of $M=\{1,...,m\}$ inputs and a set of $S=\{1,...,s\}$ outputs. Let $X_j=(x_{1j},x_{2j},...,x_{mj})\in\Re_+^m$ and $Y_j=(y_{1j},y_{2j},...,y_{sj})\in\Re_+^s$ be nonnegative and nonzero vectors of inputs and outputs of DMU*j*, $j\in N$. The efficiency of a DMU$_o$ under evaluation – shown by Eff_o – is defined as the optimal objective function value of the following problem (Charnes et al. 1978):

$$\text{Eff}_o=\max \sum_{r\in S}u_{ro}y_{ro} \Big/ \sum_{i\in M}v_{io}x_{io} \tag{1.1}$$

$$\text{Subject to} \quad \sum_{r\in S}u_{ro}y_{rj} \Big/ \sum_{i\in M}v_{io}x_{ij}\le 1, \quad j\in N \tag{1.2}$$

$$u_{ro},v_{io}\ge 0. \tag{1.3}$$

Remark 1. In program (1), v_{io} and u_{ro} are the multipliers (or weights) of the *i*th input and *r*th output. We note that this program – and also the other programs being developed in the following – could be modified by restricting the input/output weights to be strictly positive rather than non-negative. This ensures that these programs lead to Pareto efficient solutions (see, e.g., Cooper et al. 2007 for more details).

Charnes et al. (1978) showed that a linear programming problem equivalent to the fractional programming problem in (1) is:

$$\text{Eff}_o = \max \sum_{r \in S} u_{ro} y_{ro} \tag{2.1}$$

$$\text{Subject to } \sum_{i \in M} v_{io} x_{io} = 1, \tag{2.2}$$

$$\sum_{r \in S} u_{ro} y_{rj} - \sum_{i \in M} v_{io} x_{ij} \leq 0, \quad j \in N \tag{2.3}$$

$$u_{ro}, v_{io} \geq 0. \tag{2.4}$$

Remark 2. The program in (2) has an input orientation and assumes constant returns to scale (CRS). One could alternatively define the output-oriented minimization problem with other returns to scales (for a comprehensive overview of DEA models and their features see, e.g., Bogetoft and Otto 2010; Zhu 2014). It is straightforward to extend our results to the case of output orientation with other forms of returns to scale.

One of the most common methods to identify outstanding DMUs in DEA is to rely on super-efficiency (see, e.g., Zhu 2014, Ch. 10). Super-efficiency was originally introduced by Andersen and Petersen (1993) to rank the efficiency of fully-efficient frontier DMUs. It was later suggested as a means for, e.g., incentive regulation (see, e.g., Bogetoft 1997), acceptance decision rules (see, e.g., Seiford and Zhu 1998), detecting exceptional pupils (see, e.g., Thanassoulis 1999), sensitivity analysis in DEA (see, e.g., Zhu 2001) and outlier identification (see, e.g., Banker and Chang 2006).

The basic idea is to capture how the efficient frontier varies with and without each unit under evaluation in order to determine the impact. This can be done by applying the program in (2), but under the assumption that the DMU$_o$ under evaluation is excluded from the efficient frontier. This modification can be incorporated in program (2) by replacing the set of constraints in (2.3) by the following set of constraints:

$$\sum_{r \in S} u_{ro} y_{rj} - \sum_{i \in M} v_{io} x_{ij} \leq 0, \quad j \in N, \quad j \neq o. \tag{3}$$

The resulting so-called "super-efficiency program" will provide the same scores as before for the inefficient units, while the efficient ones will normally have efficiency scores greater than one. Hence, utilizing these scores, one may rank the DMUs and identify the most efficient units in the system (for a fuller discussion of super-efficiency DEA models, we refer to Zhu 2014, Ch. 10).

3. Discussions and the proposed method

Two key elements of the conventional super-efficiency program outlined in the previous section are:

1) it focuses on the local impact on the frontier in the neighbourhood of the DMU under evaluation,
2) it captures the impact of the exclusion of one DMU at a time from the efficient frontier.

The former indicates that a particular DMU that does not serve as a peer for many other DMUs might have a high super-efficiency and thereby be considered eligible for being as outstanding. The latter reveals that the approach may not characterise DMUs that are near neighbours in inputs and outputs (e.g. two units that "shade" each other) as outstanding, even if they collectively have a significant impact on efficiency. In the following, we discuss these issues in detail and extend the super-efficiency approach to overcome them:

The conventional super-efficiency computes an "individual" efficiency score for each DMU under evaluation. Hence, it measures how much an individual DMU pushes the frontier locally. Ideally, however, we would like to reward the impact of a DMU on the whole system. Therefore, we look for DMUs that have a large impact on the overall performance of the whole industry. This also seems a more relevant indicator in our context of centralized management if the aim is to incentivize efficient DMUs to innovate and hereby push the frontier. A DMU should ideally create frontier improvements that the whole organization and not just some subset of the DMUs can benefit from.

Consider the following program, which determines an overall efficiency score $\mathrm{Eff}_{\mathrm{Overall}}$ for the whole system of n DMUs.

$$\mathrm{Eff}_{\mathrm{Overall}} = \max \sum_{j \in N} \sum_{r \in S} u_r y_{rj} \tag{4.1}$$

$$\text{Subject to} \quad \sum_{j \in N} \sum_{i \in M} v_i x_{ij} = 1, \tag{4.2}$$

$$\sum_{r \in S} u_r y_{rj} - \sum_{i \in M} v_i x_{ij} \leq 0, \quad j \in N \tag{4.3}$$

$$u_r, v_i \geq 0. \tag{4.4}$$

In the objective function (4.1) and the normalization constraint (4.2), the aggregate levels of each output and of each input across the units are used, respectively. The constraints in (4.3) are also those of a standard DEA program (see program (2) in Section 2), which form the efficient frontier of the system of n DMUs. On this basis, this program – which has an input orientation – essentially seeks to reduce the total amount of inputs while producing at least the current amount of outputs. Therefore, the central management can examine to what extent the current allocation of resources in the system is efficient. Variations of this program have been applied in different contexts of measuring efficiency (see, e.g, Lozano and Villa 2004; Asmild et al. 2009; Varmaz at al. 2013; Afsharian et al. 2017).

Remark 3. As has been argued in the literature (see, e.g., Asmild et al. 2009), the program in (4) can equivalently be interpreted as the measure of performance for a "virtual unit" that possesses the mean value of inputs and outputs computed across all units in the system. With this particular interpretation, the program in (4) coincides with those DEA programs in the literature that capture the efficiency of an "average-unit" as the measure of the overall performance (or the structural efficiency) against a frontier (or an industry frontier) of all units (see, e.g., Farrell 1957; Førsund and Kittelsen 1998). We note, however, that this approach should be employed cautiously. As argued by Ylvinger (2000), the average-unit can be defined to evaluate the efficiency for a system of units (or at industry level) when a reallocation of inputs across the units is allowed. Otherwise, the use of the average-unit may bias the measure of the overall (or industry) efficiency. We refer the reader to Ylvinger (2000) for a discussion of this bias. Hence, the application of the program in (4) – and accordingly the approach being developed in the following – is only advised where the outlined requirement can be fulfilled.

One may now modify the program in (4) to capture how much an individual DMU$_o$ pushes the industry frontier of all units by measuring the super-efficiency of the whole system of units under the assumption DMU$_o$ is not allowed in the frontier. This modification can be incorporated into the above program by replacing the set of constraints in (4.3) by the following set of constraints:

$$\sum_{r\in S} u_r y_{rj} - \sum_{i\in M} v_i x_{ij} \leq 0, \quad j \in N, \quad j \neq o \tag{5}$$

The resulting super-efficiency program[1], however, still suffers from the second issue outlined above: it can miss the collective impact of DMUs on the overall efficiency of the system. This leads to the need for an appropriate extension by which we can look for a subset of DMUs that have a large impact on the overall performance of the whole organization. This can be done as follows:

We define the problem of identifying (a user-defined constant) k outstanding units in a formal manner as:

> Of the n existing DMUs, find the subset of k DMUs, which when removed from the efficient frontier, yields the greatest impact on the overall efficiency of the whole system of units. [2]

In order to find the subset of k DMUs with a largest impact, we define a binary decision variable that represents the selection of DMUs:

$$\tau_j = \begin{cases} 1 & \text{if DMU}_j \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

We now suggest the following formulation for our problem of identifying k outstanding units with the largest impact as a mixed integer linear programming problem:

[1] If the super-efficiency score of DMU$_o$ under evaluation is shown by $\mathrm{Eff}^{\mathrm{Super}}_{\mathrm{Overall}}(\mathrm{DMU}_o)$, a value of $\mathrm{Eff}^{\mathrm{Super}}_{\mathrm{Overall}}(\mathrm{DMU}_o)/\mathrm{Eff}_{\mathrm{Overall}}$ greater than one shows that the unit has an impact on the efficiency of the whole system. One may also rank the units according to their impact.

[2] We note that the resulting approach will characterise a subset of units as outstanding, if they collectively have a significant impact on efficiency. Hence, the k chosen outstanding units may or may not necessarily be near neighbours in inputs and/or outputs.

$$\text{Eff}^{k}_{\text{Overall}} = \max \sum_{j \in N} \sum_{r \in S} u_r y_{rj} \tag{7.1}$$

$$\text{Subject to} \quad \sum_{j \in N} \sum_{i \in M} v_i x_{ij} = 1, \tag{7.2}$$

$$\sum_{r \in S} u_r y_{rj} - \sum_{i \in M} v_i x_{ij} \le (1 - \tau_j)\omega, \quad j \in N \tag{7.3}$$

$$\sum_{j \in N} \tau_j = n - k, \tag{7.4}$$

$$\tau_j \in \{0,1\}, \tag{7.5}$$

$$u_r, v_i \ge 0, \tag{7.6}$$

where ω must be a large enough positive constant to make constraint (7.3) redundant when $\tau_j = 0$.

This program is a modification of (4) in the sense that those *k* DMUs are selected whose impact on the efficiency of the entire system is the largest. The modification is reflected in the set of constraints in (7.3) by which the selected *k* DMUs are excluded from the efficient frontier of the system of units.

We note that this exclusion is not from the definition of the aggregate inputs and outputs of the system of units reflected in the objective function (7.1) and the normalization constraint (7.2). As it was argued in Afsharian et al. (2017), the system of units (which can equivalently be seen as the "average-unit") should be defined in a stable manner across alternative DEA assessments. The authors showed – by graphical examples (see page 590) – that an approach in which a unit under evaluation is removed from both the frontier and the average-unit can lead to counter-intuitive results, incompatible with incentivizing units to improve their performance. Following Afsharian et al. (2017), in our approach, we also keep the definition of the system of units across all instances of program (7). With this, we measure the impact on the average-unit by reference to how the efficient frontier of the system varies with and without "the subset of units" under consideration. These units may then receive an incentive in proportion to the degree by which they impact the performance of the system.

Remark 4. The program in (7) assumes CRS. As the context requires, one could alternatively impose, e.g., variable returns to scale (VRS). We note, however, that the impact of the efficiency of the average-unit will be affected by mix as well as scale under VRS.

Remark 5. Depending on the context, one may alternatively construct a "median-unit" or "weighted average-unit" as the representative of the whole system of units. A study in which these alternatives have been applied is the one by Pastor et al. (1997). One may also first project inefficient DMUs onto the frontier in order to control for historical inefficiency in the definition of any of these possible "virtual units". With this projection, the approach is immune to the problem that inefficiencies are incorporated in determining the virtual unit. Similar approaches have been applied in other contexts of measuring efficiency (see, e.g., Afsharian et al. 2019 and Bogetoft and Wang 2005).

Remark 6. Any feasible solution to program (4) satisfies the constraints in (7). Thus, $\mathrm{Eff}_{\mathrm{Overall}} \leq \mathrm{Eff}_{\mathrm{Overall}}^{k}$. This also indicates that the objective function value of the program in (7) does not decrease with an increase for *k*. Hence, the subset of *k* units will have an impact if $\mathrm{Eff}_{\mathrm{Overall}}^{k} / \mathrm{Eff}_{\mathrm{Overall}}$ becomes greater than one. This means that the proposed program in (7) can also be seen as an extended super-efficiency approach, where the objective is to capture the collective impact of DMUs on the overall efficiency of the system. Clearly, the parameter *k* is subjective. The larger *k*, the higher number of units which will be recognized as outstanding with an impact determined by $\mathrm{Eff}_{\mathrm{Overall}}^{k} / \mathrm{Eff}_{\mathrm{Overall}}$. Therefore, with an appropriate *k*, the central management or regulator can strike a balance between not spending too much on incentives on the one hand and encouraging the units to operate as efficiently as possible.

4. An illustrative example using the data from a German retail bank

In order to illustrate the proposed approach, we apply it to the data of 16 branches of a small German retail bank, which comprises two inputs and two outputs. This data set (given in Table 1 in the Appendix) has been originated from Varmaz et al. (2013) and also used by Afsharian et al. (2017). The two inputs are *personnel expenses* and *expenses on interest payments*, while the two outputs are *interest income* and all *other income*. For a detailed description of these inputs and outputs, see Varmaz et al. (2013) or Afsharian et al. (2017).

Let us assume that we wish to find the subset of *k* branches, which have the greatest impact on the overall efficiency of the whole system of units. Since our objective is to illustrate the proposed method, we consider six alternative values of *k*, namely *k*=1,…, 6. For any *k*, the problem can be solved by the proposed program in (7) in which *m*=2, *s*=2 and *n*=16. The results are summarized in Table 2.

Table 2. The overall (super)efficiency scores and the resulting outstanding bank branches

	$\mathrm{Eff}_{\mathrm{Overall}}^{k}$	*Selected units*		$\mathrm{Eff}_{\mathrm{Overall}}^{k}$	*Selected units*
k=1	0.856	{12}	*k*=4	1.023	{1,2,6,7}
k=2	0.861	{6,12}	*k*=5	1.222	{1,2,7,8,14}
k=3	0.997	{1,2,7}	*k*=6	1.274	{1,2,6,7,8,14}

The overall efficiency of the whole system can be determined by program (4) or equivalently by the proposed program in (7) where *k*=0. Our computation shows that the overall efficiency of these 16 bank branches is $\mathrm{Eff}_{\mathrm{Overall}} = \mathrm{Eff}_{\mathrm{Overall}}^{k=0}$=0.820. As an example, consider *k*=3. As can be seen in Table 2, the efficiency of the system becomes $\mathrm{Eff}_{\mathrm{Overall}}^{k=3}$=0.997. In other words, $\mathrm{Eff}_{\mathrm{Overall}}^{k=3} / \mathrm{Eff}_{\mathrm{Overall}}$=1.216. This means that this subset of three branches has an impact and this impact is the largest compared to any other subset of three branches which could have been selected. Having extracted the results of binary variable τ_j, we observe that branches 1, 2 and 7 are those with outstanding performance.

As another example, compare two scenarios in which *k*=4 and 5. For *k*=4, the set of outstanding branches are 1, 2, 6 and 7, and for *k*=5, the outstanding branches are 1, 2, 7, 8 and 14. Although branch 6 has been included in the set of outstanding branches when *k*=4, it has not been chosen for the case in which *k*=5. This indicates that the proposed approach does not find it optimal to still include branch 6, when the objective is restricted to capture the collective impact of five branches on the “overall efficiency” of the system. Nevertheless, all selected branches included already in the solutions for *k*=4 and 5 appear in the solution with *k*=6.

For the sake of comparison, let us assume that this problem is also solved with the conventional super-efficiency program (Andersen and Petersen 1993) outlined in Section 2. The program in (2) is applied, but under the assumption that one branch at a time is excluded from the efficient frontier. Our computations show that there are *k*=5 outstanding branches (i.e. super-efficient units) in the system with the following rank: units 1, 6, 12, 7 and 16. For example, for *k*=3, this program suggests units 1, 6 and 12 as outstanding while our approach detects units 1, 2 and 7.

In order to detect more outstanding branches within the conventional super-efficiency approach, we have removed these five branches from the data set and applied once again program (2) on the remaining 11 branches. The program recognizes now only one new super-efficient branch: unit 2. Hence, e.g., for *k*=6, a sequential application of the conventional super-efficiency suggests branches 1, 6, 12, 7, 16 and 2 as those with outstanding performance. These results still deviate from those obtained by the proposed approach, e.g., for *k*=6. The reason is as explained that the conventional super-efficiency focuses on local impact of single DMU instead of global impact of a combination of DMUs.

Let us also compare our results to those obtained by the method developed by Afsharian et al. (2017). The program in (4) is applied, but under the assumption that one branch at a time is excluded from the efficient frontier. Our computations show that there are *k*=3 outstanding branches in the system with the following rank: units 12, 1 and 7. If *k*=1, this program suggests the same branch 12 as outstanding, but for *k*=2 (i.e. units 12 and 1) and *k*=3 (i.e. units 12, 1 and 7), the results differ from those in Table 2.

As also theoretically discussed in Section 3, the reason for the differing results is that neither the conventional super-efficiency nor the approach suggested by Afsharian et al. (2017) is able to appropriately capture the “collective impact” of branches on the “overall efficiency” of the whole system of units.

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Appendix

Table 1. Input and output data of a German retail bank

Unit	PEX (input 1)	IEX (input 2)	IIN (output 1)	OIN (output 2)
1	1532.00	2769.00	11092.00	1231.00
2	998.00	1757.00	5529.00	778.00
3	853.00	1220.00	2384.00	464.00
4	180.00	378.00	632.00	133.00
5	584.00	876.00	1847.00	297.00
6	498.00	2080.00	2689.00	524.00
7	261.00	395.00	1358.00	203.00
8	609.00	883.00	2688.00	352.00
9	222.00	528.00	791.00	149.00
10	264.00	700.00	856.00	193.00
11	1078.00	1448.00	1873.00	611.00
12	222.00	503.00	770.00	217.00
13	258.00	412.00	520.00	138.00
14	696.00	1099.00	2836.00	443.00
15	176.00	361.00	477.00	104.00
16	236.00	301.00	724.00	159.00