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Applying statistical methods to compare frontiers: Are organic dairy farms better than the conventional?

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Abstract

The Malmquist index is widely used in empirical studies of productivity change over time. The index is based on estimates of the frontier obtained from the convex envelopment of the data as in DEA. The statistical properties of the Malmquist index and its components, i.e. the frontier shift and the efficiency change, have until recently only been subject to a limited number of studies. The asymptotic properties of the geometric mean of the individual Malmquist indexes are studied in Kneip et al. (2018). Permutation tests for performing statistical inference in finite samples were recently introduced in Asmild et al. (2018) and are easily performed. In the present paper we illustrate the permutation methods by an analysis of data comprising organic and conventional dairy farms in Denmark from 2011-2015. Further, differences between the frontiers of the production possibility sets for two separate samples are studied, specifically those of the organic and the conventional producers. We suggest to use jackknife methods when estimating the differences to ensure that these are not affected by the well-known bias originating from estimation of the frontier. In summary, the paper offers an illustration of how to analyse productivity data, in particular a comparison of two independent groups, and furthermore an analysis of how the separate groups evolve over time is provided.

Keywords: Malmquist index, frontier differences, Data Envelopment Analysis (DEA), independent samples, permutation tests, organic farming.

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1 Introduction

Using the Malmquist index to measure productivity change over time was proposed by Caves et al. (1982). Following Färe et al. (1992), the productivity change is frequently calculated using non-parametric data envelopment analysis (DEA) to estimate the relevant frontiers. The Malmquist index and its components, measuring changes in efficiency and in technology between two time periods respectively, are subsequently determined, and typically the geometric means of the individual indexes are reported. However, these indexes are often interpreted without any associated measures of uncertainty. Simar and Wilson (1999) propose methods for calculating confidence intervals for the Malmquist index and its components using bootstrapping. Kneip et al. (2018) note that this bootstrap method is not based on theoretical results, and provide methods for calculating asymptotic confidence intervals for the Malmquist index. However, this method is only applicable for the Malmquist index itself and not for the frontier shift nor the efficiency change components. To the best of our knowledge, Asmild et al. (2018) are the first to suggest exact statistical tests to assess the significance of the Malmquist index as well as of its components. The present paper reviews the permutation tests recently developed by Asmild et al. (2018) and provides an application hereof. Furthermore, where the Malmquist index is used to analyze changes over time for balanced panel data, comparisons of frontiers for separate groups in terms of the relative location of the group specific frontiers, provide information about which group technology offers superior production possibilities. Comparison of frontiers can be performed by calculating an index almost similar to the frontier change index for two time periods which is also presented in the present paper.

The various approaches are used to analyze the case of dairy farms in Denmark, over a number of years, with focus on comparison of the performance over time of organic and conventional dairy farming. The development of the dairy industry over time is, of course, relevant to practitioners and policy makers alike. Furthermore, it is of particular importance to distinguish between organic and conventional farms, not only with respect to the relative locations of their frontiers, but also concerning the development over time of the productivity within each of the groups. This has implications for, for example, policy interventions.

2 Methodology

Using standard notation, let input and output quantities be denoted by $(x, y) \in \mathbb{R}_+^{p+q}$. Under the usual assumptions of closedness, convexity and strong disposability in both inputs and outputs, the production possibility set is given as

$$\Psi = \{(x, y) \in \mathbb{R}_+^{p+q} \,|\, x \text{ can produce } y\},$$

and the efficient frontier of Ψ is defined as

$$\Psi^\delta = \{(x, y) \in \Psi \,|\, (\gamma^{-1}x, \gamma y) \notin \Psi, \ \forall \gamma > 1\}.$$

The technical input efficiency index of Farrell (1957) is defined as

$$\theta(x, y) = \inf\{\theta > 0 | \,(\theta x, y) \in \Psi\}.$$

The production possibility set Ψ is unobserved and can in empirical applications be estimated from a set of n observations of random variables, $(X_i, Y_i), i = 1, \ldots, n$, which are assumed to be independent and identically distributed, such that (X_i, Y_i) has distribution F for all $i = 1, \ldots, n$. Denoting $\mathbf{X} = (X_1, \ldots, X_n)$ and $\mathbf{Y} = (Y_1, \ldots, Y_n)$, the estimate, $\hat{\Psi}$, of the production possibility set assuming constant return to scale (CRS) is

$$\hat{\Psi} = \{(x, y) \in \mathbb{R}_+^{p+q} \,|\exists \omega \in \mathbb{R}_+^n : x \geq \mathbf{X}\omega, y \leq \mathbf{Y}\omega\},$$

and the input efficiency for (x, y) can be estimated as $\hat{\theta}(x, y) = \inf\{\theta \in \mathbb{R}_+ | \,(\theta x, y) \in \hat{\Psi}\}$ or equivalently using the standard DEA linear programming formulation:

$$\hat{\theta}(x, y) = \min_{\theta, \omega}\{\theta \,|\, \theta x \geq \mathbf{X}\omega, y \leq \mathbf{Y}\omega, \, \omega \in \mathbb{R}_+^n\}.$$

Consider a situation, where each unit is observed in two time periods, t_1 and t_2, such that we are given observations from the random variables $(X_i^{t_1}, Y_i^{t_1})$ and $(X_i^{t_2}, Y_i^{t_2})$ for the two time periods, respectively. We will allow the possibility set and the distribution of the variables to differ between the two time periods, and therefore we introduce the notation

Ψ_t, Ψ_t^δ, and F_t for the possibility set, the frontier, and the distribution of the random variables (X_i^t, Y_i^t) in time period $t \in \{t_1, t_2\}$. We shall allow dependence between variables from the same unit in different time periods, i.e. $(X_i^{t_1}, Y_i^{t_1})$ and $(X_i^{t_2}, Y_i^{t_2})$, while there is (still) independence between variables concerning different units.

The traditional Malmquist index of productivity change (see e.g. Färe et al. 1992), from one period t_1 to another t_2 for unit i observed in both time periods is defined as

$$M(X_i^{t_1}, Y_i^{t_1}, X_i^{t_2}, Y_i^{t_2}) = \prod_{t \in \{t_1, t_2\}} \left(\frac{\hat{\theta}^t(X_i^{t_2}, Y_i^{t_2})}{\hat{\theta}^t(X_i^{t_1}, Y_i^{t_1})} \right)^{\frac{1}{2}},$$

where $\hat{\theta}^t$ denotes the efficiency estimated relative to the frontier for time t, i.e.

$$\hat{\theta}^t(x, y) = \min_{\theta, \omega} \{\theta \mid \theta x \geq \mathbf{X}^t \omega, y \leq \mathbf{Y}^t \omega, \, \omega \in \mathbb{R}_+^n\}, \; t \in \{t_1, t_2\}.$$

We consider the geometric mean of the calculated Malmquist indices i.e.

$$T_M = \prod_{i=1}^{n} M(X_i^{t_1}, Y_i^{t_1}, X_i^{t_2}, Y_i^{t_2})^{\frac{1}{n}}, \tag{1}$$

which can be interpreted as the productivity change for the whole technology, similar to the logic of the global indexes of Asmild and Tam (2007). The Malmquist index is often decomposed into two effects; the frontier shift and the efficiency change. The frontier shift for an individual unit (X_i^t, Y_i^t) is defined as

$$FS_i^t = \frac{\hat{\theta}^{t_1}(X_i^t, Y_i^t)}{\hat{\theta}^{t_2}(X_i^t, Y_i^t)}, \; t \in \{t_1, t_2\},$$

and the frontier shift component of the Malmquist index is the geometric mean of FS_i^t over $t \in \{t_1, t_2\}$,

$$FS(X_i^{t_1}, Y_i^{t_1}, X_i^{t_2}, Y_i^{t_2}) = \left(FS_i^{t_1} \times FS_i^{t_2}\right)^{\frac{1}{2}}.$$

The efficiency change between t_1 and t_2 for unit i is given as

$$EC(X_i^{t_1}, Y_i^{t_1}, X_i^{t_2}, Y_i^{t_2}) = \frac{\hat{\theta}^{t_2}(X_i^{t_2}, Y_i^{t_2})}{\hat{\theta}^{t_1}(X_i^{t_1}, Y_i^{t_1})}.$$

With the above notation, the geometric mean of the frontier shift component can be written

$$T_{FS} = \prod_{i=1}^{n} FS(X_i^{t_1}, Y_i^{t_1}, X_i^{t_2}, Y_i^{t_2})^{\frac{1}{n}}, \tag{2}$$

and similarly the geometric mean of the efficiency change is

$$T_{EC} = \prod_{i=1}^{n} EC(X_{t_1}^{i}, Y_{t_1}^{i}, X_{t_2}^{i}, Y_{t_2}^{i})^{\frac{1}{n}}. \tag{3}$$

Note that $M(X_i^{t_1}, Y_i^{t_1}, X_i^{t_2}, Y_i^{t_2}) = FS(X_i^{t_1}, Y_i^{t_1}, X_i^{t_2}, Y_i^{t_2}) \times EC(X_i^{t_1}, Y_i^{t_1}, X_i^{t_2}, Y_i^{t_2})$ and $T_M = T_{FS} \times T_{EC}$. All these statistics are positive.

The statistic T_{FS} can be used as a measure of how the two possibility sets Ψ_{t_2} and Ψ_{t_1} are placed relative to each other. A T_{FS} greater than 1 indicates that the possibility set generally is smaller in time t_1 than in time t_2. Since intersections of the frontiers are possible, when $T_{FS} > 1$ we can not generally conclude that Ψ_{t_1} is a subset of Ψ_{t_2}. For a more thorough discussion of the properties and the interpretation of the statistic see Asmild et al. (2018).

While the Malmquist Index and the efficiency change component are only defined for balanced panel data sets, the (geometric mean of the) frontier shift can also be estimated for unbalanced panels, with n_{t_1} and n_{t_2} observations in the two time periods respectively:

$$T_{FS}^{u} = \prod_{t \in \{t_1, t_2\}} \prod_{i=1}^{n_t} (FS_i^t)^{\frac{1}{n_{t_1} + n_{t_2}}}. \tag{4}$$

If, like in the present analysis, there are separate (independent) groups within the data set, the above analysis can be done within each of the groups, letting $(X_i^{g,t}, Y_i^{g,t})$, $i = 1., \ldots, n_g$ denote observations from group $g, g = 1, \ldots, G$ in time period t. In the empirical example we consider $G = 2$ separate groups of organic and conventional farms respectively, as well as analysis done on the full data set.

Besides comparing frontiers over time (possibly within a given group), we are here also interested in comparing the frontiers for the two independent groups (organic and conventional producers) at a fixed time. Similar to the definitions above, the geometric mean of the difference between the two groups' frontiers is defined as the ratio of the efficiencies relative to each of the two frontiers

$$T_{FD}^{g1,g2} = \prod_{g \in \{g_1,g_2\}} \prod_{i=1}^{n_g} \left(\frac{\hat{\theta}^{g_1}(X_i^g, Y_i^g)}{\hat{\theta}^{g_2}(X_i^g, Y_i^g)} \right)^{\frac{1}{n_{g_1}+n_{g_2}}}, \qquad (5)$$

which for subsequent use can be decomposed as

$$T_{FD}^{g1,g2} = \left(\prod_{i=1}^{n_{g_1}} \left(\frac{\hat{\theta}^{g_1}(X_i^{g_1}, Y_i^{g_1})}{\hat{\theta}^{g_2}(X_i^{g_1}, Y_i^{g_1})} \right)^{\frac{1}{n_{g_1}}} \right)^{\frac{n_{g_1}}{n_{g_1}+n_{g_2}}} \times \left(\prod_{i=1}^{n_{g_2}} \left(\frac{\hat{\theta}^{g_1}(X_i^{g_2}, Y_i^{g_2})}{\hat{\theta}^{g_2}(X_i^{g_2}, Y_i^{g_2})} \right)^{\frac{1}{n_{g_2}}} \right)^{\frac{n_{g_2}}{n_{g_1}+n_{g_2}}}, \qquad (6)$$

i.e. a weighted product of geometric means of the difference between the two groups' frontiers for observations from each of the two groups, which indicates the relative location of the two possibility sets.

It is well known that the estimate of the production possibility set is downward biased and therefore the efficiency scores are biased too. The bias decreases with increasing number of observations, so with large differences between the sizes of the two groups, the numerator and the denominator in (4) and (5) are determined with quite different biases. For a review of the asymptotic properties of the efficiency estimates see Simar and Wilson (2015).

Jackknifing methods can be used to address the issue of (differences in) biases by ensuring that whenever two frontiers are compared the frontier estimates are based on groups of equal sizes: From the larger group, draw without replacement the same number of observations as in the smaller group, and calculate the relevant statistic (T_{FS}^u or $T_{FD}^{g1,g2}$). Repeat this a large number of times, say 1000, and calculate the geometric mean of the frontier difference measures (4) resp. (5) over the jackknife replications.

If the jackknifed $T_{FD}^{g1,g2}$ is greater than 1, this indicates that the possibility set for g_1 is smaller than that for g_2, implying that the g_2 technology (on average) offers better production possibilities (and similarly for T_{FS}^u). However, considering the two components of $T_{FD}^{g1,g2}$ in (6) provides additional information. Particularly, if one of the components is larger than 1 and the other smaller than 1, this implies that neither production possibility set is a subset of the other, meaning that their frontiers intersect. Furthermore, it also implies that the observations in the two groups are located differently in the production space. This can be further investigated by considering the input- and output mixes in the groups, for example represented by the dimension-specific contributions to the overall length of the input- (or output) vector, $\frac{X_i}{||X_i||}$ (respectively $\frac{Y_i}{||Y_i||}$). Analysis hereof can, for example, be performed using the methodology of Asmild et al. (2016), by transforming the contributions into angles, ϕ, using the inverse cosine.[1]

2.1 Statistical inference of the Malmquist index and its components

To test the significance of the changes over time, i.e. of the Malmquist Index and its components, within each of the separate groups, we utilize permutation tests. Overall, the hypothesis we wish to test is that $(X^{t_1}, Y^{t_1}, X^{t_2}, Y^{t_2})$ and $(X^{t_2}, Y^{t_2}, X^{t_1}, Y^{t_1})$ have the same distribution, i.e. that the distribution in time period t_1 can be interchanged with the distribution in time period t_2. For this we use three tests designed to detect different forms of deviations from this hypothesis.

We first present the test procedures for *balanced* panels as described in detail in Asmild et al. (2018). The procedure compares the observed values of the test statistics T_M, T_{FS} and T_{EC} given in (1), (2) and (3), respectively, with N similar values of the test statistics calculated based on appropriate permutations of the original dataset: Each of the permuted datasets are obtained by interchanging every pair of observations $(X_i^{t_1}, Y_i^{t_1})$ and $(X_i^{t_2}, Y_i^{t_2})$ randomly with probability 0.5 and independently for different $i = 1, \ldots, n$.

Under the hypothesis being tested, the test statistics T_M, T_{FS} and T_{EC} based on the original dataset all have the same distributions as their N permuted counterparts calculated

[1] It is here worth noting that these angles are not scale invariant. Therefore, one should ensure that all input (output) variables are measured in similar metrics, like e.g. in the present case where all inputs are costs and the outputs are revenues.

from permuted versions of the dataset. Thus, significance probabilities are obtained by finding the proportion of simulated test statistics that are further away from 1 than the observed test statistic.

As discussed in Asmild et al. (2018) a set of three tests can be used to identify the nature of any differences between the two time periods: If the value of T_M is significantly different from one, an overall deviation from the null hypothesis can be concluded i.e. that there is some difference between the distributions F_{t_1} and F_{t_2} in the two periods. If, furthermore, the test associated with T_{FS} is significantly different from one, the deviation from the null hypothesis is of such nature that the two frontiers are different or at least that the distributions F_{t_1} and F_{t_2} are different near the frontier. If, on the other hand, T_{EC} has a value significantly different from one, the the null hypothesis is rejected because the efficiency distributions are different in the two periods.

As mentioned in the previous section, the test statistic for frontier shift, T_{FS}, can be generalized to an *unbalanced* version, T^u_{FS}, as formulated in equation (4) in order to take all available information into consideration. To perform a significance test, the permutation procedure for producing N permuted datasets also has to be modified: All complete observation pairs $(X_i^{t_1}, Y_i^{t_1})$ and $(X_i^{t_2}, Y_i^{t_2})$ are randomly interchanged as before. All remaining observations, that by assumption are independent and furthermore are identically distributed under the null hypothesis, are permuted and divided randomly into the two groups such that the two group sizes remains unchanged in the permuted dataset. Finally, a significance probability is obtained by comparing the observed (original) value of T_{FS} with the empirical distribution of the test statistic when based on each of the N permuted datasets.

It should be noted, that while there is unequal bias when estimating the frontiers for the two groups of different sizes, this will not give problems in the described test procedure as long as the sizes of the groups are fixed in all permutations of the dataset. Thereby the observed test statistic is still comparable with the permuted counterparts.

3 Danish dairy farms

The dataset is provided by SEGES (who amongst other things provide specialist advisory services to the Danish agricultural sector) and contains annual farm-level accounting data from the years 2011-2015. For the current analysis, only full-time farmers specialized in dairy production and with at least 100 dairy cows and at least 25 hectares of cultivated land are included. Observations with problematic data based on various screening criteria are excluded (as detailed in Lillethorup, 2017), resulting in an only partly balanced data set comprising between 1355 and 1567 observations in each year.

The variables included in the efficiency models are:

Inputs:

- Feed costs (costs of purchasing grains and fodder)
- Labour costs (estimated value of family labour plus paid labour)
- Other variable costs, OVC (including energy, fuel, fertilizer, veterinary costs etc.)
- Fixed costs, FC (including costs of maintenance, taxes, insurances etc.)
- Capital costs (defined as 4% of the value of the tangible assets, including land)

Outputs:

- Milk revenue
- Other (output) revenue, OO (revenue from all other outputs)

Descriptive statistics of the variables in each year, for both the conventional and the organic producers, are provided in Table 1.

Insert Table 1 about here

In Section 4 below we illustrate how the approaches outlined in Section 2, when used together can provide various insights on the development over time of the Danish dairy producers, as well as on the differences between conventional and organic farms.

4 Results

4.1 Frontier differences

Comparison of the organic and conventional farms is performed within each of the five years. The frontier differences are here defined as the efficiency scores relative to the frontier for the organic farms divided by the efficiency scores relative to the frontier for the conventional farms. Frontier difference measures (T_{FD}) larger than 1 means that the observations on average are closer to the organic frontier than to the conventional frontier, implying that the conventional technology (on average) offers better production possibilities.

First, consider the geometric mean frontier difference within each of the two subgroups, accounting for different sample sizes using the described jackknife technique. The results are shown in the upper part of Table 2 where it is seen, that during the study period the organic farms on average are located nearer the production frontier for the conventional farms than that for the organic farms, implying that the organic technology (on average) offers better production possibilities in the directions determined by the locations of the organic farms. Conversely, for the conventional farms the frontier difference measures are larger than one in 2012-2014, implying that the conventional technology (on average) offers better production possibilities in the directions determined by the locations of the conventional farms in 2012-2014. However, in 2015 the frontier difference measure for the conventional farms is smaller than one, implying that the organic technology now offers better possibilities for the conventional farms (as well as for the organic farms). The (geometric) average of the frontier differences for the organic and the conventional farms, is in all years smaller than or equal to one, indicating that the organic technology overall tend to be superior (after controlling for sample size biases).

Calculating the frontier differences without jackknifing yields the results in the lower part of Table 2, which give substantially different (and misleading) conclusions. In particular, that the mean frontier difference across all the observations is larger than one all years besides 2015, would lead to the (wrong) conclusion that the conventional technology is superior in those years. This highlights the importance of controlling for sample size biases using e.g. jackknifing.

As the organic technology (on average) offers better production possibilities for the organic farms, but the conventional technology (on average) offers better production possibilities for the conventional farms in 2012-2014, is evidence of the two frontiers intersecting in (at least) those years. That the organic farms tend to be located with an input-output mix where the organic technology offers better possibilities, and similarly for the conventional farms, makes perfect sense from an economic point of view.

Insert Table 2 about here

4.2 Mix differences

To investigate the differences in input-output mix between the organic and the conventional farms we express the dimension specific contributions to the overall length of the input and the output vectors by the angles ϕ. The average angles for the organic and the conventional farms in 2015 for each dimension are shown in the top part of Table 3. In the bottom part of the table test statistics and corresponding p-values for equality of the mean direction in the truncated ($[0, \pi/2]$) approximative normal distribution of the angles (c.f. Asmild et al. 2016).[2] are shown.

Insert Table 3 about here

From the mean angles we observe that the average ϕ_{Milk} is much smaller than the average ϕ_{OO} (for both conventional and organic farms), which implies that the share of revenue

[2]Note that since there are only two outputs, the angles are complementary and therefore the test statistics and p values for the two output angles are identical.

from milk is much larger than that from other outputs.

In terms of the comparison of the organic and the conventional farms we note that there are significant differences on four out of the five inputs. The difference on labour and on the outputs are not significant. The organic farms have a larger angle on feed than the conventional farms, meaning a smaller contribution from feed to the overall length of the input vector. Correspondingly, the organic farms have a larger contribution from capital than the conventional farms. This is likely due to the fact that being classified as an organic dairy farm in Denmark entails requirements for animal welfare, which means that the organic dairy cows in Denmark must be on pasture for around 6 month during the summer. This requires additional farmland, but results in saving on hard feed, thus more capital but less feed costs are necessary for the organic farmers compared to the conventional. Furthermore, the organic farms have a larger angle thus smaller contribution from other variable costs than the conventional farms, which is likely because of less veterinary costs and/or costs associated with pesticides etc.

Performing similar analysis in the other years shows, that the main changes over time is on the contribution from milk to the overall length of the output vector. Specifically it was large in 2013 and 2014 (for both organic and conventional farms) but dropped substantially in 2015. The latter is likely due to a drop in the raw milk prices in the European Union in 2015, as also discussed below.

4.3 Permutation tests for productivity change and its components

To further investigate the changes over time the T_M (1), T_{FS} (2), and T_{EC} (3) are calculated for the balanced subsets of the organic, respectively the conventional, farms as well as T_{FS}^u (4) for the unbalanced groups. Further, permutation tests as described in section 2.1 are performed and shown in Tables 4 and 5.

Considering the Malmquist indexes for all the year-on-year shifts, we note that the test statistics for comparisons are extreme, so the null hypothesis is rejected, meaning that the distributions F_{t_1} and F_{t_2} are not interchangeable for either the organic or the conventional farms. This can be interpreted as significant productivity changes within both groups between all consecutive time periods. To understand the nature of the productivity changes we next consider its components i.e. T_{FS} and T_{EC}. For T_{FS} we do not need a balanced dataset and more information is included when considering the unbalanced version T^u_{FS} shown in the last rows of the tables. As these all are significant, the frontiers are significantly different for all time shifts (or the distribution of points near the frontiers are different). Furthermore, when the efficiency changes T_{EC} are insignificant, we conclude that the productivity changes are likely to be due to frontiers movements.

The change from 2014 to 2015 is particularly interesting: The conventional farms exhibit worse production possibilities in 2015 compared to 2014, whereas the organic farms experienced significantly better productions possibilities in 2015 than in 2014. This explains the findings from Table 2, where the conventional farms in 2015 found the organic technology to be superior, unlike earlier years. It is here worth noting that this does not imply that the organic frontier strictly dominates the conventional frontier in 2015, since the frontiers might still intersect.

The likely reason for the change in 2015 that made organic farming superior to conventional farming for most input-output mixes is the abolishment of the milk quotas in Europe on March 31, 2015. While it (especially in the long run) enables an increased production, it in the short run led to a 25% drop in EU raw milk prices from 40 € per 100 kg in January 2014 to 30 € per 100 kg in January 2016, c.f. e.g. the EU Milk Market Observatory (2018). This price drop had a relatively larger impact on the conventional farms since the premium paid for the organic milk in Denmark is fixed/independent of the price level.

This result is also supported by the more standard profitability analysis in Jørgensen (2017) which show that the profitability of the organic farms became much higher than that of the conventional farms in Denmark in 2015.

Insert Table 4 about here

Insert Table 5 about here

5 Final remarks

This paper has focused on measures of productivity changes over time as well as frontier differences between independent groups. Statistical inference for productivity change measured by the Malmquist index, and the corresponding measures of frontier shift and efficiency change can be performed as permutation tests. These are exact tests and are in a recent paper by Asmild et al. (2018) found to be very powerful. We also suggest a method to measure frontier differences for separate independent groups, which accounts for the inherent bias in DEA estimated frontiers by using a jackknife method to minimize the effect of differences in sample sizes.

Formal tests for the significance of the differences between independent groups can be implemented in line with the methods in Asmild et al. (2018) using permutations. These methods, as well as the power of the tests, will be presented in a forthcoming paper.

The types of analyses presented here can have important policy implications, since the Danish government is focused on enhancing the competitiveness of the agricultural sector in Denmark at the same time as aiming at doubling the organic production between 2007 and 2020. Thus formal analysis comparing the economic production possibilities associated with organic and with conventional (dairy) farming is important as are the analysis of their respective productivity changes over time.

The results of the analysis presented in this paper showed that there might not have been a compelling argument for organic dairy production up until 2014, since the conclusion in terms of which production technology is superior differed depending on the input-output mix. However, in 2015 both the organic and the conventional farmers on average agreed to the organic technology being superior. An explanation for this pattern could be that the conventional farms have been "protected" by high milk prices, partly caused by the quota system. After the abolishment of the milk quotas and the corresponding drop in milk prices, which had a relatively larger impact on the conventional farms than on the organic, the frontier for the organic farming became superior to that of the conventional. This is also evident from the Malmquist index results which showed a large and significant productivity decrease for the conventional farms from 2014 to 2015, likely caused by a significant deterioration of the frontier, but a significant productivity (and frontier) improvement for the organic farms in the same period.

If subsequent analysis find that the difference between the organic and the conventional frontier is indeed significant (once the permutation based tests for comparisons of the frontiers for independent groups are fully investigated and can be applied), and persistent over the subsequent years, the business case for conversion to organic farming may be straightforward (at least if ignoring transition costs). This could also potentially be a solution to the lack of competitiveness for Danish dairy farming identified by Asmild et al. (2019).

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Type	Year	Feed	Labour	OVC	FC	Capital	Milk	OO
Conv	2011	1353	976	1585	1393	1640	4590	1083
		(833)	(448)	(749)	(826)	(794)	(2268)	(953)
	2012	1494	1006	1658	1414	1641	4652	1182
		(992)	(481)	(822)	(776)	(791)	(2468)	(1010)
	2013	1798	1057	1722	1456	1681	5558	1001
		(1133)	(531)	(841)	(732)	(819)	(2922)	(1013)
	2014	1703	1116	1788	1568	1726	5811	952
		(1081)	(576)	(870)	(860)	(850)	(3056)	(1017)
	2015	1702	1137	1757	1571	1768	4968	1183
		(1154)	(615)	(874)	(956)	(902)	(2824)	(916)
Org	2011	1096	1002	1352	1493	1925	4490	1076
		(803)	(472)	(600)	(833)	(846)	(2246)	(728)
	2012	1271	1036	1413	1498	1983	4542	1138
		(1010)	(463)	(644)	(639)	(861)	(2314)	(794)
	2013	1561	1087	1488	1561	2019	5324	957
		(1357)	(543)	(759)	(729)	(902)	(2964)	(701)
	2014	1569	1163	1558	1679	2079	5815	954
		(1422)	(629)	(807)	(842)	(990)	(3463)	(728)
	2015	1486	1168	1482	1656	2106	5504	1257
		(1272)	(608)	(677)	(796)	(974)	(3067)	(826)

Table 1: Averages and standard deviations (in parenthesis) within year and group (in 1000 DKK)

	2011	2012	2013	2014	2015
			With jackknife		
Organic	0.840	0.900	0.864	0.934	0.786
Conventional	0.988	1.023	1.041	1.071	0.920
Average	0.911	0.959	0.948	1.000	0.850
			Without jackknife		
Organic	0.949	1.008	0.994	1.084	0.872
Conventional	1.070	1.115	1.167	1.206	0.991
All	1.054	1.102	1.144	1.191	0.976

Table 2: Group frontier differences, $T_{FD}^{g_1;g_2}$ and its components (as in (6)).

Type	ϕ_{Feed}	ϕ_{Lab}	ϕ_{OVC}	ϕ_{FC}	ϕ_{Cap}	ϕ_{Milk}	ϕ_{OO}
Conv	1.098	1.249	1.060	1.125	1.057	0.239	1.332
Org	1.177	1.244	1.146	1.097	0.950	0.229	1.342
LR	40.26	0.758	123.74	12.71	120.68	1.835	1.835
p	0.0000	0.384	0.0000	0.0004	0.0000	0.176	0.176

Table 3: Average input- and output angles ϕ in 2015, and test statistics for comparisons (conventional and organic) and corresponding p-values.

	2011-12	2012-13	2013-14	2014-15
No. obs. balanced	1373	1348	1332	1220
No. obs. first year	1567	1530	1499	1454
No. obs. second year	1530	1499	1454	1355
	Balanced data set			
T_M	0.970 (0.000)	1.050 (0.000)	1.026 (0.000)	0.871 (0.000)
T_{EC}	1.002 (0.836)	0.991 (0.510)	1.000 (0.982)	1.059 (0.002)
T_{FS}	0.968 (0.000)	1.059 (0.000)	1.026 (0.145)	0.823 (0.000)
	Unbalanced data set			
T_{FS}^u	0.971 (0.000)	1.081 (0.000)	1.031 (0.063)	0.811 (0.000)

Table 4: Test statistics and significance probabilities for the subset of conventional farms (based on 1000 permutations).

	2011-12	2012-13	2013-14	2014-15
No. obs. balanced	200	186	178	166
No. obs. first year	223	214	206	196
No. obs. second year	214	206	196	179
	Balanced data set			
T_M	0.9511 (0.000)	1.0383 (0.000)	1.0152 (0.009)	1.0172 (0.002)
T_{EC}	1.0119 (0.477)	1.0159 (0.228)	1.0477 (0.006)	0.9928 (0.412)
T_{FS}	0.9399 (0.000)	1.0220 (0.123)	0.9689 (0.191)	1.0246 (0.009)
	Unbalanced data set			
T_{FS}^u	0.926 (0.000)	1.089 (0.000)	0.961 (0.027)	1.021 (0.023)

Table 5: Test statistics and significance probabilities for the subset of organic farms (based on 1000 permutations).