The impact of collateral on CVA under general wrong-way risk

A study of collateral and its usage to mitigate counterparty risk for an interest rate swap in international recessions

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Abstract

The purpose of this thesis is to investigate whether postings of collateral is an effective mitigation tool against counterparty risk for an interest rate swap between Bank of America and JPMorgan Chase in a general wrong-way risk scenario, namely the COVID-19 crisis.

To do this, the thesis will seek to quantify the value of counterparty risk (known as CVA) through a two-part analysis. First, CVA is calculated on two almost identical interest rate swaps, the only difference being that one is collateralized, on their settlement date. This settlement date predates the beginning of the COVID-19 Pandemic. These CVA calculations are based on simulated interest rates and market data, as it would have been on the initial settlement date. Second, an empirical analysis back-tests these CVA calculations using actual data obtained from the COVID-19 Pandemic. The thesis will then compare the results and analyse the effectiveness of collateral.

Throughout the thesis both the theory and importance of counterparty risk management is explained. Additionally, both the characteristics of interest rate swaps and the global derivatives market are described. This thesis will also seek to describe, model and calculate the components of counterparty risk: exposure, loss given default, probability of default, collateral calls and general wrong-way risk. The modelling and calculations will be done using the open source coding language, Python.

The project concludes that collateral is a great tool for counterparty risk mitigation as it was able to mitigate between 49.9-53% of CVA. However, the empirical CVA remained at an elevated level even after collateralization. This was partially driven by a large spike in probability of default, which collateral was not able to mitigate. This led the thesis to conclude that collateral might be most effective when combined with other mitigation methods.

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1. Introduction

1.1 Introduction

Over the past decades there has been an increase in the flow of cross-border financial transactions driven by institutional investors and big banks. These transactions have led to increased globalization of financial markets, which is happening through technological advances as well as financial innovations (Otmar, I. (2000) Introduction).

A lot of benefits have been seen as a result of the globalization process, such as institutional investors having easier access to global investing opportunities, which in turn means easier access to capital for companies. Also, market participants have an easier time getting in contact with each other, which means possibilities of arbitrage are reduced, market efficiency is increased, and asymmetry of information is reduced. However, one major risk related to the increased globalization is that financial recessions have a much easier time turning into global crises (Häusler, G. (2002) Forces driving globalization).

Especially the increased importance of the category of financial securities known as 'derivatives' has played an important role in this trend (Otmar, I. (2000) Introduction).

The global derivatives market is used by a wide variety of market participants such as sovereigns, global corporations, banks and institutional investors and has in recent decades grown exponentially in both size and complexity (Lindstrøm, M. D. (2013) Introduction). During the great recession of 2008-2009 the negative aspect of the increased financial globalization was seen as a US housing crisis turned into the worst global recession since the great depression. What enabled the initial crisis to turn into an international economic disaster was the use of complex derivatives without proper risk management. Especially a particular type of risk turned out to be very dangerous: counterparty credit risk (Gregory, J. (2015) Ch 2).

Because of the huge importance of derivatives on today's economy, it is deemed important for anyone interested in international business to understand the risks, opportunities and drivers of these instruments. Therefore, this thesis will seek to analyse counterparty credit risk for the most popular type of derivative[1], the interest rate swap, in relation to a modern global economic crisis, the COVID-19 Pandemic.

1.2 Problem

Following the great recession, new regulations on the management of counterparty credit risk was introduced. These were the so-called Bassel III accords. The hope was that these new regulations would ensure that the issues the global market faced during the great recession would never happen again (Gregory, J. (2015) Ch 2). In the eyes of many experts, the COVID-19 Pandemic was the first real test of the financial system and its regulations since the great recession (Khwaja, A. (2020)).

In the months of February and March 2020, the financial markets were hit by a storm. This storm was the COVID-19 pandemic. As governments closed down local economies and the central banks cut interest rates to their zero-lower bound the financial markets experienced huge volatility. This was especially true for the interest rate swap market. As the international crisis hit the world's economies, banks experienced substantial losses as counterparty credit risk increased. Usually banks would have just hedged their risk, but as the crisis hit, the derivatives markets experienced liquidity issues. This was especially true for the credit derivatives market. Thus, many of the banks' hedges proved unable to cover the potential losses (Becker, L. (2020)).

KPMG made a market study (2020) in which they warned market participants in the derivatives market of the risk of so-called wrong-way risk. Wrong-way risk is explained as a negative correlation between the credit quality of the counterparty and the exposure towards the counterparty[2]. This type of risk can enhance losses substantially and is therefore something market participants should always be vary of (Gregory, J. (2015) Ch 17).[3]

[1] Measured by notional amount (Lindstrøm, M. D. (2013))
[2] If the negative correlation is dependent on macroeconomic factors, it is called general wrong-way risk (Gregory, J. (2015))
[3] This will be further touched upon later

In academic literature there has been multiple articles explaining the methods for wrong-way risk calculation. Examples of these include authors such as G. Cespedes et al. (2010), who elaborated upon specific models to calculate wrong-way risk. Furthermore, A. Memartoluie, et al. (2016) has elaborated on much of the work of G. Cespedes et al. in relation to the Basel III regulations.

However, this earlier work has primarily discussed wrong-way risk from a theoretical perspective, and according to an article made in collaboration with A. Aziz, B. Boetcher J. Gregory, A. Kreninin & IBM (2014) wrong-way risk models should often be tested through the use of stressed data. Therefore, it is believed that this thesis will fit well into the existing literature, as it will seek to apply the theory in a global crisis scenario and seek to draw conclusions from the results of this comparison.

1.3 Research Question

This thesis will seek to answer the following research question:

- *Is collateral an efficient mitigation method for counterparty credit risk for interest swaps between Bank of America and JPMorgan Chase in a general wrong-way risk scenario?*

This thesis will seek to answer the research question by creating a model to calculate a value for counterparty credit risk.[4] This model will then calculate a value for two nearly identical interest rate swaps[5] in a market scenario both with and without general wrong-way risk. Lastly, the results will be compared to an empirical analysis based on interest rates and credit data from the COVID-19 Pandemic.

1.4 Methodology

This thesis aims to describe the concept of counterparty credit risk for two interest rate swaps between Bank of America and JPMorgan Chase both mathematically, theoretically and practically. To describe the concept as well as answer the research question, this thesis uses a variety of methods, which will be discussed in this section.

[4] This value is known generally as CVA (Gregory, J. (2015)). This will be discussed later
[5] The only difference will be that one swap is collateralized, and one is uncollateralized

To understand the concepts and key terms of counterparty credit risk, the book "The xVA Challenge" by J. Gregory (2015) is used. Gregory is a very acknowledged author in the field of counterparty credit risk, and it does therefore make sense to build the theoretical explanations around his work. Additional authors are used for deep dives into specific subjects in the area of counterparty credit risk. Especially wrong-way risk is described using authors such as G. Cespedes et al. (2010) and A. Memartoluie et al. (2016).

Furthermore, the thesis will seek to explain the COVID-19 Pandemic from a financial markets' point-of-view using news articles as well as reports made by known companies such as KPMG and Standard & Poor's.

The analysis will be presented as a two-part case study: the CVA modelling analysis and an empirical analysis. Initially the CVA model will be constructed based on assumptions from the previously mentioned theory as well as historical data from The Federal Reserve Bank of St. Louis and Bloomberg.

The CVA model for an interest rate swap is generally build around three steps:

1) Interest rate simulations
2) Component calculations
3) CVA calculation

The first step of the CVA model creation is a simulation of interest rate movements used in later calculations. The simulations of the interest rate are performed using the so-called Vasicek model. The Vasicek model is a stochastic model framework that can simulate interest rate movements based on a single market risk factor.

The second step of the CVA model is the calculations of the components: expected exposure, loss given default, probability of default, collateral and wrong-way risk. Here the previously simulated interest rates, market data and the assumptions based in the theory will be used. To calculate wrong-way risk this thesis will seek to calculate the so-called alpha multiplier, which is a multiplier that is added on top of the expected exposure.

Lastly, the components will be inserted into the CVA formula and CVA will be calculated for both swaps.

After the creation of the CVA model, this model will be tested using real data during the COVID-19 Pandemic. Thus, the empirical analysis is based on actual interest rate developments, whereas the first part of the analysis was based on interest rate simulations. This is done so as to stress test collateral as a mitigation method during the biggest most volatile period for the financial markets since the great recession.

The CVA model and the following empirical analysis will both be created and performed using the programming language Python. Usually when talking CVA modelling there are three standard tools that can be used: Excel, R Studio and Python. Excel was initially discarded as a viable tool in this thesis because of the limitations of the program. When setting up a CVA model it is a necessity to be able to run tens of thousands of simulations, and this would not be viable via Excel on a normal computer. The handling of such large datasets is much easier through a programming language such as R Studio or Python. The reason Python was selected over R Studio is based on the strengths of each language. R Studio is an excellent tool for statistical tasks, whereas Python is better suited for machine learning and simulations. Therefore, Python is used to carry out all data management as well as calculations in this thesis. All python code has been written by the author, however, source code from locations such as GitHub has been used for specific technical solutions and syntax corrections.

1.5 Delimitations

CVA modelling is a very complex financial modelling task. Therefore, it is important to find the right balance between simplicity and accuracy of results. It is not realistic for a master's thesis to seek out to create a fully functional market standard CVA model incorporating wrong-way risk. However, that does not mean that it is not possible to create a model that yields interesting results for the purpose of this thesis. This thesis will seek to build a CVA model with a focus on aligning with general market practice whenever possible. However, simplifying assumptions has been made to assure completion of the model.

CVA is generally bilateral in nature since exposure can turn negative as well as positive and both the party and the counterparty are at risk of default. This thesis calculates CVA as unilateral, which means the party only need to consider the counterparty's credit risk and not their own. This is a major simplification, but it does not directly hinder the thesis in analysing and answering the research question. The unilateral CVA assumption basically means that the party, whose perspective this thesis takes is risk-free, which of course is not realistic, but a necessity for simplifying purposes.

In this analysis rehypothecation and segregation are ignored.[6] This assumption is fair as two almost identical interest rate swaps[7] are analysed with the exclusion of unnecessary outside factors. This simplifying assumption means that it is also natural to ignore all other swaps in the portfolio of the banks. This is of course unrealistic, but without the exclusion of rehypothecation and segregation and the choice to ignore additional financial transactions of the banks, unnecessary noise would cloud the analysis, which in turn would cloud the results.

Furthermore, both collateral funding and operational costs are assumed to be zero. There are so-called funding costs associated with the posting of collateral, which makes it less attractive to initiate collateralized positions. However, the inclusion of funding costs would be a lengthy process that in the end does not affect the CVA estimate but is more of a reporting issue. The same can be said for operational costs that are associated with how often collateral is posted. The more often collateral is posted the more expensive it is, however, again this does not affect the CVA estimate, and will therefore be set to zero to simplify the model.

Lastly, the incorporation of wrong-way risk in the CVA model will be done through the copula approach. The copula approach is the simplest modelling approach for wrong-way risk and is therefore not generally seen as market practice. However, based on the complexity of the other modelling approaches, the copula approach was deemed the most suitable for this thesis.

1.6 Structure of Thesis

This section will give a brief description of the structure and the content of each major section of this thesis following the introduction.

This thesis wills start off with a section describing the theory and the practical case. Here the initial theoretical foundation for the thesis is introduced and elaborated upon. The over-the-counter market is introduced, interest rate swaps and how to value them is described. Also, the two interest rate swaps used in this thesis is introduced. Furthermore, one of the key terms, 'wrong-way risk', is explained. Lastly, an in-depth explanation of the COVID-19 Pandemic's

[6] Rehypothecation and segregation will be explained in greater detail later
[7] the only difference being one is collateralized, and one is uncollateralized

implications for the global interest rate swap markets as well as the presence of general wrong-way risk is explained.

Afterwards comes the counterparty credit risk section. This section further elaborates upon the theory from the previous section but moves further in-depth. The three components of counterparty credit risk (exposure, probability of default and loss given default) are explained. Then collateral as a mitigation method for counterparty credit risk is introduced and described. Lastly, the value of counterparty credit risk (CVA) is defined both theoretically and mathematically and wrong-way risk is incorporated into the CVA formula.

The thesis then moves on to the component modelling & calculations section. Here the interest rates are simulated, and the three counterparty credit risk components are modelled. Furthermore, a model for general wrong-way risk is created and the alpha multiplier is calculated for all periods of the swaps' durations.

Then the CVA calculation section follows. Here the model components are combined in a Python script and CVA is calculated for both swaps with and without the presence of general wrong-way risk. Then the results are compared, and initial conclusions are drawn.

After this it is time for the empirical analysis. Here the COVID-19 data for both interest rate movements and credit market movements are introduced. Then the empirical CVA will be calculated and the empirical results discussed and explained. Lastly, the modelled results will be compared with the empirical results and the usability of the CVA model can thereby be discussed.

Then comes the discussion section. Here points of critique will be discussed. Also, the main limiting delimitations for the thesis will be touched upon. Also, methods to expand upon the thesis and ideas for future research will be discussed. All of this will be discussed relative to the results of the analysis.

Finally, the conclusion will summarize the findings and conclude upon the research question.

2. Theory & Practical Case

This section will describe the basic theory that supports and lays the foundation for the later sections of this thesis. Key concepts such as the derivatives market, interest rate swaps and wrong-way risk will be introduced and explained in a simple way that enables the introduction of future complexity. Furthermore, the impact of COVID-19 on financial markets and the monetary reaction by central banks will also be introduced.

2.1 The Derivatives Market

This section will describe the derivatives markets as well as touch upon different types of derivatives and their associated risks.

2.1.1 Types of Derivatives

A derivative is a financial contract to either make/receive payments based on the movements in an underlying asset or to make/receive a delivery of an underlying asset. Basically, a financial derivative is a synthetic position in an asset. This means you, as an owner of a derivative, do not actually own the underlying asset, but your position moves based on the moves of the underlying. Some financial contracts are exercisable, which means that at a time of expiry the holder of the derivative can choose to either buy or sell the underlying, thus becoming the owner of the underlying asset (Gregory, J. (2015) Ch 3). Market standard for trading derivatives is that they are traded "at market". This means that the net present value of the derivative is zero. This is done to avoid credit risk. If you initiated a derivatives position with a positive net present value, one side would have to make an upfront payment against receiving a positive expected cashflow and is thus immediately exposed to the other side defaulting. Another reason for initiating trades with a net present value of zero is that making an upfront payment generates funding costs either through borrowing unsecured money and paying a borrowing spread or using money that could have been used elsewhere (Lindstrøm, M. D. (2013) Ch 3).

In the last couple of decades, the derivatives market has grown a lot in both size and complexity. One of the main drivers behind this is the use of derivatives as hedging tools (Gregory, J. (2015) Ch 3). When market participants open derivative positions, they have to monitor a key factor: leverage. Leverage is one of the biggest potential threats in the derivatives market, as only a fraction of the notional of a derivative is needed to trade said derivative. This means that the derivative can quickly rack up large losses if the market moves against the owner (Gregory, J. (2015) Ch 3). Even before the great recession, but especially after, it has become increasingly important to monitor leverage and ensure capital requirements are met. This has been done through extensive regulations – most notably the Basel Accords (Basel I, Basel II, Basel III) (Gregory, J. (2015) Ch 1 & 2). Usually the holder of a derivative has a counterparty, which means a party on the other side of the trade. In case the counterparty defaults, the derivative contract is voided. The risk of the counterparty defaulting is called counterparty credit risk, or just counterparty risk (Gregory, J. (2015) Ch 3). This thesis will go into greater detail on counterparty risk in section 3 of this thesis.

2.1.2 The Over-the-counter Market

There exist two markets for derivatives trading. The first one is the exchange-traded market and the second is the over-the-counter (OTC) market. The exchange-traded market consist of financial centers (exchanges), were parties are able to trade standardized products such as futures and options. Exchanges increase market efficiency and liquidity by making the market easier to enter and exit (Gregory, J. (2015) Ch 3). The OTC market is a more complicated structure. The contracts traded on the OTC market are private, non-reported contracts that are usually initiated between a party and a counterparty. This basically means that when you engage in a derivatives trade in the OTC market you are exposed to counterparty risk. The OTC market holds a lot of different products that are generally less standardized than the exchange-traded products. The fact that the OTC market does not trade on exchanges also mean that the market is typically less liquid and less efficient (Gregory, J. (2015) Ch 3). However, the OTC derivatives market is by far the largest derivatives market measured in notional amount, with over 91% of all notional being traded OTC in 2014 (Gregory, J. (2015) Ch 3). The OTC market contains derivatives with many different underlying assets such as fixed income, forex, commodities, credit derivatives, and equities. Fixed income derivatives are by far the largest part of the OTC derivatives market and will be the main focus of this thesis (Lindstrøm, M. D. (2013) Ch 3). In the below figure the size of the derivatives market spread out among types of underlying from 1998 to 2015 is seen:

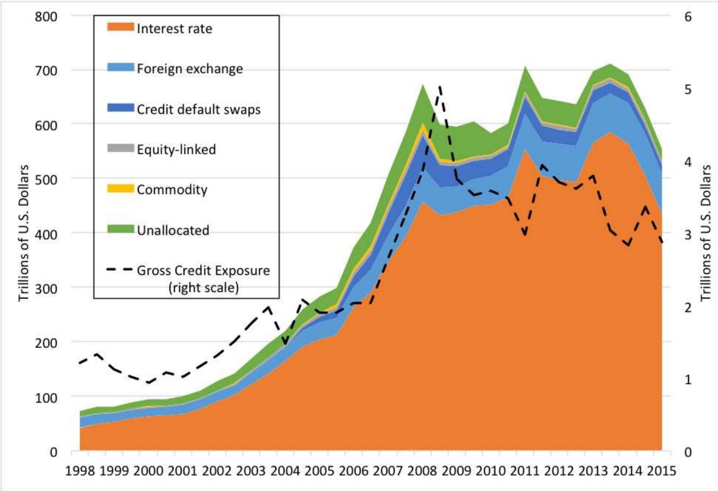

Figure 1: The OTC derivatives market by asset class. Source: Fixed Income Derivatives Lecture 1

Derivatives on the OTC market can either be collateralized or uncollateralized. This in short means that fluctuations in the value of the derivative can either be compensated by the losing party with cash or securities (collateralized), or not (uncollateralized) (Gregory, J. (2015) Ch 3). The uses of collateral will be touched upon later in this thesis.

2.1.3 Application of OTC Derivatives

All of the different market participants have different reasons for their presence in the OTC market. By generalizing a bit, it is possible to create two different roles that parties in the OTC market can take – investors and speculators. Speculators usually take a position in the market to "gamble" on a movement in the underlying. This could be a hedge fund that believes interest rates will fall and thus enters into an interest rate receiver swap, were the hedge fund will receive a fixed rate and pay a floating rate. This means that if the rates fall the hedge fund pays less to receive the fixed rate, thus making a profit. Investors usually take a position in the OTC market as a way to remove unwanted risk or hedge specific positions (Tuckman, B. & Serrat, A. (2012)). This can include corporates that has a floating loan and fears rate hikes. The corporate will buy an interest rate payer swap, were the corporate will pay a fixed rate and receive a floating rate. By entering into this swap position the corporate has effectively converted the floating rate loan to a fixed rate loan thus removing interest rate risk. It is possible to hedge a wide variety of risk via derivates, such as interest rate risk, FX risk, commodity risk, credit risk and more (Gregory, J. (2015) Ch 3).

2.1.4 The Dangers of Derivatives

Derivatives are, as previously mentioned, very helpful in hedging and risk-removing. However, derivatives carry significant risk themselves. These include market risk, credit risk, operational and legal risk, liquidity risk, and counterparty risk. This thesis will primarily be focused on counterparty risk, but other risk types will be briefly mentioned when they are relevant. An example of the effect of excessive counterparty risk is the collapse of Lehman Brothers.

The American investment bank Lehman Brothers filed for a chapter 11 bankruptcy after suffering heavy counterparty credit losses as counterparties could not pay the collateral calls that followed a downgrade in their credit ratings during the great recession. As will be explained later, the size of collateral postings can sometimes be determined based on credit ratings. The excessive risk taking, and lack of efficient risk management led to the collapse of Lehman Brothers, which now stands as a terrifying example of the lack of counterparty risk management (Gregory, J. (2015) Ch 3).

2.2 Interest Rate Swaps

This section will seek to introduce and explain the OTC derivative type known as an interest rate swap. The interest rate swap will be the instrument that the calculations of this thesis will be based around. Therefore, this section will seek to introduce some important interest rate concepts, define the derivative, showcase the valuation method of the derivative as well as introduce the specific swaps used in this thesis.

2.2.1 Interest Rate Concepts

This section will seek to introduce the concept of the so-called ‘discount factor of time’, which is used to a large extent in later calculations.

The explanation of this factor begins with the assumption of a risk-free zero-coupon bond that pays its face value at maturity time T. The price of said bond at time $t \leq T$ is denoted by $P(t,T)$. The price of the bond at maturity is logically $P(T,T) = 1$, which then means that $P(t,T)$ is the discount factor of time at time t for all cash flows at time T. This can be written as (Missouri University of Science & Technology. (n.y.)):

$$e^{y(t,T)(T-t)}P(t,T) = 1$$

Where $y(t,T)$ is the continuously compounded spot interest rate.

Because of the above formula the relation between the spot interest rate and the price of a zero-coupon bond can be explained by (Missouri University of Science & Technology. (n.y.)):

$$P(t,T) = e^{y(t,T)(T-t)}$$

Where the continuously compounded spot interest rate is defined by (Missouri University of Science & Technology. (n.y.)):

$$y(t,T) = -\frac{\log{(P(t,T))}}{(T-t)}$$

The last two important concepts to briefly explain is the simply compounded spot interest rate and the forward rate. The simply compounded spot interest rate is the constant rate an investment has to have in order to produce a single unit of cash at time T (Missouri University of Science & Technology. (n.y.)):

$$L(t,t,T) = \frac{1}{T-t} * \left(\frac{1}{P(t,T)} - 1\right)$$

The forward rate is an interest rate that can be locked in today for a cash flow in a future time period. Forward rates are generally seen as a way to view future beliefs in the movement of the spot rate and is therefore often used as a forecasting tool. The simply compounded forward rate is given by (Missouri University of Science & Technology. (n.y.)):

$$L(t,T,S) = \frac{1}{S-T} * \left(\frac{P(t,T)}{P(t,S)} - 1\right)$$

The above formulas for the forward rate and the spot interest rates are not used directly in this thesis, but the rates they represent are used in future calculations, which is why it is briefly explained here. The calculation of the price of the zero-coupon bond will be touched upon again in section 4.

2.2.2 Definition

A swap is a bilateral OTC derivatives contract were two parties agree to exchange cashflows based on the movements in an underlying asset. For an interest rate swap the underlying asset is a specific interest rate. An interest rate swap has two so-called “legs”, one leg for each party/type of cashflow. One leg pays cashflows based on the movements in the underlying asset (a floating leg based on a floating rate). The other leg pays cashflows based on a fixed rate (a fixed leg based on a fixed rate). The parties "swap" their cashflows – fixed for floating/floating for fixed (Hull, J. C. (2018) Ch 7). This is explained graphically in the figure below:

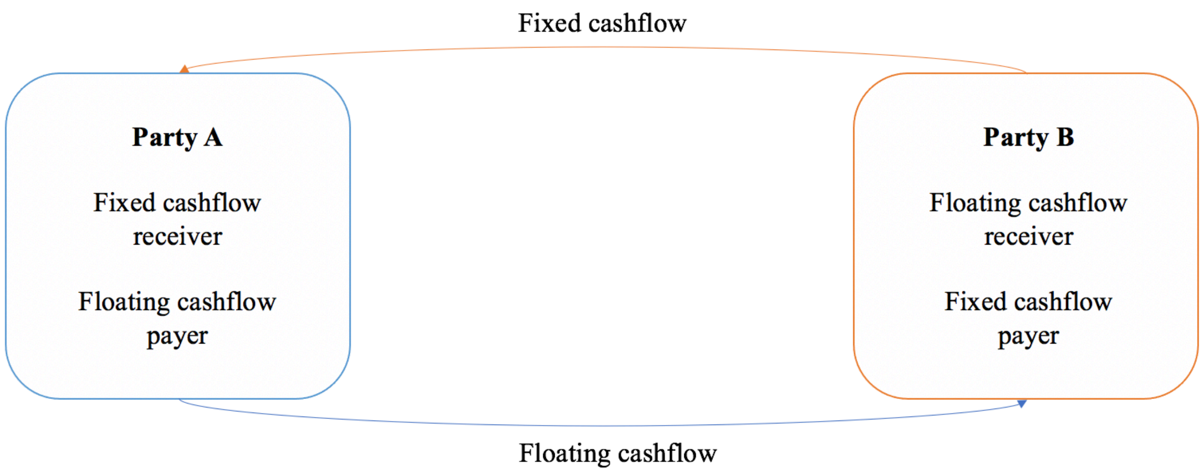

Figure 2: Interest rate swap depicted. Source: Own creation

Interest rate swaps can be either 'payer swaps' or 'receiver swaps' and are named based on the fixed leg. If you are the one paying a floating rate and receiving a fixed rate, you have entered into a receiver swap. On the other hand, if you are paying a fixed rate and receiving a floating rate, you have entered into a payer swap. The most popular interest rate swaps are abiding by some common conventions, known as the interest rate swap conventions. These conventional swaps are also known as plain vanilla swaps. The conventions that determines whether a swap is plain vanilla usually focus around the way days are counted, what to do when a payment date falls on a weekend and when payments should be made (Lindstrøm, M. D. (2013) Ch 3). These conventions can be seen in the below table for different types of interest rate swaps:

				Floating Leg			Fixed Leg	
Currency	Index name	Spot start	Roll	Term	Freq.	Day count	Freq.	Day count
EUR	Euribor	2B	MF	6M	S	Act/360	A	30/360
USD	USD Libor	2B	MF	3M	Q	Act/360	S	30/360
GBP	GBP Libor	0B	MF	6M	S	Act/365	S	Act/365
JPY	JPY Libor	2B	MF	6M	S	Act/360	S	Act/365
SEK	Stibor	2B	MF	3M	Q	Act/360	A	30/360
NOK	Nibor	2B	MF	6M	S	Act/360	A	30/360
DKK	Cibor	2B	MF	6M	S	Act/360	A	30/360

Table 1: Interest rate swap conventions. Source: Lindstrøm, M. D. (2013)

As mentioned, an interest rate swap has an underlying rate, which determines the cash flows being paid on the floating leg. As can be seen in table 1, most swaps have historically followed the so-called Libor rates (London Interbank Offered Rate), which are the rates that so-called 'Libor panel banks' can borrow funds to from each other (Hull, J. C. (2018) Ch 4). The Libor rate is slowly in the progress of being replaced, but for the sake of simplicity and data access, this thesis will assume that the floating rate for the swaps in this thesis will be a Libor rate (Cox, J. (2020)).

2.2.3 Interest Rate Swap Valuation

An interest rate swap is, as with most OTC derivatives, usually initiated with a net present value (NPV) of zero. However, during the existence of the swap, changes in the floating rate will affect the cashflows on the floating leg, which means the value of the entire swap will change. To monitor these value changes, it is important to know how to valuate an interest rate swap. There are two ways to perform this valuation, the first method is by treating the swap as a difference between a floating rate bond and a fixed rate bond, the second regards the swap as a portfolio of forward-rate agreements (FRAs). This thesis will focus on the valuation method that treats the swap as the difference between two bonds (Lindstrøm, M. D. (2013) Ch 3).

This method dictates that a receiver swap can be regarded as a long position in a fixed rate bond and a short position in a floating rate bond:

$$V_{swap} = B_{fix} - B_{float}$$

Where V_{swap} is the value of the swap, B_{fix} is the value of the fixed rate bond and B_{float} is the value of the floating rate bond (Hull, J. C. (2018) Ch 7).

The method for finding the value of the swap is now to value each of the legs, i.e. each of the bonds separately. To value the floating leg, the following formula is used:

$$PV_t^{float} = \sum_{i=S+1}^{E} \delta_i^{float} F(t, T_{i-1}, T_i) N_i P(t, T_i)$$

Where PV_t^{float} is equal to B_{float}, δ_i^{float} is the payment tenor, $F(t, T_{i-1}, T_i)$ is the floating rate (defined as $L(t, t, T)$ in section 2.3.1), N_i is the notional amount and $P(t, T_i)$ is the discount factor of time explained in section 2.3.1. Since the floating rate is calculated on the same zero-coupon curve as the cash flows are discounted on, PV_t^{float} is a telescoping series. A telescoping series is a mathematical series in which all terms cancel out except the first and last term. Telescoping series are generally very difficult to explain, so this thesis will not seek to deep dive into an explanation of this phenomenon. However, since this is a telescoping series, PV_t^{float} can be simplified to the following expression (Aabo, L. P. (2019)):

$$PV_t^{float} = N_i(1 - P(t, T_i)$$

For the fixed leg, the following formula is used:

$$PV_t^{fixed} = \sum_{i=s+1}^{E} \delta_i^{fixed} K N_i P(t, T_i)$$

Where PV_t^{fixed} is equal to B_{fixed}, δ_i^{fixed} is the payment tenor, K is the fixed rate N_i is the notional amount, and $P(t, T_i)$ is the discount factor of time.

Now it is possible to calculate the value of the swap from the perspective of the receiver (Lindstrøm, M. D. (2013) Ch 3):

$$PV_t^{receiver} = \sum_{i=s+1}^{E} \delta_i^{fixed} K N_i P(t, T_i) - N_i(1 - P(t, T_i) \Leftrightarrow$$

$$PV_t^{receiver} = N_i * \left(\sum_{i=s+1}^{E} \delta_i^{fixed} K P(t, T_i) - \big(1 - P(t, T_i)\big) \right)$$

2.2.4 Interest Rate Swaps in This Thesis

This thesis will seek to answer the problem formulation and research questions using two nearly identical interest rate receiver swaps. The only difference between the two swaps will be that one swap will be collateralized and the other will be uncollateralized (this will be explained in greater detail later). The two swaps will be assumed to be initiated between Bank of America (BAC) and JPMorgan Chase (JPM). with this thesis taking the point of view of BAC with JPM as the counterparty. The reason for the choice of banks is because they are both Libor panel banks, which means they are both able to lend interbank to the Libor rate. The following table displays the two swaps used in this thesis:

Instrument	Swap 1	Swap 2
Type	Receiver	Receiver
Settlement date	06/01/2019	06/01/2019
Maturity	2Y	2Y
Notional	100,000,000.00	100,000,000.00
Fixed rate	2.0475%	2.0475%
Fixed tenor	6M	6M
Float rate	3M LIBOR	3M LIBOR
Float tenor	6M	6M
CSA	Yes	No

Table 2: Market instruments. Source: own creation

As can be seen in the table above the swaps are receiver swaps with notional values of 100,000,000 USD, a runtime of two years with start from January 6th, 2019. The floating rate used for the swaps is the 3-month USD Libor rate.

The swaps are set to $NPV = 0$, which means that future cashflows (both fixed and floating) have been discounted to time 0 and $PV_t^{receiver} = 0$. The floating cashflows have been calculated using forward rates for the 3M USD Libor rate. This is done as the forward rate, as previously mentioned, displays the market consensus of future interest rate movements.

It is seen that the above swaps are not plain-vanilla USD swaps as that would have entailed that floating payments be made every 3 months, whereas fixed payments be made every 6 months. The reason for both payments being made every 6 months is to avoid asymmetry in the payments. Since this thesis seeks to analyze general wrong-way risk and the way collateral can be used to mitigate said risk, it is deemed most interesting to remove this payment asymmetry and thereby remove unnecessary "noise" from the calculations.

2.3 Wrong-way Risk

In this section wrong-way risk will be introduced and explained. This section is very important as wrong-way risk is a key concept in this thesis and will shape the entire analysis and conclusion. This section will seek to first give a general definition of the term followed by a deep-dive into two different kinds of wrong-way risk: specific and general. Lastly this section will introduce some of the modelling challenges usually faced when trying to model wrong-way risk.

2.3.1 Definition

The definition of wrong-way risk (from now on just WWR) is explained as a negative correlation between two risk management components: exposure and credit quality. Exposure is the loss incurred in the event of the counterparty defaulting. Credit quality is the ability of a counterparty to pay their credits and is often explained through the probability of counterparty default (Gregory, J. (2015) Ch 17). This thesis will dig deeper into both exposure and credit quality later in this thesis.

The definition means that when the credit quality of the counterparty decreases, which means the probability of counterparty default increases, the exposure, i.e. the loss incurred in case of default, increases (Gregory, J. (2015) Ch 17). Obviously, this is a toxic scenario that has to be avoided whenever possible. In academic literature WWR is generally ignored, but in practice WWR can have huge implications from a counterparty risk perspective. There also exists an opposite term to WWR called right-way risk. Right-way risk is a positive correlation between credit quality and exposure and is therefore a positive term (Gregory, J. (2015) Ch 17). This thesis will, however, only focus on WWR. There exist two types of WWR: specific and general, which will be explained next.

2.3.2 Specific Wrong-way Risk

specific wrong-way risk (from now on SWWR) can be defined as a type of WWR driven by factors relevant to the specific counterparty or market. SWWR is basically when a counterparty or industry has a specific situation that leads to WWR in a derivatives position for the party. It is very difficult to model and capture SWWR as it obviously varies greatly from counterparty to counterparty. Unless you have extensive knowledge related to the specific industry or counterparty, it is very hard to find any correlation that can explain the SWWR (Gregory, J. (2015) Ch 17). SWWR will not be discussed further in this thesis, as it does not cover the desired type of WWR that is sought analyzed.

2.3.3 General Wrong-way Risk

General wrong-way risk (from now on GWWR) is a type of WWR driven by macroeconomic factors that affect the entire economy. This type of WWR is usually present during major economic crises. GWWR relationships are often captured through historical data and can be incorporated through models. However, the capturing of GWWR through historical data is a very tedious and complicated task, so even though GWWR is easier to capture than SWWR, it is still not easy to locate. Since this thesis will seek to evolve around the COVID-19 Pandemic and its implications on interest rate derivative markets, GWWR is the preferred type of WWR to focus on (Gregory, J. (2015) Ch 17).

2.3.4 Wrong-way Risk Modelling Challenges

There are multiple challenges related to the modelling of WWR. One of these is the often-encountered issue that historical data does not capture WWR correctly. Since WWR is so difficult to capture in general, it often requires substantial correlation analysis, and even then, it is not guaranteed that the actual relationship is found (Gregory, J. (2015) Ch 17).

Furthermore, the misspecification of relationships is also an often-incurred challenge. This misspecification means that the relationships between different factors that cause WWR may be wrong – basically it is very difficult to prove independency between factors in some cases and dependency between factors in other cases (Gregory, J. (2015) Ch 17).

At last the direction of WWR is often also a challenge for models. This is best explained through an example: if interest rates decrease, it often means that the economy is in a financial crisis with widening credit spreads and increasing default rates. However, sometimes an adverse credit environment can be possible, thus implying a reverse direction of WWR than what would be expected (Gregory, J. (2015) Ch 17).

This thesis will seek to create relatively simple assumptions to minimize the above-mentioned modelling challenges an avoid having to create complex analytical models to capture relationships in historical data.

2.4 Practical Case: The COVID-19 Pandemic

The COVID-19 Pandemic started out with an unknown branch of pneumonia and ended up as a global pandemic leading to lockdowns on a massive scale with huge economic repercussions on both a macroeconomic as well as a microeconomic scale (Taylor, D. B. (2021)). This thesis will seek to compare the initial analysis with an empirical analysis based on financial data from the COVID-19 Pandemic. Therefore, it is deemed important to highlight the effect of the COVID-19 Pandemic on the interest rate swap market, the intervention from central banks and the presence of GWWR during COVID-19. If one wishes to read more in depth regarding the progress of the COVID-19 Pandemic, as well as the governmental responses, see appendix 1 and 2 respectively.

2.4.1 Initial Effects on The Interest Rate Swap Market

The COVID-19 Pandemic proved to be the biggest challenge for OTC market liquidity since the Great Recession. Traders expected a rate cut, but it came earlier than expected on March 3rd. This early rate cut let to an increased sell-off in risk assets, which was seen from the volumes on interest rate swap markets that went from an average of 250 billion USD per day to a peak of a staggering 720 billion USD on March 4. After March 4 daily volumes stayed at an elevated level of around 400 billion USD before spiking again on March 13 with a daily volume of around 600 billion USD. These increased volumes were especially seen for short term swaps such as 2Y and 5Y swaps (Khwaja, A. (2020)). Even though the increase in daily volumes intuitively makes you think that it means liquidity held up, the case is not that simple. According to a global study made by the International Swaps and Derivatives Association (2020) 96% of UK-based swap market actors reported a decline in market liquidity before central bank intervention. This can be seen in the below figure:

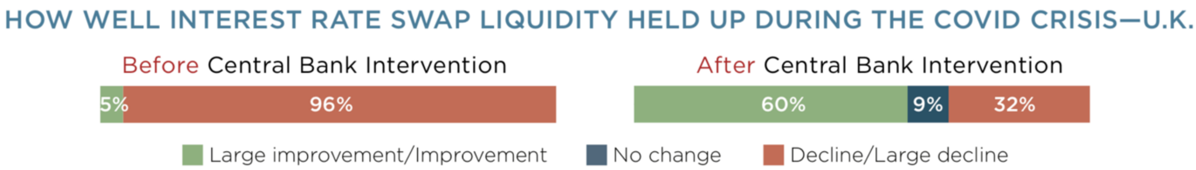

Figure 3: IRS liquidity during the COVID-19 crisis. Source: ISDA (2020)

The participants also reported that even though the pandemic was the main driver of the crisis, the expected economic impact was the main force behind the increased volatility and decreased liquidity. Market actors from both the buy-side and the sell-side pointed to two reasons behind the disruption in liquidity. These were 1) a reduced risk appetite from banks and 2) corporates in sudden need of short-term funding as revenue decreased (ISDA. (2020)). The reduced risk appetite from banks is explained in the study as being a product of the reforms implemented after the great recession to ensure a larger degree of financial stability during recessions. However, market actors did not just criticize the regulations on the banks, they also felt that the reforms led to a safer and stronger banking system during COVID-19. So even though the banks were not able to intervene and take on risk, thus fueling liquidity, in the same way they used to, they were better suited to deal with the credit losses suffered during COVID-19 (ISDA. (2020)).

So, to recap, the interest rate swap market experienced initial liquidity issues and increased volatility due to the economic fallout of the COVID-19 Pandemic – this was especially true for shorter term swaps. The decrease in credit quality of market actors, the sudden need for short-term funding as well as the lack of risk-taking from global banks due to reforms are believed to be the main driving forces behind the liquidity issues and volatility increase experienced during the initial stages of the economic crisis.

2.4.2 Global Monetary Intervention

Seeing the initial consequences of the COVID-19 Pandemic, global central banks were quick to act in the biggest act of monetary intervention in at least the last quarter of a century. The four largest central banks in key currencies (Japanese Yen, Euro, US dollars and British Sterling) expanded their balance sheet with 10 points of GDP, which can be seen in the figure below:

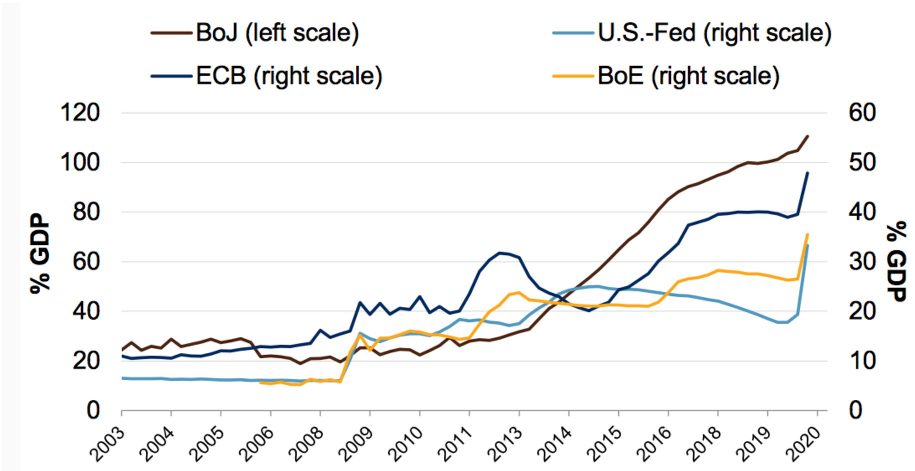

Figure 4: Total assets on key central bank balance sheets. Source: Standard & Poor's 1. (2020)

To look at this in relation to the number of USD, it means that in four months the four central banks had injected 2.4 trillion USD into the economy (Standard & Poor's 1. (2020)).

The intervention started with the central banks using traditional monetary methods such as rate cuts. All the four central banks cut their rates to their effective zero lower bound, which for the FED meant a 150-bps rate cut. As explained previously, the early rate cuts were one of the factors that are believed to have spooked traders and thus led to liquidity constraints. To deal with this the central banks added liquidity to the market by increasing the size and duration of funding operations as well as easing credit conditions for certain industries to incentivize bank lending and risk-taking. This proved to not be effective enough, so the central banks began the aforementioned major increase in their balance sheet. This asset purchase program led to the central banks even buying corporate assets as well as debt of so-called fallen angels (Standard & Poor's 1. (2020)).[8]

The monetary intervention was generally deemed relatively successful – especially in the US – were bond issuance across all credit ratings were increased compared to 2019, meaning that corporations in need of short-term funding was able to obtain said funding. The reason that the global monetary intervention is only deemed relatively successful is because one of the biggest threats to the current market situation in 2021, according to S&P, is if central banks decide to stop their monetary programs (Standard & Poor's 1. (2020)).

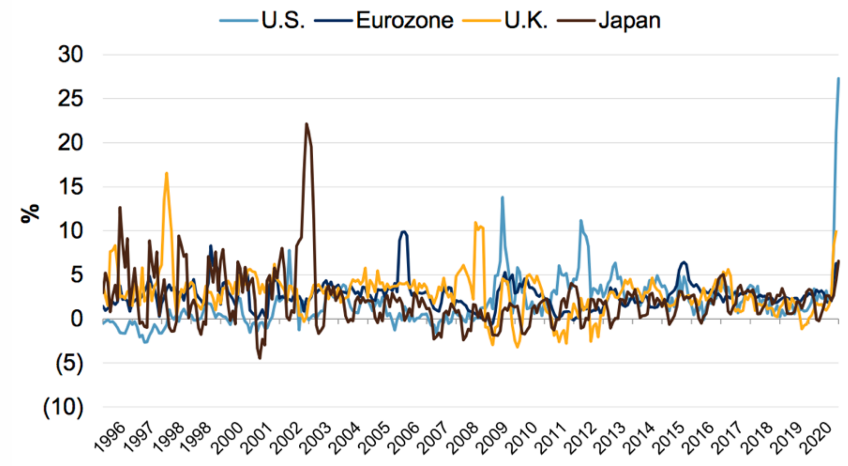

Figure 5: Money supply year-on-year in %. Source: Standard & Poor's 1. (2020)

Furthermore, the vast increase in the money supply – especially in the US – is also very worrying in regard to inflation (see the figure above). So, there may be repercussions, such as the risk of inflation, in the not-too-distant future (Standard & Poor's 1. (2020)). However, according to the ISDA study mentioned earlier, 67% of study participants found the FED's intervention to be effective in curving market liquidity issues and increased volatility (ISDA. (2020)).

[8] A fallen angel is a corporation with a credit rating being downgraded from higher than BB+ to BB+ or below (Standard & Poor's 1. (2020))

2.4.3 GWWR during COVID-19

The COVID-19 Pandemic has been hitting banks (such as Bank of America & JPMorgan Chase) and other lenders hard. As many industries has had their revenue streams either reduced or cut completely, their risk of defaulting on loans has increased. Therefore, banks have had to set aside capital buffers to bear the brunt of the expected credit losses. This tendency of industry specific risk and increased economic risk is noted in the US BICRA score, which shows a negative trend for the US economy (Standard & Poor's 2. (2020)):

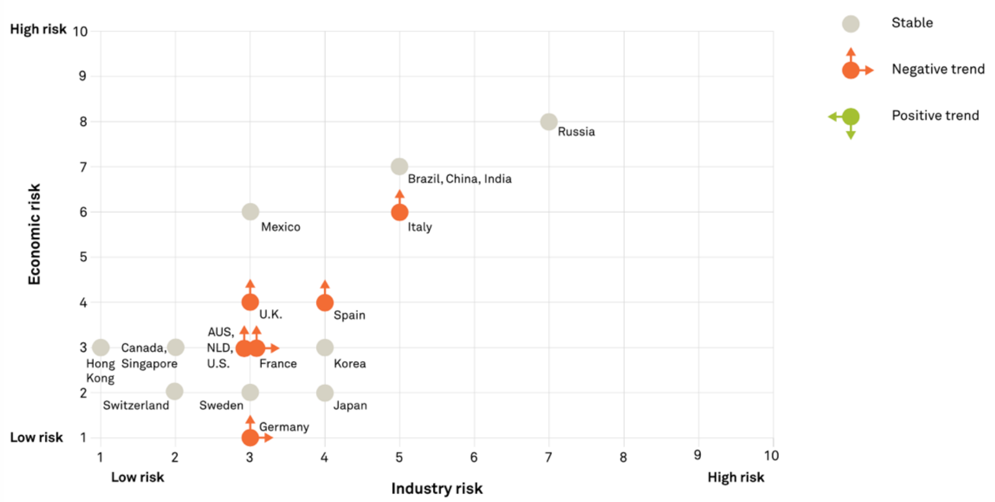

Figure 6: BICRA scores and economic and industry risk trends. Source: Standard & Poor's 2. (2020)

The BICRA (Banking Industry Counter Risk Assessment) score is a score of 1-10 ranging from the lowest risk banking systems to the highest risk banking systems. Generally, it is seen in the above figure that the risk to the US banking system is still generally low, but the COVID-19 Pandemic has started a negative trend for US banks. S&P furthermore expects that the year 2021 will not prove any easier for banks, as credit recoveries are not expected at this point in time (Standard & Poor's 2. (2020)).

Credit conditions has, as previously mentioned, been an ongoing concern during COVID-19 as default rates have been at its highest level since 2009 with the US leveraged loan index at 4.48% in October 2020. Credit conditions will probably also be an issue in 2021 as the projected default rate for 2021 edges higher to 5.47%. It is still a much lower default rate than during the great recession, were it peaked at 10.81% (Standard & Poor's 3. (2020)). This can be seen in the figure below:

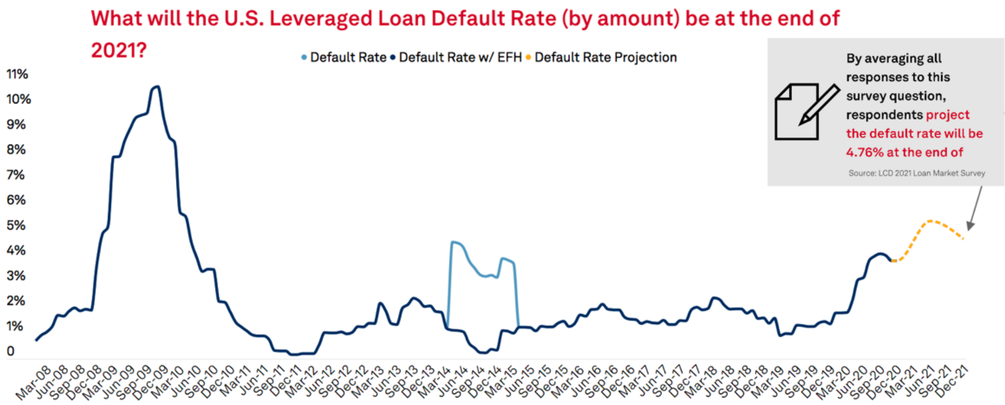

Figure 7: Historical leveraged loan default rates (US). Source: Standard & Poor's 3. (2020)

Since the great recession, banks have held substantial capital buffers intended to ensure strong balance sheets during the next global crisis. However, even with stronger balance sheets global banks still faced a revision of their credit ratings, with multiple banks facing either direct downgrades our negative outlook revisions as seen in the below figure (Standard & Poor's 2. (2020)):

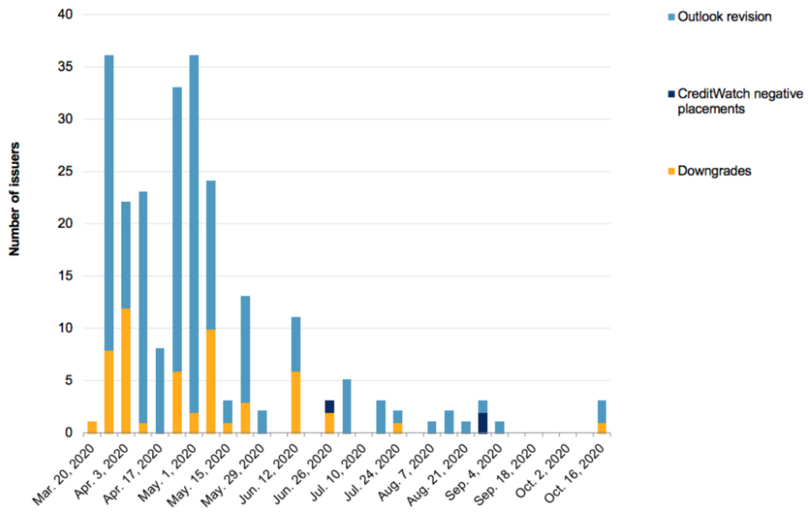

Figure 8: Weekly distribution of banks affected by COVID-19. Source: Standard & Poor's 2. (2020)

The substantial fear of credit losses suffered by banks that was manifested in the negative change in credit ratings has led to a widening of credit default swap spreads, which combined with a falling interest rate environment has given rise to fears of WWR in the market according to a market outlook by KPMG (KPMG. (2020)).

Now that the possibility of WWR in the market during COVID-19 has been introduced, this thesis will discuss how this WWR may exist for the specific instruments in this thesis.

When a receiver swap experiences a decrease in interest rates, the floating payments decrease while fixed payments remain fixed. This leads to an increase in the present value of the receiver swap, which also leads to an increase in the exposure. The exposure is, as briefly mentioned, the total amount of money the party stands to lose if the counterparty defaults (for more information see section 3.1.1).

To recap, the definition of WWR was that there exists a negative correlation between exposure and credit quality. It was explained earlier that due to credit losses sustained as well as an increase in expected credit losses, credit outlooks for bank turned negative thus leading to an increase in the risk of default. This combination of increased exposure due to falling interest rates as well as a decreased credit quality for banks such as JPMorgan Chase, aligns well with the definition of WWR. Since this assumed WWR is created by macroeconomic events it is possible to categorize it as GWWR. The presence of GWWR during the COVID-19 Pandemic will be further elaborated on in the empirical analysis, when data relevant to the specific swaps and the specific counterparty (JPMorgan Chase) is analyzed.

3. Counterparty Credit Risk

When a market participant enters into an OTC derivative position it is usually a bilateral contract. This means that the position will have a counterparty on the other side. This can best be explained as a zero-sum game, in which two actors “play” against each other. If one side of the OTC derivative increases, it must mean that the other side decreases. If the counterparty fails to fulfil the contractual agreements, which for OTC derivatives usually are agreed upon exchanges of future cashflows, it usually leads to a loss for the party. The risk of the counterparty failing to fulfil these contractual agreements is called counterparty credit risk (or simply “counterparty risk”). Counterparty risk is mainly present in two markets: the OTC derivatives market and the securities financial transactions market (Gregory, J. (2015) Ch 4). This thesis focusses solely on the OTC market and will not spend any time explaining the securities financial transactions market.

To understand counterparty risk, it is important to first understand credit risk. Traditionally when credit risk is explained, it is assumed to be the same as lending risk. Lending risk is characterized through two factors: the notional amount at risk, which is usually known beforehand, and a unilateral risk profile. However, counterparty risk varies a lot from this traditional definition. Counterparty risk usually has a very uncertain notional amount at risk, and since the contract is bilateral the value of the contract can move to be both positive and negative (Gregory, J. (2015) Ch 4).

This section will focus on explaining the components that make up counterparty risk, explaining the mitigation method of collateral and introduce the so-called credit value adjustment (CVA) through a definition as well as a mathematical derivation.

3.1 Components

Counterparty risk consists of three main components that all play an important role. The three components are the exposure, the probability of default and the loss given default. All three components will be explained in the following sections.

3.1.1 Exposure

A key determinant in the calculation of counterparty risk is the exposure. Exposure represents the core value that may be at risk in default scenarios. In the event of counterparty default the surviving party can close-out the relevant position and stop the contractual payments. When doing so the party has to look at the net amount between the party and the counterparty. This net amount is the exposure (Gregory, J. (2015) Ch 7).

In case the value of the exposure is positive for the party, it means that the defaulted counterparty owed money to the party, and the party has to try and recover as much of the exposure as possible. It is however never expected that a party can recover the full exposure.[9] In case the exposure is negative it means that the party is owing money to the defaulted counterparty. In this case the party is still legally bound to settle the total negative exposure. This means that if the exposure is positive the party will incur a loss, and if the exposure is negative the party will not achieve a gain. This can be summarized in the following expression (Gregory, J. (2015) Ch 7):

$$Exposure = \max(value, 0)$$

[9] The realistic amount of the exposure that can be recovered will be discussed later, when loss given default is introduced

A key feature of counterparty risk that has already been discussed is the bilateral nature of the risk profile. This also counts for the exposure of a position, since both the party and the counterparty are at risk of a default and thus both the party and the counterparty can incur losses. This means that from the party's point-of-view their own default will cause a loss to the counterparty. This is called negative exposure, and can be described using the following expression (Gregory, J. (2015) Ch 7):

$$Negative\ Exposure = \min\,(value, 0)$$

However, since this thesis assumes a unilateral exposure, the negative exposure will not be explained further.

In general, the calculation of the exposure is relatively simple. One has to calculate the mark-to-market (from now on known as MtM) value, which is the value of the derivative in the market for a specific period (Gregory, J. (2015) Ch 7). The practical calculations of the exposure as well as further details will be discussed in section 4 of this thesis.

3.1.2 Probability of Default

Another important factor in measuring counterparty risk is the credit quality of the counterparty, and therefore also the probability of counterparty default. The term 'probability of default' covers two aspects: 1) the probability of default during a known time horizon and 2) the probability of the counterparty suffering a decline in credit quality (Gregory, J. (2015) Ch 12).

When a market actor engages in a derivatives position with a counterparty, they usually have an idea as to the short-term default probability of said counterparty (e.g. based on credit ratings). However, it is also important to consider future default probabilities when considering engaging in OTC derivatives positions with a specific counterparty. When considering future probabilities of default, one has to consider the relationship between credit quality and financial health. Future default probabilities will have a tendency to either increase or decrease over time based on the current financial health of the company. Consider a company with bad financials. Here default is expected to occur early on, which means the initial probability of default will be high and then decrease over time. The opposite is true for a company with strong financials. (Gregory, J. (2015) Ch 12).

Furthermore, it has been empirically proven that there is a mean-reversion effect in credit quality (Gregory, J. (2015) Ch 12). Mean reversion is a theory that suggests that asset price volatility and historical returns eventually will revert back to a long-term mean (Chen, J. (2021)). This means that companies with an above-average credit quality will tend to experience a decrease in credit quality over time, and companies with a below-average credit quality will tend to experience an increase in credit quality over time. This does not sound intuitive at first, but it makes sense when analysing it: Assume a company with weak financials that do not default in the short-term. If a company close to default does not default, it usually indicates an ability to turn the business around. This change will lead to an increase in credit quality over time (Gregory, J. (2015) Ch 12).

When calculating the probability of default there are generally two methods: the 'real-world' method and the 'risk-neutral' method. The real-world method relies upon historical data to estimate default probabilities (usually credit ratings). This is a very static method and has generally been criticised for lack of sufficiency on multiple levels. The risk-neutral method is when the probability of default is derived from market data such as bonds or credit default swap spreads. This method is generally seen as market practice and usually yields a higher probability of default (Gregory, J. (2015) Ch 12). This thesis will use the risk-neutral method, which will be discussed again in section 4.

3.1.3 Loss Given Default

The final component of counterparty risk is the loss given default (from now on LGD). LGD is the percentage of the outstanding claim that is lost when a counterparty goes into default. As previously mentioned, it is not realistic to assume that a party can get 100% of the exposure back in case of a counterparty default, and therefore LGD plays an important role in estimating the expected amount of the exposure that can be reclaimed. LGD is calculated using the following formula (Gregory, J. (2015) Ch 4):

$$LGD = 1 - R$$

Where R is the recovery rate, which is the percentage of the outstanding that can be recovered in case of counterparty default. LGD is generally highly uncertain as it varies a lot from case to case (Gregory, J. (2015) Ch 4).

In the event of default an OTC derivatives holder gains the same status as senior bondholders. This means that the party's claim is treated as senior unsecured debt and therefore it is often assumed that LGD for an OTC derivative position is the same as for senior bondholders. However, there is a major issue with this assumption, which is time and market liquidity. Since an OTC derivative in general cannot be freely traded, and especially not when the counterparty is in default, this can lead to a substantially different LGD (Gregory, J. (2015) Ch 4). However, for the sake of simplicity, this thesis will assume the same LGD as for a senior bondholder. What this exactly means will be discussed in greater detail in section 4.

3.2 Mitigation of Counterparty Risk: Collateral

One of the ways to manage counterparty risk is through mitigation methods. This section will deep dive into collateral as a counterparty risk mitigation method, as this is the mitigation method of choice for this thesis. Furthermore, this section will briefly mention other well-known mitigation methods, and at last this section will discuss the collateral assumptions that will be made in this thesis for modelling purposes.

3.2.1 Definition of Collateral

The basic definition of collateral is:

> *"an asset supporting a risk in a legally enforceable way"* – J. Gregory. (2015) Ch 6

This means that collateral functions as postings of an asset (either cash or securities) to reduce the exposure of a bilateral contract and thereby diminish counterparty risk. By looking at collateral in relation to an interest rate swap, it can be said that if the swap moves in-the-money (ITM) and the present value of the swap goes up, the party will receive collateral from the counterparty, whose present value is now negative. Thus, it is a posting of assets from the "losing" side to the "winning" side of a bilateral derivative position.

It is important to remember that the transfer of collateral does not mean that the posted collateral belongs to the receiving party. The posted collateral still belongs to the party, who originally posted it. Only in case of counterparty default is the ownership of the collateral changed, in which case the collateral is used to pay out (some) of the owed money from the position's exposure (Gregory, J. (2015) Ch 6).

Collateral is posted at specific periods; this thesis will refer to these as 'collateral calls'. The modelling of collateral calls will be discussed in greater detail later in this section.

The hypothetical collateral amounts posted in the collateral calls can be determined based on the following formula:

$$Collateral = \max(MtM - threshold_C, 0) - \max(-MtM - threshold_I, 0) - C$$

Where MtM represents the current mark-to-market value of the swap, $threshold_C$ and $threshold_I$ represents the thresholds for the counterparty and the institution/party respectively and C represents the amount of collateral already held (Gregory, J. (2015) Ch 6). What these terms mean will be explained in greater detail in section 3.2.2.

3.2.2 The Credit Support Annex

As previously discussed, OTC derivatives can either be uncollateralized or collateralized. Thus, there is no standard obligation to post collateral for any OTC derivative. This means that to engage in collateralized positions, both parties have to sign what is known as a credit support annex (from now on known as a CSA). A CSA is a signed contract between both parties of an OTC derivative that states all the rules regarding the collateral calls. After the CSA has been agreed upon and signed, the only way to change it, is if both parties can agree on the change. The specifics of the CSA are often dictated by the party with the strongest credit quality. The reason for this is that collateral is generally dependent on credit quality, with a lower credit quality corresponding to higher collateral demands due to the increased counterparty risk (Gregory, J. (2015) Ch 6). There are however risks associated with this linkage between credit quality and collateral requirements. During the great recession the American International Group (AIG) faced liquidity problems due to increased collateral postings after their subsidiary AIGFP faced a credit rating downgrade (Gregory, J. (2015) Ch 2).

There generally exist two types of CSAs: one-way and two-way CSA. A one-way CSA is a unilateral collateral arrangement, which means only one party agrees to post collateral. A two-way CSA is a bilateral collateral agreement, which means both parties agree to post collateral dependent on MtM movements. One-way CSAs are common when one party has a very high credit rating (such as a sovereign), whereas a two-way CSA is market practice in the interbank market. The CSA help with defining the two main types of collateral: the initial margin and the variation margin. The initial margin is an initial collateral posting made up-front when the derivative position is initiated. The use of initial margin creates the possibility of over-collateralization. The variation margin is the collateral posted to compensate for MtM movements and can be said to be defined through a variety of factors: threshold, minimum transfer amount and haircut (Gregory, J. (2015) Ch 6).

The threshold is the size of the exposure below which collateral is not required to be posted. The higher the threshold value, the larger the risk of under-collateralization, which means that the collateral is not as effective since there is not enough of it to effectively remove the risk from the exposure. The minimum transfer amount is the smallest amount of collateral that can be posted. Minimum transfer values are used to avoid operational expenses associated with insignificant changes in the amount of collateral. Minimum transfer amount is very helpful in keeping operational costs down when collateral is posted on a regular basis. Haircuts are an extra amount of collateral added on top of the required amount to compensate for decreases in the value of the collateral. Haircuts are only used when the collateral posted is securities, as these can fluctuate in value in a way that cash cannot (Gregory, J. (2015) Ch 6).

3.2.3 Collateral in Practice

In practice not all actors in the OTC derivatives market engage in collateral agreements, and there are specific tendencies depending on the type of actor in the market. Sovereigns usually never post collateral, but they can receive collateral. This bias is based on the excellent credit ratings that most sovereigns have. Non-financial corporations rarely engage in collateralized positions as they are generally not interested in committing to the resulting operational and liquidity requirements. However, financial firms usually engage in collateralized positions and it is, as previously mentioned, market practice to initiate a two-way CSA when both firms are related to the financial services industry (Gregory, J. (2015) Ch 6). Many financial firms actually face issues in the market, as they sometimes initiate an uncollateralized position with a counterparty, and then to hedge this position, they initiate another position with a financial firm, which will then be collateralized. This means that as their initial position moves ITM, they have to post collateral to their hedge trade while receiving no collateral from their initial trade. (Gregory, J. (2015) Ch 6).

One of the main factors to consider when discussing collateral in practice is the re-use of collateral, or so-called rehypothecation. Rehypothecation is when a company can use the same collateral for different financial contracts with different counterparties. Rehypothecation proved a major issue during the great recession as companies that defaulted had posted the same collateral for multiple financial contracts, thus leading to the counterparties not getting near enough of the collateral that was posted. This meant that collateral turned into a fake safety which proved unable to live up to the expected coverage of risk. The reason rehypothecation exist is because it reduces funding costs and therefore enables more market participants to enter into collateralized positions. Some parties in the OTC market refuses to allow rehypothecation and usually demands so-called segregation. Segregation dictates that when collateral is posted it is placed in the hands of a third party to legally protect it in case of default. When comparing rehypothecation and segregation, it is clear that segregation works better at reducing counterparty risk, but also leads to increased funding costs, while rehypothecation enables more actors access to the OTC market, but also increases the risk of inefficient collateral postings. Therefore, before deciding between any of the two methods, a cost/benefit analysis should be performed (Gregory, J. (2015) Ch 6).

Lastly, when discussing collateral in practice the margin period of risk (MPR) is important. MPR is the part of the duration of a derivative, in which collateral is no longer posted. This is usually a period spanning the final time of the derivative before expiry, when the parties are certain that default will not happen (Gregory, J. (2015) Ch 6).

3.2.4 Pros and Cons of Collateral

Collateral is generally seen as a very effective way to mitigate counterparty risk. Collateral enables easier access to the OTC market; it improves pricing and it helps make markets more efficient (Bloomberg (2016)). However, there are also some issues related to the usage of collateral. Depending on how often collateral is posted, it may not be able to mitigate counterparty risk fast enough. If the underlying asset is very volatile, collateral may not be posted fast enough to combat major moves in the market. However, the interest rate market is generally seen as one of the most stable markets, which is why sudden moves in the underlying should theoretically not be an issue (Gregory, J. (2015) Ch 6).

Another important factor to remember when discussing counterparty risk mitigation methods (such as collateral), is that they do not remove risk. Mitigation methods are usually only seen in relation to their effect on exposure and counterparty risk, but they also affect the companies in ways not related to counterparty risk. Much like any other mitigation method, collateral transforms the mitigated counterparty risk into another type of risk such as liquidity risk (in case the collateral has to be liquidated in the event of default or in case of segregation), market risk (since residual counterparty risk is dependent on market movements), operational risk (the risk of errors, fraud and failed deliveries due to the large operational burden of maintaining collateral calls) and legal risk (arises due to either rehypothecation or segregation) (Gregory, J. (2015) Ch 6). This means that even though mitigation methods and specifically collateral is thought of as a viable method to reduce counterparty risk, this risk does not disappear, and managing it is not as simple as one might initially assume.

3.2.5 Other Mitigation Methods

There are two other well-known mitigation methods aside from collateral that is relevant to briefly mention. These are hedging and netting.

Hedging is one of the most popular ways among general market participants to mitigate risk such as counterparty risk. Hedging is the usage of financial instruments to remove a specific kind of risk based on a specific factor or variable. An obvious way to generally hedge counterparty risk is by hedging with credit derivatives, since credit quality is of detrimental importance when discussing counterparty risk. One of the most well-known credit derivatives is the credit default swap (CDS), which works as a kind of insurance on the counterparty that kicks in in case of default (Hull, J. C. (2018) Ch 25).

Another traditional mitigation method used by market participants is netting. Netting works when there are multiple financial transactions with both positive and negative value (Gregory, J. (2015) Ch 5). Netting helps to reduce risk by bundling financial obligations into a net financial obligation. Netting can be used both for bundling cashflows, but also in events of default (close-out netting) (Hargrave, M. (2020)).

3.2.6 Collateral Assumptions and Limitations in This Thesis

This thesis seeks to investigate the impact of collateral on counterparty risk, and whether it works during periods of financial distress. This means that this thesis seeks to see the full effects of collateral and the factors that make up the collateral calls. Therefore, even though both banks in this thesis have high credit ratings, it is still assumed that initial margin is posted. Initial margin is one of the factors that is usually only used for parties of lower credit quality, but from an academic point-of-view it is definitely interesting to see the different factors and how they affect the collateral calls and thereby also the exposure (Gregory, J. (2015) Ch 6). As mentioned, both parties of these positions will be banks, which means this is an interbank trade, and therefore it would usually be safe to assume that the CSA is a two-way CSA. However, it was stated earlier in this thesis that counterparty risk would be analysed unilaterally, which means that collateral will also be unilateral. This means that for simplicity's sake only the counterparty (JPM) will post collateral in this thesis, and if the swap runs out-of-the-money (OTM) for BAC, then collateral will be set to zero.

Furthermore, this thesis will assume that the collateral posted will be cash. The reason for this is both to make it easier to compute as collateral specific market movements and haircuts are taken out of the picture, but also because cash is by far the most used type of collateral, with 75% of all collateral posted being cash (Gregory, J. (2015) Ch 6). Lastly, for simplicity this thesis will not assume the existence of either rehypothecation or segregation, neither will funding costs or the margin period of risk (MPR). The reason for this is that it would simply become a far too complex assignment to be able to carry out in a master's thesis. Also, by excluding the MPR it will be clearer whether collateral is an effective mitigation method.

3.3 CVA

In this section the credit value adjustment (CVA) is introduced and explained through a definition as well as a mathematical derivation. Lastly, GWWR will be incorporated into the CVA formula. This section will seek to explain how the theory from section 2 and 3 will fit together in the modelling and calculations of section 4.

3.3.1 Definition

The idea of the credit value adjustment (from now on known as CVA) was for it to function as an adjustment to the risk-free value of a derivative to compensate for the risk of default. This can be summed up in the following simple formula (Gregory, J. (2015) Appendix 14A):

$$Risky\ value = Riskfree\ value - CVA$$

Based on this formula, it can be said that the CVA value is the quantification of counterparty risk. This is further proved by looking into the standard CVA formula:

$$CVA = LGD \sum_{i=1}^{m} EE(t_i) * PD(t_{i-1}, t_i)$$

Where LGD is the loss given default, EE is the expected exposure in the future given by time t_i and PD is the probability of default. Here it is noted that both the expected exposure and the probability of default is assumed to be independent, which means that WWR is assumed to be non-existent (Gregory, J. (2015) Appendix 14A). This assumption will be touched upon later and it will be shown how WWR can be incorporated into the CVA formula.

Now that CVA has been defined as an adjustment made to the risk-free value of a derivative as well as the quantification of counterparty risk, it is now possible to mathematically derive the CVA formula.

3.3.2 Deriving CVA

To derive the CVA formula under the risk-neutral measure[10] it is natural to start with finding an expression for the risky value $\tilde{V}(t,T)$ of a derivative position with a maturity date T. Denote the risk-free value of the derivative as $V(t,T)$ and default time of the counterparty as τ. This denotation of the risk-free value is very important, but this will be touched upon later. For now, two scenarios are considered:

1) The counterparty does not default before time T

In case the counterparty does not default before time T, the payoff function can be described as follows:

[10] A probability measure used in mathematical finance to account for risk aversion. Theoretically the current value of an asset equals the future cashflows discounted to period 0. This does not function in reality as investors are risk averse (more afraid to lose money than eager to gain money). Therefore, the risk-neutral measures function as an adjustment to compensate for this (Chen, J. (2020))

$$I(\tau > T)V(t,T)$$

Where $I(\tau > T)$ is the so-called indicator function, which denotes counterparty default. The indicator function has a value of 1 if default has not occurred and a value of 0 if default has occurred.

2) The counterparty defaults before time T

In case the counterparty defaults before time T, the payoff function consists of two terms: 1) the value of the position that had been paid out prior to default, as well as 2) the payoff at default.

The value of the position that had been paid out prior to default can be described as follows:

$$I(\tau \leq T)V(t,\tau)$$

The payoff function at default is a bit more intricate than the two prior payoff functions. This is because of the MtM value of the position at the time of default. If the MtM value is positive, the party can expect to receive a recovery fraction of the risk-free value based on the recovery rate (R). In case the MtM value is negative the party will have to settle this amount (this relationship was also described in the exposure section of this thesis). The payoff function can be described as follows:

$$I(\tau \leq T)(RV(\tau,T)^{+} + V(\tau,T)^{-})$$

Where $x^{-} = \min(x, 0)$ and $x^{+} = \max(x, 0)$

If the three payoff functions above are put together, the following expression for the risky value is calculated:

$$\tilde{V}(t,T) = E^{Q}[I(\tau > T)V(t,T) + I(\tau \leq T)V(t,\tau) + I(\tau \leq T)(RV(\tau,T)^{+} + V(\tau,T)^{-})]$$

By using the relationship of $x^{-} = x - x^{+}$ the following expression can be constructed:

$$\tilde{V}(t,T) = E^{Q}[I(\tau > T)V(t,T) + I(\tau \leq T)V(t,\tau) + I(\tau \leq T)(RV(\tau,T)^{+} - V(\tau,T)^{+})]$$

The above expression can then be iterated further upon and re-arranged into:

$$\tilde{V}(t,T) = E^{Q}[I(\tau > T)V(t,T) + I(\tau \leq T)V(t,\tau) + I(\tau \leq T)((R-1)V(\tau,T)^{+} + V(\tau,T))]$$

Now, the two terms in the third payoff function can be combined since $V(t,\tau) + V(\tau,T) \equiv V(t,T)$:

$$\tilde{V}(t,T) = E^Q[I(\tau > T)V(t,T) + I(\tau \leq T)V(t,T) + I(\tau \leq T)((R-1)V(\tau,T)^+)]$$

At last, it is known that $I(\tau > T)V(t,T) + I(\tau \leq T)V(t,T) \equiv V(t,T)$, which can lead to a further simplification of the formula:

$$\tilde{V}(t,T) = V(t,T) - E^Q[(1-R)I(\tau \leq T)V(\tau,T)^+)]$$

It is remembered from the start of this derivation that the risk-free value was denoted as $V(t,T)$, which means that the above formula is now corresponding to the basic formula of the risky value, and it can now be re-written as:

$$\tilde{V}(t,T) = V(t,T) - CVA(t,T)$$

Where:

$$CVA(t,T) = E^Q[(1-R)I(\tau \leq T)V(\tau,T)^+)]$$

This is however not the entire derivation. If the standard CVA formula is sought to be derived, there are still a few more steps to complete. First of all, the CVA formula above has to be re-written like so:[11]

$$CVA(t,T) = -(1-\bar{R})E^Q[I(u \leq T)V^*(u,T)^+]$$

Where $\bar{R}$ is the expected recovery rate, and $V^*(u,T) = V(u,T)$. This is key for the derivation of the standard CVA formula, as this step means that WWR is ignored throughout the rest of the derivation. In the following section it will be demonstrated how to simply introduce WWR to the CVA formula, but for the derivation of the standard formula, WWR will be ignored.

[11] (1-R) becomes negative when leaving the brackets, as CVA is negative per definition (see the simple formula in section 3.3.1). Thus, there is a "hidden" minus in front of the calculation all the time. The remaining calculations will hide this minus again, as CVA will be calculated as a positive value (although it is in fact negative)

Since CVA is calculated over all times before the maturity date, it is necessary to integrate over all possible default times. This leads to:

$$CVA(t,T) = (1-\bar{R})E^Q\left[\int_t^T B(t,u)V(u,T)^+ dF(t,u)\right]$$

Where $B(t,u)$ is the risk-free discount factor and $F(t,u)$ is the cumulative probability of default for the counterparty.

It is known that under the risk-neutral measure the expected exposure is calculated using the following formula:

$$EE_d(u,T) = E^Q[B(t,u)V(u,T)^+]$$

By assuming deterministic default probabilities[12], it is possible to re-write the CVA formula and include expected exposure like this:

$$CVA(t,T) = (1-\bar{R})\left[\int_t^T EE_d(u,T)dF(t,u)\right]$$

Finally, the above formula can be computed via an integration scheme[13], which leads to the following standard CVA formula:

$$CVA(t,T) \approx (1-\bar{R})\sum_{i=1}^{m} EE_d(u,T)\,[F(t,t_i) - F(t,t_{i-1})]$$

Remember that the above expression is only a CVA approximation, but as long as m (number of periods) is reasonably large, then the approximation should be fairly accurate (Gregory, J. (2015) Appendix 14A).

[12] Deterministic models stand opposite to stochastic models. Were deterministic models yield an output that is fully determined by parameter values and initial assumptions, stochastic models possess some inherent randomness (NC State University. (2013))
[13] Integration is the process of finding a function when its derivative is given (Vedantu. (n.y.))

3.3.3 Incorporation of GWWR

As mentioned earlier, the above derivation led to the standard CVA formula which is written as:

$$CVA = LGD \sum_{i=1}^{m} EE(t_i) * PD(t_{i-1}, t_i)$$

This formula assumes that the expected exposure is unconditional of counterparty default. However, as determined earlier, GWWR demands that the exposure of a position is positively correlated to the probability of default, so that a decrease in credit quality would lead to both an increase in probability of default and exposure. The simplest way to incorporate GWWR into the CVA formula is therefore by making the expected exposure conditional upon the default of the counterparty.[14] This changes the formula to look like this (Gregory, J. (2015) Ch 17):

$$CVA = LGD \sum_{i=1}^{m} EE(t_i | \mathrm{t_i} = \tau_c) * PD(t_{i-1}, t_i)$$

Here $EE(t_i | \mathrm{t_i} = \tau_c)$ represents the expected exposure at time t_i conditional on that time being the default time of the counterparty. The conditional relationship for the expected exposure opens up for two ways of quantifying GWWR (Gregory, J. (2015) Ch 17):

1) A qualitative assessing linked with the use of stressed data
2) Modelling of the relationship between default probabilities and expected exposure

This will be further discussed in the modelling section of GWWR.

4. Component Modelling & Calculations

This section will explain the modelling approaches for the CVA components and the results thereof. The components that are being modelled and calculated in this section is the simulated interest rates, exposure, probability of default, loss given default, the collateral calls as well as the GWWR model.

[14] This theoretically ensures that a linkage between PD and EE has been created

4.1 Interest Rates

The following section will discuss the approach used to forecast the 3M USD Libor rate that is being used as the floating rate for the floating leg of the two interest rate swaps. The simulation has been performed using a model known as the Vasicek model, which will be introduced and elaborated on. When the model has been introduced, the calculation of zero-coupon bond prices under the Vasicek model will be discussed, as this is relevant for the later exposure calculations. Afterwards, the results of the interest rate simulation will be explained.

4.1.1 The Vasicek Model

To forecast the interest rates used for the two interest rate swaps in this thesis the Vasicek model will be used. The Vasicek model is a model that has a history of being applied as a forecasting tool for developments in the price of financial instruments. This model is composed of market risk, time, volatility and a mean reversion factor (Hull, J. C. (2018) Ch 31). When a model is only composed of a single risk factor (such as the Vasicek model), it is called a one-factor model. This means that a single factor (the market risk factor) is used to explain the movements in an interest rate (Corporate Finance Institute. (n.y.)). This initially sounds very restrictive, as it is expected that movements in financial instruments is explained through multiple complex factors. However, in practice a one-factor model basically implies that rates move in the same direction over any short time interval, but that they do not move by the same amount (Hull, J. C. (2018) Ch 31). Therefore, a single-factor model is not as restrictive as one might come to believe, and by only using a single factor, unnecessary complexity is removed from the calculation.

Two main assumptions in the Vasicek model are:

1) Interest rates can be negative
2) Interest rates will not increase or decrease to extreme levels

The assumption of negative interest rates was previously a major issue for the model, but with recent years interest rates having been below zero, this is now a practical assumption. The assumption of interest rates not increasing or decreasing to extreme levels is enforced by the mean reversion factor in the model. As previously mentioned, mean reversion is a quantitatively proven factor in financial markets, and is therefore a sound assumption for an interest rate model. To recap, mean reversion ensures that a financial instrument in the long-term will move towards a mean (usually based on historical data).

The function for the Vasicek model is:

$$dr = a(b - r)dt + \sigma dz$$

Where a is the mean reversion speed, b is the long-term mean, r is the rate of the previous period, dt is a time factor, σ is the standard deviation (and therefore displays market volatility) and dz is the market risk factor. $a(b - r)dt$ is also referred to as the drift. The market risk factor is explained in the Vasicek model through a so-called Wiener process (or Brownian motion), which is a stochastic (random) process (Hull, J. C. (2018) Ch 31). A Wiener process is a standard component in a wide array of industries such as engineering, physical sciences and finance (Probability Course. (n.y.)).

To forecast the interest rates in this thesis a script has been set up in the open source coding language 'Python'. This is done as it enables a large number of interest rate simulations, and for CVA risk management purposes it is advised to maximize the amount of simulations to properly account for all possible market outcomes. This thesis will run 25,000 simulations of the 3M USD Libor rate.

4.1.2 Zero-coupon Bond Prices

The computation of $P(t,T)$, which is used in the calculation of MtM values of the swaps, will be explained now.

As explained in section 2, $P(t,T)$ is the price of a zero-coupon bond at time t with maturity at time T, and can therefore be used as a discount factor of time. In the Vasicek model $P(t,T)$ is calculated as follows (Hull, J. C. (2018) Ch 31):

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

Where $A(t,T)$ is computed as (Hull, J. C. (2018) Ch 31):

$$A(t,T) = \exp\left(\frac{(B(t,T) - T + t)\left(a^2 b - \frac{\sigma^2}{2}\right)}{a^2} - \frac{\sigma^2 B(t,T)^2}{4a}\right)$$

Where a is the mean-reversion spread, b is the long-term mean, σ is the standard deviation (volatility) and $B(t,T)$ is given by (Hull, J. C. (2018) Ch 31):

$$B(t,T) = \frac{1 - e^{-a(T-t)}}{a}$$

The calculations of the factors that comprise $P(t,T)$ can be seen in appendix 3.

4.1.3 Results

All simulations will start in period 0, which is the start date of the swap. This means that the interest rate of period 0 will have a fixed value of 2.7968% across all simulations. From the Vasicek formula it can be explained that the long-term mean will take a constant value of 3.7737%, which is the historical mean of the 3M USD Libor rate from its inception until period 0. This means that the simulated interest rates should have a tendency to deviate towards this mean through the mean-reversion parameter. The volatility is described as the historical standard deviation of the 3M USD Libor rate, and takes the value of 0.0427. All of this is can be seen in appendix 4: 'The Vasicek Script', which contains the Python code that yielded the results.

When running the Monte Carlo simulation, a total of 25,000 times, the following graph is created in Python:

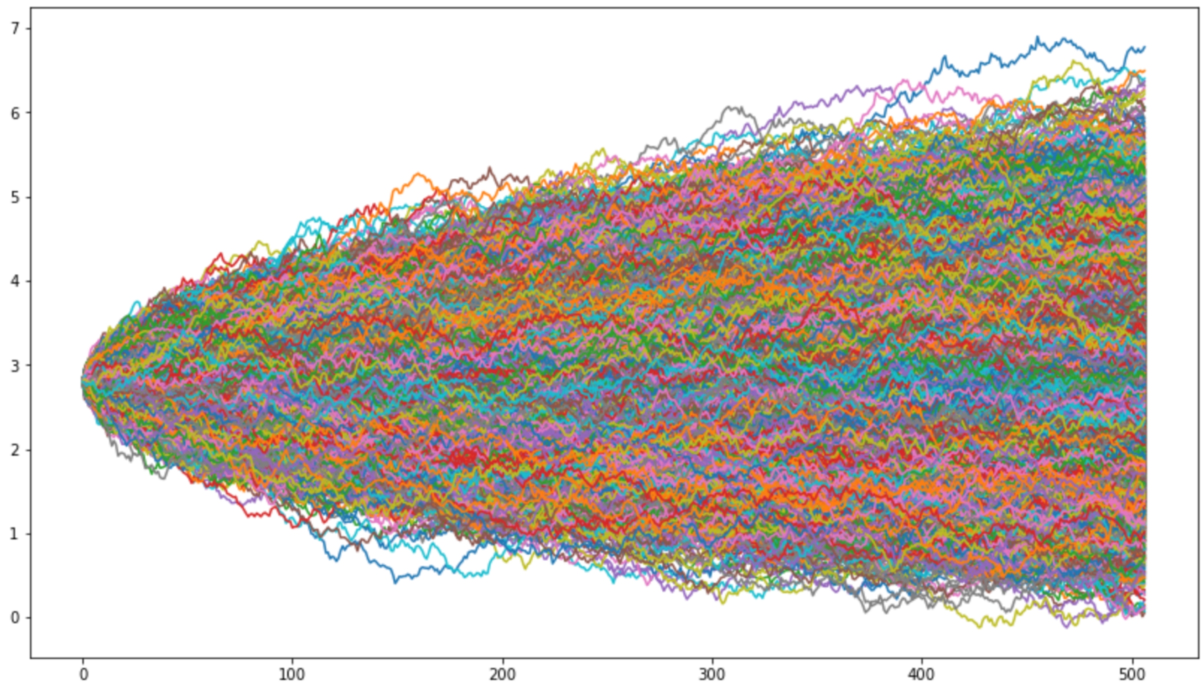

Figure 9: 25,000 interest rate simulations. Source: Own creation

It can be a bit difficult to read the above chart, however, it can be seen that the maximum simulation has a value around 7%, while the smallest simulation value lies just around 0%. It was previously mentioned that the Vasicek model enabled negative interest rates, and it can be seen that some of the simulations has dipped below 0% at a few points in time. The reason there are not a lot of negative rate simulations is primarily explained through the mean-reversion. As interest rates deviate towards the long-term mean (which is positive), it is difficult for the simulation to end up negative. However, when looking at historical 3M USD Libor rates it is not necessarily an issue as these rates has never been below 0% for prolonged periods, however, they have reached the zero-lower bound on multiple occasions after the great recession (see appendix 5: 'Historical Libor Rates').

The above interest rate simulations can now be used to calculate daily exposure paths in the following section of this thesis.

4.2 CVA Components

This section will seek to explain the modelling approaches and the results for the CVA components. The components explained in this section is the expected exposure, collateral, loss given default and probability of default. The Python code that have modelled the expected exposure and the collateral can be seen in appendix 6. The calculations of probability of default can be seen in appendix 7.

4.2.1 Expected Exposure

As explained in section 3, the exposure is the core value that may be at risk in a default scenario. When calculating the MtM values based on the interest rates simulated earlier in this section the following values are seen:

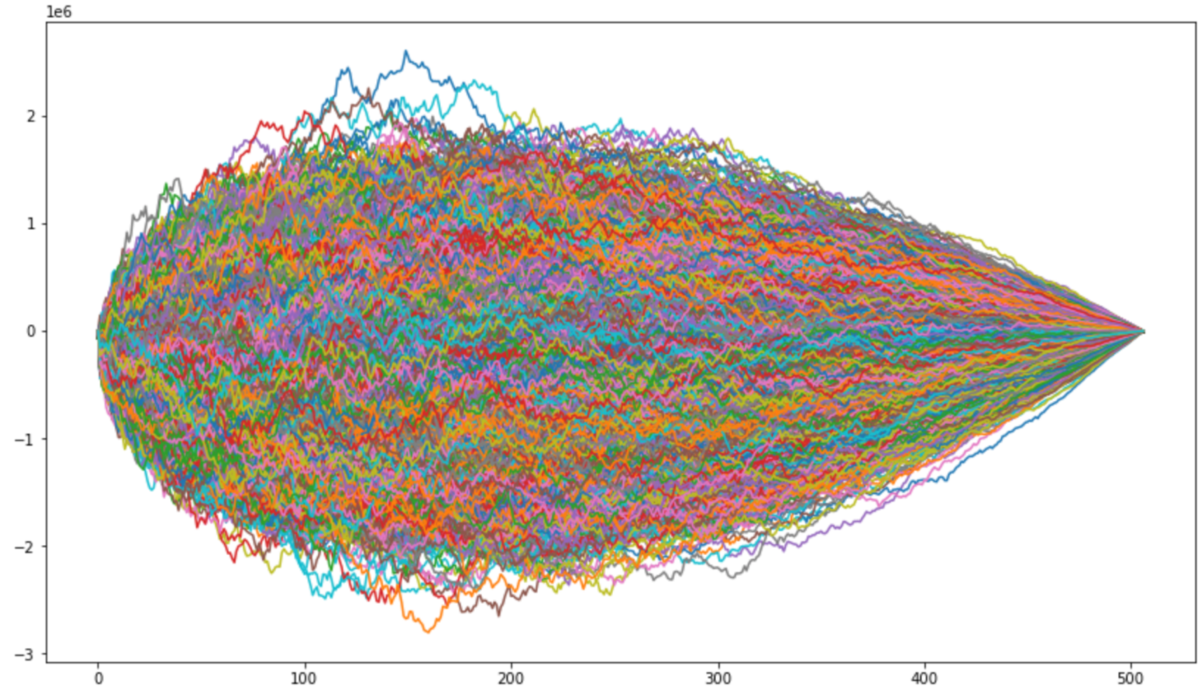

Figure 10: Simulated MtM values. Source: Own creation

It is seen that some of the MtM values are negative, which does not abide by the unilateral assumption made for simplifying reasons. Therefore, it is important to use the exposure calculation introduced in section 3 of this thesis, which stated that exposure was calculated as the maximum value of either the MtM of the swap or zero. This leads to the following graph:

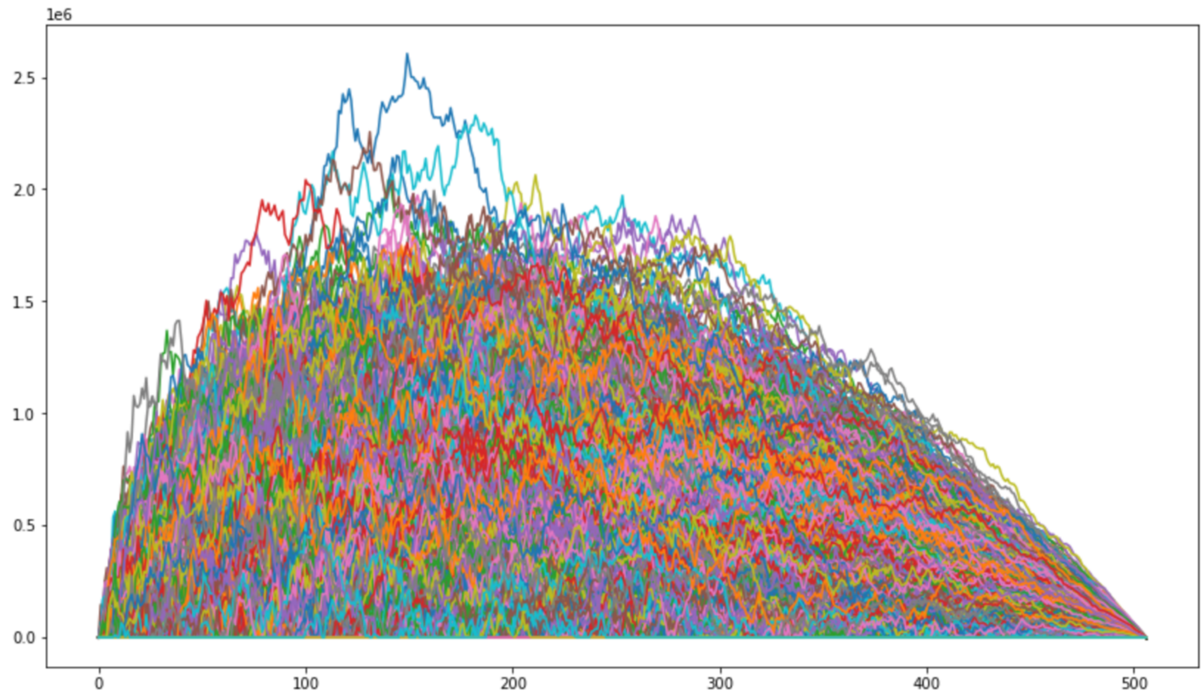

Figure 11: Calculated exposure paths. Source: Own creation

When calculating the CVA value, one does not perform a calculation for each individual exposure, instead it is market practice to calculate the 'expected exposure'. The expected exposure is a mean of all the calculated exposures based on all the interest rate simulations. The expected exposure therefore displays the expected value that may be at risk during the duration of the swaps (Gregory, J. (2015) Ch 7).

Based on the above exposure paths, it is now possible to calculate the expected exposure via the following formula:

$$Expected\ Exposure = \frac{\sum Exposures}{N}$$

It is expected that the graphical depiction of expected exposure will be a sort of slightly skewed bell curve with an initial sharp increase followed by a slow decline. The reason for this is the risk associated with periodic cashflows.

Consider a forward rate agreement (FRA). This derivative consists of a single exchange of cashflows in the future. This means that the exposure profile of such a derivative is a simple decreasing function that reflects that, as time passes uncertainty of future value decreases. However, interest rate swaps are not as simple as FRAs. An interest rate swap consists of multiple exchanges of cashflows in the future. This affects the risk profile, as risk will increase in the beginning as there is a lot of time until the contract expires (and therefore a lot of uncertainty about future cashflows). However, as time passes the risk decreases as cashflows are met and uncertainty disappears (Gregory, J. (2015) Ch 7).

Another impact of periodic cashflows on the exposure profile is caused by the asymmetry in payments. As previously mentioned, plain vanilla swaps will usually have a divergence in payments on the floating and fixed legs (for example 3-month payments on the floating leg and 6-month payments on the fixed leg). This asymmetry can lead to increased risk from a receiver swap owner's point of view, as this party would make floating payments with a larger frequency than the counterparty's fixed payments (Gregory, J. (2015) Ch 7). In this thesis it has, however, been decided to assume equal payment frequencies. This not only makes it less complicated to calculate the exposure profile, but it also ensures that unnecessary noise in regard to the problem formulation is filtered out.

The below graph displays the expected exposure calculated for the uncollateralized interest rate swap, and as can be seen, it has the expected shape:

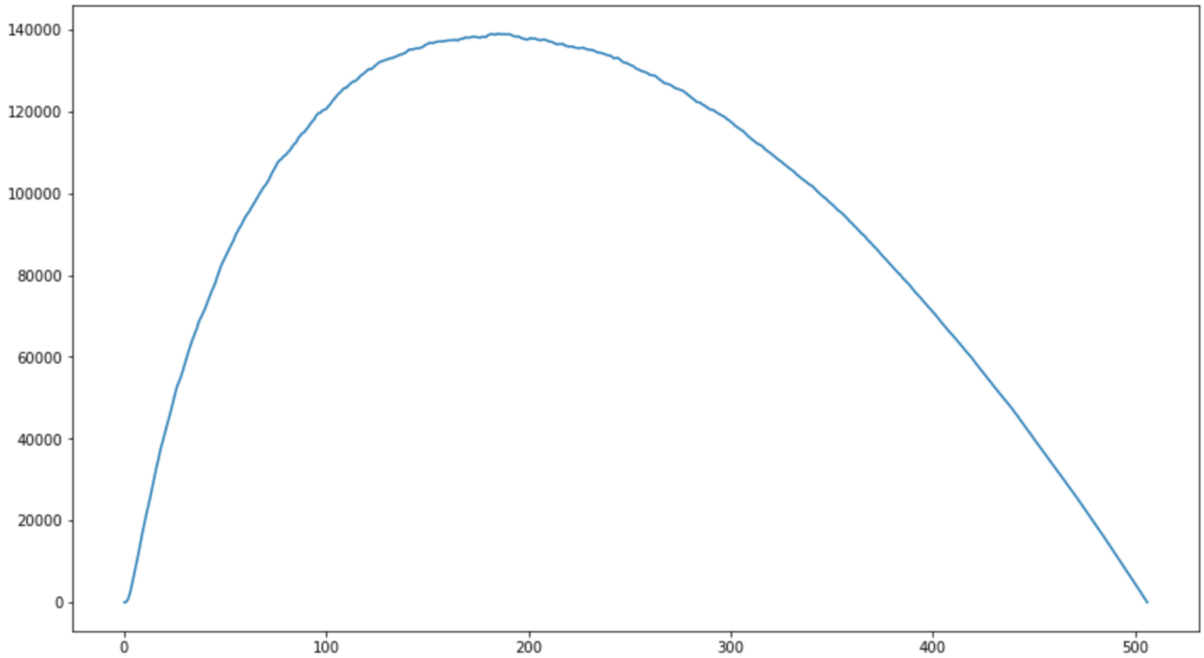

Figure 12: Expected exposure for the uncollateralized swap. Source: Own creation

It can be seen that the expected exposure of the swap appears much lower than many of the exposure paths in figure 11. The reason for this is that all the exposure paths that had negative values and were set to zero are impacting the value of the expected exposure.

The next section will focus on the calculation of the collateral calls as well as the expected exposure of the collateralized swap.

4.2.2 Collateral Calls

As discussed in section 3, collateral is used as a method to mitigate counterparty risk. More specifically it is used as a method to decrease the exposure of a derivative position. However, collateral can never be expected to fully remove an exposure, but it should be able to significantly decrease it given the right conditions in the CSA. This thesis assumes daily collateral calls, which is normal among big banks such as JPM and BAC (Gregory, J. (2015) Ch 6). Daily collateral calls enhance the ability to mitigate large volatilities in the market, but it also comes with increased operational costs. However, for the sake of simplicity this thesis will not assume operational costs and will therefore not be looked anymore into (Gregory, J. (2015) Ch 6). Furthermore, the existence of rehypothecation and segregation will not be covered in this thesis. Additionally, funding costs are assumed to be zero, which is a big assumption when dealing with cash collateral. However, as the funding costs are not important for the analysis of the research question, it is not deemed necessary to analyse. However, funding costs will be briefly touched upon again in section 7.

As mentioned in section 3, the CSA in this thesis will include both an initial margin, threshold as well as a minimum transfer amount. This is done to hopefully showcase their usability and also the risks of both under- and overcollateralization that can arise. As mentioned earlier, an initial margin is usually not posted between two parties with credit ratings as high as the ones of JPMorgan Chase and Bank of America. However, it is an efficient tool to introduce the chance of overcollateralization, which is why it is deemed important for an academic paper, even though it may deviate from real world scenarios. The below table showcases the values of the initial margin, threshold and minimum transfer amount selected for this thesis:

CSA Values		
Basic Collateral Rules	**Values**	**% of Notional**
Initial Margin	15,000.00	0.0150%
Threshold	5,000.00	0.0050%
Minimum Transfer Amount	5,000.00	0.0050%

Table 3: CSA values. Source: own creation

It can be seen in the above table that the values of the initial margin, threshold and minimum transfer amount are very small compared to the notional value of the swap. However, due to the low value of the expected exposure it is assumed to be realistic.

The collateral in this thesis will be calculated from the perspective of one party (adherent to the assumption of unilateral CVA) and will therefore be calculated as follows:

$$Collateral = \max(MtM + initial\ margin - threshold - minimum\ transfer\ amount, 0) - C$$

Where MtM represents the current mark-to-market value of the swap and C represents the amount of collateral already held (Gregory, J. (2015) Ch 6). The daily collateral calls are expected to lead to minor daily fluctuations in the collateral value and therefore also in the value of the expected exposure. These collateral fluctuations are actually one of the reasons why daily collateral calls are a costly affair from an operational point of view. However, if a big bank decided to not go with daily collateral calls, under-collateralization and market risk is a larger threat, which is why it is usually deemed to be worth it from a risk management perspective (Gregory, J. (2015) Ch 6).

When subtracting the collateral postings from the expected exposure of the uncollateralized swap, the following expected exposure for the collateralized swap is calculated:

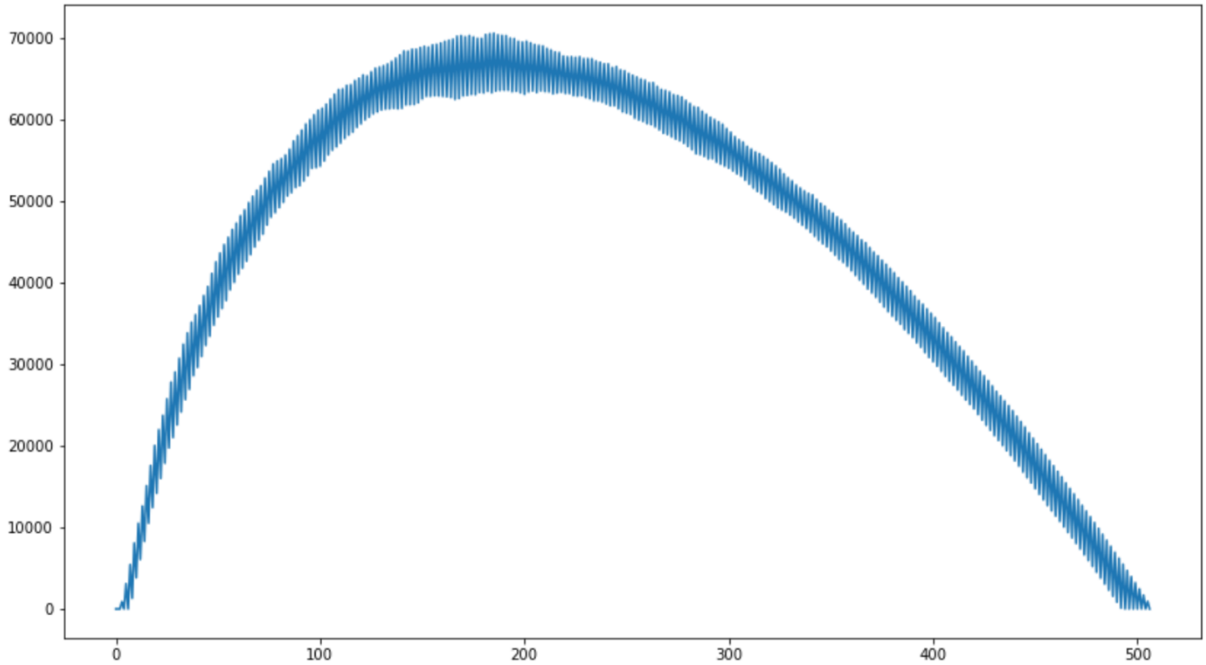

Figure 13: Expected exposure for the collateralized swap. Source: Own creation

Now both expected exposures can be put in a plot together, which will graphically showcase the impact of collateral:

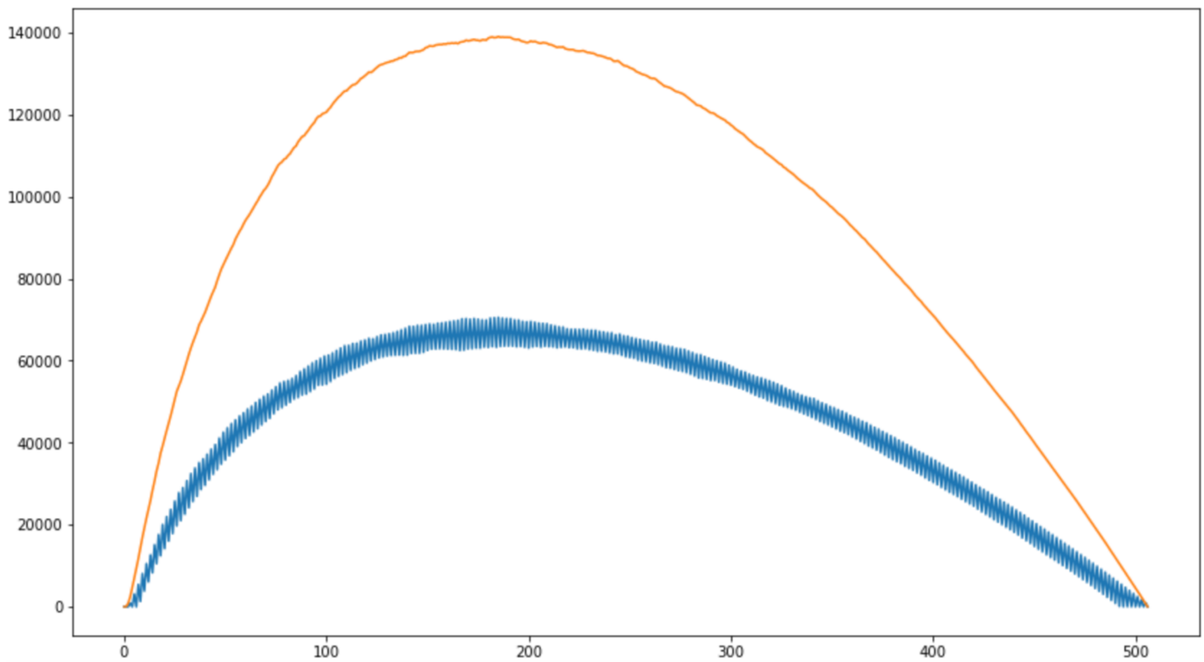

Figure 14: Expected exposures for both swaps. Source: Own creation

As can be seen above, collateral has had a huge impact on the expected exposure, with the uncollateralized swap's peak exposure having a value close to 140,000 and the collateralized swap's peak exposure having a value close to 70,000. All code related to the calculation of collateral calls and collateralized expected exposure can be seen in appendix 6.

4.2.3 Loss Given Default

LGD is a component that is very difficult to accurately model as it depends on the specific default scenario and therefore on the specific counterparty. Since default scenarios are so dependent on the specific defaulting party, no two default scenarios can be expected to be identical. Given that default scenarios can never be totally identical the LGD will naturally vary a lot. For simplification purposes, it is generally deemed appropriate in most cases to assume a fixed LGD throughout the entire runtime of the swaps, as long as the LGD is based on valid assumptions (Gregory, J. (2015) Ch 4). As previously mentioned, OTC derivatives are, in the case of counterparty default, treated as senior unsecured debt. This means that the recovery rate for an interest rate swap such as the two in this thesis would approximate the same recovery rate as for senior unsecured bonds (Gregory, J. (2015) Ch 4). The recovery rate for senior unsecured bonds has a mean of 38% according to Moody' (2007):

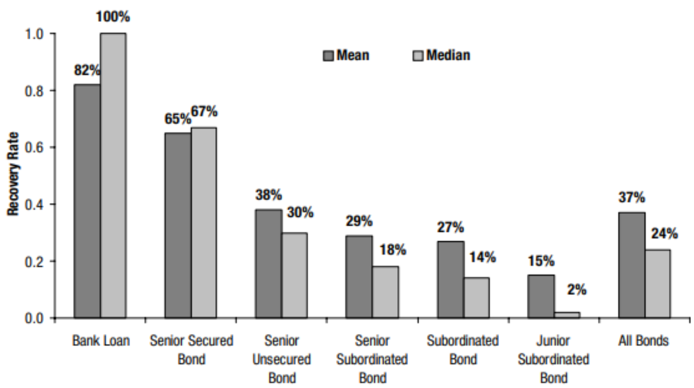

Figure 15: Discounted ultimate recovery rates by debt type. Source: Moody's (2007)

Therefore, this thesis will assume a recovery rate for the swaps of 38% and thus the LGD can be calculated as:

$$LGD = 1 - 0.38$$

$$LGD = 0.62$$

The LGD of the swaps in this thesis is thereby assumed to be 62%.

4.2.4 Probability of Default

In section 3 of this thesis the probability of default was briefly touched upon. Here the risk-neutral method was introduced and decided upon as the calculation method for the probability of default going forward. The move from previous calculation methods to the risk-neutral method was driven by regulations following the great recession, as previous calculations proved unable to properly capture the probability of default. To re-iterate, the risk-neutral method is based upon market data such as credit default swap spreads (from now on CDS spreads). Usually CDS spreads are the preferred instrument to capture credit risk, but for companies that do not have associated CDSs, it is possible to use a proxy spread such as bond spreads (Gregory, J. (2015) Ch 12). The counterparty in this thesis is JPMorgan Chase, and since JPMorgan Chase is a major company, they have associated CDSs, which will be used in the calculations going forward.

The start date for the 2-year interest rate swaps was January 6th, 2019. However, due to a lack of data it was not possible to obtain the CDS spread curve for this date. Instead the CDS spread curve that is used is downloaded from Bloomberg on January 6th, 2020. This is of course a source of error, but this thesis will nevertheless assume that the CDS spread curve can still be used as a proxy for the CDS spread curve on the start date. Even though it will add some error to the results, it is deemed better than just assuming values.

According to the CVA formula derived in section 3, the probability of default should be calculated as the so-called marginal probability of default, which is a probability of default between two periods. This probability of default is calculated using the following formula (Gregory, J. (2015) Appendix 12A):

$$PD(t_1, t_2) = F(t_2) - F(t_1)$$

Where t is a time period and $(t_1 \leq t_2)$ and $F(t)$ is calculated like so (Gregory, J. (2015) Appendix 12A):

$$F(t) = 1 - \exp(-ht)$$

Where h is the hazard rate, which is the conditional probability of default in a very small period. The hazard rate is calculated using the following formula (Gregory, J. (2015) Appendix 12A):

$$h = \frac{s}{LGD}$$

Where s is the CDS spread and LGD is the loss given default. By writing the above formulas into the formula for the marginal probability of default the following expression is achieved:

$$PD(t_1, t_2) = \left(1 - \exp\left(-\frac{s}{LGD} t_2\right)\right) - \left(1 - \exp\left(-\frac{s}{LGD} t_1\right)\right)$$

In the below table the calculated marginal probabilities of default are showcased for JPM:

Maturities	Marginal PD JPM
6M	0.1872%
1Y	0.2480%
2Y	0.5531%
3Y	0.6721%
4Y	0.8143%
5Y	1.0344%
7Y	2.6106%
10Y	4.3240%

Table 4: Marginal probabilities of default for JPM. Source: own creation

The above table showcases the marginal PD for the maturities on the CDS spread curve. However, this thesis seeks to calculate the CVA estimate for the interest rate swaps on a daily basis to account for the daily collateral calls. To ensure that daily probabilities of default is calculated based on table 4, the daily values will be found using the method of interpolation. Interpolation is the process of constructing a full curve based on a set of discrete observations, such as the ones above. There are generally four methods of interpolation: 1) constant, 2) linear, 3) log-linear and 4) hermite spline. Of these four, only a single method assumes no arbitrage and is therefore usable in this thesis – the hermite spline interpolation method. Hermite spline is calculated using the following formula (Hagan, S. P. & West, G. (2006)):

$$r(t) = a_i + b_i(t - t_i) + c_i(t - t_i)^2 + d_i(t - t_i)^3$$

The above formula has been implemented using the FidInterpolate function in VBA (see appendix 8) and can be summarized in the following figure for the total duration of the swaps in this thesis:

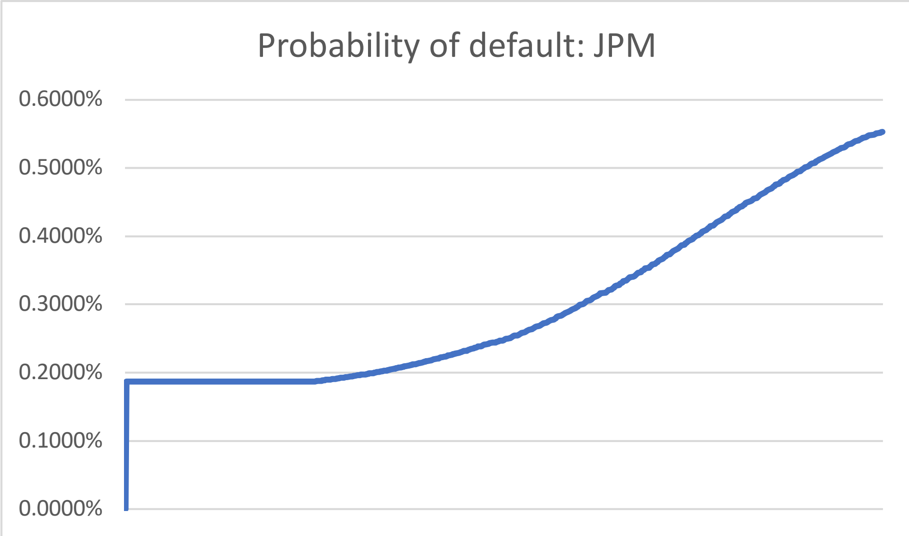

Figure 16: Marginal probability of default interpolated over the duration of the swaps. Source: own creation

The above probabilities of default are increasing over time, which is as expected. As previously mentioned, it is expected that companies with a good credit rating will have a low probability of default in the short run, which will increase over time. Thus, the interpolated results are aligned with the expectations. As mentioned in the start of section 4, all calculations related to the probability of default can be seen in appendix 7.

4.3 General Wrong-Way Risk

The following section will describe the considerations and methodology behind the modelling of GWWR in this thesis. This section will cover the different methods as well as the importance of the so-called alpha multiplier. Afterwards the method that was deemed most suitable for this thesis will be discussed. Then the limitations applied to the selected model will be touched upon. Lastly the results will be discussed.

4.3.1 Modelling Methods & The Alpha Multiplier

According to the Basel Accords an accepted way to compensate for GWWR is by adding a "buffer" on top of the position exposure. This buffer is calculated using the so-called alpha multiplier and is added in the following way (Cespedes, J. C. G. et al. (2010)):

$$Expected\ Exposure_{GWWR} = Expected\ Exposure * \alpha$$

According to the Basel Accords banks can decide to not model their alpha multiplier, in which case alpha takes a default value of 1.4. This means that the exposure in a GWWR-scenario increases by 40%. The reasoning behind the alpha multiplier is that the capital buffer ensures that losses in case of counterparty default does not exceed liquidity (Aziz, A. et al. (2014)). However, banks can use their own models in which alpha can deviate from 1.4. In an internal GWWR-model, alpha can take the minimum value of 1.2, which is the lower bound according to regulations (Cespedes, J. C. G. et al. (2010)). A key factor when discussing GWWR modelling is the exposure-at-default (EAD). EAD is calculated using the following formula (Cespedes, J. C. G. et al. (2010)):

$$EAD = \alpha * Effective\ EPE$$

And the Effective EPE (effective expected positive exposure) is calculated as the weighted mean of the maximum values of the expected exposure during certain time intervals. By iterating on this expression, the following formula for alpha can be calculated (Cespedes, J. C. G. et al. (2010)):

$$\alpha = \frac{EAD}{Effective\ EPE}$$

To model EAD and thereby also the alpha multiplier there are different approaches. The most common include the hazard rate approach, the copula approach and the structured approach. However, after the great recession many new approaches for computing GWWR has been created (Aziz, A. et al. (2014)). This thesis will focus on the three most well-known approaches.

The hazard rate approach is an approach based around the quantification of default scenarios based on developments in so-called hazard rates. Basically, a correlation between the specific hazard rate paths and the exposure of the position is created. This is done through a simulation of hazard rate paths with related exposure paths. All hazard rate simulations are then filtered based on whether default has occurred or not (this is usually approximated using a threshold for the hazard rate value). The hazard rate approach is generally seen as one of the simpler approaches to calculating WWR, but it generally only yields very weak dependencies. Therefore, it is not deemed as a suitable approach for this thesis, as a relatively strong dependency is preferred to fully analyse GWWR-scenarios (Aziz, A. et al. (2014)).

Another known approach to the quantification of WWR is the copula approach. The copula approach is generally seen as the simplest method used to model WWR. The copula approach specifies a direct dependency between counterparty default and exposure, which is an effective way to create a strong dependency. This basically means that one assumes a fixed correlation between market risk and credit quality, which ensures that theoretical GWWR-scenarios can be created. This method relies upon the use of a distribution (copula) to create the results. Usually the Gaussian (normal) distribution is used as it is the simplest and also the easiest to implement. There are multiple issues related to the use of the copula approach in real-world scenarios. These include most notably the fixed correlation between counterparty default and market risk. However, there are also positive aspects linked to the use of the copula approach, such as the strong dependency that can be created. Furthermore, the copula approach basically adds the WWR on top of pre-computed exposures, which means that it limits complexity and time consumption (Aziz, A. et al. (2014)).

The structured approach is the final of the WWR modelling approaches that will be discussed in this thesis. The structured approach generally seeks to deal with some of the weak points of the copula approach. The structured approach deals with the fixed correlation of market risk and counterparty default risk by quantifying the correlation through large amounts of historical data. A default boundary (a boundary that defines default as when a firm value reaches a certain lower bound) is defined and linked to the firm value of the counterparty, which is dependent on the market risk factor. The structured approach is definitely the best model of the three to model GWWR, however, due to the level of complexity as well as the high demand for historical data, it can be very difficult to apply (Aziz, A. et al. (2014)).

As have been mentioned previously, this thesis seeks to be as close to market practice as possible in both modelling and calculations. However, it is also important to keep in mind what is actually realistic to accomplish. Based on this, the modelling of GWWR using the structured approach is deemed far too complex for this thesis. Therefore, GWWR will be modelled using the copula approach as this is the least complex approach to model GWWR. In the discussion section of this thesis it will be discussed how improvements on the GWWR model could make for interesting later research.

4.3.2 The Gaussian Copula Approach

For simplicity the copula approach in this thesis will be built as a single-factor model. This means that much like the interest rate simulation described in section 4.1, market risk will be explained through a single factor. This factor will be denoted by Z, which is a normally distributed stochastic variable. To achieve the goal of this modelling approach, and be able to compute the alpha multiplier, two inputs are needed: 'EAD' and 'Effective EPE'. To be able to calculate EAD on a daily basis throughout the duration of the swaps, default scenarios has to be simulated. Default scenarios are generally sought out via a simulation of the so-called creditworthiness index. The creditworthiness index is calculated using the following formula (Mermatoluie, A. et al. (2016)):

$$CWI_k = \sqrt{\rho_k} * Z + \sqrt{1 - \rho_k} * \varepsilon_k$$

Where ρ_k is known as the sensitivity of obligor (or simply the correlation parameter), Z is the factor describing market risk and ε_k is the counterparty default risk, which because of the lack of data combined with the need for simplicity are both described as normally distributed stochastic variables (Cespedes, J. C. G. et al. (2010)).

To specify whether a default scenario has occurred or not, a so-called default indicator has to be specified. The default indicator is a binary variable that can take either a value of 1 or 0 based on whether or not a default has occurred in the simulation of the creditworthiness index. If $CWI_k \leq PD$ the default indicator takes a value of 1, which means default has occurred. If $CWI_k > PD$ the default indicator takes a value of 0, which means default has not occurred (Mermatoluie, A. et al. (2016)). This default indicator is then used to calculate the exposure for default scenarios using the following formula (Mermatoluie, A. et al. (2016)):

$$L_m = \sum_{k=1}^{K} y_{km} * 1\{CWI_k \leq PD\}$$

Where y_{km} is the LGD adjusted exposure.

Since defaults are rare it is important to make a lot of credit simulations to ensure that there are enough default scenarios to calculate EAD from. This thesis will make as many credit simulations as there are interest rate simulations. This corresponds to a total of 25,000 credit simulations, which is in the low end of what is normal for a GWWR model, but since it is demanding for a regular computer to run many simulations, it is not viable to simulate more (Mathworks. (n.y.)).

Following the quantification of the default scenarios in the credit simulation, it is possible to calculate EAD. EAD will in this thesis be described as the mean of all exposures with a corresponding default scenario. Furthermore, it is also possible to calculate the Effective EPE, which is calculated using the following formula:

$$EEPE = \frac{1}{t_E}(\max(EE))$$

Where the max values are gathered over a number of dates. It can be seen that EEPE is calculated as a weighted average of the max values (FinCad. (2014)).

In this thesis this will be implemented by calculating the maximum expected exposure for 8 periods, each corresponding to 1 quarter for the total 2-year period. The weights will be identical for all periods.

4.3.3 Model Limitations

This thesis has had to implement multiple limitations to the GWWR model in order to achieve a functional model. Many of these limitations has of course had a negative effect on the model's ability to function in a real-world scenario. However, as mentioned previously in this thesis there has to be a balance between complexity and usability. One of the main limitations is the correlation/sensitivity of obligor in the calculation of the creditworthiness index. This correlation is assumed to be a constant with a value of 0.75. This affects the credit simulation and therefore also the alpha multiplier. This assumption is not generally realistic to assume, but nevertheless the assumption enables the model to function without having to implement unnecessary complexity. Another limitation is the very small amount of simulations as it creates results with a risk of potential error from lack of data. Lastly, the use of normally distributed stochastic variables functions as a great placeholder instead of using real data, but it is not expected to yield the same results as a complex statistical data-based model (Aziz, A. et al. (2014)).

4.3.4 Results

The GWWR script started by simulating 25,000 CWIs, which each related to one of the calculated exposure paths. These CWIs were compared to the summed PD for JPMorgan Chase, and all exposure paths with a CWI with a value higher than the summed PD was dropped. Since this GWWR model is stochastic, a certain degree of randomness will always influence the results. Some of this randomness was seen as OTM exposures or very little ITM exposures being marked as default scenarios. It was mentioned in the introduction that WWR modelling should generally test on stressed data, to ensure a bias towards the worst-case scenario. To ensure this holds true the model filters summed exposure values by a lower bound. If an exposure path has a summed value below 25,000,000, it is dropped. The reason for this is to remove exposure paths in which the swap is only a little ITM or even OTM.

After this, all remaining exposure paths are used in the calculation of EAD:

$$EAD = \frac{LGD * Exposure_n}{N}$$

The calculated EAD is showcased in the below graph:

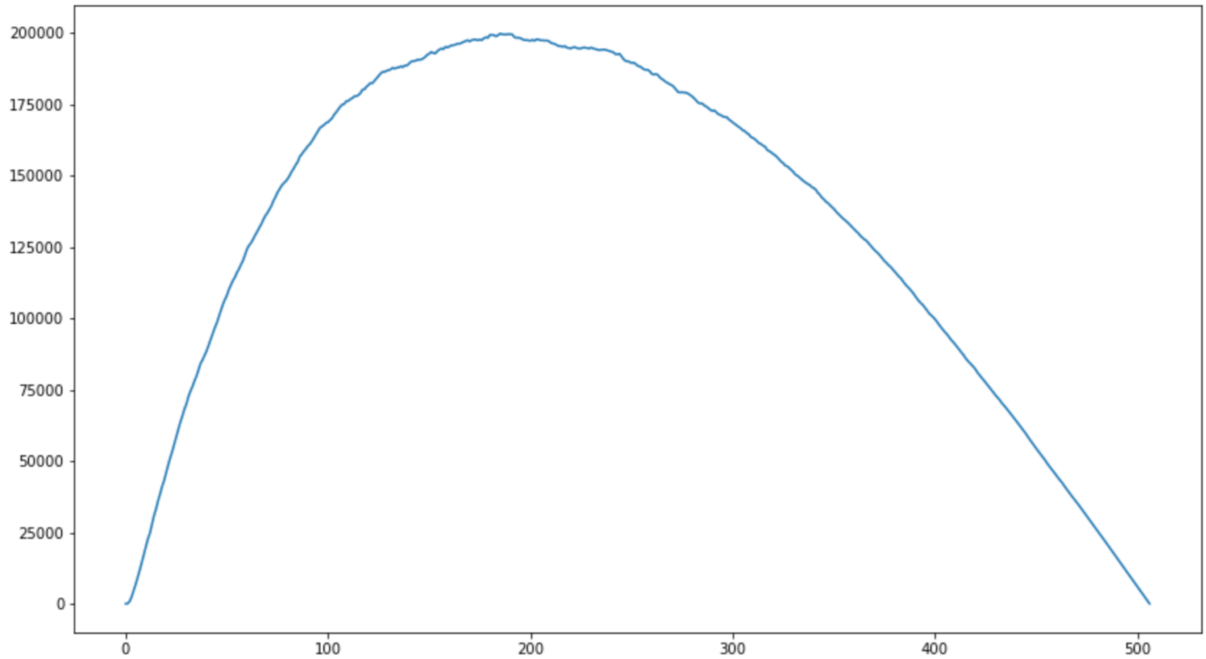

Figure 17: Exposure at default. Source: Own creation

The next step is to calculate the Effective EPE, which as recalled is calculated as a weighted average of maximum expected exposure values over 8 periods, each corresponding to 1 quarter. The 8 quarterly max values are seen in the below table:

	Expected_Exposure	EPE
0	96520.070279	96520.070279
1	131944.456525	131944.456525
2	139024.519955	139024.519955
3	138821.397065	138821.397065
4	130535.146386	130535.146386
5	111330.793825	111330.793825
6	82590.808520	82590.808520
7	45456.314505	45456.314505

Table 5: Quarterly maximum EE values. Source: Own creation

It can be seen that the max values are taken from the expected exposure of the uncollateralized swap as the exposure paths used in the calculation of EAD are also uncollateralized. The use of the collateralized exposure should yield the same alpha multiplier, if one calculated collateral calls for all exposure paths. Using the above max values, the Effective EPE is calculated to be 109,527.8384, which can be seen relative to the expected exposure of the uncollateralized swap in the following graph:

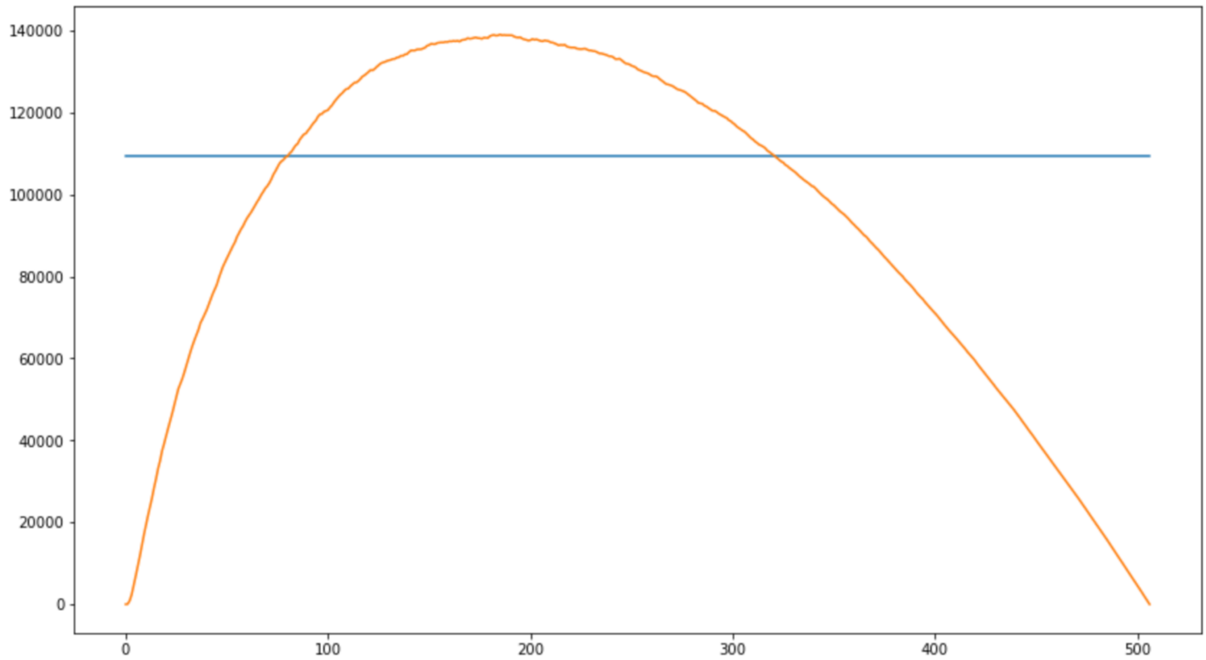

Figure 18: EEPE and EE. Source: Own creation

Now the components are ready for the calculation of the alpha multiplier, which is calculated for each day. This daily alpha will then be used to calculate a mean of all alpha multipliers, which will be used as the modelled alpha. The reason for this is that if not then the only periods that will yield an alpha over 1.2 will be the periods in which the EAD is larger than the EEPE.

The average alpha multiplier value calculated as a mean of daily alpha values are **1.4390**. This means that all expected exposure values in a GWWR scenario will increase by 43.9% compared to the normal expected exposure. The effects of this will be seen in the following section.

5. Credit Value Adjustment

This section will display the calculation of CVA both with and without GWWR for both the collateralized and the uncollateralized swap between Bank of America and JPMorgan Chase. Lastly, this section will compare the results and initial conclusions will be drawn. All calculations in this section has been performed using the Python code seen in appendix 10.

5.1 Calculation of CVA for Swaps without GWWR

This sub-section will calculate the CVA estimate for both swaps without the presence of GWWR. This subsection will compare the CVA estimate of both swaps, thus seeing the effect of collateral a scenario not trying to incorporate the risk of financial recessions.

Since this CVA estimate assumes that GWWR does not exist, the formula used will be the classic CVA formula:

$$CVA = LGD \sum_{i=1}^{m} EE(t_i) * PD(t_{i-1}, t_i)$$

Here the only difference between the two swaps will be the expected exposure, which will be collateralized for one swap and uncollateralized for the other swap.

When calculating the CVA estimate it is normal to summarize the value over all periods. This thesis will do the same. Hence the sum of all daily CVA estimates and its value in percent relative to the notional for both the collateralized and the uncollateralized swap can be seen in the below table:

Swap portfolio without GWWR		
	Swap 1	Swap 2
CVA	36,323.16	76,938.23
% of Notional	0.0363%	0.0769%

Table 6: CVA results without GWWR. Source: Own creation

The table above showcases swap 1 (collateralized) and swap 2 (uncollateralized). It can be seen in the above results that CVA has been reduced by nearly 53% through the use of collateral. This is a substantial reduction in the CVA estimate, which definitely backs up the claim that collateral is a very efficient mitigation method for counterparty risk.

5.2 Calculation of CVA for Swaps with GWWR

This sub-section will calculate the CVA estimate for both interest rate swaps between Bank of America and JPMorgan Chase under the assumption of GWWR. Much like the previous sub-section, the CVA estimates will be compared to see the effectiveness of collateral in a scenario incorporating the risk of financial recessions.

From section 3, in which CVA was derived and WWR was incorporated, the following formula was presented:

$$CVA = LGD \sum_{i=1}^{m} EE(t_i|\mathrm{t_i} = \tau_c) * PD(t_{i-1}, t_i)$$

The only difference between this CVA calculation and the previous calculation is that this calculation will use the uncollateralized expected exposure times the alpha multiplier and then calculate a new collateral call with the same CSA values.

The summarized CVA estimates for swap 1 (collateralized) and swap 2 (uncollateralized) are seen in the below table:

Swap portfolio with GWWR		
	Swap 1	Swap 2
CVA	36,323.21	110,716.45
% of Notional	0.0363%	0.1107%

Table 7: CVA results with GWWR. Source: Own creation

It can be seen that the use of collateral has a significant impact on the CVA estimate in this model. In fact, collateral is able to reduce the CVA estimate with above 67% of the total value. This appears to be a bit excessive, and the reason for this is might be that the GWWR model only adds GWWR to the CVA estimate through the exposure and not through any of the other CVA components. This means that at a glance it looks like collateral is very effective, but whether or not this is actually a realistic outcome will be discussed later in the empirical analysis.

5.3 Comparison of Results

The results of both the collateralized and the uncollateralized interest rate swap between Bank of America and JPMorgan Chase both with and without GWWR scenarios has been plotted in the below graph:

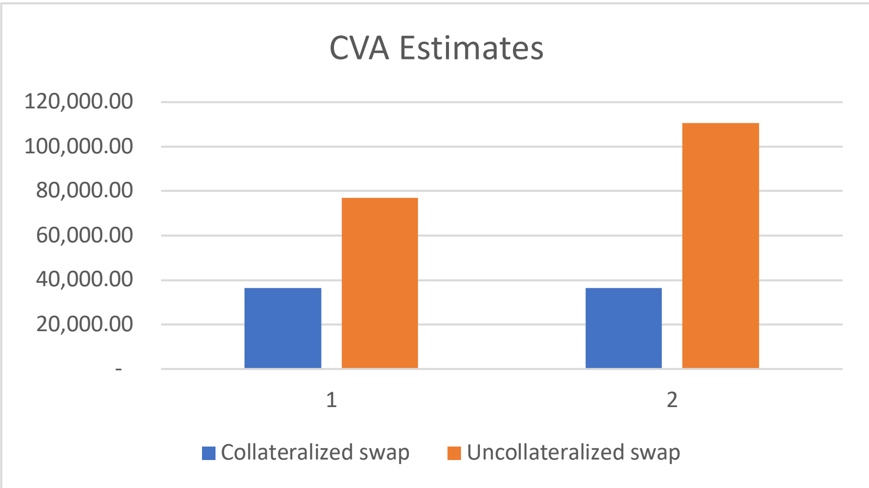

Figure 19: CVA results with/without GWWR. Source: Own creation

Market scenario 1 describes a scenario without GWWR and market scenario 2 describes a scenario with GWWR.

It can be seen that for the uncollateralized swaps the GWWR scenario is substantially higher, which is expected since the alpha multiplier has increased the expected exposure with over 43% while the remaining CVA components has remained unchanged. However, what initially seems a bit weird is that the collateralized swaps have (almost) the same CVA estimate. This means that collateral was able to remove all of the extra risk in the GWWR scenario. This initially seems really positive. However, as mentioned, the GWWR model assumes that the presence of GWWR is only displayed in the expected exposure and not actually in the PD of the counterparty. Therefore, it is only natural to expect collateral to be able to mitigate the extra risk. If, on the other hand the GWWR model had been more complex, then collateral would not have been able to remove all the extra risk, thus leading to a higher (and more realistic) CVA estimate for the collateralized swap.

The empirical analysis will seek to shed more light on this point of critique for the model, and if there is a tendency that the copula approach with the alpha multiplier is not efficient enough to capture the entirety of market risk, then this will be touched upon later in the discussion section.

6. Empirical Analysis

This section will conduct an empirical analysis with the intention of calculating an empirical CVA value that can be compared to the calculated CVA estimates of the previous section. The empirical analysis will be performed using a combination of market data and necessary assumptions. The first part of this empirical analysis section will be an introduction to the data and the assumptions. Afterwards the empirical CVA will be calculated for both of the original 2Y swaps. Lastly, the results of the empirical analysis are compared to the results of the CVA calculation from section 5. This comparison will then draw conclusions on the usability of the CVA model as well as the ability of the collateral model in a real-world scenario.

The Python script that has modelled all the calculations in this analysis can be seen in appendix 11.

6.1 Data Preparation

This section will seek to present the data used in the empirical analysis as well as the necessary assumptions made in the calculation of the empirical CVA. All data used will stretch from the settlement date of the 2Y swaps until the date of expiry.

6.1.1 Interest Rates

For this empirical analysis the actual 3M USD Libor rate will be used instead of the simulation paths used in the CVA model.

The actual 3M USD Libor rate developments during the duration of the swaps are showcased in the following chart (Federal Reserve Bank of St. Louis. (n.y.)):

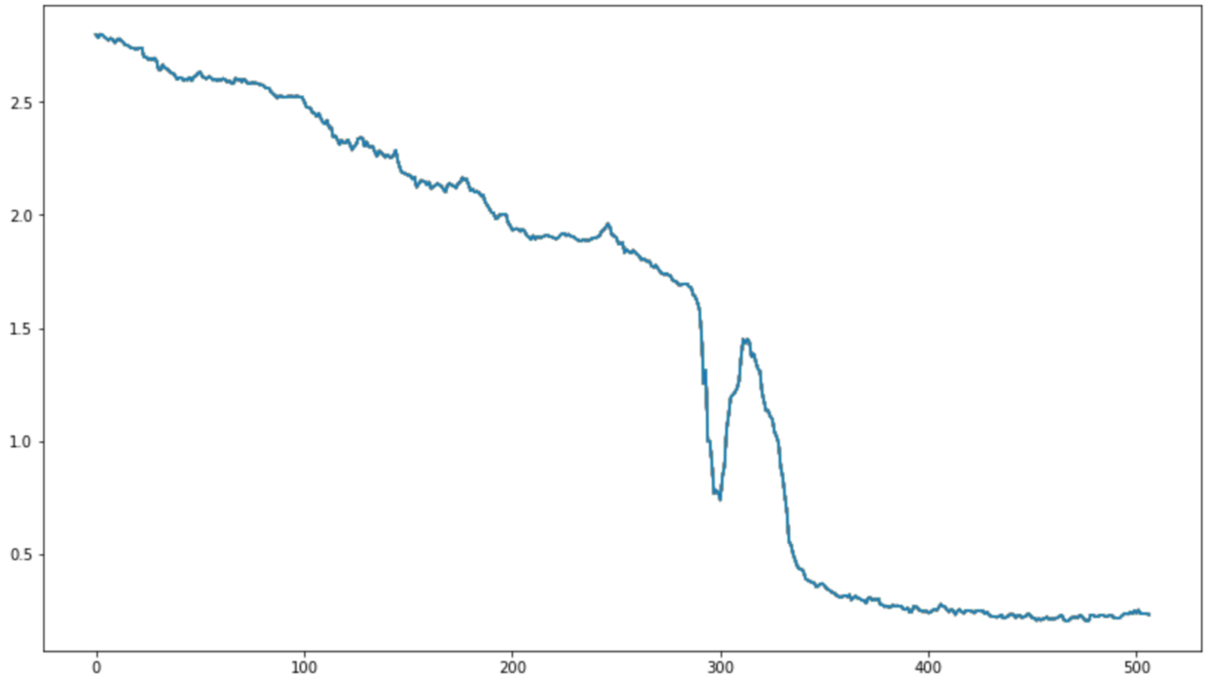

Figure 20: 3M USD Libor rates. Source: Own creation

As can be seen in the figure above, the interest rates have decreased slowly from the start date of January 6th, 2019 until February 26th, 2020 after which a period of increased market volatility ensued before the interest rate bottomed out at around 0.2-0.3%. The period of increased volatility is the initial phase of the COVID-19 Pandemic described in section 2.3. This period of increased market volatility is expected to cause some divergence between the empirical results and the CVA model, as this sharp drop in interest rates is irregular and therefore difficult to model. However, it is also expected that the GWWR model should be able to somewhat compensate for this divergence through the addition of the alpha multiplier in the CVA model.

6.1.2 Exposure

Based on the above interest rate it is now possible to calculate a single exposure path, which is the real exposure for the two swaps. The exposure is calculated as a day by day development instead of discounting back to the first day. This is done as the calculations take place after the expiry of the swaps instead of the settlement date. The exposure is based on the real 3M USD Libor rate and can be mapped in the following graph:

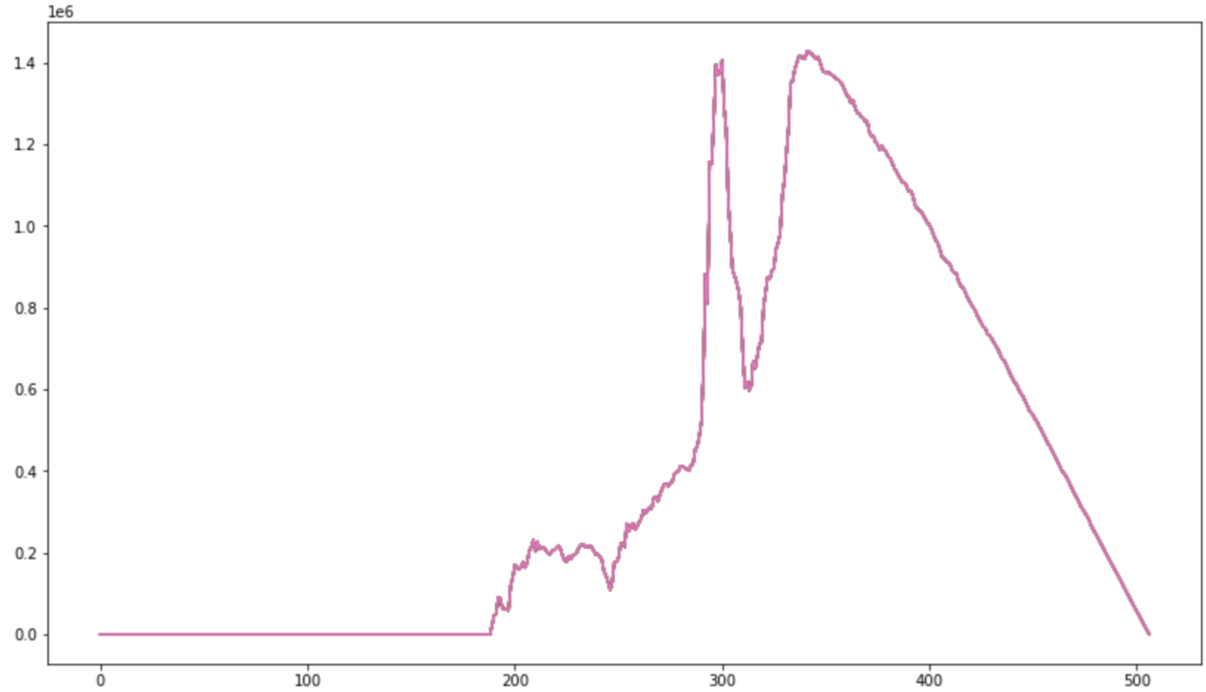

Figure 21: Empirical Exposure. Source: Own creation

As can be seen in the above figure, the swap starts OTM as the floating rate is higher than the fixed rate. Therefore, the exposure value is 0. When the floating rate falls below the fixed rate the exposure turns above 0. The COVID-19 Pandemic's influence on the interest rate swap market is clearly visible as the two major spikes in exposure.

6.1.3 Collateral

Since this empirical analysis seeks to analyze the effectiveness of collateral during a major international recession, it is detrimental that the components making up the CSA remains the same as for the previous calculations. This means that to ensure the results of the empirical analysis are actually comparable to the results of the CVA model, the collateral agreement has to be identical for the collateralized swaps in both the empirical and theoretical calculations.

When applying the identical CSA to the exposure shown in figure 21, the following collateralized exposure is calculated:

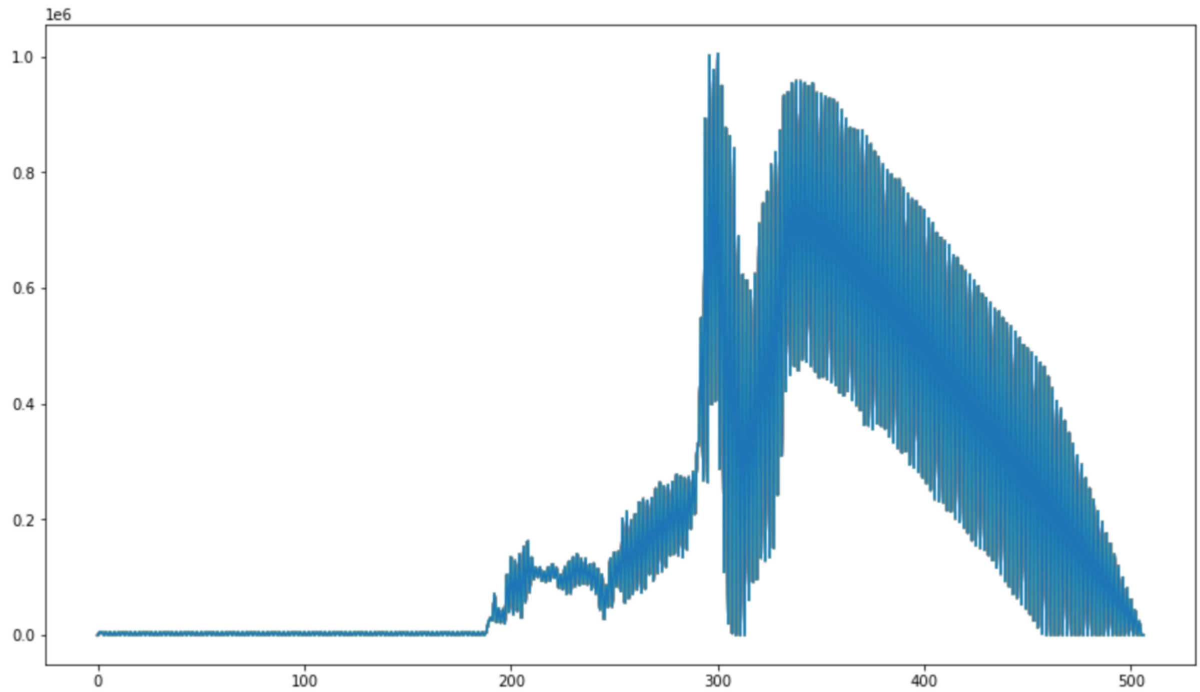

Figure 22: Collateralized exposure. Source: Own creation

As can be seen in the above figure, the collateral postings have decreased the exposure peaks substantially. The daily fluctuations appear larger, but this can be explained through the larger exposure value compared to the expected exposure from previous calculations. By comparing the empirical exposure values for both swaps with the expected exposures in section 4, it can be seen that the exposures are substantially larger in the empirical analysis. This divergence between results can be attributed to the interest rate simulation, which generally expected much higher interest rates than the rate levels caused by COVID-19. So, while the simulated interest rates gave a sort of best-case scenario the empirical data displays a worst-case scenario.

6.1.3 LGD

As discussed earlier in this thesis, it is very difficult to model LGD, which is why the CVA model assumed a constant LGD of 62%, as this was the average LGD of senior unsecured bonds. Since there was no additional market data to be had to contradict this assumption, and since it would be too complex a task to model an LGD value, LGD will still be assumed to have a constant value of 62%. This will ensure that the only factors that can create differences between the empirical CVA and the CVA model will be the interest rate movements and the probability of default.

6.1.4 Probability of Default

The probability of default in the empirical analysis will be based on market data, but not in the same fashion as for the CVA model. The CVA model used a CDS curve to calculate a future expected marginal PD. The empirical analysis will calculate the marginal PD based on actual CDS spreads covering the duration of the swaps.

The actual data used are week-by-week CDS spreads that has then been interpolated using the hermite spline method to get daily spreads (much like in the PD calculation in the CVA model). The CDS spread data available for JPMorgan Chase was 5-year CDS spreads, which does not align with the 2-year runtime of the swaps, but nevertheless it is still viable market data as it should be able to capture the COVID-19 Pandemic, which is expected to create a divergence in results. The CDS curve is represented in the following chart:

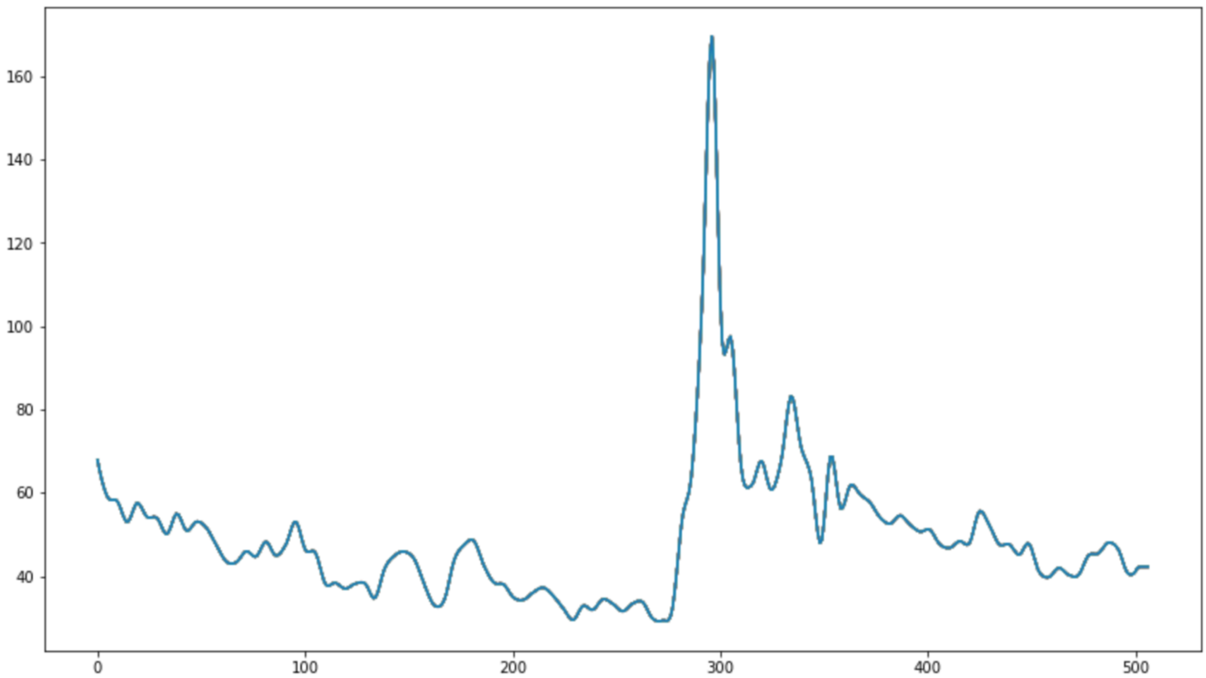

Figure 23: JPM CDS spreads. Source: Own creation

As can be seen in the above figure, the COVID-19 Pandemic led to an extreme increase in CDS spreads from a low of 29.2997 to a high of 167.5042. As discussed in section 2.3, the crisis led to a drying up in liquidity[15] before central bank intervention, which led to credit hedging strategies being unable to function properly. This drying up in liquidity is seen as the extreme spike in CDS spreads. This should lead to yet another reason for divergence between the CVA

[15] especially in credit markets

model and the empirical results as the CDS curve had a much slower, gradual increase over time.

The CDS spreads have been used to calculate the marginal PD using the same formula as for the CVA model:

$$PD(t_1, t_2) = \left(1 - \exp\left(-\frac{s}{LGD} t_2\right)\right) - \left(1 - \exp\left(-\frac{s}{LGD} t_1\right)\right)$$

This calculation has led to the following graph showcasing the marginal PD for the counterparty, JPMorgan Chase:

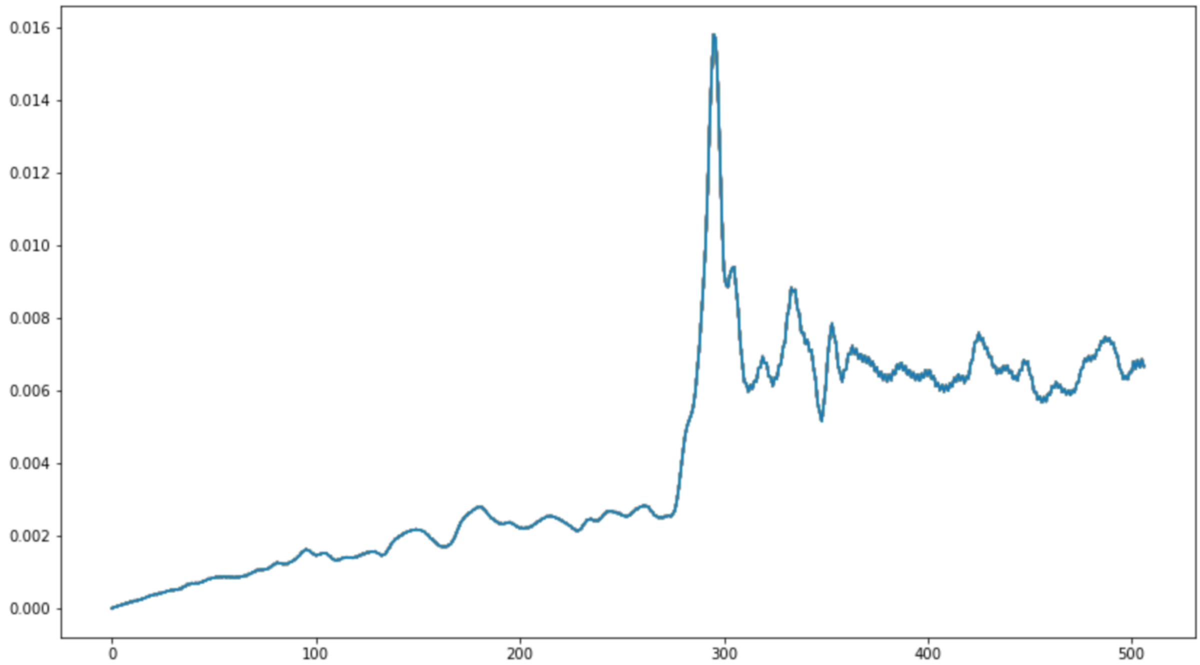

Figure 24: JPM marginal PD. Source: Own creation

As expected, the CDS spike during the COVID-19 Pandemic has led to a max value of marginal PD of 1.5676%. This is well above the expected 2-year marginal PD, which only reached a max value of 0.5531%.

6.1.5 GWWR

In the COVID-19 Pandemic section of this thesis, it was explained how GWWR was expected to be present for receiver swaps due to a combination of: 1) decreases in interest rates leading to swaps moving ITM and thereby also increasing exposures and 2) decreased credit quality of counterparties due to economic losses that showcased itself in widening CDS spreads.

To check whether or not this expected trend is true, and thereby if this is a case of GWWR the interest rate developments can be compared to the change in PD for JPM. This comparison is displayed graphically here:

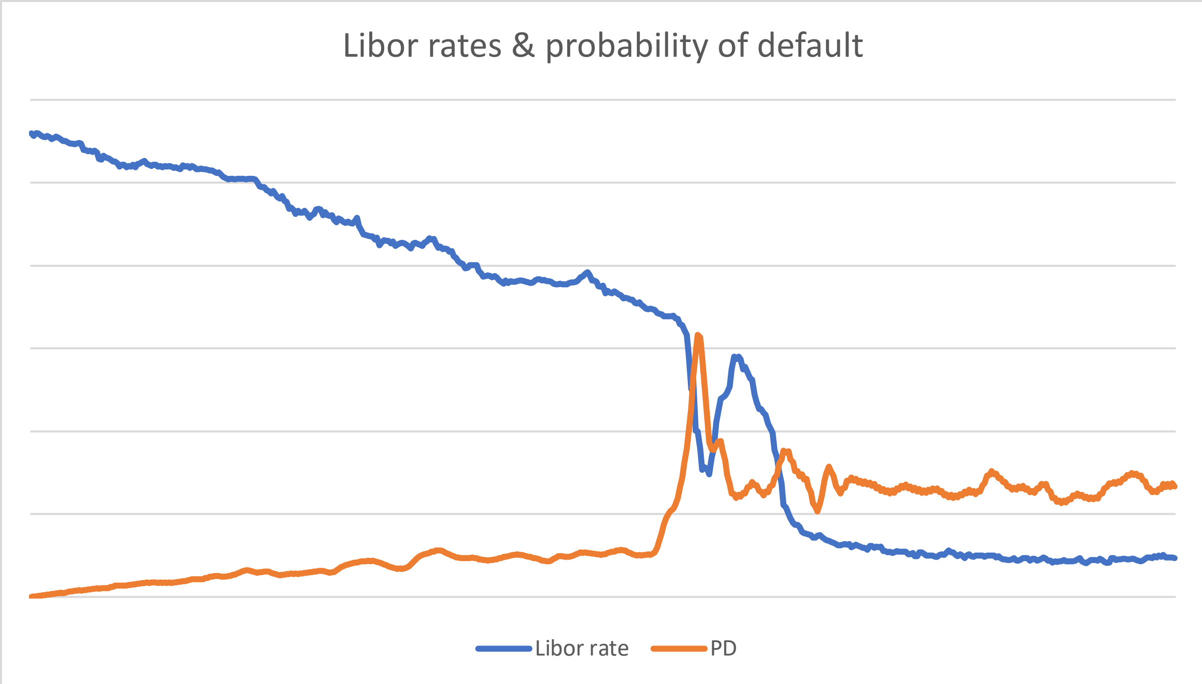

Figure 25: PD for JPM & 3M Libor. Source: Own creation

When looking at the above figure, there appears to be a relatively consistent correlation between the movement of the CDS spreads and the movements in the interest rate during COVID-19. It can be seen that during the volatile period of the COVID-19 Pandemic, interest rates decreased at the same time as a rapid increase in PD began. Afterwards as interest rates remained low the probability of default of JPM remained at an elevated level.

Basically, the tendency seen in the above graph means that when interest rates decreased, and the receiver swaps moved deep ITM the credit quality of the counterparty (JPMorgan Chase) decreased. Thereby, it can be assumed that GWWR was present in the market during the COVID-19 Pandemic for the interest rate receiver swaps in this thesis.

This assumption that was first introduced in the COVID-19 Pandemic section and now described using data means that the premise this thesis has been built on is realistic.

6.2 Empirical CVA Calculation

The calculation of the empirical CVA value will be based around the same formula as for the CVA model:

$$CVA = LGD \sum_{i=1}^{m} EE(t_i | \mathrm{t_i} = \tau_c) * PD(t_{i-1}, t_i)$$

This is done as to ensure the ability of comparison between the empirical analysis and the CVA model. This does in practice mean that the calculation will be carried out in the exact same way (of course without the alpha multiplier as it was proved that GWWR is present in the market).

By using the empirically calculated exposures as well as PD and the assumed LGD, it is then possible to calculate the empirical CVA for both the uncollateralized and the collateralized swaps, which have been showcased in the below table:

Empirical swap portfolio		
	Swap 1	**Swap 2**
CVA	404,773.38	809,130.19
% of Notional	0.4048%	0.8091%

Table 8: Empirical CVA results. Source: Own creation

As can be seen the CVA values are very large at 0.8091% of notional for the uncollateralized swap and 0.4048% for the collateralized swap. This is of course driven by the exposure as well as the PD. It was seen in figure 24 that the COVID-19 Pandemic led to a huge spike in PD for JPMorgan Chase. Also, the sudden rate cuts led to a large increase in exposure. It is seen that the collateral postings were able to mitigate 49.9% of the empirical CVA. This is great, as that means it has been able to greatly reduce the risk, however, elevated CVA levels are still seen even with the collateral postings.

6.3 Conclusion: Differences between Empirical and Forecasted Outcomes

CVA estimates both with and without GWWR has been calculated and can now be compared to the empirical CVA values. The comparison can be seen graphically in the figure below:

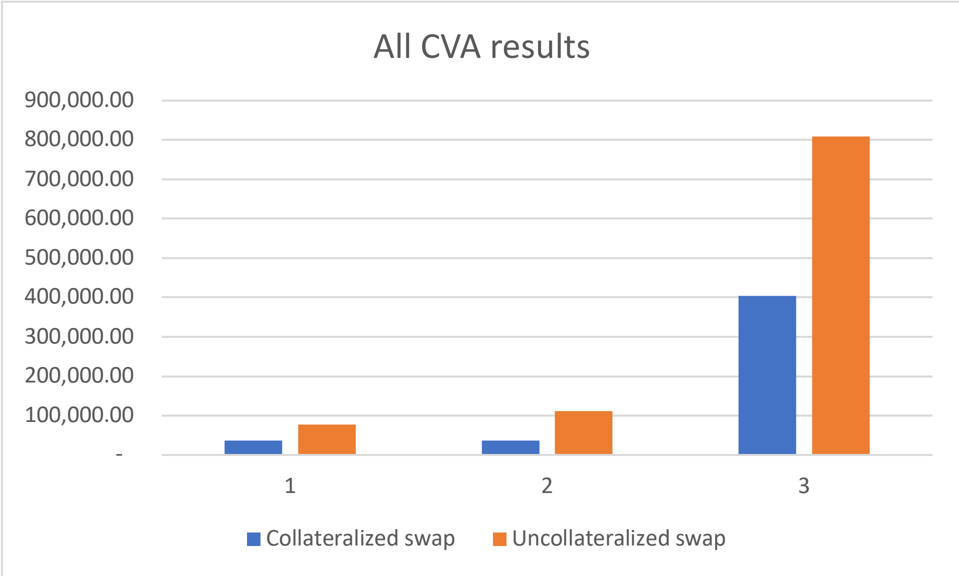

Figure 26: All CVA results. Source: Own creation

The above figure displays 1) the two swaps without GWWR, 2) the two swaps with GWWR and 3) the two swaps in the empirical analysis.

It can be seen that there is a very large divergence between the empirical CVA values and the CVA estimates (both with and without GWWR). This is of course sad to see, as it means the CVA model with GWWR was not able to compensate for the volatility of the COVID-19 Pandemic. There are multiple assumed reasons behind this:

1) The interest rate simulations are biased towards rate increases

The Vasicek model used historical Libor rates to create the mean-reversion component, which the simulations would deviate around. The mean-reversion component was in general biased towards higher rates, which meant the model already from the start predicted wrong. To compensate for this, one could have used a more restricted dataset with a larger emphasis on lower rates. One could also have made a weighted average, which applied more weight on recent interest rate developments.

Since the interest rate simulation were biased to predict OTM exposures, these results will have influenced the expected exposure to end up with a lower value.

2) **The enormous spike in CDS spreads for JPMorgan Chase during the COVID-19 Pandemic**

During the initial phase of COVID-19, the CDS spreads widened by an enormous amount due to financial uncertainty as well as the lack of liquidity. This enormous spike meant that for certain periods the PD of JPMorgan Chase was nearly triple the PD that was calculated in the CVA model. This has of course had a major impact in the elevated CVA levels in the empirical analysis.

3) **COVID-19 was a unique crisis**

Based on the COVID-19 related economic data from section 2.3, it was seen that this crisis was very unique in that it was characterized by a lot of uncertainty as well as very bleak outlooks for the world economy. The very nature of this crisis was after all the primary reason some experts deemed this the biggest test of market liquidity since the great recession. This means that it is natural to expect some level of divergence between the CVA estimates and the empirical calculations. After all, what is the likelihood that a model can capture the risk of a very unique and severe financial recession?

However, even though the CVA results are much higher than the CVA estimates, it can still be seen that the use of collateral mitigated 49.9% of the empirical CVA. This is almost as high as the regular CVA model without GWWR that mitigated just above 52%. The mitigation percentage of the CVA model with GWWR was over 67%, but this is deemed unrealistic as the main driver behind this is that GWWR was added as an additional layer on the expected exposure.

Thus, it can be concluded that the CVA model both with and without GWWR was not particularly accurate in predicting and compensating for the economic fallout of the COVID-19 Pandemic. However, it can be seen that collateral functioned well in mitigating counterparty risk both in the model and in the empirical analysis.

7. Discussion

This section will discuss both points of critique as well as ideas for future research relevant to the thesis.

7.1 Points of Critique

This section will focus on discussing the thesis and its eventual pitfalls. Here the focus will be on discussing the usability of collateral to mitigate counterparty risk. Furthermore, the GWWR model and its usability in this thesis will be discussed. Lastly, this section will discuss some of the potential issues related to the runtime of the swap in relation to the empirical analysis and its results.

7.1.1 Collateral in Relation to Probability of Default

It has been seen in both the results from the CVA model as well as the results of the empirical analysis that collateral functions very well at reducing the exposure of a position. For the CVA model it was seen how CVA without GWWR was reduced by nearly 53% and empirical CVA was reduced by 49.9% because of the collateral posted. Therefore, one might draw the conclusion that collateral is a very efficient mitigation method of counterparty risk, but there are facets that has to be addressed.

It is at the very core of collateral that it helps to reduce CVA by mitigating the exposure of a position, which is fine in normal market scenarios, but when the market is hit by a financial recession such as the COVID-19 Pandemic then the probability of counterparty default becomes a key influence on the MtM CVA value. By looking at the empirical analysis it can be seen that PD nearly tripled during the initial market response to COVID-19. Even though collateral calls were able to reduce the empirical CVA by 49.9%, it was still substantially larger than the CVA estimates both with and without GWWR

This could indicate that collateral as a standalone mitigation method is not efficient. Perhaps collateral could be used in combination with an additional mitigation method that focused more on the mitigation of PD. Here a theoretical recommendation would be hedging through credit derivatives. However, one of the issues with the COVID-19 Pandemic was that liquidity in especially the credit derivatives market dried up, which meant that credit hedges proved inefficient. Another issue with hedging as a mitigation method that could harmonize with collateral is that, as mentioned earlier, many market participants refuse to initiate collateralized positions. Thus, a party can end up hedging a collateralized position with an uncollateralized one. This would then create the risk of asymmetry in the collateral calls.

7.1.2 Pitfall of GWWR: The Alpha Multiplier

When looking at the results of the CVA estimates with GWWR compared to the estimates without GWWR, it is seen that the collateralized CVA estimate is close to identical. This is definitely not realistic in a real market scenario, which is backed up by the results of the empirical analysis. The reason for overly effective collateral calls in the GWWR scenario is that the GWWR model only incorporates GWWR through the use of the alpha multiplier, which adds an additional value on top of the expected exposure. As could be seen in the empirical analysis, financial recessions do not only reflect in the development of the interest rates, but also in the PD of the counterparty. This simplification of GWWR means that the effect of GWWR can be removed through the use of collateral in the CVA model, but not in the empirical analysis. If one wanted to compensate for this, then one would have to incorporate GWWR in the PD of the counterparty. This will be touched upon again later in this discussion, when ideas for future research is discussed.

7.1.3 Swap Duration & The Empirical Analysis

The swaps that are at the center of both the CVA model and the empirical analysis are both 2Y swaps. This means that approximately 50% of the empirical analysis uses data from a normal market scenario and 50% from the COVID-19 Pandemic. This is an issue for the validity of the empirical analysis since it can cloud the results. This does however not mean that it is impossible to use 2Y swaps for the empirical analysis. It does however mean that it could have been prudent to build a portfolio of different swaps for this thesis. E.g. one could have analyzed a 2Y, 5Y and 10Y swap. This would also have been useful to further the analysis of GWWR scenarios across duration.

Initially, this thesis had planned to analyze a 5Y swap, as this would mean approximately 80% of the data in the empirical analysis would be from normal market conditions. However, there was not enough available data (such as CDS spreads) to be able to create an analysis of a 5Y swap. And since the market analysis made by ISDA (2020) pointed to 2Y swaps being especially hard hit by the COVID-19 Pandemic, it was still deemed relevant to analyze the 2Y swap.

7.2 Ideas for Future Research

This section will look into how the thesis could be expanded upon in case of future research. Here the focus will primarily be on technical improvements on the calculations as well as more theoretical improvements. The subjects that will be covered in this section is GWWR, collateral and CVA.

7.2.1 GWWR: The Structured Approach

One way to improve upon this thesis from a more technical point of view would be through the implementation of a more advanced GWWR model. It was discussed in section 4.3 that the copula approach, which is the model of choice for this thesis, is not particularly advanced. There are a few challenges related to the use of the copula approach, which include the extensive use of simplifying assumptions. The reason for the use of the copula approach instead of the more advanced structured approach is because of the extensive need for data as well as complex calculations. However, future research could implement the structured approach and thereby gain a better ability to model GWWR correctly, thus coming closer to a more accurate CVA model.

Future research could even take it the step further and create multiple GWWR models inspired by the new methods introduced after the great recession. This could turn the research in the direction of an exercise in analyzing and comparing the effectiveness of WWR models. The first step to do this would be to gain access to the necessary data, which would be able to link counterparty default to market movement. Also, one would need to gain a broad knowledge of market practice in WWR modelling – preferably through discussion with personnel from a CVA-desk in a bank of some size or through new research papers on the subject.

7.2.2 Collateral: Operational Cost and Funding Costs

One of the main simplifications made for the collateral model was the exclusion of operational cost and funding costs. The exclusion of the two main sources of cost related to collateral makes collateral seem as a better mitigation tool than it actually is. If a party is able to remove around

50% of their CVA without suffering any drawbacks, obviously it is an optimal solution. By introducing funding costs as well as operational cost (a cost incurred based upon the frequency of collateral calls) it would enable the thesis not just to conclude whether collateral can be used to mitigate the risks of financial recessions, but also if it is an efficient tool to do so.

By implementing operational cost and funding costs it could be possible to analyze the efficiency of collateral based upon multiple factors such as notional amount and frequency of collateral calls. Perhaps an efficient collateral model could be created based upon the notional amount and call frequency – it could already be assumed that the larger the notional amount the more frequent collateral calls should be posted. Perhaps it could even be analyzed if swaps with very small notional amounts are worth the funding costs and operational cost to collateralize. Collateral is in general the most expensive mitigation method, and this could be an interesting aspect to bring into an analysis for future research.

7.2.3 Combining Mitigation Methods

As mentioned earlier in this discussion, collateral proved an efficient tool to mitigate the exposure of a position, however it does not impact changes in PD. PD proved a real challenge in the empirical analysis, as major CVA spikes was seen during the COVID-19 Pandemic, even though the collateral model removed a substantial amount of the exposure. This indicates that collateral might be more efficient when combined with additional mitigation methods that can focus directly on PD. It would be interesting for future research to analyze optimal combinations of mitigation methods. As previously mentioned, it would be expected that hedging would prove a theoretically efficient tool to combine with collateral, but this is not necessarily true based upon the research on the COVID-19 Pandemic, which explained that a dry up in liquidity for traditional hedging tools (such as CDSs) meant that the hedges were inefficient.

Furthermore, it could be expected that the optimal combination of mitigation methods would deviate from financial recessions, as different macroeconomic factors are to blame for individual recessions (e.g. the great recession starting of as a housing crisis were as COVID-19 was a global pandemic). To put it differently, there is no guarantee that financial recessions behave the same way, so why would the same combination of mitigation methods function for all types of recessions? This viewpoint could prove very interesting to analyze as one could test mitigation methods in combination with each other to find the most efficient combination in theory, and then test these combinations with historical data built around different financial recessions. This would obviously require a very large amount of data from different time periods, and it would also require a very broad knowledge surrounding multiple mitigation methods such as collateral, netting, hedging and more.

8. Conclusion

The purpose of this thesis is to investigate whether postings of collateral is an effective mitigation tool against counterparty risk for an interest rate swap between Bank of America and JPMorgan Chase in a general wrong-way risk scenario, namely the COVID-19 crisis.

The two swaps used in this thesis are two 2Y interest rate swaps with a settlement date on the 6th of January 2019 and an expiration date on the 7th of January 2021. The notional amount of both swaps is 100 million USD. The only difference between the two swaps is that one is collateralized, and one is uncollateralized.

To analyze the effectiveness of collateral, three scenarios has been created: 1) a normal market scenario calculated from the settlement date looking forward, 2) a GWWR market scenario that includes the risk of a financial recession calculated from the settlement date looking forward and 3) an empirical analysis calculated after the expiry of the swaps that back-tests the previous results. The results are summarized in the table below:

Summarized results as % of notional			
	Uncollateralized CVA	**Collateralized CVA**	**CVA reduction**
Normal	0.0769%	0.0363%	52.7892%
GWWR	0.1107%	0.0363%	67.1926%
Empirical	0.8091%	0.4048%	49.9743%

Table 9: Summarized results as % of notional. Source: Own creation

In the above table it can be seen that both CVA calculations performed from the settlement date looking forward are a bit optimistic on the effectiveness of collateral. It is seen that the normal market scenario expects a reduction in CVA of 53% and the GWWR market scenario expects a reduction in CVA of 67%. The large expected CVA reduction in the GWWR scenario was explained earlier as a result of the very simple GWWR model that only incorporated GWWR in the expected exposure, which is the CVA component that collateral seeks to mitigate.

It can also be seen that the empirical CVA calculation performed after the expiry of the swap looking back yields a reduction of CVA of 50%, which is a very good result, and definitely proves the effect of collateralization. However, it can be seen that compared to both the normal and GWWR market scenario the CVA value as a percentage of notional is still highly elevated even after the posting of collateral. This is believed to be a result of the presence of GWWR during the COVID-19 Pandemic, as a spike in the exposure of the swaps was combined with major spikes in the PD of the counterparty. This can lead one to believe that even though collateral is a helpful tool, it might not be efficient as a standalone tool in GWWR scenarios such as the COVID-19 Pandemic.

Based on the calculations performed in this thesis, it can be concluded that collateral is a good mitigation method for counterparty risk for the analyzed interest rate swaps. However, during real GWWR scenarios, such as financial recessions, it might not be prudent to only use collateral as it only mitigates exposure and not the probability of counterparty default. Both of which increased drastically during COVID-19 as a result of the presence of GWWR.

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10. Appendices

Appendix 1: Progress of the pandemic during 2020

The following appendix has a focus on COVID-19 related events happening in especially the US, but also the EU as this is deemed most relevant to the thesis.

In the end of December, the first cases of an unknown type of pneumonia (COVID-19) appeared in the Wuhan province of China. In January the first reports of deaths at the hand of COVID-19 came in from China. At the same time the first cases began to appear outside of China – including in the US (Taylor, D. B. (2021)).

In the end of January Wuhan was put under a severe lockdown and the WHO declared a global health emergency, which led to travel bans from multiple western countries towards Chinese tourists travelling from China (Taylor, D. B. (2021)).

In February the first deaths outside of China due to Covid-19 was confirmed (including in the US), Italy experienced a surge in cases and multiple members of the EU bloc began to enter into lockdowns (Taylor, D. B. (2021)).

In March the US became the hardest hit country in the world and the CDC therefore recommended that people stopped attending gatherings of more than 50 people to contain the spread. To relieve the waning economy, president Trump signed the CARES act as nearly 10 million American workers applied for unemployment benefits (the highest number of applicants in the history of the US). Furthermore, the EU countries imposed a complete travel ban on all countries outside of the bloc (Taylor, D. B. (2021)).

In April and May the global death toll rose to over 200,000 with US death tolls rising to over 100,000. Furthermore, both the Japanese and German economies entered into official recessions (Taylor, D. B. (2021)).

June saw a bit of recovery and the EU launched their reopening plans and prepared to lift the lockdowns (Taylor, D. B. (2021)).

In July and August however, it was published that around 5 million Americans had lost their health insurance. Furthermore, the EU passed their 857 billion USD stimulus package (Taylor, D. B. (2021)).

In September and October global deaths rose above 1,000,000 and US deaths rose above 200,000 while the US unemployment rate increased to 7.9% (Taylor, D. B. (2021)).

In November the UK re-entered lockdown and the US surpassed 10,000,000 cases and 250,000 deaths (Taylor, D. B. (2021)).

In December, the year 2020 ended with the UK beginning their vaccination program against COVID-19 and US deaths rose to 300,000 while the FDA approved the Moderna vaccine (Taylor, D. B. (2021)).

Appendix 2: Global fiscal stimulus

This section will focus on the US CARES act as this is the biggest fiscal stimulus package and it is deemed most relevant for this thesis.

The CARES act injected 2 trillion USD into the private economy of the US. It was split, so 560 billion USD was reserved to individuals, 500 billion USD to major corporations, 377 billion to small businesses, 340 billion to states and local governments, 154 billion to public health and 44 billion to education and other posts. One of the most discussed posts in the CARES act was a one-time cash payment of 1,200 USD to individuals that qualified as well as a 600 USD increase in unemployment benefits per week. Furthermore, the CARES act offers up to a 13-week extension of payments for qualified individuals as well as a deferral of student loans without adding interest. This economic help was supposed to do two things: 1) relieve financial distress for households suffering due to the economic consequences of the COVID-19 Pandemic, and 2) increase consumer spending and stimulate the economy. The money reserved to relieve businesses was not a direct cash payment as for the households. Instead that money would be used as a way for businesses to secure short-term funding at low interest rates, which as mentioned earlier was one of the reasons the financial markets struggled during the initial days of the COVID-19 Pandemic (Law, T. J. (2020)). Of course, the short-term funding provided to small businesses would not directly impact the swap market, but short-term funding for larger corporations might, as these are, as previously mentioned, typical actors in the swap market.

Appendix 3: Calculation of P(t,T) factors

See attached MS Excel file.

Appendix 4: The Vasicek script

The following code has also been attached as an ipynb file with the same name as this appendix. This appendix contains screen dumps corresponding to the entire code from the ipynb file. All code has been written by the author with additional explanations (blue writing) in the code to explain certain processes.

NB: All code has been written in the Python environment, JupyterLab, using Anaconda Navigator.

```
#Import libraries - libraries are used to perform tasks in python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from matplotlib import dates
%matplotlib inline
from math import *
from pylab import *
from scipy.stats import norm
from sklearn import linear_model

#All the necessary data is downloaded
df = pd.read_csv('Quantitative Data/Test_data_2Y.csv', delimiter = ',', usecols=['Dates', 'Rates', 'Libor rates',
                                                                                  'Pct_Diff'])
sim_df = pd.read_csv('Quantitative Data/Forecasting_dates_2Y.csv', delimiter = ';', usecols=['Dates'])

#Here the components of the Vasicek model is defined
u = df['Rates'].mean() #long-term mean defined as the mean of the historical rates
stdev = df['Pct_Diff'].std() #volatility defined as the stdev of the historical rates
T = len(sim_df) #length of dataframe
dt = (1/T) #time factor

#Here the Vasicek model is created as a function
def Vasicek(sim_df, r0, K, u, stdev, N, seed=777):
    sim_df['t'] = list(range(len(sim_df)))

    #make a number of simulation colls - 25,000 chosen
    for col_idx in range(25000):
        rates = [r0] #interest rates
        sim_df['sim_{}'.format(col_idx + 1)] = 0
```

```
        #run Vasicek model for each row for each col
        for i in range(len(sim_df)):
            if i == 0:
                continue #skip first row as this rate is already given
            w = np.random.normal() #define the Wiener process / Brownian motion

            dr = K * (u - rates[-1]) * dt + stdev * w
            rates.append(rates[-1] + dr)

        sim_df['sim_{}'.format(col_idx + 1)] = rates
    return sim_df

Vasicek(sim_df, 2.7968, 0.30, u, stdev, 507)

sim_df = Vasicek(sim_df, 2.7968, 0.30, u, stdev, 507)

#Create a plot of all simulations and save it as a .png file
def create_plot(sim_df):
    plt.figure(figsize=(14,8))
    sim_df = sim_df.drop(['t'], axis = 1)
    for column_idx in sim_df:
        if column_idx != "Dates":
            plt.plot([i for i in range(len(sim_df))] , list(sim_df[column_idx]))

    plt.savefig('Monte Carlo simulations.png')
    plt.show()

create_plot(sim_df)

#Save the results as a .csv file, which can be used in the exposure calculation
sim_df.to_csv("Interest rate simulation_test.csv")
```

Appendix 5: Historical Libor rates

See attached MS Excel file.

Appendix 6: The Exposure & Collateral script

The following code has also been attached as an ipynb file with the same name as this appendix. This appendix contains screen dumps corresponding to the entire code from the ipynb file. All code has been written by the author with additional explanations (blue writing) in the code to explain certain processes.

NB: All code has been written in the Python environment, JupyterLab, using Anaconda Navigator.

```
#Import libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
from math import *
from pylab import *
from scipy.stats import norm

#Import simulated interest rates and discount factor parts and prepare dataset
df = pd.read_csv('Quantitative Data/Interest rate simulation.csv', delimiter = ',',
                 index_col = None, float_precision='round_trip')
df = df.drop(['Unnamed: 0', 'Dates'], axis = 1)

df.rename(columns={col: "exposure_{}".format(col) for col in df} , inplace = True)
df = df.drop(['exposure_t'], axis = 1)

rates_df = pd.read_csv('Quantitative Data/Interest rate simulation.csv', delimiter = ',',
                 index_col = None, float_precision='round_trip')
rates_df = rates_df.drop(['t', 'Unnamed: 0', 'Dates'], axis = 1)

time_df = pd.read_csv('Quantitative Data/Time_DF_2Y.csv', delimiter = ',',
                 index_col = None, float_precision='round_trip')

#Calculate the 'discount factor of time': P(t,T). The discount factor of time is the price of the zero-coupon bond
A = time_df['A']
B = time_df['B']

for rates_idx,col_name in enumerate(rates_df.columns):

    time_df['P_{}'.format(rates_idx+1)] = 0

    Float_r_column = (rates_df[col_name])/100
```

```
    for idx,Float_r in enumerate(Float_r_column):
        P = exp(A[idx] - B[idx] * Float_r)
        time_df.loc[idx,'P_{}'.format(rates_idx+1)] = P

#Define the necessary inputs for the exposure calculation
N = 100000000
delta = 0.5
fixed_r = 0.0204

#calculate all exposure path - one for each interest rate sim
for sim_idx,exposure_sim in enumerate(df.columns):

    sim_idx += 1

    for idx,P in enumerate(time_df[f'P_{sim_idx}']):
        sum_P = (sum(time_df.loc[idx: , f'P_{sim_idx}']) - 1) #-1 to compensate for final period, when P = 1
        if (idx == 0) or (idx == len(df) - 1):
            df.loc[idx, exposure_sim] = 0
        else:
            df.loc[idx, exposure_sim] = N * (delta * fixed_r * sum_P - (1 - P))

        #This should be greyed out, if you want to see exposures that can turn negative.
        #By enabling below code, the lower bound is set to 0
        if (df.loc[idx, exposure_sim] <= 0):
            df.loc[idx, exposure_sim] = 0

#Create a plot of all exposure paths
def create_plot(df):
    plt.figure(figsize=(14,8))
    for column_idx in df:
        plt.plot([i for i in range(len(df))] , list(df[column_idx]))

    plt.show()

create_plot(df)
```

```
#Calculate the expected exposure as a mean of all exposure paths
def expected_exposure(df):
    df['Expected_Exposure'] = [df.iloc[idx,:].mean() for idx in range(len(df))]

expected_exposure(df)

#Plot expected exposure
def plot_ee(df):
    plt.figure(figsize=(14, 8))
    plt.plot([i for i in range(len(df))] , list(df['Expected_Exposure']))
    plt.show

plot_ee(df)

#Calculate collateral calls as a function
def run_collateral_call(df):
    #Define components
    Initial_margin = 15000
    Threshold = 5000
    Minimum_transfer_amount = 5000

    df['collateral'] = 0
    df['EE_coll'] = 0

    for idx in range(len(df)):
        #first and last period collateral is set to zero
        if (idx == 0) or (idx == len(df)-1):
            df.loc[idx,'collateral'] = 0
            continue # breaks iteration and skips to the next one.
```

```python
        #if the expected exposure falls below zero it is set to zero
        if df.loc[idx,'Expected_Exposure'] < 0:
            df.loc[idx,'Expected_Exposure'] = 0
            df.loc[idx,'collateral'] = Initial_margin
        EE = df.loc[idx,'Expected_Exposure']

        #when we are in period 1 (second period) we calculate without subtracting previous periods collateral call.
        #For all other periods we subtract with previous collateral
        if (idx == 0) or (idx == 1):
            df.loc[idx,'collateral'] = max(EE + Initial_margin - (Threshold + Minimum_transfer_amount), 0)
        else:
            df.loc[idx,'collateral'] = max(EE + Initial_margin -
                                           (Threshold + Minimum_transfer_amount), 0) - df.loc[idx-1,'collateral']

        #collateralized EE is calculated below as EE minus collateral
        df.loc[idx,'EE_coll'] = EE - df.loc[idx,'collateral']

        #if the collateralized EE falls below zero, it is set to zero - collateral held is set to 0
        if df.loc[idx,'EE_coll'] < 0:
            df.loc[idx,'EE_coll'] = 0
            continue
        if df.loc[idx,'collateral'] <= 0:
            df.loc[idx,'collateral'] = 0

    return df

df = run_collateral_call(df)

#plot the collateral
def plot_coll(df):
    plt.figure(figsize=(14, 8))
    plt.plot([i for i in range(len(df))] , list(df['collateral']))
    plt.show

plot_coll(df)
```

```python
#Plot expected exposure with collateral
def plot_ee_coll(df):
    plt.figure(figsize=(14, 8))
    plt.plot([i for i in range(len(df))] , list(df['EE_coll']))
    plt.show

plot_ee_coll(df)

#Plot expected exposure with coll and without coll
def plot_ee_coll(df):
    plt.figure(figsize=(14, 8))
    plt.plot([i for i in range(len(df))] , list(df['EE_coll']))
    plt.plot([i for i in range(len(df))] , list(df['Expected_Exposure']))
    plt.show

plot_ee_coll(df)

#Gather Expected_Exposure, Initial Margin, Expected_Exposure_with_margin, collateral and EE_coll in an excel file
cva_df = pd.read_csv('Quantitative Data/PD_2Y.csv', delimiter = ',',
                     index_col = None, float_precision='round_trip')

cva_df['Expected_Exposure'] = df['Expected_Exposure']
cva_df['collateral'] = df['collateral']
cva_df['EE_coll'] = df['EE_coll']

#Save all exposure paths to a .csv file to be used in the GWWR model
cva_df.to_csv("CVA_Components.csv")

wwr_df = df.copy
wwr_df = df.drop(['Expected_Exposure', 'collateral', 'EE_coll'], axis = 1)
wwr_df.to_csv("WWR_Components.csv")
```

Appendix 7: FidInterpolate function

The code in this appendix is acquired from the FidAnalytics library used in the CBS Master's elective course: 'Fixed Income Derivatives and Risk Management for Financial Institutions'.

This code is the only code in this thesis that has not been written by the author.

```
Public Function fidInterpolate(KnownX As Variant, KnownY As Variant, OutputX As Variant, Method As String) As Variant
Dim n As Integer, j As Integer, i As Integer
KnownX = CVar(KnownX)
KnownY = CVar(KnownY)

   n = UBound(KnownX) ' Count the number of rows

If n <> UBound(KnownY) Then ' Check if KnownX and KnownY has the same # or rows.
   GoTo ErrHandler
End If

' Regardless of interpolation method, the extrapolation is set to flat (constant)
' First, we need to check for extrapolation:
If OutputX < KnownX(1, 1) Then
      fidInterpolate = KnownY(1, 1)
   ElseIf OutputX >= KnownX(n, 1) Then
      fidInterpolate = KnownY(n, 1)
   Else ' if we are not extrapolating, we need to find the
       'relevant points in the interpolation gird
      For j = 1 To n
         If KnownX(j, 1) <= OutputX Then
             i = i + 1
         Else
             i = i
         End If
      Next


   Select Case LCase(Method)
      Case "constant":
      fidInterpolate = KnownY(i, 1)

      Case "linear":
      fidInterpolate = ((OutputX - KnownX(i, 1)) * KnownY(i + 1, 1) _
               + (KnownX(i + 1, 1) - OutputX) * KnownY(i, 1)) / (KnownX(i + 1, 1) - KnownX(i, 1))


      Case "loglinear":
      fidInterpolate = KnownY(i + 1, 1) ^ ((OutputX - KnownX(i, 1)) / (KnownX(i + 1, 1) - KnownX(i, 1))) _
               * KnownY(i, 1) ^ ((KnownX(i + 1, 1) - OutputX) / (KnownX(i + 1, 1) - KnownX(i, 1)))

      Case "hermite":
      ' We need to distinguish between i=1, i=N and all other i's
      ' when calculating b(i):
      ' (Note that since we cannot use bi+1 as a variable name, we are using bk instead)
      Dim bi As Double, bk As Double, hi As Double
      Dim mi As Double, ci As Double, di As Double
      Dim K As Integer
      K = i + 1
      If i = 1 Then
         bi = ((KnownX(3, 1) + KnownX(2, 1) - 2 * KnownX(1, 1)) * (KnownY(2, 1) - KnownY(1, 1)) / (KnownX(2, 1) - KnownX(1, 1)) _
           - (KnownX(2, 1) - KnownX(1, 1)) * (KnownY(3, 1) - KnownY(2, 1)) / (KnownX(3, 1) - KnownX(2, 1))) * _
           (KnownX(3, 1) - KnownX(1, 1)) ^ -1
         bk = ((KnownX(K + 1, 1) - KnownX(K, 1)) * (KnownY(K, 1) - KnownY(K - 1, 1)) / (KnownX(K, 1) - KnownX(K - 1, 1)) _
           + (KnownX(K, 1) - KnownX(K - 1, 1)) * (KnownY(K + 1, 1) - KnownY(K, 1)) / (KnownX(K + 1, 1) - KnownX(K, 1))) _
           * (KnownX(K + 1, 1) - KnownX(K - 1, 1)) ^ -1

      ElseIf i = n - 1 Then
         bi = ((KnownX(i + 1, 1) - KnownX(i, 1)) * (KnownY(i, 1) - KnownY(i - 1, 1)) / (KnownX(i, 1) - KnownX(i - 1, 1)) _
           + (KnownX(i, 1) - KnownX(i - 1, 1)) * (KnownY(i + 1, 1) - KnownY(i, 1)) / (KnownX(i + 1, 1) - KnownX(i, 1))) _
           * (KnownX(i + 1, 1) - KnownX(i - 1, 1)) ^ -1
         bk = -((KnownX(n, 1) - KnownX(n - 1, 1)) * (KnownY(n - 1, 1) - KnownY(n - 2, 1)) / (KnownX(n - 1, 1) - KnownX(n - 2, 1)) _
          - (2 * KnownX(n, 1) - KnownX(n - 1, 1) - KnownX(n - 2, 1)) * (KnownY(n, 1) - KnownY(n - 1, 1)) / (KnownX(n, 1) - KnownX(n - 1, 1))) _
          * (KnownX(n, 1) - KnownX(n - 2, 1)) ^ -1
      Else
         bi = ((KnownX(i + 1, 1) - KnownX(i, 1)) * (KnownY(i, 1) - KnownY(i - 1, 1)) / (KnownX(i, 1) - KnownX(i - 1, 1)) _
           + (KnownX(i, 1) - KnownX(i - 1, 1)) * (KnownY(i + 1, 1) - KnownY(i, 1)) / (KnownX(i + 1, 1) - KnownX(i, 1))) _
           * (KnownX(i + 1, 1) - KnownX(i - 1, 1)) ^ -1
         bk = ((KnownX(K + 1, 1) - KnownX(K, 1)) * (KnownY(K, 1) - KnownY(K - 1, 1)) / (KnownX(K, 1) - KnownX(K - 1, 1)) _
           + (KnownX(K, 1) - KnownX(K - 1, 1)) * (KnownY(K + 1, 1) - KnownY(K, 1)) / (KnownX(K + 1, 1) - KnownX(K, 1))) _
           * (KnownX(K + 1, 1) - KnownX(K - 1, 1)) ^ -1
      End If

      hi = KnownX(i + 1, 1) - KnownX(i, 1)
      mi = (KnownY(i + 1, 1) - KnownY(i, 1)) / hi
      ci = (3 * mi - bk - 2 * bi) / hi
      di = (bk + bi - 2 * mi) * hi ^ -2
      fidInterpolate = KnownY(i, 1) + bi * (OutputX - KnownX(i, 1)) + ci * (OutputX - KnownX(i, 1)) ^ 2 + di * (OutputX - KnownX(i, 1)) ^ 3

   End Select

End If

Exit Function
ErrHandler:
   fidInterpolate = "Error: Unidentical # of rows in KnownX and KnownY"

End Function
```

Appendix 8: Probability of default calculation

See attached MS Excel file.

Appendix 9: The GWWR script

The following code has also been attached as an ipynb file with the same name as this appendix. This appendix contains screen dumps corresponding to the entire code from the ipynb file. All code has been written by the author with additional explanations (blue writing) in the code to explain certain processes.

NB: All code has been written in the Python environment, JupyterLab, using Anaconda Navigator.

```
#Import libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
from math import *
from pylab import *
from scipy.stats import norm

#Import data
df = pd.read_csv('Quantitative Data/WWR_Components.csv', delimiter = ',',
                 index_col = None, float_precision='round_trip')
df = df.drop(['Unnamed: 0'], axis = 1)

cva_df = pd.read_csv('Quantitative Data/CVA_Components.csv', delimiter = ',',
                 index_col = None, float_precision='round_trip')
cva_df = cva_df.drop(['Unnamed: 0'], axis = 1)

df_eee = cva_df.copy()
df_eee = df_eee.drop(['t', 'Dates', 'Marginal_PD', 'PD', 'collateral', 'EE_coll'], axis = 1)

#calculate summed PD for JPM. This will be used as a benchmark for default scenarios
sum_pd = sum(cva_df['PD'])
sum_pd

from tqdm.notebook import tqdm

#Simulate CWI once for each exposure (total of 25,000 simulations)
#Correlation between credit risk factor and macroeconomic/market risk is set at 0.75
rho = 0.75
```

```
#Calculate EAD for all simulations that are left.
df['EAD'] = sum(df, axis = 1)/df.shape[1]

#Plot EAD
def plot_ead(df):
    plt.figure(figsize=(14, 8))
    plt.plot([i for i in range(len(df_eee))] , list(df['EAD']))
    plt.show

plot_ead(df)

#Here the boundaries for EPE values are created. 8 boundaries are created (1 for each quarter)
#Also EPE is calculated here using the Expected Exposure
df_eee_2_years = df_eee.drop(range(len(df_eee)))

year_boundaries = [(len(df_eee)//8)*i for i in range(1,9)]
print("year_boundaries: ", year_boundaries)

for idx,upper_bound in enumerate(year_boundaries):
    if idx == 0:
        lower_bound = -1
    else:
        lower_bound = year_boundaries[idx-1]
    series = df_eee.loc[lower_bound +1 : upper_bound, :].reset_index(drop=True) #this code takes from start index,
                                                                                #up to and including end index
    lower_bound += 1  # fixing the +1 in the series

    print(f"idx: {idx}  lower_bound_idx: {lower_bound}")
    print(f"idx: {idx}  upper_bound_idx: {upper_bound}")
    print("")
    df_eee_2_years = df_eee_2_years.append( series.agg(['max']), ignore_index = True)

df_eee_2_years['EPE'] = df_eee_2_years.iloc[:, :].mean(axis=1)
```

```
for col_idx in tqdm(range(25000)):
    col_idx = col_idx%25000
    Z = list(norm.ppf(np.random.rand(507))) #Macroeconomic risk factor
    epsilon = list(norm.ppf(np.random.rand(507))) #Specific counterparty credit risk factor

    df['CWI_{}'.format(col_idx+1)] = 0
    for idx in range(len(cva_df)):
        df.loc[idx,'CWI_{}'.format(col_idx+1)] = (rho * Z[idx] + math.sqrt(1 - rho**2) * epsilon[idx])

#Set simulations with CWI > sum(PD) to 0 and simulations with CWI <= sum(PD) to 1
#Simulations with a value of 1 are default scenarios
for col in df.columns:
    if "CWI" in col:
        if sum(df[col]) <= sum_pd:
            pass
        else:
            sim_num = col.replace("CWI_", "")
            df[f"exposure_sim_{sim_num}"] = 0
        df.drop(columns = [col] , inplace = True) #use the inplace parameter to change the dataframe object,
                                                  #rather than returning a new dataframe.
# drop all columns where sum = 0
for col in df.columns:
    if sum(df[col]) == 0:
        df.drop(columns= [col], inplace = True)

#remove exposures that are deeply OTM or not that much ITM for the party
#it is assumed that the counterparty should not be at risk of default here
for col in df.columns:
    if sum(df[col]) < 25000000:
        df.drop(columns= [col], inplace = True)

#Adjust all exposures with LGD as per the EAD-formula
LGD = 0.62
df = df*LGD
```

```
#Calculate Effective EPE as a mean of quarterly EPE values
df_eee['EEPE'] = (96520.070279 + 131944.456525 + 139024.519955 + 138821.397065
                  + 130535.146386 + 111330.793825 + 82590.808520 + 45456.314505) / 8

#Plot EEPE
def plot_epe(df_eee):
    plt.figure(figsize=(14, 8))
    plt.plot([i for i in range(len(df_eee))] , list(df_eee['EEPE']))
    plt.plot([i for i in range(len(df_eee))] , list(df_eee['Expected_Exposure']))
    plt.show

plot_epe(df_eee)

#Calculate daily alpha values
df['alpha'] = df['EAD'] / df_eee['EEPE']

for idx,alpha in enumerate(df['alpha'].copy()):
     if alpha < 1.2:
         df.loc[idx,'alpha'] = 1.2

#Calculate alpha as a mean of daily values.
#Insert alpha multiplier in CVA_Components_with_alpha.csv and save to drive
cva_df['alpha'] = df['alpha'].mean()
cva_df.to_csv("CVA_Components_with_alpha.csv")
```

Appendix 10: The CVA script

The following code has also been attached as an ipynb file with the same name as this appendix. This appendix contains screen dumps corresponding to the entire code from the ipynb file. All code has been written by the author with additional explanations (blue writing) in the code to explain certain processes.

NB: All code has been written in the Python environment, JupyterLab, using Anaconda Navigator.

```
#Import libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
from math import *
from pylab import *
from scipy.stats import norm

#Import data
df = pd.read_csv('Quantitative Data/CVA_Components_with_alpha.csv', delimiter = ',',
                 index_col = None, float_precision='round_trip')
df = df.drop(['Unnamed: 0'], axis = 1)

disc_df = pd.read_csv('Quantitative Data/disc_factor.csv', delimiter = ',',
                 index_col = None, float_precision='round_trip')

#Define the CVA components
EE = df['Expected_Exposure']
EE_coll = df['EE_coll']
PD = df['PD']
LGD = 0.62
disc_factor = disc_df['Disc factor']

#Calculate CVA without GWWR - both without and with collateral
df['CVA_uncoll'] = 0
for idx in range(len(df['CVA_uncoll'])):
    df.loc[idx, 'CVA_uncoll'] = LGD * ((EE[idx] * disc_factor[idx]) * PD[idx])

df['CVA_coll'] = 0
for idx in range(len(df['CVA_coll'])):
    df.loc[idx, 'CVA_coll'] = LGD * ((EE_coll[idx] * disc_factor[idx]) * PD[idx])
```

```python
#Calculate a new EE + EE_coll with GWWR
#Here calculate EE_GWWR as EE times alpha
EE = df['Expected_Exposure']
alpha = df['alpha']

df['EE_GWWR'] = 0
for idx in range(len(df['EE_GWWR'])):
    df.loc[idx, 'EE_GWWR'] = EE[idx] * alpha[idx]

#Make a new collateral call to calculate a new EE_coll
#Use the new values in the below calculations
def run_collateral_call(df):
    #Define the same CSA factors as earlier
    Initial_margin = 15000
    Threshold = 5000
    Minimum_transfer_amount = 5000

    df['collateral_GWWR'] = 0
    df['EE_GWWR_coll'] = 0

    for idx in range(len(df)):
        #first and last period collateral is set to zero
        if (idx == 0) or (idx == len(df)-1):
            df.loc[idx,'collateral_GWWR'] = 0
            continue # breaks iteration and skips to the next one.

        #if the expected exposure falls below zero it is set to zero
        if df.loc[idx,'EE_GWWR'] < 0:
            df.loc[idx,'EE_GWWR'] = 0
            df.loc[idx,'collateral_GWWR'] = Initial_margin
        EE = df.loc[idx,'EE_GWWR']
```

```python
        #when in period 1 (second period) calculate without subtracting previous periods collateral call.
        #For all other periods we subtract with previous collateral
        if (idx == 0) or (idx == 1):
            df.loc[idx,'collateral_GWWR'] = max(EE + Initial_margin - (Threshold + Minimum_transfer_amount), 0)
        else:
            df.loc[idx,'collateral_GWWR'] = max(EE + Initial_margin - (Threshold + Minimum_transfer_amount), 0)
                                                     - df.loc[idx-1,'collateral']

        #Define collateralized EE as EE minus collateral
        df.loc[idx,'EE_GWWR_coll'] = EE - df.loc[idx,'collateral_GWWR']
        #if the collateralized EE falls below zero, it is set to zero - so is collateral held
        if df.loc[idx,'EE_GWWR_coll'] < 0:
            df.loc[idx,'EE_GWWR_coll'] = 0
            continue
        if df.loc[idx,'collateral_GWWR'] <= 0:
            df.loc[idx,'collateral_GWWR'] = 0

    return df

run_collateral_call(df)

#Define CVA components with GWWR
EE = df['EE_GWWR']
EE_coll = df['EE_GWWR_coll']
PD = df['PD']
LGD = 0.62
```

```python
#Calculate CVA with GWWR and both with/without collateral
df['CVA_uncoll_GWWR'] = 0
for idx in range(len(df['CVA_uncoll_GWWR'])):
    df.loc[idx, 'CVA_uncoll_GWWR'] = LGD * ((EE[idx] * disc_factor[idx]) * PD[idx])

df['CVA_coll_GWWR'] = 0
for idx in range(len(df['CVA_coll_GWWR'])):
    df.loc[idx, 'CVA_coll_GWWR'] = LGD * ((EE_coll[idx] * disc_factor[idx]) * PD[idx])

#Plot all cva values
cva_df = df
cva_df = cva_df.drop(['t', 'Dates', 'Marginal_PD', 'PD', 'Expected_Exposure',
                      'collateral', 'EE_coll', 'alpha', 'EE_GWWR', 'collateral_GWWR', 'EE_GWWR_coll'], axis = 1)
def create_plot(cva_df):
    plt.figure(figsize=(14,8))
    for column_idx in cva_df:
            plt.plot([i for i in range(len(cva_df))] , list(cva_df[column_idx]))

    plt.show()

create_plot(cva_df)

#Sum all CVA values
sum_cva_uncoll = sum(df['CVA_uncoll'])
sum_cva_coll = sum(df['CVA_coll'])
sum_cva_gwwr_uncoll = sum(df['CVA_uncoll_GWWR'])
sum_cva_gwwr_coll = sum(df['CVA_coll_GWWR'])

#Save to Excel
df.to_excel("CVA_results.xlsx")
cva_df.to_csv("CVA_results.csv")
```

Appendix 11: The empirical analysis script

The following code has also been attached as an ipynb file with the same name as this appendix. This appendix contains screen dumps corresponding to the entire code from the ipynb file. All code has been written by the author with additional explanations (blue writing) in the code to explain certain processes.

NB: All code has been written in the Python environment, JupyterLab, using Anaconda Navigator.

```
#Import libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
from math import *
from pylab import *
from scipy.stats import norm

#All the necessary data is downloaded
df = pd.read_csv('Quantitative Data/Empirical_Analysis_Inputs_2Y.csv', delimiter = ',',
                 index_col = None, float_precision='round_trip')

cva_df = pd.read_csv('Quantitative Data/CVA_results.csv', delimiter = ',',
                 index_col = None, float_precision='round_trip')

time_df = pd.read_csv('Quantitative Data/Time_df_2Y.csv', delimiter = ',',
                 index_col = None, float_precision='round_trip')

disc_df = pd.read_csv('Quantitative Data/disc_factor.csv', delimiter = ',',
                 index_col = None, float_precision='round_trip')

rates_df = df['Rates']

#Here the empirical exposure is calculated based on the historical libor rate
df['Exposure'] = 0

N = 100000000 #USD

fixed_r = 0.0204
delta = 0.5 #6M
P = df['P']
```

```
for idx in range(len(df['Exposure'])):

    sum_P = (sum(df.loc[idx: , 'P']) - 1)
    if (idx == 0) or (idx == len(df) - 1):
        df.loc[idx, "Exposure"] = 0
    else:
        df.loc[idx, "Exposure"] = max(N * ((delta * fixed_r * sum_P) - (delta * (df['Rates'][idx]/100) * sum_P)),0)

    if (df.loc[idx, "Exposure"] <= 0):
        df.loc[idx, "Exposure"] = 0

#Calculate empirical CVA without collateral
LGD = 0.62
EE = df['Exposure']
PD = df['PD_JPM']
disc_factor = disc_df['Disc factor']

df['CVA_historical_uncoll'] = 0
for idx in range(len(df['CVA_historical_uncoll'])):
    df.loc[idx, 'CVA_historical_uncoll'] = LGD * ((EE[idx]*disc_factor[idx]) * PD[idx])

#Collateral calls are calculated the same way as for the CVA model
#Collaterealized exposure is also calculated
def run_collateral_call(df):
    Initial_margin = 15000
    Threshold = 5000
    Minimum_transfer_amount = 5000

    df['collateral'] = 0
    df['EE_coll'] = 0

    for idx in range(len(df)):
```

```python
        #first and last period collateral is set to zero
        if (idx == 0) or (idx == len(df)-1):
            df.loc[idx,'collateral'] = 0
            continue # breaks iteration and skips to the next one.

        #if the expected exposure falls below zero it is set to zero
        if df.loc[idx,'Exposure'] < 0:
            df.loc[idx,'Exposure'] = 0
            df.loc[idx,'collateral'] = Initial_margin
        EE = df.loc[idx,'Exposure']

        #when we are in period 1 (second period) we calculate without subtracting previous periods collateral call.
        #For all other periods we subtract with previous collateral
        if (idx == 0) or (idx == 1):
            df.loc[idx,'collateral'] = max(EE + Initial_margin - (Threshold + Minimum_transfer_amount), 0)
        else:
            df.loc[idx,'collateral'] = max(EE + Initial_margin - (Threshold + Minimum_transfer_amount), 0)
            - df.loc[idx-1,'collateral']

        df.loc[idx,'EE_coll'] = EE - df.loc[idx,'collateral']
        #if the collateralized EE falls below zero, it is set to zero - collateral held is set to 0
        if df.loc[idx,'EE_coll'] < 0:
            df.loc[idx,'EE_coll'] = 0
            continue
        if df.loc[idx,'collateral'] <= 0:
            df.loc[idx,'collateral'] = 0

    return df

df = run_collateral_call(df)

#Collateralized empirical CVA is calculated
```

```python
LGD = 0.62
EE = df['EE_coll']
PD = df['PD_JPM']
disc_factor = disc_df['Disc factor']

df['CVA_historical_coll'] = 0
for idx in range(len(df['CVA_historical_coll'])):
    df.loc[idx, 'CVA_historical_coll'] = LGD * ((EE[idx]*disc_factor[idx]) * PD[idx])

#Save to Excel
df.to_excel("Empirical_results.xlsx")
```

Appendix 12: Bloomberg: CDS data for JPM & BAC

CONTRIBUTED CDS SPREADS

1 Enter the corporate ticker, currency and debt type to search.
Corporate Ticker: BAC Currency:USD Debt Type:1 99) Search

1 Contributed CDS Spreads 98) RED Info

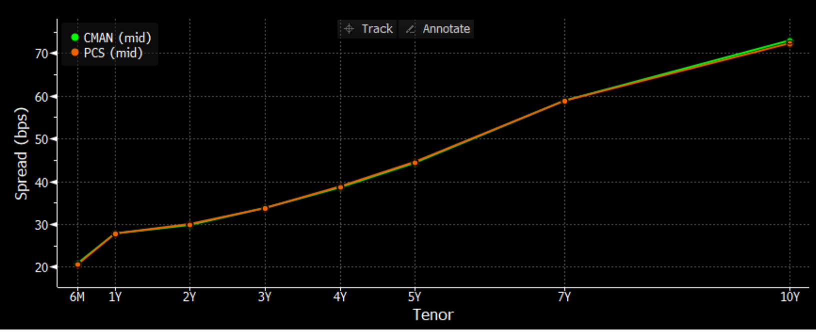
	Term	Ticker	Contributor	Bid (bps)	Ask	Update Time
2)	6 mo	CBAC1UF	CBIN	17.394	23.608	10:01
3)	1 yr	CBAC1U1	CBIN	22.721	32.672	10:01
4)	2 yr	CBAC1U2	CBIN	25.122	34.841	10:01
5)	3 yr	CBAC1U3	CBIN	29.146	38.422	10:01
6)	4 yr	CBAC1U4	CBIN	35.037	42.354	10:01
7)	5 yr	CBAC1U5	CBIN	42.271	46.807	10:01
8)	7 yr	CBAC1U7	CBIN	51.988	65.645	10:01
9)	10 yr	CBAC1U10	CBIN	65.188	79.499	10:01

Day Count: ACT/360
Frequency: Q
Contributor Preferences: CBBT MSG1 CBIN CMAN * Recovery Rate: 0.4000
Reco Override: 0.4000

11) Pricing Sources CDS Curve Screen

Bank of America Corp - BOFA CDS USD SR

Tenor	Name	MSG Quotes Bid	MSG Quotes Ask	MSG Quotes Time	CMAN Bid	CMAN Ask	CMAN Time	My PCS (Pricing Sources) Bid	My PCS Ask	My PCS Time	My PCS Source
1) Curve	BOFA CDS USD SR CRV										
2) 6 Mo	BOFA CDS USD SR 6M D14				16.61	24.86	01/06/21	17.39	23.61	10:01:08	CBIN
3) 1 Yr	BOFA CDS USD SR 1Y D14				23.11	32.62	01/06/21	22.72	32.67	10:01:08	CBIN
4) 2 Yr	BOFA CDS USD SR 2Y D14				25.20	34.49	01/06/21	25.12	34.84	10:01:08	CBIN
5) 3 Yr	BOFA CDS USD SR 3Y D14				29.12	38.18	01/06/21	29.15	38.42	10:01:08	CBIN
6) 4 Yr	BOFA CDS USD SR 4Y D14				35.07	42.23	01/06/21	35.04	42.35	10:01:08	CBIN
7) 5 Yr	BOFA CDS USD SR 5Y D14				42.10	46.60	01/06/21	42.27	46.81	10:01:08	CBIN
8) 7 Yr	BOFA CDS USD SR 7Y D14				52.13	65.61	01/06/21	51.99	65.64	10:01:08	CBIN
9) 10 Yr	BOFA CDS USD SR 10Y D14				65.89	80.04	01/06/21	65.19	79.50	10:01:08	CBIN



CONTRIBUTED CDS SPREADS

1 Enter the corporate ticker, currency and debt type to search.
Corporate Ticker: JPM Currency:USD Debt Type:1 99) Search

1 Contributed CDS Spreads 98) RED Info

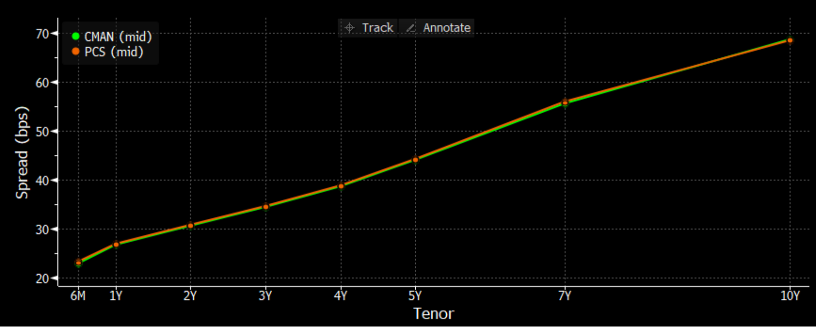
Term	Ticker	Contributor	Bid (bps)	Ask	Update Time
2) 6 mo	CT662282	CBIN	19.793	26.668	10:01
3) 1 yr	CJPM1U1	CBIN	22.151	31.926	10:01
4) 2 yr	CJPM1U2	CBIN	26.006	35.568	10:01
5) 3 yr	CJPM1U3	CBIN	30.137	39.065	10:01
6) 4 yr	CT385936	CBIN	35.237	42.440	10:01
7) 5 yr	CJPM1U5	CBIN	41.793	46.793	10:01
8) 7 yr	CJPM1U7	CBIN	49.011	62.852	10:01
9) 10 yr	CJPM1U10	CBIN	61.233	75.541	10:01

Day Count: ACT/360
Frequency: Q
Contributor Preferences: CBBT MSG1 CBIN CMAN * Recovery Rate: 0.4000
Reco Override: 0.4000

11) Pricing Sources CDS Curve Screen

JPMorgan Chase & Co - JPMCC CDS USD SR

Security		MSG Quotes			CMAN			My PCS (Pricing Sources)			
Tenor	Name	Bid	Ask	Time	Bid	Ask	Time	Bid	Ask	Time	Source
1) Curve	JPMCC CDS USD SR CRV										
2) 6 Mo	JPMCC CDS USD SR 6M D14				19.50	26.33	01/06/21	19.79	26.67	10:01:59	CBIN
3) 1 Yr	JPMCC CDS USD SR 1Y D14				22.10	31.60	01/06/21	22.15	31.93	10:01:59	CBIN
4) 2 Yr	JPMCC CDS USD SR 2Y D14				25.93	35.22	01/06/21	26.01	35.57	10:01:59	CBIN
5) 3 Yr	JPMCC CDS USD SR 3Y D14				30.01	38.78	01/06/21	30.14	39.06	10:01:59	CBIN
6) 4 Yr	JPMCC CDS USD SR 4Y D14				35.04	42.15	01/06/21	35.24	42.44	10:01:59	CBIN
7) 5 Yr	JPMCC CDS USD SR 5Y D14				41.52	46.52	01/06/21	41.79	46.79	10:01:59	CBIN
8) 7 Yr	JPMCC CDS USD SR 7Y D14				48.76	62.50	01/06/21	49.01	62.85	10:01:59	CBIN
9) 10 Yr	JPMCC CDS USD SR 10Y D14				61.57	75.77	01/06/21	61.23	75.54	10:01:59	CBIN



Appendix 13: Interest rate swap calculation

See attached MS Excel file.

Appendix 14: CVA Components

See attached MS Excel file.

Appendix 15: CVA Results

See attached MS Excel file.

Appendix 16: Empirical Results

See attached MS Excel file.

Appendix 17: Forward Libor rates

See attached MS Excel file.

Appendix 18: Discount Factor

See attached MS Excel file.