Tail-Risk Hedging

An Empirical Study

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Abstract

All investors are interested in avoiding losses, as doing so can dramatically increase accumulated returns in their portfolios. Put options can effectively mitigate tail risk in equity portfolios but are often found to be too expensive, imposing a substantial drag on portfolio performance. It is however common in the current literature to focus on passive buy and hold put option strategies, i.e. buying options and holding them until maturity. In our paper we implement a rule-based monetization put option strategy, where we allocate a certain percentage of an equity portfolio's capital to buy put options and combine this with a strategy where the put options are sold if their market price reaches a pre-determined target. The proceeds are then re-invested into equites and new put options. We compare the results to an unhedged position in the underlying equities, represented by the S&P500 TR index, and a position in a constant volatility strategy.

We find little evidence that the put option monetization strategies reduce portfolio drawdowns, and we find that all the monetization strategies have lower total portfolio returns and Sharpe ratios as compared to the index. The results are conflicting with other claims and research from option practitioners and can possibly be explained by our inclusion of real life data and differences in methodology. The tested constant volatility strategy reduces drawdowns more effectively as compared to the put option monetization strategies, and it also earns a higher total return and Sharpe ratio as compared to the index. The constant volatility strategy utilizes a negative relationship between market volatility and market returns, and the persistence of market volatility.

Our results suggest that simple rule-based monetization strategies are not able to adequately reduce drawdowns and enhance portfolio returns, and that other actively managed strategies, utilizing additional techniques, could be needed to make put option strategies profitable. Investors and portfolio managers may find it easier to achieve the goals of better risk adjusted returns by the implementation of a constant volatility strategy.

Keywords: tail-hedge, put options, option monetization strategy, constant volatility.

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1 Introduction

Stock markets are a fundamental piece of capitalism and enable entrepreneurs to get capital to their ventures from return seeking individuals and organizations. They are often seen as places where wealth is created, but there are also those who have seen, and experienced, the destruction of wealth in the markets. This is because markets are inherently risky. No one can predict the future, and thus no one can be certain that the entrepreneur's company will be successful and earn money for its stockholders; the future is open, as Karl Popper put it.

There are many sources of risk in the stock markets and they can be divided into company specific (idiosyncratic) risks, e.g., product failures and fraudulent management, and market (systemic) risks, e.g., interest rate changes and wars. A common way for investors to reduce risk in their portfolios is diversification, which effectively reduces idiosyncratic risk as it is less likely that twenty companies will have failing products, compared to one single company failing. In order to reduce systemic risks, investors can diversify further by investing in different asset classes, such as debt, real estate, commodities and cryptocurrencies. Individuals and organizations can do this by themselves, but more commonly, they outsource the investment decisions to investment professionals that invests the capital using passive or active strategies.

Booms and busts are characteristics of the financial markets, and they affect investors and portfolio managers regardless of their strategies being passive or active. Classical and frightening examples of market crashes, often called *tail events*, include Black Monday in October 1987, when the Dow Jones Industrial Average index fell over 20 percent in a single day, the financial crisis in the fall of 2008, and the recent COVID-19 crisis[1]. Plunging stock prices are usually accompanied by higher market volatility, and in these periods, stock correlations tend to increase which limits the benefits of diversification (Junior & Franca, 2012). These types of large, broad drawdowns are thus negative for all investors with long positions in equities and raises the question: can they be avoided?

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[1] The term *tail events* refers to returns being distributed in form of a bell shape, where large negative returns are placed in the left tail of such a distribution.

The answer to that question is yes, they can. Investors can protect themselves from large drawdowns through the inclusion of various portfolio insurance techniques, but this is not a free lunch. The arguably most solid protection, excluding holding cash, comes from buying put options. Put options can easily be used on a passive index portfolio to create a floor value of the portfolio's holdings during a certain time period. However, put options are often considered as being too expensive and they often impose a substantial drag on portfolio performances (Ilmanen, Thapar, Tummala & Villalon, 2020).

Some practitioners, such as Nassim Nicholas Taleb, Mark Spitznagel and Vineer Bhansali are, however, frequently and sometimes loudly arguing for the opposite (Bhansali, 2013; Spitznagel & Paul, 2013; Taleb, 2013)[2]. Bhansali criticizes the literature on put option returns for often applying a passive buy and hold strategy, and he argues that there is nothing pristine about doing so and draws an analogy to bonds, an instrument investors frequently do not hold until maturity. Instead, he suggests four different ways a portfolio manager actively could manage put options as a part of an equity portfolio in a way that he argues not only would limit drawdowns, but also enhance returns. A strategy involving selling the put options after they have increased in value and reached a target multiple, or *monetizing* them, is one simple rule-based approach. Bhansali shows this strategy to be effective in achieving those goals in comparison with a passive put option strategy and a no hedging strategy.

Our purpose with this thesis is to add to the currently limited amount of literature on strategies that involves applying active management of put options as a means to hedge tail risk. We do this by employing the monetization strategy, proposed by Bhansali, on real data. The research question of this paper is consequently:

Can a rule-based put option monetization strategy protect an equity portfolio from large drawdowns and enhance portfolio returns?

To test this, we implement eight different put option monetization strategies, in combination with the S&P 500 TR index, and compare the results to an unhedged position in the index itself, and to a position in a constant volatility strategy. The latter is a portfolio allocation method that

[2] It is worth noting that all three of them have personal interests in tail hedging funds.

repeatedly has been empirically proven to mitigate tail risk and enhance returns compared to passively holding equity indices.

We implement the monetization strategies buy buying three-month S&P500 put option, where the annual allocated budget to the tail hedge strategy, 1.5 or 3.0 percent of the portfolio, decides the moneyness of the options. If the put option price hits the pre-defined target multiple, either 2.5x, 5.0.x, 7.5x or 10.0x, the put options are sold and the proceeds reinvested in the index and in new put options. The constant volatility strategy is implemented by forecasting the future volatility, using a GARCH(1,1) model, and then allocating the portfolios capital in the S&P500 TR index based on a pre-determined target volatility. This involves the use of leverage when the forecasted volatility is lower than the target, and de-leveraging when the forecasted volatility is higher than the target.

All strategies are tested during the time period 1996 to 2020, which includes the burst of the dotcom bubble, the financial crisis, and the COVID-19 crisis. We compare the strategies' performance by looking at a variety of different portfolio metrics, where we focus the discussion on maximum drawdowns, the total returns, and the Sharpe ratios. In our 25 year dataset, there are several interesting time periods we look closer at. These periods are the three earlier mentioned market crises in addition to the years 2010-2017, which is a period characterized by a longer, calmer bull run.

We find little evidence that the put option monetization strategies reduce portfolio drawdowns, and all the monetization strategies have lower total portfolio returns and Sharpe ratios, as compared to the S&P500 TR index. Our results indicate that simple, rule-based monetization strategies are not able to adequately reduce drawdowns and enhance returns of an equity portfolio, compared to an unhedged equity portfolio. The findings are in line with most earlier research on put options as tail hedges but is conflicting with the results of Bhansali (2013). The conflicting results may be a direct consequence of the proposed strategy being tested on real life data, but also due to a difference in the choice of the time to expiry of the options used. It is also a possibility that targeting further out of the money options could be better hedges as proposed by Taleb (2013).

Our tested constant volatility strategy reduces drawdowns more effectively compared to the put option monetization strategies, and the strategy also have a higher total return and Sharpe ratio as compared to the index. The strategy benefits from a negative relationship between market volatility and market returns, effectively leveraging up during low volatility periods, where returns tend to be positive, and to deleveraging during volatile periods, where returns tend to be large and negative. It also manages to reduce the fat tails of the return distribution, and the results are consistent across a wide range of volatility targets.

While our tested monetization strategies were not successful, it is important to note that we are not able to rule out actively managed put options as unprofitable. We especially encourage future research to investigate the use of indirect hedges in order to exploit increasing correlations during market crises, and strategies involving buying further out of the money put options.

The rest of the paper is structured as follows: section two introduces the reader to modern portfolio theory, the Black-Scholes model for option pricing, and GARCH-modelling. Section three takes a closer look at some of the existing literature on passive vs active investing, put options as tail hedges, and constant volatility strategies. In section four, we present our data and methodology, section five subsequently presents our results which then are discussed in section six. Section seven concludes the paper.

2 Theory

This section presents the foundation for modern portfolio theory, introduces the Black-Scholes option pricing model, and describes the GARCH model used to model volatility.

2.1 Mean Variance Framework

It is almost a law of nature that investors want high return and low risk investments. The most famous framework in finance for modelling this tradeoff is the mean variance framework developed by Harry Markowitz (1952, 1959). He showed that, given the previously mentioned preferences, it is possible to compute an optimal portfolio for any given time period. To create this optimal portfolio, one needs the information on risky assets' expected returns, variances and covariances, and to assume that the returns follow a normal distribution in addition to assuming that volatility and correlations are constant. It is then possible to find a portfolio of assets that maximizes the expected return for the investor for every level of risk, measured as volatility. This is normally referred to as the efficient frontier and is visualized in Figure 2.1, where the green squares represent the individual assets used to create the frontier.

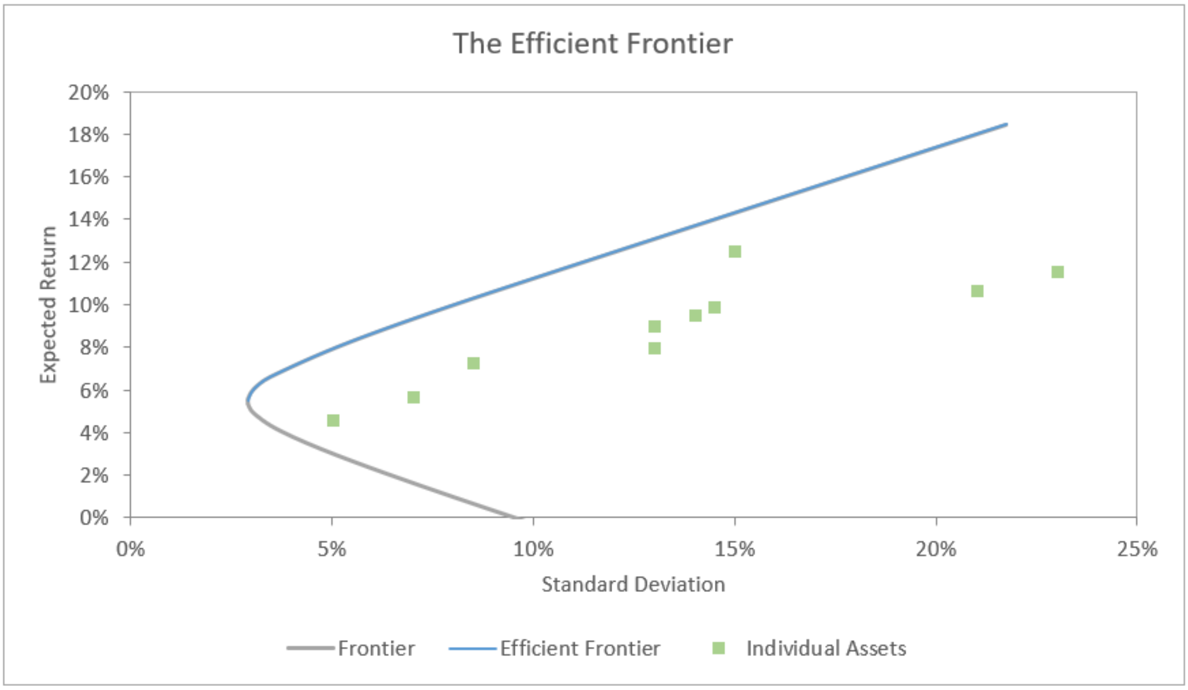

Figure 2.1: The Efficient Frontier

By adding a risk-free asset to the other risky assets, it becomes optimal for all investors to hold the same mix of risky assets. This particular portfolio is called the tangency portfolio and is, in any combination with the risk-free asset, the portfolio with the highest risk adjusted return, or *Sharpe ratio*, out of all portfolios found in the efficient frontier[3]. The investors can combine this portfolio together with the risk-free asset according to their own risk preference, and the different combinations results in a new, straight, efficient frontier called the Capital Allocation Line (CAL) as seen in Figure 2.2.

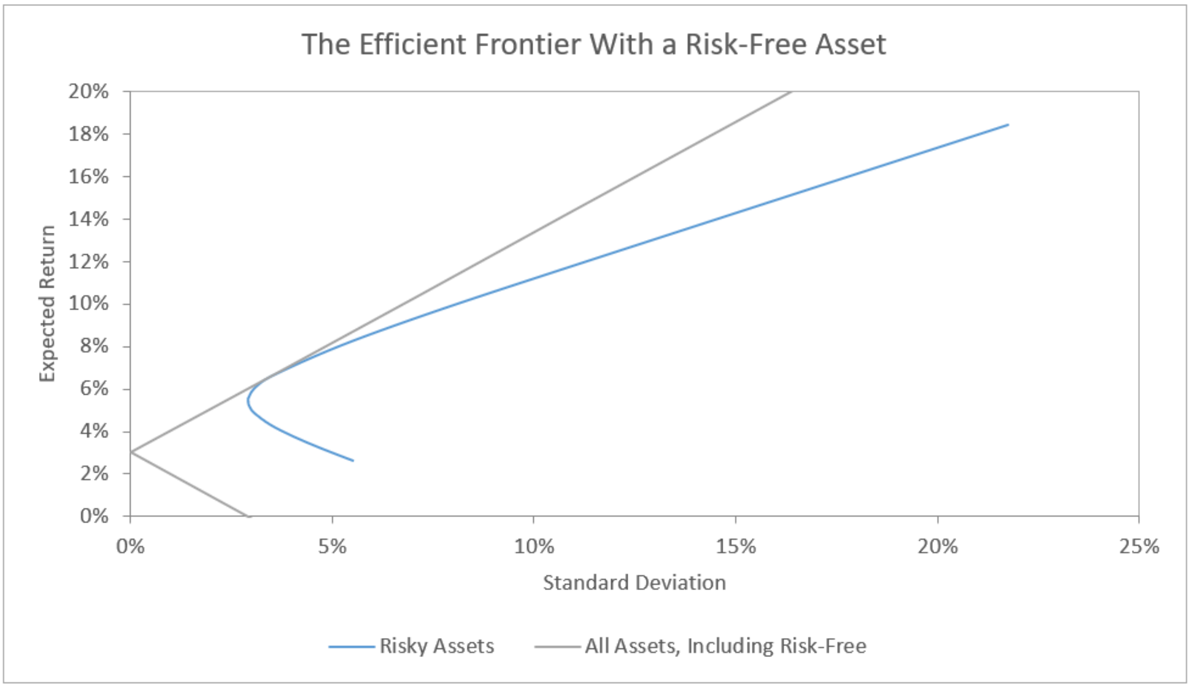

Figure 2.2: The Efficient Frontier Including a Risk-Free Asset

2.2 CAPM

A further development in financial theory which builds on the mean variance theory is the Capital Asset Pricing Model (CAPM), derived by Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966).

If one assumes that investors:

[3] See p. 50 for a formal description of the Sharpe ratio.

- Have mean variance preferences and share the same investment horizon.
- Have homogenous beliefs regarding the expected returns, volatilities, and correlations of securities.
- Can borrow and lend at the risk free rate.
- Are price takers.
- Can trade all assets without frictions, e.g., transaction costs and taxes.

A consequent is that all investors should hold the tangency portfolio, and since every security is owned by someone, the conclusion that the tangency portfolio equals the market portfolio follows. Given that all investors are well diversified holding the market portfolio, an investor gets no compensation for bearing idiosyncratic (i.e. firm specific) risk and an asset's risk premium is therefore derived solely from how it covariates with the market portfolio and thus the systemic risk it bears. (Berk & DeMarzo, 2019)

The CAPM-equation that describe a single asset's return is:

$$R_i = R_f + \beta_i \times \left(R_m - R_f\right)$$

Where Beta is given by:

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)}$$

The equations imply that the market portfolio has a beta of one, less risky investments have lower betas and subsequently lower expected returns, while more risky investments have higher betas and a higher expected returns. It is easy to see that this results in a linear relationship between an asset's beta and its expected return. This relationship is called the Security Market Line (SML) and is plotted in Figure 2.3. Jensen (1967) tested the CAPM and found no evidence of portfolio managers' ability to produce higher returns than predicted by CAPM, or generate *alpha,* as it is called after the publication. If CAPM holds, that should also be the case since all assets are to be found on the SML. If an event would lead to a stock getting a higher expected return, investors would quickly start to buy it, causing the price of the stock to go up, thus lowering the expected return until it is back on the SML and the market again is in equilibrium. As visualized in the figure, the beta of an asset can also be negative. Such an asset has a negative correlation with the market portfolio and will likely have a negative expected return. It will however pay off when the market portfolio declines in value and could as such be seen as a form of insurance.

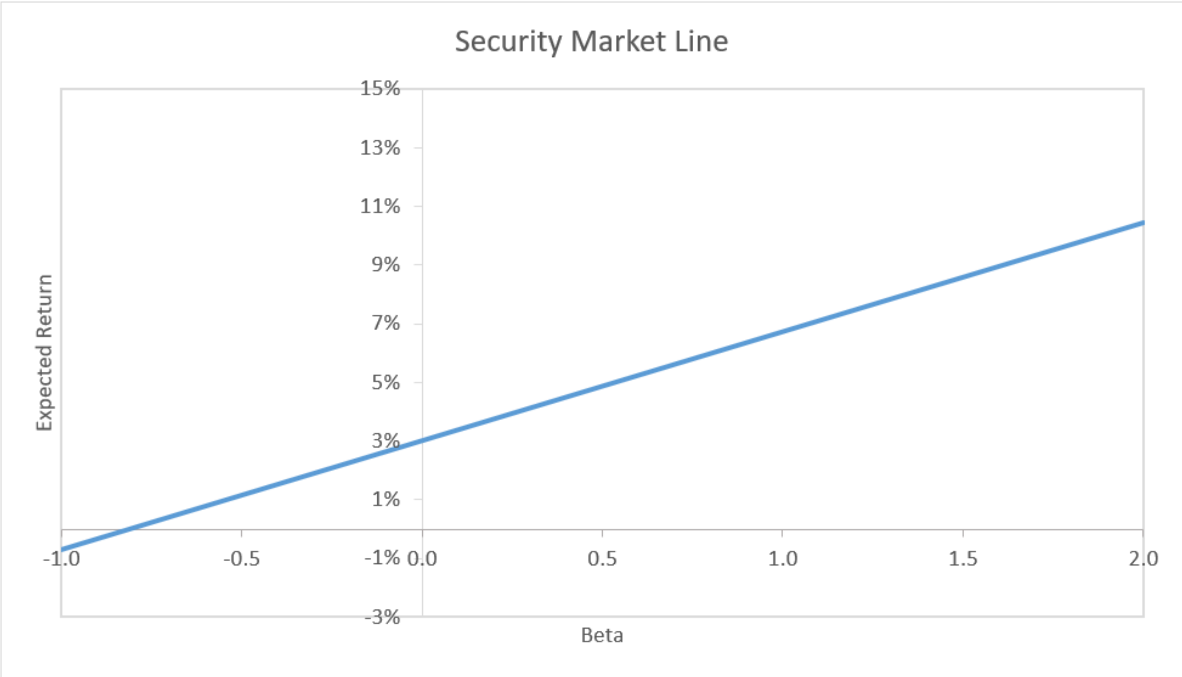

Figure 2.3: The Security Market Line

In practice, the market portfolio is not observable since there is no competitive price data on all assets that should be included in it, e.g. art and jewelry. It is therefore not uncommon to use an index such as the S&P500 as a proxy for it, but this feature of the CAPM theory makes it impossible to reject, since advocators for CAPM could always argue that it was not tested with the true market portfolio. (Berk & DeMarzo, 2019)

2.3 Efficient Market Hypothesis

A further development of financial theory following the mean variance and CAPM frameworks is the Efficient Market Hypothesis (EMH) laid out by Fama (1970). If investors have homogenous beliefs about the future, they must have access to the same information today, thus prices in the financial markets must reflect all available information. Therefore, stock prices are correctly priced today and will only react to new information, i.e., markets are efficient. Fama defined three different forms of efficiency:

- Weak form: prices reflect all information from historical returns. This implies that it is not possible to outperform the market with technical analysis.
- Semi-strong form: prices reflect all publicly available information. This implies that it is not possible to outperform the market with the help of technical or fundamental analysis.
- Strong form: prices reflect all available information. This adds to the above, that outperformance is not possible even with the use of insider information.

The most plausible form of market efficiency out of the three is the semi strong one, given the many actors, such as analysts, conduct all forms of analyses, and that usage of insider information is restricted by law (Berk & DeMarzo, 2019). However, all forms of market efficiency assumes that the acquiring of information is costless for all market participants. Grossman and Stiglitz (1980) argue that since there is a cost associated to the obtaining of information, e.g. the wage of an analyst, efficient markets are not possible. Thaler (2015) also claims that the market is not efficient, and especially points to the underlying assumption of the EMH that investors are completely rational, which he proves that they are not. Thaler and Fama are although in agreement that markets are unpredictable (Chicago Booth Review, 2016).

Evidence against the EMH, or the CAPM, comes in form of numerous market anomalies[4]. Fama and French (1993) introduced the three factor model where two factors were added to the CAPM. The added factors were SMB (Small-Minus-Big) and HML (High-Minus-Low). SMB is a factor relating to companies' market capitalization, where small companies were found to outperform big companies. The HML factor relates to the valuation of the companies, where low valued stocks (value stocks) were found to outperform high valued stocks (growth stocks). Another well-known anomaly is the momentum factor, where buying past winners and selling past losers have been found the be a profitable strategy (Jegadeesh & Titman, 1993).

[4] To test the EMH, a perfect model for explaining returns is needed. If one conducts a test of the EMH with the CAPM and detect anomalies, it is impossible to know if it is the EMH or the CAPM that should be rejected.

2.4 Black Scholes Option Pricing Model

There is an ocean of derivative products, i.e. securities that derive their value from an underlying asset or benchmark, in the financial markets today. Among the most common ones are options. An option gives the buyer the right, but not the obligation, to buy or sell an asset at a certain price in the future. Since the scope of the paper only involves the use of European put options, i.e., a contract that allows the investor to sell an asset at a specified price and time, we will not discuss call options or American options in this section. We will also focus on giving the reader intuitive knowledge about the payoff and the pricing of put options, thus advanced readers who seek further knowledge about options are directed to Hull (2018).

We will provide examples of the payoff and pricing of a European put option on a non-dividend paying stock as presented in Hull (2018), as the intuition do not differ significantly between having a dividend paying stock or a stock market index as the underlying asset.

Five different variables affect the pricing of a European put option:

- The price of the underlying asset (*S*).
- The strike price of the option, i.e., at what price can the underlying be sold (*K*).
- The expiration date of the option, i.e., at what day can de underlying by sold (*T*).
- The volatility of the stock price (σ).
- The risk-free interest rate (*r*).

The buyer of such an option gets the choice to sell the stock at time T, for the price of K and the payoff to the buyer can therefore be specified as:

$$Payoff = Max(K - S(T), 0)$$

Where S(T) is the stock price at time T. The buyer of the contract will thus earn money if the price of the underlying stock is lower than the strike price, i.e., the put ends up *in the money*, at the end of the put option’s life. If the stock price at time T is higher than the strike price, i.e., the option ends up *out of the money*, the owner of the option will of course choose not to exercise the option, since the price of the stock is higher in the market and he can sell it there instead. A put option's payoff, with respect to different stock prices at time T, is visualized in Figure 2.4, where K = 95. A put option can never be worth more than its strike price and is usually substantially cheaper to buy or sell compared to the underlying stock itself. As seen in the figure, it is however similar to the payoff an investor would get from shorting the stock and as it is cheaper, it is therefore to be seen as a leveraged product.

Figure 2.4: The Payoff of a Put Option with Strike 95

Due to the put option's ability to generate a payoff when stock prices fall, it can serve as an insurance for stock declines. If an investor combines an investment in a stock with the purchase of a put option, a payoff structure, often called a *protective put*, like the one pictured in Figure 2.5, is created. It can clearly be seen in the figure that there is a "floor" in the payoff structure of these positions, created by the gains from the put option when the stock price is lower than K (95). An insurance like this is of course not acquired for free, and it is important to note, as can be seen in Figure 2.6, that the initial cost of buying the option affects an investor's profit negatively if it ends up out of the money.

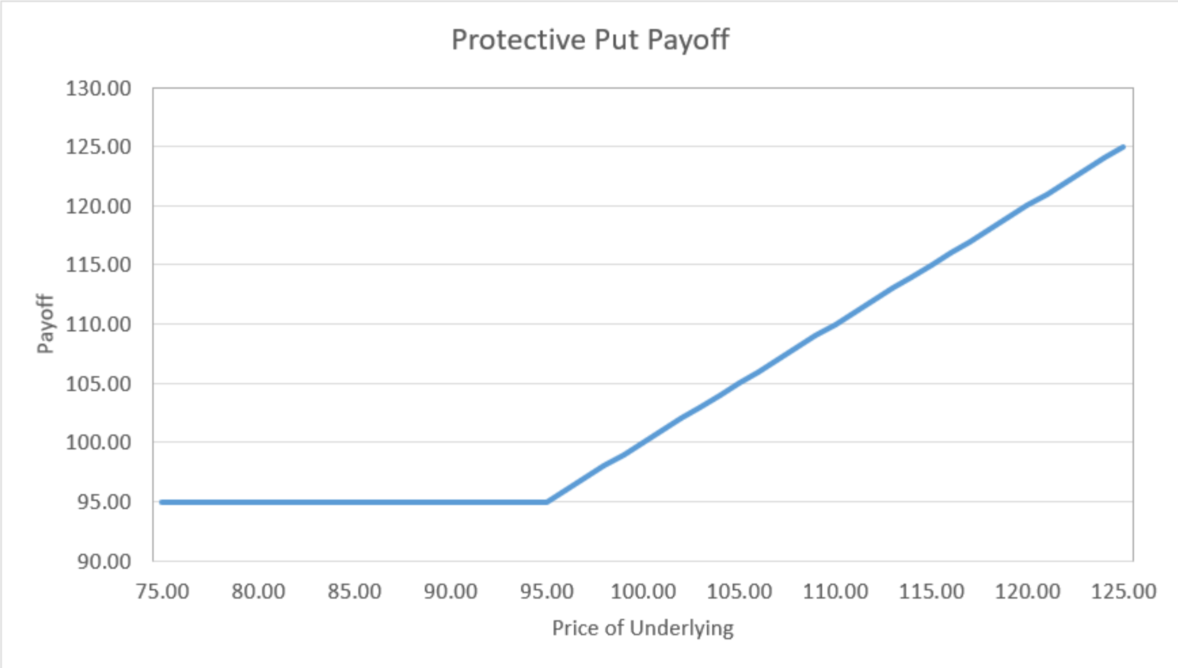

Figure 2.5: The Protective Put Payoff with Strike 95

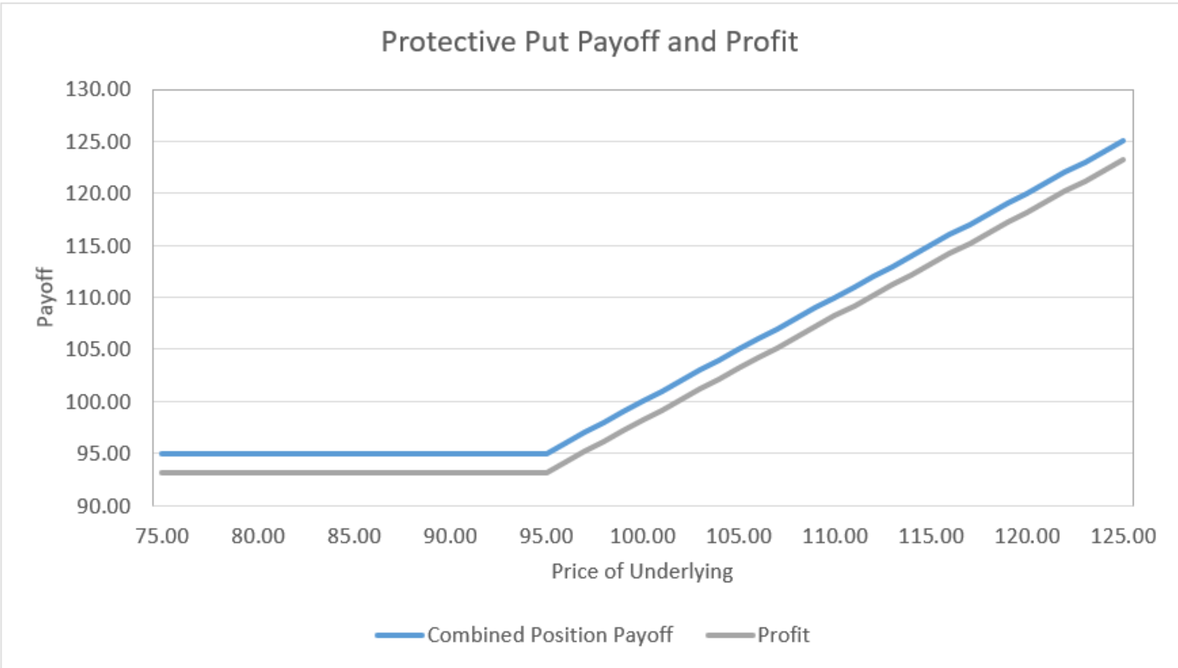

Figure 2.6: The Protective Put Payoff and Profit with Strike 95

The Black-Scholes model, also sometimes referred to as the Black-Scholes-Merton model, developed in the beginning of the 1970s by Black and Scholes (1973) and Merton (1973), is usually used to price options. One of the key insights is that option payoffs can be created by mixing positions in a risk-free asset and a stock. To avoid arbitrage, the option and the synthetically created option, consisting of the risk-free asset and the stock, must then be priced the same. The model uses some simplifying assumptions about the stock price characteristics and trading environment, and the resulting pricing formula for a put option is:

$$p = Ke^{-rT}N(-d_2) - SN(-d_1)$$

Where:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

and N(d1/d2) is the cumulative probability function for a variable with a standard normal distribution. The formula needs the previously mentioned five inputs to calculate the put option price, and it is fairly straightforward to understand how changes in the variables affect the price. If we assume all other variables to stay the same, an increase in:

- The stock price would lead to a decreased chance of the put option ending up in the money, i.e., the put option price decreases.
- The strike price would lead to an increased chance of the put option ending up in the money, i.e., the option price increases.
- The time to expiry lead to an increased chance of the put option ending up in the money due to an increased window of possible events affecting the stock price negatively, i.e., the option price increases.
- The volatility would lead to an increased chance of the put option ending up in the money due to an increased probability of large declines of the stock price, i.e., the option price increases.
- The discount rate would decrease the net present value of the strike price which is the price the owner of the option could sell the stock for, i.e., the option price decreases.

Applying the pricing formula with the inputs S = 100, K = 95, T = 0.25 (i.e., three months), σ = 0.20 and r = 0.01 results in a put option price of 1.813. These specifications are used in the following examples where we show how the pricing of put options is affected by changes in the stock price, time to expiry and volatility, again keeping all other variables constant.

An option's sensitivity to changes in the stock price is measured by the option's delta, which is the first derivative of the put option price with respect to the underlying stock price. As can be seen in Figure 2.7, the delta for a put option is always between -1 and 0 due to the negative correlation between the underlying stock price and the value of the put option. It can intuitively be understood that the delta of a far out of the money put option is close to zero, since it is of little importance to the value of a put option with a strike price of 95 if the stock price is 114 or 115. On the other hand, a change in the underlying stock price has a very tangible effect on the put option value that is deep in the money, since the stock price movements directly affects the size of the payoff at expiry, and in these cases delta approaches -1.

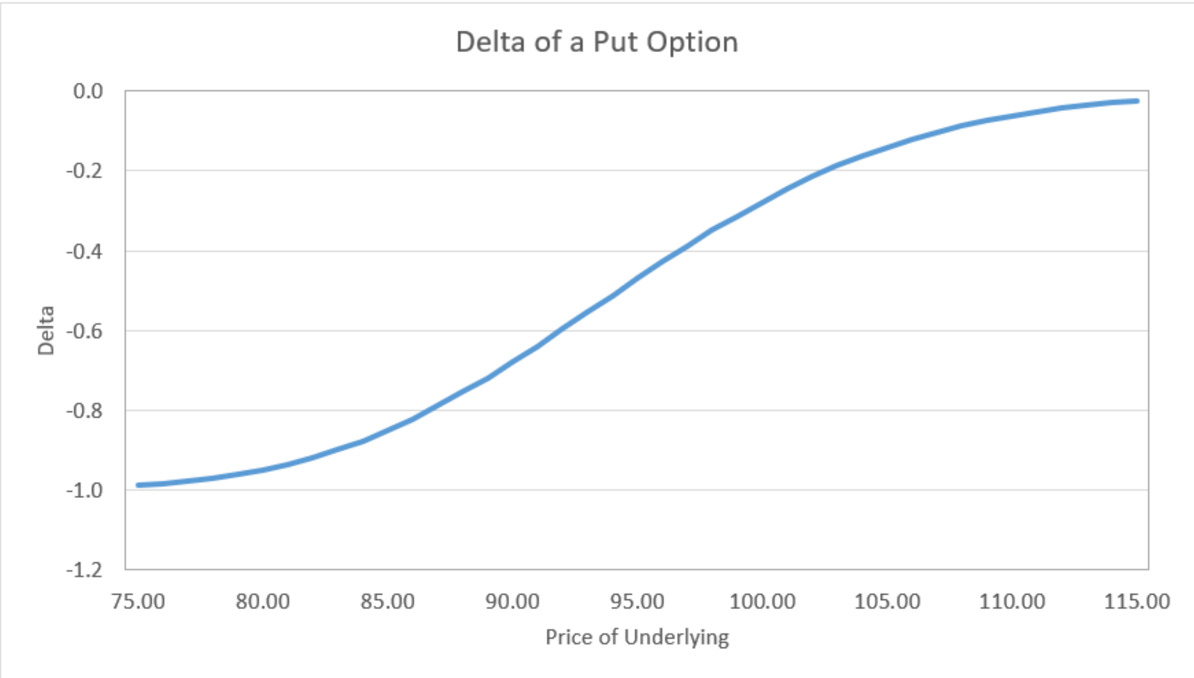

Figure 2.7: The Delta of a Put Option with Strike 95

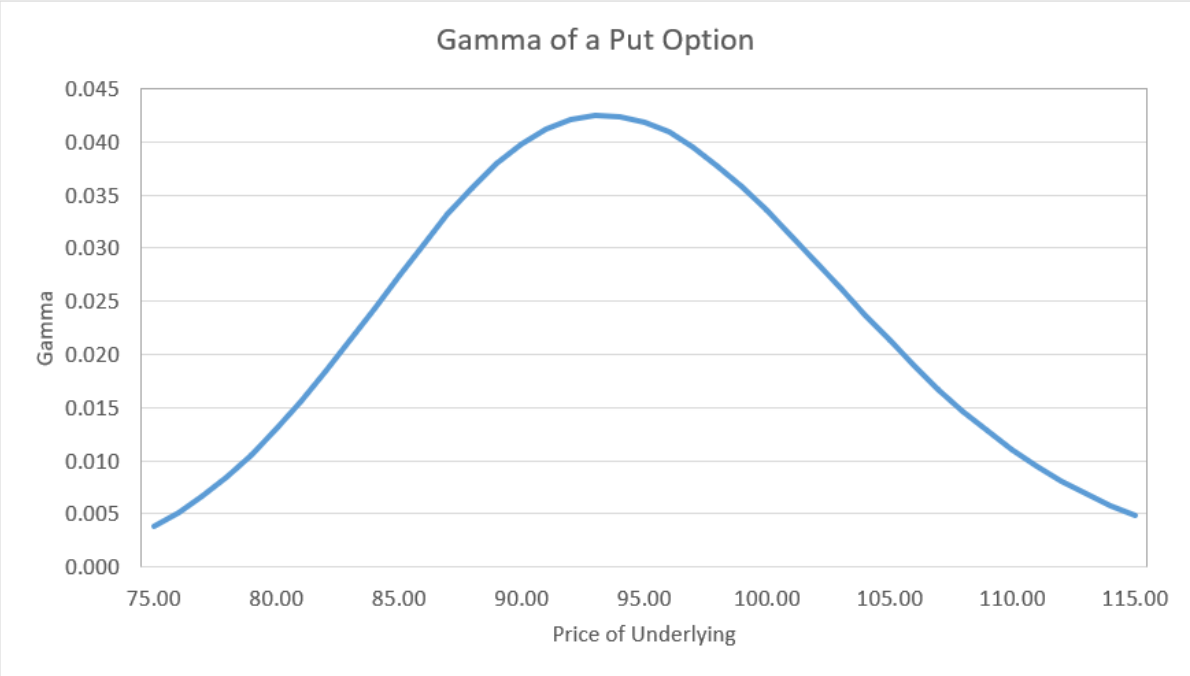

Figure 2.8: The Gamma of a Put Option with Strike 95

The speed in which delta increases or decreases, i.e., the second derivative of the put option price with respect to the stock price, is called the option's gamma and is pictured in Figure 2.8. Given the earlier laid out intuition, it follows that the rate of change of delta, i.e., gamma, should be low when the option is either deep in or far out of the money, and this is indeed pictured in Figure 2.8, where gamma is highest close to the option's strike price.

An option's sensitivity to passage of time, also referred to as *time decay*, is measured by its theta, which is the first derivative of the option price with respect to time to expiry. As can be seen in Figure 2.9, theta is usually negative and more so with shorter time to expiry and Figure 2.10 shows theta to be most negative when the option is at the money.

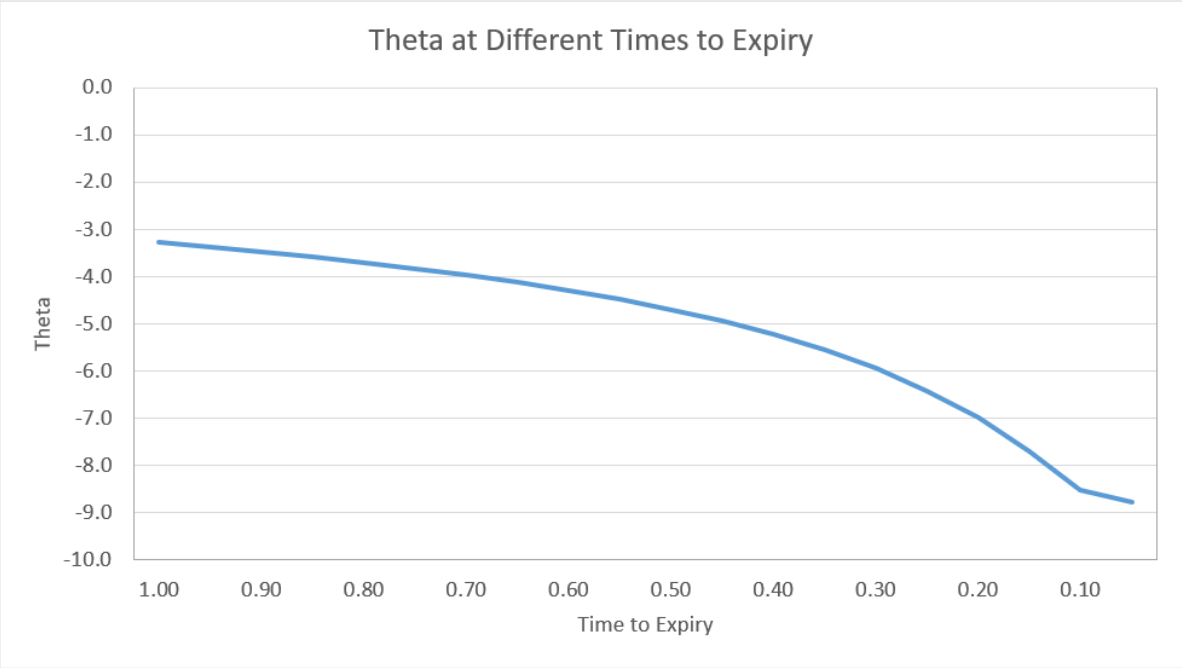

Figure 2.9: The Theta of a Put Option Based on the Time to Expiration

Figure 2.10: The Theta of a Put Option with Strike 95

Option prices are also sensitive to changes in volatility. This is measured by vega which is the first derivative of the option price with respect to volatility. In Figure 2.11 we can again see that changes in volatility affects the price of the option the most when the option is at the money. Figure 2.12 depicts a fictional scenario where an investor buys the put option previously specified at day zero. At day one, an event in the markets causes the volatility to permanently rise to 0.6 without affecting the underlying stock price or interest rates. In the figure, we can see the effects of this event during the life of the option, where the rise in volatility first causes a sharp increase in the option price, which slowly decreases as time passes to eventually end up worthless at expiry.

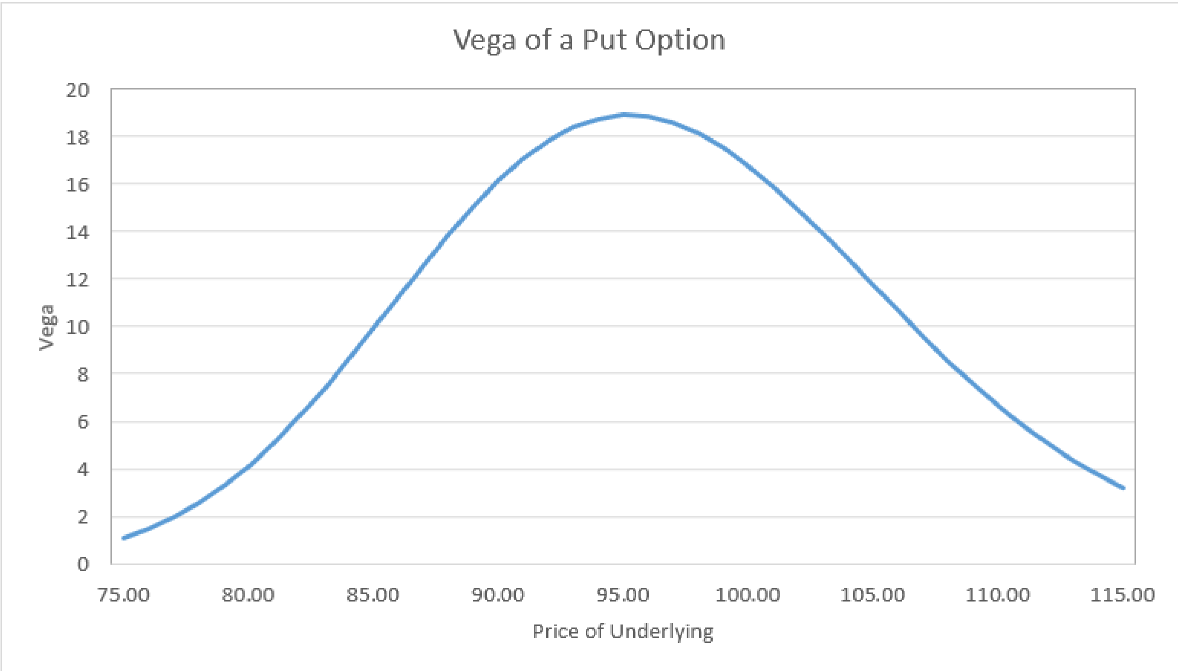

Figure 2.11: The Vega of a Put Option with Strike 95

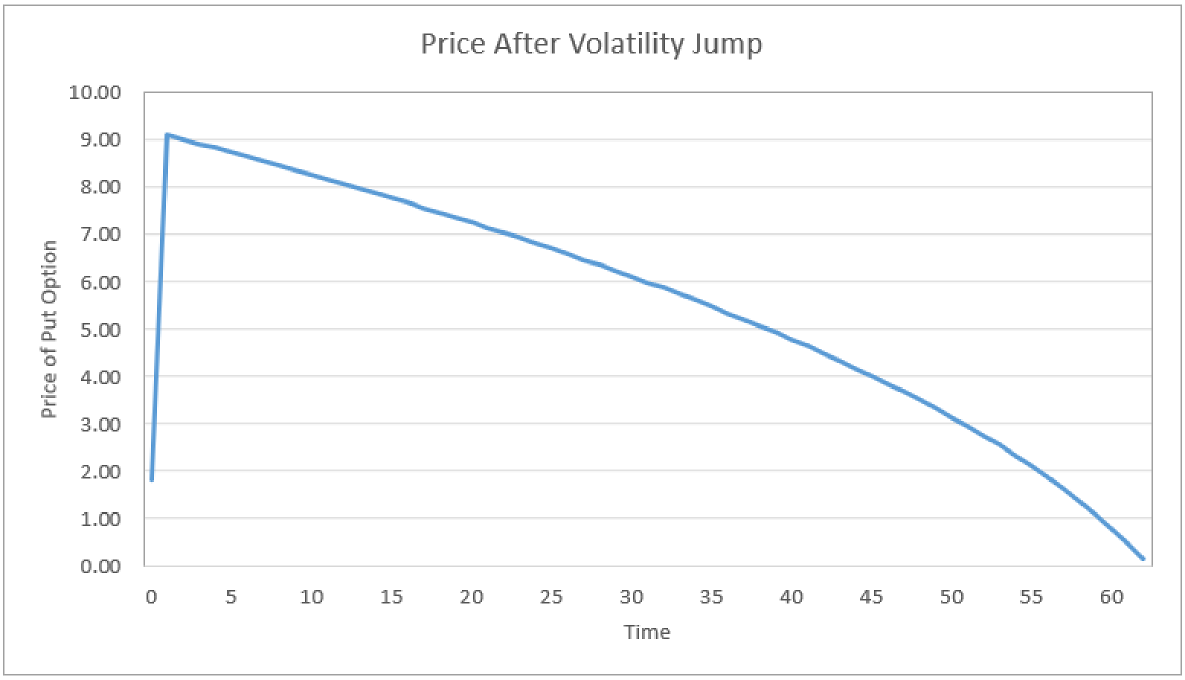

Figure 2.12: The Price of a Put Option After Volatility Jump

In practice, investors are only able to observe four out of the five required inputs in the Black-Scholes model: the stock price, the risk-free rate, the strike price, and the time to expiry. Since options are traded and their prices are observable, it is however possible to use option prices as an input in the Black-Scholes model and back-solve for the value of volatility. This measure is called the implied volatility. Given the assumptions underlying the Black-Scholes model, the implied volatility should be the same regardless an option's strike price, or *moneyness*, but this is not observed in real option prices. The implied volatility from option prices instead reveals a *volatility smile*, where the implied volatility is higher for far out of the money call options, and even higher for far out of the money put options, i.e., they are more expensive than the Black-Scholes model suggest them to be. This stems from the fact that stock returns are not entirely normally distributed, instead they have fatter tails and display a negative skewness compared to normal distributions, they also have more observations in the middle than the normal distribution would suggest. (Hull, 2018; Berk & DeMarzo, 2019)

2.5 GARCH

We mentioned in the earlier section how implied volatilities could be back-solved from observed option prices using the Black-Scholes formula. This is the approach the Chicago board options exchange (Cboe) uses to calculate its famous VIX index, which is presented in Figure 2.13 below.

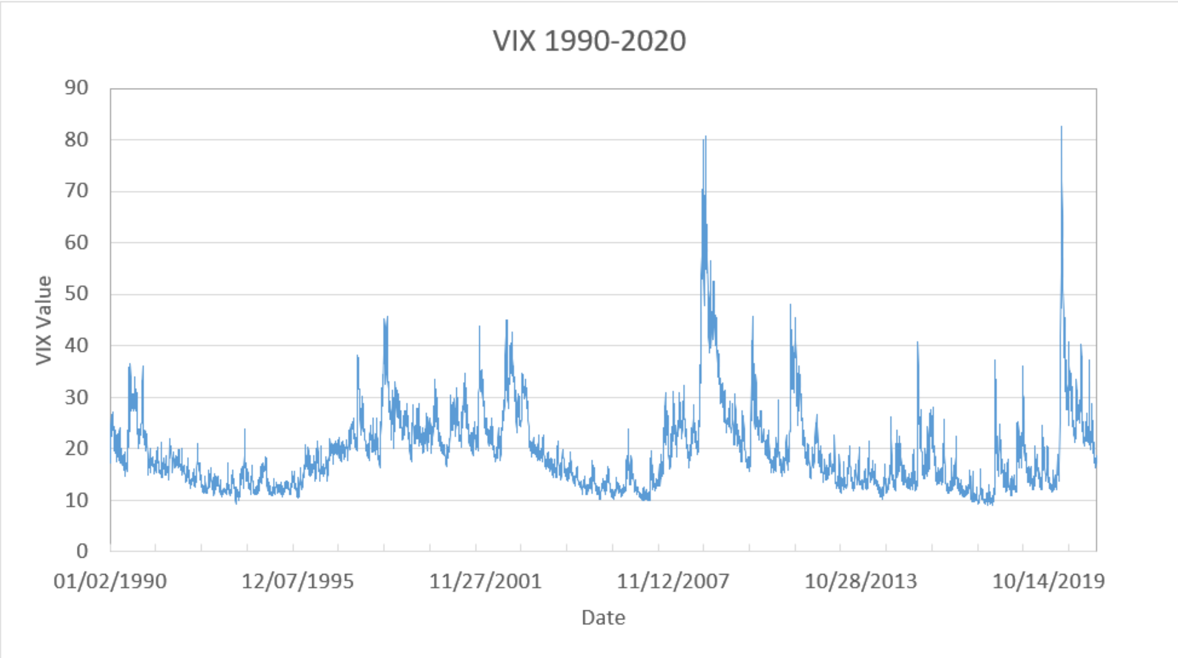

Figure 2.13: VIX Index Historical Level, 2nd January-30th April 2020. Data retrieved from Cboe.

One of the assumptions in the mean variance framework is that the volatility of returns is constant. It only takes one look at the VIX index to find this assumption to be inaccurate. Looking at the index, it is also possible to notice that the time series seems to be quite persistent, meaning that periods with low volatility are likely to be followed by periods with low volatility and vice versa, this phenomenon is called volatility clustering. The time series in the figure also seems to revert to a long term mean after periods of low or high volatility.

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model was proposed by Bollerslev (1986) after expanding on the work of ARCH models by Engle (1982). It is today the standard method for modelling the observed behavior of financial assets, where the GARCH(1,1) model seen in equation below is the most widely used (Hull, 2018):

$$r_t = \mu + \epsilon_t$$

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

The GARCH(1,1) variance estimate can be described as a weighted average of three parts:

- The unexpected return the previous period.
- The estimated variance the previous period.
- The long term mean variance level.

The alpha in the equation represents the weight given to the unexpected return, calculated as $\epsilon_t = \sigma_t e_t$, and is a reflection how big impact news has on the variance estimate. The beta represents how lasting the news effects are on the variance level and omega represents the long term mean variance level. The mentioned weights are usually found by applying a maximum likelihood method on historical data. Other GARCH(p,q) models can be applied where weights are given to p lags of the unexpected returns and q lags of the previous estimated variance. There are also models that limits outliers' impact on the variance estimates which can improve the results from using GARCH models (Carnero, Pena & Ruiz, 2012).

3 Literature Review

If taken by face value, the EMH implies that there is not much to be gained from actively trading a portfolio of assets, but empirically there are a lot of actively managed portfolios and research that would suggest otherwise. The following section first provides an overview of passive vs active investing, and subsequently explores two actively managed strategies that aims to reduce portfolio risk and increase portfolio returns.

3.1 Passive vs Active Investing

A sometimes heated debate in finance today is the one of passive vs active investing. Advocates for passive investing can be said to be believers of the Efficient Market Hypothesis (EMH) as laid out by Fama (1970). A passive strategy is most often implemented by replicating the holdings of an index such as the S&P 500. Passive portfolio managers therefore have an easy task, only having to buy the stocks that are included in the index in the right proportion.

For active managers, the investment opportunities are quite different. There are approximately 3,500 publicly traded stocks only in the U.S. today (Wilshere, 2021), and combined with the stocks in the rest of the world, together with investment possibilities in all other asset classes, there are endless possibilities for active portfolio managers to construct portfolios that accompany their goal of outperforming the market. There is doubt, however, whether portfolio managers are actually able to do this. Fama and French (2010) fail to verify active managers' outperformance, and the arithmetic of active management, argued by Sharpe (1991), explains how all active managers cannot perform above average, and that after fees, actively managed portfolios must be a net negative sum game to investors.

In the light of findings like the one presented by Fama and French, the total share of passive investments has seen a substantial rise the recent years and has been called out as a bubble by the famous short seller Michael Burry (Stevenson, 2019). AQR (2018) instead finds some support in favor for active managers[5], and a reasonable middle ground between passive and active investing is proposed by Pedersen (2015), who argues that markets are inefficient to the

[5] Michael Burry is one of the investors pictured in the famous movie ”The Big Short” and currently manages an active fund. AQR is a global investment management firm with actively managed funds. It is fair to say they both have an interest in promoting active management.

extent where portfolio managers make enough money on their strategies to cover their costs of implementing them, while the profits simultaneously not encourage further active investing.

3.2 Tail Risk Hedging

Israelov (2017) writes that put options, as presented in the earlier Theory section, are often seen as the gold standard of tail hedging. However, he argues that put options are an ineffective way of mitigating drawdowns and that in order to actually be successful with such a strategy, the timing of the implementation is a crucial component, since an untimed implementation could lead to increased drawdowns. The study focuses on the CBOE S&P500 5% Put Protection index and Israelov argue the equity put option strategy to be unsatisfactory, and finds a simple equity reducing strategy to be superior in order to mitigate large negative returns. Similar results are found by Asvanunt, Nielsen and Villalon (2015), who also argue that indirect approaches to reduce equity tail risk are better for most investors compared to the direct method of buying put options.

One indirect approach to manage equity risk in portfolios, is the method of overlaying a synthetic put option by mixing a risk-free asset and risky assets, as proposed by Leland and Rubinstein (1981). This is a kind of dynamic replication strategy that makes practical use of the findings associated to the creation of the Black-Scholes option price model described earlier. Another method for managing equity risk with changes in a risk-free asset and risky assets. is the Constant Proportion Portfolio Insurance (CPPI) strategy proposed by Black and Jones (1987) and Black and Perold (1992). Here the investor reduces the exposure to risky assets and invests in the risk-free asset after suffering losses, thus ensuring a certain level of capital in the portfolio.

Hocquard, Ng and Papageorgiou (2013) criticize the use of both equity put options as well as the above mentioned dynamic strategies, and argues that all portfolio insurance techniques result in a significant drag on a portfolio’s performance. Litterman (2011) even proposes that long term investors should consider selling tail risk insurance rather than buying it. He argues that the average return per unit of risk is considerably higher selling the insurance, due to the fact that eventual losses would occur in the worst possible time, making the insurance premium very expensive.

Ilmanen (2012) finds empirical support for Litterman's conclusion and states that his findings are consistent with an investor preference for positively skewed payoffs, meaning that both buying insurance and buying "lottery tickets", could be attractive activities for investors, leading them to be overpriced in the markets. He also argues that a timed selling strategy is likely to do better than a static approach that always sells insurance, and notes that good entries for selling hedges often are found after adverse events in the market.

Taleb (2013) argues in a response to Ilmanen's paper that Ilmanen is not accounting for the convexity of value increases in far out of the money put options in severe crises, and that it would take approximately 900 years of market data to make a statement about tail risk hedging being too expensive. However, Ilmanen do recognize the value of the eventual gains from a tail hedge strategy since it could provide positive returns during bad times, which is when payoffs are most valuable to investors. He further highlights the importance of an active manager's timing and selection of options if such a strategy is to be employed.

A more positive view on tail risk hedging with options are presented by Bhansali and Davis (2010). They write that it is a big challenge for investment officers to convince investment committees to commit to an insurance cost which potentially would lead to relative underperformance compared to peers over certain periods of time. They also argue this observation to harmonize well with the prospect theory presented by Kahneman and Tversky (1979), which provides a behavioral framework that make the projection that people are willing to take a risk of bigger losses to avoid guaranteed smaller losses. Bhansali and Davis argue and show theoretically that investors with risk preferences similar to a 60/40 stock and bond portfolio, get a better return distribution and downside risk protection by the inclusion of put options as a tail risk hedge. The inclusion of such a hedge would in their case allow for a higher allocation in the riskier asset, thus earning a higher risk premium on that part of the portfolio while having a more solid hedge toward negative returns in form of the option instead of a larger part of bonds.

Bhansali (2013) strongly advocates an actively managed tail risk hedge. He argues that the current literature on put options, as a mean of hedging most of the times, only consider a passive buy and hold strategy which fails to capture the full use case of options as a hedging tool. Bhansali further argues that a tail hedging strategy should be seen as a strategic asset allocation decision, therefore warranting its own budget in the portfolio, and that it also should be incorporated as an always on strategy, i.e., the portfolio should always be insured. He further suggests that a portfolio manager should be flexible and gives four examples of different means to actively manage a tail hedge position. Following is a brief description of them:

- *Extension* is a method where the manager keeps track of the term structure of volatility and forward prices of the underlying assets, in order to extend hedges when it is cheap to do so. There is a tradeoff here between shorter and longer contracts. Shorter contracts may quickly rise in value but will then also have a high time decay (high theta) of the option value. A longer contract might not rise as sharply in value, but can, due to lower time decay, preserve a higher option value for a longer time.
- *Conversion* is the act of switching to purchasing put spreads instead of put options, and the technique is preferably used in times of high volatility when put options are expensive.
- *Rotation* could also more explanatory be called indirect hedging. Bhansali writes that portfolio tail risk is almost always systemic risk, thus tail risk becomes a macro risk and it is therefore important to calculate with and try to imagine improbable and high severity scenarios that would affect the financial markets. Bhansali argues that since absolute correlations between assets rise in times of systemic crises, a free lunch is almost served since it enables the manager to buy cheaper hedges on assets that during normal times are less correlated with the portfolio. In the case of a later systemic downturn, the assets do become correlated with the portfolio and are able to serve as an adequate, although not perfect, hedge.
- *Monetization* is a procedure in which the portfolio manager makes the choice to sell the option before expiry due to a recent price increase of the option. This way, the manager is sure of extracting value from the option, which, if kept, could still end up out of the money and be of zero value, much like pictured in Figure 2.12 in the previous Theory section.

Bhansali uses the previously introduced Black-Scholes option pricing model to test several simple active trading strategies based on different arbitrary monetization rules and portfolio budgets during 1928-2010. Since no data of implied volatilities exists for such a long time period, he first creates a model to estimate implied volatilities back in time. The other inputs in the model are the historical prices of the S&P500 index and the one-month U.S. Treasury-bill. The strategies buy one year put options with a strike price set to exactly match the chosen annual option budget and invests the rest in the S&P500 index. The simple monetization rule is that whenever the price of the bought option hits a pre-defined target price specified in terms of the original cost of the option, e.g., five times the purchase price, the option is sold, and the proceeds are reinvested in the index and a new put option. Since the strategy sells options when they are expensive, an implication is that new options bought afterwards also are also expensive, they will therefore typically be bought further out of the money to compensate for this. This is a direct consequence of the strike price being a function of the chosen budget. Bhansali's results are presented in Table 3.1 and 3.2 below.

Average Time to Payoff

		Monetization Multiple			
		2.5	*5.0*	*7.5*	*10.0*
Annual Budget (bps)	*50*	1.16	2.68	4.23	5.36
	100	1.52	4.01	6.69	8.92
	150	1.91	3.83	7.3	8.93
	200	1.96	5.35	8.93	14.64

Table 3.1: Copy of Table from Bhansali (2013)

Profit/Loss in bps After Payoff

		Monetization Multiple			
		2.5	*5.0*	*7.5*	*10.0*
Annual Budget (bps)	*50*	67	116	164	232
	100	98	99	81	108
	150	89	176	30	161
	200	108	-70	-286	-928

Table 3.2 Own Calculation of Profit and Loss Based on Bhansali Data

The strategies are profitable whenever the multiple is higher than the average time it takes for the multiple to be reached, which can be seen to be standard outcome. Only three strategies result in negative returns, and these occur when the budget is 200 basis points and for the multiples 5.0x, 7.5x and 10.0x.

It is a clear trend in the data that further out of the money options hit the multiples more often, as compared to the more expensive options less far out of the money. Of course, it is also a natural result that the average time to payoff is lower for lower multiples. The longest average waiting time for a payoff is almost 15 years, the shortest is slightly more than 1 year. The average times to payoff for all the strategies that apply a multiple of 5.0x or higher, are all longer than 2.5 years.

Bhansali compares the monetization strategy with an annual budget of 100 bps and target multiple of 5.0x, to a position in the S&P500 Index and to a passive put option strategy where the one year put options are held until expiry. The comparison favors the active tail hedging strategy and Bhansali attributes the good performance of the strategy to the strategy's ability to invest proceeds from the sold options into equities during times of market stress, thus buying on dips and exploiting an increased risk premium in the market.

Bhansali, Chang, Holdom and Rappaport (2020) also demonstrate how monetization rules can add value to option strategies. Here they compare three different monetization strategies, where they sell 50% of the put position if it hits a multiple of 3.0x, 5.0x or 8.0x of its original purchase price, with a passive buy and hold put option strategy. The comparison is made over three different time periods that experienced drawdowns, 1999-2003, 2008-2009 and 2018-2019, and the main finding is that all tested strategies outperformed the passive strategy. However, it is difficult to say whether one of the 3.0x, 5.0x or 8.0x multiple strategies are superior compared to the others.

3.3 Constant Volatility

An important assumption in the mean variance framework is that volatility of returns is constant over time, but as we saw earlier in Figure 2.13, this is not an accurate description of reality. As Hocquard et al. (2013) rightfully points out, and as can be seen in Figure 3.1, the risk of experiencing a large drawdown is very different in a portfolio with a volatility of 30 percent compared to a portfolio with a volatility of 15 percent. Given the time varying nature of volatility in stock prices, they argue that the mean variance framework with a static measure of volatility is a rather useless tool of risk management for a portfolio. In their paper, they put this in perspective with the monthly return of October 2008 as an example, which would be considered as four-standard-deviation event given the historical average standard deviation of S&P 500, but only a one-standard-deviation event with the use of the prevailing volatility level at the time. It would therefore make sense to actively manage the volatility of the assets under management in order to keep the risk for large drawdowns in the portfolio at, for the portfolio manager, a pre-defined desired level.

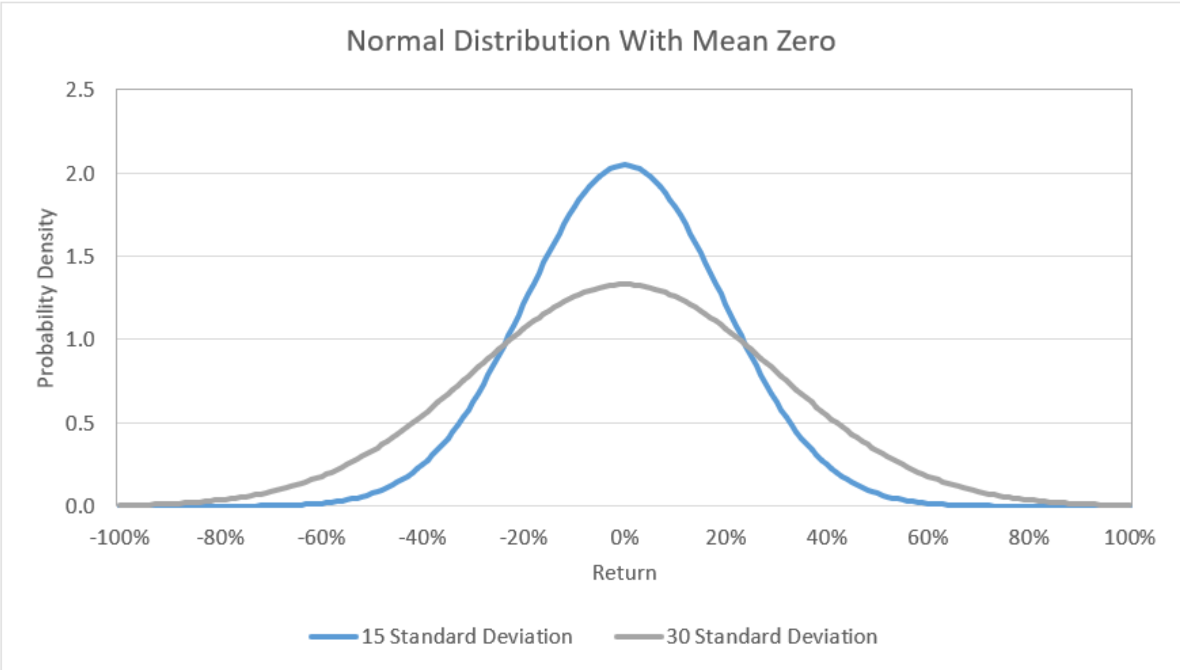

Figure 3.1: Example of Tails from Hocquard Ng Pap. (2013)

There is empirical evidence of a negative correlation between volatility and stock returns, see Bollerslev, Litvinova and Tauchen, (2006), Harvey, Hoyle, Korgaonkar, Rattray, Sargaison and Van Hemert (2013), and Hocquard et al. (2013). This behavior might be attributable to the leverage effect, i.e., a negative return on equity leads to an increased debt to equity ratio which makes the equity stake of a firm more volatile, as proposed by Black (1976). This is the relationship usually pictured by the old saying that stocks take the stairs up, but the elevator down. Together with the previously mentioned characteristics of stock market returns, i.e., time-varying and persistent volatility, it plays an important role in strategies where the allocation decision to risky assets is a function of the risky assets' volatility. Two attractive features that emerges from applying a strategy where the portfolio manager aims to keep a constant level of volatility in the portfolio are:

- The ability to avoid big left tail events that are more likely to occur when the volatility of returns is high.
- The benefits from taking on leverage during times when volatility is low, and returns are higher.

These two features of the strategies suggest better risk adjusted returns compared to just passively holding the same equities during the same time period. The ability to do so naturally depends on the ability to accurately forecast volatility (Hallerbach, 2012) and generating returns that compensate for implementation costs.

Perchet, Carvalho, Heckel and Moulion (2014) conduct several Monte Carlo simulations with different GARCH models and in line with the previous reasoning, find it beneficial to rebalance equity portfolios to target a constant volatility. They also find the key effects behind the result to be the persistence of volatility and fait tails of negative returns. They also provide evidence that constant volatility approaches would lead to the same Sharpe ratio as a buy and hold strategy if stock returns follow a normal distribution with constant volatility.

Hocquard et al. (2013) targets a constant volatility by implementing a method based on Dybvig's (1988) payoff distribution model, and they model the daily returns as a GARCH (1,1) process. The trading in the paper is done on a daily basis and they present their results net of financing and implementation costs of 25 basis points. The strategy is tested on numerous indices and it successfully mitigates drawdowns among all of the tested indices compared to buying and holding the indices themselves. However, their strategy only manages to produce higher Sharpe ratios in 3 out of the 6 tested equity indices, compared to the passive buy and hold strategy.

Furthermore, Harvey et al. (2018) show that a constant volatility model effectively can reduce large negative returns in a range of different asset classes and improve Sharpe ratios for equity and credit assets. Moreira and Muir (2017) similarly find their constant volatility model, which instead target a constant variance, to effectively increase Sharpe ratios in several U.S. based equity allocation strategies, in addition to equity investments in 20 OECD stock market indices. The two papers apply a relatively simple allocation rule where the equity exposure approximately is chosen such that:

$$r_t^{scaled} = r_t \times \frac{\sigma^{target}}{\hat{\sigma}_{t-1}}$$

and both strategies survive the introduction of trading costs.

Doan, Papageorgiou, Reeves and Sherris (2018) also apply a constant volatility strategy with successful results. They use an outlier corrected GARCH (1,1) to model the daily return volatility and the allocation decision to the risky asset is made in the same way as described in the above equation. The strategy is implemented with a long position in the underlying equity index consisting of 100 percent of the capital, and then use futures on the index to lever or de-lever the exposure to equity risk. The use of futures is done to decrease transactions costs. They also implement a threshold the forecasted volatility must exceed the current level of volatility with before changing the exposure, thus further reducing the transaction costs of the strategy. The strategy manages to increase risk-adjusted performances in the U.S., U.K., German, and Australian markets compared to their respective index. The greatest outperformance was found in the two latter markets and attribute the outperformance due to higher average annual returns and reduced drawdowns. They conclude that over time, the strategy produces substantial improvements in cumulative returns, and that the results are stable over long and short term horizons as well as for different countries. Their constant volatility model gets a significant alpha on a 1 percent significance level tested with the Fama French three-factor model in the U.S. markets between June 1929 to December 2013. When adding a momentum factor to the model, the size of the alpha drops and instead becomes significant on a 10 percent level, due to their strategy being strongly positively correlated with the momentum factor.

4 Data and Methodology

This section goes into detail on the data and methodology used to create and test our strategies. First, we will describe the data used throughout the thesis, mainly the S&P 500 index, the 3-month U.S. Treasury rate, and S&P 500 option contracts and prices. Second, we will look at the strategy of using put options to manage the portfolio tail risk. In this section, we will look at an actively managed strategy that is based on allocating a certain percentage of an overall portfolio to buy out-of-the (OTM) put options. Third, we will describe the process of the constant volatility strategy. Here we will primarily describe how we estimated future volatility, and how we used this estimate as the foundation of the investment strategy. Finally, we will describe the method of how we will evaluate the results and the metrics used.

4.1 Data

Throughout the paper, our focus is on the U.S. stock market. The reason for focusing on the U.S. market is primarily due to the availability of data and due to the liquidity of the major U.S. market indices. More specifically, we focus on the S&P 500 index, an index chosen due to its representation of U.S. market at large. To estimate the risk-free rate, we use the 3-month U.S. Treasury rate. For the tail-hedge strategy, we use actual prices of put options on the S&P 500. Our interest in basing our research on real data, have dictated the timeframe for which we are testing and evaluating the strategies. As we only have option prices from 1996 and onwards, our timeframe for our analysis is 2nd January 1996 through 30th December 2020. Both strategies will be limited to this timeframe as we want to compare the two, and due to the availability of existing research on constant volatility going further back in time, we find this to be reasonable.

4.1.1 The S&P 500

The Standard and Poor’s 500 (S&P 500) is an index comprised of approximate the 500 largest public U.S. companies. The index was created in 1957 and is considered the leading indicator for large-cap U.S. equities (S&P 500, 2021). To become a part of S&P 500 there are several criteria, and it is therefore not exactly the 500 companies with the largest market capitalization, but it is close. The index itself is calculated as the sum of the market cap for all 500 chosen companies, adjusted for such things as issuance of new shares and mergers, divided by an unknown divisor (Kenton, 2021).

As the S&P 500 index does not account for dividends, we chose to use the S&P 500 Total Return index as the basis for estimating returns. The total return index assumes that dividends are reinvested, and thereby provides a more accurate estimates of the returns one would receive if one were to invest in a portfolio replicate of the S&P 500. Figure 4.1 depicts the relative performance of both the S&P 500 index and the S&P 500 Total Return (S&P 500 TR) index. From the graph it is clear that the total return index outperforms the standard index, which makes sense as dividends are reinvested. The average annual excess return for the S&P 500 and the S&P 500 TR is 5.3% and 7.3%, respectively.

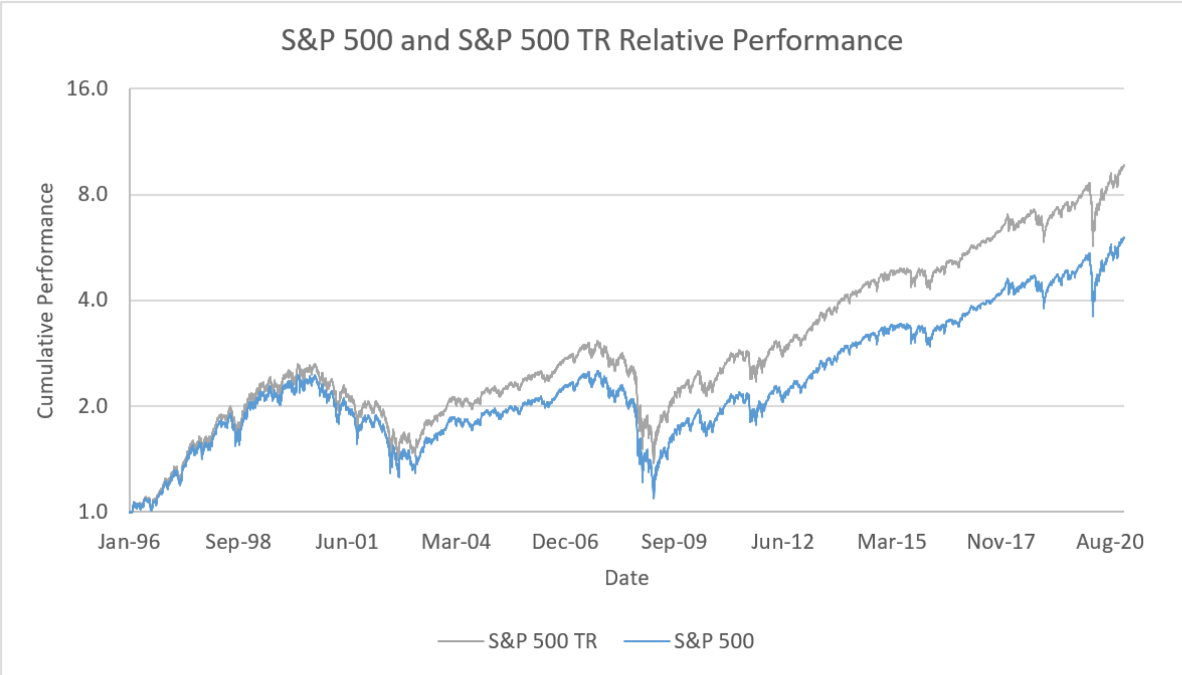

Figure 4.1: The S&P 500 and the S&P 500 TR Relative Performance 1996-2020

The S&P 500 data is collected from Yahoo Finance, using ticker GSPC for the S&P 500 index and ticker SP500TR for the S&P 500 TR index (Yahoo!, 2021). Throughout the paper, we use daily closing prices as the basis for return calculations. Further, to display the characteristics of the data used, we have displayed the observed distribution of excess daily returns, together with the expected normal distribution in the Figure 4.2 below. Here one clearly observes that the returns are not normally distributed.

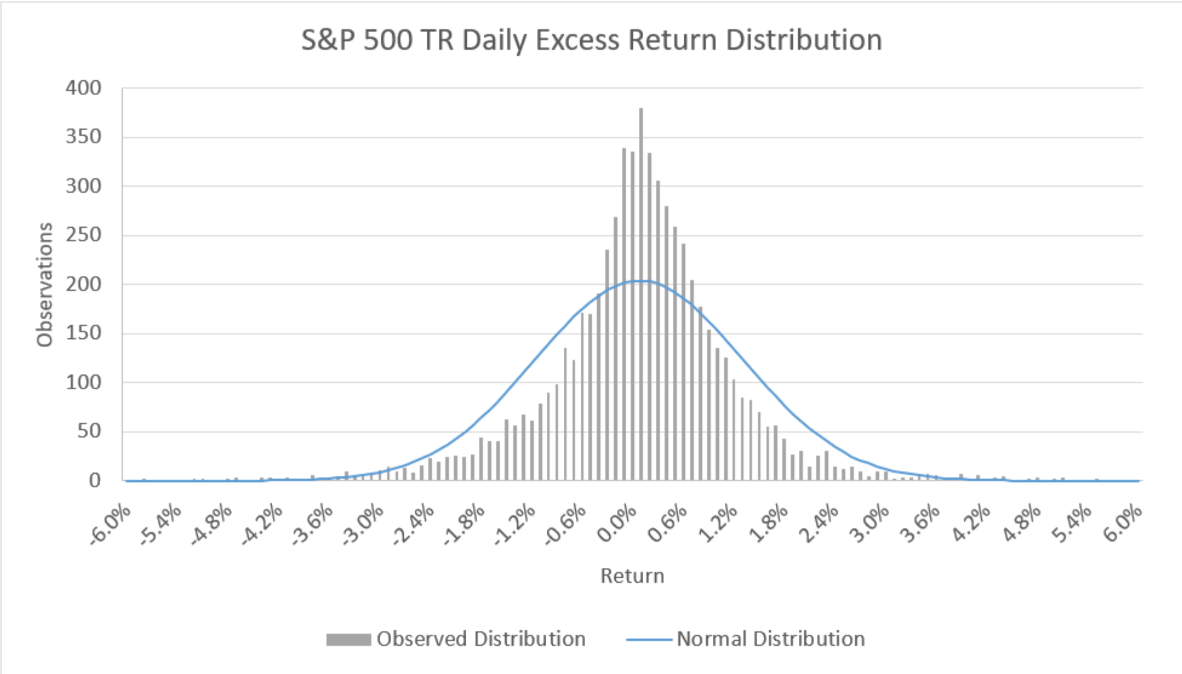

Figure 4.2: The S&P 500 TR Daily Excess Return Distribution 1996-2020

Further, to showcase the time-varying and persistent volatility, Figure 4.3 illustrates the rolling annual standard deviation of excess S&P 500 TR returns.

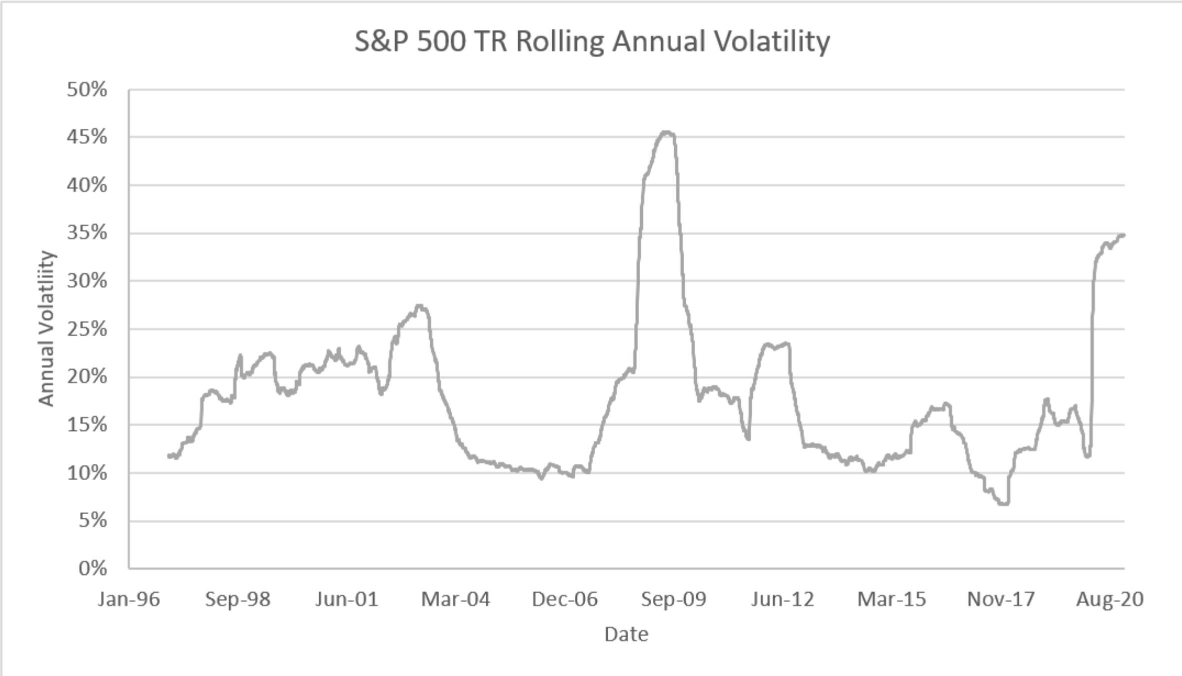

Figure 4.3: The S&P 500 TR Rolling Annual Volatility 1996-2020

In the result section, we look at various time horizons that are based on the change in standard deviation in the underlying S&P 500 TR. Looking more detailed at periods with low realized standard deviation and periods with large increases in standard deviation.

4.1.2 Risk-Free Rate

Throughout the paper, we assume that the risk-free rate is equal to the rate at which the three-month U.S. Treasury bill trades at in the secondary market. We use the daily rate retrieved from the Federal Reserve Economic Database (FRED), using the identifier DTB3 (Board of Governors of the Federal Reserve System (US), 2021). For S&P 500 trading dates where there are no data on the risk-free rate, we assume the risk-free rate to be equal to the most recently available data point.

The risk-free rate is the assumed rate of return one would get on a zero-risk investment, and despite no investment being entirely without risk, the three-month Treasury bill is a good approximation (Chen, 2021). In the paper, we use the risk-free rate primarily to estimate excess returns on the two strategies. Figure 4.4 below displays the variation in the rate over the past 25 years.

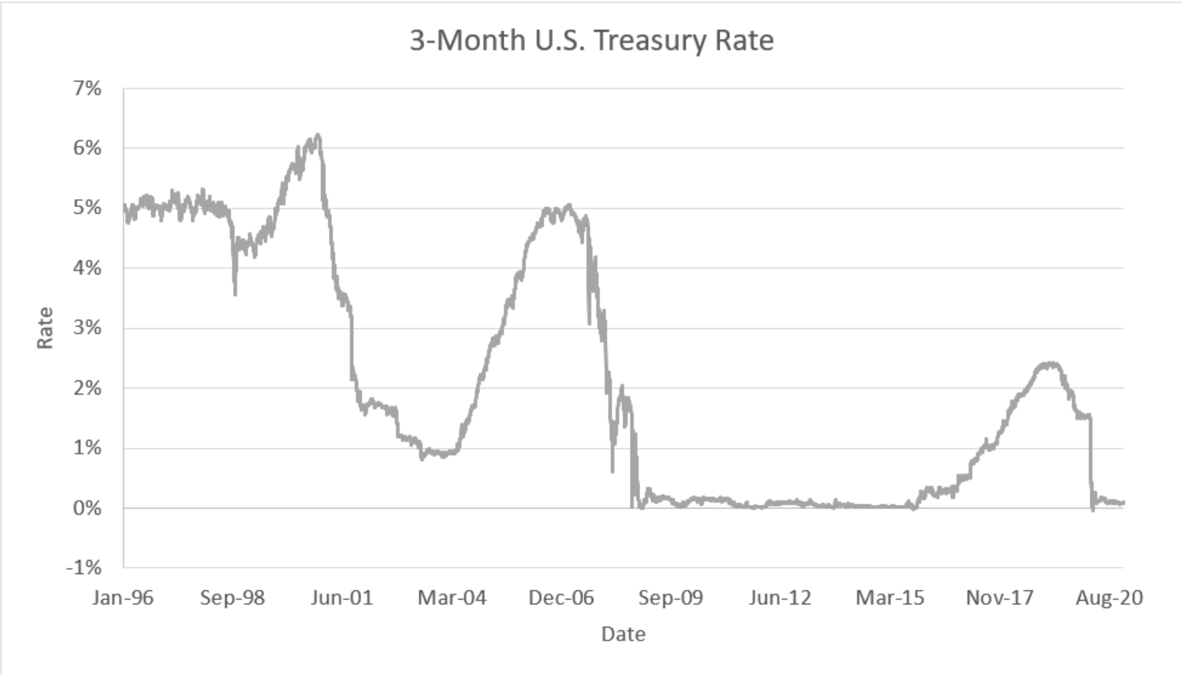

Figure 4.4: The Three-Month U.S. Treasury Rate 1996-2020

4.1.3 S&P 500 Option Prices

For the tail-hedge strategy we use put options on the S&P 500 index. The necessary historical S&P 500 (SECID: 108105) option data was retrieved from Option Metrics through the Wharton Research Data Service (WRDS). From OptionMetrics we retrieved all put option contracts, together with associated date, expiration date, strike price, best bid, best offer, volume, open interest, and option-id, from 1996 through 2020. The best bid is the highest bid at close, while the best offer is the lowest closing ask.

Throughout the period we only use the traditional S&P 500 index (ticker: SPX) option contracts. These contracts now expire the third Friday every month, while prior to February 15, 2015 they expired the third Saturday of every month. The exercise style of the contract is European, which means the option can only be exercised on the expiration day. Further, the last trading day is the business day prior to the day settlement value is calculated. These option contract have AM-settlement which means the datapoint used to determine if the option expires in-the-money is based upon the open price of the S&P 500 index on expiration date. For data points prior to the change in expiration day, the settlement value is based on the open price for the preceding Friday. Potential cash will be paid the first business day following expiration day (Cboe Global Markets, 2021). Our reasoning for only using this option contract is that these are the most available and liquid options, especially early in the dataset, in addition we wanted to be consistent throughout the strategy.

4.2 Methodology

In this subsection we will outline our approach to creating and testing our strategies. First, we outline general background and assumptions. Looking at backtesting in general, assumptions common for both strategies, and calculations used throughout the paper. Second, we outline the methodology for each of the two strategies. Third, we go into detail on how we assess the performance and results of the strategies. Finally, we describe some of the limitations to our method.

4.2.1 Overall Assumptions and Calculations

The basis for testing our strategies and performance is performing a backtest, which gives us an indication on how the strategies would have worked historically. This does not give any guarantee for how it will work going forward, but it may provide an indication. To perform a backtest it is important to create clear trading rules. One needs to consider available information at the time of trading, possible limitations in non-hypothetical data, and how to enter and exit trades (Pedersen, 2015). Our goal when testing the strategies has been to, where possible, use realistic assumptions and methods. Methods that are possible to implement in reality.

Common for both strategies, is the trading cost associated with buying and selling the underlying S&P 500 TR. Transaction costs is the difference between the price right before trading and the price you paid, plus any fees and commission. The costs include bid-ask spread, possible market impact, and, as mentioned, fees and commission. These costs wary based on the securities one buys, the size of trades, and the type of investor (Pedersen, 2015). For our analysis we use a transaction cost of 4.9 bps when trading in the underlying S&P 500. This estimate is based on findings by Frazzini, Israel and Moskowitz (2012), here they found that the median transaction cost for U.S. stock trades, by large institutional money managers, was 4.9 bps in 2011 (Pedersen, 2015). We found this estimate to be reasonable for our type of strategy, where most of the trades are not significantly large, and considering the costs we have seen used in similar research, this estimate is on the conservative side.

Throughout the paper, we use lognormal returns for intermediate calculation, using the following formula:

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right)$$

We primarily use this for simplicity as log returns are additive, meaning that the log return over a period is equal to the sum of the individual returns. This simplifies the process of calculating averages and returns over longer periods, especially as we use geometric average. Our reason for choosing geometric average is that it is more appropriate when capital is neither added nor withdrawn throughout the investment period, which corresponds to how we tested our strategies (Pedersen, 2015). When we refer to final returns in the paper, we have calculated them as: $e^{r_t} - 1$.

Another assumption made throughout the paper, is that we can buy and sell at the daily S&P 500 close price. In addition, we also assume that we can buy and sell securities at the same time. These assumptions are made due to data availability and for simplification. We believe these assumptions are reasonable as one could closely replicate this in reality by only trading in the last minutes of the trading day. Further, all analyses are done using Python.

4.2.2 Put Option Monetization Strategy

The primary strategy we investigate in this paper, is the put option monetization tail-hedge strategy of buying OTM put options. The foundation and inspiration for our execution of the strategy is Vineer Bhansali and his book *Tail risk hedging: creating robust portfolios for volatile markets*. From the book, our strategy is based on what he refers to as active tail risk management, which implies including an active monetization rule (Bhansali, 2013). Compared to Bhansali, we perform the strategy on the S&P 500 Total Return index, while he uses the S&P 500 index. Also, we only look at the period 1996 to 2020, while he includes data from 1928 and up until 2010. More importantly, we include transaction cost and use actual prices from actual option contracts. Bhansali uses historical volatility surfaces, together with the Black-Scholes model, to get option price estimates.

Throughout the strategy, we follow a couple fundamental assumptions. First, we only buy contracts where the open interest is at least 1,000 at the time of purchase. Open interest for option contracts, informs of how many active contracts there are for the specific option contract (Norris, 2021). This provides some indication on the contract liquidity, and as we actively buy and sell, it is important to have some minimum liquidity standards. The reason for choosing 1,000 is to some degree arbitrary, but it is mainly to weed out the most illiquid contracts. For one period in 2000, we found that there were no option contracts with positive open interest. For the analysis, we assumed this to be a mistake. WRDS does disclose that there are some errors in the open interest variable from OptionMetrics, and we assume that to be the instance here (Wharton WRDS, n.d.). Further we only target contracts expiring every third Friday or Saturday, as described in the data section. We also, to make the strategy more conservative, buy at the bid-ask spread. We assume that the purchasing price is equal to the best ask at close, and that the value and selling price is equal to the best bid at close. Finally, we assume that the transaction cost for purchasing and selling options is 20 bps. This is a conservative estimate based on the *CBOE U.S. Options Fee Schedule*, where the fee for customers on SPX option contracts (one contract is 100 positions in the underlying) with a premium above $1.00 is $0.36 (CBOE Global Markets, 2021). Most of options in this strategy has a premium well above $1.00, so an average cost of 20 bps is conservative. We primarily look at the transaction fee, as we account for the bid-ask spread in our calculations.

4.2.2.1 Calculations

When performing the strategy, we start with $1,000 in total portfolio value, and allocate a percentage of this to the option strategy. We test with both an annual allocation of 1.5 percent and 3.0 percent. Further, we target period lengths, option contracts time to expiration, of three months. Our reasoning for choosing three months as compared to longer term options, is mainly liquidity reasons, as shorter maturity options are more liquid compared to longer maturity ones. We estimate the period length to the closest half month in times where we have sold the option prior to expiration. The below formula describes our period allocation calculation:

$$Period\ allocation = \frac{Period\ length\ in\ months}{12} \times Annual\ allocation$$

We use the period allocation to calculate how much we should spend on options for the period. As an example, using a 1.5 percent annual allocation with a period length of three months and a total portfolio value of $1,000, we have: $\frac{3}{12} \times 0.015 \times \$1{,}000 = \$3.75$. We then account for the transaction costs of buying the option contracts and get the final estimate we can use to purchase options:

$$Cash\ allocation_t = Total\ portfolio\ value_{t-1} \times Period\ allocation$$

$$Cash\ allocation\ after\ fees_t = \frac{Cash\ allocation_t}{1 + Option\ transaction\ cost\ in\ \%}$$

After we have the cash allocation, we calculate how many options to buy. Here we assume that we can buy fractional contracts. We aim for a direct hedge, where we buy one option (1/100 of a contract) per position in the underlying. To calculate the hedge, we use the value of our position in the underlying S&P 500 TR, referred to as the equity value, and we use the S&P 500 close price, both from the previous day.

$$Hedge\ ratio_t = \frac{Equity\ value_{t-1}}{S\&P\ 500\ close_{t-1}}$$

To find the desired option contract that matches our budget and desired hedge, we calculate the target price. This target is used to find the most appropriate option contract to buy. A higher target price, everything else equal, means higher moneyness.

$$Target\ price_t = \frac{Cash\ allocation\ after\ fees_t}{Hedge\ ratio_t}$$

We accept all option prices that are within a range of plus-minus 30 percent of our target price. If we do not find a contract within this constraint, we look for shorter dated option contracts, and calculate a new cash allocation based on the shorter time to expiration. We start by reducing the time to expiration by one month, i.e., looking for two-month option, until we find an appropriate contract. If none of the lengths matches our specific constraints, we buy the closest one-month option contract. If we buy a two-month option contract for one period, we target a four-month or one-month the next period. Similar, if we buy a one-month option, we target a two-month option the next period.

The number of options we buy for the period is based on the cash allocation, adjusted for fees, and the price of the option we are buying. We buy as many options as possible based on the cash allocation we have, again assuming we can buy fractional options. The option price is the best ask price at the time.

$$Number\ of\ options_t = \frac{Cash\ allocation\ after\ fees_t}{Option\ price_t}$$

Throughout the holding period of the option, we look if the option price increases above the option cost at a predetermine multiple. We test multiples of 2.5x, 5.0x, 7.5x and 10.0x, in addition we also do one with no multiple for comparison (i.e. never selling the option and holding until expiration). This is the active monetization part of the strategy, where we sell the option if the price increases beyond a predetermined point. This point is everyday based on the relationship between what we paid for the option, and the current market price. We use the below formula to show if the price reaches the determined multiple:

$$if(Option\ cost \leq Option\ price \times Multiple, 1{,}0)$$

If the multiple is reached at some point in the period, we calculate the end value of the option based on the price when the multiple is reached, and the number of options we own. We also take into account the cost of selling the options:

$$End\ value_t = \frac{Price_t \times Number\ of\ options_t}{1 + Option\ transaction\ cost\ in\ \%}$$

On the other hand, if the multiple is not reached, we calculate the end value based on whether the option expires in-the-money. We check if the strike price is higher than the open S&P 500 price on the day of expiration (day prior to expiration for Saturday options). If that is the case, we multiply the difference by the number of options we own, and account for transaction costs:

$$End\ value_t = \frac{Max(Strike_t - S\&P\ 500\ open_t, 0) \times Number\ of\ options_t}{1 + Option\ transaction\ cost\ in\ \%}$$

At the end of each period, we calculate the net costs based on the cash we spend on new options and based on the end value of the old option. In cases where the option expires worthless, the net costs will be equal to the cost of the options.

$$Net\ costs_t = Cash\ allocation_t - End\ value_t$$

The value of our S&P 500 TR position, at the end of each day, is the return in the underlying times the previous day position, adjusted for any costs and income. We also include transaction costs associated with buying the underlying index when we have negative net costs (positive returns from options), and costs associated with selling the underlying to finance the buying of options.

$$\begin{aligned} &Equity\ value_t \\ &\qquad = Equity\ value_{t-1} \times e^{S\&P\ 500\ TR\ return} - Net\ costs_t \\ &\qquad - Abs(Net\ costs_t \times S\&P\ 500\ transaction\ cost\ in\ \%) \end{aligned}$$

The daily value of our option position is simply the daily price of the option multiplied with the number of options we own:

$$Option\ value_t = Number\ of\ options_t \times Option\ price_t$$

The daily total portfolio value is the sum of our position in the underlying and the options value. It is this estimate we use to calculate our returns of the strategy.

$$Total\ portfolio\ value_t = Equity\ value_t + Option\ value_t$$

4.2.3 Constant Volatility

The second strategy we are testing, is the strategy where we use the next-day volatility estimate to increase or decrease exposure. The primary component in this strategy is in the volatility estimation, and we use a GARCH model to get this estimation. Further, our strategy uses the paper *Portfolio management with targeted constant market volatility* by Doan et al. (2018), as the basis for our approach.

The main differences from Doan et al. are the specifics of the volatility estimation, the underlying asset, and the time-period. In their paper, they highlight the importance of an accurate volatility forecast, which is in line with the papers by Hallerbach (2012) and Carnero et al. (2012). This points to that the most important aspect of this strategy is to have an effective volatility estimate. We also used the GARCH(1,1) model like the paper, but we did not go into the same detail of creating the absolute best model. Doan et al. include strategies such as eliminating outliers to improve the estimate, however, we do not go to the same lengths. The reason being that there is sufficient research on the topic of optimizing volatility estimation, and on constant volatility strategies being profitable and effective. Therefore, we are not looking to dispute this, but instead use this strategy for a comparison to our tail-hedge strategy. Furthermore, where their paper uses the CRSP value-weighted market returns as the underlying equities, we use the S&P 500 TR. In addition, our paper focuses on 1996 through 2020, whereas their paper looks at the period 1926 through 2013. Our underlying equites and time-period matches that of our tail-hedge strategy.

4.2.3.1 Calculations

The first step in performing this strategy, is to estimate next-day volatility. We use the GARCH (1,1) model described in the Theory section to get these estimations, as research indicates this to be the best model for estimating daily volatility. In the model, the returns are daily log returns multiplied by 100. The specifics of the model include using a constant mean, GARCH volatility and using a Skewed Student's t-distribution. In the model we use the log returns as our input, and we use a 252 day rolling window to estimate next day variance. We used 252 days, 1 year, in our model as we found it to be a fitting length, not too long and not too short. Also, in hindsight, this worked well with our data.

After obtaining the variance estimate, we convert it to an annualized standard deviation, annual being 252 days. For comparison reasons, we chose a target volatility of 19 percent, which is similar to realized volatility of the S&P 500 TR over the period. Thereafter, we calculated the daily scaled returns using the returns, calculated from close to close, and multiplying it by the ratio volatility target divided by the volatility estimate for the previous day, the day we "bought" the underlying.

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right)$$

$$r_t^{scaled} = r_t \times \frac{\sigma^{target}}{\hat{\sigma}_{t-1}}$$

Next, we calculate the transaction costs associated with increasing and decreasing exposure. Daily, the transaction costs vary based on the absolute difference between the position prior to making the transaction and the size of the transaction. Here we assume that the transaction cost is equal for both buying and selling, and that it is constant across time and across order sizes. The S&P 500 transaction costs is as previously mentioned, estimated to be 4.9 bps.

$$Transaction\ costs_t = abs\left(S\&P\ 500\ transaction\ cost\ in\ \% \times \left(\frac{\sigma^{target}}{\hat{\sigma}_{t-1}} - \frac{\sigma^{target}}{\hat{\sigma}_{t-2}}\right)\right)$$

We also account for the cost of borrowing and what we receive for holding cash, or cash equivalent. We assume that the cost of borrowing and the rate we get for holding cash is the same, and that it is equal to the risk-free rate (the three-month T-bill). Here we assume everything above 1 is borrowed money, and everything below is held in cash which yield the risk-free rate.

$$Financing\ costs_t = \left(\frac{\sigma^{target}}{\hat{\sigma}_{t-1}} - 1\right) \times rf_{t-1}$$

Finally, we calculate the net return by subtracting both the transaction cost and the financing cost from the scaled daily returns:

$$r_t^{net} = r_t^{scaled} - Transaction\ costs_t - Financing\ costs_t$$

It is this net return we use in our results, and they are referred to as the absolute returns.

4.2.4 Results and Performance

To evaluating the performance of our strategies, we will look at different measures that capture the returns, volatility, risk and characteristics of the returns for the various strategies. We calculate and evaluate the alpha, beta, average annual excess return, annual standard deviation, Sharpe ratio, downside standard deviation, Sortino ratio, maximum drawdowns, total absolute returns, employed leverage, skewness, and excess kurtosis. First, we use these metrics to evaluate variations to our strategies, second, we use it compare the S&P 500 TR index, the constant volatility strategy, and the tail-hedge strategy over multiple time periods.

4.2.4.1 Performance and Risk Measures

For the first performance measure, we estimate the alpha and beta of the various strategies. The beta describes the market exposure, while the alpha describes the excess returns not explained by the market. The reason it is interesting to look at these estimates, is that it tells how the returns are generated. Also, alpha is one estimate that potentially can describe the returns that are generated due to skill, as opposed to market exposure (Pedersen, 2015). We obtain the necessary daily market returns and risk-free rate from the Kenneth R. French website (French, 2021). In the alpha and beta calculation, we assume the risk-free rate to be equal to the one-month Treasury bill. To calculate the alpha and beta, we run a regression using the strategies excess returns as the predictive values and the market excess returns as the descriptive:

$$R_t^e = \alpha + \beta R_t^{M,e} + \varepsilon_t$$

Another variation to calculating alpha and beta, is to include other risk exposures. In this calculation we use the three-factor model by Fama and French. This model includes, in addition to the market exposure, the high-minus-low factor and the small-minus-big factor. The high-minus-low factor describes returns where you are long stocks with high book-to-market ratio and short low book-to-market stocks. The small-minus-big factor describes return where you are long small market cap stocks and short big (Pedersen, 2015). We also retrieve the necessary data for this estimation from the Kenneth R. French website. Similar to the one-factor model, we run a regression using the excess returns of our strategies as the predictive values:

$$R_t^e = \alpha + \beta^M R_t^{M,e} + \beta^{HML} R_t^{HML} + \beta^{SMB} R_t^{SMB} + \varepsilon_t$$

For both the one-factor and the three-factor model, we must check if the alphas and betas are significant. To do this, we use the associated p-value we obtain when performing the regressions to test for significance. We consider a p-value below 0.05 to be significant.

Next, we look at the average annual excess returns, annual standard deviation, and the Sharpe ratio. To calculate the average annual excess returns, we subtract the daily risk-free rate (three-month T-bill) from the daily log returns for the strategy and use the follow formula: $e^{r^{average} \times 252} - 1$, where the $r^{average}$ is the average excess returns over the period. For the standard deviation calculation, we calculate the sample standard deviation on the daily excess strategy returns for the period, and then multiply it by the square root of 252: $\sigma^{sample} \times \sqrt{252}$. Then, we calculate the Sharpe ratio to look at the relationship between standard deviation and returns. The Sharpe ratio measure the reward per standard deviation, or risk, and is calculated by dividing the expected excess returns by the standard deviation (Pedersen, 2015):

$$Sharpe\ ratio = \frac{E(R - rf)}{\sigma(R - rf)}$$

Another risk-reward measure is the Sortino ratio. Whereas the Sharpe ratio looks at both upside and downside risk, the Sortino ratio is only concerned about the downside risk. This estimate is interesting as the main concern for most investors is downside risk, and the Sortino ratio measures the reward per risk to the downside. To calculate the ratio, we first need to estimate the downside standard deviation. We do this by calculating the standard deviation of the returns where they are below some minimum acceptable return (MAR). For our analysis, we assume the MAR to be equal to the daily risk-free rate. The calculation is done be using the below formula, where we only calculate the standard deviation for the returns below the risk-free rate:

$$\sigma^{downside} = \sqrt{E\left((MAR - R)^2 1_{\{R<MAR\}}\right)}$$

When we have the downside standard deviation, we use the same annual excess returns as in the Sharpe ratio, and divide it by the downside standard deviation (Pedersen, 2015).

$$Sortino\ ratio = \frac{E(R - rf)}{\sigma^{downside}}$$

Next, we look at the drawdowns. This measure compares the highest value of the portfolio until that point in time with the current value of the portfolio. The drawdown describes the losses of the portfolio and is interesting to look at from a risk perspective. To calculate the drawdown, we first calculate the daily high water mark (HWM), which is equal to the highest portfolio value achieved up until that point in time. Then we subtract today's price from the high water mark, and then divide it by the high water mark:

$$DD_t = \frac{HWM_t - P_t}{HWM_t}$$

In our result section, we look at the maximum drawdown for the period, which displays the highest drop in the portfolio over the time period (Pedersen, 2015).

We also look at the implied leverage employed in each of the strategies. This is calculated by looking at price change in the strategy relative to the price change in the underlying. For our results, we look at the average daily leverage employed during the time period.

$$Leverage\ employed_t = \frac{r_t^{strategy}}{r_t^{underlying}}$$

Finally, we look at the skewness and excess kurtosis of the returns. Both estimates describe the distribution of the returns and is interesting at it provides information on how the strategies affect the distribution. In our calculation, we use normal returns as opposed to log returns when we make these estimates.

The skewness tells us if the returns skew right or left compared to a normal distribution. A positive skew means that the returns skew to the left and that we have fatter tails on the right. While a negative skew means that the returns skew to the right, and that we have fatter tails to the left (Chen, 2021). This is interesting to know, as we get information on whether the fat tails are more to the upside or to the downside.

The kurtosis on the other hand describes the tails to both the right and to the left. We focus on excess kurtosis, which is obtained by subtracting three from the kurtosis estimate. A higher kurtosis means that there are fatter tails and more extreme returns. For our analysis this is an interesting metric, as we wish to minimize extreme returns, at least to the downside.

4.2.4.2 Time Periods

In addition to looking at the full period 1996 through 2020, we will also look at the performance for some specific time periods. We have chosen the periods based on the characteristics of the returns. First, we will look at a period with few major drawdowns, then we look at three periods with substantial drawdowns. Looking at these specific periods will provide information on how the strategies perform under various market conditions.

The first additional time period we will analyze, is the period 2010 through 2017. This period is characterized by few and only minor drawdowns. During this period, it was mainly an uptrend in the index value, as can be seen in Figure 4.5 below. By analyzing this period, we will see how the strategies work when there are no large drawdowns.

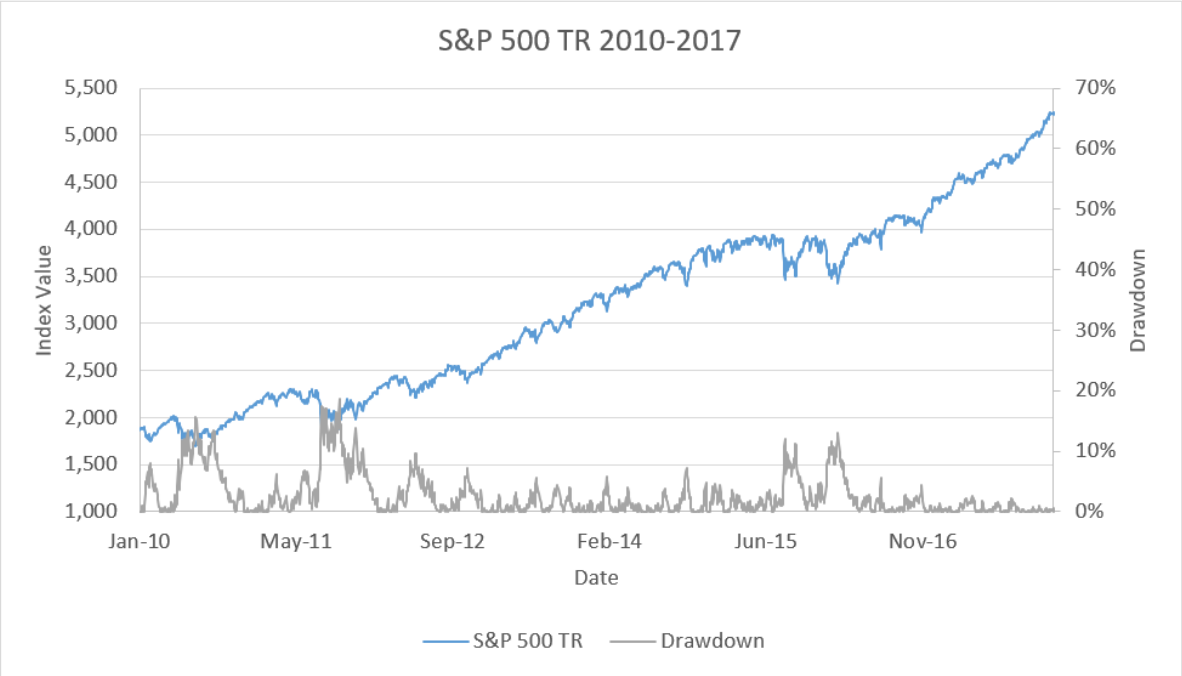

Figure 4.5: The S&P 500 TR Performance and Drawdowns 2010-2017

Next, we look at the first of three periods with large drawdowns visualized in Figure 4.6. During the period 2000 through 2003, the max drawdown was close to 50 percent. This period describes the aftermath of the dotcom bubble, where tech companies saw great valuation gains followed by large losses. This period represents a period with a prolonged downturn, as it took around two years for the index to hit its lows.

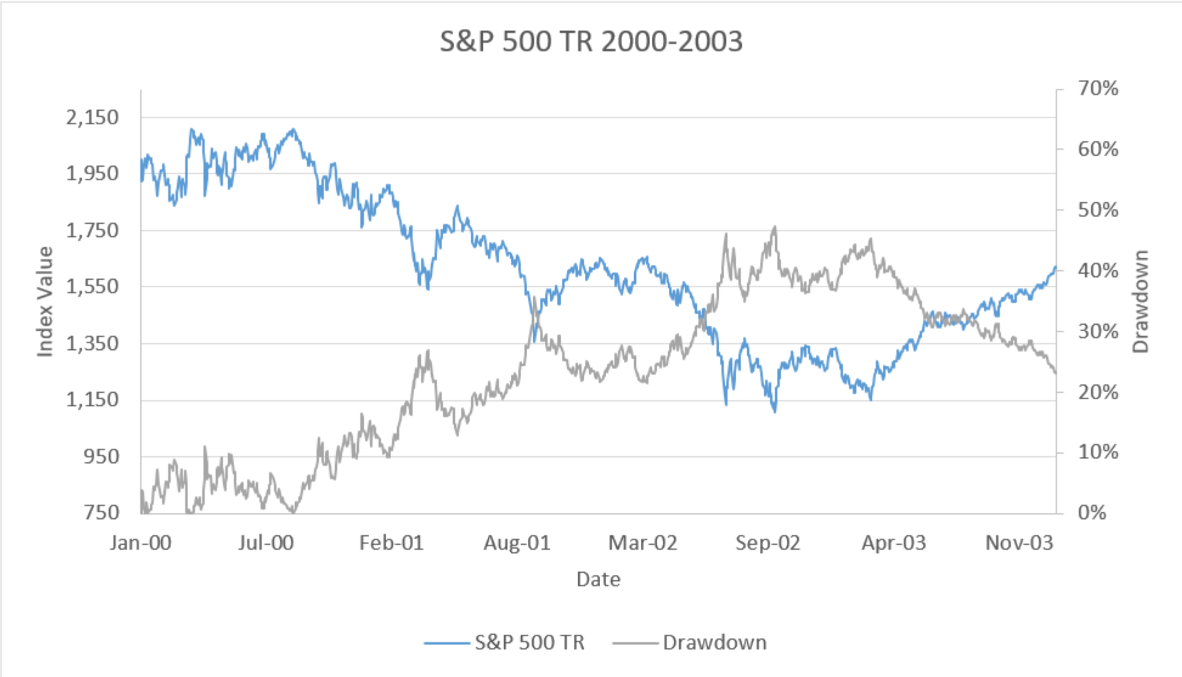

Figure 4.6: The S&P 500 TR Performance and Drawdowns 2000-2003

We also look at the period 2007 through 2009, pictured in Figure 4.7. This period shows how the S&P 500 TR performed under the global financial crisis. Here the maximum drawdown was above 50 percent, and the downturn was sharper than what we saw after the dotcom bubble. That makes this period interesting, as we see how the strategies performed during a steeper and more immediate downturn.

Figure 4.7: The S&P 500 TR Performance and Drawdowns 2007-2009

Finally, we will look at a more recent down period in Figure 4.8. During 2020 we saw a very steep downturn in the S&P 500 TR early in the year, before the market recovered quickly thereafter. The performance during this period will provide information on how the strategies work with extremely steep downturns, followed by a speedy recovery.

Figure 4.8: The S&P 500 TR Performance and Drawdowns 2020

4.2.5 Limitations

It is important to note that there are potential limitations to our method and thereby also our results. Primarily, the strategies we are testing are inspired by other studies that have successfully shown the strategies to be profitable. We do not know if these studies have purposefully cherrypicked inputs and variations that provide such results. Thereby, it could be the case that the inputs and variables we base our strategies on, could be subject to this basis and that in reality the results would be different without hindsight and cherry-picking.

Further, we make several assumptions regarding transaction costs. In reality, these costs can vary greatly based on the type of investor you are, the underlying market you trade, and the position sizes you trade. Therefore , these costs might not be representable for all investors and this could impact the returns and performance. Specifically, the assumption on financing costs might be too liberal. We assume that we can borrow and invest at the risk-free rate (three-month Treasury bill rate). It might be more appropriate to use a more conservative estimate, where the cost of borrowing is some basis-points above the risk-free rate, and the return for cash investments is some basis-points below. In a low interest-rate environment as we have had for the last many years, this is less relevant, but it could still change the results.

A final limitation to consider, is the scalability of the strategies. Scalability refers to how large the positions in the strategy can be without significantly impacting the results. In the constant volatility, the only underlying is the S&P 500, which is very liquid. Here scalability most likely is not a great problem. However, the tail-hedge strategy with a direct hedge might have issues with scalability. If the portfolio is too large, both the availability of options could be a problem in addition to the price impact of buying and selling large quantities of options. Therefore, the results we found for the tail-hedge strategy might not be equally representable for a large portfolio.

5 Results

In this section we will objectively look the results of our strategies and their variations. First, we will look at the performance of variations in the put option monetization strategy, where we vary both the multiple at which we sell, and the portfolio allocation to the strategy. Second, we will look at the constant volatility strategy and show performance sensitivity to variations in transaction costs and volatility target. Finally, we will compare the strategies with each other and with the underlying S&P 500 TR index over multiple time periods.

5.1 Put Option Monetization Tail-Hedge Strategy

We test the put option monetization tail-hedge strategy with variations in the portfolio allocation and in the target multiple. The portfolio allocation decides how much of the overall portfolio we spend on options. A higher allocation means that we buy options with higher moneyness, while a lower allocation means that the options we buy are further out of the money. A higher multiple, means that we hold on to the options longer, and only sell if the price increases substantially. We have for comparison reasons, also included the results for where there are no multiple. These tests are performed over the full period 1996 through 2020.

For the strategy where we allocate 1.5 percent of our portfolio to buying options, we only see one significant alpha. This alpha is -0.72 percent and is found when we use a multiple of 2.5x. All the alpha and beta estimates can be found in the appendix.

From Figure 5.1, we can see that the multiple that performs the best varies over the period. The 7.5x multiple looks to perform best after the dotcom bubble and up until the global financial crisis, while 2.5x multiple seems to outperform after 2010. Regardless, all strategies fall short of outperforming the underlying S&P 500 TR index.

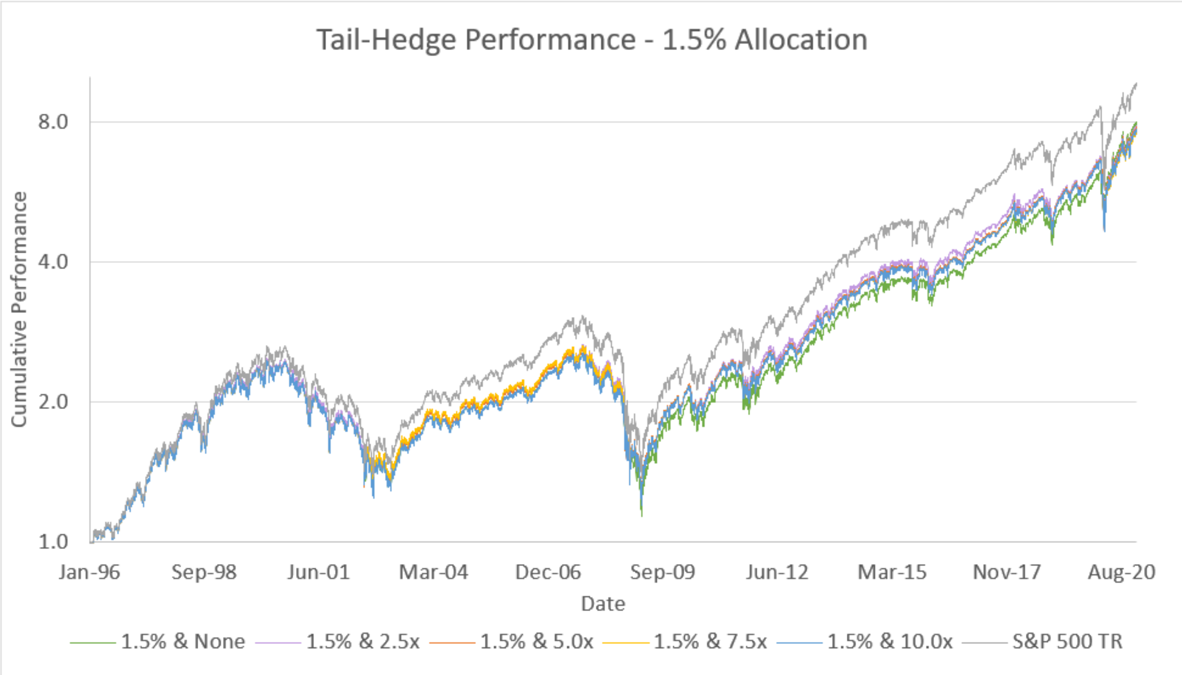

Figure 5.1: Performance of Variations in the Tail-Hedge Multiple

Looking at the various performance measures in Table 5.1, we see that on average the strategies with a multiple performed very similar. They all had similar Sharpe and Sortino ratios. Also, the maximum drawdowns and returns were similar. One observation worth noting, is that using no multiples, no monetization, performed better on average. The skewness for all is very similar and slightly negative, while the kurtosis varies more. The strategy with no multiple and with a multiple of 10, had lower kurtosis.

	Portfolio Allocation = 1.5%					
Multiple	**None**	**2.5x**	**5.0x**	**7.5x**	**10.0x**	**Index**
Average Excess Return	6.5%	6.4%	6.4%	6.3%	6.3%	7.3%
Annual Standard Deviation	17.3%	18.9%	18.7%	18.3%	18.5%	19.5%
Sharpe Ratio	0.37	0.34	0.34	0.34	0.34	0.37
Downside Standard Deviation	13.0%	15.0%	14.7%	14.3%	14.5%	15.7%
Sortino Ratio	0.50	0.43	0.43	0.44	0.43	0.46
Maximum Drawdown	55.4%	54.7%	53.8%	54.6%	52.9%	55.3%
Maximum Drawdown Date	09-Mar-09	09-Mar-09	09-Mar-09	09-Mar-09	09-Mar-09	09-Mar-09
Total Absolute Return	700%	686%	680%	664%	671%	872%
Leverage Employed	0.95	0.95	0.95	0.95	0.95	1.00
Skewness	-0.06	-0.08	-0.07	-0.09	-0.05	-0.18
Excess Kurtosis	1.95	6.95	6.97	4.86	6.33	7.18

Table 5.1: Performance Measures of Variations in the Tail-Hedge Multiple

Next, we look at the tail-hedge strategy with a 3.0 percent allocation. Also here we find negative alpha for some of the strategies. Both the 2.5x and 5.0x multiple has negative significant alpha using both the one and three-factor model. The full table can be found in the appendix.

Based on Figure 5.2 below, we see that the results are similar to that of the 1.5 percent allocation. The tail-hedge strategies underperform compared to the index, and different multiples seems to work for different time periods. Again, we see that the 2.5x multiple performs well in the years following 2010.

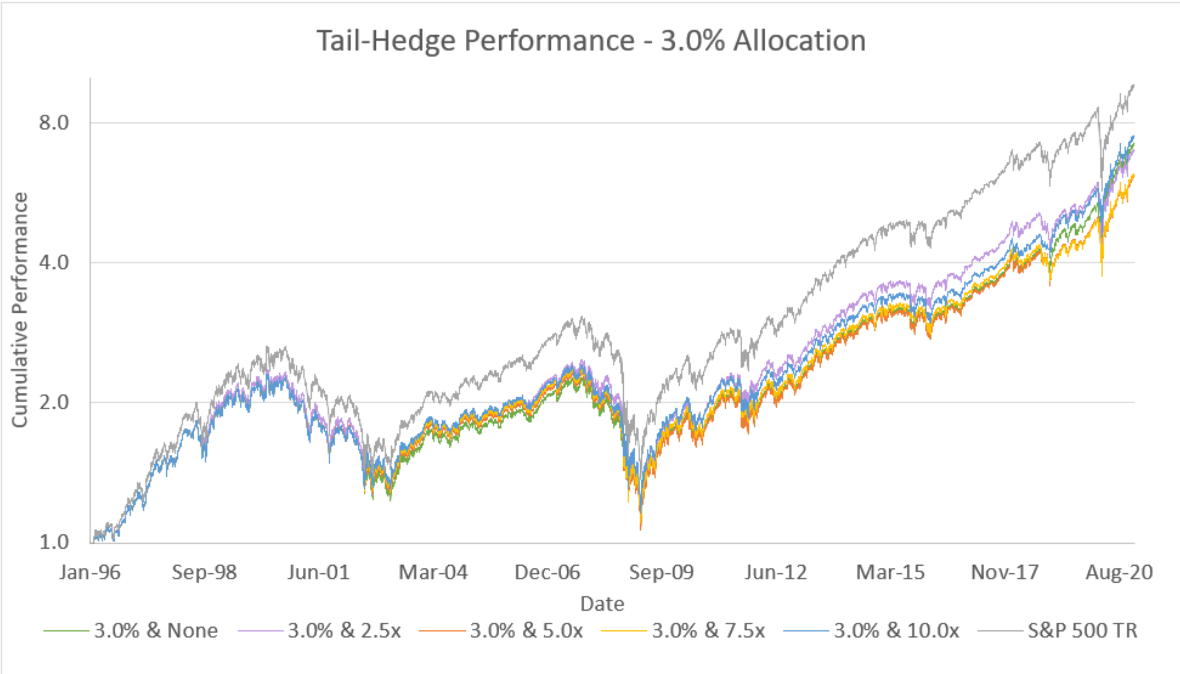

Figure 5.2: Performance of Variations in the Tail-Hedge Allocation

The performance measures in Table 5.2 tell a similar story. We see that returns are similar except for the no multiple and the 10 multiple. The no multiple outperforms the index on a risk-adjusted basis. The 10x multiple performs best when looking at absolute returns, and we also see higher Sharpe and Sortino ratios compared to the other strategies with a multiple. Again, we see that the no multiple strategy has low kurtosis compared to the rest.

Portfolio Allocation = 3.0%						
Multiple	**None**	**2.5x**	**5.0x**	**7.5x**	**10.0x**	**Index**
Average Excess Return	6.0%	5.9%	5.4%	5.4%	6.2%	7.3%
Annual Standard Deviation	16.0%	18.0%	17.4%	17.4%	17.4%	19.5%
Sharpe Ratio	0.38	0.33	0.31	0.31	0.36	0.37
Downside Standard Deviation	11.9%	14.2%	13.5%	13.5%	13.4%	15.7%
Sortino Ratio	0.51	0.42	0.40	0.40	0.46	0.46
Maximum Drawdown	52.9%	54.0%	54.3%	53.4%	51.8%	55.3%
Maximum Drawdown Date	09-Mar-09	09-Mar-09	09-Mar-09	09-Mar-09	09-Mar-09	09-Mar-09
Total Absolute Return	625%	603%	519%	523%	653%	872%
Leverage Employed	0.85	0.90	0.87	0.87	0.86	1.00
Skewness	0.01	-0.01	-0.02	0.07	0.09	-0.18
Excess Kurtosis	2.12	7.02	6.59	6.97	7.73	7.18

Table 5.2: Performance Measures of Variations in the Tail-Hedge Allocation

Comparing only the variation in portfolio allocation from the performance tables, and keeping the multiples constant, we see that on average the 1.5 percent allocation outperform the 3.0 percent allocation. There is some variation in how the multiples perform under the various allocation. We see that the 2.5x multiple has the best absolute returns for the 1.5 percent allocation, while the 10x multiple has the best performance for the 3.0 percent allocation.

5.2 Constant Volatility

For the constant volatility strategy, we test performance with variations in the transaction costs and the volatility target. We test variation in the transaction costs to see how sensitive the strategy is to changes in this variable, as the costs can vary significantly based on the investor. In addition, we test variations in the volatility target to see how flexible the model is.

For both changes in the transaction costs and volatility targets, we do not find any significant alphas for any variation in the model. All alpha and beta results can be found in the appendix.

First, we look at variations in the transaction costs. From Figure 5.3 below we can see that the higher the cost, the worse the return. We also clearly see that the strategy is very sensitive to the transaction costs, nevertheless all variations outperform the index.

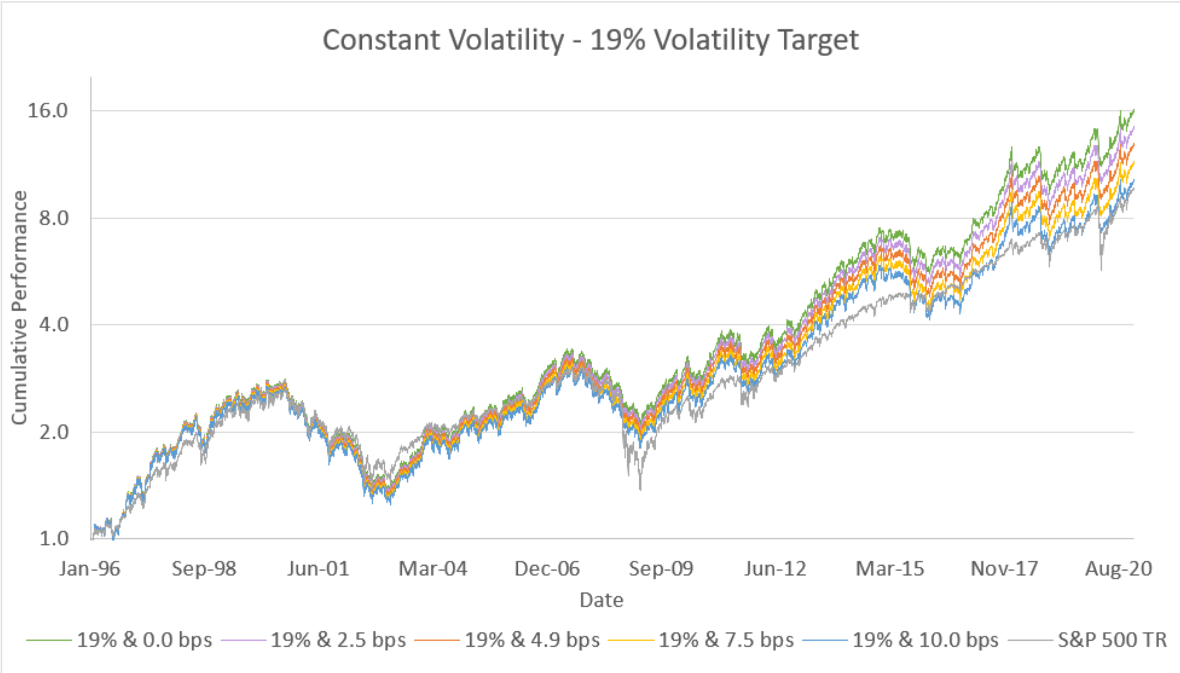

Figure 5.3: Performance of Variations in the Constant Volatility Transaction Cost

Looking at the performance measures in Table 5.3 confirms what we saw from the graph. Higher transaction costs reduce returns. All strategies performed better than the index on all the metrics, especially interesting to see is that the kurtosis is significantly lower for the constant volatility strategy as compared to the index.

	Volatility Target = 19%					
Transaction Cost	**0.0 bps**	**2.5 bps**	**4.9 bps**	**7.5 bps**	**10.0 bps**	**Index**
Average Excess Return	9.5%	9.0%	8.5%	8.0%	7.6%	7.3%
Annual Standard Deviation	19.8%	19.8%	19.8%	19.8%	19.8%	19.5%
Sharpe Ratio	0.48	0.46	0.43	0.41	0.38	0.37
Downside Standard Deviation	14.9%	14.9%	14.9%	15.0%	15.0%	15.7%
Sortino Ratio	0.64	0.60	0.57	0.54	0.50	0.46
Maximum Drawdown	52.3%	52.6%	52.9%	53.3%	53.6%	55.3%
Maximum Drawdown Date	11-Mar-03	11-Mar-03	11-Mar-03	11-Mar-03	11-Mar-03	09-Mar-09
Total Absolute Return	1519%	1346%	1197%	1053%	930%	872%
Leverage Employed	1.37	1.37	1.37	1.37	1.37	1.00
Skewness	-0.59	-0.59	-0.59	-0.59	-0.59	-0.18
Excess Kurtosis	-0.23	-0.22	-0.22	-0.22	-0.21	7.18

Table 5.3: Performance Measures of Variations in the Constant Volatility Transaction Cost

For variation in volatility targets, we see a similar story. Looking at Figure 5.4, we see that targeting a higher volatility, while keeping the transaction costs constant, results in higher returns.

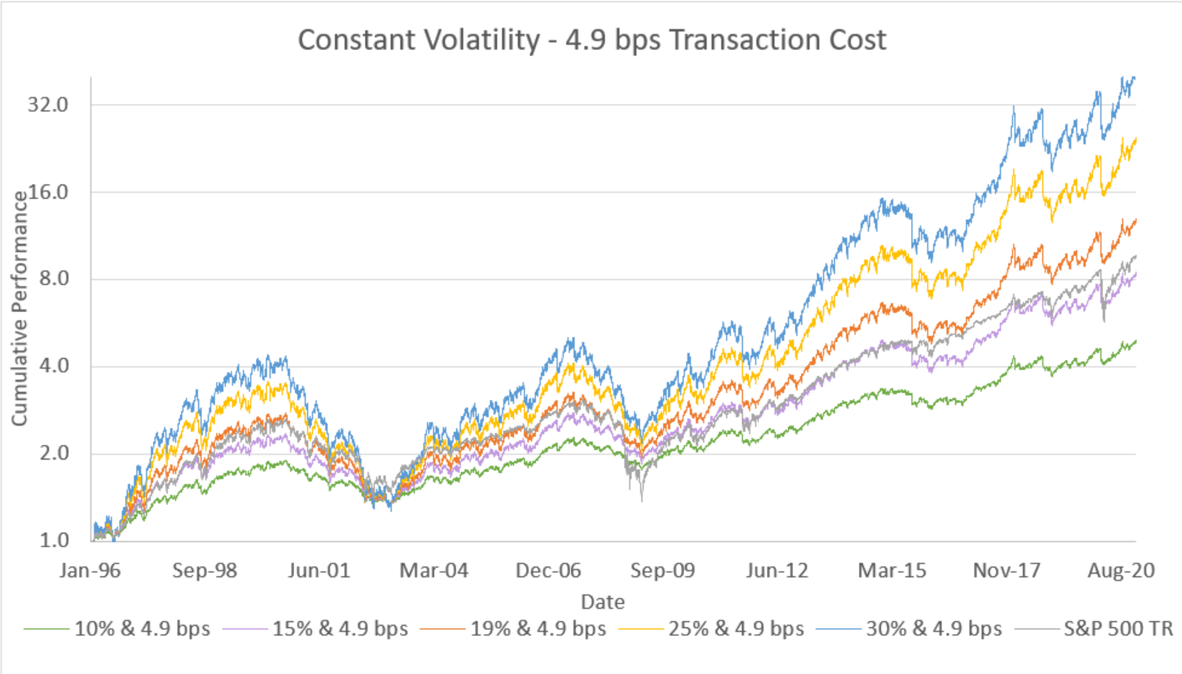

Figure 5.4: Performance of Variations in the Constant Volatility Target

Comparing the target volatility to the realized volatility in Table 5.4 below, we see that the model does a good job at estimating volatility. The realized volatility is a bit higher than the target for all volatility targets, and we see that the absolute difference between target volatility and realized volatility increases as the target increases.

Transaction Cost = 4.9 bps						
Volatility Target	**10%**	**15%**	**19%**	**25%**	**30%**	**Index**
Average Excess Return	4.4%	6.7%	8.5%	11.4%	13.8%	7.3%
Annual Standard Deviation	10.4%	15.6%	19.8%	26.1%	31.3%	19.5%
Sharpe Ratio	0.42	0.43	0.43	0.44	0.44	0.37
Downside Standard Deviation	7.9%	11.8%	14.9%	19.7%	23.6%	15.7%
Sortino Ratio	0.56	0.57	0.57	0.58	0.59	0.46
Maximum Drawdown	30.5%	44.0%	52.9%	63.9%	71.1%	55.3%
Maximum Drawdown Date	11-Mar-03	11-Mar-03	11-Mar-03	11-Mar-03	11-Mar-03	09-Mar-09
Total Absolute Return	391%	742%	1197%	2376%	4146%	872%
Leverage Employed	0.73	1.08	1.37	1.80	2.16	1.00
Skewness	-0.63	-0.61	-0.59	-0.56	-0.54	-0.18
Excess Kurtosis	-0.06	-0.15	-0.22	-0.32	-0.40	7.18

Table 5.4: Performance Measures of Variations in the Constant Volatility Target

From the above results, we see that the constant volatility strategy is very sensitive to the transaction costs assumed. We also see that the model does a good job at predicting volatility over a large range of volatility targets.

5.3 Comparison

In this subsection, we compare the two strategies and the underlying index. We will use the constant volatility model with a target of 19 percent and 4.9 bps in transaction costs as the representative for the constant volatility strategy. To represent the put option monetization tail-hedge strategy, we use two estimates: the model with 1.5 percent allocation and a multiple of 2.5x together with the model with a 3.0 percent allocation an a multiple of 10x. We chose these two tail-hedge estimates as they represent the two extremes, highest and lowest multiple, and one of each allocation. We will compare these strategies over several time periods.

5.3.1 Period: 1996 to 2020

The first period we look at, is the full period from 1996 through 2020. This is the same period we have looked at in the previous two subsections, where we look at variation to the constant volatility and tail-hedge model.

Looking at the return distribution of the index, the constant volatility strategy and the put option monetization tail-hedge strategy, visualized in Figure 5.5, 5.6 and 5.7, respectively, tells us how the strategies have worked at limiting extreme returns. From the below histograms, we can see that the S&P 500 TR index and the tail-hedge strategy has similar return distributions. The 1.5 percent and 2.5x multiple tail-hedge distribution can be found in the appendix and is very similar to the 3.0 percent and 10x multiple. Contrasting, we see that the constant volatility model returns very well resembles a normal distribution.

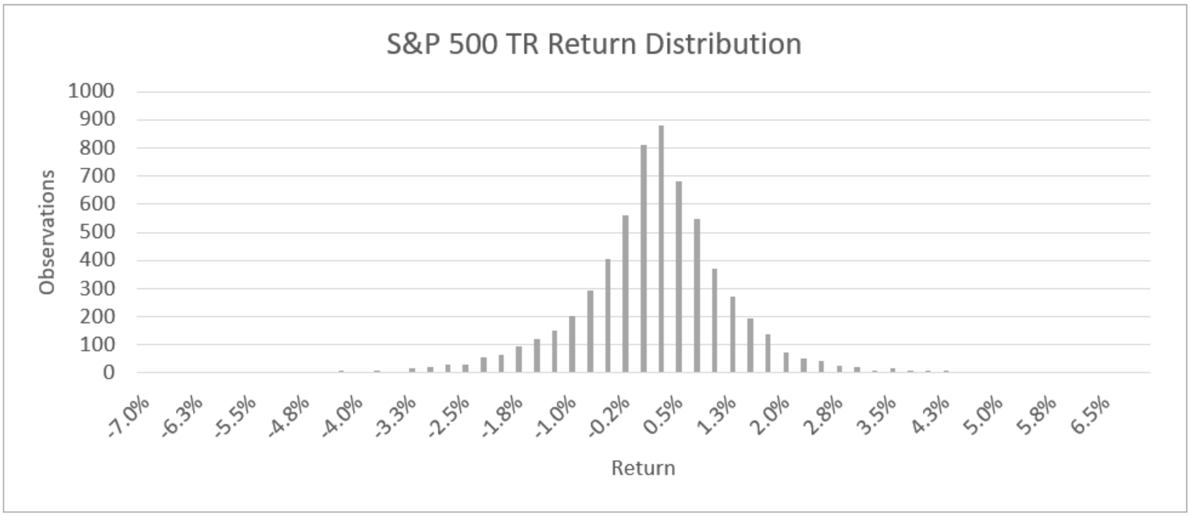

Figure 5.5: Return Distribution for the S&P 500 TR Index

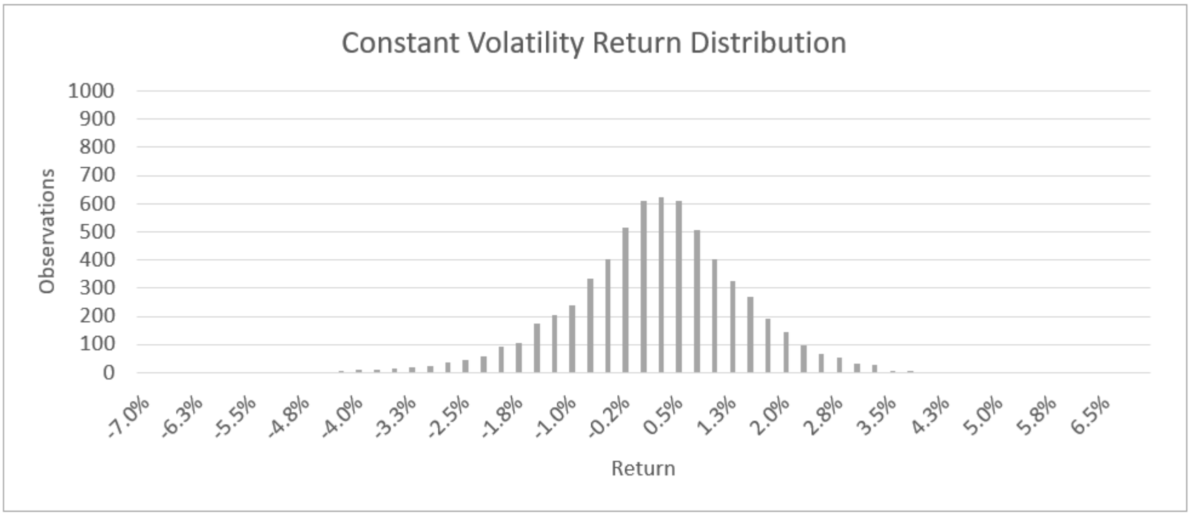

Figure 5.6: Return Distribution for the Constant Volatility Strategy

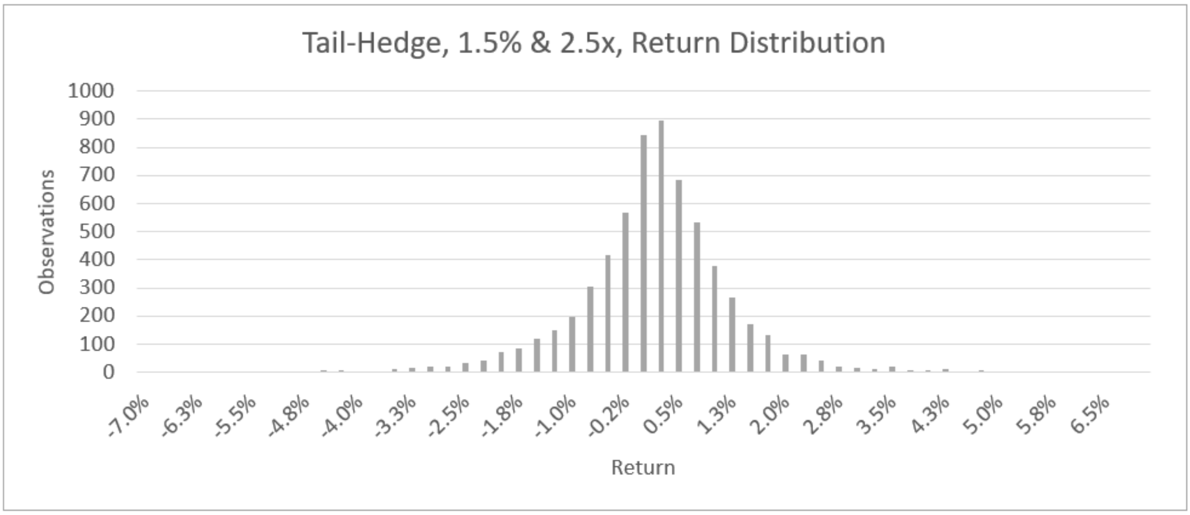

Figure 5.7: Return Distribution for the One of the Tail-Hedge Strategies

Comparing the performance of the strategies over the time period, we see in Figure 5.8 that the constant volatility strategy outperforms all other strategies. Despite the index outperforming both tail-hedge strategies, it is interesting to note that 1.5 percent allocation and 2.5x multiple clearly outperforms the 3.0 percent allocation and 10x multiple throughout the period, until the very last year.

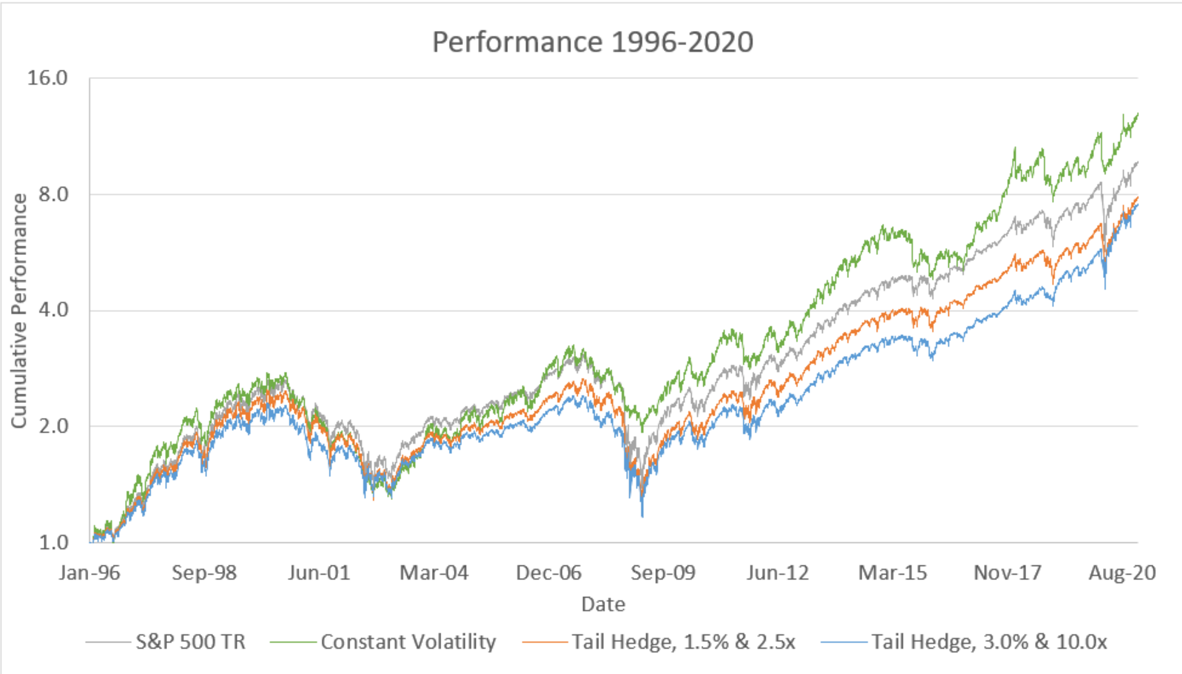

Figure 5.8: Performance of the Strategies 1996-2020

The outperformance of the constant volatility strategy is also evident in performance metrics displayed in Table 5.5 below. We see that the put option monetization tail-hedge strategies had lower standard deviation that the index, but also substantially lower returns, which resulted in no outperformance in the Sharpe or Sortino ratio as compared to the market. The constant volatility strategy had a Sharpe ratio of 0.43 and a Sortino ratio of 0.57, well above the index that had a ratio of 0.37 and 0.57, respectively. Both the tail-hedge strategies had lower drawdown than the index, and the 3.0 percent and 10x multiple had a lower drawdown than the constant volatility strategy as well.

Looking at the leverage employed, we see that the constant volatility strategy had a much higher average leverage, 1.37, than the tail-hedge strategies, 0.95 and 0.86. Confirming what we saw in the return distributions, the distribution of the constant volatility strategy returns had much lower kurtosis. It had an excess kurtosis of -0.22 compared to 7.18 for the index and 6.95 and 7.73 for the tail-hedge strategy.

	1996-2020			
	S&P 500 TR	**Constant Volatility**	**Tail-Hedge 1.5% & 2.5x**	**Tail-Hedge 3.0% & 10.0x**
Average Excess Return	7.3%	8.5%	6.4%	6.2%
Annual Standard Deviation	19.5%	19.8%	18.9%	17.4%
Sharpe Ratio	0.37	0.43	0.34	0.36
Downside Standard Deviation	15.7%	14.9%	15.0%	13.4%
Sortino Ratio	0.46	0.57	0.43	0.46
Maximum Drawdown	55.3%	52.9%	54.7%	51.8%
Maximum Drawdown Date	09-Mar-09	11-Mar-03	09-Mar-09	09-Mar-09
Total Absolute Return	872%	1197%	686%	653%
Leverage Employed	1.00	1.37	0.95	0.86
Skewness	-0.18	-0.59	-0.08	0.09
Excess Kurtosis	7.18	-0.22	6.95	7.73

Table 5.5: Performance Measures for the Period 1996 to 2020

To summarize, we see that the constant volatility strategy clearly outperforms the index over the period 1996 through 2020, while the tail-hedge strategies underperform.

5.3.2 Period: 2010 to 2017

The second period we look at is the period of 2010 through 2017, a period where we saw few drawdowns and mainly an upward trend in the S&P 500. For this period, we did not find any significant alphas in any of the periods, except for significant alpha of -1.47 percent for the 3.0 percent allocation tail-hedge when using the three-factor model. All numbers can be found in the appendix.

In Figure 5.9 below, we see that the constant volatility strategy again outperforms both the index and the tail-hedge strategy. It does, however, seem that the constant volatility strategy has higher volatility than the others over this period. The tail-hedge strategies perform the worst, but we do see that the tail-hedge strategy with 1.5 percent allocation and a multiple of 2.5x clearly outperforms the tail-hedge with 3.0 percent allocation and 10x multiple.

Figure 5.9: Performance of the Strategies 2010-2017

From Table 5.6 below, we do see that the constant volatility strategy had higher standard deviation and higher drawdowns than the index and tail-hedge strategy. During this period, the index had both higher Sharpe ratio and Sortino ratio than both strategies. The tail-hedge strategies had the lowest volatility and drawdowns over this period, and lower employed leverage. The constant volatility had an average leverage of 1.60, while the tail-hedge had 0.98 and 0.89. All strategies and index had negative skewness over the period, and the both the constant volatility and tail-hedge strategies had lower kurtosis than the index.

	2010-2017			
	S&P 500 TR	**Constant Volatility**	**Tail-Hedge 1.5% & 2.5x**	**Tail-Hedge 3.0% & 10.0x**
Average Excess Return	13.5%	15.8%	12.3%	10.2%
Annual Standard Deviation	14.7%	19.1%	14.2%	13.0%
Sharpe Ratio	0.91	0.82	0.87	0.78
Downside Standard Deviation	11.8%	14.9%	11.1%	10.0%
Sortino Ratio	1.15	1.06	1.12	1.02
Maximum Drawdown	18.6%	27.8%	17.2%	18.5%
Maximum Drawdown Date	03-Oct-11	11-Feb-16	03-Oct-11	03-Oct-11
Total Absolute Return	179%	228%	157%	120%
Leverage Employed	1.00	1.60	0.98	0.89
Skewness	-0.37	-0.60	-0.25	-0.27
Excess Kurtosis	1.67	-0.45	0.98	-0.06

Table 5.6: Performance Measures for the Period 2010 to 2017

Summing up the period 2010 through 2017, we see that the constant volatility strategy had higher returns than the index and tail-hedge strategies, but also higher volatility, while the tail-hedge had the lowest volatility, but also the lowest returns.

5.3.3 Period: 2000 to 2003

Third, we look at the period 2000 to 2003. This is the first of three periods with large drawdowns we will investigate. This period is characterized by a long but slow downturn, where the drawdown lasted around two years. For this period, we did not find any significant alphas, and the estimates can be found in the appendix.

Looking at Figure 5.10, we see that the constant volatility performed worst, while the tail-hedge strategies performed very similar to the index. The best performing tail-hedge strategy early-on was the 1.5 percent allocation, while the 3.0 percent allocation worked better in the second half of the period.

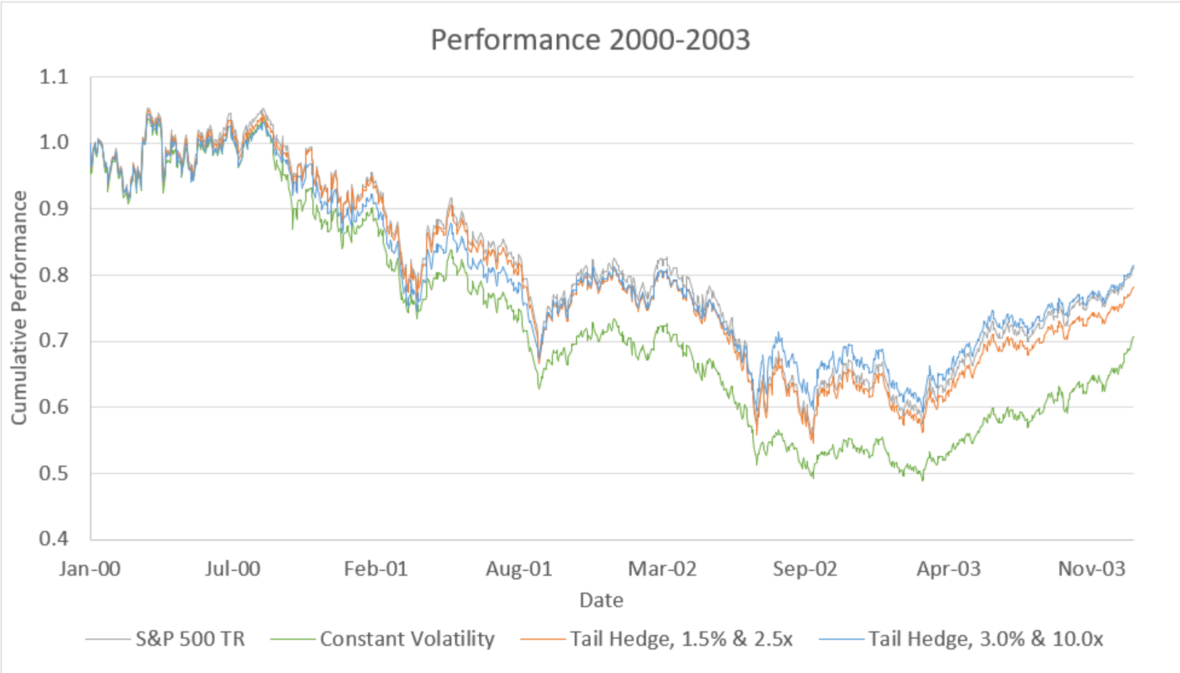

Figure 5.10: Performance of the Strategies 2000-2003

From the Table 5.7, we see that all strategies and the index had negative returns. The constant volatility had the lowest standard deviation, followed by the tail-hedge strategies. Overall, the 3.0 percent tail-hedge performed best, having the highest return, and the lowest volatility. It also had the lowest drawdown in 43.2%, and the lowest leverage employed. The constant volatility strategy had the highest drawdown, while the 1.5 percent tail-hedge had next highest, barely exceeding the index's drawdown. The kurtosis is similar for all estimates, while the constant volatility strategy had the lowest skewness.

	2000-2003			
	S&P 500 TR	Constant Volatility	Tail-Hedge 1.5% & 2.5x	Tail-Hedge 3.0% & 10.0x
Average Excess Return	-7.8%	-10.9%	-8.7%	-7.7%
Annual Standard Deviation	21.9%	19.6%	21.2%	19.7%
Sharpe Ratio	-0.36	-0.56	-0.41	-0.39
Downside Standard Deviation	14.0%	12.8%	13.3%	12.0%
Sortino Ratio	-0.56	-0.86	-0.65	-0.65
Maximum Drawdown	47.4%	52.9%	48.0%	43.2%
Maximum Drawdown Date	09-Oct-02	11-Mar-03	09-Oct-02	11-Mar-03
Total Absolute Return	-19%	-29%	-22%	-19%
Leverage Employed	1.00	0.98	0.97	0.86
Skewness	0.21	-0.04	0.25	0.31
Excess Kurtosis	-1.66	-1.93	-1.68	-1.64

Table 5.7: Performance Measures for the Period 2000 to 2003

For the period 2000 to 20003, the constant volatility strategy period performed worst, while the 3.0 percent allocation tail-hedge strategy performed the best.

5.3.4 Period: 2007 to 2009

Fourth, we take a look at the time period 2007 to 2009. Here we see a sharper drawdown, as compared to the 2000 to 2003 period. For the period, we did not find any significant alphas, and the table containing this information can be found in the appendix.

Looking at the 2007-2009 price chart in Figure 5.11, we see that the constant volatility strategy had the best performance and significantly lower drawdown than the index and the tail-hedge strategies. Looking at the two tail-hedge strategies, we find that the 1.5 percent allocation strategy worked best prior to the steepest index drawdown, while the 3.0 percent allocation strategy worked best during and after this drawdown.

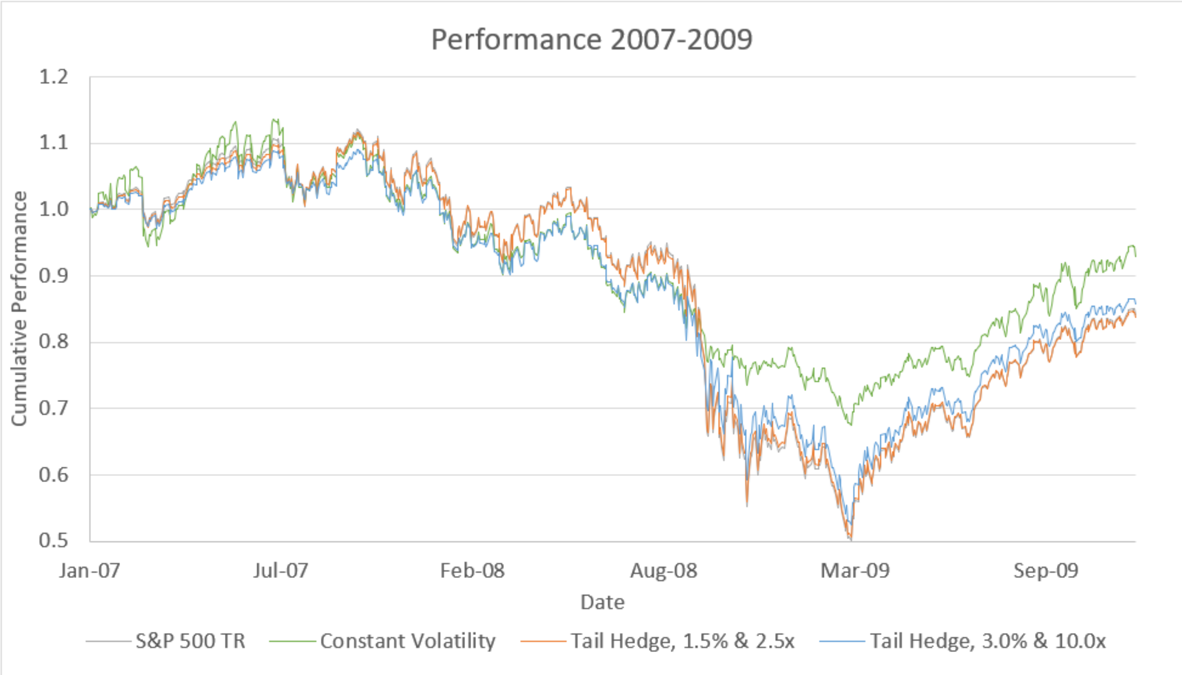

Figure 5.11: Performance of the Strategies 2007-2009

From Table 5.8, we clearly see that the constant volatility strategy performed best. It had the highest returns, -4.3 percent, and lowest volatility of 20.3 percent, resulting in the highest Sharpe and Sortino ratio. The tail-hedge strategies performed similar to the index, while having slightly lower standard deviation. The tail-hedge strategies had maximum drawdowns of 54.7 and 51.8 percent, while the constant volatility strategy had 40.6 percent, and the index 55.3 percent. The constant-volatility strategy had significantly lower skewness and kurtosis than both the tail-hedges and the index, which both had similar estimates. The leverage employed was lowest for the tail-hedge strategies, while it was slightly higher than the index for the constant volatility strategy.

	2007-2009			
	S&P 500 TR	Constant Volatility	Tail-Hedge 1.5% & 2.5x	Tail-Hedge 3.0% & 10.0x
Average Excess Return	-7.4%	-4.3%	-7.5%	-6.8%
Annual Standard Deviation	30.0%	20.3%	29.3%	27.4%
Sharpe Ratio	-0.25	-0.21	-0.26	-0.25
Downside Standard Deviation	24.1%	15.9%	23.5%	21.7%
Sortino Ratio	-0.31	-0.27	-0.32	-0.31
Maximum Drawdown	55.3%	40.6%	54.7%	51.8%
Maximum Drawdown Date	09-Mar-09	09-Mar-09	09-Mar-09	09-Mar-09
Total Absolute Return	-16%	-7%	-16%	-14%
Leverage Employed	1.00	1.02	0.96	0.93
Skewness	0.07	-0.68	0.10	0.21
Excess Kurtosis	3.34	-0.29	3.31	3.96

Table 5.8: Performance Measures for the Period 2007 to 2009

To summarize, during the 2007 to 2009 period, the constant volatility strategy performed the best, followed by the tail-hedges on average, while the index performed worst.

5.3.5 Period: 2020

Finally, we look at the year 2020. This year was characterized by a very steep drawdown followed by a quick recovery. The drawdown lasted only a few months, and the index ended positive for the year. We did not find any significant alphas for any of the strategies during this time period, and the full table with this information can be found in the appendix.

By looking at Figure 5.12, we see that 3.0 percent tail-hedge strategy greatly outperformed the other strategies and the index. The constant volatility strategy looks to have had the worst returns, but a lower volatility. The 1.5 percent tail-hedge performed very similar to the index throughout the period.

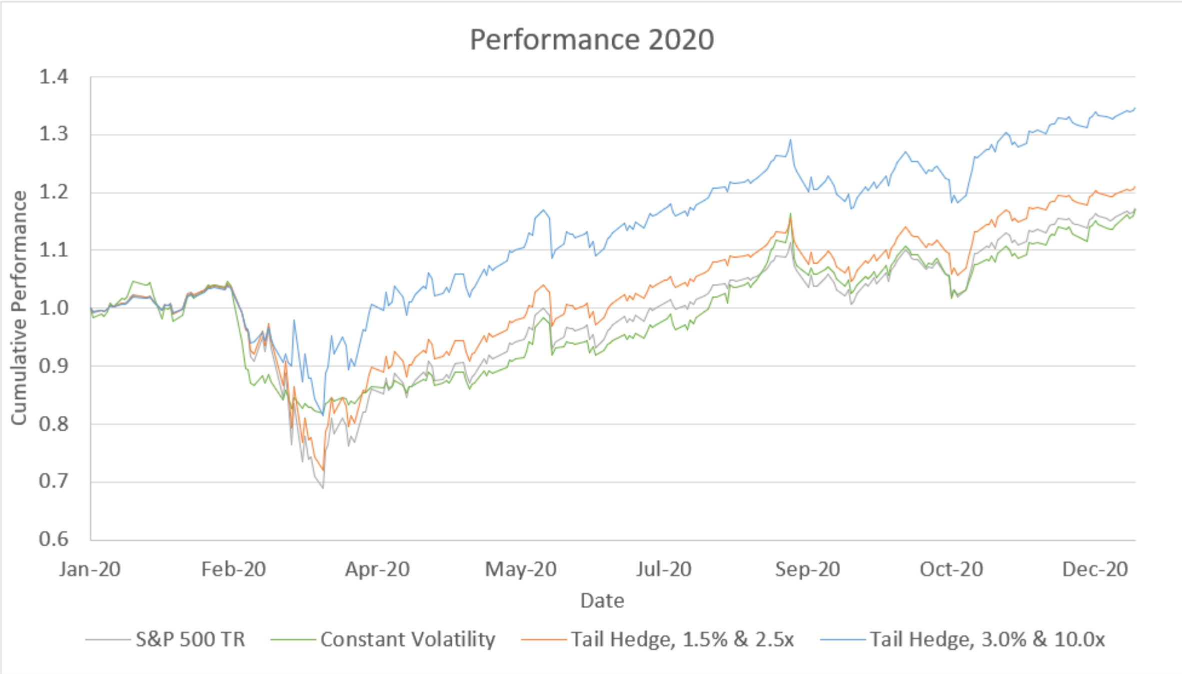

Figure 5.12: Performance of the Strategies 2020

From Table 5.9 below we see that the 3.0 percent tail-hedge performed the best, with an average excess return of 34.4 percent and a volatility of 29.3 percent. This strategy also had the highest Sharpe and Sortino ratio, in addition second lowest drawdown. The 1.5 percent tail-hedge performed better than the index, with a return of 20.9 percent compared to 17.0 percent for the index, and a slightly lower standard deviation. It did however, perform worse than the constant volatility based on the risk-adjusted estimates. The 3.0 percent tail-hedge and the constant volatility had similar maximum drawdowns, 21.5 and 21.7 percent, respectively. The constant volatility strategy had an average employed leverage of 1.1, while the tail-hedge strategies had below 1.0.

	2020			
	S&P 500 TR	**Constant Volatility**	**Tail-Hedge 1.5% & 2.5x**	**Tail-Hedge 3.0% & 10.0x**
Average Excess Return	17.0%	16.7%	20.9%	34.4%
Annual Standard Deviation	34.7%	22.5%	33.1%	29.3%
Sharpe Ratio	0.49	0.74	0.63	1.17
Downside Standard Deviation	31.7%	20.4%	29.2%	24.2%
Sortino Ratio	0.54	0.82	0.72	1.42
Maximum Drawdown	33.8%	21.7%	30.7%	21.5%
Maximum Drawdown Date	23-Mar-20	23-Mar-20	23-Mar-20	23-Mar-20
Total Absolute Return	17%	17%	21%	35%
Leverage Employed	1.00	1.10	0.98	0.94
Skewness	-0.54	-1.49	-0.32	0.00
Excess Kurtosis	5.00	2.11	4.56	6.19

Table 5.9: Performance Measures for the Period 2020

To summarize the 2020 period, we see that the tail-hedge strategy performs very well, especially the 3.0 percent allocation. The constant volatility strategy had the lowest returns, but also substantially lower standard deviation.

6 Discussion

We find none of our tested put option monetization strategies, incorporated in an equity portfolio, to consistently reduce max drawdowns in any meaningful way. Nor do they enhance returns compared to the passive index holding strategy. The Sharpe ratios for all monetization strategies are lower compared to the index's, and they are also outperformed by applying a passive buy and hold put option strategy by the same measures. Our tested constant volatility strategy on the same index has a better ability to reduce drawdowns and substantially increases total returns compared to passively holding the index.

6.1 Put Option Monetization Strategies

We have tested eight different monetization strategies, consisting of four different multiple targets, 2.5x, 5.0x, 7.5x, and 10.0x, with a 1.5 percent budget allocation and a 3 percent budget allocation. We see no clear pattern in the strategies' performances during the 1996-2020 time period, where the average annual excess returns ranges from 5.4 - 6.4 percent and the Sharpe ratios to range from 0.29 to 0.36. The best performing monetization strategy with the 1.5 percent budget was the 2.5x multiple strategy with Sharpe ratio 0.34, and the best performing strategy with the 3 percent budget was the 10.0x multiple strategy with Sharpe ratio 0.36. Given the results it is difficult to call one strategy superior to the others, but it should be noted that both of the no monetization strategies get slightly higher Sharpe ratios compared to the monetization strategies.

Given that put options are leveraged assets with negative correlations with the market and therefore have a negative beta, it should not be seen as a surprise from a CAPM perspective that the results from incorporating put options in a portfolio are negative. However, as can be seen in 2020, incorporating a put option strategy with a position in equities can potentially also produce great payoffs. During that year we also see a clear difference between the 2.5x multiple 1.5 percent budget allocation strategy and the 10.0x multiple 3 percent budget allocation strategy, where the latter performs considerably better and returns 34 percent, compared to 21 percent for the other hedging strategy and 17 percent for the S&P 500 TR Index. Here it is clear that the 10.0x multiple strategy benefited greatly from not taking the profits from the put options too early, whereas during the full 25 year period, the lower multiple strategy still managed to earn a higher excess return.

A varying result, like the one described above due to different target multiples, raises an obvious question about timing and showcases how important it can be for at least the short term results (remember the both strategies still performed very similarly during the 25 year time period). Because it is also a fact, exactly as Israelov (2017) argues, that an untimely implementation of a put option strategy actually can lead to increased portfolio drawdowns. This happens in the 3 percent budget strategy for the target multiples 2.5x, 5.0x and 7.5x. In general, i.e., except for the 2020 example, we see little evidence of the put option strategies' ability to limit drawdowns. During both the dotcom bubble and the financial crisis, none of the 2.5x multiple 1.5 percent budget and the 10.0x multiple 3 percent budget strategies reduced portfolio drawdowns adequately and they did not reduce the smaller maximum drawdown during the 2010-2017 period either.

Ilmanen (2012) also discuss the importance of timing for an active manager of put options and does not rule out all put option strategies as unprofitable. To time the market is nevertheless a notoriously difficult task. Imagine for example if some investors in October 2019, let us say by an act of God, got information about the upcoming COVID-19 virus and the nationwide lockdowns in 2020. What would they have predicted would happen to stock prices during 2020? Would they have predicted the S&P 500 to return over 15 percent? We are doubtful about that and while it may not be a proof of market inefficiency, it is a display of just how unpredictable the financial markets can be. In that sense, it actually gives support to Bhansali's suggestion about an "always on" tail hedging strategy but it still remains an issue on how to implement one with the use of put options.

While our results are in line with the prevailing view of put options as an insurance, the argumentation of Litterman (2011) and the findings of Ilmanen (2012) in the way that it seems like an unprofitable idea to buy portfolio insurance in the long run, the critique from Taleb (2013) might still be valid also to our findings. To implement our strategy on further out of the money put options, instead of purchasing our put options at the highest moneyness our budget allowed, could of course change our results and conclusions. The results from Bhansali's strategy, displayed in Table 3.1, also show a tendency that further out of the money options performs better compared to more at the money options over time. However, when reading the results of the table, it is important to be aware of the Black-Scholes model's inability to correctly model option prices that are far out of the money, i.e., real option prices display a volatility smile. It could therefore be the case that Bhansali's results in favor for the put option strategy would be different if it was implemented on real option prices. If an investor still would want to implement a further out of the money strategy, it is also not unlikely that he would stumble on liquidity constraints since far out of the money options are less liquid compared to at the money options.

Another difference of our tested strategy, compared to the one Bhansali (2013) presents, is the time to maturity of the options in the strategies. Whereas he implements the strategy with the use of one year options, we use three months options for liquidity reasons. This can be another reason for our conflicting results and also a reason as to why we find the passive buy and hold to maturity put option strategy to perform better compared to the monetization strategies. It could be the case because compared to a one year put option, a three month put option has less time to first increase in price, say due to it suddenly being in the money, and then also to decrease in price due to the underlying asset's price increase before it matures. Assuming the underlying asset has a positive expected return, the shorter maturity option has a larger chance of earning returns by ending up being in the money than the longer maturity option in scenarios similar to the one described. The probability of a price journey like the one demonstrated in Figure 2.12, where the put option price spikes before maturity, only to later decrease and end up out of the money, is thus larger for a longer maturity option. It follows that it may make more sense and be more profitable to apply a monetization strategy when implementing it with longer maturity options.

It should also be noted that, despite our findings, we are like Ilmanen (2012) not able to rule out put options as a means of tail risk hedging. We have only tested one of the four different methods Bhansali proposes for actively managing put options as an insurance, and implemented it in a rigid, rule-based way. It is not unreasonable to believe that it could prove to be efficient to implement an indirect hedging technique and aim at exploiting increasing correlations in times of crisis. An active manager might also be able to make more use of the monetization technique while managing options with different maturities. All the methods Bhansali proposes can be put together in a single tail risk hedging strategy, but such a strategy would be very opportunistic and flexible in its nature, making it very difficult to empirically backtest. A way to try and determine whether those kind of strategies are profitable or not could instead be to look at portfolio managers' performance applying such a method. But doing so would then instead raise the question whether the eventually successfully portfolio managers were lucky or not, and the debate of passive vs active investing would get new fuel.

6.2 Constant Volatility

We find our implemented constant volatility strategy to be able to reduce drawdowns in crises, and enhance portfolio returns over the investigated 25 year period. The constant volatility strategy gets an average annual excess return is 8.6 percent compared to the index's 7.3 percent, and a Sharpe ratio of 0.43 compared to the index's 0.37. The strategy performs worse compared to the index during the burst of the dotcom bubble but effectively reduces the max drawdowns in the two later crises.

Measured in volatility, the strategy has not been riskier compared to the S&P500 TR index, and it also has a beta smaller than one, indicating it has been less risky compared to the broader market factor used in the data from the Kenneth French website, seen from a CAPM perspective. To investigate why we see these results is beyond our scope of the paper, but it is nevertheless reasonable to attribute the results to its ability to reduce tail risk and usage of leverage during calmer, steadily increasing markets and that this stems from the negative correlation of volatility and market returns and the persistence of volatility.

Looking at the strategy's performance during crises, we see that it has the largest drawdown of all compared portfolios during the burst of the dotcom bubble. It subsequently outperforms all of them during the financial crisis, where compared to the index the maximum drawdown is reduced from 55 to 41 percent, and during the 2020 market crisis it is reduced from 34 to 22 percent respectively. This result might be an effect of market volatility being higher in the latter two crises, as can be seen in Figure 2.13. It also worth noting that the strategy effectively decreases the excess kurtosis, i.e., the fat tails, of the return distribution compared to both the index, and the put option monetization strategies that are not able to achieve that.

Throughout the bull market period of 2010-2017, the strategy is on average highly levered and delivers a 2.3 percentage annual outperformance while also experiencing a larger max drawdown compared to the index. During this time period the strategy takes on more risk, and while the total return is better, the standard deviation gets higher and the Sharpe ratio is lower compared to the index. This is however compensated in the long run by the strategy's capability of reducing risk in worse market conditions and the impressive improvement in total returns over time. Because while the Sharpe ratio may be a fair metric of risk adjusted returns, it is not unreasonable to think that investors are happier with a 20 percentage annual excess return with a 20 percentage annual standard deviation, compared to a 5 percentage annual excess return with a 4 percentage annual standard deviation, thus Sharpe ratios should not be the only performance metric of a portfolio and must be put into perspective.

A target volatility of 19 percent may be too high for risk averse investors and too small for risk seeking investors and was arbitrarily chosen to match the average standard volatility of the S&P 500 TR index during the 25 year period. Therefore, we also implemented our strategy with volatility targets of 10, 15, 25 and 30 percent. These strategies show similar results as the 19 percentage volatility target strategy and behaves as expected with respect to their leverage. The Sharpe ratios range from 0.42 to 0.44 and the average annual excess returns range from 4.4 percent for the 10 percent target to 13.8 percent for the 30 percent target.

The results are in line with previous research on constant volatility models conducted by Hocquard et al. (2013), Harvey et al. (2018) and Doan et al. (2018). Our findings are still very interesting given that our strategy is arguable a less complex one compared to the ones implemented in the previously mentioned research. For example, in our strategy we are not trading futures to reduce costs, we have no threshold for when we choose not to trade a specific day, and our GARCH(1,1) model is not corrected for outliers. Optimizing on these aspects ought to increase the performance of our tested strategy. This is partly demonstrated in Table 5.3, where it can be seen that the trading costs have a material impact on our constant volatility strategy. Our simplified model can also be the reason as to why our strategy is not getting significant alphas when tested with the Fama French three-factor model like the strategy applied by Doan et al. achieved.

Assuming a consistent negative relationship between volatility and market returns, a constant volatility strategy could be said to become a momentum strategy, where the strategy levers up during times of market prosperity and levers down during market declines. The momentum factor is one of the known anomalies to the EMH as demonstrated by Jegadeesh and Titman (1993) and seen in that perspective constant volatility strategies' success may be somewhat anticipated. This thought is also demonstrated in the paper by Doan et al. (2018) where they also highlight that momentum strategies have been shown not to survive transaction costs, whereas their strategy, like ours, do.

7 Conclusion

In this paper, we contributed to the limited literature on using actively managed put options as tail hedges. We investigated whether a put option monetization strategy can protect an equity portfolio from drawdowns and enhance its returns. We have done so by applying eight different monetization strategies, using S&P 500 put options and an underlying equity portfolio represented by the S&P 500 Total Return index, during the time period 1996 to 2020. We have compared the results from the strategies with an unhedged position in the chosen index, and with a constant volatility strategy applied on the same underlying.

We find our monetization strategies to only effectively reduce portfolio drawdowns in one of the three periods we investigated closer. Over the course of the 25-year period, all the strategies' total returns and Sharpe ratios are inferior to the unhedged index position. The observed results suggest that rule-based monetization strategies are not able to adequately reduce drawdowns, less, enhance returns of the portfolio compared to the unhedged position. These observations harmonize with the argumentation of Litterman (2011) and the findings of Ilmanen (2012), but critique from Taleb (2013), regarding the moneyness of the options employed in put option strategies, is also applicable to our study. Moreover, we are only testing one of four active put option trading techniques Bhansali (2013) suggests a portfolio manager could use. The conducted study is, nevertheless, a fair test of a simple monetization strategy using real life option prices that Bhansali shows to be profitable using Black-Scholes modelled option prices.

The observed results might be sensitive to the chosen representation of the equity portfolio and the investigated time period. Furthermore, the results could be sensitive to the choices of time to expiry and moneyness of the bought options in the tested strategies. Active money managers could also try and exploit increasing correlations in swift market declines with the use of indirect hedges. Future research on actively traded put option strategies could therefore reasonably target the use of longer maturity options, further out of the money options and the purchasing of indirect hedges.

A more certain way to achieve the goals of reduced tail risk, and increased returns, seems to be the implementation of a constant volatility strategy. Our simple employment of a volatility targeting strategy managed to adequately reduce portfolio drawdowns in two out of the three examined crises, improving the excess kurtosis of the return distribution and generating a higher total return and Sharpe ratio compared to the index. These results are impressive given that we implement a less complex version of the strategy compared to earlier successful implementations in research papers by Hocquard et al. (2013) and Doan et al. (2018), among others. Improvements in the model includes the usage of futures and trading thresholds to keep down trading costs, and the use of an outlier corrected GARCH(1,1) model to make better volatility forecasts. Constant volatility strategies exploit a frequently observed negative relationship between market volatility and market returns, enabling it to lever up and earn returns in calm and prosperous markets, while also being able to lever down and avoid larger negative returns in times of market stress.

Comparing our put option monetization strategies with a constant volatility strategy, that often makes use of leverage, could be thought of as unfair. However, they are both two different means to the same goal: decreased drawdowns and enhanced returns. Both strategies should be of interest to essentially all investors since risk management is of utmost importance in all portfolios. While the usage of options and leverage can require a certain amount of capital and sophistication, and therefore might be most effectively applied by fund managers, insurance companies and pension funds. The products available in the modern financial markets also make these strategies implementable for private investors.

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Appendix

Portfolio Allocation = 1.5%										
Multiple	**None**		**2.5x**		**5.0x**		**7.5x**		**10.0x**	
	One-Factor Model									
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	-0.01%	0.99	-0.88%	0.07	-0.84%	0.10	-0.79%	0.15	-0.82%	0.13
Beta	0.84	0.00	0.96	0.00	0.95	0.00	0.92	0.00	0.93	0.00
	Three-Factor Model									
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	0.12%	0.91	-0.72%	0.03	-0.69%	0.07	-0.64%	0.15	-0.67%	0.12
Beta	0.84	0.00	0.96	0.00	0.95	0.00	0.93	0.00	0.94	0.00
SMB	-0.13	0.00	-0.16	0.00	-0.16	0.00	-0.16	0.00	-0.16	0.00
HML	-0.02	0.00	0.03	0.00	0.03	0.00	0.02	0.00	0.02	0.00

Table A.1: Alpha and Beta Estimates for the Tail-Hedge Strategy Using an Allocation of 1.5%

Portfolio Allocation = 3.0%										
Multiple	**None**		**2.5x**		**5.0x**		**7.5x**		**10.0x**	
	One-Factor Model									
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	-0.22%	0.63	-1.06%	0.05	-1.36%	0.03	-1.28%	0.07	-0.49%	0.52
Beta	0.99	0.00	0.91	0.00	0.88	0.00	0.87	0.00	0.87	0.00
	Three-Factor Model									
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	-0.06%	0.84	-0.92%	0.04	-1.22%	0.02	-1.13%	0.07	-0.35%	0.62
Beta	0.99	0.00	0.91	0.00	0.88	0.00	0.87	0.00	0.87	0.00
SMB	-0.17	0.00	-0.16	0.00	-0.15	0.00	-0.15	0.00	-0.15	0.00
HML	0.02	0.00	0.03	0.00	0.03	0.00	0.03	0.00	0.03	0.00

Table A.2: Alpha and Beta Estimates for the Tail-Hedge Strategy Using an Allocation of 3.0%

Volatility Target = 19%										
Transaction Cost	**0.0 bps**		**2.5 bps**		**4.9 bps**		**7.5 bps**		**10.0 bps**	
	One-Factor Model									
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	3.05%	0.14	2.59%	0.20	2.16%	0.29	1.69%	0.41	1.24%	0.55
Beta	0.86	0.00	0.86	0.00	0.86	0.00	0.86	0.00	0.86	0.00
	Three-Factor Model									
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	3.16%	0.12	2.71%	0.18	2.27%	0.26	1.80%	0.37	1.35%	0.51
Beta	0.87	0.00	0.87	0.00	0.87	0.00	0.87	0.00	0.87	0.00
SMB	-0.06	0.00	-0.06	0.00	-0.06	0.00	-0.06	0.00	-0.06	0.00
HML	-0.12	0.00	-0.12	0.00	-0.12	0.00	-0.12	0.00	-0.12	0.00

Table A.3: Alpha and Beta Estimates for the Constant Volatility Strategy Varying Transaction Costs

Transaction Cost = 4.9 bps										
Volatility Target	**10%**		**15%**		**19%**		**25%**		**30%**	
	One-Factor Model									
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	0.63%	0.56	1.37%	0.40	2.16%	0.29	3.67%	0.17	5.23%	0.10
Beta	0.46	0.00	0.68	0.00	0.86	0.00	1.14	0.00	1.36	0.00
	Three-Factor Model									
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	0.69%	0.52	1.46%	0.36	2.27%	0.26	3.82%	0.15	5.40%	0.09
Beta	0.46	0.00	0.68	0.00	0.87	0.00	1.14	0.00	1.37	0.00
SMB	-0.03	0.00	-0.05	0.00	-0.06	0.00	-0.08	0.00	-0.10	0.00
HML	-0.06	0.00	-0.10	0.00	-0.12	0.00	-0.16	0.00	-0.19	0.00

Table A.4: Alpha and Beta Estimates for the Constant Volatility Strategy Varying Volatility Target

1996-2020								
	S&P 500 TR		**Constant Volatility**		**Tail-Hedge 1.5% & 2.5x**		**Tail-Hedge 3.0% & 10.0x**	
	One-Factor Model							
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	-0.22%	0.63	2.16%	0.29	-0.88%	0.07	-0.49%	0.52
Beta	0.99	0.00	0.86	0.00	0.96	0.00	0.87	0.00
	Three-Factor Model							
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	-0.06%	0.84	2.27%	0.26	-0.72%	0.03	-0.35%	0.62
Beta	0.99	0.00	0.87	0.00	0.96	0.00	0.87	0.00
SMB	-0.17	0.00	-0.06	0.00	-0.16	0.00	-0.15	0.00
HML	0.02	0.00	-0.12	0.00	0.03	0.00	0.03	0.00

Table A.5: Alpha and Beta Estimates for the Period 1996-2020

2010-2017								
	S&P 500 TR		**Constant Volatility**		**Tail-Hedge 1.5% & 2.5x**		**Tail-Hedge 3.0% & 10.0x**	
	One-Factor Model							
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	0.21%	0.63	0.53%	0.85	-0.34%	0.52	-1.29%	0.09
Beta	0.97	0.00	1.14	0.00	0.93	0.00	0.84	0.00
	Three-Factor Model							
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	-0.10%	0.65	-0.09%	0.98	-0.62%	0.12	-1.47%	0.04
Beta	0.99	0.00	1.18	0.00	0.95	0.00	0.86	0.00
SMB	-0.14	0.00	-0.15	0.00	-0.13	0.00	-0.09	0.00
HML	0.00	0.17	-0.13	0.00	0.00	0.39	0.01	0.05

Table A.6: Alpha and Beta Estimates for the Period 2010-2017

2000-2003	S&P 500 TR		Constant Volatility		Tail-Hedge 1.5% & 2.5x		Tail-Hedge 3.0% & 10.0x	
	One-Factor Model							
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	-0.56%	0.76	-5.24%	0.09	-1.82%	0.32	-1.53%	0.48
Beta	0.96	0.00	0.83	0.00	0.93	0.00	0.85	0.00
	Three-Factor Model							
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	0.55%	0.63	-4.29%	0.14	-0.85%	0.47	-0.10%	0.95
Beta	0.97	0.00	0.83	0.00	0.94	0.00	0.85	0.00
SMB	-0.22	0.00	-0.15	0.00	-0.21	0.00	-0.21	0.00
HML	0.04	0.00	0.02	0.31	0.05	0.00	0.02	0.11

Table A.7: Alpha and Beta Estimates for the Period 2000-2003

2007-2009	S&P 500 TR		Constant Volatility		Tail-Hedge 1.5% & 2.5x		Tail-Hedge 3.0% & 10.0x	
	One-Factor Model							
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	-0.54%	0.63	-0.87%	0.89	-0.96%	0.42	-0.89%	0.69
Beta	1.01	0.00	0.57	0.00	0.99	0.00	0.92	0.00
	Three-Factor Model							
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	-0.44%	0.54	-0.88%	0.89	-0.86%	0.35	-0.78%	0.71
Beta	1.01	0.00	0.56	0.00	0.99	0.00	0.92	0.00
SMB	-0.13	0.00	0.08	0.02	-0.11	0.00	-0.10	0.00
HML	-0.02	0.00	0.03	0.28	-0.01	0.05	0.00	0.87

Table A.8: Alpha and Beta Estimates for the Period 2007-2009

2020	S&P 500 TR		Constant Volatility		Tail-Hedge 1.5% & 2.5x		Tail-Hedge 3.0% & 10.0x	
	One-Factor Model							
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	-4.59%	0.04	5.38%	0.72	-0.67%	0.81	12.86%	0.20
Beta	1.00	0.00	0.48	0.00	0.95	0.00	0.80	0.00
	Three-Factor Model							
	Value	*p-value*	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
Annual Alpha	-2.01%	0.19	4.30%	0.78	2.03%	0.36	15.35%	0.13
Beta	1.00	0.00	0.48	0.00	0.95	0.00	0.79	0.00
SMB	-0.13	0.00	0.02	0.74	-0.13	0.00	-0.08	0.08
HML	0.03	0.00	-0.02	0.61	0.03	0.00	0.04	0.15

Table A.9: Alpha and Beta Estimates for the Period 2020

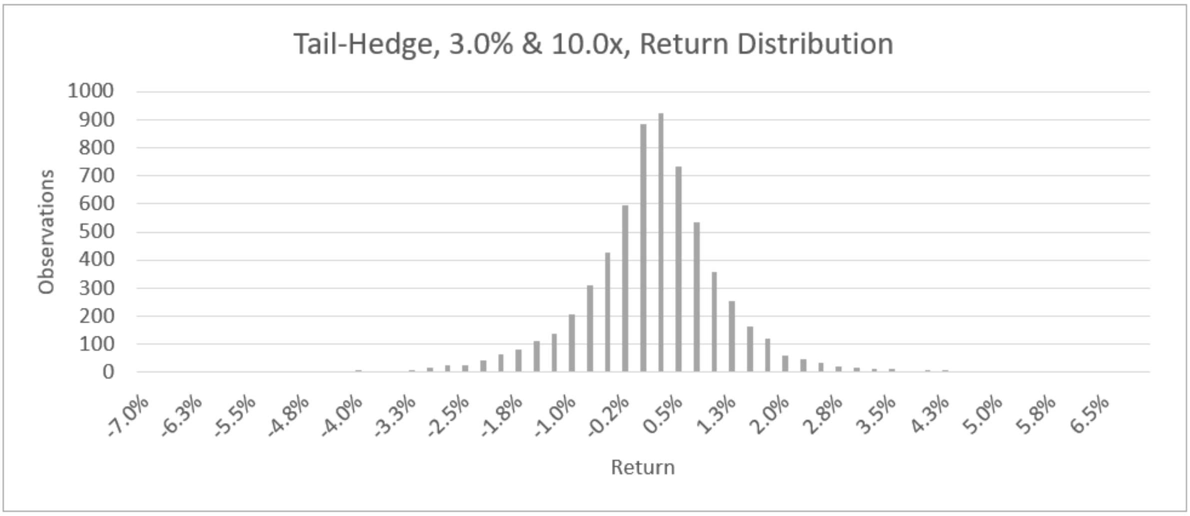

Figure A.1: Return Distribution for the Second Tail-Hedge Strategy Variation