Essays on Empirical Asset Pricing and Private Equity

Jørgensen, Rasmus

COPENHAGEN BUSINESS SCHOOL
SOLBJERG PLADS 3
DK-2000 FREDERIKSBERG
DANMARK

WWW.CBS.DK

ISSN 0906-6934

Print ISBN: **978-87-7568-057-3**
Online ISBN: **978-87-7568-058-0**

ESSAYS ON EMPIRICAL ASSET PRICING AND PRIVATE EQUITY

PhD Series 02.2022

Rasmus Jørgensen

ESSAYS ON EMPIRICAL ASSET PRICING AND PRIVATE EQUITY

CBS PhD School

PhD Series 02.2022

CBS COPENHAGEN BUSINESS SCHOOL
HANDELSHØJSKOLEN

Essays on Empirical Asset Pricing and Private Equity

Rasmus Jørgensen

A thesis presented for the degree of
Doctor of Philosophy

Supervisor: Jesper Rangvid
Ph.D. School in Economics and Management
Copenhagen Business School

Rasmus Jørgensen
Essays on Empirical Asset Pricing and Private Equity

1st edition 2022
PhD Series 02.2022

ISSN 0906-6934

Print ISBN: 978-87-7568-057-3
Online ISBN: 978-87-7568-058-0

The CBS PhD School is an active and international research environment at Copenhagen Business School for PhD students working on theoretical and empirical research projects, including interdisciplinary ones, related to economics and the organisation and management of private businesses, as well as public and voluntary institutions, at business, industry and country level.

Abstract

This thesis is the result of my PhD studies at the Department of Finance and the Pension Research Center (PeRCent) at Copenhagen Business School.

The thesis applies methods from empirical asset pricing to study the payoffs of non-traded assets, specifically private equity funds, relative to the payoffs of public market assets. Assessing the risk and return characteristics of private equity funds presents unique challenges due to the absence of quoted market prices and consequential lack of return data. This thesis addresses some of these challenges. The thesis consists of three essays, which are self-contained and can be read independently.

The first essay provides empirical evidence that the risk-adjusted value of buyout fund payoffs decreases significantly relative to current performance measures if investors account for time-varying risk prices and interest rates. The results suggest that buyout funds do not improve the risk-return trade-off of a myopic investor with time-varying required compensation for bearing risk.

The second essay shows how a variant of the widely used Public Market Equivalent measure of risk-adjusted fund performance relates to portfolio choice problems. I use the relationship to estimate the optimal allocation between buyout funds and a benchmark of publicly traded stocks directly from fund cash flows. The empirical results show that both log-utility and power-utility investors should allocate an economically significant fraction of wealth to buyout funds.

The third essay (co-authored with Nicola Giommetti) proposes a modification to existing methods for risk-adjusting private equity fund cash flows based on a decomposition of excess private equity performance. We apply and evaluate the method by studying the value of buyout, venture capital and generalist funds to CAPM and long-term investors.

Acknowledgments

This thesis was only possible due to the support of several people and organizations. I would like to acknowledge the support of Arbejdsmarkedets Tillægspension (ATP) and the Ministry of Higher Education and Science for funding my PhD studies.

I would furthermore like to thank my supervisors Jesper Rangvid at Copenhagen Business School and Christian Kjær at ATP.

I am grateful for many interesting discussions with my co-author Nicola Giommetti, and for Ludovic Phalippou facilitating my visit at University of Oxford, Saïd Business School. I would also like to thank my fellow PhD students and faculty at the Department of Finance, and my colleagues at ATP.

Finally, I am grateful to my family for their support throughout my studies.

Rasmus Jørgensen

Copenhagen, November 2021

Summaries in English

Conditional Risk Adjustment of Buyout Funds

The first essay studies the risk-adjusted performance of buyout funds. Economically valid measures of private equity fund performance can be expressed in terms of stochastic discount factors. The stochastic discount factors defining past performance measures, however, do not appear sufficiently flexible to account for the time-varying risk prices and short-term interest rates characterizing financial markets. This essay examines the value of buyout fund cash flows to investors with time-varying required compensation for bearing risk.

I propose a general stochastic discount factor with time-varying coefficients which can account for investors' required compensation for bearing risk changing with time. The stochastic discount factor specification is motivated by the literature on conditional performance evaluation of mutual and hedge funds. The specification takes into consideration how stock market predictability and time-varying market risk affect investors' marginal utility of receiving a cash flow. The proposed stochastic discount factors imply heterogeneous required returns for individual buyout funds depending on the economic environment during a given fund's lifetime, which is not the case for past performance measures.

I specify several models which impose different restrictions on conditional expected market returns and conditional market risk and apply the stochastic discount factors to discount buyout fund cash flows. I find that buyout funds, on average, exhibit negative or zero abnormal profits. This result is in contrast to the positive risk-adjusted performance found in studies not accounting for time-varying risk prices and interest rates. The results suggest that the value of buyout fund payoffs are significantly lower than implied by previous performance measures when investors' required compensation for bearing risk varies as a function of market valuations. Altogether, the time-varying coefficient stochastic discount factor specifications imply that buyout funds do not improve the risk-return trade-off of a myopic investor with time-varying required compensation for bearing risk.

Private Equity Performance: Implications for Portfolio Choice

The second essay examines the applicability of a particular Public Market Equivalent (PME) measure, of risk-adjusted performance, for portfolio choice problems. Despite the widespread use of PMEs among academics and practitioners, applicability for portfolio choice has only attracted little scrutiny.

I set forth a simple portfolio choice problem and show a relation between the allocation problem and performance evaluation using PMEs. Specifically, I consider an investor maximizing expected utility of terminal wealth. The investor allocates between a buy-and-hold investment in a public market benchmark and a private equity strategy. The allocation problem's first-order conditions reveal that the expected PME can be interpreted as a log-utility investor's marginal expected utility obtainable from a marginal allocation to the PE strategy, given that the investor has no initial private equity allocation. The portfolio choice problem also reveals that the distribution of excess private equity performance, represented by the PME, provides sufficient information to determine a log-utility investor's optimal private equity allocation. I furthermore provide an analytical expression for the log-utility investor's optimal allocations by solving an approximate expected utility optimization problem.

The optimal private equity allocation for investors with general power-utility preferences and relative risk aversion larger than one, on the other hand, does not depend exclusively on PME moments. The solution to an approximate problem reveals that power-utility investors should also consider the covariance between excess private equity performance and benchmark strategy returns.

I exploit the empirical tractability of the simple allocation problem to estimate optimal buyout fund allocations relative to several public benchmarks. I estimate allocations directly from fund cash flows and benchmark returns using an expected utility maximization criterion, thereby bypassing the need to estimate buyout fund alphas and betas. Empirically, I find that both log-utility and power-utility investors should allocate a significant fraction of wealth to buyout funds when the alternative investment is a broad portfolio of public equities. Log-utility investors, however, refrain from investing in buyout funds when the benchmark is a portfolio of small-value stocks. Buyout allocations generally decrease in risk aversion such that power-utility investors allocate less to buyout funds than log-utility investors.

I furthermore estimate optimal allocations conditional on the state of the economy at fund inception and find that the public market valuation level is the most important determinant of conditional allocations. Finally, I modify the expected utility framework and show that investors with fund selection skills should allocate a considerably larger fraction of wealth to buyout funds than investors randomly selecting buyout funds. Investors with limited access to top quartile funds should allocate less to buyout funds.

Risk Adjustment of Private Equity Cash Flows

with Nicola Giommetti

The third essay proposes a modification of existing stochastic discount factor-based methods for risk-adjusting private equity fund cash flows. Existing methods do not explicitly distinguish between the two operations performed by stochastic discount factors, (1) risk adjustment and (2) time-discounting. We propose a modification based on a decomposition of excess private equity performance, which differentiates between time-discounting and risk adjustment, thereby providing a theoretical foundation for a set of additional restrictions on stochastic discount factors used to risk-adjust fund cash flows. Specifically, our implementation restricts investors' subjective term structure of interest rates such that it is determined by market data. The additional restrictions ensure that the term structure is constant across different stochastic discount factor models, which allows for a more accurate comparison of performance across models.

We apply and evaluate our method empirically by examining the value of buyout, venture capital and generalist funds to log-utility and CAPM investors. We furthermore provide a new perspective on fund performance by considering the risk-adjusted value of private equity cash flows to long-term investors distinguishing between permanent and transitory wealth shocks. We find that certain stochastic discount factor specifications, estimated using existing methods, lead to economically implausible performance estimates due to unrealistic time-discounting. Specifically, the inclusion of an additional risk factor in the long-term investor's stochastic discount factor leads to particularly unrealistic time-discounting and implausibly high risk-adjusted performance for long-term investors. In contrast, our implementation effectively fixes the term structure of interest rates which ensures time-discounting in line with market data. Fixing the term structure furthermore ensures that differences in performance across models arise exclusively from differences in cash flow risk adjustment. We moreover find that the additional restrictions our method imposes reduce cross-sectional variation in fund performance, even though it does not use information about private equity cash flows.

Summaries in Danish

Conditional Risk Adjustment of Buyout Funds

Det første essay undersøger kapitalfondes risikojusterede afkast. Økonomisk valide mål for fondes risikojusterede afkast kan udtrykkes igennem stokastiske diskonteringsfaktorer. Stokastiske diskonteringsfaktorer, der definerer tidligere mål, afspejler dog ikke kapitalmarkeds karakteristika såsom tidsvarierende risikopriser og korte renter. I artiklen undersøger jeg derfor værdien af kapitalfondes pengestrømme for investorer med tidsvarierende afkastkrav for at bære markedsrisiko.

Jeg specificerer en generel stokastisk diskonteringsfaktor model, hvor underliggende risikofaktorer indgår med tidsvarierende koefficienter. Modellen tager højde for at investorers afkastkrav, for at bære risiko, varierer over tid og den er inspireret af modeller der, ofte benyttes til betinget risikojustering af investeringsforeningers og hedge-fondes afkast. Specifikt afspejler modellen, at forudsigelige markedsafkast og tidsvarierende markedsrisiko påvirker investorers marginale nytte og derved værdien af pengestrømme til/fra kapitalfonde. I modsætning til tidligere mål for fondes risikojusterede afkast pålægger modellen, med tidsvarierende koefficienter, kapitalfonde forskellige afkastkrav afhængig af økonomiens tilstand i løbet af fondenes levetid.

Empirisk anvender jeg stokastiske diskonteringsfaktorer, der på forskellig vis begrænser, hvorledes det betingede markedsafkast og den betingede markedsrisiko varierer over tid, til at tilbagediskontere kapitalfondes pengestrømme. De empiriske resultater indikerer, at kapitalfonde i gennemsnit udviser negativ eller ingen overnormal profit. Dette resultat står i modsætning til tidligere studier, der finder overnormal profit for kapitalfonde. Disse studier tager dog ikke i samme grad højde for tidsvarierende risikopriser og renter. Resultaterne viser således, at den risikojusterede værdi af kapitalfondes pengestrømme er betydelig lavere end tidligere mål antyder, såfremt investorers afkastkrav for at bære risiko varierer over tid, som en funktion af markedets prisfastsættelse.

Private Equity Performance: Implications for Portfolio Choice

Det andet essay analyserer, hvorledes et specifikt Public Market Equivalent (PME) mål, for kapitalfondes risikojusterede afkast, kan anvendes til at løse portefølje allokeringsproblemer. På trods af PME-målets udbredelse blandt både forskere og praktikere, er målets anvendelighed med hensyn til at løse portefølje allokeringsproblemer kun i begrænset grad blevet undersøgt.

Jeg opstiller et simpelt allokeringsproblem og viser en sammenhæng mellem problemet og risikojustering af kapitalfondes afkast via PME-målet. Allokeringsproblemet er karakteriseret ved, en investor der maksimerer den forventede nytte af hans slutformue ved at allokere mellem en investering i et noteret benchmark og en strategi, der investerer i kapitalfonde. Optimeringsproblemets førsteordensbetingelser viser, at kapitalfondes forventede risikojusterede afkast, repræsenteret ved den forventede PME, kan fortolkes som en log-nytte investors forventede nytte forøgelse ved en marginal allokering til kapitalfondsstrategien givet, at investoren ikke har en eksisterende allokering til kapitalfonde. Allokeringsproblemet viser derudover, at marginalfordelingen for kapitalfondes risikojusterede afkast er tilstrækkelig til, at en log-nytte investor kan bestemme optimale allokeringer. Løsningen på et approksimativt allokeringsproblem fører desuden til et analytisk udtryk for den optimale allokering til kapitalfonds strategien for en log-nytte investor. For investorer med isoelastisk nyttefunktion, og relativ risiko aversion større end én, afhænger den optimale allokering til kapitalfonde ikke kun af den marginale fordeling for kapitalfondes risikojusterede afkast. Et approksimativt allokeringsproblem viser, at denne type investorer også bør tage hensyn til kovariansen mellem kapitalfondes overnormale profit og afkast på benchmark strategien.

Empirisk anvender jeg allokeringsproblemet til at estimere den optimale allokering til kapitalfonde, relativt til forskellige porteføljer af noterede aktier. Jeg benytter et forventet nytte maksimerings kriterium til at estimere optimale allokeringer direkte fra fondes pengestrømme og benchmark afkast, hvorfor estimering af kapitalfondes alfa og beta ikke er påkrævet. Jeg finder, at både log-nytte investorer og investorer med isoelastisk nytte bør allokere en signifikant andel af deres formue til kapitalfonde, såfremt alternativinvesteringen er en bred portefølje af noterede aktier. Investorer med log-nytte præferencer bør derimod afholde sig fra at investere i kapitalfonde, såfremt alternativinvesteringen er en portefølje af små aktier med lave værdiansættelser. Derudover viser jeg, at den optimale allokering til kapitalfonds strategien typisk er faldende i investorens risiko aversion. Investorer med isoelastisk nyttefunktion allokerer således en mindre andel af deres formue til kapitalfonde relativt til investorer med log-nytte. Jeg estimerer desuden en log-nytte investors optimale allokering til kapitalfonds strategien betinget på økonomiens tilstand på fondenes starttidspunkt. De empiriske resultater viser, at værdiansættelsen af noterede aktier er den vigtigste tilstandsvariabel med hensyn til at bestemme optimale betingede allokeringer. Afslutningsvis viser jeg, at investorer med kompetencer til at udvælge fonde med de højeste risikojusterede afkast bør allokere en betydelig større andel af deres formue til kapitalfonde, relativt til investorer der udvælger fonde tilfældigt. Investorer med begrænset adgang til de bedste fonde bør til gengæld allokere en betydelig mindre andel af deres formue til kapitalfonde.

Risk Adjustment of Private Equity Cash Flows

med Nicola Giommetti

Det tredje essay introducerer en modifikation af eksisterende metoder der anvender stokastiske diskonteringsfaktorer til at værdiansætte kapitalfondes pengestrømme. Nuværende metoder skelner ikke eksplicit mellem stokastiske diskonteringsfaktorers to funktioner, (1) risikojustering og (2) diskontering af tid. Vi foreslår en modifikation, der differentierer mellem risikojustering og tidsdiskontering. Den modificerede metode dekomponerer værdien af kapitalfondes pengestrømme, hvilket resulterer i et sæt ekstra, teoretisk begrundede, begrænsninger på stokastiske diskonteringsfaktorer, der anvendes til at risikojustere pengestrømmene. Specifikt begrænser vores metode den stokastiske diskonteringsfaktors implicitte rentekurve således, at rentekurven er i overensstemmelse med markedsdata. De ekstra begrænsninger garanterer således, at rentekurven er konstant på tværs af forskellige modeller for den stokastiske diskonteringsfaktor, hvilket foranlediger en mere præcis sammenligning af pengestrømmenes værdi på tværs af modeller.

Vi implementerer og evaluerer vores metode empirisk ved at undersøge værdien, for log-nytte og CAPM-investorer, af at investere i forskellige typer kapitalfonde. Derudover giver vi et nyt perspektiv på kapitalfondes risikojusterede afkast ved at undersøge værdien af fondes pengestrømme for en langsigtet investor, der skelner mellem permanente og transitoriske formuestød. Vores resultater viser, at visse stokastiske diskonteringsfaktor modeller resulterer i økonomisk usandsynlige risikojusterede afkast, som følge af urealistisk diskontering af tid, hvis eksisterende metoder anvendes. Inklusionen af en ekstra risikofaktor i den langsigtede investors stokastiske diskonteringsfaktor resulterer i usandsynlig tidsdiskontering, hvilket leder til en usandsynlig høj værdi af kapitalfondes pengestrømme for langsigtede investorer. Vores implementering sikrer derimod, at den langsigtede investors tidsdiskonteringen er i overensstemmelse med markedsdata. Vores metode medfører, at forskelle i værdien af pengestrømme på tværs af modeller, alene skyldes forskelle i risikojusteringen af kapitalfondenes pengestrømme. De empiriske resultater viser derudover, at de ekstra begrænsninger, vores metode foreskriver, reducerer tværsnitsvariationen i risikojusterede afkast, på trods af at metoden ikke anvender information om fondes pengestrømme.

Contents

Chapter 1

Conditional Risk Adjustment of Buyout Funds

Rasmus Jørgensen*

Abstract

This paper studies the risk-adjusted payoffs of buyout funds in the presence of time-varying risk prices and interest rates. Using a sample of buyout funds from Preqin, I risk-adjust fund cash flows using stochastic discount factors (SDFs) with time-varying coefficients known from the literature on performance evaluation of mutual and hedge funds. I find positive risk-adjusted performance of 20-30 cents, per dollar of committed capital, for constant market price of risk SDF models. Letting the SDF implied market price of risk and short-term interest rate vary over time, such that they are consistent with market data, results in a significant decrease in the risk-adjusted value of buyout fund cash flows. The results suggest that investing in buyout funds do not improve the risk-return trade-off of investors with time-varying requirements for bearing risk.

The author is affiliated with Pension Research Center (PeRCent) and the Department of Finance, Copenhagen Business School, Solbjerg Plads 3, DK-2000 Frederiksberg, Denmark as well as ATP. Thanks to Nicola Giommetti, Jesper Rangvid and Paul Whelan for helpful comments.

1 Introduction

Assessing the risk-adjusted performance of private market assets, such as private equity (PE) funds, has proven to be challenging due to the nature of these assets' payoffs. PE funds are privately held and fund shares rarely trade in secondary transactions. As a result, quoted returns are generally not available. Returns constructed from fund valuations (Net Asset Values) are furthermore not representative of true fund returns because NAVs typically are stale and prone to manipulation.[1] The absence of reliable return data means that the methods, usually applied to risk-adjust the returns of publicly traded assets, such as linear factor models, cannot immediately be applied to PE fund payoffs. These issues have resulted in numerous performance measures derived from fund cash flows. Recently Sorensen and Jagannathan (2015) and Korteweg and Nagel (2016) relates the widely used Public Market Equivalent (PME)[2] measure of risk-adjusted fund performance to performance evaluation with stochastic discount factors (SDFs). SDF-based methods for risk-adjusting PE cash flows are generally valid provided that (1) an SDF exists which correctly risk-adjust the returns of a set of publicly traded assets, (2) PE fund payoffs are spanned by the traded assets and (3) the law of one price holds. The PME provides the correct risk adjustment of cash flows for log-utility investors while the Generalized PME (GPME) with the public equity market as the underlying risk-factor, proposed by Korteweg and Nagel (2016), amounts to valuing payoffs using a log-return CAPM.[3] Both measures specify the required returns and risk-free interest rates that PE cash flows are evaluated against through restrictions on the underlying SDFs.

The restrictions imposed by those SDFs are, however, not necessarily consistent with stylized facts of capital markets. In this paper, I propose a general SDF specification that can accommodate time-varying market price of risk and variation in short-term interest rates and that generates prevalent features of financial markets such as counter-cyclical conditional maximal Sharpe ratios. I apply the SDF to examine the risk-adjusted performance of buyout funds in the presence of time-varying market prices of risk and interest rates. I consider a time-varying coefficients SDF specification well-known from the literature on stock market predictability and conditional perfor-

[1] Jenkinson, Sousa, and Stucke (2013) and Brown, Gredil, and Kaplan (2019) study manipulation of reported returns.

[2] PME measures are constructed by either (1) replicating fund cash flows in a benchmark of publicly traded stocks and comparing PE and benchmark performance or (2) by discounting cash flows with benchmark returns.

[3] The GPME introduced by Korteweg and Nagel (2016) discounts cash flows with a general SDF. Referring to the GPME in this paper, I refer to the specific exponentially affine SDF specification with constant coefficients and a single public market benchmark as the underlying risk factor.

mance evaluation of mutual and hedge funds:

$$M_{t+1} = \exp(a_t - b_t f_{t+1}) \tag{1}$$

The time-varying coefficient SDF resembles the linear models often used to risk-adjust hedge fund returns in the presence of time-varying expected returns, see for example Ferson and Schadt (1996) and Farnsworth, Ferson, Jackson, and Todd (2002).

I study average buyout fund performance by discounting fund cash flows with SDF realizations. I denote the average value of fund cash flows, discounted using an SDF with time-varying coefficients, as Conditional PMEs (CPME) to distinguish this performance measure from previous (G)PME measures. The CPME nests both the PME and GPME. The PME is defined by the exponentially affine SDF, with coefficients $a_t = 0$ and $b_t = 1$, corresponding to a log-utility investor's SDF. The GPME is defined by a_t and b_t being constant over time. Restrictions on the SDFs ensure that the SDF coefficients characterize the underlying risk factor's conditional moments and the return on a conditionally risk-free asset. The log-utility SDF imposes a factor risk premium corresponding to the risk factor's log return variance. The GPME enforces a constant market price of risk. Empirically the equity market premium implied by the log-utility model appears too low, and the literature on equity market predictability suggests that expected market returns, and potentially the market price of risk, vary over time.[4] The time-varying coefficients SDF incorporates these stylized facts into the risk adjustment of fund cash flows.

I operationalize the time-varying coefficients SDF by modeling coefficient dynamics in different ways. In a preliminary analysis, I consider a semi-parametric approach to construct a_t and b_t from SDF coefficients estimated in short windows within a larger sample. The main analysis considers three parametric models. Two log-normal models in which conditional risk factor moments determine the SDF coefficients. The log-normal models exploit predictability in public equity market returns and specify conditional expected log market returns as a function of the market valuation level. The first log-normal model imposes constant conditional market variance such that changes in the expected excess log market returns lead to changes in the market price of risk. The second specification models the conditional variance as a GARCH process such that the conditional market price of risk also changes with variation in conditional market risk. Lastly, I consider an instrumental variables model, along the lines of Cochrane (1996) and Lettau and Ludvigson (2001), which relaxes the log-normality assumption and directly models coefficients as affine functions of

[4]See for example, Keim and Stambaugh (1986), Fama and French (1989), Ferson and Harvey (1991) and Lamont (1998), among others.

state variables.

In the empirical analysis, I use buyout fund cash flows from Preqin, S&P 500 returns as the risk factor and the dividend to price ratio as the state variable governing expected log market returns. I find a positive risk-adjusted performance of approximately 20 cents per dollar of committed capital using the PME, consistent with previous studies. Evaluating cash flows using the GPME, which imposes a constant market price of risk, also results in positive risk-adjusted performance. I, however, find that GPME estimates are sensitive to the method used to estimate SDF coefficients. Specifically, the GPME depends critically on the SDF coefficients' ability to accurately reflect average market returns and short-term interest rates in periods with the most buyout fund activity. Despite differences in GPME point estimates, the estimates are positive regardless of the estimation method.

To assess the relevance of using time-varying SDF coefficients, I construct time-varying coefficients from constant coefficients estimated in shorter windows within the sample period. The short-window coefficients display significant variation across time, suggesting that SDF coefficients should vary over time to conform with market data. Constructing an aggregate stochastic discount factor from the semi-parametric coefficients results in risk-adjusted performance decreasing relative to the constant coefficients (GPME) specification. The semi-parametric method produces average abnormal buyout profits of -2 to 3 cents per dollar of committed capital.

The conditional log-normal SDF specification, with constant conditional market variance and modeling expected log market returns as a function of the market dividend to price ratio, results in significant market price of risk variation. The specification produces risk-adjusted performance of -31 cents to -12 cents depending on the buyout fund sample. The negative performance is partly due to the 2008 financial crisis during which the model implied price of risk increases considerably and partly due to the period following the crisis, a period in which the SDF is relatively low while buyout fund payoffs are high. The conditional log-normal model incorporating time-varying market volatility produces risk-adjusted buyout fund performance of -11 cents to 2 cents per dollar of committed capital. I furthermore find risk-adjusted performance of zero to negative five cents using the instrumental variables model. A few simple economic checks suggest that the conditional SDF specifications are more economically credible. The conditional specifications, for instance, imply counter-cyclical conditional Sharpe ratios. Finally, I show that the results are robust to using different valuation ratios and to potential manipulation of terminal Net Asset Values (NAVs) for partially liquidated funds.

This paper contributes to the literature studying PE fund risk and return characteristics. I show that the risk-adjusted value of buyout fund cash flows decreases significantly once accounting for time-varying market prices of risk and time variation in short-term interest rates. This result implies that investors with time-varying required compensation for bearing risk do not improve their risk-return trade-off from a marginal allocation to buyout funds, meaning that this type of investor should not allocate to buyout funds. Investors with constant required compensation for bearing risk should, on the other hand, add a marginal allocation to buyout funds. Provided that long-term institutional investors, such as pension funds and endowments, do not consider short-term fluctuations in risk prices, the results suggest that these investors should demand buyout fund payoffs.

The following section provides an overview of related literature. Section 2 presents measures of risk-adjusted performance and relates time-varying coefficient SDFs to previous measures of risk-adjusted performance. Section 3 presents data used in the empirical analysis. Section 4 outlines the time-varying coefficient SDF models in detail. Section 5 presents results of the empirical analysis. Section 6 reports robustness tests and Section 7 concludes.

1.1 Related Literature

The literature on private equity fund performance is extensive. The absolute performance of funds has been studied to a great extent using IRRs and cash flow multiples. Kaplan and Schoar (2005), Harris, Jenkinson, and Kaplan (2014) among others, find persistence in fund-level performance, while Robinson and Sensoy (2016) show that payoffs vary with the business cycle. The literature on risk-adjusted performance is less developed despite tracing back to Gompers and Lerner (1997), Kaplan and Schoar (2005) and Cochrane (2005) among others. This is partly due to the structure of private equity fund investments, which complicates using the methods traditionally used for evaluating public equity, hedge fund and mutual fund returns. The structure of private equity fund payoffs has led to a multitude of risk-adjusted performance measures. A subset relies on replicating private equity investments using public equity market indices and comparing the absolute performance of the benchmark investment to the performance of private equity funds. Long and Nickels (1996) proposes the Index Comparison Method (ICM), Rouvinez (2003) introduces the PME+ measure while Cambridge Associates recommends the mPME.

Arguably the most widely used measure of risk-adjusted performance is the PME introduced by Kaplan and Schoar (2005), defined as the present value of distributions divided by the present value of contributions, discounting cash flows with the realized returns of a benchmark index. A similar measure, a profitability index, is introduced by Phalippou and Gottschalg (2009). Phalippou and Gottschalg (2009) proposes a transformation of the profitability index to determine alpha, while Gredil, Griffiths, and Stucke (2014) proposes "The Direct Alpha Method" as an alternative measure of private equity alpha. Driessen, Lin, and Phalippou (2012) put forward a methodology for estimating the beta of private equity based on cash flows. Given the beta estimate, PE alpha is recovered. Ang, Chen, Goetzmann, and Phalippou (2018) take yet another approach and use a Bayesian framework to extract a time series of private equity returns from cash flows, from which PE alpha and betas can be estimated. Brown, Ghysels, and Gredil (2020) apply mixed frequency data analysis techniques to nowcast NAVs, which can then be used to estimate the systematic and total risk of PE.

Recent papers have given economic meaning to the PME measure. Sorensen and Jagannathan (2015) establish the connection between the PME measure and the stochastic discount factor of a log-utility investor, thereby identifying the assumptions under which the PME is a valid measure of risk-adjusted performance. Korteweg and Nagel (2016) proposes the GPME, which provides the correct risk adjustment of cash flows when the marginal investor's relative risk aversion is different from one. Korteweg and Nagel (2016) shows empirically that the abnormal performance of venture capital funds becomes zero when performance is measured using the GPME. Gredil, Sørensen, and Waller (2020) examines private equity performance using consumption-based asset pricing models, specifically the long-run risk model of Bansal and Yaron (2004) and the external habit model of Campbell and Cochrane (1999). Gupta and Van Nieuwerburgh (2021) consider a reduced form no-arbitrage stochastic discount factor to account for the term structure of equity risk premia and utilize a replicating portfolio of public equity strips to estimate the expected returns of private equity fund categories. They conclude that risk-adjusted profits across private equity categories are negative.

Korteweg (2019) synthesizes the literature on risk-adjusting private equity payoffs and concludes that investments in venture capital funds do not provide abnormal returns in aggregate. Buyout funds, however, appear to provide positive risk-adjusted performance relative to broad public equity indices. Korteweg (2019) also provides a perspective on the interpretation of stochastic discount factor-based measures of performance. Because a given stochastic discount factor represents the marginal rate of substitution, of some investor, the GPME measures whether the agent can increase expected utility by a marginal allocation to private equity. From a benchmark perspective specifying a stochastic discount factor, which is a function of traded assets, amounts to specifying an ex-ante mean-variance efficient portfolio.

The benchmark perspective bridges the gap to a rich literature using stochastic discount factors to evaluate the performance of public equity portfolios, mutual funds and hedge funds, see for instance Ferson and Harvey (1999) and Ferson and Schadt (1996). This paper is related to a part of this literature centered around risk-adjusting the returns of mutual and hedge funds by comparing fund returns to the returns of dynamic trading strategies. The paper is also related to the literature using conditional asset pricing models to study the cross-section of stock returns. Jagannathan and Wang (1996), Cochrane (1996) and Lettau and Ludvigson (2001), for instance, show improvements in cross-sectional pricing when time variation in conditional moments are instrumented with state variables known to predict future market returns.

The literature on risk-adjusted private equity fund performance conditional on the macroeconomic environment is relatively sparse. Brown, Harris, Hu, Jenkinson, Kaplan, and Robinson (2021) reports predictability in aggregate cash flows and finds that distributions and contributions are differentially exposed to fluctuations in macroeconomic variables. Robinson and Sensoy (2016) risk-adjust cash flows using the PME and finds that it is predictable using financial variables. Despite the extensive literature examining aggregate PE performance, the direct effect on risk-adjusted performance of time-varying discount and interest rates is not widely studied. The papers closest to the spirit of this paper are Korteweg and Nagel (2016), Gredil et al. (2020) and Gupta and Van Nieuwerburgh (2021).

2 Risk-Adjusting Cash Flows

To study the risk-adjusted value of PE fund payoffs, I use the realized Net Present Value (NPV) of fund cash flows defined as:

$$NPV_j = \sum_{h=0}^{H} M_{t,t+h} \cdot C_{j,t+h} \tag{2}$$

Where t is the time of the first fund cash flow such that t depends on j. $M_{t,t+h}$ is a multi-period stochastic discount factor realization discounting cash flows at horizon h, $C_{j,t+h}$, back to fund inception. The cross-sectional average fund NPV measures aggregate risk-adjusted performance, which is zero for an appropriately chosen SDF.

The specification of the stochastic discount factor provides the content of a given performance evaluation model. Specifically, the contents of a particular model comes from the risk factor realizations driving variation in the SDF and the restrictions placed on the SDF via the coefficients. The PME measure amounts to discounting cash flows using the stochastic discount factor of a log-utility investor, which in the one-period case is $M_{t+1} = \exp(-r^w_{t+1})$. The underlying risk factor return r^w_{t+1} is the log-return on the investor's wealth. In benchmark applications, the wealth return is commonly replaced by the return on a broad index of public equities. The GPME applies the exponentially affine constant coefficients SDF, $M_{t+1} = \exp(a - br^m_{t+1})$ to discount cash flows. Where r^m_{t+1} denotes the log return on a public equity market index. These SDFs impose specific restrictions on risk factor moments. The log-utility SDF restricts the market risk premium to equal the underlying risk factor's variance. The constant coefficients stochastic discount factor alleviates the risk premium restriction but instead imposes a constant price of risk. The constant price of risk restriction is clear in a conditionally log-normal model. Choosing the coefficients in the constant coefficients SDF such that $\mathrm{E}_t[\exp(m_{t+1})\exp(r^m_{t+1})] = 1$ and $\mathrm{E}_t[\exp(m_{t+1})\exp(r^f_t)] = 1$ holds, results in the coefficients:

$$a = -r^f_t + b\mathrm{E}_t[r^m_{t+1}] - \frac{1}{2}b^2\sigma^2_t[r^m_{t+1}] \tag{3}$$

$$b = \frac{-r^f_t + \mathrm{E}_t[r^m_{t+1}] + \frac{1}{2}\sigma^2_t[r^m_{t+1}]}{\sigma^2_t[r^m_{t+1}]} \tag{4}$$

The SDF slope, b, represents the market price of risk, which is constant in the SDF underlying the GPME. Empirically the quantities determining the coefficients, however, appear to vary over time. As an example, short-term Treasury bill yields, a proxy for a conditionally risk-free asset, vary over time while the market dividend to price ratio also fluctuates over time, reflecting variation in long-run expected returns according to Cochrane (2011). This time variation means that the market price of risk, implied by capital market data, might fluctuate over time. The constant coefficients condition might thus be too restrictive. To alleviate the constant coefficients constraint, I consider the general time-varying coefficients SDF:

$$M_{t+1} = \exp(a_t - b_t r^m_{t+1}) \tag{5}$$

For conditionally log-normal returns the coefficients are given by $a_t = -r^f_t + b_t\mathrm{E}_t[r^m_{t+1}] - \frac{1}{2}b^2_t\sigma^2_t[r^m_{t+1}]$ and $b_t = (-r^f_t + \mathrm{E}_t[r^m_{t+1}] + \frac{1}{2}\sigma^2_t[r^m_{t+1}])(\sigma^2_t[r^m_{t+1}])^{-1}$, implying that the coefficients can vary with time. The dynamics of the conditional risk-free rate, the conditional market return and variance determine whether coefficients vary over time empirically.

To see the difference between the constant and time-varying coefficient stochastic discount factors, consider the maximal Sharpe ratios generated by the SDFs. In the log-normal model with time-varying coefficients, the maximal Sharpe ratio fluctuates with changes in b_t and the conditional market variance:[5]

$$\frac{\sigma_t[M_{t+1}]}{\mathrm{E}_t[M_{t+1}]} = \sqrt{\exp\left(b_t^2\sigma_t^2[r_{t+1}^m]\right) - 1} \tag{6}$$

The maximal Sharpe ratio is constant for the SDFs underlying the PME and GPME provided that the conditional variance is constant. The constant coefficients stochastic discount factor allows the maximal Sharpe ratio to scale with the market price of risk such that the model spans assets with higher Sharpe ratios. The time-varying coefficients model allows the maximal Sharpe ratio to fluctuate with variation in the market price of risk. In the reduced form time-varying coefficients SDF model, the market price of risk can be interpreted as arising from either time-varying risk aversion or changing investor sentiment. Periods with high market price of risk represent periods with high risk-aversion or pessimistic investors.

The maximal Sharpe ratio is related to expected excess asset returns through the Euler equation $\mathrm{E}_t[M_{t+1}R_{t+1}^i] = 1$.[6] For the time-varying coefficients SDF in Equation 5, and conditionally log-normal returns, expected excess asset returns are given by:

$$\mathrm{E}_t[R_{t+1}^i] - R_t^f = -\sigma_t[R_{t+1}^i]\frac{\sigma_t[M_{t+1}]}{\mathrm{E}_t[M_{t+1}]}\rho_t[M_{t+1}, R_{t+1}^i] \approx b_t\sigma_t[r_{t+1}^m]\sigma_t[R_{t+1}^i]\rho_t[R_{t+1}^m, R_{t+1}^i] \tag{7}$$

Where $\sigma_t[M_{t+1}]/\mathrm{E}_t[M_{t+1}] \approx b_t\sigma_t[r_{t+1}^m]$.[7] Expected excess returns scales with the conditional maximal Sharpe ratio for a given conditional asset volatility and correlation, between the asset and the factor underlying the SDF. A time-varying maximal conditional Shape ratio thus lets an asset's required (model implied) return vary over time.

The intuition carries through for cash flows. To see how the SDF models affect the value of cash flows, consider discounting cash flows with two different realized SDFs. Specifically, consider a constant coefficients SDF $M_{t+1}^c = \exp(a^c - b^c r_{t+1}^m)$, and a time-varying coefficients SDF, $M_{t+1} = \exp(a_t - b_t r_{t+1}^m)$ for $b_t > b^c$ and $a_t = a^c$ at time t. In this case, a positive risk factor realization results in a lower realized SDF in the time-varying coefficients model because of higher required risk compensation. Discounting an exogenous cash flow using the time-varying coefficients SDF consequently results in a lower realized payoff value. The payoff value is lower because investors' marginal utility from a receiving payoff in a good state is lower. The difference in the value investors ascribe to private equity fund cash flows depends on how cash flows and fundraising vary with the SDF. The difference between the constant and time-varying coefficients SDFs becomes

[5] Appendix A presents expressions for the conditional Sharpe ratios in the models with time-varying coefficients.
[6] See for example Lettau and Uhlig (2002).
[7] See Appendix A.

apparent by expressing the time-varying coefficients SDF as a function of the constant coefficients SDF. Using $a_t = a^c + (a_t - a^c)$ and $b_t = b^c + (b_t - b^c)$ and substituting the coefficients into the time-varying coefficients SDF leads to the realized multi-period SDF:

$$M_{t,t+h} = \exp\left(a^c h + \sum_{i=1}^{h}(a_{t-1+i} - a^c) - b^c \sum_{i=1}^{h} r_{t+i} - \sum_{i=1}^{h}(b_{t-1+i} - b^c) r_{t+i}\right) = M^c_{t,t+h} Z_{t,t+h} \quad (8)$$

Where $Z_{t,t+h} = \exp\left(\sum_{i=1}^{h}(a_{t-1+i} - a^c) - \sum_{i=1}^{h}(b_{t-1+i} - b^c) r_{t+i}\right)$. The time-varying coefficients SDF is equivalent to the constant coefficients SDF times an additional discount factor, $Z_{t,t+h}$, capturing the cumulative effects of time-varying time-discounting and conditional risk during a given period. Conceptually, the time-varying coefficients model extends the constant coefficients model in the same way that preference-based stochastic discount factors are augmented by a recession-related state variable in the macro-finance literature, see for example Cochrane (2017).

3 Data

I use data from Preqin, constituting US buyout funds, incepted during the period 1984-2008, to estimate the risk-adjusted value of buyout fund cash flows. The cash flow data contains net of fees contributions and distributions as well as NAVs.[8] I split the sample into two subsamples, (1) a sample of fully liquidated funds and (2) a full sample. Partially liquidated funds are included in the full sample if cumulative fund contributions exceed 80 % of committed capital and the fund has a life of at least eight years. For fully liquidated funds, I use realized cash flows to estimate the realized NPV. For partially liquidated funds, the last cash flow includes the latest available NAV. I normalize cash flows to equal a commitment of one dollar. Table 1 reports summary statistics for TVPIs by vintage year and gives an overview of the samples.

Data on public markets are from Amit Goyal's data library and Center for Research in Security Prices (CRSP). The return on the market is the monthly market capitalization weighted S&P 500 Total Return Index from CRSP. The dividend to price ratio is defined as the sum of the last 12 months dividends on the S&P 500, divided by the current price. The risk-free interest rate is from Kenneth French's data library.

[8] Brown, Harris, Jenkinson, Kaplan, and Robinson (2015) show that the cash flow data from Preqin is generally consistent with cash flow data from other well-established providers and representative of aggregate performance.

4 Empirical Implementation

This section outlines different methods to estimate the coefficients in the constant coefficients SDF model. The section furthermore presents models for the time-varying coefficients SDF specification.

4.1 Constant Coefficients

Valuing cash flows with the constant coefficients SDF corresponds to benchmarking buyout fund payoffs against the average equity market premium during the SDF coefficient estimation period. For investments with quoted prices and consequently observable returns, it is reasonable to compare investment returns to the average equity market premium in the investment holding period. This is, in fact, in accordance with the literature risk-adjusting the returns of hedge and mutual funds using SDFs, for instance Ferson and Schadt (1996) and Farnsworth et al. (2002). Benchmarking is, however, more complicated for PE funds due to the absence of reliable return data and because PE funds call and distribute capital at stochastic times during a fund's life. The fund life cycle means that funds are not fully invested throughout their life, which means that the definition of an investment holding period is ambiguous for PE funds. The nature of private markets thus complicates the estimation of appropriate SDF coefficients.

Korteweg and Nagel (2016) identify SDF coefficients from cross-sectional variation in the discounted cash flows of artificial public market funds mimicking the timing of cash flows for a sample of PE funds. The artificial funds invest contributions in benchmark assets, while distributions are determined by a payout function, which specifies the fraction of NAV to distribute each time a fund distributes capital. This methodology gives rise to two effects, in comparison to estimating SDF coefficients from period returns using standard Generalized Method of Moments (GMM) moment restrictions, (1) a duration effect and (2) a weighting effect. The duration effect arises because artificial fund distributions is the result of compounding returns over a fund's life. This effect means SDF coefficients are estimated approximately at the average duration of the artificial funds' payoffs. Since the horizon is not explicit, the duration effect somewhat confounds the interpretation of the SDF coefficients. The weighting effect arises because more weight is placed on periods with more funds. The SDF coefficients are essentially estimated in periods with more funds.

The weighting effect can be illustrated in a standard GMM framework. The one-period Euler equation, $\mathrm{E}_t[M_{t+1}R^m_{t+1}] = 1$, states that the price of investing one dollar in the market at time t and receiving payoff R^m_{t+1} at time $t+1$ is one, in expectation, for an appropriately chosen SDF. Let z_t be an instrument representing aggregate PE fund activity. As an example, let z_t denote the number of funds available for investment at time t.[9] Instrumenting the Euler equation with z_t results in the moment condition, $\mathrm{E}_t[M_{t+1}R^m_{t+1}z_t] = z_t$, which states that the price of receiving $R^m_{t+1}z_t$ at $t+1$ is z_t. Applying this instrument is equivalent to risk-adjusting a managed strategy investing one dollar in the market for each active PE fund at time t. This strategy increases notional exposure to the market when more funds are available. If the instrument covaries with market returns, the SDF coefficients obtained by using the unconditional instrumented moment condition will differ from coefficients estimated using the constant instrument, $z_t = 1$. If these coefficients differ, it implies that PE funds are more (less) exposed to periods with high (low) average market returns relative to the sample average. Risk-adjusting such a managed strategy corresponds to risk-adjusting a cross-section of artificial funds investing period-by-period in the public market in the same periods as the PE funds being evaluated. This standard GMM approach differs from Korteweg and Nagel (2016) due to the payout function, which accounts for cash flow timing and compounding. Gredil et al. (2020) show that the Korteweg and Nagel (2016) methodology is a special case of the instrumented Euler equation approach using a particular instrument.

The weighting effect can alternatively be interpreted in terms of the moments determining SDF coefficients. Using an instrument capturing PE activity essentially localizes moment estimation to periods with more fund exposure. A potential issue arising from localizing the SDF coefficient estimation is that they can become excessively sensitive to market shocks in some periods. The sensitivity to market shocks can, in turn, lead to unreasonable estimates in small samples, such as a negative unconditional market price of risk. Using sufficiently long asset return time series generally means that a negative unconditional market price of risk does not occur. Replacing the equally weighted moments in Equation 3 with weighted moments, using weight z_t, localizes moments to periods with more fund activity. Differences in equally weighted and fund activity weighted moments directly reveal whether moments differ in periods with more fund exposure relative to a longer sample. Differences can furthermore indicate moments changing with time. The empirical analysis in Section 5 uses weighted moments to evaluate the effect of localizing the SDF coefficient estimation to periods relevant for the average buyout fund.

4.2 Time-Varying Coefficients

Individual Fund Estimates

To directly examine whether SDF coefficients vary over time, I consider a semi-parametric method, based on the constant coefficients SDF. Specifically, I estimate constant SDF coefficients

[9] Alternatively aggregate NAV can be used as the instrument.

in shorter windows within the full sample period. To see the intuition for estimating coefficients in shorter windows, consider risk-adjusting the cash flows of a single PE fund. The appropriate benchmark for the fund's payoffs is the average market risk premium in the investment period. I, therefore, estimate bespoke SDF coefficients for each fund using (1) the period from fund inception until the last cash flow and (2) the ten years succeeding fund inception. This guarantees that the SDF coefficients reflect the average equity premium in each fund's investment period. The fund-specific coefficients reveal whether SDF coefficients vary over time and how this affects risk-adjusted performance. Discounting fund cash flows with the fund-specific stochastic discount factors, $M^i_{t+1} = \exp(a^i - b^i r^m_{t+1})$, shows the effect on the value of cash flows of time-varying time discounting and risk price.

Using bespoke SDF coefficients for every fund is inconsistent with the existence of a single SDF correctly risk-adjusting cash flows. I, therefore, construct time-varying SDF coefficients from the individual fund coefficients. Specifically, I construct a_t and b_t as weighted averages of a^i and b^i for funds active at time t, by weighting individual fund coefficients with the inverse coefficient variances, estimated using GMM, thereby reducing the weight on poorly measured coefficients. The aggregate coefficients are essentially weighted averages of moments estimated in shorter windows within the full sample period.

Conditional Log-Normal Models

In addition to the semi-parametric models, I consider a log-normal model where the time-varying coefficient SDF satisfies the moment conditions $\mathrm{E}_t[M_{t+1}R^m_{t+1}] = 1$ and $\mathrm{E}_t[M_{t+1}R^f_t] = 1$. In this setting the coefficients are given by:[10]

$$a_t = -r^f_t + b_t\mathrm{E}_t[r^m_{t+1}] - \frac{1}{2}b^2_t\sigma^2_t[r^m_{t+1}] \tag{9}$$

$$b_t = \frac{-r^f_t + \mathrm{E}_t[r^m_{t+1}] + \frac{1}{2}\sigma^2_t[r^m_{t+1}]}{\sigma^2_t[r^m_{t+1}]} \tag{10}$$

The coefficients are identified by specifying a functional form for the conditional moments provided that a conditional risk-free asset exists. Constructing a_t and b_t from conditional moments ensures that the SDF correctly risk-adjusts the conditionally risk-free asset and market returns period by period. The SDF specification can be interpreted as a conditional log return CAPM. Figure 1 illustrates, together with the solution for b_t, that the SDF slope changes with the three quantities; (1) the conditional expected log market return, (2) the level of the conditional risk-free asset and

[10]See Appendix B for the derivation.

(3) the conditional variance.

I consider two log-normal models, both specifying the realized market return as an autoregressive process with additional state variables predicting future returns. The first specification imposes constant conditional variance. The second model uses time-varying conditional variance following a GARCH(1,1) process. The constant conditional variance log-normal model is defined by dynamics:

$$r_{t+1} = \mu + \boldsymbol{\theta}^\top \boldsymbol{z_t} + \epsilon_{t+1} \tag{11}$$

$$\epsilon_{t+1} = \sigma_{t+1} e_{t+1} \tag{12}$$

$$\sigma^2_{t+1} = \omega \tag{13}$$

Where $e_{t+1} \sim N(0,1)$ and $\boldsymbol{z_t}$ is a vector of state variables governing the conditional expected log market return. The state vector, $\boldsymbol{z_t}$, consists of the lagged market return as well as state variables predicting future market returns. The dynamics imply that the next period conditional expected log return is given by, $\mathrm{E}_t[r_{t+1}] = \mu + \boldsymbol{\theta}^\top \boldsymbol{z_t}$ while the conditional variance is constant. The coefficients a_t and b_t are defined given an observable proxy for the conditional risk-free return. The expected equity market return specification is consistent with a large literature modeling stock market returns using Vector Autoregressions such as Campbell (1996) and Campbell and Viceira (1999). I keep the estimation of expected log market returns simple, but more elaborate models for the expected market returns, such as those considered in Kelly and Pruitt (2013) and Kelly and Pruitt (2015), can also be applied.

The second log-normal specification models expected log-returns similarly, and the conditional variance is modeled as follows:

$$\epsilon_{t+1} = \sigma_{t+1} e_{t+1} \tag{14}$$

$$\sigma^2_{t+1} = \omega + \alpha \epsilon^2_t + \beta \sigma^2_t \tag{15}$$

The time-varying conditional variance captures how changing risk affects the SDF coefficients. An increase in the conditional market risk premium results in a higher market price of risk while an increase in conditional risk implies a decrease in the market price of risk. The market price of risk can thus be constant despite fluctuations in expected market returns.

Instrumental Variables Model

The log-normal models rely on strong distributional assumptions as well as the existence of an observable risk-free asset. As an alternative, I model coefficients directly as affine functions of state variables.[11] This model requires minimal assumptions but is less transparent. For instance, obtaining the conditional risk premium from the conditional market price of risk requires the conditional variance, which the model does not specify. Conceptually, the specification is a conditional log-return CAPM in the spirit of Black (1972). In the simple case, where coefficients are functions of a single state variable, the conditional one-factor stochastic discount factor has an equivalent three-factor representation:

$$M_{t+1} = \exp(a_t - b_t f_{t+1}) = \exp(a_0 + a_1 z_t - b_0 f_{t+1} - b_1 f_{t+1} z_t) \tag{16}$$

The three-factor representation of the conditional model illustrates the connection to the constant coefficients SDF. The conditional model collapses to the constant coefficients model when a_1 and b_1 are zero.

I estimate the SDF coefficients by imposing that the underlying factor and the returns of Treasury bills are risk-adjusted unconditionally in-sample. I furthermore impose orthogonality between the pricing errors and the instrument driving SDF coefficient variation, which leads to two additional restrictions and an exactly identified model. Because the moment conditions are satisfied in-sample dynamically rebalanced portfolios formed based on the state variable of the two basis assets, the public equity market and Treasury bills have zero average pricing error.

4.3 State Variables

I use a single state variable to keep the time-varying coefficient SDF models parsimonious. The asset pricing literature provides several candidate state variables predicting equity market returns. Aggregate valuations ratios are perhaps the most theoretically valid in terms of capturing variation in conditional expected market returns. Campbell and Shiller (1988) propose the dividend to price ratio as a natural candidate for modeling expected returns. I use the transformed public equity market dividend to price ratio, $z_t = \log(1+\frac{D_t}{P_t})$ proposed by Gao and Martin (2019) which provides

[11] See for example Cochrane (1996), Jagannathan and Wang (1996) and Lettau and Ludvigson (2001) for similar SDF specifications.

information on weighted future log market returns in excess of future log dividend growth:[12]

$$z_t = (1-\rho)\sum_{j=0}^{\infty}\rho^j[r_{t+1+j} - \Delta d_{t+1+j}] \tag{17}$$

If expected returns follow an AR(1) process with autocorrelation ϕ_z the expected log return in excess of expected dividend growth relative to the unconditional means can be expressed as:[13]

$$\mathrm{E}_t[r_{t+1} - g_{t+1}] - (\overline{r_{t+1}} - \overline{g_{t+1}}) = \frac{1-\rho\phi_z}{1-\rho}(z_t - \bar{z}_t) \tag{18}$$

The expression illustrates that conditional expected log market returns, relative to log dividend growth, are above the long-run average when the log dividend to price ratio is above its long-run average. Conditional expected log returns can thus be higher (lower) than long-run expected log returns for extended periods of time since the dividend to price ratio is relatively slow-moving. The relation implies that the price of risk in the stochastic discount factor models can diverge from the unconditional price of risk for long periods of time. This, in turn, affects the value of buyout fund cash flows realized during different periods.

4.4 NPV Inference

I consider aggregate risk-adjusted performance defined as the cross-sectional average fund NPV:

$$\mathrm{E}[NPV_j] = \frac{1}{N}\sum_{j=1}^{N}\sum_{h=0}^{H} M_{t,t+h}\cdot C_{j,t+h} \tag{19}$$

The null hypothesis for a given SDF specification is $\mathrm{E}[NPV_j] = 0$. I test the null hypothesis using a J-statistic which is asymptotically chi-square distributed with one degree of freedom. I take SDF coefficients as given and apply the spectral density matrix estimator from Korteweg and Nagel (2016) to adjust the moment standard error for cross-correlation in fund pricing errors.[14] The spectral density matrix estimator places additional weight on funds with overlapping investment periods. The weight on adjacent funds is determined by $\bar{d}$, which I set to 1.5 in the empirical analysis. I report both unadjusted and adjusted standard errors, but the reported J-statistics use the adjusted standard errors. I also report bootstrap confidence intervals for $\mathrm{E}[NPV_j]$.[15]

[12] $\rho = \exp(\bar{z}_t)$.

[13] See Gao and Martin (2019) for details.

[14] Boyer, Nadauld, Vorkink, and Weisbach (2021) follow a similar approach and evaluate the PE moment condition, taking model parameters as given.

[15] Appendix C provides details on the spectral density matrix estimator and bootstrap methodology.

5 Results

5.1 Public Market Equivalent

Table 2 presents average NPVs for the PME and GPME models. The first row in each panel reports the average NPV and the second row reports bootstrap confidence intervals. The third row reports the NPV standard error, assuming uncorrelated fund pricing errors. The fourth row reports standard errors corrected for fund cross-correlation and the last row reports the J-statistic using the corrected standard errors.

Table 2 column 1 reports PMEs for the two samples. The PME is approximately 0.20 in both samples. A PME of 0.20 means that buyout funds provide investors with a risk-adjusted profit of 20 cents for every dollar of committed capital. The PME estimates are significantly different from zero at the one percent level, which means that a marginal allocation to buyout funds provides significant value to a log-utility investor. The PMEs are similar to estimates in Gredil et al. (2020) using a larger sample of buyout funds. Since the PME is defined in differences it is not directly comparable to the Kaplan and Schoar (2005) PME. The average Kaplan and Schoar (2005) PME is approximately 1.23 which is in agreement with Harris et al. (2014) finding an average PME of 1.22 for buyout funds and Robinson and Sensoy (2016) reporting a PME of 1.19. Aggregate risk-adjusted performance, measured using the PME, is thus consistent with previous studies.

The unconditional maximal Sharpe ratio implied by the log-utility SDF provides some insight into why the log-utility model is not able to account for the performance of buyout funds. The annualized unconditional maximal Sharpe ratio is 0.15 in the sample period. The Sharpe ratio of buyout fund returns cannot be recovered from PMEs, but Ang et al. (2018) estimate buyout fund returns from cash flows which yields an annualized Sharpe ratio of approximately 0.50 in the period 1996 to 2014. The maximal Sharpe ratio is approximately 0.16 for the log-utility SDF in this period. The low maximal Sharpe ratio implies that buyout fund payoffs are outside the log-utility SDFs volatility bound.

5.2 Generalized Public Market Equivalent

Table 2 reports GPMEs using different methods to estimate the SDF coefficients. Column 2 and 3 report GPMEs using the log-normal solutions in Equation 3. Column 2 uses the sample average risk-free return, equity market return and variance to estimate coefficients. Column 3 uses weighted averages, weighting periods with the number of active buyout funds. The last column presents estimates using the methodology from Korteweg and Nagel (2016).

Column 2 shows that the GPME of fully liquidated funds is approximately 1.0 and 0.75 in the full sample. The risk-adjusted performance is seemingly considerably larger than implied by the PME. The difference between the PME and GPME is, however, primarily due to the SDF not accurately reflecting the average market return and the risk-free rate in periods important for the average buyout fund. In the section below, I show that the in-sample GPME becomes large when the SDF coefficients are jointly high. The SDF coefficients in column 2 are high relative to the other SDF specifications, which results in a high GPME.

Column 3 reports the GPME for the weighted moments specification. The GPME is 0.26 in the fully liquidated sample and statistically different from zero at the five percent level. The GPME is 14 cents per dollar of committed capital in the full sample but not statistically different from zero. The GPME difference, relative to the equally weighted coefficients, highlights the importance of weighting periods. Localizing coefficient estimates to periods with more buyout fund activity leads to a better fit to cash flows. Weighting periods, however, implicitly reduces the number of independent observations leading to less precise coefficient estimates, which is evident from the standard errors.

Column 4 presents GPMEs using the methodology from Korteweg and Nagel (2016) for comparison. The GPME is 0.27 and 0.30 in the fully liquidated and full samples, respectively. Both estimates are significantly different from zero at the five percent level. The fully liquidated sample GPME is almost identical to the methodology using weighted moments, while the difference is larger for the full sample.

GPME Sensitivity

To see precisely how changing SDF coefficients changes GPMEs, it is informative to evaluate the risk adjustment performed by the constant coefficients SDF. The average GPME can be expressed as:[16]

$$\mathrm{E}[GPME_j] = \sum_{h=0}^{H} \frac{1}{N} \sum_{j=1}^{N} M_{t,t+h} C_{j,t+h} = \sum_{h=0}^{H} \mathrm{E}[M_{t,t+h}]\mathrm{E}[C_{j,t+h}] + \mathrm{cov}(M_{t,t+h}, C_{j,t+h}) \qquad (20)$$

[16]Intuitively the moments $\mathrm{E}[\cdot]$ and $\mathrm{cov}(\cdot)$ are cross-sectional in the empirical implementation in Section 5, but $\mathrm{E}[C_{j,t+h}]$ can be interpreted as an estimate of the expected buyout fund cash flow at horizon h.

Suppose cash flows and cumulative log market returns are jointly normal at every horizon. Applying Stein's lemma to linearize the covariance at each horizon then leads to:[17,18]

$$\mathrm{E}[GPME_j] = \sum_{h=0}^{H} \mathrm{E}[M_{t,t+h}] \Big(\mathrm{E}[C_{j,t+h}] - b \cdot \mathrm{cov}(r^m_{t,t+h}, C_{j,t+h}) \Big) \tag{21}$$

The determinants of risk-adjusted performance are (1) the average SDF, (2) the market price of risk, (3) average buyout fund cash flows and (4) the covariance between cash flows and the market at each horizon. The average GPME represents the difference between average (expected) cash flows, $\mathrm{E}[C_{j,t+h}]$, and required (model implied) cash flows, $b \cdot \mathrm{cov}(r^m_{t,t+h}, C_{j,t+h})$, scaled by the average SDF and summed across horizons. The difference between average cash flows and required cash flows can be considered a certainty equivalent cash flow. Risk-adjusted performance is positive when scaled risk-adjusted cash flows, summed across horizons, are positive.

The SDF coefficients determine (1) the average SDF and (2) the price of risk. The SDF intercept changes the SDF mean at different horizons while the price of risk changes required cash flows and consequently the difference between expected and required cash flows, as well as the SDF mean. Average cash flows and the covariance between cash flows and the risk factor is determined directly by the data and not affected by changes in SDF coefficients. To determine how the average GPME changes with the SDF coefficients, under constraints imposed by public market moments, I (1) draw blocks of public market returns, (2) estimate SDF coefficients and (3) discount fund cash flows with an SDF using the resampled coefficients but the original sample risk factor realizations. Using resampled coefficients but in-sample factor realizations, alter the SDF's average value and the market price of risk across samples but preserves the original sample covariance and expected cash flows.[19]

Table 3 reports percentiles for average GPMEs resulting from discounting cash flows using the resampled coefficients. I resample public market returns and estimate SDF coefficients for different return frequencies to assess the effect of estimating SDF coefficients at different horizons. Column 1 reports the return frequency used to estimate the coefficients. A 1-month frequency means I use monthly returns to estimate coefficients, while one-year overlapping returns are used for a 1-year frequency. I use a long sample, starting in 1949, of public market returns and risk-free rates to accommodate long-horizon returns. Columns 2 to 4 report selected percentiles for the GPME.

[17] Stein's lemma states that for jointly normally distributed variables (Y, X) and a function $f(Y)$ the covariance can be written as, $\mathrm{cov}(f(Y), X) = \mathrm{E}[\frac{\partial f}{\partial Y}]^\top \mathrm{cov}(Y, X)$, if $f(Y)$ is a differentiable function such that $\mathrm{E}[|f'(Y)|] < \infty$.

[18] Stein's lemma implies the linearization, $\mathrm{cov}(M_{t,t+h}, C_{j,t+h}) = -b\mathrm{E}[M_{t,t+h}]\mathrm{cov}(r^m_{t,t+h}, C_{j,t+h})$ at each horizon, h. A similar linearization can be attained from a first-order Taylor approximation of the SDF at each horizon.

[19] Appendix D outlines the methodology in detail.

Column 5 reports the number of samples remaining after disregarding economically implausible SDF coefficients. Economically implausible samples are defined as samples resulting in an average SDF greater than one as this implies a negative risk-free interest rate. An average SDF greater than one will tend to grow cash flows over time as opposed to discount cash flows. Because buyout fund cash flows are negative in the first years of a fund's operations but positive towards the end of a fund's life, an average SDF greater than one will ascribe exceedingly high value to distributions. The last column reports the number of samples resulting in a negative average GPME.

Column 2 shows that the first percentile is positive across all frequencies. GPMEs range from approximately 0.17 to 0.20. The first percentile decreases moderately as the horizon increases. The same pattern holds across horizons for the 2.5th and 5th percentiles. The resampled GPMEs suggest that the duration (horizon) effect outlined in Section 4.1, which partly distinguishes the methodology used by Korteweg and Nagel (2016) from the methods used in this paper, only has a minor effect on average GPMEs. The results in Section 5.2, on the other hand, suggest that the average GPME changes significantly once the time-period weighting is modified.

The last column in Table 3 shows positive average GPME in every sample for horizons from 1 month to 10 years. The predominantly positive resampled GPMEs suggest that the covariance between cash flows and the public market is insufficient for equating required and expected cash flows in the constant coefficients SDF specification.

Figure 2 plots average GPMEs as a function of the SDF coefficients. The top figures show average GPMEs using monthly returns to estimate SDF coefficients. The bottom plots show the same using eight-year returns. The figure shows that both high and low coefficients result in high average GPMEs. The restriction imposed on the average SDF leads to samples with jointly high coefficients being disregarded. The U-shape means that the average GPME is especially high in these samples. The large average GPME reported in Section 5.2 arises as a consequence of this effect.

The figure furthermore illustrates that the PME is a special case of the GPME. The GPME is equivalent to the sample PME of approximately 0.20 for coefficient values $a = 0$ and $b = 1$. The number of samples in the region around these SDF coefficient values is relatively small, which means that the restrictions imposed by the log-utility SDF are satisfied only in a small number of samples. This result suggests that the log-utility SDF is not able to account for the sample average market return and risk-free interest rate.

Altogether the PME and GPME estimates suggest that buyout funds exhibit positive risk-adjusted performance. While Korteweg and Nagel (2016) find that positive risk-adjusted performance for venture capital funds using the PME and zero abnormal performance using the GPME, the results above suggest that changing SDF does not reduce the risk-adjusted performance of buyout funds.

5.3 Conditional Public Market Equivalent

Table 4 reports NPVs using fund specific coefficients to discount the cash flows of each fund. Column 1 reports NPVs arising from estimating the individual fund coefficients in the period from each fund's inception to the fund's last cash flow. Column 2 reports NPVs arising from estimating coefficients in the period from each fund's inception to the fund's last cash flow if it is less than ten years, otherwise using a ten-year estimation period. This estimation window adjusts for some funds having long lifetimes but economically insignificant cash flows at long horizons. Column 3 reports the NPV arising from using an aggregate SDF with time-varying coefficients constructed from the individual fund coefficients in the period from each fund's inception to the fund's last cash flow.

Column 1 shows NPVs of approximately 12 cents in the fully liquidated sample and 5 cents in the full sample. The NPV estimates are not significantly different from zero. The estimates show that letting coefficients reflect average market returns in each fund's investment period lowers NPVs.

Figure 3 plots fund specific coefficients for the full sample. The solid line represents within vintage year averages. Focusing on the second plot, the market price of risk is high for funds raised from 1986 to 1992 while the price of risk decreases substantially from 1993 to 2001 followed by an increase back to pre-1993 levels.[20] The price of risk essentially quantifies expected excess market returns per unit of variance and effectively represents the average Sharpe ratio in the ten years after fund inception. The plot shows variability in market Sharpe ratios across vintage years consistent with investors' required compensation for bearing risk changing with time.

Column 2 presents NPVs for an adjusted estimation period placing more weight on earlier periods in a fund's life. The NPV is -0.04 in the fully liquidated sample and -0.07 in the full sample however not statistically different from zero. It is an open question how to estimate risk prices for proper benchmarking of cash flows. It is, therefore, interesting that applying fund-specific SDF coefficients results in NPVs closer to zero as this implies that fund-specific risk prices result in a better fit to fund cash flows. The lower cross-sectional NPV variance compared to the GPME estimates supports this result.

[20]The coefficient time series are highly correlated because the SDF intercept depends on the market price of risk and average equity market return.

Column 3 reports NPVs for the SDF with time-varying coefficients constructed from individual fund coefficients. The NPV is 3 cents in the fully liquidated sample and negative 2 cents in the full sample. The estimates are not significantly different from zero. The results highlight that allowing coefficients to change across time lowers the risk-adjusted value of cash flows.

Conditional Log-Normal Models

Table 5 presents the forecasting equations determining the conditional log market return and market variance used to determine the SDF coefficients in the log-normal models. CPME denotes the conditional log-normal model with constant conditional variance and CPME GARCH denotes the log-normal model with time-varying variance. The mean equation for the log-normal model with constant conditional variance shows that the log dividend to price ratio positively predicts future market returns with a statistically significant coefficient of 0.72. Using $\log(1 + \frac{D_t}{P_t}) \approx \frac{D_t}{P_t}$ a 1 percentage point increase in the dividend to price ratio is approximately associated with a 0.72 percentage point increase in next period log market returns. The expression for b_t means that the conditional market price of risk is high when the dividend to price ratio is high, assuming the risk-free rate is unchanged.

Figure 4 plots the SDF coefficients during the sample period. The dynamics implied by the mean equation and the variability in the dividend to price ratio result in a high but decreasing market price of risk from the 1980s until the dot-com bubble in 2001. The declining market price of risk is a consequence of expected market returns declining more than the risk-free interest rate, which compresses the equity market premium. The decline results in a negative conditional market price of risk, and consequently a negative conditional risk premium, in this period. A negative conditional risk premium is not necessarily desirable, but it is a common property of linearly forecasting future market returns, see for example Lettau and Ludvigson (2001). The implied in-sample unconditional risk premium is positive since the conditional market price of risk is positive on average, and the conditional variance is strictly positive. After the year 2000 the price of risk increases and it spikes immediately after the 2008 financial crisis.

Table 6 column 1 reports NPVs for the constant conditional variance model. The NPV is negative 12 cents in the fully liquidated sample and statistically significant at the one percent level. The NPV means an investor with time-varying required compensation for risk loses 12 cents on a risk-adjusted basis by investing in buyout funds. The full sample NPV is negative 31 and statistically different from zero. The lower risk-adjusted performance in the full sample stems partly from funds with substantial investments during the 2008 financial crisis having lower risk-adjusted performance than earlier vintages, which I show in Section 5.4.

The constant and time-varying coefficients SDF specifications are reduced-form models, which makes it challenging to evaluate the models' economic credibility. Some basic economic checks can, however, provide insights into each model's credibility. Consider the SDF implied conditional maximal Sharpe ratio. Figure 5 plots the maximal Sharpe ratios for the log-normal model with constant variance (dotted line), the instrumental variables model (solid line) as well as the GPME (dashed line), and PME (dash-dotted line) models assuming constant conditional market variance. Two things are worth noticing. First, the market price of risk is higher for the GPME relative to the PME because $b > 1$. Secondly, the conditional log-normal model generates a time-varying conditional maximal Sharpe ratio, which is higher than the GPME and PME model implied Sharpe ratios in the first part of the sample and the period during and immediately after the financial crisis. The conditional models' maximal Sharpe ratios vary due to time variation in b_t leading to counter-cyclical conditional Sharpe ratios. The counter-cyclicality is consistent with models based on specifications of investor preferences, such as the external habit model of Campbell and Cochrane (1999) in which the conditional Sharpe ratio varies as a function of investor's surplus consumption ratio. The aggregate surplus consumption ratio is typically low in recessions leading to high conditional Sharpe ratios in these periods.

The variation and magnitude of the conditional SDF models' Sharpe ratios are moreover consistent with data-driven estimates of the market's conditional Sharpe ratio. The solid red line plots the Sharpe ratio implied by the lower bound on the equity premium introduced in Martin (2016).[21] The line plots the implied equity premium $\mathrm{E}_t[R^m_{t+1}] - R^f_t = R^f_t \cdot SVIX^2_{t+1}$ normalized by the sample standard deviation of market returns, which makes it comparable to the SDF implied Sharpe ratios. The plot shows that the variation in the SVIX implied Sharpe ratio is comparable with the variation in the SDF implied Sharpe ratios. The magnitudes are furthermore reasonable relative to the SVIX implied Sharpe ratio, which can be interpreted as a lower bound. One exception is the period from 2012 to 2016, where the SVIX implied Sharpe ratio is below the conditional models' Sharpe ratios. The implied Sharpe ratio of the conditional log-normal model appears particularly high relative to the SVIX in this period.

Table 5 reports the mean equation for the time-varying volatility model. The forecasting equation differs from the constant variance model because the dividend to price ratio correlates with changes in the time-varying variance. The dividend to price ratio coefficient decreases compared to the constant variance model. The coefficient is positive and statistically significant at the ten

[21] Thanks to Paul Whelan for providing an updated SVIX time series.

percent level. The forecasting equation implies that a one percentage point increase in the log dividend to price ratio leads to a 0.54 percentage point increase in the next period's log market return.

Figure 6 plots SDF coefficients. Focusing on the conditional market price of risk both the constant and time-varying variance specifications imply declines from 1986 to 2001 and increases immediately after 2001. The conditional risk prices differ in 2008-2009, where the market price of risk in the constant variance model increases due to a dividend to price ratio increase. Accounting for time-varying variance, the dividend to price ratio increase is offset by an increase in market variance resulting in an essentially unchanged price of risk during the financial crisis, in contrast to the constant variance model.

Column 2 in Table 6 reports the NPV for the time-varying variance specification. The top panel reports a 2 cents NPV for the sample of fully liquidated funds. The J-statistic is low because the point estimate is approximately zero. Based on the NPV, the time-varying volatility model appears to fit average buyout fund cash flows better than the constant variance model. The second panel reports an NPV of -0.11 for the full sample. The J-statistic is low in this sample due to a large NPV standard error.

The results suggest that buyout funds provide more value to investors if the market price of risk is partly determined by time-varying market variance. The differences in the dynamics of the market price of risk in the constant and time-varying volatility models suggest the NPV differences arise partly from the 2008 financial crisis. Section 5.4 shows the differences in the discounted value of cash flows during this period. Altogether the log-normal models imply negative or zero risk-adjusted performance.

Instrumental Variables Model

Table 5 panel 3 reports coefficients for the instrumental variables model. To ease coefficient interpretation, I demean the transformed dividend to price ratio such that the conditional market price of risk is given by, $b_t = b_0 + b_1(z_t - \overline{z})$. The unconditional market price of risk, b_0, is 3.14. The coefficient, b_1, on the interaction between the market return and the state variable is 247.037. The approximate relationship between the dividend to price ratio and $\log(1 + \frac{D}{P})$ means that a one percentage point increase in the dividend to price ratio is associated with a conditional market price of risk increase of 2.47. The conditional market price of risk is 1.91 higher than the unconditional market price of risk when the log dividend to price ratio is one standard deviation above the sample mean.

Figure 7 plots coefficients during the sample period. The evolution across time is smoother relative to the conditional log-normal models because the market price of risk only is a function of the dividend to price ratio. The conditional market price of risk is positive throughout the sample period. Interestingly, the variation in coefficients is qualitatively similar to the variation in individual fund coefficients. Figure 8 plots individual fund coefficients for each vintage year. The dotted line shows the average within vintage coefficient values. The black line plots yearly values for the fitted coefficients in the instrumental variables model using the first state variable observation in each year. The figure shows the similarity between the market price of risk dynamics in the instrumental variables model and the vintage year averages. The price of risk for each vintage is essentially the market Sharpe ratio in the subsequent 10-year period. The similarity between the time series reflects the dividend to price ratio's ability to predict 10-year realized Sharpe ratios and that the instrumental variable model captures these dynamics.

Table 6 column 3 reports NPVs. The average risk-adjusted profit in the fully liquidated sample is negative 5 cents and negative 1 cent for the full sample. Both estimates are significantly indistinguishable from zero. The NPV estimates suggest that myopic investors, requiring time-varying compensation for bearing risk, value buyout fund payoffs fairly.

5.4 NPV Decomposition

Figure 9 plots the average value of discounted cash flows, distributions and contributions across the sample period for the (G)PME and CPME specifications. The top plot shows the contribution to the average NPV from each period. Aggregating the average discounted cash flows across time periods results in the PME and GPME (KN) specifications in Table 2 Panel B and the CPME specifications in Table 6 Panel B.

The plot shows that variation in discounted cash flows during and immediately after the 2008 financial crisis leads to lower average NPVs in the CPME models relative to the (G)PME models. The lower CPMEs are due to higher SDF realizations in the conditional models coupled with large contributions, relative to distributions, in the period. The second plot shows that the value of distributions is low in the CPME models from 2004 to 2008 and moderately high around 2012. The last figure shows that the value of contributions is higher (more negative) for the CPME models around the 2008 financial crisis. The net effect of the differences in discounted distributions and contributions is lower CPMEs compared to (G)PMEs.

Figure 9 also illustrates the differences between the conditional models. The NPV produced by the log-normal model with constant variance (CPME) is low compared to the other conditional models. The top figure shows that the discounted cash flows resemble the other conditional specifications until 2008. Immediately after the financial crisis, the NPV drops more in the log-normal model, relative to the other conditional models, while the rebound in NPV in 2012-2016 is absent in the log-normal constant volatility model. These dynamics produce the lower average NPV in the constant volatility specification. The conditional maximal Sharpe ratio implied by the log-normal specification, depicted in Figure 5 (dotted line), is significantly higher than the other SDF specifications in the period 2012-2016 and the maximal Sharpe ratio is markedly higher than the SVIX implied maximal Sharpe ratio. The maximal Sharpe ratios implies that the log-normal specification perhaps overly discounts cash flows compared with the other conditional models.

6 Robustness

6.1 Valuation Ratios

To assess robustness, I consider two alternative valuation ratios for the state variable used in the CPME specifications, the book to market ratio and the log cyclically adjusted price to earnings ratio (CAPE). The book to market ratio is from Amit Goyal's data library, and the price to earnings ratio is constructed following the CAPE methodology using nominal prices and earnings from Robert Shiller's data library. The price to earnings ratio is log-transformed.

Table 7 reports NPVs. The CPME estimates in the top panel are approximately zero and statistically indistinguishable from zero. The robustness results resemble the main results except for the constant variance log-normal model in column one. The risk-adjusted performance is approximately zero for the cyclically adjusted price to earnings ratio, whereas the CPME is significantly negative using the dividend to price ratio.

The NPVs in the bottom panel also resemble the main results. CPMEs are significantly negative for the price to earnings ratio and the book to market ratio for the log-normal model with constant variance. The NPV arising from using the price to earnings ratio is approximately the same as the NPV in the specification using the dividend to price ratio. The book to market ratio results in slightly higher, albeit negative CPME, which is significantly different from zero. The CPME estimates are not significantly different from zero in the conditional log-normal model with time-varying variance or the instrumental variables model. The point estimates for the instrumental variables model are slightly higher than the estimates in the main analysis. The NPV difference between the dividend- and earnings-based valuation ratios is small compared to the difference relative to the book-to-market ratio. The difference is partly due to book to market ratio dynamics during the 2008 crisis. The dividend- and earnings-based valuation ratios increase markedly immediately after the crisis, which signals higher expected returns. The book to market ratio increase is, on the other hand, more moderate and followed by a reversal.

6.2 Net Asset Values

To assess the impact of including NAVs in the terminal cash flow of partially liquidated funds, I consider the possibility that NAVs are under- or overstated. Table 8 reports average NPVs for the PME and CPME specifications. The top panel reports NPVs applying a discount of 90 percent to the terminal NAV, while the second panel reports risk-adjusted performance assuming NAVs are 10 percent higher than reported. The top panel shows a small decrease in NPVs, verifying that terminal NAV changes do not significantly affect the main conclusions. The second panel shows that a 10 percent increase in NAVs leads to a minor increase in average risk-adjusted performance. Altogether, the robustness tests support the conclusions of the main analysis.

7 Conclusion

This paper uses stochastic discount factors with time-varying coefficients to study the risk-adjusted performance of buyout funds. Conceptually, the stochastic discount factors represent the marginal rate of substitution of a myopic investor timing the public equity market based on conditional expected market returns and the short-term risk-free interest rate. The stochastic discount factors incorporate a time-varying market risk price and short-term interest rate such that buyout funds incepted in different vintage years are faced with different required rates of return.

Empirically, I find significantly positive risk-adjusted buyout fund performance for stochastic discount factor models with constant market price or risk. I, however, show that market prices vary considerably over time which validates the relevance of using time-varying coefficient stochastic discount factors. Incorporating time variation in market risk prices, I find significantly lower risk-adjusted performance relative to the constant price of risk models. Valuing buyout fund cash flows using a conditional log-normal SDF, in which the market risk price changes proportionally to expected excess market returns, results in significantly negative risk-adjusted buyout fund performance. Letting the risk price vary with conditional market risk results in buyout fund performance consistent with public markets. Applying a conditional log-return CAPM model, I furthermore find that buyout funds exhibit zero abnormal profits.

Despite some variation in the risk-adjusted performance estimates across time-varying coefficients stochastic discount factor specifications, the results suggest that risk-adjusted buyout fund performance decreases compared to measures previously considered in the literature. Altogether, the time-varying coefficient SDF specifications imply that buyout funds do not improve the risk-return trade-off of a myopic investor with time-varying required compensation for bearing risk.

Table 1
Summary Statistics

The table reports TVPIs across vintage years. The total number of funds in the fully liquidated sample is $N = 162$ and $N = 387$ for the full sample. The table reports the TVPI mean, standard deviation and selected percentiles for each vintage year.

	Fully Liquidated						All Funds					
Vintage	No. Funds	Mean	Std.	25%	50%	75%	No. Funds	Mean	Std.	25%	50%	75%
1984	1	3.7060		3.7060	3.7060	3.7060	1	3.7060		3.7060	3.7060	3.7060
1985	2	1.9610	0.9914	1.6105	1.9610	2.3115	2	1.9610	0.9914	1.6105	1.9610	2.3115
1986	1	2.9440		2.9440	2.9440	2.9440	1	2.9440		2.9440	2.9440	2.9440
1987	5	4.1918	1.8288	3.2190	4.4859	5.5930	5	4.1918	1.8288	3.2190	4.4859	5.5930
1988	4	2.0065	0.6396	1.5346	1.8245	2.2964	4	2.0065	0.6396	1.5346	1.8245	2.2964
1989	2	2.8785	1.7416	2.2628	2.8785	3.4943	2	2.8785	1.7416	2.2628	2.8785	3.4943
1990	5	2.3139	0.7630	2.3551	2.3800	2.4710	5	2.3222	0.7758	2.3551	2.3800	2.4710
1991	2	2.2105	0.5855	2.0035	2.2105	2.4175	2	2.2692	0.5025	2.0915	2.2692	2.4468
1992	6	1.9262	1.5567	0.8067	1.7540	3.1955	6	1.9262	1.5567	0.8067	1.7540	3.1955
1993	8	2.5238	0.9164	1.9816	2.2485	3.4197	8	2.5238	0.9164	1.9816	2.2485	3.4197
1994	14	2.0140	0.8370	1.3725	2.0252	2.2555	14	2.0140	0.8370	1.3725	2.0252	2.2555
1995	11	1.2566	0.6575	0.8165	1.2024	1.6410	11	1.2566	0.6575	0.8165	1.2024	1.6410
1996	16	1.4616	0.7142	0.7878	1.7350	1.9295	16	1.4616	0.7142	0.7878	1.7350	1.9295
1997	18	1.4873	0.4851	1.1173	1.5197	1.8110	20	1.4487	0.4855	1.0306	1.4781	1.7491
1998	22	1.3457	0.6307	0.8768	1.2655	1.5410	34	1.4019	0.5912	0.9145	1.3278	1.6715
1999	14	1.5197	0.6751	1.1499	1.4448	1.9080	26	1.5396	0.6153	1.1513	1.5425	1.9952
2000	15	2.1191	0.6391	1.6380	2.3361	2.5308	32	1.9634	0.5824	1.5005	2.0206	2.4253
2001	7	1.6468	0.7126	1.2647	1.7194	1.8399	19	1.8875	0.5452	1.6622	1.7593	2.1932
2002	1	2.0681		2.0681	2.0681	2.0681	15	1.8530	0.6780	1.3035	1.8807	2.2658
2003	2	0.6119	0.8434	0.3137	0.6119	0.9101	14	1.7186	0.6719	1.4847	1.7443	1.9625
2004	2	0.8199	0.6725	0.5821	0.8199	1.0577	20	1.7103	0.6483	1.4589	1.6875	1.8657
2005	3	1.0832	0.0797	1.0402	1.0690	1.1190	37	1.4852	0.4391	1.1770	1.4173	1.7389
2006	0						46	1.5186	0.4495	1.3347	1.5507	1.7459
2007	1	0.9414		0.9414	0.9414	0.9414	44	1.6022	0.4711	1.3850	1.5159	1.7451
2008	0						3	1.2150	0.3085	1.0889	1.3069	1.3870

Table 2
Buyout Fund Performance: PME and GPME

The table reports average NPVs. PME denotes average discounted cash flows using a log-utility SDF. GPME (Equal) utilize equal weighted moments to estimate SDF coefficients using the log-normal coefficient solution in Equation 3. GPME (Weighted) uses weighted moments and the number of active buyout funds in each time period as weights. GPME (KN) denotes the NPV resulting from using the methodology from Korteweg and Nagel (2016). Standard errors are in square brackets. Coefficient standard errors in the last column follows the methodology in Korteweg and Nagel (2016). For GPME (Equal) and GPME (Weighted) coefficient standard errors are estimated from a non-parametric block bootstrap with a block size of 5 years and 5000 samples. The unadjusted NPV standard errors assume uncorrelated funds, while the adjusted standard errors account for cross-correlation using $\bar{d} = 1.5$ for fund overlap. The J-statistics use the adjusted standard errors and are reported in brackets. The confidence intervals are bootstrap confidence intervals taking into account cross-correlation between funds. Confidence intervals are estimated using the percentile method. ***,**,* denotes significance at the 1, 5 and 10-percent level.

	PME	GPME (Equal)	GPME (Weighted)	GPME (KN)
		Panel A: Fully Liquidated Funds ($N = 162$)		
NPV	0.2049***	1.0620***	0.2644**	0.2676***
C.I 95%	[0.045 , 0.367]	[0.616 , 1.537]	[0.064 , 0.471]	[0.071 , 0.476]
Unadjusted	[0.0390]	[0.2000]	[0.0640]	[0.0510]
Adjusted	[0.0399]	[0.3910]	[0.1201]	[0.0994]
J-statistic	(26.35)	(7.38)	(4.84)	(7.25)
Parameters				
a	0	0.0160	0.0059	0.0056**
		[0.0136]	[0.0156]	[0.0026]
b	1	3.5211**	2.3039	2.1719***
		[1.6171]	[2.1129]	[0.4206]
		Panel B: All Funds ($N = 387$)		
NPV	0.2021***	0.7580***	0.1437	0.2984**
C.I 95%	[0.027 , 0.397]	[0.350 , 1.176]	[-0.058 , 0.353]	[0.042 , 0.579]
Unadjusted	[0.0227]	[0.1055]	[0.0639]	[0.0469]
Adjusted	[0.0350]	[0.2811]	[0.1201]	[0.1393]
J-stat	(33.34)	(7.27)	(2.31)	(4.59)
Parameters				
a	0	0.0160	0.0060	0.0103**
		[0.0136]	[0.0142]	[0.0041]
b	1	3.5211**	2.4799	3.0179***
		[1.6171]	[2.2758]	[0.5889]

Table 3
GPME Sensitivity

The table reports selected percentiles for the distribution of average GPMEs resulting from discounting fund cash flows with resampled SDF coefficients following the methodology in Appendix D.1. The initial number of bootstrap samples is 25000. The restriction on the average SDF described in Appendix D.1 results in the lower sample size reported. The column "Frequency" reports the frequency of returns used to estimate the SDF coefficients. Columns two to four report percentiles of the average GPME distribution. Column four shows the number of samples and column five the number of samples in which the average GPME is negative

	E[$GPME$]				
Frequency	1st percentile	2.5th percentile	5th percentile	N	$N < 0$
1 month	0.1978	0.2001	0.2043	2179	0
1 year	0.2001	0.2024	0.2049	4697	0
2 years	0.2006	0.2024	0.2052	5236	0
3 years	0.1990	0.2022	0.2057	4456	0
4 years	0.1974	0.2020	0.2061	3679	0
5 years	0.1923	0.1988	0.2045	2845	0
6 years	0.1899	0.2001	0.2053	2213	0
7 years	0.1882	0.1970	0.2035	1737	0
8 years	0.1825	0.1912	0.2012	1541	0
9 years	0.1783	0.1893	0.1979	1705	0
10 years	0.1803	0.1916	0.1996	1804	0
11 years	0.1730	0.1851	0.1961	1723	1
12 years	0.1730	0.1872	0.1966	1464	2

Table 4
Buyout Fund Performance using Individual Fund SDFs

The table reports average discounted cash flows. Column one (Full Period) reports the NPV allowing SDF coefficients to vary across funds. The time period from fund inception to the last cash flows is used as the estimation window for the SDF coefficients. The second column uses an adjusted estimation window, specifically a ten year period starting from each fund's inception. The third column reports the results of combining the individual fund parameters from the first column at every point in time using inverse variance weights. Standard errors are in square brackets. The NPV standard errors assumes independent fund pricing errors, while the adjusted standard errors take into account cross-correlation with $\bar{d} = 1.5$. The J-statistics for the NPV estimates are reported in brackets and use the adjusted standard errors. Confidence intervals are estimated by bootstrapping discounted cash flows. ***,**,* denotes statistical significance at the 1, 5 and 10-percent level.

	Full Period	Adj. Period	CPME IVW
	Panel A: Fully Liquidated Funds ($N = 162$)		
NPV	0.1220	-0.0364	0.0302
C.I 95%	[-0.047 , 0.292]	[-0.174 , 0.108]	[-0.115 , 0.178]
Unadjusted	[0.0445]	[0.0328]	[0.0404]
Adjusted	[0.0401]	[0.0397]	[0.0682]
J-statistic	(2.31)	(0.83)	(0.20)
	Panel B: All Funds ($N = 387$)		
NPV	0.0469	-0.0749	-0.0196
C.I 95%	[-0.137 , 0.235]	[-0.235 , 0.084]	[-0.174 , 0.140]
Unadjusted	[0.0297]	[0.0248]	[0.0256]
Adjusted	[0.0798]	[0.0535]	[0.0645]
J-statistic	(0.35)	(1.96)	(0.09)

Table 5
Time-Varying Coefficient SDFs Estimation

The table reports results for the mean and conditional variance estimation used in the conditional log-normal SDF models. The table also reports SDF coefficients for the instrumental variables model. The top panel reports the mean equations for the conditional log-normal models. The log market return r^m_{t+1} is the dependent variable and the transformed dividend to price ratio $\log(1 + \frac{D_t}{P_t})$ is the independent variable. The CPME regression is estimated by OLS using monthly returns in the buyout fund sample period. Standard errors are Newey-West standard errors with 12 lags. The CPME GARCH mean equation is estimated using Maximum Likelihood with normally distributed errors, the standard errors are robust. The middle panel reports the conditional variance estimates. Standard errors are in square brackets and t-statistics are in brackets. The bottom panel reports coefficients for the instrumental variable SDF $M_{t+1} = \exp(a_0 + a_1(z_t - \overline{z}) - b_0 r^m_{t+1} - b_1 r^m_{t+1}(z_t - \overline{z}))$. The coefficients are estimated in the buyout fund sample period using GMM. Standard errors are Newey-West standard errors with 12 lags. Standard errors are in square brackets and t-values are in brackets. ***,**,* denotes significance at the 1, 5 and 10-percent level.

	Mean Equation				
	Constant	r^m_t	z_t	Adj R^2	N
CPME	-0.0081	0.0651	0.7200***	1.4279%	376
	[0.0062]	[0.0766]	[0.2266]		
	(-1.31)	(0.85)	(3.18)		
CPME GARCH	-0.0029	-0.0075	0.5448*	0.7921%	376
	[0.0071]	[0.0636]	[0.3139]		
	(-0.41)	(-0.12)	(1.74)		
	Conditional Variance				
	ω	α	β		
CPME	0.001908*				
	[0.002314]				
	(8.25)				
CPME GARCH	0.000049	0.1490*	0.8435*		
	[0.000040]	[0.0425]	[0.0409]		
	(1.25)	(3.50)	(20.62)		
	Instrumental Variables Model				
	a_0	a_1	b_0	b_1	
Parameters	0.01998	3.2738**	3.1447**	247.0370*	
	[0.0125]	[1.6378]	[1.4401]	[142.3330]	
	(1.59)	(2.00)	(2.18)	(1.74)	

Table 6
Buyout Fund Performance: CPME

The table reports average NPVs for the conditional SDF models. CPME denotes the conditional log-normal model with constant variance, CPME GARCH denotes the model with time-varying variance while GPME Ins. denotes the instrumental variables model. Standard errors assuming funds are uncorrelated are reported and adjusted standard errors taking into account cross-correlation from overlapping funds are also reported, using $\bar{d} = 1.5$ for fund overlap. Standard errors are in square brackets. The J-statistic for the NPV is reported in brackets, using the adjusted standard errors. ***,**,* denotes statistical significance at the 1, 5 and 10-percent level.

	CPME	CPME GARCH	CPME Ins.
	Panel A: Fully Liquidated Funds ($N = 162$)		
NPV	-0.1232^{***}	0.0192	-0.0452
C.I 95%	[-0.228 , -0.021]	[-0.128 , 0.172]	[-0.158 , 0.071]
Unadjusted	[0.0273]	[0.0487]	[0.0390]
Adjusted	[0.0162]	[0.0818]	[0.0577]
J-statistic	(57.76)	(0.06)	(0.62)
	Panel B: All Funds ($N = 387$)		
NPV	-0.3125^{***}	-0.1061	-0.0039
C.I 95%	[-0.465 , -0.193]	[-0.428 , 0.212]	[-0.310 , 0.295]
Unadjusted	[0.0241]	[0.0580]	[0.0373]
Adjusted	[0.0822]	[0.1427]	[0.0661]
J-statistic	(14.46)	(0.55)	(0.00)

Table 7
Other Valuation Ratios

The table reports NPVs using the conditional stochastic discount factors with alternative state variables. CPME denotes the conditional log-normal model with constant variance, the CPME GARCH denotes the model with time-varying variance while GPME Ins. denotes the instrumental variables model. The table reports NPVs using the cyclically adjusted price earnings ratio and the book-to-market ratio from Welch and Goyal (2008) as the state variables. Standard errors assuming fund pricing errors are uncorrelated are reported and adjusted standard errors taking into account cross-correlation from overlapping funds using $\bar{d} = 1.5$ are also reported. Standard errors are in square brackets. The J-statistic is reported in brackets using the adjusted standard errors. ***,**,* denotes statistical significance at the 1, 5 and 10-percent level.

	CAPE			**BM**		
	CPME	CPME GARCH	CPME Ins.	CPME	CPME GARCH	CPME Ins.
	Panel A: Fully Liquidated Funds ($N = 162$)					
NPV	-0.0142	-0.0056	0.0262	0.0073	0.0139	-0.0602
C.I 95%	[-0.139 , 0.116]	[-0.149 , 0.142]	[-0.106 , 0.169]	[-0.114 , 0.134]	[-0.121 , 0.156]	[-0.163 , 0.048]
Unadjusted	[0.0394]	[0.0449]	[0.0497]	[0.0327]	[0.0382]	[0.0339]
Adjusted	[0.0395]	[0.0614]	[0.0852]	[0.0362]	[0.0544]	[0.0596]
J-statistic	(0.129)	(0.01)	(0.09)	(0.04)	(0.07)	(1.02)
	Panel B: All Funds ($N = 387$)					
NPV	-0.3149^{***}	-0.1265	0.0691	-0.1867^{**}	-0.0454	0.1131
C.I 95%	[-0.515 , -0.162]	[-0.537 , 0.280]	[-0.285 , 0.413]	[-0.275 ,-0.102]	[-0.288 , 0.212]	[-0.132 , 0.404]
Unadjusted	[0.0333]	[0.0723]	[0.0454]	[0.0201]	[0.0467]	[0.0314]
Adjusted	[0.1214]	[0.1564]	[0.0858]	[0.0796]	[0.0941]	[0.0726]
J-statistic	(6.73)	(0.65)	(0.65)	(5.49)	(0.23)	(2.42)

Table 8
Net Asset Values

The table reports average NPVs for the PME and CPME specifications using the full sample of funds. Panel A reports NPVs for terminal NAVs which are 90 percent of the reported NAVs. Panel B shows the same when terminal NAVs are 110 percent of reported NAVs. Standard errors assuming fund pricing errors are uncorrelated are reported and adjusted standard errors taking into account cross-correlation from overlapping funds using $\bar{d} = 1.5$ are also reported. Standard errors are in square brackets. The J-statistic is reported in brackets using the adjusted standard errors. ***,**,* denotes statistical significance at the 1, 5 and 10-percent level.

	PME	CPME	CPME GARCH	CPME Ins.
		Panel A: 90%		
NPV	0.1919***	−0.3141***	-0.1115	-0.0169
C.I 95%	[0.0266 , 0.387]	[-0.472 , -0.193]	[-0.444 , 0.206]	[-0.322 , 0.278]
Unadjusted	[0.0228]	[0.0241]	[0.0487]	[0.0376]
Adjusted	[0.0375]	[0.0827]	[0.1445]	[0.0681]
J-statistic	(26.26)	(14.41)	(0.06)	(0.06)
		Panel B: 110%		
NPV	0.2122***	−0.3109***	-0.1006	0.0091
C.I 95%	[0.035 , 0.413]	[-0.468 , -0.189]	[-0.428 , 0.225]	[-0.304 , 0.331]
Unadjusted	[0.0226]	[0.0240]	[0.0577]	[0.0370]
Adjusted	[0.0328]	[0.0816]	[0.1409]	[0.0645]
J-statistic	(41.83)	(14.52)	(0.51)	(0.02)

Figure 1
Conditional Log-Return CAPM

The figure illustrates conditional expected log asset returns as a function of the covariance between the log asset return and the public equity market; the risk factor underlying the stochastic discount factor. The example uses an annualized log risk free rate, r_t^f of 2 percent, which defines the intercept. The annualized market standard deviation is 20 percent and the expected log market return plus the Jensen's inequality correction is 14 percent. The figure also plots an asset with a conditional correlation of one and the same conditional variance as the market, resulting in the conditional asset covariance being equal to the market variance. The conditional expected log return of the asset is 16 percent. The risk factor exposure leads to required return of 14 percent and an annualized conditional alpha of 2 percent.

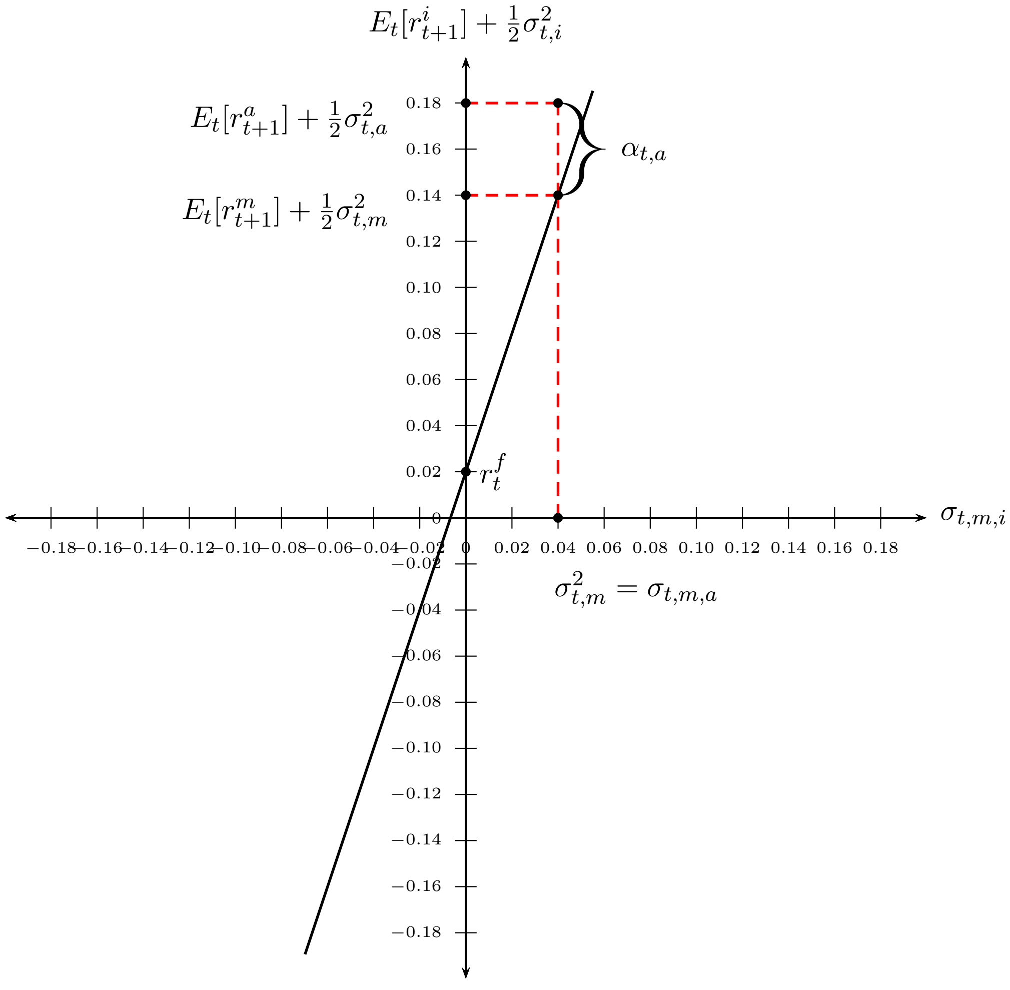


Figure 2
GPME Sensitivity

The figure plots the average GPME for the fully liquidated sample using resampled stochastic discount factor coefficients. Coefficients are estimated via GMM using returns resampled from a long sample starting in 1949. Samples where the fund-weighted average stochastic discount factor is above one are discarded. See Appendix D for details on the methodology. The top figure plots the average GPME as a function of a and b, using monthly returns to estimate parameters. The bottom plots shows the same using eight year returns to estimate the SDF coefficients.

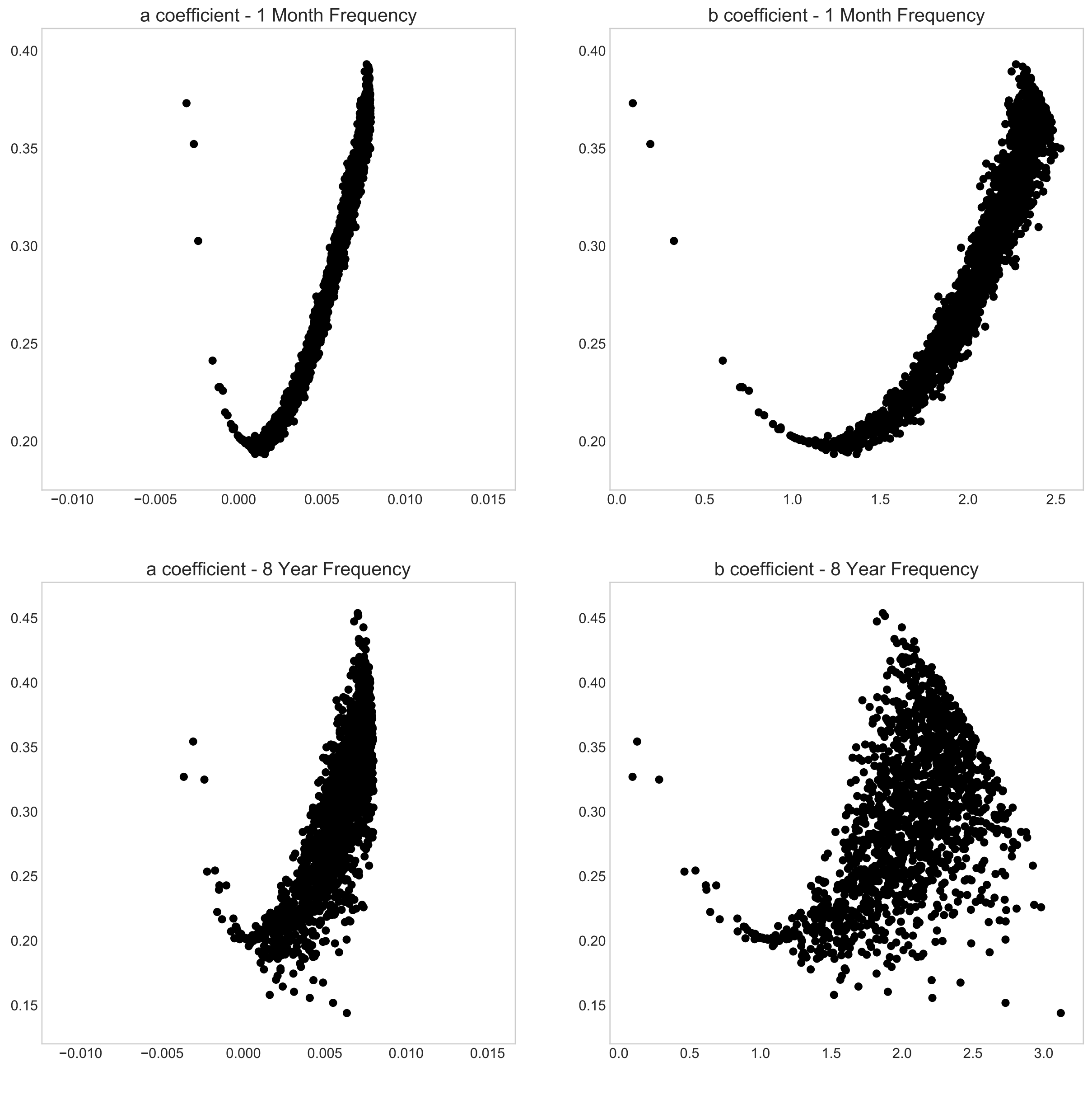


Figure 3
Stochastic Discount Factor Coefficients for Individual Funds

The figure shows stochastic discount factor coefficients for individual funds. Each dot represents the coefficient resulting from estimating the constant coefficients SDF parameters in $M_{t+1} = \exp(a - br^m_{t+1})$ using a 10-year period starting from the inception of the given fund. The figure shows coefficient estimates for the full sample of buyout funds grouped by vintage year. The x-axis shows fund vintage year and the solid line plots within vintage year averages.

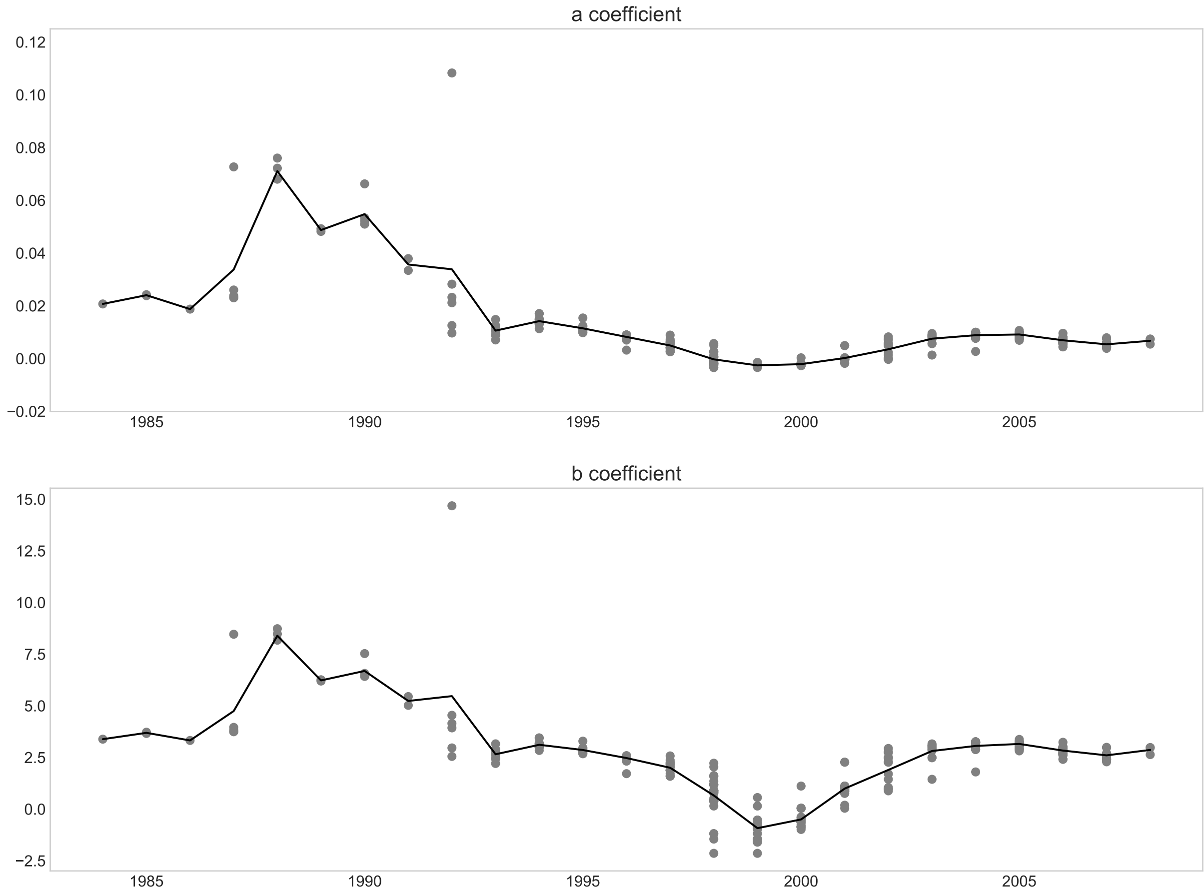


Figure 4
Log-Normal Constant Volatility SDF Coefficients

The figure plots the time variation in the stochastic discount factor coefficients throughout the sample period for the log-normal model with constant conditional variance. The figures plot the stochastic discount factor intercept and slope:

$$a_t = -r_t^f + b_t \mathrm{E}_t[r_{t+1}^m] - \frac{1}{2} b_t^2 \sigma_t^2 [r_{t+1}^m]$$
$$b_t = \frac{-r_t^f + \mathrm{E}_t[r_{t+1}^m] + \frac{1}{2}\sigma_t^2[r_{t+1}^m]}{\sigma_t^2[r_{t+1}^m]}$$

The conditional market return is given by $\mathrm{E}_t[r_{t+1}] = \mu + \boldsymbol{\theta}^\top \boldsymbol{z_t}$ in Table 5 and the conditional variance is constant. The top figure plots the SDF intercept a_t and the bottom plot shows the SDF slope, b_t.

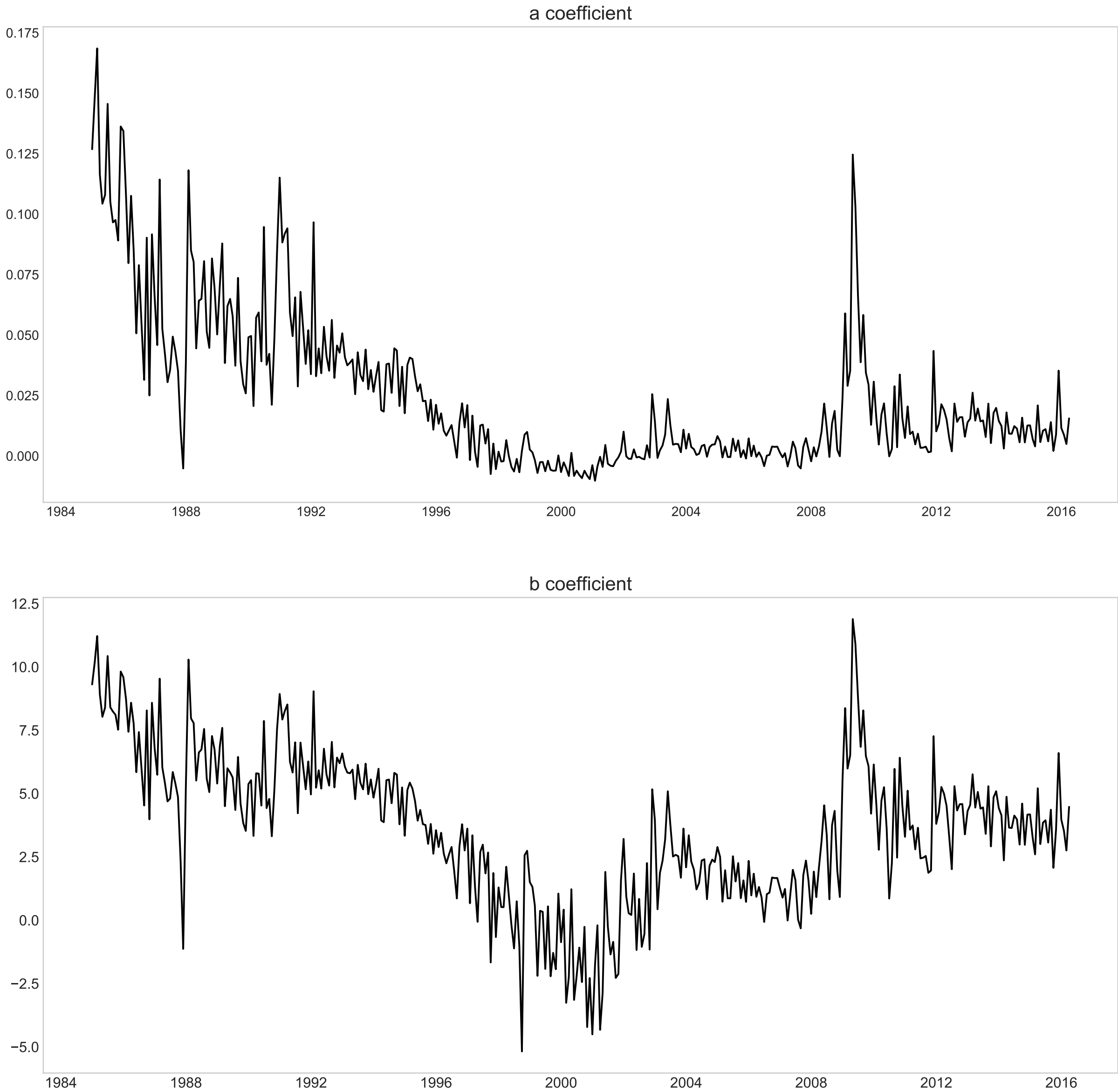


Figure 5
Maximal Sharpe Ratios

The figure plots the annualized maximal Sharpe ratio:

$$\frac{\sigma_t[M_{t+1}]}{\mathrm{E}_t[M_{t+1}]} = \sqrt{\exp\left(b_t^2 \sigma_t^2[r_{t+1}^m]\right) - 1}$$

The figure plots the maximal Sharpe ratios implied by the PME, GPME and CPME stochastic discount factors for constant variance, $\sigma_t^2[r_{t+1}^m]$, equal to the sample market variance. The solid line plots the maximal Sharpe ratio for the instrumental variables model and the dotted line plots the same for the conditional log-normal model with constant variance. The dashed and dash dotted lines plots the maximal Sharpe ratios for the PME and GPME models with b_t equal to 1 and 3.02 respectively. The solid red line plots the market Sharpe resulting from using the equity premium, $\mathrm{E}_t[R_{t+1}^m] - R_t^f = R_t^f \cdot SVIX_{t+1}^2$, from Martin (2016) and the sample equity market variance.

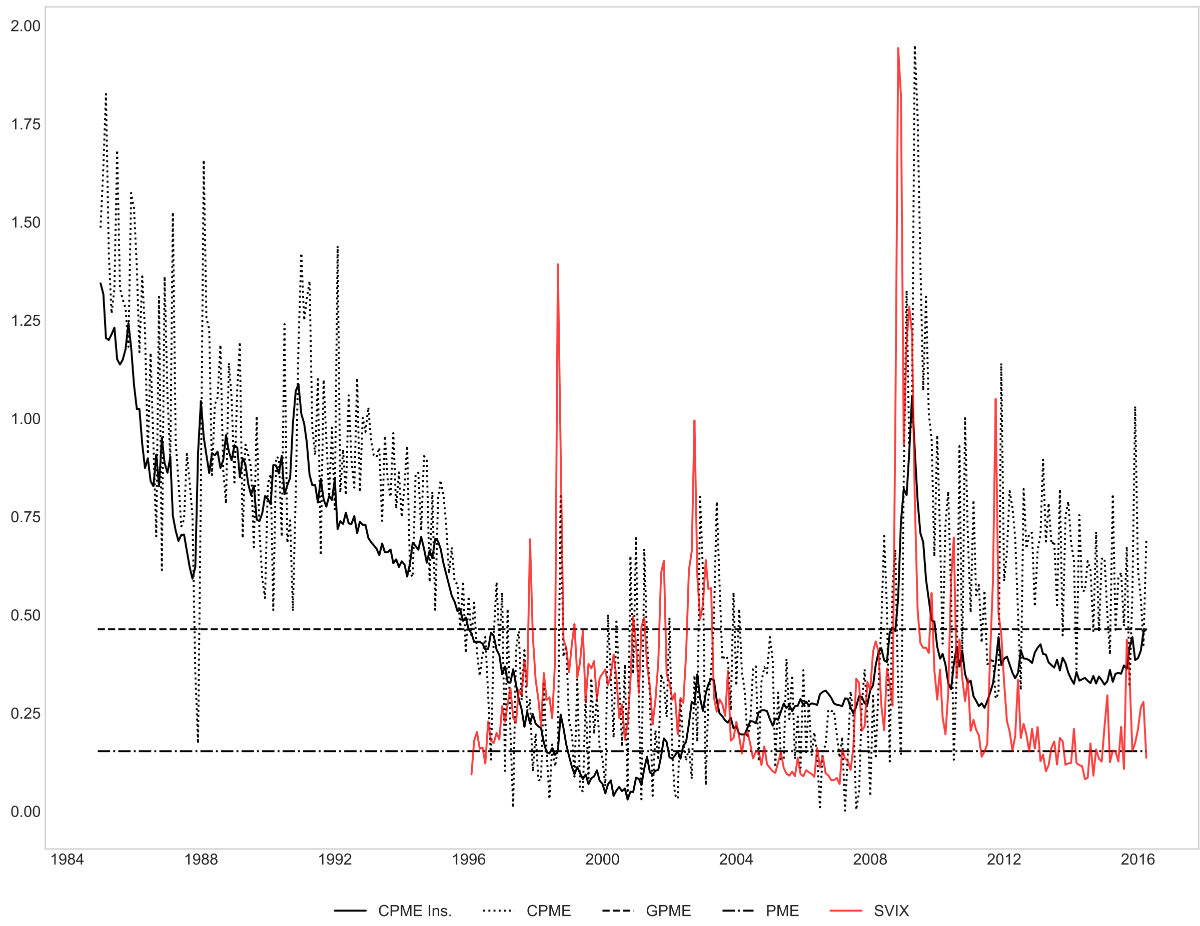


Figure 6
Log-Normal Time-Varying Volatility SDF Coefficients

The figure plots time variation in the stochastic discount factor coefficients throughout the sample period for the log-normal model with time-varying conditional variance. The conditional market return is given by $\mathrm{E}_t[r_{t+1}] = \mu + \boldsymbol{\theta}^\top \boldsymbol{z_t}$ and the conditional variance follows a GARCH(1,1) process given by the coefficients in Table 5. The top figure plots the SDF intercept, a_t, and the bottom plot shows the SDF slope, b_t.

Figure 7
Instrumental Variables Model SDF Coefficients

The figure plots time variation in the stochastic discount factor coefficients for the instrumental variables specification using stochastic discount factor:

$$M_{t+1} = \exp(a_t - b_t f_{t+1}) = \exp(a_0 + a_1 z_t - b_0 f_{t+1} - b_1 f_{t+1} z_t)$$

The top figure plots $a_t = a_0 + a_1 z_t$ and the bottom plot shows $b_t = b_0 + b_1 z_t$ using the coefficient estimates in Table 5.

Figure 8
Individual Fund and Instrumental Variables Model Coefficients

The figure plots the individual fund SDF coefficients depicted in Figure 3 and the coefficients implied by the stochastic discount factor:

$$M_{t+1} = \exp(a_t - b_t f_{t+1}) = \exp(a_0 + a_1 z_t - b_0 f_{t+1} - b_1 f_{t+1} z_t)$$

The dots represent individual fund coefficients plotted by vintage year. The dotted line plots within vintage year average SDF coefficients. The solid line plots the SDF coefficients implied by the instrumental variables stochastic discount factor at the beginning of each vintage year.

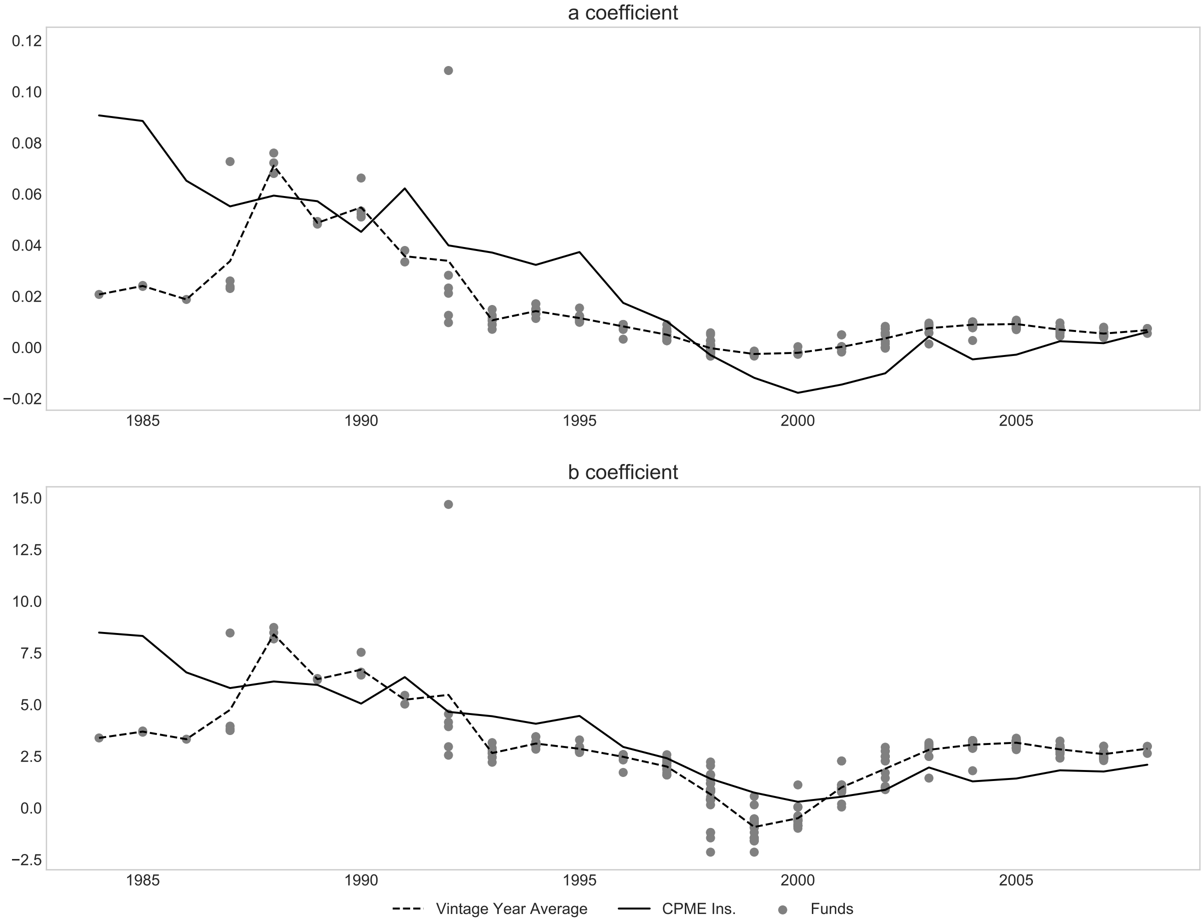
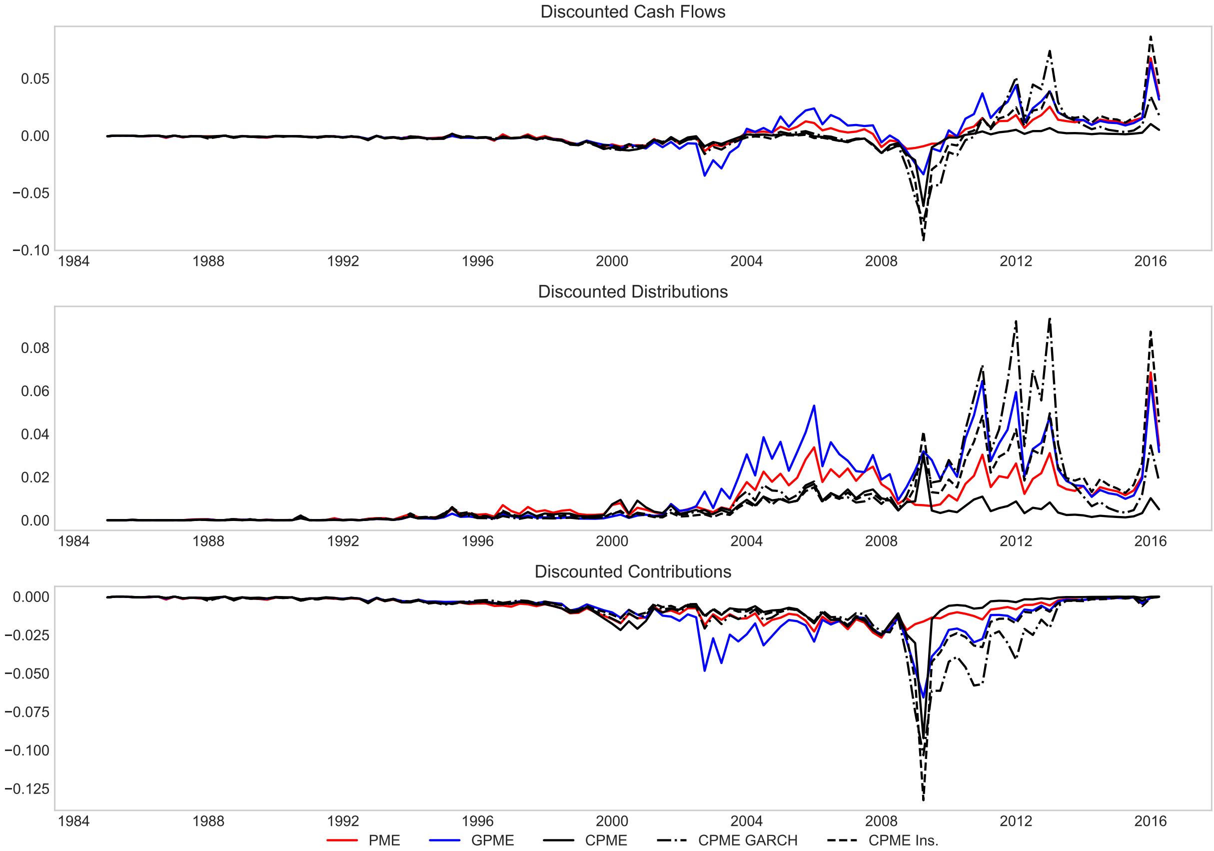


Figure 9
Average NPVs

The figure plots contributions from different time periods to average NPVs for the full sample of buyout funds. The sum across time periods in the top plot yields the average risk-adjusted value of cash flows. The sum across time periods in the second plot results in the average value of distributions while last plot yields the average value of contributions. The top plot is given by the sum of the second and third figure. The solid red line plots discounted values for the PME, the solid blue line for the GPME (KN) specification in Table 2 Panel B. The solid black line (CPME) represents the conditional log-normal model with constant volatility, the dash-dotted line (GPME GARCH) the conditional log-normal model with time-varying volatility and the dashed line (CPME Ins.) represents the instrumental variables specification.



Appendix

A Maximal Sharpe Ratio

Consider the stochastic discount factor with time-varying coefficients:

$$M_{t+1} = \exp(a_t - b_t r^m_{t+1}) \tag{A.1}$$

Assuming conditional log-normality and pricing the market and risk-free asset, the coefficients are given by (See Appendix B):

$$a_t = -r^f_t + b_t \mathrm{E}_t[r^m_{t+1}] - \frac{1}{2} b_t^2 \sigma_t^2 [r^m_{t+1}] \tag{A.2}$$

$$b_t = \frac{-r^f_t + \mathrm{E}_t[r^m_{t+1}] + \frac{1}{2}\sigma_t^2[r^m_{t+1}]}{\sigma_t^2[r^m_{t+1}]} \tag{A.3}$$

$$\tag{A.4}$$

Inserting into the stochastic discount factor:

$$M_{t+1} = \exp\Big(- r^f_t - \frac{1}{2} b_t^2 \sigma_t^2 [r^m_{t+1}] - b_t (r^m_{t+1} - \mathrm{E}_t[r^m_{t+1}])\Big) \tag{A.5}$$

Using the properties of the log-normal distribution, for $X \sim \mathcal{N}(\mu, \sigma^2)$:

$$\mathrm{E}[\exp(X)] = \exp(\mu + \frac{1}{2}\sigma^2) \tag{A.6}$$

$$\sigma^2[\exp(X)] = (\exp(\sigma^2) - 1) \exp(2\mu + \sigma^2) \tag{A.7}$$

Conditional log-normality implies that the conditional mean and conditional standard deviation of the stochastic discount factor is given by:

$$\mathrm{E}_t[M_{t+1}] = \exp(-r_t^f) \tag{A.8}$$

$$\sigma_t[M_{t+1}] = \sqrt{\exp(b_t^2\sigma_t^2[r_{t+1}^m]) - 1}\sqrt{\exp(-2r_t^f)} \tag{A.9}$$

The maximal Sharpe ratio is therefore:

$$\frac{\sigma_t[M_{t+1}]}{\mathrm{E}_t[M_{t+1}]} = \frac{\sqrt{\exp(b_t^2\sigma_t^2[r_{t+1}^m]) - 1}\sqrt{\exp(-2r_t^f)}}{\exp(-r_t^f)} = \sqrt{\exp(b_t^2\sigma_t^2[r_{t+1}^m]) - 1} \tag{A.10}$$

And the squared maximal Sharpe ratio is:

$$\left(\frac{\sigma_t[M_{t+1}]}{\mathrm{E}_t[M_{t+1}]}\right)^2 = \exp(b_t^2\sigma_t^2[r_{t+1}^m]) - 1 \tag{A.11}$$

A.1 Constant Coefficients Model

The constant coefficient stochastic discount factor is the special case where b does not vary with time:

$$\left(\frac{\sigma_t[M_{t+1}]}{\mathrm{E}_t[M_{t+1}]}\right)^2 = \exp(b^2\sigma_t^2[r_{t+1}^m]) - 1 \tag{A.12}$$

A.2 Instrumental Variables Model

The instrumental variables model is the case where $b_t = b_0 + b_1 z_t$. The maximal squared Sharpe ratio is therefore:

$$\left(\frac{\sigma_t[M_{t+1}]}{\mathrm{E}_t[M_{t+1}]}\right)^2 = \exp\left(b_0^2\sigma_t^2[r_{t+1}^m] + b_1^2 z_t^2\sigma_t^2[r_{t+1}^m] + 2b_0b_1z_t\sigma_t^2[r_{t+1}^m]\right) - 1 \tag{A.13}$$

In the instrumental variables model time variation in the conditional maximal Sharpe ratio is driven by time variation in the instrument as well as the conditional variance of the factor return and scales with b_0 and b_1.

A.3 Expected Excess Returns

The maximal Sharpe ratio is related to expected asset returns through the Euler equation:

$$1 = \mathrm{E}_t[M_{t+1}R^i_{t+1}] \tag{A.14}$$

$$1 = \mathrm{E}_t[M_{t+1}]\mathrm{E}_t[R^i_{t+1}] + \mathrm{cov}_t(M_{t+1}, R^i_{t+1}) \tag{A.15}$$

Using $\mathrm{E}_t[M_{t+1}] = 1/R^f_t$:

$$\mathrm{E}_t[R^i_{t+1}] - R^f_t = -\frac{\mathrm{cov}_t(M_{t+1}, R^i_{t+1})}{\mathrm{E}_t[M_{t+1}]} \tag{A.16}$$

$$\mathrm{E}_t[R^i_{t+1}] - R^f_t = -\sigma_t[R^i_{t+1}]\frac{\sigma_t[M_{t+1}]}{\mathrm{E}_t[M_{t+1}]}\rho_t[M_{t+1}, R^i_{t+1}] \tag{A.17}$$

Where ρ_t denotes, a potentially time-varying, correlation and $\sigma_t[M_{t+1}]/\mathrm{E}_t[M_{t+1}]$ denotes the conditional maximal Sharpe ratio for a given stochastic discount factor. The expected excess asset return depends on the asset's conditional volatility, the maximal Sharpe ratio and the correlation between the asset and the stochastic discount factor. Using the maximal Sharpe ratio in Equation A.11 for the SDF, $M_{t+1} = \exp(a_t - b_t r^m_{t+1})$, expected excess returns are given by:

$$\mathrm{E}_t[R^i_{t+1}] - R^f_t = -\sigma_t[R^i_{t+1}]\sqrt{\exp(b^2_t\sigma^2_t[r^m_{t+1}]) - 1}\,\rho_t[M_{t+1}, R^i_{t+1}] \tag{A.18}$$

Using the first-order Taylor approximation $\exp(x) \approx 1 + x$, arising from approximating $\exp(x)$ around zero, leads to the following approximation of the maximal Sharpe ratio:

$$\sqrt{\exp(b^2_t\sigma^2_t[r^m_{t+1}]) - 1} \approx b_t\sigma_t[r^m_{t+1}] \tag{A.19}$$

Approximating the function x^{-b} around one yields $x^{-b} \approx 1 - b(x-1)$. The SDF can then be approximated as:

$$M_{t+1} = \exp(a_t - b_t r^m_{t+1}) \approx \exp(a_t)(1 - b_t(R^m_{t+1} - 1)) \tag{A.20}$$

The correlation between the SDF and the asset can therefore be approximated as:

$$
\begin{aligned}
\rho_t[\exp(a_t - b_t r^m_{t+1}), R^i_{t+1}] &\approx \rho_t[\exp(a_t)(1 - b_t(R^m_{t+1} - 1)), R^i_{t+1}] && \text{(A.21)}\\
&\approx \frac{\mathrm{cov}_t(\exp(a_t)(1 - b_t(R^m_{t+1} - 1)), R^i_{t+1})}{\sigma_t[R^i_{t+1}]\sigma_t[\exp(a_t)(1 - b_t(R^m_{t+1} - 1))]} && \text{(A.22)}\\
&\approx \frac{-\exp(a_t) b_t \mathrm{cov}_t(R^M_{t+1}, R^i_{t+1})}{\sigma_t[R^i_{t+1}]\exp(a_t) b_t \sigma_t[R^m_{t+1}]} && \text{(A.23)}\\
&\approx -\rho_t[R^i_{t+1}, R^m_{t+1}] && \text{(A.24)}
\end{aligned}
$$

Using A.19 and A.24 in Equation A.18 leads to the following expected excess returns:

$$
\mathrm{E}_t[R^i_{t+1}] - R^f_t \approx \underbrace{b_t \sigma_t[r^m_{t+1}]}_{\text{Approx. Max SR}} \ \sigma_t[R^i_{t+1}]\rho_t[R^i_{t+1}, R^m_{t+1}] \tag{A.25}
$$

For the same conditional asset risk and correlation between the asset and the risk-factor underlying the SDF, the asset's conditional expected excess return fluctuates with the conditional maximal Sharpe ratio. The asset's Sharpe ratio is proportional to the maximal Sharpe ratio.

B Conditional Log-Return CAPM

Let the one-period stochastic discount factor with time-varying coefficients be given by:

$$M_{t+1} = \exp(a_t - b_t r^m_{t+1}) \tag{B.1}$$

Using $\mathrm{E}_t[M_{t+1}] = 1/R^f_t$, the natural logarithm of the expected stochastic discount factor is given by:

$$\mathrm{E}_t[m_{t+1}] + \frac{1}{2}\sigma^2_t[m_{t+1}] = -r^f_t \tag{B.2}$$

Assuming the underlying risk factor is conditionally log-normally distributed the conditional expected stochastic discount factor is given by:

$$a_t - b_t\mathrm{E}_t[r^m_{t+1}] + \frac{1}{2}b^2_t\sigma^2_t[r^m_{t+1}] = -r^f_t \tag{B.3}$$

$$a_t = -r^f_t + b_t\mathrm{E}_t[r^m_{t+1}] - \frac{1}{2}b^2_t\sigma^2_t[r^m_{t+1}] \tag{B.4}$$

Imposing that the stochastic discount factor risk-adjusts the underlying asset return results in:

$$1 = \mathrm{E}_t[M_{t+1}R^m_{t+1}] \tag{B.5}$$

$$0 = \mathrm{E}_t[m_{t+1} + r^m_{t+1}] + \frac{1}{2}\sigma^2_t[m_{t+1} + r^m_{t+1}] \tag{B.6}$$

$$0 = a_t - b_t\mathrm{E}_t[r^m_{t+1}] + \mathrm{E}_t[r^m_{t+1}] + \frac{1}{2}(b^2_t\sigma^2_t[r^m_{t+1}] + \sigma^2_t[r^m_{t+1}] - 2b_t\sigma^2_t[r^m_{t+1}]) \tag{B.7}$$

Using a_t leads to:

$$b_t = \frac{\mathrm{E}_t[r^m_{t+1}] - r^f_t + \frac{1}{2}\sigma^2_t[r^m_{t+1}]}{\sigma^2_t[r^m_{t1}]} \tag{B.8}$$

Resulting in the stochastic discount factor:

$$M_{t+1} = \exp\Big(- r^f_t - \frac{1}{2}b^2_t\sigma^2_t[r^m_{t+1}] - b_t(r^m_{t+1} - \mathrm{E}_t[r^m_{t+1}])\Big) \tag{B.9}$$

To see what the stochastic discount factor means for expected returns consider the restrictions simultaneously:

$$1 = \mathrm{E}_t[\exp(a_t - b_t r^m_{t+1}) R^m_{t+1}] \tag{B.10}$$

$$1 = \mathrm{E}_t[\exp(a_t - b_t r^m_{t+1}) R^f_t] \tag{B.11}$$

Leading to the set of equations:

$$0 = a_t - b_t \mathrm{E}_t[r^m_{t+1}] + \mathrm{E}_t[r^m_{t+1}] + \frac{1}{2}(b_t^2 \sigma_t^2[r^m_{t+1}] + \sigma_t^2[r^m_{t+1}] - 2b_t \sigma_t^2[r^m_{t+1}]) \tag{B.12}$$

$$0 = a_t - b_t \mathrm{E}_t[r^m_{t+1}] + r^f_t + \frac{1}{2} b_t^2 \sigma_t^2[r^m_{t+1}] \tag{B.13}$$

In vector form:

$$\begin{bmatrix} \mathrm{E}_t[r^m_{t+1}] + \frac{1}{2}\sigma_t^2[r^m_{t+1}] \\ r^f_t \end{bmatrix} = (b_t \mathrm{E}_t[r^m_{t+1}] - a_t - \frac{1}{2} b_t^2 \sigma_t^2[r^m_{t+1}]) \begin{bmatrix} 1 \\ 1 \end{bmatrix} + b_t \begin{bmatrix} \mathrm{cov}_t(r^m_{t+1}, r^m_{t+1}) \\ 0 \end{bmatrix} \tag{B.14}$$

This is a cross-sectional regression with $R^2 = 100\%$ of the expected log market return and the conditional risk-free asset onto (1) a constant and (2) the covariances between the risk factor underlying the stochastic discount factor and the two test assets. Since the first test asset is the underlying factor itself the covariance equals the factor variance. The second test asset is the conditional risk-free asset, which has zero conditional covariance with the factor. The regression coefficients are $(b_t \mathrm{E}_t[r^m_{t+1}] - a_t - \frac{1}{2} b_t^2 \sigma_t^2)$ and b_t respectively. Using the expression for a_t expected log returns are expressed as a function of the conditional risk-free rate and the conditional market price of risk b_t:

$$\begin{bmatrix} \mathrm{E}_t[r^m_{t+1}] + \frac{1}{2}\sigma_t^2[r^m_{t+1}] \\ r^f_t \end{bmatrix} = r^f_t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + b_t \begin{bmatrix} \mathrm{cov}_t(r^m_{t+1}, r^m_{t+1}) \\ 0 \end{bmatrix} \tag{B.15}$$

In the simple two dimensional case, expected log-returns are given by a regression equation with intercept r^f_t and slope b_t, with the conditional covariance between the risk factor and asset returns being the explanatory variable. Figure 1 shows an example.

Consider a test asset with log-return r^i_{t+1}. The conditional required return of the asset, given the exposure to the stochastic discount factor, is $y^i_{t+1} = r^f_t + b_t \mathrm{cov}_t(r^m_{t+1}, r^i_{t+1})$. The conditional expected (average) log return is, $\mathrm{E}_t[r^i_{t+1}] + \frac{1}{2}\sigma_t^2[r^i_{t+1}]$. The conditional pricing error is the difference between the expected (average) return and the required return given the asset's exposure to the market and is given by, $\alpha_{i,t} = \mathrm{E}_t[r^i_{t+1}] + \frac{1}{2}\sigma_t^2[r^i_{t+1}] - y^i_{t+1}$. The moment restrictions ensures the conditional pricing error of the market is zero at each point in time. The conditional stochastic discount factor therefore price all asset for which the conditional covariance times the conditional price of risk lines up with conditional expected (average) log-returns.

C Standard Errors and Confidence Intervals

I estimate the spectral density matrix of fund pricing errors following Korteweg and Nagel (2016). They show that the spectral density matrix can be decomposed into a matrix, Γ, of correlations and a diagonal matrix, Λ, of pricing error variances:

$$S = \Lambda^{\frac{1}{2}} \Gamma \Lambda^{\frac{1}{2}} \tag{C.1}$$

The dimensions of the matrices equals the number of assets being priced. The general case of multiple assets is illustrated below but the dimension is one in the main analysis. In the general case the estimator for Γ is given by:

$$\hat{\Gamma} = \left[\frac{1}{N} \sum_{i=1}^{N} \operatorname{diag}(u_i \circ u_i) \right]^{-\frac{1}{2}} \left(\frac{1}{N} \sum_{i=1}^{N} u_i' u_i \right) \left[\frac{1}{N} \sum_{i=1}^{N} \operatorname{diag}(u_i \circ u_i) \right]^{-\frac{1}{2}} \tag{C.2}$$

Where u_i is the pricing error of fund i and $\circ$ denotes the Hadamard product. For the variance, Λ, the degree of overlap between funds matter. The overlap between funds i and k is given by the distance:

$$d(i,k) = 1 - \frac{\min[t(i) + h(i), t(k) + h(k)] - \max[t(i), t(k)]}{\max[t(i) + h(i), t(k) + h(k)] - \min[t(i), t(k)]} \tag{C.3}$$

Applying Bartlett-type weights the estimator for the variance matrix is given by:

$$\hat{\Lambda} = \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{N} \max\left(1 - d(i,k)/\bar{d}, 0\right) \operatorname{diag}(u_i \circ u_k) \tag{C.4}$$

Where $\bar{d}$ determines the weight on adjacent funds. I use $\bar{d} = 1.5$. These estimators leads to the estimator for the spectral density matrix:

$$\hat{S} = \hat{\Lambda}^{\frac{1}{2}} \hat{\Gamma} \hat{\Lambda}^{\frac{1}{2}} \tag{C.5}$$

In the empirical analysis the spectral density matrix is estimated using demeaned buyout fund pricing errors which means the dimension of the spectral density matrix is one. The standard errors reported in the main text, which are not adjusted for overlapping funds, are determined by the variance of the buyout fund moment scaled with $\frac{1}{N}$ hence the moment covariance matrix Σ is given by; $\Sigma = \frac{1}{N} S$.

In the case adjusting for correlation between overlapping funds I used the spectral density estimator above. I determine the moment covariance matrix as; $\Sigma = \frac{1}{N} S$ using the adjusted spectral density matrix.

Because I do not use artificial funds along the lines of Korteweg and Nagel (2016) the standard errors above differ from fully specified errors accounting for pricing error correlation and how moments change with changes in parameters. To see how the standard errors above differ let K denote the number of moments. Let W denote a weighting matrix with ones in the diagonal in the entries corresponding to the artificial funds and epsilon weight in the entry corresponding to the price equity moment. Let d denote the matrix of partial derivatives of moment conditions with respect to parameters and S the full spectral density matrix. In the fully specified model the moment covariance matrix is given by:

$$\Sigma = \frac{1}{N}\Big(I - d(d^\top W d)^{-1} d^\top W\Big) S \Big(I - W d(d^\top W d)^{-1} d^\top\Big) \tag{C.6}$$

Let $A_1 = d(d^\top W d)^{-1} d^\top W$ and $A_2 = W d(d^\top W d)^{-1} d^\top$ and write the covariance matrix as:

$$\Sigma = \frac{1}{N}\Big(S - A_1 S - S A_2 + A_1 S A_2\Big) \tag{C.7}$$

The expression shows the difference between the fully specified moment covariance matrix and using the spectral density matrix, taken parameters as given, is determined by, $-A_1 S - S A_2 + A_1 S A_2$. The fully specified standard error is higher than the standard error taking coefficients as given if the entry corresponding to the private equity moment is positive.

C.1 Bootstrap

The bootstrap procedure used to estimate NPV confidence intervals is as follows. First, the parameters for a given stochastic discount factor are estimated in the original sample. The cash flows are then discounted using these parameter values, resulting in a T by N panel of discounted fund cash flows. To preserve cross-correlation between funds the entire cross-section is resampled. Rows of the panel of discounted cash flows are sampled with replacement K times. The cross-sectional average E$[NPV_k]$ in each bootstrap sample yields the bootstrap distribution for E$[NPV]$. This distribution provides the basis for the confidence intervals of the average NPV. The confidence interval is therefore conditional on the estimated parameter values. The percentile method is used to estimate the 95 percent confidence interval for a given stochastic discount factor specification.

D GPME Sensitivity Methodology

The methodology samples public market returns and risk free interest rates resulting in K artificial samples. In each sample stochastic discount factor parameters are estimated using GMM and the moment conditions:

$$1 = \mathrm{E}[M_{t+h} R^{m}_{t+h}] \tag{D.1}$$

$$1 = \mathrm{E}[M_{t+h} R^{f}_{t-1+h}] \tag{D.2}$$

M_{t+h} is the multi-period stochastic discount factor and R_{t+h} denotes the exponential cumulative log return on the market and a one-period risk-free asset. Overlapping observations are used to estimate parameters for $h > 1$. I use returns of different frequencies to evaluate whether stochastic discount factor parameters differ across horizons. Specifically returns are resampled at a monthly frequency ($h = 1$) as well as at lower frequencies for horizons from one to twelve years. Using long-horizon returns requires resampling large blocks of returns to maintain possible long-horizon autocorrelation. A block-length equal to the frequency of returns is used, i.e the block length is 12 months for a yearly horizon, for instance. To ensure observations are resampled with equal probability I use non-parametric circular block resampling.

D.1 Economically Infeasible Coefficients

The non-parametric nature of the procedure leads to extreme stochastic discount factor parameters in some samples. To exclude economically unreasonable parameter values, the stochastic discount factor mean is used to discard infeasible samples. I estimate the mean stochastic discount factor for each fund in a sample using the given funds' investment period and I deem a sample admissible if the cross-sectional average of the stochastic discount factor means is below one, otherwise the sample is discarded. A mean below one means the implied risk-free rate is positive on average.

E Supporting Output

Table E.1
Robustness: Log Price to Earnings Estimation

The table reports the mean and conditional variance estimations for the conditional log-normal models and coefficients for the instrumental variables model. The top panel reports the mean equations for the conditional log-normal models. The log market return r_{t+1}^m is the dependent variable and the log cyclically adjusted price to earnings ratio is the state variable. The regression is estimated by OLS using monthly returns in the buyout fund sample period. Standard errors are Newey-West standard errors with 12 lags. The CPME GARCH mean equation is estimated using Maximum Likelihood with normally distributed errors, the standard errors are robust. The middle panel reports the conditional variance estimates. Standard errors are in square brackets and t-statistics are in brackets. The bottom panel reports coefficient estimates for the instrumental variable stochastic discount factor $M_{t+1} = \exp(a_0 + a_1 z_t - b_0 r_{t+1}^m - b_1 r_{t+1}^m z_t)$. The parameters are estimated using GMM and standard errors are Newey-West standard errors with 12 lags, these are in square brackets and t-values are in brackets. ***,**,* denotes statistical significance at the 1, 5 and 10-percent level.

	Mean Equation				
	Constant	r_t^m	z_t	Adj R^2	N
CPME	0.0640**	0.0612	−0.0171**	1.04%	376
	[0.0242]	[0.0767]	[0.0075]		
	(2.64)	(0.80)	(-2.28)		
CPME GARCH	0.0686*	-0.0139	-0.0181	0.45%	376
	[0.0390]	[0.0640]	[0.0117]		
	(1.78)	(-0.22)	(-1.54)		
	Conditional Variance				
	ω	α	β		
CPME	0.001916*				
	[0.002320]				
	(8.26)				
CPME GARCH	0.0000463	0.1532*	0.8426*		
	[0.000038]	[0.0428]	[0.0409]		
	(1.23)	(3.58)	(20.58)		
	Instrumental Variables Model				
	a_0	a_1	b_0	b_1	
Parameters	0.0181	−0.0763*	3.1402**	-5.8800	
	[0.0124]	[0.0455]	[1.4518]	[3.6874]	
	(1.46)	(-1.68)	(2.16)	(-1.59)	

Table E.2
Robustness: Book to Market Estimation

The table reports the mean and conditional variance estimations for the conditional log-normal models and coefficients for the instrumental variables model. The top panel reports the mean equations for the conditional log-normal models. The log market return r^m_{t+1} is the dependent variable and the book to market ratio from Welch and Goyal (2008) is the state variable. The CPME regression is estimated by OLS using monthly returns in the buyout fund sample period. Standard errors are Newey-West standard errors with 12 lags. The CPME GARCH mean equation is estimated using Maximum Likelihood with normally distributed errors, the standard errors are robust. The middle panel reports the conditional variance estimates. Standard errors are in square brackets and t-statistics are in brackets. The bottom panel reports the parameter estimates for the instrumental variable stochastic discount factor $M_{t+1} = \exp(a_0 + a_1 z_t - b_0 r^m_{t+1} - b_1 r^m_{t+1} z_t)$. The parameters are estimated using GMM. Standard errors are Newey-West standard errors with 12 lags, these are in square brackets and t-values are in brackets. ***,**,* denotes statistical significance at the 1, 5 and 10-percent level

	Mean Equation				
	Constant	r^m_t	z_t	Adj R^2	N
CPME	-0.0045	0.0640	0.0389***	1.04%	376
	[0.0064]	[0.0751]	[0.0143]		
	(-0.70)	(0.85)	(2.72)		
CPME GARCH	-0.2676	-0.0076	3.6143	0.52%	376
	[0.7754]	[0.0629]	[2.2539]		
	(-0.35)	(-0.12)	(1.60)		
	Conditional Variance				
	ω	α	β		
CPME	0.001916*				
	[0.002296]				
	(8.35)				
CPME GARCH	0.000045	0.1523*	0.8432*		
	[0.000037]	[0.0417]	[0.0389]		
	(1.24)	(3.65)	(21.70)		
	Instrumental Variables Model				
	a_0	a_1	b_0	b_1	
Parameters	0.0203*	0.2409**	3.3214**	16.4505*	
	[0.0109]	[0.1078]	[1.3830]	[8.7909]	
	(1.86)	(-2.24)	(2.40)	(1.87)	

Chapter 2

Private Equity Performance: Implications for Portfolio Choice

Rasmus Jørgensen*

Abstract

I study the connection between a particular Public Market Equivalent (PME) measure of risk-adjusted private equity (PE) performance and portfolio choice. I present a simple expected utility maximization problem and show that the PME is applicable for allocating between a PE trading strategy and a single risky benchmark. Log-utility investors' allocations are characterized entirely by moments of excess PE performance, represented by the PME. Power-utility investors must, however, also consider the covariance between excess performance and benchmark returns. I estimate optimal PE allocations directly from cash flows by maximizing an expected utility criterion for a sample of US buyout funds. Empirically, log-utility investors should allocate their entire wealth to buyout funds, if the alternative investment is the broad public equity market, while risk averse investors allocate 25-35% of wealth to buyout funds. Buyout fund allocations are markedly lower, across levels of investor risk aversion, if the alternative investment is a portfolio comprised of small or value stocks. I extend the base framework to study conditional buyout fund allocations and allocations for investors with fund selection skills or limited access to top quartile funds.

The author is affiliated with Pension Research Center (PeRCent) and the Department of Finance, Copenhagen Business School, Solbjerg Plads 3, DK-2000 Frederiksberg, Denmark as well as ATP. Thanks to Jesper Rangvid and Peter Feldhütter for helpful comments.

1 Introduction

Institutional investors are increasing allocations toward private market assets such as private equity (PE) funds. While these assets now constitute a substantial portion of pension and endowment fund's investment portfolios, evaluating PE fund risk and return characteristics is challenging due to the absence of quoted market prices and consequential lack of return data.[1] Public Market Equivalents (PMEs) have proven particularly useful for evaluating risk-adjusted performance from fund cash flows.[2] The fundamental idea is to either replicate fund cash flows in a benchmark of publicly traded stocks, and compare the performance of these investments, or to discount cash flows with benchmark returns. Sorensen and Jagannathan (2015) show that the PME introduced by Kaplan and Schoar (2005)[3] is a valid economic measure of risk-adjusted performance by establishing a connection between the PME and a log-utility investor's stochastic discount factor. Despite the connection between the PME and stochastic discount factors, the PME's applicability for portfolio choice problems has attracted little scrutiny.

This paper studies the applicability of a specific PME measure for portfolio allocation.[4] I show a direct connection between the PME and a simple portfolio choice problem where a limited partner (LP) maximizes expected utility of terminal wealth by allocating initial wealth between two risky trading strategies; (1) a buy-and-hold investment in a benchmark of publicly traded stocks and (2) a private equity strategy. Because the PME evaluates performance relative to a pre-specified benchmark, using the PME for portfolio choice problems only allows for pairwise comparison of trading strategies and consequently allocation between two risky assets, which is undoubtedly a limitation for portfolio choice applications. The simple problem is nevertheless interesting because a few restrictions make the expected utility maximization problem empirically tractable. I exploit the tractability to estimate optimal allocations to a random buyout fund and a public benchmark directly from cash flows and benchmark returns using a sample of US buyout funds from Preqin. Specifically, I estimate allocations from the terminal wealth of the two trading strategies by maximizing an expected utility criterion, thereby bypassing the direct estimation of buyout fund alpha and beta.

[1] Da Rin and Phalippou (2017) report PE allocations of up to 25% for pension funds and endowments.

[2] Papers using PMEs include, Kaplan and Schoar (2005), Axelson, Jenkinson, Strömberg, and Weisbach (2013), Phalippou (2014), Harris et al. (2014), Sorensen and Jagannathan (2015), Robinson and Sensoy (2016), Korteweg and Nagel (2016), Gredil et al. (2020) and Harris, Jenkinson, Kaplan, and Stucke (2020) among others, industry reports include, McKinsey & Company (2020) and Preqin (2020).

[3] Other PME definitions are used in the literature, Long and Nickels (1996) for instance introduce the Index Comparison Method (ICM), Rouvinez (2003) suggests the PME+ measure and Cambridge Associates proposes the mPME.

[4] I study the PME introduced in Korteweg and Nagel (2016) (defined in Section 2.3) which is a variant of the Kaplan and Schoar (2005) PME.

The paper's main findings are five-fold. (1) PME moments[5] are sufficient to determine allocations for investors with log-utility preferences. (2) Investors with general power-utility preferences, using PMEs to allocate wealth, must also consider the covariance between excess PE performance and benchmark returns, and the covariance becomes increasingly important as risk aversion increases. (3) Empirically, both log- and power-utility investors should allocate a substantial fraction of wealth to buyout funds if the alternative investment is the broad equity market, but a smaller fraction for benchmark portfolios comprised of small and value stocks. (4) The valuation level of the public equity market is the most important state variable for conditional buyout fund allocations. (5) Optimal buyout allocations for LPs with fund selection skills are comparable with the allocations of an unskilled LP when the expected excess performance from selecting a random buyout fund is high. On the other hand, a skilled LP's optimal allocation is considerably larger than an unskilled LP's if the expected excess performance from selecting a random buyout fund is low.

The paper presents a simple portfolio allocation problem. Investors choose between a buy-and-hold investment in a publicly traded benchmark and a private equity strategy by maximizing expected utility of terminal portfolio wealth. The private equity strategy is an active trading strategy investing in a single PE fund, financing capital calls by selling shares of the publicly traded benchmark, and reinvesting distributions from the PE fund in the benchmark. The PE strategy is fully invested throughout the PE fund's investment period making terminal wealth comparable to the terminal wealth of the buy-and-hold benchmark investment. The trading strategy definitions, in conjunction with the expected utility maximization criterion, produce a connection between the PME and portfolio allocations.

The expected (average) PME, commonly used to evaluate risk-adjusted performance, is directly related to the first-order condition of a log-utility investor's expected utility maximization problem. The expected PME represents the first-order condition with respect to the PE allocation evaluated at 100 percent of wealth invested in the benchmark strategy. From the portfolio choice perspective, the expected PME can be interpreted as the marginal expected utility increase obtainable from a marginal allocation to the PE strategy, provided that the investor has no initial PE allocation. The portfolio choice perspective thus highlights that the average PME does not measure marginal value for investors with substantial PE allocations in place, which is the practically important case for most PE investors such as pension and endowment funds. Harvard's endowment, for instance, has PE allocations of 23 percent in place (Harvard, 2020). The portfolio choice perspective highlights

[5]The first moment represents a fund's expected excess performance, the second moment excess performance variance etc.

that these investors cannot rely solely on average risk-adjusted performance to determine whether to change their PE allocation.

A log-utility investor's allocation problem reveals that the optimal PE allocation generally depends on all PME moments. The solution to an approximate optimization problem leads to the optimal fraction of wealth allocated to PE as a function of the first and second PME moments:

$$\theta_p^\star = \frac{\mathrm{E}[\mathrm{PME}]}{\mathrm{E}[\mathrm{PME}^2]} \tag{1}$$

The optimal allocation illustrates that investors should, in addition to considering average risk-adjusted performance, consider excess performance variability. The portfolio allocation problem thus provides a new perspective on the risk and returns of PE funds. As an example, the average risk-adjusted performance of buyout and venture capital relative to a given public market benchmark might be similar. The excess performance variability of venture capital funds is, however, considerably larger than that of buyout funds. The variability means that the two fund categories are quite different from a portfolio allocation perspective. The expected utility framework highlights the variance of excess performance as an important measure of PE risk due to the direct connection to portfolio choice.[6]

The solution to an approximate portfolio choice problem for an investor with general power-utility preferences, and risk aversion greater than one, shows that these investors penalize PME variance proportionally to the level of risk aversion. Power-utility investors should furthermore consider the covariance between excess PE performance and benchmark returns, which becomes increasingly important as risk aversion increases. The covariance means that conservative LPs derive additional value from the PE investment if excess performance correlates negatively with the benchmark. Provided that the correlation is sufficiently negative power-utility investors allocate a positive fraction of wealth to the PE strategy despite zero expected excess performance.

Empirical analyses reveal that log-utility investors should allocate 100 percent of wealth to buyout funds if the alternative investment is a broad portfolio of publicly traded stocks. A log-utility LP, in fact, shorts the benchmark strategy to obtain more buyout fund exposure, if allowed. The large buyout allocation reflects the high average risk-adjusted performance relative to the broad equity market. The average buyout fund PME is negative compared to a benchmark portfolio of small-value stocks. The negative average PME results in log-utility investors shorting the buyout fund strategy for this benchmark. Conservative LPs with $\gamma = 8$ allocate approximately 25 percent

[6] PME variance has not been extensively studied, notable exceptions are Harris, Jenkinson, Kaplan, and Stucke (2018) and Brown et al. (2021).

of wealth to buyout relative to a market benchmark and 13-16 percent for benchmark portfolios of small and value stocks.

I furthermore examine allocations conditional on the state of the economy at fund inception using approximate solutions to the portfolio choice problems. Estimating PME moments conditional on public market valuations, corporate credit conditions and the yield curve reveals that the valuation level of the public equity market is the most important state variable to consider for a log-utility investor.

I extend the expected utility framework and analyze allocations of (1) LPs with fund selection skills and (2) LPs with limited access to top-performing funds. Specifically, I estimate allocations for LPs selecting top quartile funds with higher probability (skilled LPs) and for LPs selecting funds below the top quartile with higher probability (LPs with limited access). Unsurprisingly LPs with fund selection skills should allocate a larger fraction of wealth to buyout funds, and allocations are increasing in skill. Interestingly, optimal buyout fund allocations for LPs with fund selection skills are comparable with the optimal allocations of unskilled LPs when the expected excess performance from simply selecting a random buyout fund is high. If, on the other hand, the expected excess performance from selecting a random buyout fund is low, a skilled LP's optimal allocation is far greater than an unskilled LP's allocation. I furthermore find that optimal buyout fund allocations are considerably lower for LPs with limited access to top quartile funds.

The paper contributes to the literature on the risk and return of PE funds by connecting performance evaluation and portfolio choice. Related studies on PE performance include Kaplan and Schoar (2005), Cochrane (2005), Phalippou and Gottschalg (2009), Driessen et al. (2012), Harris et al. (2014), Sorensen and Jagannathan (2015), Korteweg and Nagel (2016) and Gredil et al. (2020) among others. Papers closest to the spirit of this paper are Gredil, Liu, and Sensoy (2021) using an expected utility framework to examine diversification in portfolios of PE funds and Brown, Hu, and Kuhn (2021) studying the effect of including PE funds into diversified portfolios. This paper differs by establishing a direct connection between the PME, two specific trading strategies and portfolio choice problems. The relation between the PME and portfolio choice offers a new perspective on the advantages and disadvantages of the PME for performance evaluation. Specifically, investors with large PE allocations in place cannot rely exclusively on the average PME to determine whether they should adjust their current PE allocation.

The paper also adds to the literature by estimating optimal PE allocations directly from fund cash flows and benchmark returns without explicitly modeling period-by-period returns. Traditional methods of portfolio choice generally require (1) modeling asset returns and (2) solving for portfolio weights. However, using the connection between the PME and portfolio choice, and an average utility maximization criterion inspired by Brandt (1999), Aït-Sahalia and Brandt (2001), Brandt and Santa-Clara (2006) and Brandt, Santa-Clara, and Valkanov (2009), the construction of returns from Net Asset Values and the estimation of PE alpha and beta can be bypassed.

The organization of the paper is as follows. Section 2 presents the trading strategies and the relation to the PME. Section 3 presents the portfolio allocation problem and illustrates the connection between portfolio choice and the average PME commonly used to assess the risk-adjusted performance of PE funds. This section also presents solutions to approximate portfolio allocation problems. Section 4 presents the buyout fund and benchmark portfolio data and outlines the empirical methodology used to estimate optimal allocations. Section 5 presents empirical estimates of optimal buyout fund allocations and illustrates applications of the expected utility framework by analyzing conditional allocations and optimal allocations for LPs with fund selection skills or LPs with restricted access to top quartile funds. Section 6 concludes.

2 Setup

I consider the problem of allocating between two risky strategies by maximizing expected utility of terminal wealth. Investors have access to two trading strategies (1) A buy-and-hold investment in a portfolio of publicly traded stocks and (2) a private equity strategy.

2.1 Benchmark Strategy

The benchmark strategy is a buy-and-hold investment in a portfolio of publicly traded stocks. Limited partners (LPs) invest at time 0 and receive a payoff at terminal date T with no intermediate distributions.[7] Consider the strategy as a fund investment with fund shares that cannot be traded in a secondary market or redeemed before the terminal date. From the LP's perspective, the benchmark strategy is consistent with the microstructure of private markets where secondary transactions are uncommon. The payoff at the terminal date is the gross total return from fund inception:

$$W_T^b = W_0 \exp\left(r_{0:T}^b\right) \tag{2}$$

Where W_0 denotes the initial investment and $r_{0:T}^b$ denotes the cumulative log-return from inception until the terminal date.

[7]Dividends are reinvested.

2.2 Private Equity Strategy

The private equity trading strategy actively trades in a single PE fund and the benchmark. Consider the strategy as being managed by an intermediary investing in an underlying PE fund. At fund inception, the intermediary invests the LP's commitment in the benchmark. The intermediary finances capital calls from the underlying PE fund during the fund's life by selling benchmark shares. When the underlying PE fund distributes capital the intermediary reinvests distributions in the benchmark until the terminal date. The LP thus invests at fund inception and receives a single distribution at the terminal date. Reinvesting proceeds in the benchmark makes terminal wealth comparable to the terminal wealth in the benchmark strategy.[8] The exact timing is as follows. In each period the managed fund earns the benchmark return on uncalled capital. The underlying PE fund then calls capital from or distributes capital to the intermediary. Capital calls reduce the next period's uncalled capital while distributions are reinvested in the benchmark until the end of the fund. The intermediary observes capital calls (C_t) and distributions (D_t) per dollar of committed capital but not the underlying PE fund's Net Asset Value (NAV). [9] The dynamics lead to the following terminal wealth (see Appendix A for details on the timing of payoffs in the PE strategy):

$$W_T^P = W_0\Big(\exp(r^b_{0:T}) + \sum_{t=1}^{T}(D_t - C_t)\exp(r^b_{t:T})\Big) \tag{3}$$

The cumulative return $\exp(r^b_{t:T})$ carries cash flows forward from the cash flow realization date until the terminal date.[10] The PE trading strategy is mix of payoffs from a pure private equity investment and investments in the benchmark. The strategy payoffs coincide with the underlying PE fund's payoffs if the underlying fund calls the full amount of committed capital in the first period and distributes all capital at the end of the fund's life.

[8] The structure deviates from that of a standard private equity fund where LPs commit capital at fund inception, which is called during the first 4-5 years, and distributed throughout the fund's lifetime. One drawback of the structure in this paper is that it can lead to negative wealth. Consider a fund where the first capital draw is 50 percent of committed capital, leaving 50 percent to be invested in the benchmark. If at the next date the benchmark index decreases by 25 percent and at the subsequent date the private equity fund calls the remaining committed capital, the LP will not be able to meet the capital call without borrowing. Depending on the outcome of the PE investment terminal wealth can become negative.

[9] It is inconsequential for the portfolio choice problem presented in subsequent sections that NAVs are not observed since LPs maximize expected utility of terminal wealth which only depends on fund cash flows and benchmark returns.

[10] Note that $\exp(r^b_{i:j}) = 1$ for $i = j$.

2.3 Relation to Public Market Equivalents

The ratio of PE to benchmark strategy terminal wealth is a measure of fund performance. From the trading strategy definitions the terminal wealth ratio is:

$$\frac{W_T^p}{W_T^b} = \frac{W_0\Big(\exp(r_{0:T}^b) + \sum_{t=1}^T (D_t - C_t)\exp(r_{t:T}^b)\Big)}{W_0 \exp(r_{0:T}^b)} = 1 + \frac{\sum_{t=1}^T (D_t - C_t)\exp(r_{t:T}^b)}{\exp(r_{0:T}^b)} \tag{4}$$

The relative return at a time t, $\exp(r_{t:T}^b)/\exp(r_{0:T}^b)$, carries cash flows forward to the terminal date and back to fund inception to obtain the present value of cash flows realized at time t. Carrying cash flows forward to the terminal date and then back is equivalent to carrying cash flows back from the realization date to fund inception using discount factor $\exp(-r_{0:t}^b)$. The equivalency provides a relation between the terminal wealth ratio and the Generalized PME of Korteweg and Nagel (2016). The GPME of a fund is the sum of fund cash flows discounted using some general multi-period stochastic discount factor. Sorensen and Jagannathan (2015) and Korteweg and Nagel (2016) show that the PME of Kaplan and Schoar (2005) corresponds to discounting cash flows with a log-utility investor's stochastic discount factor, which is the inverse of the log-utility investor's return on wealth. Using the return on a traded benchmark portfolio in place of the wealth return the one-period stochastic discount factor is given by $M_{t+1} = \exp(-r_{t+1}^b)$. Discounting cash flows with the multi-period stochastic discount factor results in the equivalence between the wealth ratio and the PME:[11]

$$\frac{W_T^p}{W_T^b} = 1 + \sum_{t=1}^{T}(D_t - C_t)\exp(-r_{0:t}^b) = 1 + \mathrm{PME} \tag{5}$$

The equivalence means that the PME can also be interpreted as the percentage of additional wealth an investor obtains by investing in a strategy which finances the PE investment by selling the benchmark, relative to a buy-and-hold investment in the same benchmark. The relation between the PME and the trading strategies furthermore shows the reinvestment assumption implicit in the PME.

The wealth ratio interpretation highlights a shortcoming of evaluating a single fund's performance using the fund's realized PME. A wealth ratio greater than one means, that actively trading the benchmark and the PE fund provides higher terminal wealth than a buy-and-hold investment in the benchmark. Larger terminal wealth can, however, be achieved in several ways. Consider, for example, a fund manager levering up the benchmark instead of investing in an underlying PE fund. Levering up the benchmark leads to a wealth ratio larger than one, provided that benchmark

[11] Following the definition in Korteweg and Nagel (2016) the PME in Equation 5 is the GPME for a log-utility investor.

returns are higher than the funding rate on average. Such a strategy will, nevertheless, also entail more risk, which is generally not observable from the realized terminal wealth ratio of a single fund.

3 Portfolio Choice

Using the trading strategy definitions, and the relation to the PME, a connection between portfolio choice and evaluation of average fund performance can be established. Consider the problem of allocating initial wealth W_0 between the two trading strategies at time 0 by maximizing expected utility of terminal portfolio wealth.[12] After the initial allocation investors are not allowed to rebalance portfolio allocations, they cannot allocate to a risk-free asset, but they are allowed to short either strategy.[13] Investors have no intermediary consumption or outside income.

These restrictions essentially transform the problem into a static expected utility maximization problem over terminal wealth with portfolio wealth given as a linear combination of the two strategies' terminal wealth, $W_T^I = \theta_b W_T^b + \theta_p W_T^p$, with θ_i denoting the fraction of initial wealth allocated to strategy i.[14] The portfolio choice problem essentially becomes a one-period problem where the period is a long time span. One can think of the time span as the lifetime of a typical PE fund. LPs solve the portfolio choice problem by maximizing expected utility over terminal wealth.

$$\begin{aligned} &\max_{\boldsymbol{\theta}} \mathrm{E}\big[u(W_T^I)\big] \\ &\text{s.t. } \theta_b + \theta_p = 1 \end{aligned} \tag{6}$$

Where $u(\cdot)$ is a concave utility function. I consider power utility preferences over terminal wealth:

$$u(W_T) = \begin{cases} \frac{W_T^{1-\gamma}}{1-\gamma} & \text{if } \gamma > 1 \\ \ln(W_T) & \text{if } \gamma = 1 \end{cases} \tag{7}$$

γ denotes investor's relative risk aversion.

[12] I set $W_0 = 1$ in subsequent sections.

[13] In practice, the PE strategy cannot be sold short.

[14] Note that the fraction of wealth, allocated to the private equity trading strategy presented Section 2.2, does not necessarily equal the optimal allocation to a pure private equity investment. The difference is due to the reinvestment of cash flows at the benchmark rate of return. The reinvestment makes the PE strategy return a mix of private equity and benchmark payoffs. If PE exhibits positive risk-adjusted performance, and investors can reinvest in PE, the optimal allocation to this strategy will differ from the allocation to the PE strategy presented.

3.1 Performance Evaluation using PMEs

The expected utility maximization problem of an LP with log-utility preferences is as follows:

$$\max_{\boldsymbol{\theta}} \mathrm{E}\big[\ln(\theta_b W_T^b + \theta_p W_T^p) - \lambda(\theta_b + \theta_p - 1)\big] \tag{8}$$

The first-order condition with respect to the PE allocation is:

$$\mathrm{E}\big[W_T^p(\theta_b W_T^b + \theta_p W_T^p)^{-1}\big] = \lambda \tag{9}$$

Evaluating the first-order condition at $\theta_b = 1$ and $\theta_p = 0$ leads to:[15]

$$\mathrm{E}\left[\frac{W_T^p}{W_T^b}\right] = \mathrm{E}\big[1 + \mathrm{PME}\big] \tag{10}$$

The first-order condition provides the connection between the allocation problem and the expected (average) PME and highlights two important points.[16] (1) The expected PME represents the marginal expected utility increase a log-utility investor, already 100 percent invested in the benchmark, obtains from a marginal allocation to the PE strategy. (2) The expected PME is insufficient for determining optimal PE allocations unless the allocation is zero, which results in an optimal allocation of 100 percent in the benchmark.

Collectively Equations 9 and 10 show that the average PME does not measure the expected utility increase obtainable by investors with substantial initial allocations to PE. The difference amounts to a difference in the portfolio in the denominator in Equations 9 and 10. The portfolio in Equation 9 is a combination of the PE and benchmark strategies whereas the PME in Equation 10 uses the benchmark portfolio in the denominator.[17] This difference is important for PE investors such as pension funds and endowments because these types of investors typically have large PE allocations in place. The average PME consequently does not measure the value of a marginal allocation to PE for these investors.

[15]In the following PME represents a random variable. E[PME] represents investors' expected excess PE performance. This differs slightly from the ex-post use of PMEs for performance evaluation.

[16]The relation between the allocation problem and the PME is a reflection of the connection between the PME and the stochastic discount factor of a log-utility investor established by Sorensen and Jagannathan (2015). Conceptually, the first-order condition is analogous to the one used to evaluate the average risk-adjusted value of cash flows in Korteweg and Nagel (2016) for a set of transformed payoffs.

[17]Conceptually the denominator portfolio acts as a stochastic discount factor. The difference in Equations 9 and 10 amounts to whether the private equity strategy payoff enters the stochastic discount factor pricing the trading strategy payoffs.

3.2 Solving Approximate Problems

The interaction between additive wealth and power utility preferences means that the portfolio choice problems cannot be solved analytically. Approximating the solutions to the expected utility maximization problem, however, leads to analytical solutions, which provide insights into the determinants of optimal allocations.

Log-Utility

Investors with log-utility preferences solve the maximization problem:

$$\max_{\boldsymbol{\theta}} \mathrm{E}\big[\ln(W_T^I)\big] \tag{11}$$

Using the restriction $\theta_b + \theta_p = 1$ and the relation between the terminal wealth ratio and and PME in Equation 5, portfolio wealth can be expressed as $W_T^I = W_T^b(1 + \theta_p \mathrm{PME})$ (see Appendix B). Portfolio wealth is given by benchmark wealth, the fraction of initial wealth allocated to the PE strategy and excess PE performance represented by the PME. Using the expression for portfolio wealth in the maximization problem leads to:

$$\max_{\theta_p} \mathrm{E}\Big[\ln(W_T^b) + \ln(1 + \theta_p \mathrm{PME})\Big] \tag{12}$$

This maximization problem highlights an interesting property of the log-utility specification. The optimal PE allocation is separable from benchmark wealth. This separability means the PME distribution is sufficient for determining the optimal PE allocation. It also means the optimal allocation is invariant to transformations of terminal wealth which do not alter the PME distribution. Reinvesting both strategies' terminal wealth in the benchmark to extend the investment horizon, for instance, changes strategy wealth distributions but not the PME distribution, which means it does not change a log-utility investor's optimal allocation.

Linearizing the maximization problem by a second-order Taylor expansion of $\ln(1 + \theta_p \mathrm{PME})$ around $\mathrm{PME} = 0$ results in:

$$\max_{\theta_p} \mathrm{E}\Big[w_T^b + \theta_p \mathrm{PME} - \frac{1}{2}\theta_p^2 \mathrm{PME}^2\Big] \tag{13}$$

Where w_T^b denotes the log of benchmark wealth. The optimal PE allocation is given by:

$$\theta_p^\star = \frac{\mathrm{E}[\mathrm{PME}]}{\mathrm{E}[\mathrm{PME}^2]} = \frac{\mathrm{E}[\mathrm{PME}]}{\mathrm{E}[\mathrm{PME}]^2 + \sigma^2_{\mathrm{PME}}} \tag{14}$$

The expected PME normalized by the second moment determines the PE allocation. The expression illustrates the difference between evaluating optimal allocations and evaluating average risk-adjusted performance. For performance evaluation, it is sufficient to assess the average PME while determining optimal allocations also requires accounting for PME variability. This is an important difference between the performance evaluation and the portfolio choice perspectives on the risk and return of PE funds.

Power-Utility

Investors with power-utility preferences and $\gamma > 1$ solves the problem:

$$\max_{\theta_p} \mathrm{E}\left[\frac{1}{1-\gamma}(W_T^I)^{1-\gamma}\right] \tag{15}$$

Subject to the constraint, $\theta_b + \theta_p = 1$. Applying the approximation from the previous section does not immediately result in an analytical solution. However, using an alternative approximation, presented in Appendix B, the optimization problem can be expressed as:

$$\max_{\theta_p} \mathrm{E}\left[\frac{1}{1-\gamma}\exp\Big((1-\gamma)w_T^b + (1-\gamma)\theta_p \mathrm{PME} - \frac{1}{2}\theta_p^2(1-\gamma)\mathrm{E}[\mathrm{PME}^2]\Big)\right] \tag{16}$$

Assuming benchmark log wealth and the PME are jointly normally distributed the properties of the log-normal distribution leads to optimal PE allocation (see Appendix B):

$$\theta_p^\star = \frac{\mathrm{E}[\mathrm{PME}] - (\gamma-1)\mathrm{cov}(w_T^b, \mathrm{PME})}{\gamma\sigma_{\mathrm{PME}}^2 + \mathrm{E}[\mathrm{PME}]^2} \tag{17}$$

Equation 17 shows the optimal allocation increases in the expected PME and decreases in PME variance and γ.[18] For LPs with $\gamma > 1$, positive covariance leads to an optimal allocation lower than implied by the expected excess PE performance. The optimal allocation is larger than implied by the expected PME if the covariance is negative. The covariance accounts for power-utility LPs trading off average portfolio wealth with portfolio variance. LPs like positive covariance between log-wealth and PMEs because it increases average portfolio wealth for a given average log-wealth and PME. On the other hand, LPs dislike positive covariance because it increases total risk. The $\gamma-1$ term captures these opposing effects. As risk aversion increases, LPs care relatively more about portfolio risk and consequently place more weight on the covariance. Conservative LPs care a great deal about portfolio variance and therefore about the covariance between log-wealth and excess

[18] Appendix C provides a portfolio choice example with a benchmark asset and a secondary asset which provides some intuition for the expression for the optimal PE allocation.

performance. The effects exactly offset for log-utility preferences. The covariance represents an additional diversification benefit to conservative LPs if excess PE performance correlates negatively with the benchmark.

The covariance means a power-utility investor's optimal PE allocation can be negative (positive) despite a positive (negative) expected PME, which means that conservative LPs can optimally choose not to allocate to the PE strategy despite positive expected PME. This effect is not present for investors with log-utility preferences. For this type of investor, the sign of the optimal allocation is consistent with the expected PME's sign.

4 Empirical Implementation

4.1 Data

Funds Data

To estimate optimal allocations, I construct the trading strategies outlined in Section 2 for a sample of buyout funds. Constructing the strategies requires fund cash flows and benchmark returns. I use net of fees fund-level cash flows for 359 US buyout funds from Preqin incepted from 1980 to 2008. Fund cash flows are realized during the period from 1980 to 2016. The sample contains contributions and distributions to and from the funds as well as fund valuations (NAVs) throughout each fund's life. I include fully and partially liquidated funds to preserve the largest possible sample. For partially liquidated funds, residual NAV from unrealized investments is added to the funds' last cash flow which is standard practice. Funds are required to be at least eight years old and have called at least 80 percent of committed capital, which mitigates the influence of excessively large residual NAVs. Cash flows are normalized such that payoffs are per dollar of committed capital. Table 1 reports summary statistics for the funds data.

Benchmarks

I consider several different benchmark portfolios. The benchmark returns are from Kenneth French's data library. The public equity market is represented by the value-weighted return on CRSP firms. I also consider three long-only characteristic portfolios. (1) A value portfolio, which is a portfolio of the 20 percent highest ranked stocks in the CRSP universe based on their book to market ratio. (2) A portfolio of small stocks represented by the 20 percent smallest stocks in the CRSP universe measured by market capitalization. (3) A small-value portfolio, which is the result of a double sort on market capitalization and the book-to-market ratio, and contains the smallest stocks in the universe with the highest book-to-market ratio. The benchmark portfolios are market capitalization weighted. I consider characteristic sorted portfolios as previous literature shows that buyout funds payoffs covary with small and value stocks, see for example Ang et al. (2018) and Stafford (2020).

4.2 Estimation

Objective Function

I relate the theoretical portfolio choice problem to the data by constructing the trading strategies, outlined in Section 2, for each buyout fund in the sample. The benchmark investment for fund i produces one realization of terminal wealth $W^b_{T,i}$ which is the gross return on a strategy investing one dollar at the inception of fund i and ending at the date of the fund's last cash flow, T, which depends on i. As an example, if a fund is incepted in 2001Q4 and the last cash flow occurs in 2011Q4, the benchmark terminal wealth is the cumulative return in these ten years. The private equity strategy for fund i produces realized terminal wealth, $W^p_{T,i}$ which is the terminal wealth of a trading strategy that reinvests the realized cash flows of fund i until the date of the fund's last cash flow. The ratio of realized terminal wealth equals one plus the PME of that fund. Each fund in the sample, therefore, leads to a set of three payoffs, $\{W^b_{T,i}, W^p_{T,i}, \text{PME}_i\}$, one realization of benchmark strategy wealth, one realization of buyout fund strategy wealth and one realization of excess buyout fund performance in the historical sample.[19] The buyout fund sample from Preqin consequently yields $N = 359$ terminal wealth realizations for each trading strategy and thus $N = 359$ portfolio realizations for a given candidate allocation $\boldsymbol{\theta} = [\theta_b, \theta_p]^\top$.

For a given candidate allocation, I estimate the expected utility of terminal portfolio wealth from the cross-sectional sample average of terminal portfolio wealth utils. I thus estimate optimal allocations by maximizing the average utility an investor would have obtained from implementing a strategy allocating the same fraction of a dollar between the buyout fund strategy and benchmark strategy every time a new buyout fund is incepted in the historical sample. The resulting optimal allocations can be interpreted as the optimal allocations of an LP selecting a random buyout fund

[19]Section 4.2 outlines how the investment horizon is equalized across buyout funds, such that T is the same across funds.

from the universe of funds.[20] The expected utility criterion is:

$$\mathrm{E}\Big[u(W_{T,i}^{I})\Big] = \frac{1}{N}\sum_{i=1}^{N} u(W_{T,i}^{I}) = \sum_{i=1}^{N} u(W_{T,i}^{I})p \tag{18}$$

Where $W_{T,i}^{I}$ denotes realized terminal portfolio wealth of fund i for candidate allocation $\boldsymbol{\theta} = [\theta_b, \theta_p]^{\top}$.[21] Estimating optimal allocations by maximizing sample average utility is conceptually similar to the approach in Brandt (1999), Aït-Sahalia and Brandt (2001), Brandt and Santa-Clara (2006) and Brandt et al. (2009). The approach in this paper differs as it exploits cross-sectional variation in fund payoffs to estimate allocations. This modification is necessary due to the structure of private equity fund data.[22] The objective function is conceptually similar to the expected utility definition in Gredil et al. (2021)[23] while identification via cross-sectional variation in payoffs is consistent with studies such as Driessen et al. (2012) estimating PE alpha and beta, and Korteweg and Nagel (2016) estimating Generalized PMEs from cross-sectional variation in payoffs. Identification of the optimal allocations relies on assumptions similar to these studies. Fund payoffs must be generated from the same underlying process and fund investment periods should be sufficiently scattered across time. The requirement that funds are spread across time ensures that cross-sectional variation in payoffs reveals possible states of wealth for a given trading strategy.

A simple example illustrates how the cross-sectional method works. Suppose (1) a new PE fund is incepted in every time period, (2) all funds invest one dollar at inception and distribute capital after 11 years and (3) PE fund cash flows are generated from the same underlying process. With these assumptions estimating expected utility of terminal wealth using cross-sectional units is equivalent to estimating expected utility using a time-series of 11 year overlapping returns, from the processes generating returns (Appendix E presents a Monte Carlo simulation showing that the time-series and cross-sectional approaches result in similar optimal allocations for random fund inception dates). The time-series method is equivalent to the maximum expected utility estimator Brandt (1999) applies to estimate optimal unconditional buy-and-hold allocations for long-horizon

[20] I examine the optimal allocation to PE in aggregate and therefore consider fund-level observations. For a single fund or fund manager, the optimal allocation to the manager can be determined from deal-level data, provided a sufficient number of deals, by constructing the PE and benchmark strategies for each deal.

[21] Conceptually, terminal portfolio wealth realizations, $W_{T,i}^{I}$, across funds reveal the possible states of terminal wealth for a given trading strategy (θ_b, θ_p).

[22] The method can be applied to time-series of returns if quoted returns were available for PE funds. Appendix D provides an example using buyout fund returns derived from a NAV-based index and returns from PE companies listed on global exchanges.

[23] Expressing expected utility in terms of probability weighted utils, $\sum_{i=1}^{N} u(W_{T,i})p_i$, Equation 18 resembles the expected utility definition used in Gredil et al. (2021) however with different probability weights and using terminal strategy wealth as opposed to Multiples of Invested Capital (MOICs).

portfolio choice problems.

The maximum expected utility problem can be expressed as a Generalized Method of Moments (GMM) estimator. The GMM formulation provides a distribution theory for the estimated allocations. The first-order condition with respect to the PE allocation of the portfolio choice problem in Section 3 leads to the moment condition:[24]

$$\mathrm{E}\Big[\big(W^p_{T,i} - W^b_{T,i}\big)\big(W^b_{T,i} + \theta_p(W^p_{T,i} - W^b_{T,i})\big)^{-\gamma}\Big] = 0 \tag{19}$$

Where the average is across cross-sectional units.

Plug-in Estimators

The GMM framework provides a method for estimating allocations implied by the data without modeling the distribution of terminal wealth. The additional restrictions imposed to derive the approximate optimal allocations in Section 3.2 however means that these can be estimated directly from payoff moments. The analytical solutions thus provide convenient plug-in estimators for optimal allocations by estimating moments from cross-sectional sample moments. The solution for a log-utility investor has a particularly convenient form that facilitates estimation via OLS regressions. The log-utility investor's PE allocation can be estimated from a cross-sectional regression without an intercept of a vector of ones onto PME realizations:[25]

$$1 = \hat{\theta}_p \mathrm{PME}_i + \epsilon_i \tag{20}$$

Where $\hat{\theta}_p$ is the estimated approximately optimal allocation for a log-utility investor. Expressing the estimation problem as a linear regression provides standard errors for the optimal allocation.[26] I bootstrap standard errors for the optimal allocations of investors with $\gamma > 1$ since approximate allocations cannot be estimated via OLS regressions for these investors.

Restrictions

Even though buyout funds have a similar contractual investment horizon of 10-12 years, the actual investment horizon is stochastic such that the realized investment horizon varies across

[24]Appendix F presents details.

[25]To see this, consider the OLS estimator; $\hat{\theta} = (\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{X}^\top \boldsymbol{y}$. When the dependent variable equals a vector of ones and the independent variable is a vector of fund PMEs, $\hat{\theta}$ is an estimate of the log-utility investor's approximate optimal allocation, $\frac{\mathrm{E[PME]}}{\mathrm{E[PME^2]}}$.

[26]The OLS approach to estimate the standard errors is introduced by Britten-Jones (1999) for mean-variance optimal allocations.

buyout funds in the sample. To equalize investment horizons across funds, I reinvest the two trading strategies' terminal wealth in the benchmark such that the investment horizon equals the investment horizon of the fund with the longest sample lifetime. Funds late in the historical sample cannot be extended with realized market returns for the full duration since market returns have not yet been realized for a part of these funds. I extend the investment horizon of these funds using the average benchmark return in the individual fund's investment period. The scaling does not affect the PME or the allocation for log-utility LPs, but it has a small effect on allocations for LPs with $\gamma > 1$. Appendix G Table G.1 presents optimal allocations for an alternative assumption on funds' investment horizons. Appendix G Table G.2 presents allocations for a methodology reinvesting terminal wealth at a risk-free interest rate to equalize investment horizons. Both alternatives produce optimal allocations similar to those presented in Section 5.

The PE trading strategy definition means that negative terminal wealth can occur.[27] I discard funds for which the PE strategy results in negative terminal wealth since expected utility is not defined in this case. Negative terminal wealth occurs for one fund in the sample when the small and small-value portfolios are used as benchmarks. I discard this fund across specifications such that optimal allocations are comparable across benchmarks. The effect of removing the fund on the average PME is negligible, but discarding the worst performing fund does affect optimal allocations to a small extent.[28] I furthermore restrict solutions to the domain of allocations such that terminal portfolio wealth is strictly positive.

5 Empirical Results

Table 2 reports average PMEs for the buyout fund sample. Column 1 reports the PME relative to the broad equity market. The average PME is 0.19, which means that, on average, investing 100 percent of initial wealth in the buyout fund strategy results in approximately 19 percent higher terminal wealth compared to investing in the public equity market. Columns 2 and 3 report PMEs for the value and size benchmarks. The PMEs are 0.09 and 0.11 respectively and statistically different from zero. The PME is -0.01 for the small-value benchmark. This means that LPs achieve 1 percent lower wealth by investing in buyout funds compared to a portfolio of small-value stocks.

[27] The framework can accommodate benchmark strategies that do not result in negative wealth. One such benchmark strategy invests uncalled capital in a risk-free asset in contrast to the risky benchmark as implied by the PME. This alternative benchmark is potentially more reasonable given the commitment risk of PE funds. Using this benchmark gives rise to another measure of risk-adjusted performance similar to the "Profitability Index" in Ljungqvist and Richardson (2003).

[28] The optimal PE allocation for the market benchmark and $\gamma = 1$ is 110% after the fund is removed and 100% including the fund. The optimal allocation is 24.9% and 24.8% for LPs with $\gamma = 8$.

5.1 Buyout Fund Allocations

Table 3 reports GMM estimates for optimal buyout fund allocations. Each row reports allocations for different levels of investor risk aversion. The first row reports allocations for investors with log-utility preferences. The high average PME relative to the broad equity market is reflected in a large buyout allocation for log-utility LPs. Log-utility LPs optimally allocate 110 percent of wealth to buyout funds and 100 percent if allocations are restricted to be non-negative. Column 3 reports an optimal allocation of 78 percent for the value benchmark, while Column 5 reports a buyout fund allocation of 73 percent relative to small stocks. The buyout fund allocation is larger for the benchmark of small stocks compared to the value benchmark despite higher average PME for the latter. This discrepancy illustrates the difference between the performance evaluation and portfolio choice perspectives on the risk and return of PE funds. Columns 7 and 8 report allocations for the small-value benchmark. The average PME is negative, which results in a benchmark allocation of 108 percent for log-utility LPs and a negative buyout fund allocation.

The results suggest that log-utility investors should allocate a considerable fraction of wealth to buyout funds for three of the four benchmarks. The large allocations are partly due to the risk-willingness of investors with log-utility preferences. To see this consider a one-period i.i.d. log-normal economy. With one risky and a risk-free asset the optimal risky asset allocation is $\theta = (\mathrm{E}[r] - r_f + \frac{1}{2}\sigma^2)(\gamma\sigma^2)^{-1}$ for power-utility preferences. Setting $\gamma = 1$ the return-to-risk ratio is approximately 2-3 empirically for the broad equity market portfolio, which means a log-utility investor optimally invests 200-300% of wealth in the risky asset. The large buyout fund allocations thus partly reflect the log-utility investor's risk-willingness and the larger average wealth obtainable from investing in buyout funds as indicated by the positive average PMEs.

Table 4 presents average PMEs for levered equity market benchmarks, which are occasionally used to account for the high average PME of buyout funds relative to the equity market. Average PMEs decrease with leverage, β, supporting findings in Robinson and Sensoy (2016). PMEs are, however, positive and significantly different from zero across the different levels of leverage. Figure 1 plots buyout fund allocations relative to the equity market benchmark for different leverage levels. The figure shows that log-utility investors maintain a substantial allocation to buyout funds for $\beta = 1.3$ despite allocations decreasing in leverage. Interestingly, a log-utility investor's allocation at a given level of leverage approximately corresponds to the allocation of a power-utility investor with risk aversion matching the level of leverage. The buyout allocation is 0.93 for a log-utility LP using a benchmark with leverage $\beta = 1.3$ and the allocation is 0.93 for a power-utility investor with $\gamma = 1.3$. This similarity suggests that, at low levels of leverage, using a levered benchmark approximately corresponds to increasing risk aversion by an equivalent amount [29] From a portfolio perspective using levered benchmarks may therefore not provide risk averse power-utility investors with as valuable information as previously thought. As an example, the optimal buyout fund allocation of an investor with $\gamma = 4$ is around 40-41 percent regardless of benchmark leverage.

Table 3 column 1 shows that buyout fund allocations, relative to the equity market benchmark, decreases to 25 percent for $\gamma = 8$. Figure 2 plots the optimal buyout allocations as a function of investor risk aversion. The grey shaded area represents γ between 2-3, levels of risk aversion empirically consistent with a power-utility investor holding the market. Buyout fund allocations are positive and decreasing in risk aversion. Because the portfolio choice problem does not incorporate frictions, such as the illiquidity of buyout fund partnership interests, the allocations represent upper bounds for the optimal allocation to the buyout fund strategy. Intuitively illiquidity can be introduced by raising the LP's effective risk aversion. Ang, Papanikolaou, and Westerfield (2014) establish the relation to risk aversion by introducing illiquidity induced by investors not being able to trade an asset. Raising risk aversion leads to lower optimal buyout fund allocations empirically, as evident from Figure 2.[30]

Table 3 columns 3 and 5 report allocations for the value and small stock benchmarks. Allocations decrease from 0.73-0.78 for log-utility LPs to 0.13-0.18 for LPs with $\gamma = 8$. Despite lower allocations, LPs maintain economically significant allocations to buyout funds. For the small-value benchmark, log-utility LPs short buyout funds while LPs with risk aversion of 2-3 allocate approximately 23 percent to buyout funds. The allocation decreases for risk aversion larger than three resulting in a 5 percent allocation for the most conservative LP. The allocations for the small-value benchmark highlight an important point. LPs with $\gamma > 1$ can expect negative PME and optimally allocate a positive amount of wealth to the buyout strategy.

Table 3 shows considerable differences in optimal allocations across benchmarks. A log-utility investor, for instance, allocates 110 percent to the buyout strategy for the market benchmark but shorts buyout funds for the small-value benchmark. The large differences across benchmarks are not specific to using PMEs for portfolio choice but are a consequence of the expected utility maximization criterion. To see this consider a simple example with a benchmark asset and a secondary asset with some α and β relative to the benchmark. Appendix C presents the optimal allocation to the secondary asset.[31] A log-utility investor choosing between the benchmark and

[29] Korteweg (2019) also notes the link between levered PMEs and the degree of investor risk aversion.

[30] Interestingly, Giommetti and Sorensen (2021) show, in a more elaborate portfolio choice model, that optimal PE allocations are not necessarily decreasing in investor risk aversion.

[31] The example uses an annualized risk-free rate of 2 percent, benchmark asset log return volatility of 15 percent, benchmark asset risk premium of 4.5 percent and secondary asset log-return volatility of 15 percent.

a secondary asset with annualized $\alpha = 2\%$ and $\beta = 1.3$, optimally allocates 99 percent of wealth to the secondary asset. The investor allocates negative 78 percent for annualized $\alpha = -2\%$. This allocation difference is of the same magnitude as the differences across benchmarks for the PME implied allocations. Differences between the PME implied allocations thus reflect implicit differences in buyout fund α and β relative to the respective benchmarks.[32]

5.2 Plug-in Estimators

Table 5 reports plug-in estimates for optimal allocations. Table 6 presents bootstrap statistics for the allocations and Appendix H reports OLS estimates for $\gamma = 1$. Table 5 panel A reports allocations for the equity market benchmark. The optimal buyout fund allocation is 91 percent for log-utility LPs which is slightly below the GMM estimate. Buyout fund allocations for the log-utility investor are 72 and 59 percent for the value and small stock benchmarks. The average PME is similar for these benchmarks, meaning that the lower allocation relative to the small stock benchmark is due to differences in PME variance. Turning to the small-value benchmark the plug-in estimate is negative 9 percent and approximately the same as the GMM estimate. The allocation difference for this benchmark illustrates that the estimator is more accurate for average PMEs around zero, which is a consequence of the approximation in Section 3.2.[33]

Buyout allocations decrease for the market, value and small stock benchmarks for LPs with risk aversion larger than one. The results for the small-value benchmark are more complicated. The GMM and approximate estimates are similar for $\gamma = 2$. The approximate allocation increases for $\gamma > 2$ while the allocation estimated via GMM decreases for conservative investors with risk aversion larger than three. The differences illustrate a shortcoming of the plug-in estimator, specifically that it does not account for higher-order moments. The allocation differences suggest that higher-order moments affect optimal allocations relatively more for the small-value benchmark.

The last two columns in each panel decompose the optimal buyout fund allocation into two components. The first component is the allocation attributable to the average PME. The second component is due to the covariance between the PME and the benchmark. Separating the two components shows the optimal allocation for a covariance of zero. The results highlight the importance of the covariance for risk averse LPs. The covariance component constitutes 1/3 to 1/4 of the optimal allocation for the market, value and size benchmarks at levels of risk aversion of 2-3. The optimal buyout fund allocation would furthermore be negative, at all levels of risk aversion,

[32]In the example allocation differences is due to α and β differences. In the empirical implementation, allocation differences also reflect differences in higher-order moments.

[33]Appendix I illustrates the difference in approximation accuracy for the market and small-value benchmarks.

for the small-value benchmark if excess performance and the benchmark were uncorrelated.

PME Variance Adjustment

Estimating expected utility of wealth in the cross-section requires fund observations sufficiently spread across time. Individual fund observations do, however, not necessarily provide independent information on PME variance if fund investment periods overlap. Overlapping investment periods can lead to underestimating the variance, which leads to overstated buyout fund allocations if the average risk-adjusted performance is positive. To adjust the variance for overlapping investment periods, I apply the methodology introduced by Korteweg and Nagel (2016) to adjust the variance of fund pricing errors.[34]

Table 7 reports allocations using adjusted PME variances and Table 8 reports bootstrap statistics. Adjusting for overlapping funds increases the variance across benchmarks. The increase is smallest for the market benchmark. The adjustment, nevertheless, leads to a decrease in the optimal buyout allocation for a log-utility investor from 91 to 73 percent. The allocation decreases from 28 to 22 percent for the most conservative LP with $\gamma = 8$. Turning to the value and small benchmarks in Panel B and C variances increase significantly. The allocation for the value benchmark decreases from 72 percent to 28 percent, while the allocation decreases from 58 to 24 percent for the benchmark of small stocks for a log-utility investor. LPs with $\gamma = 8$ optimally allocate 7-8 percent to the buyout fund strategy for the value and small stock benchmarks.

Other Fund Category

The effect of PME variance on optimal allocations also has implications across fund categories. Table 9 reports average risk-adjusted performance, PME variances and optimal allocations for a log-utility investor for buyout (BO), venture capital (VC) and funds of funds (FoF) using an alternative data set from Preqin.[35]

The average buyout fund PME is 0.14 in comparison with 0.19 in the full sample. PME variance is marginally higher in the alternative sample producing an optimal allocation of 64% compared to 91% for the full sample of buyout funds. The average venture capital PME (0.13) is slightly lower than that of buyout funds (0.14), while the variance is almost nine times higher. The

[34]Korteweg and Nagel (2016) account for overlapping funds in the estimation of the spectral density matrix to adjust standard errors. I use the spectral density estimator to penalize PME variance for overlapping funds. I use $\bar{d} = 1.5$. See Korteweg and Nagel (2016) for details on the estimator.

[35]The data contain Kaplan and Schoar (2005) PMEs relative to the S&P 500 index for US funds. Appendix J describes how the PMEs are transformed to approximate PMEs in differences. The variances are not adjusted for overlapping funds because the alternative data do not include cash flow realization dates which means fund overlap cannot be estimated.

average risk-adjusted performance and variance result in an optimal venture capital allocation of 7% compared to 64% for buyout funds.

Funds of funds display low risk-adjusted performance (0.02) and PME variance (0.06) compared to the other fund categories. The lower risk-adjusted performance is partly due to the additional layer of fees charged by fund of funds[36] while the lower variance partly reflects the risk reduction provided by funds of funds through diversification. Harris et al. (2018) shows that funds of funds investments reduce PME variance relative to direct fund investments in the same fund category. They report a PME standard deviation for buyout FoFs of 0.24 compared to 0.55 for direct funds while for venture capital the standard deviation is 0.57 for FoFs relative to 1.78 for direct fund investments.[37] The analytical expressions for the optimal PE allocations show that the additional cost of funds of funds decreases investor's optimal allocations while the diversification benefits increase optimal allocations. From a portfolio choice perspective fund of funds thus provide valuable intermediation if the variance reduction is greater than the reduction in average risk-adjusted performance.[38] Table 9 shows a log-utility investor optimally allocates 36% of wealth to funds of funds despite relatively low average risk-adjusted performance.

Figure 3 further illustrates the importance of PME variance. The figure depicts PE allocations as a function of average risk-adjusted performance for different levels of PME variance. Each line represents the PME variance of a fund category. The figure shows significantly lower allocations to venture capital relative to buyout funds for the same average risk-adjusted performance. The portfolio choice perspective thus highlights that similar average risk-adjusted performance may imply markedly different optimal allocations depending on the variability of excess performance.

5.3 Extensions

The expected utility framework presented above can be extended along multiple dimensions. In this section, I apply the plug-in estimators to study optimal buyout fund allocations conditional on the state of the economy at fund inception. I furthermore examine how optimal allocations differ for investors with fund selection skills or limited access to top quartile funds.

Conditional Allocations

The plug-in estimators provide a mapping between PME moments and allocations such that optimal allocations can be estimated from conditional payoff moments. I estimate the conditionally

[36] See Harris et al. (2018) for information on the fees charged by funds of funds.

[37] Harris et al. (2018) defines the PME as a ratio, the standard deviations are therefore not directly comparable to standard deviations based on the PME defined in differences.

[38] Andonov (2020) provides a general study of delegated investment management for alternative assets.

expected PME from the cross-sectional regression; $\text{PME}_\text{i} = \boldsymbol{Z}\beta_m + \varepsilon_i$, where $\boldsymbol{Z}$ contains state variables, or functions of state variables, observable one period prior to each fund's inception date. The regression implies conditional expectation $\text{E}[\text{PME}|\boldsymbol{Z}] = \boldsymbol{Z}\beta_m$. I estimate the conditional variance by the predicted value of the regression $(\text{PME} - \text{E}[\text{PME}|\boldsymbol{Z}])^2 = \boldsymbol{Z}\beta_v + \eta$. Using the conditional moments in the plug-in estimator for investors with log-utility preferences produces allocations as a function of regression coefficients and state variables at fund inception, $\theta_p(\beta_m, \beta_v, \boldsymbol{Z})$. I summarize investment opportunities using four variables. (1) The S&P 500 dividend yield, (2) a corporate bond credit spread, (3) a government bond term premium and (4) a long-term government bond yield.[39] I consider each state variable separately to evaluate the relative importance of the state variables for optimal allocations. I use a second-order polynomial function for each state variable.

Figure 4 plots buyout fund allocations as a function of state variables for a log-utility investor using the market as the benchmark. The solid line depicts allocations using the state vector $\boldsymbol{z_0} = [1, z_0, z_0^2]^\top$ to estimate the conditional PME mean and variance. The dotted line shows allocations assuming constant conditional variance. The histograms in the background depict state variable realizations across funds. The figure shows that the allocation policies are not constant, which implies that the state of the economy at fund inception affects buyout funds' excess performance.

The top left figure plots conditional allocations for the dividend yield. LPs allocate the largest fraction of wealth to buyout funds when the dividend yield is especially low or high. The smallest allocations occur in the middle of the state variable domain. The dividend yield provides the largest variation in conditional allocations across the state variable domain among the four state variables. The variability in conditional allocations suggests investors should be particurlaly concerned with the level of public market valuations.

Considering the credit spread conditional allocations are U-shaped for constant PME variance (dotted line) but inversely U-shaped when the variance is also a function of the credit spread. Compared to the dividend yield the range of conditional allocations is considerably lower. The portfolio policy for the credit spread appears sensitive to the modeling of payoff moments since the shape of conditional allocations changes depending on how the PME variance is modeled.

Conditional allocations are inversely U-shaped in the term-spread, which is evident from the bottom left plot. The range of conditional allocations is small compared to the dividend yield.

[39] The data used to construct the state variables are from Amit Goyal's data library. The dividend yield is defined as the sum of the last 12 months dividends divided by the current price of the S&P 500 price. The credit spread is the difference between the annualized yield of BAA and AAA-rated corporate bonds. The term premium is the difference between the annualized 3-month Treasury bill yield and a 10-year yield. The long-term government bond yield is the yield on 10-year Treasury bonds.

The smallest allocation occurs for a term spread approximately equal to zero, meaning for a flat yield curve. The yield curve typically flattens in recessions. The results thus suggest that LPs should allocate less to buyout funds under these circumstances.

The last plot depicts allocations as a function of long-term interest rates. Allocations decrease in long-term interest rates when PME variance is modeled as a function of the long-term yield, such that LPs allocate more to buyout funds when long-term interest rates are low. Considering the constant variance specification allocations increase in the long-term yield at low yield levels while allocations are flat for annualized yields above 6 percent.

LP Selection Skill and Fund Access

The previous sections examine optimal allocations for an LP investing in a random buyout fund. Lerner, Schoar, and Wongsunwai (2007), however, finds that realized returns from investing in PE differ across institutional investors depending on investor sophistication and Cavagnaro, Sensoy, Wang, and Weisbach (2019) finds that certain LPs can select funds with above-average performance. Certain LPs might thus be capable of obtaining better than average performance by selecting a subset of funds or by attaining exclusive access to the best performing funds. Exclusive access to certain funds for some LPs implies limited access for other LPs. These constrained LPs may not be able to achieve the risk-adjusted performance of an average fund.

To analyze the portfolio implications of these observations, I modify the expected utility framework to incorporate fund selection skills and differential fund access. I introduce LP selection skills or better fund access by shifting the probabilities, p_i, in Equation 18 towards funds with higher ex-post PME realizations. Conversely, limited access to funds is introduced by increasing the probabilities of selecting funds below a certain performance threshold. Incorporating skill (limited access) in this way mechanically increases (decreases) the LPs' expected risk-adjusted performance, which increases (decreases) optimal PE allocations. This is evident from the approximate solution in Equation 17. The effect on optimal allocations, however, also depends on PME variance and covariance with the benchmark. LPs can, in theory, have skill in choosing funds with high PMEs, but these funds might not be the best investments if they increase variance disproportionally or correlate positively with the benchmark. From a portfolio choice perspective, LPs should choose funds providing the highest utility of wealth. I, however, consider the practically important case of LPs selecting funds with the highest risk-adjusted performance.

Figure 5 plots buyout allocations for log-utility LPs with fund selection skill. Allocations are estimated by assigning higher probabilities to funds with top quartile (top 25%) risk-adjusted performance. The horizontal axis represents LP selection skill by a probability multiple, p_Q. A probability multiple of 2 means that the LP is twice as likely to select a fund in the top quartile relative to an unskilled LP. A multiple of one represents the base case with no selection skill.

Unsurprisingly buyout allocations increase for LPs with fund selection skills. Interestingly the change in optimal allocation depends immensely on the benchmark. Considering the market benchmark the optimal allocation increase approximately 9 percentage points from 1.1 in the base case to 1.2 for a probability multiple of two. The allocation change is considerably larger for the characteristic portfolio benchmarks.[40] As an example, consider the small-value benchmark. Because the base case allocation is moderately negative, only a small level of selection skill is needed for LPs to optimally allocate a positive amount of wealth to the buyout strategy. An LP twice as likely to select a fund in the top quartile allocates 60 percent of wealth to buyout funds compared to negative 8 percent in the base case for the small-value benchmark. While the relationship between fund selection skill and allocations is unsurprising, the difference in the value of fund selection skill across benchmarks is more surprising.

Figure 6 depicts allocations for log-utility LPs with limited access to top quartile funds. Buyout fund allocations decrease across benchmarks as access to the best performing funds becomes more limited. As an example, optimal allocations for the value and small stock benchmarks decrease from approximately 75 percent to 25 percent when access to the best funds is restricted the most, illustrating the importance of LPs having access to the best performing funds. These results suggest that restricted access to the best performing funds should significantly affect investors' allocation decisions.

Figure 7 plots buyout fund allocations for restricted access to top quartile funds for LPs with different levels of risk aversion, for the market benchmark. The lines represent different levels of access. Restricting access approximately corresponds to a downward shift in the optimal buyout fund allocations. Conceptually, restricted access is a friction lowering the optimal allocation to the PE asset across all levels of risk aversion. The plot shows a relatively larger absolute change in allocations at lower levels of risk aversion. As an example, the allocation changes from approximately 50 percent in the unrestricted case to 38 percent in the most restricted specification for an LP with $\gamma = 3$ while the allocation decreases from 25 percent to 20 percent for $\gamma = 8$.

[40]Intuitively, $\frac{\Delta\theta_p^{SV}}{\Delta p_Q} > \frac{\Delta\theta_p^{Mkt}}{\Delta p_Q}$.

6 Conclusion

This paper relates the PME measure of risk-adjusted performance to a simple portfolio allocation problem between two risky trading strategies. I introduce a setting where a log-utility investor's optimal PE allocation is characterized entirely by moments of excess PE performance, represented by the PME. To a second-order approximation, average excess PE performance and variance determine the optimal PE allocation. Investors with general power-utility preferences determining allocations from the PME must also account for the covariance between excess PE performance and benchmark returns. Consequently, power-utility investors can observe positive average risk-adjusted performance but optimally choose not to allocate to the PE strategy.

I relate the portfolio choice problem's theoretical formulation to cash flow data on buyout funds using a sample of funds from Preqin. Empirically buyout fund's high average excess performance and low excess performance variance, relative to a broad equity market benchmark, leads to substantial buyout fund allocations, particularly for log-utility investors. Optimal buyout fund allocations decrease in investor risk aversion, but allocations are economically significant even for highly risk averse investors. Optimal buyout fund allocations are generally lower if the alternative investment is a portfolio of value, small or small-value stocks.

The valuation level of the public market appears to be the most important state variable for conditionally optimal buyout allocations for a log-utility investor allocating between a buyout fund strategy and the public equity market. LPs should generally allocate more to buyout funds when the public market dividend yield is low (public market valuations are high). I furthermore find that buyout fund allocations for LPs with fund selection skills are comparable with the allocations of unskilled LPs if the expected excess performance from selecting a random buyout fund is high. On the other hand, the allocation of skilled LPs is far greater if the expected excess performance from selecting a random buyout fund is low. Additionally, limited access to top quartile funds markedly decreases optimal buyout fund allocations.

Applying the PME to determine allocations undoubtedly limits the variety of problems, that can be analyzed. Using PMEs only permits allocating between a pre-specified benchmark and a particular PE strategy. While this paper seeks to reconcile the PME measure with portfolio choice models, interesting questions for further research include whether other measures such as TVPIs, IRRs or GPMEs can better accommodate portfolio choice with multiple assets.

Table 1
Summary Statistics

This table reports summary statistics for the funds data from Preqin. Panel A reports descriptive statistics on fund size, number of cash flows per fund, TVPI's and PME's. Panel B reports equal-weighted and size-weighted TVPI's and PME's, using the broad equity market as the benchmark, across vintage years.

Panel A: Descriptive Statistics

	Fund Size ($M)	#CFs	TVPI	PME
Mean	1473.0214	34.7465	1.7536	0.1933
Median	576.0000	34.0000	1.6366	0.1472
Std.	2590.1832	9.5177	0.9163	0.4190

Panel B: Performance by Vintage Year

		Equal-Weighted		Size-Weighted	
Vintage Year	#Funds	TVPI	PME	TVPI	PME
1980 - 1984	3	5.9146	0.5715	6.0069	0.6001
1985 - 1986	3	2.2887	0.0897	1.5421	−0.0201
1987 - 1988	9	3.2206	0.3124	3.1743	0.3240
1989 - 1990	7	2.4752	0.1487	2.6456	0.2175
1991 - 1992	8	1.9973	−0.0248	1.8777	−0.0184
1993 - 1994	22	2.1994	0.1643	2.2131	0.1574
1995 - 1996	24	1.4780	0.0480	1.6020	0.0910
1997 - 1998	49	1.4104	0.1644	1.4305	0.1740
1999 - 2000	55	1.7421	0.4551	1.7938	0.4873
2001 - 2002	32	1.8220	0.3151	1.9769	0.4572
2003 - 2004	34	1.7142	0.2497	1.9667	0.3702
2005 - 2006	78	1.5354	0.0614	1.4900	0.0454
2007 - 2008	35	1.6430	0.0729	1.5564	0.0450
All	359	1.7536	0.1933	1.6424	0.1791

Table 2
Public Market Equivalents

The table reports average PMEs for buyout funds relative to benchmarks of public equities. The benchmarks are (1) the public equity market represented by a value-weighted portfolio of US CRSP companies, (2) a value portfolio represented by the 20 percent highest ranked stocks in the CRSP universe based on their book to market ratio, (3) a portfolio of small stocks represented by the 20 percent smallest stocks in the CRSP universe measured by market capitalization and (4) a small-value portfolio constituting the smallest stocks in the CRSP universe with the highest book-to-market ratio. Standard errors are in square brackets and t-statistics are in brackets.

	Market	**Value**	**Small**	**Small-Value**
E[PME]	0.1933	0.0882	0.1062	−0.0097
	[0.0221]	[0.0180]	[0.0219]	[0.0178]
	(8.74)	(4.91)	(4.86)	(-0.54)

Table 3
Optimal Allocations

The table reports the optimal fraction of initial wealth allocated to the buyout fund strategy and the benchmark strategy for investors with different levels of risk aversion. Allocations are estimated by maximizing $\mathrm{E}\Big[u(W_{T,i})\Big]$ across funds, for preferences:

$$u(W_T) = \begin{cases} \frac{W_T^{1-\gamma}}{1-\gamma} & \text{if } \gamma > 1 \\ \ln(W_T) & \text{if } \gamma = 1 \end{cases}$$

Allocations are estimated using GMM and standard errors are reported in square brackets.

	Market		**Value**		**Small**		**Small-Value**	
γ	θ_p	θ_b	θ_p	θ_b	θ_p	θ_b	θ_p	θ_b
1	1.101	−0.101	0.781	0.219	0.729	0.271	−0.084	1.084
	[0.082]	[0.082]	[0.146]	[0.146]	[0.133]	[0.133]	[0.154]	[0.154]
2	0.689	0.311	0.449	0.551	0.458	0.543	0.226	0.774
	[0.078]	[0.078]	[0.090]	[0.090]	[0.082]	[0.082]	[0.099]	[0.099]
3	0.508	0.492	0.327	0.673	0.360	0.641	0.230	0.770
	[0.062]	[0.062]	[0.073]	[0.073]	[0.069]	[0.069]	[0.095]	[0.095]
5	0.354	0.646	0.211	0.789	0.249	0.751	0.124	0.877
	[0.052]	[0.052]	[0.066]	[0.066]	[0.060]	[0.060]	[0.077]	[0.077]
8	0.249	0.751	0.126	0.874	0.156	0.844	0.048	0.952
	[0.044]	[0.044]	[0.057]	[0.057]	[0.048]	[0.048]	[0.055]	[0.055]

Table 4
Levered Market Benchmark PMEs

The table reports average PMEs for levered benchmark portfolios. The benchmark is the broad public equity market and the one-period levered benchmark return is given by; $R_{t+1} = R_t^f + \beta(R_{t+1}^m - R_t^f)$, where β denotes the level of leverage. Standard errors are reported in square brackets and t-statistics are in brackets.

β	**0.8**	**1.0**	**1.1**	**1.2**	**1.3**
E[PME]	0.2273	0.1933	0.1802	0.1694	0.1609
	[0.0222]	[0.0221]	[0.0223]	[0.0226]	[0.0231]
	(10.24)	(8.74)	(8.08)	(7.49)	(6.96)

Table 5
Approximate Optimal Allocations

The table reports optimal allocations to buyout funds for investors with different levels of risk aversion for different public market benchmarks. Allocations are estimated using the plug-in estimator:

$$\theta_p = \frac{\mathrm{E[PME]} - (\gamma - 1)\mathrm{cov}(w_T^b, \mathrm{PME})}{\gamma\sigma^2_{\mathrm{PME}} + \mathrm{E[PME]}^2}$$

σ^2 denotes PME variance. The last two columns in each panel separates the optimal buyout fund allocation into a component attributable to E[PME] and a component attributable to the covariance by splitting the numerator in the expression above such that the sum of the components equals the optimal allocation.

	Panel A: Market					**Panel B:** Value				
	E[PME]	σ^2	θ_p	θ_p **Mean**	θ_p **Cov**	E[PME]	σ^2	θ_p	θ_p **Mean**	θ_p **Cov**
$\gamma = 1$	0.193	0.175	0.910	0.910		0.088	0.116	0.716	0.716	
$\gamma = 2$			0.576	0.499	0.077			0.433	0.369	0.064
$\gamma = 3$			0.450	0.344	0.106			0.335	0.249	0.086
$\gamma = 5$			0.343	0.212	0.131			0.255	0.151	0.104
$\gamma = 8$			0.280	0.134	0.146			0.209	0.095	0.114
	Panel C: Small					**Panel D:** Small-Value				
$\gamma = 1$	0.106	0.171	0.583	0.583		−0.010	0.114	−0.085	−0.085	
$\gamma = 2$			0.359	0.301	0.058			0.231	−0.043	0.274
$\gamma = 3$			0.281	0.203	0.078			0.337	−0.028	0.365
$\gamma = 5$			0.217	0.123	0.095			0.422	−0.017	0.439
$\gamma = 8$			0.181	0.077	0.104			0.469	−0.011	0.480

Table 6
Approximate Optimal Allocations Bootstrap Statistics

The table reports bootstrap statistics for the optimal buyout fund allocation. Each simulation draws funds with replacement from the original sample and estimates the optimal buyout fund allocation using the plug-in estimators. I use 1000 samples for each benchmark and γ combination.

	Panel A: Market					**Panel B:** Value				
γ	1	2	3	5	8	1	2	3	5	8
Mean	0.9065	0.5749	0.4496	0.3430	0.2805	0.7191	0.4355	0.3363	0.2551	0.2087
Std.	0.0751	0.0523	0.0454	0.0422	0.0420	0.1262	0.0810	0.0716	0.0689	0.0699
2.5%	0.7537	0.4685	0.3621	0.2585	0.1961	0.4707	0.2718	0.1975	0.1165	0.0719
97.5%	1.0599	0.6786	0.5377	0.4233	0.3589	0.9627	0.5945	0.4810	0.3894	0.3462
	Panel C: Small					**Panel D:** Small-Value				
Mean	0.5827	0.3625	0.2856	0.2228	0.1870	-0.0879	0.2317	0.3386	0.4242	0.4724
Std.	0.0959	0.0636	0.0574	0.0561	0.0571	0.1597	0.0994	0.0942	0.0981	0.1032
2.5%	0.4006	0.2461	0.1821	0.1152	0.0775	-0.4169	0.0408	0.1577	0.2372	0.2720
97.5%	0.7673	0.4894	0.4004	0.3403	0.3091	0.2166	0.4337	0.5361	0.6165	0.6787

Table 7
Approximate Optimal Allocations with Adjusted PME Variance

The table reports optimal allocations to buyout funds using the plug-in estimator. PME denotes the average PME and σ^2 denotes the PME variance adjusted for overlapping funds. The last two columns in each panel separates the optimal allocation into the two components, $\frac{\mathrm{E[PME]}}{\gamma\sigma^2_{\mathrm{PME}}+\mathrm{E[PME]}^2}$ and $\frac{-(\gamma-1)\mathrm{cov}(w^b_T,\mathrm{PME})}{\gamma\sigma^2_{\mathrm{PME}}+\mathrm{E[PME]}^2}$.

	Panel A: Market					**Panel B:** Value				
	E[PME]	σ^2	θ_p	θ_p **Mean**	θ_p **Cov**	E[PME]	σ^2	θ_p	θ_p **Mean**	θ_p **Cov**
$\gamma = 1$	0.193	0.226	0.734	0.734		0.088	0.304	0.283	0.283	
$\gamma = 2$			0.456	0.395	0.061			0.168	0.143	0.025
$\gamma = 3$			0.354	0.270	0.084			0.129	0.096	0.033
$\gamma = 5$			0.268	0.166	0.102			0.097	0.058	0.040
$\gamma = 8$			0.218	0.105	0.113			0.080	0.036	0.044
	Panel C: Small					**Panel D:** Small-Value				
$\gamma = 1$	0.106	0.428	0.242	0.242		−0.010	0.676	−0.014	−0.014	
$\gamma = 2$			0.146	0.123	0.024			0.039	−0.007	0.046
$\gamma = 3$			0.114	0.082	0.032			0.057	−0.005	0.061
$\gamma = 5$			0.088	0.049	0.038			0.071	−0.003	0.074
$\gamma = 8$			0.073	0.031	0.042			0.079	−0.002	0.081

Table 8
Approximate Optimal Allocations with Adjusted PME Variance Bootstrap Statistics

The table reports bootstrap statistics for the optimal buyout fund allocation. Each simulation draws funds with replacement from the original sample and estimates the optimal buyout fund allocation using the plug-in estimators. I use 1000 samples for each benchmark and γ combination. The PME variance in each sample is adjusted for overlapping funds.

	Panel A: Market					**Panel B:** Value				
γ	1	2	3	5	8	1	2	3	5	8
Mean	0.6735	0.4159	0.3210	0.2413	0.1949	0.2948	0.1714	0.1293	0.0953	0.0760
Std.	0.2041	0.1252	0.0935	0.0663	0.0509	0.1274	0.0652	0.0447	0.0304	0.0247
2.5%	0.3682	0.2303	0.1807	0.1383	0.1133	0.1289	0.0846	0.0691	0.0508	0.0332
97.5%	1.1575	0.7134	0.5430	0.3961	0.3132	0.6254	0.3365	0.2429	0.1661	0.1343
	Panel C: Small					**Panel D:** Small-Value				
Mean	0.2283	0.1372	0.1062	0.0812	0.0670	-0.0159	0.0365	0.0540	0.0680	0.0759
Std.	0.0741	0.0396	0.0287	0.0211	0.0180	0.0284	0.0152	0.0119	0.0105	0.0103
2.5%	0.1154	0.0783	0.0629	0.0465	0.0331	-0.0804	0.0029	0.0293	0.0480	0.0561
97.5%	0.4032	0.2299	0.1756	0.1300	0.1059	0.0338	0.0646	0.0772	0.0894	0.0963

Table 9
Allocations to Other Fund Categories

The table reports PME means, variances and the optimal fraction of initial wealth invested in the PE strategy for a log-utility investor using the S&P 500 as the benchmark. The table reports optimal allocations for buyout funds ($N = 337$), venture capital funds ($N = 386$) and fund of funds ($N = 173$) using an alternative data set from Preqin.

	Buyout	**Venture Capital**	**Fund of funds**
E[PME]	0.144	0.127	0.022
σ^2_{PME}	0.204	1.786	0.060
θ_p	64.0%	7.0%	35.8%

Figure 1
Buyout Fund Allocations for Levered Benchmarks

The figure plots buyout fund allocations, as a function of investor risk aversion, for levered positions in a broad equity market benchmark. The levered benchmark one period return is given by $R_{t+1} = R_t^f + \beta(R_{t+1}^m - R_t^f)$ and the lines in the figure represents different levels of leverage, β. Allocations are estimated using GMM.

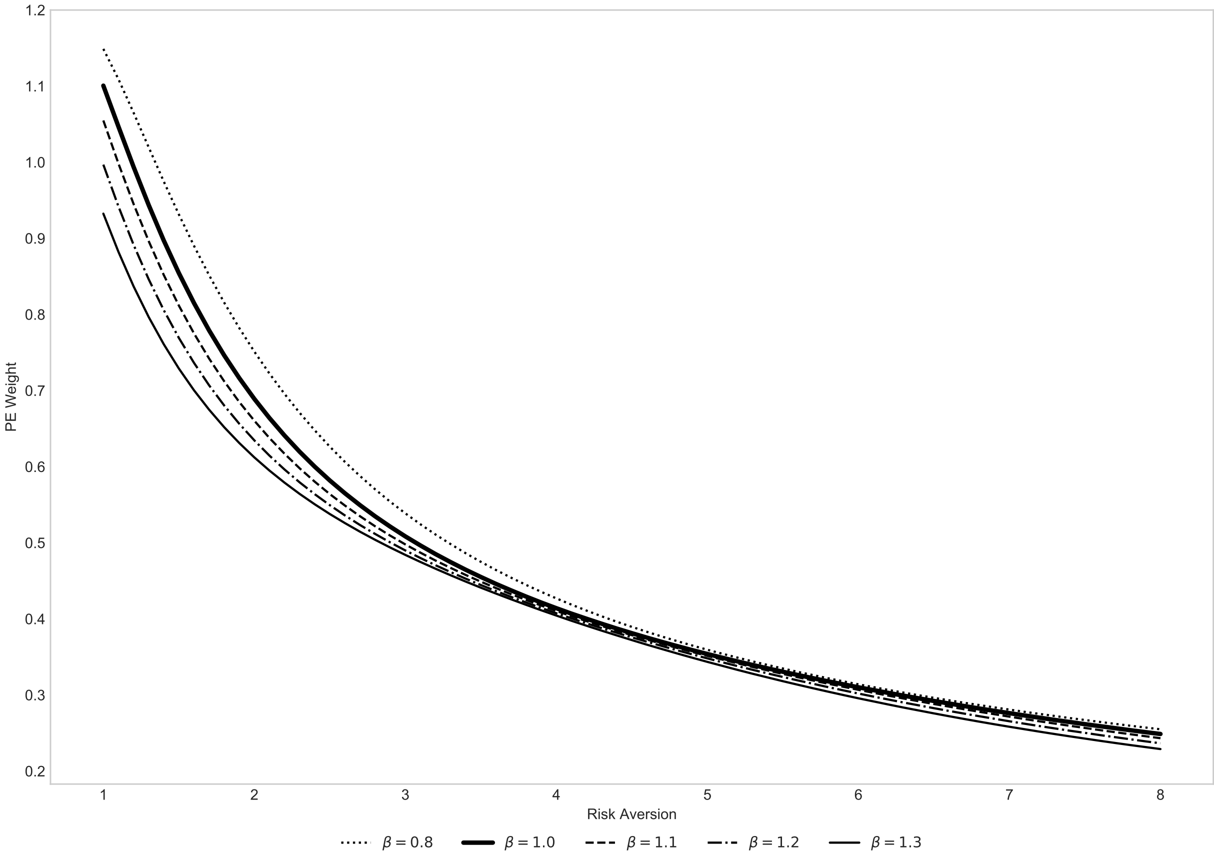


Figure 2
Optimal Allocations Relative to the Broad Equity Market Benchmark

The figure shows the optimal allocation to buyout funds and the benchmark strategy for different levels of risk aversion. The plot depicts optimal allocations, estimated using GMM, for the broad public equity market benchmark. The solid line shows the optimal allocations to the buyout fund strategy. The dashed line shows optimal allocations to the benchmark. The red dots depicts the optimal allocations of a log-utility investor. The grey shaded area highlights allocations for risk aversion between two and three.

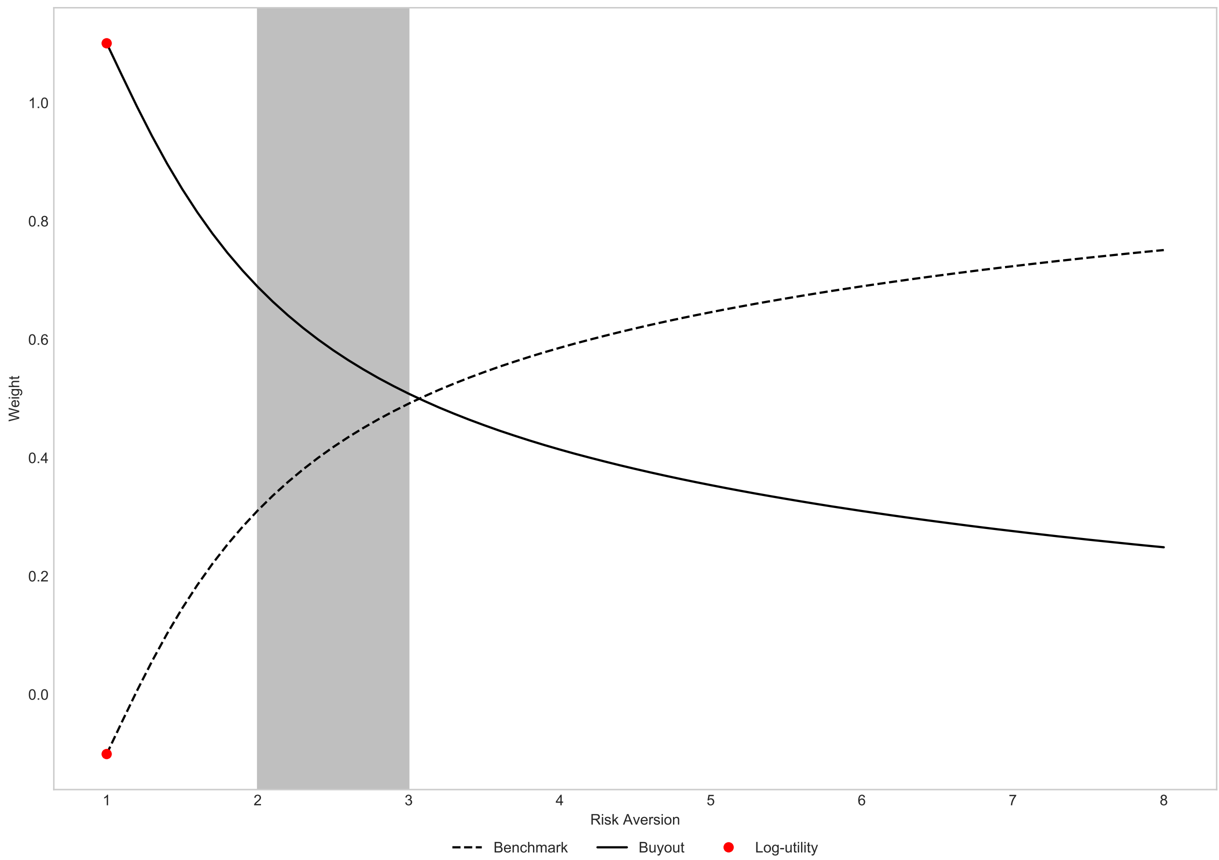


Figure 3
Optimal Allocations for Other Fund Categories

The figure plots the optimal private equity allocation for a log-utility investor estimated using:

$$\theta_p = \frac{\mathrm{E[PME]}}{\mathrm{E[PME^2]}}$$

The lines depict the optimal PE allocation as a function of the average PME for different levels of PME variance. The dotted line represents the optimal allocations corresponding to the variance of fund of funds reported in Table 9. The dash dotted and solid line plots the same for buyout and venture capital funds. The red dots represent the allocations corresponding to the average PME for the fund category reported in Table 9.

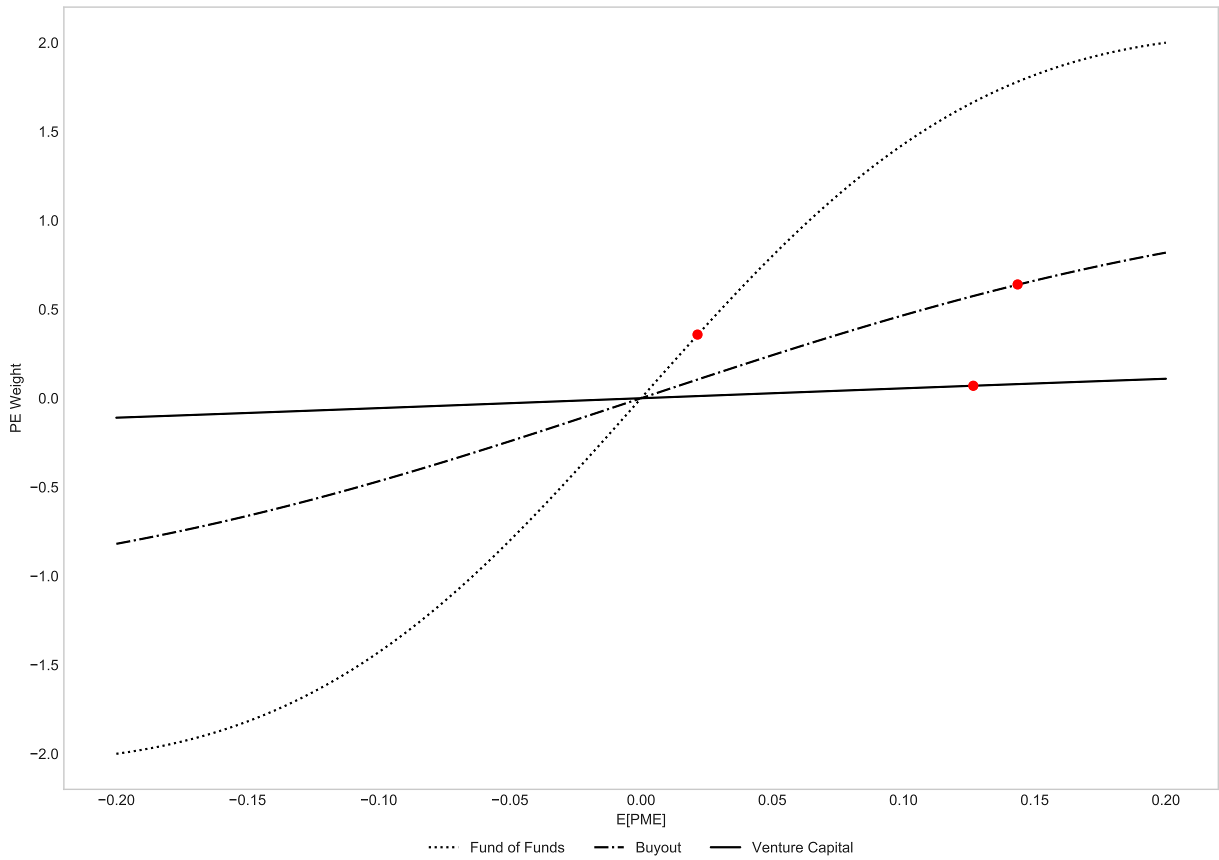


Figure 4
Conditional Allocations

The plot shows optimal buyout fund allocations, as a function of state variables at fund inception, for a log-utility LP using the broad equity market benchmark. The four state variables are the S&P 500 dividend yield, the credit spread, the term premium and a long-term Treasury bond yield. The solid line plots buyout fund allocations using state vector $\boldsymbol{z_0} = [1, z_0, z_0^2]^\top$ to estimate the conditional PME mean and variance. The dotted line plots the same for constant PME variance. Histograms of state variable realizations across funds are in the background.

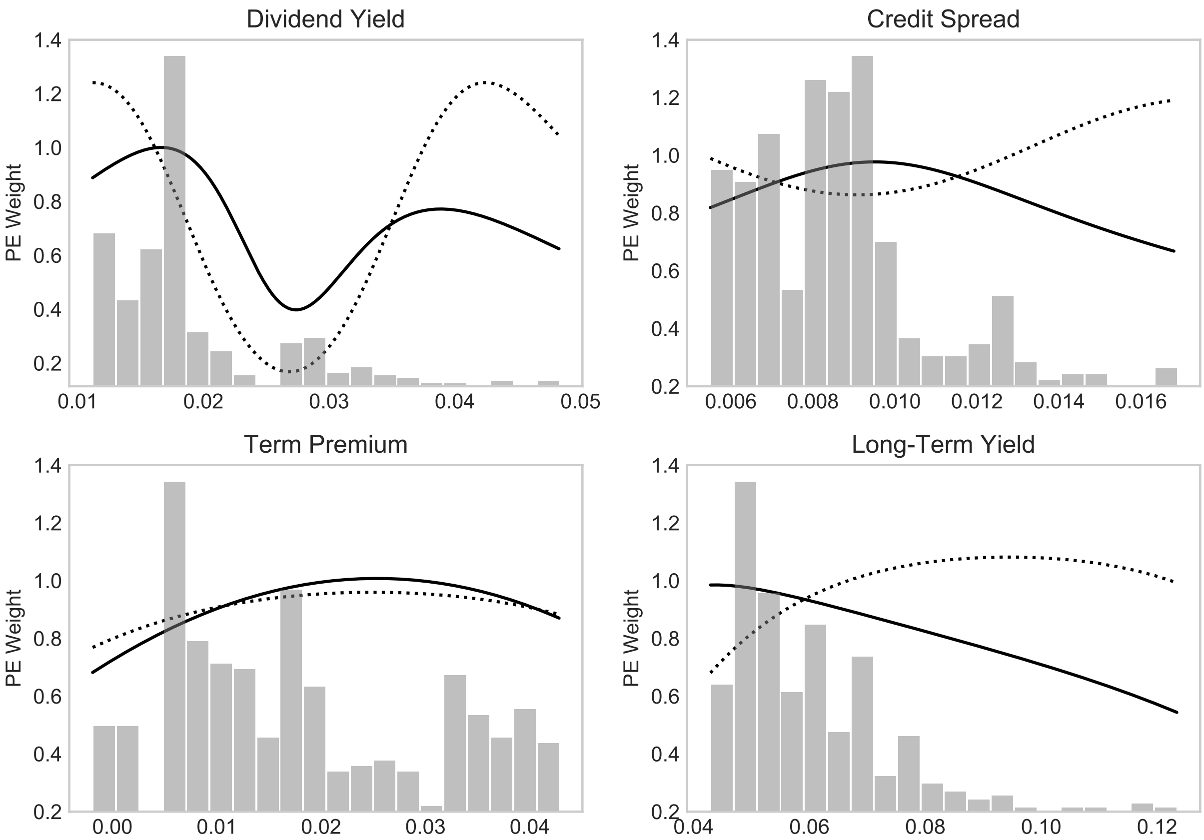


Figure 5
Optimal Buyout Fund Allocations for Skilled LPs

The figure plots optimal buyout allocations for log-utility LPs with fund selection skill. Skilled LPs select funds with risk-adjusted performance above a certain quartile, Q, with higher probability than funds below the quartile. The plot shows optimal buyout allocations relative to different public market benchmarks for LPs selecting funds in the top quartile (top 25 %) with higher probability. The x-axis shows the probability multiple p_Q. The probability of selecting a fund above Q are multiplied by p_Q while the probabilities of selecting funds below Q are maintained at the base case $p_i = \frac{1}{N}$. Weights are normalized to sum to one resulting in the shifted probabilities.

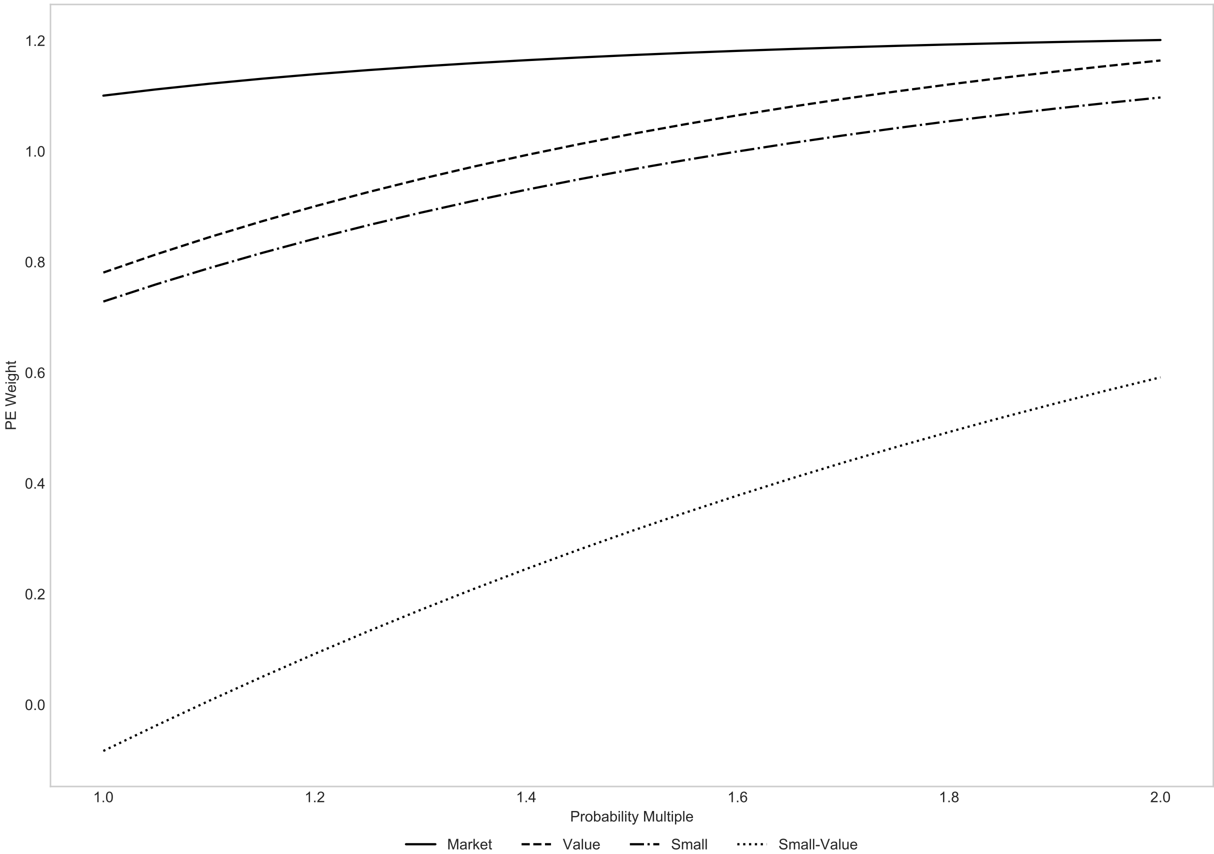


Figure 6
Optimal Buyout Fund Allocations for LPs with Limited Access to the Best Funds

The figure plots optimal allocations for log-utility LPs with limited access to the best performing buyout funds. The plots shows optimal allocations relative to different public market benchmarks for LPs selecting funds below the top quartile (top 25 %) with higher probability. The x-axis shows the probability multiple p_Q. The probability of selecting a fund below Q are multiplied by p_Q while the probabilities of selecting funds above Q are maintained at the base case $p_i = \frac{1}{N}$. Weights are normalized to sum to one leading to the shifted probabilities.

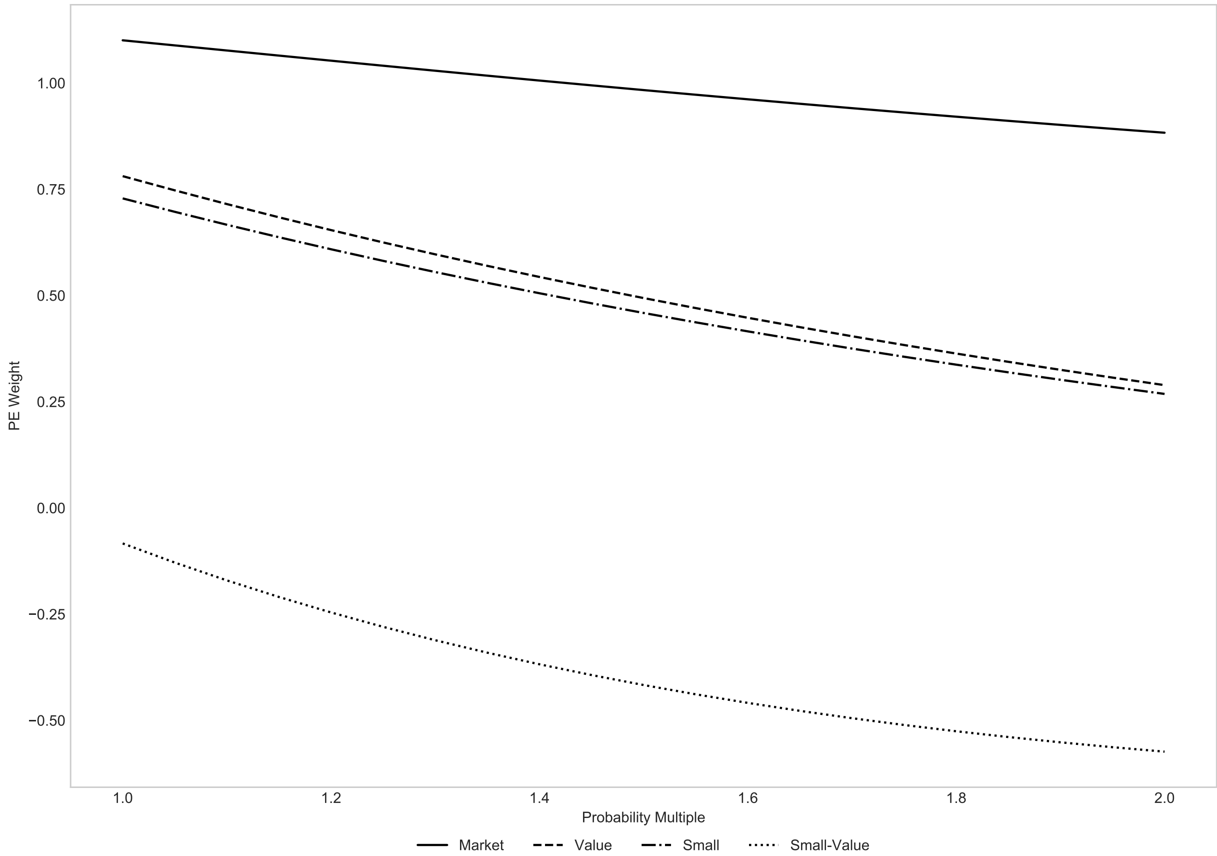
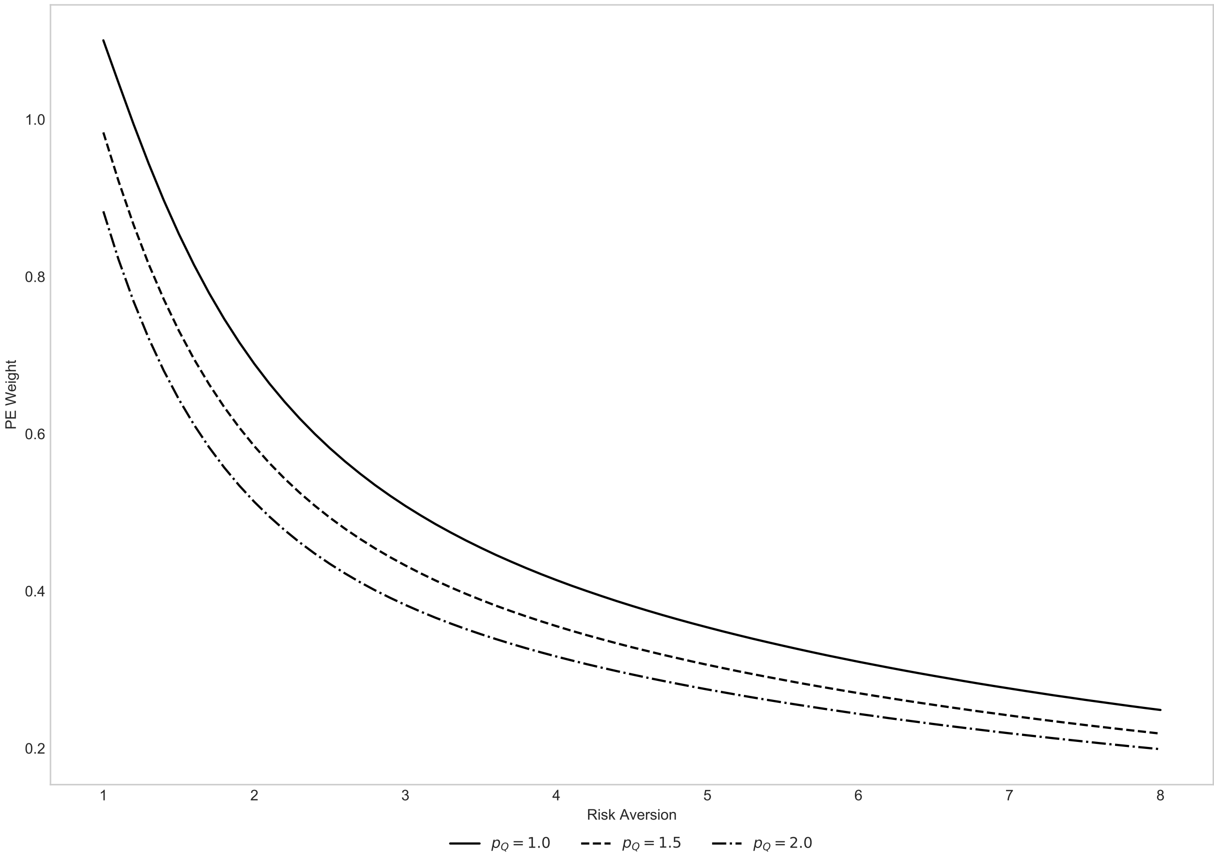


Figure 7
Optimal Buyout Fund Allocations for LPs with Limited Access Relative to the Market Benchmark

The figure plots optimal allocations for LPs with limited access to the best performing funds. The plots shows optimal allocations relative to the market benchmark for LPs selecting funds below the top quartile (top 25 %) with higher probability. The x-axis shows LP risk aversion. The solid line plots base case allocations and the remaining lines represent different levels of restrictions on top quartile fund access. A probability multiple $p_Q = 2.0$ represents the case with most restricted access to top quartile funds and $p_Q = 1.0$ represents the base case of full access.



Appendix

A Private Equity Strategy Terminal Wealth

Private equity strategy terminal wealth is given by the terminal value of two separate accounts. The first account, A, invests uncalled capital at the benchmark rate of return period by period. Capital calls from the underlying PE fund is withdrawn from the account. The second account, B, reinvests distributed capital into the benchmark until the terminal payoff date. Denote committed capital (initial wealth) by W_0. Let C_t and D_t denote capital calls and distributions per dollar of committed capital, i.e. calls and distributions are given by $W_0 C_t$ and $W_0 D_t$ respectively. Account, A, starts with initial wealth W_0 and invests uncalled capital in the benchmark period by period. Capital calls are drawn each period these occur. The timing is that the account earns the benchmark return and subsequently draws capital which leads to the following evolution for the account's value:

$$A_0 = W_0 \tag{A.1}$$

$$A_1 = A_0 \exp(r^b_{0:1}) - W_0 C_1 \tag{A.2}$$

$$A_2 = A_1 \exp(r^b_{1:2}) - W_0 C_2 \tag{A.3}$$

$$A_3 = A_2 \exp(r^b_{2:3}) - W_0 C_3 \tag{A.4}$$

Where $r^b_{0:1}$ denotes the log-return on the benchmark from period zero to one. Repeated substitution leads to the following:

$$A_3 = W_0 \exp(r^b_{0:1}) \exp(r^b_{1:2}) \exp(r^b_{2:3}) - W_0 C_1 \exp(r^b_{1:2}) \exp(r^b_{2:3}) - W_0 C_2 \exp(r^b_{2:3}) - W_0 C_3 \tag{A.5}$$

$$A_3 = W_0 \Big(\exp(r^b_{0:3}) - \sum_{t=1}^{3} C_t \exp(r^b_{t:3}) \Big) \tag{A.6}$$

Where $\exp(r^b_{0:3}$ denotes the compounded benchmark return from fund inception until time 3 and $\exp(r^b_{i:j}) = 1$ for $i = j$. For an investment with horizon T:

$$A_T = W_0\Big(\exp(r^b_{0:T}) - \sum_{t=1}^{T} C_t \exp(r^b_{t:T})\Big) \tag{A.7}$$

Account, B, starts off with zero initial capital. When the underlying private equity fund makes distributions these are put into the account and reinvested period by period at the benchmark rate of return. This leads to the following evolution for the account value:

$$B_0 = 0 \tag{A.8}$$

$$B_1 = B_0 \exp(r^b_{0:1}) + W_0 D_1 \tag{A.9}$$

$$B_2 = B_1 \exp(r^b_{1:2}) + W_0 D_2 \tag{A.10}$$

$$B_3 = B_2 \exp(r^b_{2:3}) + W_0 D_3 \tag{A.11}$$

Repeated substitution leads to:

$$B_3 = B_0 \exp(r^b_{0:1}) \exp(r^b_{1:2}) \exp(r^b_{2:3}) + W_0 D_1 \exp(r^b_{1:2}) \exp(r^b_{2:3}) + W_0 D_2 \exp(r^b_{2:3}) + W_0 D_3 \tag{A.12}$$

$$B_3 = W_0\Big(\sum_{t=1}^{3} D_t \exp(r^b_{t:3})\Big) \tag{A.13}$$

Resulting in:

$$B_T = W_0\Big(\sum_{t=1}^{T} D_t \exp(r^b_{t:T})\Big) \tag{A.14}$$

The total terminal wealth in the private equity strategy is the sum of the terminal account values, $W_T = A_T + B_T$:

$$W_T = W_0\Big(\exp(r^b_{0:T}) + \sum_{t=1}^{T} (D_t - C_t) \exp(r^b_{t:T})\Big) \tag{A.15}$$

B Solving Approximate Problems

Applying the restriction $\theta_b + \theta_p = 1$ terminal portfolio wealth, W_T^I, can be expressed as follows:

$$W_T^I = (1-\theta_p)W_T^b + \theta_p W_T^p = (1-\theta_p)W_T^b + \theta_p \frac{W_T^p}{W_T^b} W_T^b \tag{B.1}$$

$$W_T^I = (1-\theta_p)W_T^b + \theta_p(1+\text{PME})W_T^b = W_T^b(1+\theta_p\text{PME}) \tag{B.2}$$

B.1 Log-Utility

The expected utility maximization problem over terminal wealth for log-utility preferences is consequently:

$$\max_{\theta_p} \mathrm{E}\big[\ln(W_T^I)\big] = \max_{\theta_p} \mathrm{E}\Big[\ln(W_T^b) + \ln(1+\theta_p\text{PME})\Big] \tag{B.3}$$

A second-order Taylor expansion of the function $\ln(1+\theta_p\text{PME})$ around PME $= 0$ leads to:

$$\ln(1+\theta_p\text{PME}) \approx \theta_p\text{PME} - \frac{1}{2}\theta_p^2\text{PME}^2 \tag{B.4}$$

Substitution into Equation B.3 yields:

$$\max_{\theta_p} \mathrm{E}\big[\ln(W_T^I)\big] \approx \max_{\theta_p} \mathrm{E}\Big[w_T^b + \theta_p\text{PME} - \frac{1}{2}\theta_p^2\text{PME}^2\Big] \tag{B.5}$$

Where $w_T^b = \ln(W_T^b)$. This leads to the optimal PE allocation:

$$\theta_p^\star = \frac{\mathrm{E}[\text{PME}]}{\mathrm{E}[\text{PME}^2]} = \frac{\mathrm{E}[\text{PME}]}{\mathrm{E}[\text{PME}]^2 + \sigma^2_{\text{PME}}} \tag{B.6}$$

B.2 Power-Utility

LPs with power-utility preferences and $\gamma > 1$ solves:

$$\max_{\theta_p} \mathrm{E}\Big[\frac{1}{1-\gamma}(W_T^I)^{1-\gamma}\Big] = \max_{\theta_p} \mathrm{E}\Big[\frac{1}{1-\gamma}\Big(\exp(w_T^b)\exp(\ln(1+\theta_p\text{PME}))\Big)^{1-\gamma}\Big] \tag{B.7}$$

Applying the approximation of $\ln(1+\theta_p\text{PME})$ does not immediately lead to an analytical solution. The additive PME and PME^2 terms combined with power-utility complicates the problem. I therefore approximate PME^2 using a constant, specifically the least square optimal constant, i.e. the linear projection of PME^2 onto a constant. Using this projection leads to the approximation,

$\text{PME}^2 \approx \text{E}[\text{PME}^2]$, resulting in the crude approximation:

$$\ln(1+\theta_p \text{PME}) \approx \theta_p \text{PME} - \frac{1}{2}\theta_p^2 \text{E}[\text{PME}^2] \tag{B.8}$$

Substituting into Equation B.7 results in:

$$\max_{\theta_p} \text{E}\Big[\frac{1}{1-\gamma}(W_T^I)^{1-\gamma}\Big] \approx \max_{\theta_p} \text{E}\Big[\frac{1}{1-\gamma}\exp\Big((1-\gamma)w_T^b + (1-\gamma)\theta_p\text{PME} - \frac{1}{2}\theta_p^2(1-\gamma)\text{E}[\text{PME}^2]\Big)\Big] \tag{B.9}$$

Maximizing the expectation is the same as maximizing the log of the expectation since the logarithm is an increasing function. Since $\gamma > 1$, $\frac{1}{1-\gamma} < 0$ meaning that the logarithm of the constant is not defined. I therefore negate the objective function and consider the minimization problem:

$$\min_{\theta_p} \text{E}\Big[\frac{-1}{1-\gamma}\exp\Big((1-\gamma)w_T^b + (1-\gamma)\theta_p\text{PME} - \frac{1}{2}\theta_p^2(1-\gamma)\text{E}[\text{PME}^2]\Big)\Big] \tag{B.10}$$

Where the constant $\frac{-1}{1-\gamma} > 0$ for $\gamma > 1$. Assuming benchmark log wealth and the PME are jointly normally distributed the properties of the log-normal distribution leads to the following minimand after taking the logarithm of the expectation:

$$\begin{aligned}\min_{\theta_p} c + (1-\gamma)\Big(&\text{E}[w_T^b] + \theta_p\text{E}[\text{PME}] - \frac{1}{2}\theta_p^2\text{E}[\text{PME}^2] + \frac{1}{2}(1-\gamma)\sigma_b^2 \\ &+ \frac{1}{2}(1-\gamma)\theta_p^2\sigma_{\text{PME}}^2 + \theta_p(1-\gamma)\text{cov}(w_T^b, \text{PME})\Big)\end{aligned} \tag{B.11}$$

Where $c = \ln(-1/(1-\gamma))$. This minimand leads to the following first-order condition:

$$0 = \text{E}[\text{PME}] - \theta_p\text{E}[\text{PME}^2] + (1-\gamma)\theta_p\sigma_{\text{PME}}^2 + (1-\gamma)\text{cov}(w_T^b, \text{PME}) \tag{B.12}$$

$$0 = \text{E}[\text{PME}] - \theta_p(\text{E}[\text{PME}]^2 + \gamma\sigma_{\text{PME}}^2) + (1-\gamma)\text{cov}(w_T^b, \text{PME}) \tag{B.13}$$

Where the property $-\text{E}[\text{PME}]^2 = -\text{E}[\text{PME}^2] + \sigma_{\text{PME}}^2$ has been used. The first-order condition leads to optimal PE allocation:

$$\theta_p^\star = \frac{\text{E}[\text{PME}] - (\gamma-1)\text{cov}(w_T^b, \text{PME})}{\gamma\sigma_{\text{PME}}^2 + \text{E}[\text{PME}]^2} \tag{B.14}$$

C CRRA Portfolio Choice

Consider a one-period choice between two risky assets with jointly log-normally distributed gross returns F_{t+1} and R_{t+1}. The first asset is a benchmark asset which I will denotes as the market. The second asset is a private equity asset. I follow Gredil et al. (2021) and define the following processes for log-returns:

$$f_{t+1} = r_f + \gamma\sigma^2 - \frac{1}{2}\sigma^2 + \sigma\epsilon_{t+1} \tag{C.1}$$

$$r_{t+1} = \alpha + r_f + \beta(f_{t+1} - r_f) - \frac{1}{2}(\sigma^2(\beta^2 - 1) + \omega^2) + \omega\eta_{t+1} \tag{C.2}$$

Where ϵ_{t+1} and η_{t+1} are standard normally distributed. r_f is a risk-free return. γ is the risk aversion of a representative investor and σ^2 is the variance of log market returns. α is the abnormal return on the PE asset, β is the systematic risk of the PE asset and ω^2 is the idiosyncratic variance.

Campbell and Viceira (2002) provides optimal asset allocations for multiple risky assets and a risky benchmark for power-utility investors solving:

$$\max \mathrm{E}_t\left[\frac{W_{t+1}^{1-\gamma}}{1-\gamma}\right] \tag{C.3}$$

Subject to:

$$W_{t+1} = R_{p,t+1}W_t \tag{C.4}$$

Where $R_{p,t+1}$ is the gross portfolio return, which is a linear combination of gross asset returns. The optimal allocation asset allocation is given by:

$$w_t = \frac{1}{\gamma^\star}\Sigma_t^{-1}(\mathrm{E}_t[r_{t+1} - r_{0,t+1}] + \frac{1}{2}\sigma_t) + \left(1 - \frac{1}{\gamma^\star}\right)(-\Sigma_t^{-1}\sigma_{0t}) \tag{C.5}$$

Where $r_{0,t+1}$ denotes the log-return on the risky benchmark, r_{t+1} denotes the return on the second asset, σ_t denotes the variance of log-returns on the second asset, $\gamma^\star$ denotes the investor's relative risk aversion, Σ_t denotes the covariance of log-returns and σ_{0t} denotes the vector of covariances of excess log returns with the benchmark log return.

Using the return processes defined above the quantities required to determine optimal allocations are:

$$\mathrm{E}_t[f_{t+1}] = r_f + \gamma\sigma^2 - \frac{1}{2}\sigma^2 \tag{C.6}$$

$$\mathrm{E}_t[r_{t+1}] = \alpha + r_f + \beta(\gamma\sigma^2 - \frac{1}{2}\sigma^2) - \frac{1}{2}(\sigma^2(\beta^2 - 1) + \omega^2) \tag{C.7}$$

$$\mathrm{E}_t[r_{t+1} - f_{t+1}] = \alpha + \beta(\gamma\sigma^2 - \frac{1}{2}\sigma^2) - \frac{1}{2}(\sigma^2(\beta^2 - 1) + \omega^2) - \gamma\sigma^2 + \frac{1}{2}\sigma^2 \tag{C.8}$$

$$\sigma_t^2[f_{t+1}] = \sigma^2 \tag{C.9}$$

$$\sigma_t^2[r_{t+1}] = \omega^2 \tag{C.10}$$

$$\sigma_{0t} = (\beta - 1)\sigma^2 \tag{C.11}$$

Using these in the solution for the optimal PE allocation yields:[41]

$$w_t = \frac{1}{\gamma^\star}\Big(\frac{\alpha + (\gamma\sigma^2 - \frac{1}{2}\sigma^2)(\beta - 1) - \frac{1}{2}\sigma^2(\beta^2 - 1)}{\omega^2}\Big) + \Big(1 - \frac{1}{\gamma^\star}\Big)(-(\beta - 1)\frac{\sigma^2}{\omega^2}) \tag{C.12}$$

The expression is conceptually similar to the expression for optimal allocations derived based on the PME. The weight in the PE asset is given by the excess performance of the PE asset plus a term accounting for the covariance between excess performance and the benchmark, normalized by the idiosyncratic risk of the PE asset. For $\gamma^\star = 1$ and $\beta = 1$ the optimal allocation is as follows:

$$w_t = \frac{\alpha}{\omega^2} \tag{C.13}$$

In this case the optimal PE allocation is similar to the allocation for a log-utility investor derived from the PME.

[41] Note that $\gamma^\star$ is the relative risk averison of the investor solving the allocation problem while γ is the risk aversion of the marginal investor determining market returns.

D Return-Based Optimal Allocations

Let the quarterly return to buyout funds constructed from aggregate cash flows and NAVs be given by:

$$R^p_{t+1} = \frac{NAV_{t+1} + CF_{t+1}}{NAV_t} \tag{D.1}$$

Variation in the returns above arises partly from NAV variation. Allocations estimated from these returns are therefore affected by NAV variations in contrast with the allocations estimated in the main analysis which are entirely based on cash flows. For benchmark returns R^b_{t+1} define the portfolio return:

$$R^I_{t+1} = \theta_p R^p_{t+1} + \theta_b R^b_{t+1} = R^b_{t+1} + \theta_p (R^p_{t+1} - R^b_{t+1}) \tag{D.2}$$

Where the last equality holds due to $\theta_p + \theta_b = 1$. Maximizing the expected utility of the portfolio return yields:

$$\max_{\theta_p} E\Big[u(R^I_{t+1})\Big] \tag{D.3}$$

The sample equivalent is:

$$\max_{\theta_p} \frac{1}{T} \sum_{t=0}^{T-1} u(R^I_{t+1}) \tag{D.4}$$

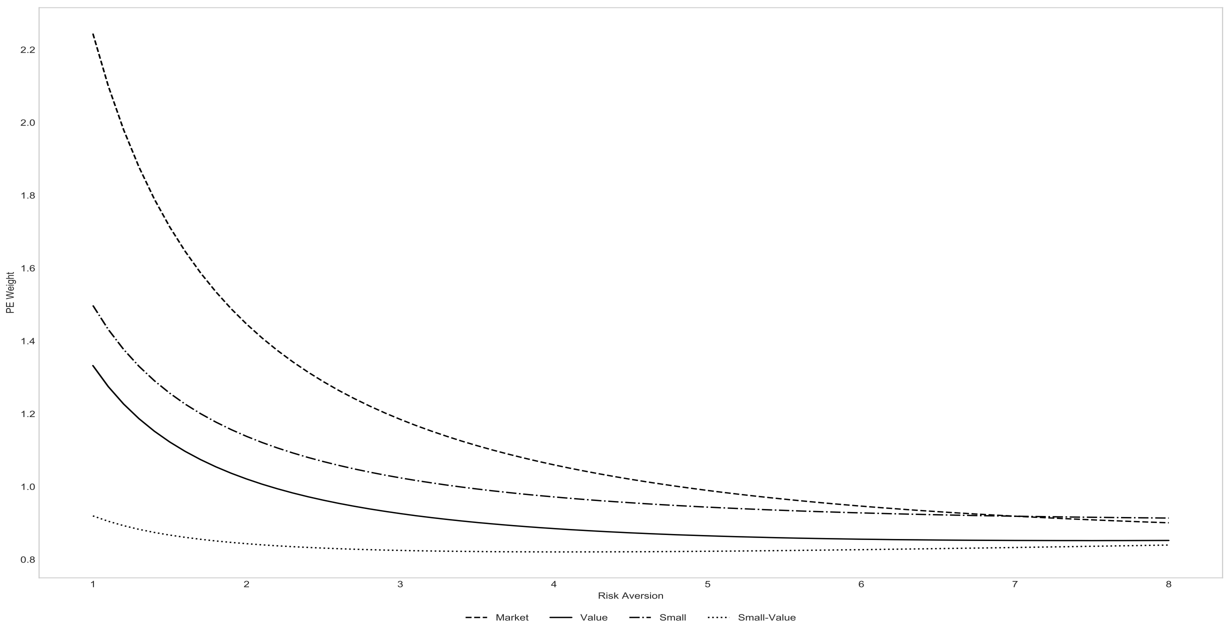
This objective function is equivalent to the criterion applied by Brandt (1999) to estimate optimal allocations for constant investment opportunities. Maximizing the sample equivalent, using aggregate buyout fund returns from Equation D.1 and benchmark returns, results in the allocations in the top plot in Figure D.1. The returns-based optimal buyout allocations are significantly larger than the cash flow implied allocations. Potential manipulation and smoothing of NAVs however mean the reliability of NAV-based returns is questionable, which is an issue the cash flow based allocations in the main analysis do not suffer from. The bottom plot depicts optimal allocations for indices of listed PE companies. Allocations to these indices are considerably lower than the allocations in the top figure.

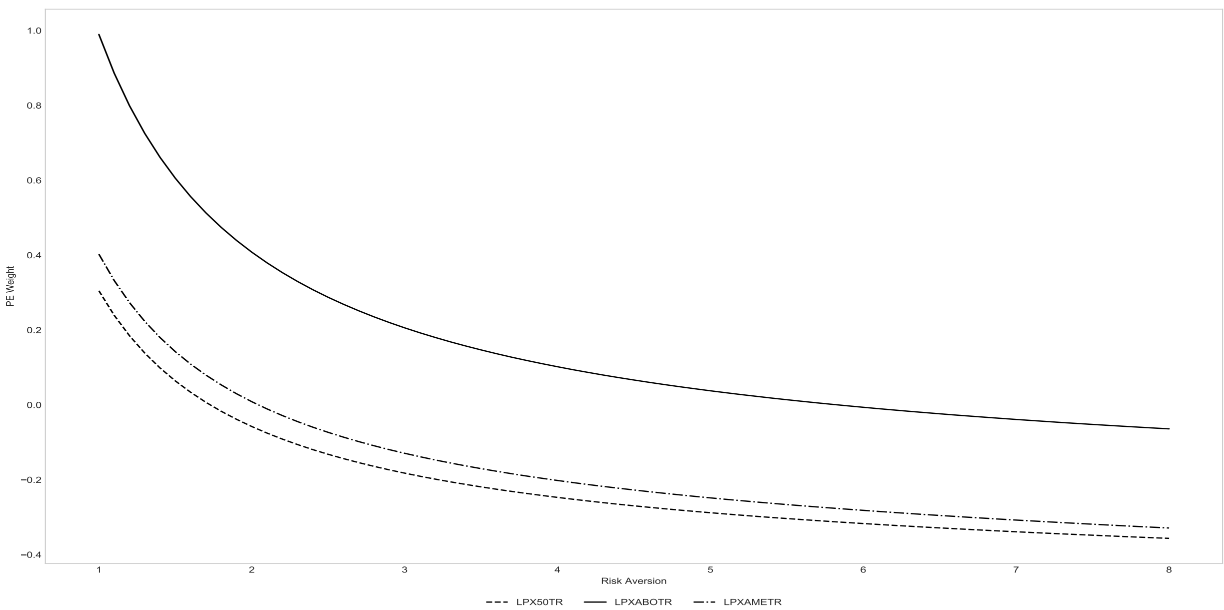
Figure D.1
Optimal Allocations

The figure plots optimal PE allocations estimated by maximizing:

$$\frac{1}{T}\sum_{t=0}^{T-1} u(R^I_{t+1})$$

The top plot shows buyout fund allocations for different benchmark portfolios and levels of investor risk aversion. Quarterly buyout fund returns are determined from NAVs and cash flows from the Preqin sample following Equation D.1. The bottom plot shows PE allocations relative to the market for a number of different listed PE indices by LPX Group sourced from Bloomberg. LPX50TR is an index of the 50 largest PE companies listed on global exchanges. LPXABOTR is an index of all major PE buyout companies listed on global exchanges. LPXAMETR is comprised of all major North American listed PE companies. The indices are converted into US dollars. Allocations are estimated on monthly returns in the period 1998-2016.





E Monte Carlo Simulation

Following Gredil et al. (2021) and Korteweg and Nagel (2016) I define the log market return, f_{t+1}, and the log PE return, r_{t+1}, processes:

$$f_{t+1} = r_f + \gamma\sigma^2 - \frac{1}{2}\sigma^2 + \sigma\epsilon_{t+1} \tag{E.1}$$

$$r_{t+1} = \alpha + r_f + \beta(f_{t+1} - r_f) - \frac{1}{2}(\sigma^2(\beta^2 - 1) + \omega^2) + \eta_{t+1} \tag{E.2}$$

Where ϵ_{t+1} is standard normal and η_{t+1} is multivariate normal with covariance matrix Γ, where $\Gamma = (\rho + I_N(1-\rho))\omega^2$ and I_N is the identity matrix. ρ denotes the correlation between PE investments generated from the second process and ω^2 denotes the idiosyncratic risk of PE investments. To simulate the processes, I use the annualized values; $r_f = 0.02$, $\sigma = 0.15$ and $\omega = 0.15$. I set $\rho = 0.1$ and consider PE assets with different α and β. I furthermore set $\gamma = 2$ which means the market risk premium is $\gamma\sigma^2 = 0.045$ per year. I set $T = 800$ quarters and set the number of funds in the sample $N = 350$ to approximately match the number of funds in the Preqin sample. I consider a fund life, H, of 11 years and draw fund inceptions dates, s, from a discrete uniform distribution on the integer interval $[1, T - H]$.

I use the return processes to estimate the aggregate (average) allocation to PE utilizing two different methods: (1) A time-series method. For each simulation run, I use the full time-series of PE returns for each simulated fund to calculate 11-year cumulative returns for the fund and the market processes. I then use the time-series of 11-year returns to estimate the allocation maximizing the expected utility of wealth for each fund. The cross-sectional average optimal PE allocation yields the optimal allocation for a single simulation run. The optimal PE allocation is thus the average optimal allocation to PE. (2) A cross-sectional approach. I use the randomly drawn inception dates of the simulated PE funds to calculate the 11-year cumulative return of the PE fund and the market covering the PE fund's investment period. I choose the PE allocation maximizing the cross-sectional expectation of portfolio wealth utils analogously to the approach used in the main analysis.

Table E.1 reports allocations for different values of α and β. $\gamma^\star$ denotes the risk aversion of the investor solving the expected utility maximization problem to distinguish it from the level of risk aversion generating market returns. The results show the time-series and cross-sectional methods results in comparable optimal allocations. Evaluating the standard deviation of optimal allocations suggests that the cross-sectional method may lead to slightly less efficient estimates of optimal allocations.

Table E.1
Monte Carlo Simulation

The table reports optimal private equity allocations resulting from simulating returns. Returns are simulated using. $r_f = 0.02$, $\sigma = 0.15$, $\omega = 0.15$ and $\rho = 0.1$. Each simulation has sample size $T = 800$ quarters and $N = 350$ funds. Panel A reports optimal private equity allocations for $\alpha = 0.02$. Panel B reports optimal allocations for $\alpha = 0.00$. The rows Time-Series denotes the methodology using time-series variation in 11-year returns to estimate allocations. Cross-Section denotes the methodology using cross-sectional variation in 11-year returns to estimate allocations. The number of simulations is 2500 and $\gamma^\star$ denotes investor risk aversion. Standard deviations are in square brackets.

	Panel A: $\alpha = 0.02$				**Panel B:** $\alpha = 0.00$			
	$\beta = 1.3$		$\beta = 1.0$		$\beta = 1.3$		$\beta = 1.0$	
	$\gamma^\star = 1$	$\gamma^\star = 2$	$\gamma^\star = 1$	$\gamma^\star = 2$	$\gamma^\star = 1$	$\gamma^\star = 2$	$\gamma^\star = 1$	$\gamma^\star = 2$
Time-Series	0.968	0.379	0.903	0.478	0.117	-0.078	-0.013	-0.030
	[0.192]	[0.128]	[0.152]	[0.096]	[0.185]	[0.099]	[0.147]	[0.088]
Cross-Section	0.951	0.347	0.896	0.446	0.131	-0.052	0.005	0.000
	[0.204]	[0.130]	[0.178]	[0.107]	[0.204]	[0.097]	[0.173]	[0.095]

F Generalized Method of Moments

Terminal portfolio wealth is given by $(1-\theta_p)W_T^b + \theta_p W_T^p$, leading to the expected utility optimization problem:

$$\max \mathrm{E}\Big[\frac{1}{1-\gamma}\big((1-\theta_p)W_T^b + \theta_p W_T^p\big)^{1-\gamma}\Big] \tag{F.1}$$

The associated first-order condition is:

$$\mathrm{E}\Big[(W_T^p - W_T^b)(W_T^b + \theta_p(W_T^p - W_T^b))^{-\gamma}\Big] = 0 \tag{F.2}$$

Using $\gamma = 1$ results in the first-order condition for a log-utility investor. Let $(W_{T,i}^p - W_{T,i}^b)(W_{T,i}^b + \theta_p(W_{T,i}^p - W_{T,i}^b))^{-\gamma} = \epsilon_i$ denote the pricing error of fund i. The first-order condition then provide a moment restriction, $\mathrm{E}[\epsilon_i] = 0$, which can be used to estimate the optimal allocation using (G)MM. The problem is exactly identified with one moment restriction and one parameter. Writing the optimization problem as a GMM estimation provides standard errors for the optimal allocations. Let $d = \frac{\partial}{\partial\theta_p}\mathrm{E}[\epsilon_i]\big|_{\theta=\hat{\theta}_p}$. Then:

$$d = -\gamma \mathrm{E}\Big[(W_{T,i}^p - W_{T,i}^b)^2(W_{T,i}^b + \hat{\theta}_p(W_{T,i}^p - W_{T,i}^b)^{-(\gamma+1)}\Big] \tag{F.3}$$

Assuming homoscedastic pricing error variance the spectral density matrix is given by the variance of pricing errors:

$$S = \sum_{j=-\infty}^{\infty} \mathrm{E}\big[\epsilon_i \epsilon_{i-j}^\top\big] = \sigma_\epsilon^2 \tag{F.4}$$

Resulting in the variance of θ_p:

$$\sigma_{\theta_p}^2 = \frac{1}{N}\big(d^\top S^{-1} d\big)^{-1} \tag{F.5}$$

An application of the delta method shows that the standard error for the benchmark allocation is the same due to the restriction $\theta_b = (1-\theta_p)$.

G Supporting Output

Table G.1
Optimal Allocations: Rescaled Wealth

The table reports the optimal fraction of initial wealth allocated to the buyout fund and the benchmark strategies. Allocations are estimated by maximizing $\mathrm{E}\Big[u(W_{T,i})\Big]$. Trading strategy terminal wealth is rescaled by reinvesting proceeds in the benchmark until the investment horizon H equals 60 quarters if the fund's last cash flows is before $H = 60$. Funds with last cash flow after 60 quarter are rescaled using $(W^p_{T,i}/W^b_{T,i})^{60/H_i} = (1 + \mathrm{PME}_i)^{60/H_i}$ where H_i is the quarter of the fund's last cash flow.

	Market		**Value**		**Small**		**Small-Value**	
γ	θ_p	θ_b	θ_p	θ_b	θ_p	θ_b	θ_p	θ_b
1	1.104	−0.104	0.823	0.177	0.750	0.250	−0.083	1.083
	[0.083]	[0.083]	[0.149]	[0.149]	[0.141]	[0.141]	[0.162]	[0.162]
2	0.692	0.308	0.486	0.514	0.450	0.550	0.169	0.831
	[0.081]	[0.081]	[0.094]	[0.094]	[0.085]	[0.085]	[0.094]	[0.094]
3	0.507	0.493	0.358	0.642	0.341	0.659	0.205	0.795
	[0.062]	[0.062]	[0.077]	[0.077]	[0.068]	[0.068]	[0.079]	[0.079]
5	0.353	0.647	0.236	0.764	0.237	0.763	0.151	0.849
	[0.047]	[0.047]	[0.063]	[0.063]	[0.055]	[0.055]	[0.065]	[0.065]
8	0.257	0.743	0.148	0.852	0.160	0.840	0.081	0.919
	[0.040]	[0.040]	[0.050]	[0.050]	[0.045]	[0.045]	[0.048]	[0.048]

Table G.2
Optimal Allocations: Wealth Reinvested at Risk-Free Interest Rate

The table reports the optimal fraction of wealth allocated to the buyout fund and the benchmark strategies. Allocations are estimated by maximizing $\mathrm{E}\Big[u(W_{T,i})\Big]$. To equalize the investment horizon across funds trading strategy wealth is reinvested such that the horizon of each fund equals the horizon of the fund with the longest sample investment horizon. For each fund strategy wealth is reinvested in the benchmark if realized benchmark returns are available. Realized benchmark returns are not available at long horizons for some funds in the sample. For long horizons wealth is reinvested at quarterly risk-free interest rate of 1.4% which equals the average in-sample risk-free rate. The reinvestment of wealth does not change the optimal allocations for $\gamma = 1$.

	Market		**Value**		**Small**		**Small-Value**	
γ	θ_p	θ_b	θ_p	θ_b	θ_p	θ_b	θ_p	θ_b
1	1.101	−0.101	0.781	0.219	0.729	0.271	−0.084	1.084
	[0.082]	[0.082]	[0.146]	[0.146]	[0.133]	[0.133]	[0.154]	[0.154]
2	0.664	0.336	0.468	0.532	0.454	0.546	0.209	0.791
	[0.079]	[0.079]	[0.090]	[0.090]	[0.082]	[0.082]	[0.093]	[0.093]
3	0.466	0.534	0.346	0.654	0.361	0.639	0.242	0.758
	[0.059]	[0.059]	[0.070]	[0.070]	[0.067]	[0.067]	[0.082]	[0.082]
5	0.302	0.698	0.228	0.772	0.257	0.743	0.162	0.838
	[0.040]	[0.040]	[0.055]	[0.055]	[0.056]	[0.056]	[0.067]	[0.067]
8	0.212	0.788	0.143	0.857	0.167	0.833	0.083	0.917
	[0.031]	[0.031]	[0.047]	[0.047]	[0.044]	[0.044]	[0.049]	[0.049]

H Log-Utility: OLS Estimation

Table H.1
Log-Utility: OLS Estimation

The table reports OLS coefficients for the regression:

$$1 = \theta_p \mathrm{PME}_i + \epsilon_i$$

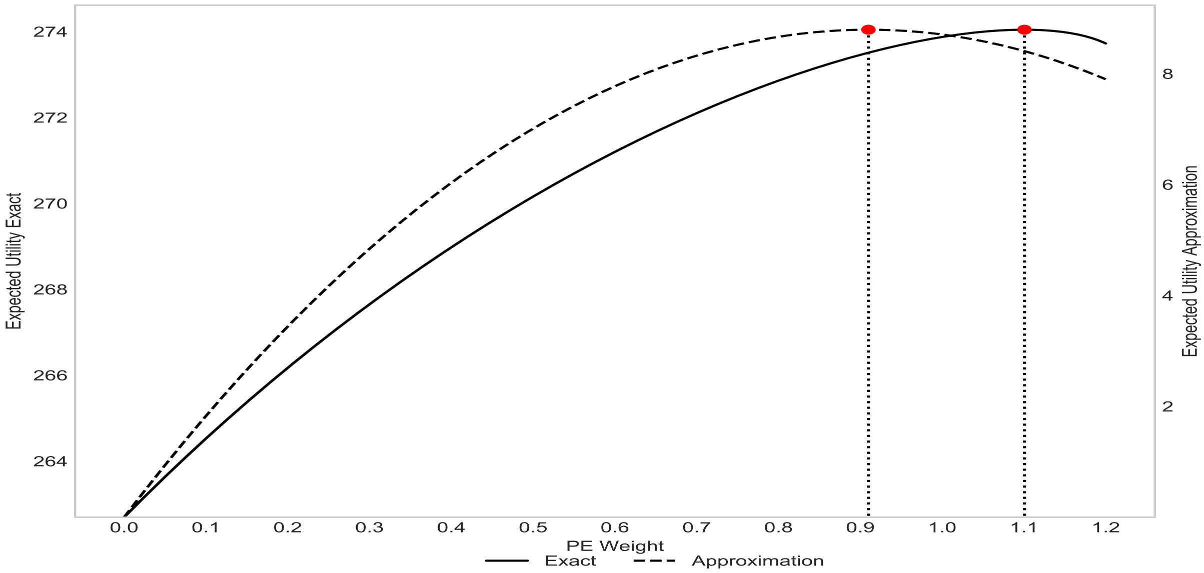
The regression coefficient equals the plug-in estimate for the optimal buyout fund allocation. Heteroscedastic OLS standard errors are reported in square brackets and t-statistics are reported in brackets.

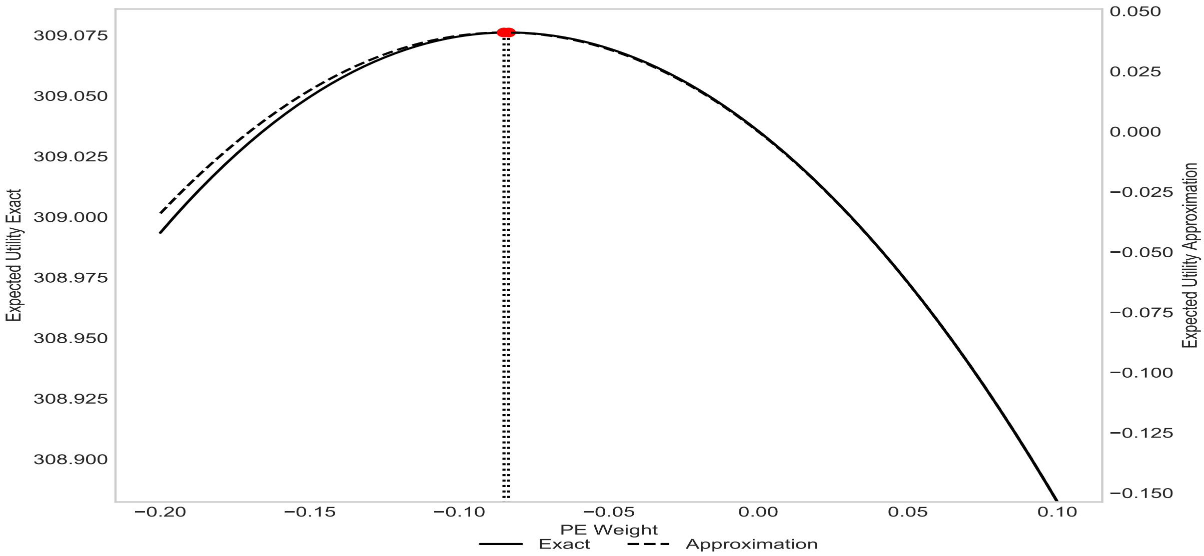
	Market	**Value**	**Small**	**Small-Value**
θ_p	0.910	0.715	0.583	-0.085
	[0.073]	[0.125]	[0.096]	[0.159]
	(12.46)	(5.71)	(6.09)	(-0.53)

I Expected Utility Approximation

Figure I.1
Expected Utility Approximation

The figure shows the expected utility of terminal portfolio wealth for a log-utility investor as a function of the buyout fund weight, θ_p, imposing $\theta_p + \theta_b = 1$. The solid line represents plots the function $\mathrm{E}[\ln(W^I_{T,i})]$. The dashed line plots the the approximation $\mathrm{E}[\mathrm{PME}_i]\theta_p - \frac{1}{2}\mathrm{E}[\mathrm{PME}_i^2]\theta_p^2$. Benchmark log terminal wealth determines the intercept of approximate function, which is set to zero without loss of generality. The left axis shows the expected utility from the exact function and the right axis plots the expected utility from the approximation. The scales of the expected utility are not comparable. The top figure plots expected utility for the market benchmark while the bottom figures shows the same for the small-value benchmark. The red dots shows the maximal expected utility and the optimal private equity allocation resulting from each function.





J PME Transformation

The PME of fund i, defined in differences is:

$$\text{PME}_i^{diff} = \sum_{t=0}^{T_i} (D_{i,t} - C_{i,t}) \frac{1}{R_{0:t}} \tag{J.1}$$

$D_{i,t}$ and $C_{i,t}$ denotes distributions and contributions, $R_{0:t}$ denotes the cumulative gross return from fund inception to the time of the cash flow. The ratio PME is:

$$\text{PME}_i^{ratio} = \frac{\sum_{t=0}^{T_i} D_{i,t} \frac{1}{R_{0:t}}}{\sum_{t=0}^{T_i} C_{i,t} \frac{1}{R_{0:t}}} \tag{J.2}$$

To see the relation between the definitions start from the definition of the PME in differences:

$$\text{PME}_i^{diff} = \sum_{t=0}^{T_i} D_{i,t} \frac{1}{R_{0:t}} - \sum_{t=0}^{T_i} C_{i,t} \frac{1}{R_{0:t}} \tag{J.3}$$

Divide each side by the present value of contributions:

$$\frac{\text{PME}_i^{diff}}{\sum_{t=0}^{T_i} C_{i,t} \frac{1}{R_{0:t}}} = \frac{\sum_{t=0}^{T_i} D_{i,t} \frac{1}{R_{0:t}}}{\sum_{t=0}^{T_i} C_{i,t} \frac{1}{R_{0:t}}} - 1$$

Which leads to:

$$1 + \frac{\text{PME}_i^{diff}}{\sum_{t=0}^{T_i} C_{i,t} \frac{1}{R_{0:t}}} = \frac{\sum_{t=0i}^{T_i} D_{i,t} \frac{1}{R_{0:t}}}{\sum_{t=0}^{T_i} C_{i,t} \frac{1}{R_{0:t}}} = \text{PME}_i^{ratio}$$

The PME in differences can therefore be recovered from the ratio PME by:

$$\text{PME}_i^{diff} = (\text{PME}_i^{ratio} - 1) \sum_{t=0}^{T_i} C_{i,t} \frac{1}{R_{0:t}} \tag{J.4}$$

Transforming the ratio PME to the PME in differences requires the discounted value of contributions. The alternative Preqin dataset does not contain cash flows. I therefore approximate PMEs in differences from ratio PMEs using the average discounted value of contributions from the buyout sample containing cash flows which equals 0.91.

Chapter 3

Risk Adjustment of Private Equity Cash Flows

Nicola Giommetti and Rasmus Jørgensen*

Abstract

Existing stochastic discount factor methods for the valuation of private equity funds result in unrealistic time discounting. We propose and evaluate a modified method. Valuation has a risk-neutral component plus a risk adjustment, and we fix the risk-neutral part by constraining the subjective term structure of interest rates with market data. We show that (i) our approach allows for economically meaningful measurement and comparison of risk across models, (ii) existing methods estimate implausible performance when time discounting is particularly unrealistic, and (iii) our approach results in lower variation of performance across funds.

Nicola Giommetti is from Copenhagen Business School (ngi.fi@cbs.dk) and Rasmus Jørgensen is from Copenhagen Business School and ATP (rjo@atp.dk). We are particularly indebted to Morten Sorensen for his guidance and support. We thank Anders Vilhelmsson (discussant) and seminar participants at Copenhagen Business School, ATP and Nordic Finance Network PhD Workshop for helpful discussion and comments. We are grateful to the Private Equity Research Consortium (PERC) and the Institute of Private Capital (IPC) for data access and support.

1 Introduction

Asset allocation to buyout, venture capital, and other private equity (PE) funds has increased consistently over the past decade.[1] It remains challenging, however, to estimate the risk and performance of these funds, especially due to their illiquid secondary market and the consequential absence of reliable return data. It is possible to extrapolate PE returns from cash flow data, as in Ang et al. (2018), but this requires restrictive assumptions on the return-generating process. To avoid those assumptions, Korteweg and Nagel (2016, KN) develops a stochastic discount factor (SDF) valuation framework that uses cash flows instead of returns and that benchmarks PE against publicly traded assets. Central to the SDF framework is a requirement for proper benchmarking: the SDF must price benchmark assets during the sample period. To satisfy this requirement, KN use a heuristic implementation. They build artificial funds invested in the benchmark assets and estimate SDF parameters by pricing these funds.

In this paper, we propose an alternative implementation. Our implementation estimates a set of SDF parameters such that the subjective term structure of interest rates is determined by market data. Theoretically, our approach is based on a decomposition of PE performance in a risk-neutral part and a risk adjustment. By construction, the risk-neutral component does not vary as we add or remove risk factors from the SDF, so we can meaningfully measure the economic cost of PE risk and compare it across SDFs. Empirically, we evaluate our approach against the KN implementation and find that the KN method results in unrealistic time discounting, which can generate implausible performance estimates. For example, a zero-coupon bond paying $1 at 3 years maturity can have discounted value up to $9, and a zero-coupon bond paying $1 at 10 years maturity can have discounted value up to $7. Our approach avoids these unrealistic implied zero-coupon bond prices. As a result, we obtain a more robust estimation of performance across SDFs and a lower variation of performance across funds.

We use our method to risk-adjust PE cash flows for two types of investors: a CAPM investor and a long-term investor who distinguishes between permanent and transitory wealth shocks. We discount net-of-fees cash flows of 1866 PE funds started in the US between 1978 and 2009, and divided into three categories: buyout, venture capital, and generalist.[2] As benchmark assets, we use the S&P 500 total return index and quarterly T-bills. For the CAPM investor, the buyout fund category has generated 30 cents of NPV per dollar of commitment, compared to 7 cents for venture capital and 21 cents for generalist funds. Unsurprisingly, venture capital has the highest

[1] Bain & Company (2021) quantify and discuss trends in PE allocation.

[2] PE data is maintained by Burgiss, and it is one of the most comprehensive PE datasets available to date.

(absolute value of) risk adjustment, equal to 65 cents per dollar of commitment and about twice as large compared to the 31 cents of buyout funds and 35 cents of generalist funds. We find only modest differences between the CAPM and the long-term investor. Compared to the CAPM, the long-term investor assigns 3 to 8 cents lower risk adjustment across the three fund categories, resulting in 3 to 8 cents higher NPV.

Comparing our method with the KN method, we find large differences for the buyout fund category. For the CAPM the risk-adjusted performance of buyout funds is similar across the two methods, but performance components differ considerably. The KN method results in a larger risk-neutral value, which is then compensated by higher risk adjustment. Furthermore, the standard deviation of performance across funds is 142 cents using the KN method, while the standard deviation is 98 cents for our method. For the long-term investor, the KN method indicates very high buyout performance, up to 80 cents of NPV per dollar of commitment, in contrast to 35 cents with our method. The high NPV is, however, not due to lower risk adjustment. The NPV is, instead, due to an increase in the risk-neutral value, which increases from 80 cents for the CAPM to 250 cents for the long-term model. The standard deviation of performance in the long-term model increases to 925 cents with the KN method, while it remains stable at 105 cents with our method.

We also find differences between the two methods for the venture capital and generalist fund categories. Relative to the KN method, our method assigns 20 to 30 cents higher NPV to venture capital and 10 to 20 cents higher NPV to generalist, depending on the type of investor. For these categories, performance estimates using the KN method are not as implausible as for buyout. Our method, however, consistently results in more realistic time discounting and lower variation of performance across funds.

We furthermore use our method to decompose the risk adjustment based on the timing of cash flows during a fund's life. For all three fund categories, cash flows have marginally negative risk exposure in the first 3 years of operations, indicating weak pro-cyclicality of contributions. After the first 3 years, the risk exposure of cash flows turns positive, and we find differences in the timing of risk across the three categories. Much of the risk adjustment is due to cash flows from year 9 to 11 for buyout funds and year 4 to 7 for venture capital. For generalist funds, the risk adjustment is spread more homogeneously between year 4 and 10.

A potential concern remaining in our approach is that our performance decomposition does not provide clear guidance on how to estimate risk prices for proper benchmarking. In practice, we restrict the SDF to price S&P 500 returns in the sample period at a 10-year horizon. This condition is heuristic, however, based on the typical horizon of PE funds. To address this concern, we study the robustness of our results by changing the price of risk exogenously. We find only weak effects on the risk adjustment and NPVs of buyout and generalist funds. Their NPV remains positive over a wide range of risk prices. Venture capital, on the other hand, has higher risk exposure and its valuation is more sensitive to the price of risk.

This paper fits into the literature studying the risk and return of PE investments. Korteweg (2019) surveys that literature, and we build on a series of studies benchmarking PE cash flows against publicly traded assets. In this context, a popular measure of risk-adjusted performance is Kaplan and Schoar (2005)'s Public Market Equivalent (PME). The PME discounts cash flows using the realized return on a portfolio of benchmark assets. Sorensen and Jagannathan (2015) show that the PME can fit into the SDF framework as a special case of Rubinstein (1976)'s log-utility model. The log-utility model, however, does not necessarily price benchmark assets. In this case, the PME applies the wrong risk adjustment. To address that issue, Korteweg and Nagel (2016) propose a generalized PME, and we build on their work.[3]

Starting with Ljungqvist and Richardson (2003), several authors study the performance of PE funds adjusting for different risk factors. Franzoni, Nowak, and Phalippou (2012) along with Ang, Chen, Goetzmann, and Phalippou (2018) estimate some of the most elaborate models considering Fama-French three factors, the liquidity factor of Pástor and Stambaugh (2003), and in some cases also profitability and investment factors. With our long-term investor, we introduce a new risk factor representing shocks to investment opportunities (discount rate news) as in the intertemporal CAPM of Campbell (1993). Closest to the spirit of our long-term investor is the work of Gredil et al. (2020), studying PE performance using SDFs of leading consumption-based asset pricing models.

Beyond the PE literature, Farnsworth et al. (2002) provide an early study of the SDF framework for the valuation of mutual funds, and Li, Xu, and Zhang (2016) apply the framework to the case of hedge funds. A main observation in those studies is that SDF models perform better when the SDF is required to price the risk-free asset, as this identifies the mean of the SDF. We extend that observation to the case of PE funds, which require multiperiod discounting. With multiperiod discounting, we obtain more robust performance estimates across SDFs and lower variation of performance across funds when the SDF is required to price the risk-free asset at all relevant horizons, as this helps identify the mean of the SDF at those horizons.

[3]Parallel effort by Gupta and Van Nieuwerburgh (2021) takes a different approach to benchmark PE cash flows. They try to replicate funds' cash flows with a portfolio of synthetic dividend strips, which is then priced with standard asset pricing techniques.

2 Risk Adjustment of Private Equity Cash Flows

We measure the risk-adjusted performance of PE funds using the Generalized Public Market Equivalent (GPME). In its most general form, the GPME of fund i is the sum of fund's cash flows, $C_{i,t}$, discounted with SDF realizations:

$$\mathrm{GPME}_i \equiv \sum_{h=0}^{H} M_{t,t+h} C_{i,t+h} \tag{1}$$

The term $M_{t,t+h}$ denotes a multi-period SDF discounting cash flows from $t+h$ to the start of the fund. Time t is the date of the first cash flow of the fund, and it depends on i despite the simplified notation. The letter H indicates the number of periods (quarters in our case) from the first to last cash flow of the fund. As a convention, we let H be the same across funds. Funds active for a lower number of periods have a series of zero cash flows in the last part of their life.

Section 3 presents functional forms of the SDF. They typically include at least one risk factor and depend on a vector of parameters. Those parameters must be estimated such that the SDF reflects realized returns on benchmark assets during the sample period. This intuitive condition is necessary for proper benchmarking, but it is unclear how it should be translated into formal statements. Korteweg and Nagel (2016) propose a heuristic approach based on the construction of artificial funds that invest in the benchmark assets. They estimate parameters by setting the NPV of those artificial funds to zero. In the rest of this section, we propose an alternative approach based on a decomposition of risk-adjusted performance.[4]

Investing in a random fund yields NPV equal to $\mathrm{E}[\mathrm{GPME}_i]$, and it is helpful to decompose this quantity in a typical asset pricing way:

$$\mathrm{E}[\mathrm{GPME}_i] = \underbrace{\sum_{h=0}^{H} \mathrm{E}[M_{t,t+h}]\mathrm{E}[C_{i,t+h}]}_{\text{risk-neutral value}} + \underbrace{\sum_{h=0}^{H} \mathrm{cov}(M_{t,t+h}, C_{i,t+h})}_{\text{risk adjustment}} \tag{2}$$

As the right-hand side of this expression illustrates, the NPV is the sum of a risk-neutral value and a risk adjustment. This decomposition suggests at least one consideration. By definition, the risk-neutral value should be determined by cash flows and risk-free rates, and it should not change as risk factors are added or removed from the SDF.

Further, an objective of a benchmarking exercise like ours is to assess the risk exposure of PE to different risk factors. In general, the GPME does not allow direct measurement of risk quantities,

[4] A related but different decomposition is discussed by Boyer et al. (2021).

and we are left with indirect evidence based on the behavior of the risk adjustment (Jeffers, Lyu, and Posenau, 2021). As we add or remove risk factors from the SDF, it is tempting to attribute differences in GPME to changes in risk adjustment, but that interpretation is robust only when the risk-neutral value is fixed. These considerations guide our choice of restrictions for the SDF.

2.1 Moment Conditions

We impose restrictions on the SDF in the form of moment conditions, which we introduce in two steps. First, we describe a set of moment conditions anchoring the mean of the SDF to risk-free rates. Those conditions are the main determinant of risk-neutral value and represent the central innovation of this paper. Second, we describe the remaining moment conditions linking the SDF to realized returns on risky benchmark assets. This second set of moment conditions is the main determinant of the risk adjustment.

To identify the mean of the SDF at different horizons, we impose standard asset pricing conditions on risk-free rates. Consider \$1 invested at time t in a risk-free asset that pays $R^{\mathrm{f}}_{t,t+h}$ at time $t+h$. This investment is priced by the SDF if $\mathrm{E}_t[M_{t,t+h}] = 1/R^{\mathrm{f}}_{t,t+h}$. Taking unconditional expectations on both sides, we get the following moment condition:

$$\mathrm{E}[M_{t,t+h}] = \mathrm{E}\left[\frac{1}{R^{\mathrm{f}}_{t,t+h}}\right] \tag{3}$$

Imposing this restriction for all horizons h from 1 to H, the risk-neutral value can be rewritten exclusively in terms of cash flows and risk-free rates, and it becomes independent of the SDF's functional form. Importantly, the risk-neutral value does not change as we add or remove risk factors from the SDF.

Empirically, we wish to impose condition (3) to the SDF. However, the practical meaning of the expectation operator inside that condition can be elusive. How does the population condition translate into a sample condition?

To address this question, it is useful to consider the sample version of $\mathrm{E}[\mathrm{GPME}_i]$. We call it simply GPME, and compute it as the mean of GPME_i across N funds in a sample:

$$\mathrm{GPME} \equiv \frac{1}{N}\sum_{i=1}^{N}\sum_{h=0}^{H} M_{t,t+h}C_{i,t+h} \tag{4}$$

This quantity can be decomposed similarly to its population counterpart. For each horizon h, we define $\overline{M}_h = \frac{1}{N}\sum_i M_{t,t+h}$ as the average SDF and $\overline{C}_h = \frac{1}{N}\sum_i C_{i,t+h}$ as the average cash flow across funds. Using these definitions, we can write

$$\text{GPME} = \sum_{h=0}^{H} \overline{M}_h \overline{C}_h + \sum_{h=0}^{H} \overline{M}_h A_h \tag{5}$$

where $A_h = \frac{1}{N}\sum_i (M_{t,t+h}/\overline{M}_h - 1)(C_{i,t+h} - \overline{C}_h)$ is the covariance between a normalized SDF and cash flows. In this decomposition, the risk-neutral value is $\sum_{h=0}^{H} \overline{M}_h \overline{C}_h$ and the risk adjustment is $\sum_{h=0}^{H} \overline{M}_h A_h$. Fixing the risk-neutral value requires restrictions on $\overline{M}_h$, and the expectation operator inside condition (3) must be implemented as a cross-sectional mean. As a result, we impose the following sample condition on the SDF:

$$\frac{1}{N}\sum_{i=1}^{N} M_{t,t+h} = \frac{1}{N}\sum_{i=1}^{N} \frac{1}{R^{\mathrm{f}}_{t,t+h}} \tag{6}$$

This expression must hold for horizons 1 to H, and it represents H moment conditions. For large h, however, we do not observe all returns on benchmark assets.[5] In that case, we rescale N to account for the missing observations.

In the GPME decomposition, risk prices determine the risk adjustment, but the decomposition does not provide clear guidance on how to identify the appropriate risk prices. In this case, we use heuristic rules. For each benchmark asset, b, with risky return $R^{\mathrm{b}}_{t,t+h}$, we impose the following condition:

$$\frac{1}{N}\sum_{i=1}^{N} M_{t,t+h} R^{\mathrm{b}}_{t,t+h} = 1 \tag{7}$$

This expression is the risky counterpart of the risk-free rate condition above. In our empirical applications, we impose it only for horizon $h = 40$ quarters, or 10 years, which represents the typical horizon of a PE fund. It is possible to impose this condition for every h between 1 and H, and we verify in unreported analysis that our empirical results are robust to that choice.

In summary, we restrict the SDF with conditions (6)-(7) to price benchmark assets. With the restricted SDF, we use expression (4) to estimate the NPV of investing in a random PE fund and decomposition (5) to measure the two sources of value, risk-neutral and risk adjustment. This procedure fits into the GMM framework with the complication that the sample size varies across moments.

[5] Some funds in our data operate longer than 15 years such that $H > 60$ quarters. However, we cannot observe returns on benchmark assets at horizon $h = 60$ for funds started in 2009, for example, because that would require knowing returns realized in 2024.

2.2 Statistical Inference

For statistical inference, a main problem is that funds of close vintages are likely to have positively correlated cash flows. This correlation can originate from exposure to the same factor shocks, and some could remain, after controlling for public factors.[6] To address this problem, Korteweg and Nagel (2016) integrate methods from spatial econometrics in their GMM framework. Below, we illustrate our inference, which is closely related to their method.

To compute the standard error of the GPME, we ignore uncertainty about SDF parameters, but we allow for correlation between overlapping PE funds. We measure the distance between funds i and k by their degree of overlap. Defining $T(i)$ and $T(k)$ as the last non-zero cash flow dates of fund i and j, we compute their distance as follows:

$$d(i,k) \equiv 1 - \frac{\min\{T(i),T(k)\} - \max\{t(i),t(k)\}}{\max\{T(i),T(k)\} - \min\{t(i),t(k)\}} \tag{8}$$

The distance is zero if the overlap is exact, and it is 1 or greater if there is no overlap. This distance is used to construct weights that account for cross-sectional correlation in the sample estimate of the asymptotic variance. Specifically, we estimate the variance of $\sqrt{N}$ GPME as

$$v \equiv \frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{N} \max\{1 - d(i,k)/\bar{d}, 0\}\; u_i u_k \tag{9}$$

where $u_i \equiv \mathrm{GPME}_i - \mathrm{GPME}$. In the sum, each product $u_i u_k$ is assigned a weight between 0 and 1, and weights decrease with the distance between two funds. In our empirical work, we set $\bar{d} = 2$, and some non-overlapping pairs of funds still get positive weight. The standard error of the GPME is estimated as $\sqrt{v/N}$.

The resulting standard error ignores parameter uncertainty and can be interpreted as a lower bound. Our primary objective remains to obtain point estimates of the GPME that are as economically robust as possible.[7]

[6] Ang, Chen, Goetzmann, and Phalippou (2018), for example, find a factor in PE returns, which is not spanned by publicly traded factors.

[7] Other than our focus on point estimates, an additional reason for ignoring parameters uncertainty is that moments with different sample sizes can complicate the derivation of GMM standard errors. To the best of our knowledge, there is only limited asymptotic GMM theory allowing for moments constructed with samples of different size. Some of that theory is developed by Lynch and Wachter (2013).

3 Stochastic Discount Factor

We focus on applications with exponentially affine SDFs. To illustrate, consider the case with a generic single factor f. The multi-period SDF can be written as follows:

$$M_{t,t+h} = \exp\left(a_h - \gamma_h f_{t,t+h}\right) \tag{10}$$

In this expression, a_h and γ_h indicate a pair of parameters per horizon. γ_h can be interpreted as the risk price of f at horizon h. In absence of other restrictions, this SDF has a total of $2H$ parameters. Korteweg and Nagel (2016) restrict $a_h = ah$ and $\gamma_h = \gamma$, working with only 2 parameters. We do not impose any functional form on a_h. This additional flexibility is necessary to satisfy moment conditions (6) and fix the subjective term structure of interest rates with market data. We maintain the restriction on risk prices, $\gamma_h = \gamma$, for two reasons. First, our main argument is about fixing the risk-neutral value of the GPME, which is not determined by risk prices, so we maintain this part of the model as simple as possible. Second, we exploit this simplicity to study the robustness of our empirical results with respect to risk prices.

The single-factor form of the SDF can easily be extended with additional factors and corresponding risk prices. Below, we describe the form used in our empirical work.

3.1 CAPM and Long-Term Investors

We consider two risk factors. One factor is the log-return on the market, $r^{\mathrm{m}}_{t,t+h} = \ln(R^{\mathrm{m}}_{t,t+h})$. The other factor is news about future expected returns on the market, also known as discount rate (DR) news in the literature. DR news arriving between t and $t+h$ is denoted $N^{\mathrm{DR}}_{t,t+h}$, and is defined as follows:

$$N^{\mathrm{DR}}_{t,t+h} \equiv (\mathrm{E}_{t+h} - \mathrm{E}_t) \sum_{j=1}^{\infty} \rho^j r^{\mathrm{m}}_{t+h+j} \tag{11}$$

In this expression, ρ is an approximation constant just below 1, and the right-hand side measures cumulative news between t and $t+h$ about market returns from $t+h$ onwards. In a simple model with homoscedastic returns, this factor summarizes variation in investment opportunities, and positive news corresponds to better opportunities (Campbell, 1993).[8]

[8]Formally, $\rho \equiv 1 - \exp(x)$ where x is the mean of the investor's log consumption-wealth ratio. In our empirical applications, one period corresponds to one quarter, and we set $\rho = 0.95^{1/4}$ corresponding to a mean consumption-wealth ratio of approximately 5% per year.

With the two risk factors, we construct the following SDF:

$$M_{t,t+h} = \exp\left(a_h - \omega\gamma\, r^{\mathrm{m}}_{t,t+h} - \omega(\gamma-1)N^{\mathrm{DR}}_{t,t+h}\right) \tag{12}$$

This is a two-factor version of (10) with risk price $\omega\gamma$ for the market return and $\omega(\gamma-1)$ for DR news. Appendix A connects this SDF to theory and shows that the parameter γ can be interpreted as the investor's relative risk aversion, while ω is the portfolio weight in the market, with $1-\omega$ invested in the risk-free asset. Throughout our main analysis and unless otherwise specified, we assume $\omega = 1$ representing an investor fully invested in the market.

This SDF recognizes that the same realized market return implies different marginal utility depending on expected returns. If expected returns are constant, $N^{\mathrm{DR}}_{t,t+h}$ is zero, and the SDF simplifies to a single-factor CAPM model. If expected returns vary over time, $N^{\mathrm{DR}}_{t,t+h}$ appears as an additional risk factor with positive risk price for investors with $\gamma > 1$. These investors are particularly averse to portfolio losses arriving jointly with negative news about expected returns. These losses are permanent in the sense that they are not compensated by higher expected returns, and a risk averse, long-term investor fears them in particular (Campbell and Vuolteenaho, 2004).

We compare GPME estimates obtained with different restrictions on (12). In one case, we impose $a_h = 0$ and $\gamma = 1$. These restrictions correspond to a log-utility investor and result in the same SDF of Kaplan and Schoar (2005)'s PME. In addition to the log-utility investor, we consider two other types, CAPM and long-term investors, differentiated only by $N^{\mathrm{DR}}_{t,t+h}$, which is zero for the CAPM and estimated below for long-term investors. The SDFs of these two investors require estimation of a_h and γ, and we compare our method with that of Korteweg and Nagel (2016). Since Korteweg and Nagel (2016) impose $a_h = ah$, we refer to their method as 'single intercept' and we refer to ours as 'multiple intercepts'.

3.2 A Model of Discount Rate News

To estimate DR news, we follow a large literature starting with Campbell (1991) that models expected market returns using vector autoregression (VAR).[9] We assume that the data are generated by a first-order VAR:

$$x_{t+1} = \mu + \Theta x_t + \varepsilon_{t+1} \tag{13}$$

[9]See for example Campbell (1991, 1993, 1996), Campbell and Vuolteenaho (2004), Lustig and Van Nieuwerburgh (2006), Cochrane (2011).

In this expression, μ is a $K \times 1$ vector and Θ is a $K \times K$ matrix of parameters. Furthermore, x_{t+1} is $K \times 1$ vector of state variables with $r^{\mathrm{m}}_{t+1} - r^{\mathrm{f}}_{t+1}$ as first element, and $\boldsymbol{\varepsilon}_{t+1}$ is a i.i.d $K \times 1$ vector of shocks with variance Σ_{ε}.

In this model, DR news is a linear function of the shocks:

$$N^{\mathrm{DR}}_{t,t+1} = \lambda \varepsilon_{t+1} \tag{14}$$

The vector of coefficients for DR news is defined as $\lambda = \rho e' \Theta (I - \rho\Theta)^{-1}$, where I is the identity matrix and $\boldsymbol{e}' = (1, 0, 0, \dots, 0)$. Those coefficients measure the long-run sensitivity of expected returns to each element of x_t.

Combining the VAR model with the definition of multi-period DR news from (11), we obtain the following result:

$$N^{\mathrm{DR}}_{t,t+h} = \lambda \varepsilon_{t+h} + \left(\lambda - e'\rho\Theta\right) \varepsilon_{t+h-1} + \left(\lambda - e'\rho\Theta - e'\rho^2\Theta^2\right) \varepsilon_{t+h-2} + \cdots + (\lambda - e' \textstyle\sum_{i=1}^{h-1} \rho^i \Theta^i) \varepsilon_{t+1} \tag{15}$$

This equation expresses multi-period DR news in terms of observables, and it constitutes the empirical specification of the risk factor. In section 4, we obtain two versions of this factor by estimating two VAR models that differ in the choice of state variables.

4 Expected Returns and Discount Rate News

4.1 Public Market Data

The VAR vector x_t contains data about publicly traded assets at a quarterly frequency from 1950 to 2018. The first element of x_t is the difference between the log-return on the value-weighted S&P 500 and the log-return on quarterly T-bills. For this element, data is from the Center of Research in Security Prices (CRSP). The remaining elements of x_t are candidate predictors of expected returns and DR news. We consider (1) the log dividend-price ratio, (2) the term premium, (3) a credit spread of corporate bond yields and (4) the value spread. The log dividend-price ratio, term premium, and credit spread are constructed using data from Amit Goyal's website. The log dividend-price ratio is defined as the sum of the last 12 months dividends divided by the current price of the S&P 500. The term premium is the difference between the annualized yield on 10-year constant maturity Treasuries and the annualized quarterly T-bill yield. Credit spread is the difference between the annualized yield on BAA-rated corporate bonds and AAA-rated corporate bonds. For the value spread, we rely on data from Kenneth French's data library. We construct the value spread as the difference in log book-to-market ratio of small-value and small-growth stock portfolios. These portfolios are generated from a double sort on market capitalization and book-to-market ratio.

Table 1 reports summary statistics for the public market data in our sample period. From Panel A, the quarterly log equity premium is 1.6%, corresponding to 6.4% annually, with a quarterly standard deviation of 8%. Further, our candidate return predictors are highly persistent, especially the log dividend-price ratio with an autocorrelation coefficient of 0.982. Panel B reports correlations between contemporaneous and lagged state variables. The first column reports univariate correlations between the one-period ahead excess market return ($r_t^{\mathrm{m}} - r_t^{\mathrm{f}}$) and lagged predictors. The market return is positively correlated with the lagged dividend-price ratio, credit spread and term premium, and negatively correlated with the lagged value spread.

4.2 VAR Estimation

We estimate the VAR model using OLS at a quarterly frequency in the post-war period from 1950 to 2018. We consider two different specifications: (1) a parsimonious specification including only the log dividend-price ratio as the predictor and (2) a specification including the full set of predictors.

Table 2 reports the two VAR estimations. Panel A reports the parsimonious specification including only the dividend-price ratio, and Panel B reports the full specification. Each row corresponds to an equation in the VAR. The first row of each panel corresponds to the market return prediction equation. Standard errors are in brackets, and the last two columns report R^2 and F-statistic for each forecasting equation. Panel A shows that dividend-price ratio significantly predicts excess market returns with a coefficient of 0.025. The R^2 is 2.8 percent, and the F-statistic is statistically different from zero, consistent with the dividend-price ratio and lagged market return jointly predicting excess market returns. Panel B also includes the value spread, credit spread, and term premium in the VAR. The first row shows that the lagged market return, dividend-price ratio and term premium positively predict excess returns. The coefficients on the dividend-price ratio and term premium are statistically significant at the five percent level. The value spread and credit spread negatively predict excess market returns, although the corresponding coefficients are statistically insignificant.

Table 3 shows properties of one-period DR news, $N_{t,t+1}^{\mathrm{DR}}$, implied by the two VAR estimations. Panel A reports the vector of coefficients, λ, measuring the sensitivity of DR news to each element of ε_t. The "DP only" column shows that shocks to the dividend-price ratio are a significant determinant of DR news. The “Full VAR” column shows that shocks to the dividend-price ratio and term premium are significant determinants of DR news in the full VAR specification. These coefficients, however, do not represent a complete picture of how much each variable affects DR news; they do not account for the fact that elements of ε_t have different variances. We, therefore, decompose the unconditional variance of DR news to compare the importance of shocks to different variables.

Panel B of Table 3 decomposes the variance of DR news, $\lambda\Sigma_\varepsilon\lambda'$, into variance contributions from each variable’s shock. The column “DP only” reports the decomposition for the parsimonious VAR. In this specification, 105% of DR news variance originates from shocks to the dividend-price ratio and negative 5% stems from the lagged market return. Shocks to DR news are almost exclusively determined by shocks to the dividend-price ratio. The “Full VAR” column shows that the dividend-price ratio is the largest contributor to DR news variance in the full specification. Even in this specification, the dividend-price ratio contributes nearly 100% percent of the variance. The value spread and term premium contribute only 3% and 4%, respectively, while the credit spread’s contribution is essentially zero. These results suggest that both specifications rely almost exclusively on shocks to the dividend-price ratio to determine DR news.

5 Private Equity Performance

5.1 Funds Data

We analyze PE data maintained by Burgiss. Our sample contains net-of-fees cash flows of 1866 PE funds started in the US between 1978 and 2009 and divided into three mutually exclusive categories: buyout, venture capital, and generalist funds. Burgiss provides at least two levels of classification for each fund. At the most general level (Tier 1), funds are primarily classified as ‘equity’, ‘debt’ or ‘real assets’. We focus exclusively on equity. At a more detailed level (Tier 2), we distinguish between equity funds classified as buyout, venture capital, and generalist. We define buyout and venture capital funds using the homonymous Tier 2 classes. In our generalist category, we include funds with Tier 2 classification of ‘generalist’, ‘expansion capital’, ‘unknown’ and ‘not elsewhere classified’. To obtain our sample, we exclude funds with less than 5 million USD of commitment. We also exclude funds whose majority of investments are not liquidated by 2019. For that, we impose two conditions. First, we only include funds of vintage year 2009 or earlier. Second, among those funds, we exclude those with a ratio of residual NAV over cumulative distributions larger than 50%. Finally, we normalize cash flows and residual NAV by each fund’s commitment.

Figure 1 plots the aggregate sum of normalized contributions, distributions, and net cash flows for the three fund categories over time. For all the categories, our sample constitutes mostly of cash flows observed between 1995 and 2019. For venture capital, we see uniquely large distributions in the year 2000, corresponding to the dot-com bubble; those distributions are almost 10 times larger than distributions and contributions observed at any other time.

Table 4 summarizes the PE data. Panel A reports descriptive statistics and shows that the sample consists of 652 buyout funds, 971 venture capital funds and 243 generalist funds. The median (average) fund size is $421 ($1099) million for buyout, $126 ($222) million for venture capital and $225 ($558) million for generalist funds. The average number of years between the first and last buyout fund cash flow is 14.16 years, 15.52 years for venture capital, and 14.42 years for generalist funds. The average number of cash flows per fund is approximately 36 for buyout, 28 for venture capital and 33 for generalists. The sample includes 311 unresolved buyout funds with an average NAV-to-Distributions ratio of 0.10, 353 unresolved venture capital funds with NAV-to-Distributions ratio of 0.14, and 88 unresolved generalist funds with a NAV-to-Distributions ratio of 0.12.

Panel B of Table 4 reports Total Value to Paid-In ratios (TVPI) across vintage years.[10] Across the three categories, TVPIs fluctuate over time and are typically higher for earlier vintages. Furthermore, venture capital shows high TVPIs between 1993 and 1996. These large multiples can be ascribed, at least in part, to funds of these vintages deploying capital in the period leading up to the 2001 dot-com bubble and exiting investments before 2001.

5.2 Buyout

Table 5 reports the GPME estimation for buyout funds. The first estimation corresponds to "Log-Utility" and uses the inverse return on the market as SDF. The resulting GPME is a reformulation of Kaplan and Schoar (2005)'s PME defined as the sum of discounted cash flows rather than the ratio of discounted distributions over contributions. The log-utility GPME is 0.20 and significantly different from zero at the one percent level; buyout funds provide log-utility investors with 20 cents of abnormal profits per dollar of committed capital.

In addition to the log-utility model, Table 5 reports GPME estimation for CAPM and long-term (LT) investors assuming they are fully invested in the market ($\omega = 100\%$). The "Single Intercept" columns estimate only one intercept parameter a, with $a_h = ah$, and the estimations use the moment conditions of Korteweg and Nagel (2016). The "Multiple Intercepts" columns use

[10] If there are less than five funds per vintage year, figures are omitted due to confidentiality.

our method as described in Section 2, which does not impose a functional restriction on a_h and estimates multiple intercepts linking the subjective term structure of interest rates to market data.

Buyout Value for CAPM Investors

Table 5 shows a GPME of 0.28 for the CAPM SDF using a single intercept. The estimate is statistically different from zero at the ten percent level. With multiple intercepts, the GPME is 0.30 and statistically significant at the one percent level. The estimates are similar and imply that buyout funds provide CAPM investors with 28-30 cents of NPV per dollar of committed capital. CAPM investors thus derive 8-10 cents more value than log-utility investors from a marginal allocation to buyout funds.

Even though the GPME estimates for the CAPM investor are similar across the two methods, we show below that the different restrictions placed on the single and multiple intercepts specifications imply markedly different SDF properties. To further explore differences between the two methods, Table 5 also reports the two GPME components from the decomposition in Section 2. The first component is $\sum_h \overline{M}_h \overline{C}_h$ and corresponds to the risk-neutral part of GPME. For each horizon, we take the average cash flow across funds and discount it with the average SDF across funds. We then sum over horizons. The second component is $\sum_h \overline{M}_h A_h$ and corresponds to the total risk adjustment inside GPME.

For the CAPM investor, the risk-neutral component is 0.61 with multiple intercepts and 0.79 with a single intercept. The risk adjustment, instead, is -0.31 with multiple intercepts and -0.51 with a single intercept. For the CAPM, the single intercept specification achieves a similar GPME estimate by assigning a higher risk-neutral value, but also more negative risk adjustment, relative to the multiple intercepts case. Differences between the two methods become increasingly evident considering long-term investors below.

Buyout Value for Long-Term Investors

In Table 5, the "LT" columns report GPME estimations for long-term investors. In particular, the "LT (DP)" columns use the VAR specification with only the dividend-price ratio as the return predictor to measure DR news. With a single intercept, the LT (DP) investor assigns a GPME of 0.80 to buyout. This point estimate is considerably higher relative to the CAPM investor, but it is not statistically significant due to the large standard error. Further, a NPV of 80 cents per dollar of commitment is large enough to appear economically implausible, and our decomposition shows that this GPME arises from summing a risk-neutral value of 2.50 with a risk adjustment of

-1.69. Compared to the CAPM, this large value is not generated by a change in risk adjustment. Instead, it stems from a large increase in the risk-neutral component. With multiple intercepts, we estimate a GPME of 0.35 for the LT (DP) investor. This point estimate is statistically significant, and it is only marginally higher compared to CAPM. By construction, the difference relative to CAPM is entirely due to risk adjustment.

The "LT (Full)" columns of Table 5 use DR news computed with the full VAR specification, which includes the dividend-price ratio, term premium, credit spread, and value spread as return predictors. With a single intercept, the resulting GPME is 0.41, although not statistically significant, and substantially lower than 0.80 obtained in the LT (DP) case. With multiple intercepts, the GPME is 0.34, statistically significant at the one percent level, and close to the 0.35 estimated in the LT (DP) case. While the single intercept methodology suggests that the two VAR specifications result in DR news which generate large GPME differences, the multiple intercepts method suggests similar GPME implications of the two VAR specifications. In Section 4, we show that both VAR estimations generate DR news that vary almost exclusively from shocks to the dividend-price ratio, suggesting that the full VAR might have similar dynamics to the parsimonious VAR. Consistent with this interpretation, the multiple intercepts method estimates virtually identical GPMEs for LT (DP) and LT (Full) investors.

Time Discounting of Buyout Funds

Figure 2 plots the average SDF across funds as a function of the horizon. At each horizon h, the average SDF measures the present value of one dollar paid for certain at that horizon by all funds in the sample. The figure compares the single intercept and multiple intercepts specifications from Table 5. By construction, multiple intercepts estimations using the same sample imply the same average SDF as a function of the horizon. Single intercept estimations impose less structure on the SDF, and the average SDF at each horizon varies depending on the risk factors considered.

The figure illustrates the unrealistic time discounting for the single intercept estimation with a peculiar pattern of negative time discounting in the first 3 years, positive discounting from year 4 to 8, and negative discounting again from year 8 to 11. This pattern is qualitatively consistent across investors and it is quantitatively strongest for the LT (DP) model, suggesting that the very large GPME estimate obtained with this model might be due to this implausible time discounting pattern resulting from the single intercept method.

With multiple intercepts, Figure 2 shows that time discounting is consistently positive and stable across horizons, and this result corresponds to more stable GPME estimates across models, as shown in Table 5. It also results in a lower variation of GPME_i across funds, as we show below.

Cross-Sectional Variation of Performance

Table 6 summarizes the cross-sectional distribution of GPME_i resulting from the different estimations. The table contains results for all three fund categories. We focus primarily on buyout, but a similar discussion applies to venture capital and generalist funds. For each estimation, we report the mean of GPME_i, which corresponds to the GPME estimates in Table 5. Below the mean, we report the standard deviation and selected percentiles for the GPME_i distribution.

Differences in the distribution of GPME_i are interesting because the multiple intercepts estimations, just like the single intercept ones, restrict the SDF using exclusively public market data, and ignore any information about PE cash flows. Thus, differences in the GPME_i distribution are a result that is not imposed by construction. We find that using multiple intercepts consistently imply lower variation of GPME_i across funds, relative to using a single intercept.

For buyout, the log-utility model generates the lowest standard deviation of GPME_i, equal to 0.64. The single intercept CAPM model implies a standard deviation of 1.42, while the multiple intercepts CAPM model implies a standard deviation of 0.98. Thus, the multiple intercepts model generates a markedly lower standard deviation for the CAPM, even though the two models have a similar mean (0.28 vs. 0.30). Further, the lower standard deviation of the multiple intercepts CAPM model comes with less extreme tail observations as indicated by the reported percentiles. For long-term investors, we see qualitatively similar differences between the single intercept and multiple intercepts methods, but with more extreme magnitudes. Especially for the LT (DP) investor, the GPME_i standard deviation of the single intercept model is extremely high, 9.25, relative to 1.05 obtained with the multiple intercepts model.

Components of Buyout Performance across Horizons

With multiple intercepts, we take the GPME decomposition one step further. Not only do we decompose GPME in a risk-neutral part and a risk adjustment, but we also decompose the risk-neutral part and the risk adjustment based on the contribution of each horizon. To illustrate, we decompose the risk-neutral part as follows:

$$\sum_{h=0}^{H} \overline{M}_h \overline{C}_h = \overline{M}_0 \overline{C}_0 + \sum_{h=1}^{4} \overline{M}_h \overline{C}_h + \sum_{h=5}^{8} \overline{M}_h \overline{C}_h + \cdots + \sum_{h=53}^{56} \overline{M}_h \overline{C}_h + \sum_{h=57}^{H} \overline{M}_h \overline{C}_h \tag{16}$$

These components correspond to values coming from year 0, year 1, year 2, ..., year 14, and year 15 or higher. A similar decomposition is done for the risk adjustment.

Figure 3 plots the resulting GPME components against horizon for selected models with multiple intercepts. The figure focuses on the CAPM and LT (DP) models. By construction, the risk-neutral component from each horizon (grey bars) is identical across models, and differences originate exclusively from components of the risk adjustment plotted as black bars for the CAPM and white bars for LT (DP).

Decomposing the risk-neutral part, the grey bars in Figure 3 show the "J-curve" typical of PE cash flows.[11] Investors contribute capital primarily in the first 4 years, corresponding to negative average cash flows at short horizons. Average cash flows turn positive from year 5, as funds distribute capital.

Decomposing the risk adjustment, Figure 3 shows that the CAPM and LT (DP) models are similar not only on the overall risk adjustment but also on its components across horizons. Surprisingly perhaps, risk adjustment is moderately positive in the first years of fund operations and turns negative only after the third year. At short horizons, net cash flows are dominated by contributions, and a positive risk adjustment suggests that buyout funds tend to call less capital in bad times with high SDF realizations. This tendency decreases risk and has a small but positive effect on GPME. This result is consistent with Robinson and Sensoy (2016), also finding pro-cyclicality in contributions.

Components of the risk adjustment turn negative at longer horizons after year 3, and they are the most negative between year 9 and 11. Interestingly, the risk adjustment is small for years 6 to 8, even though average cash flows are high during those years. This result appears consistent with Gupta and Van Nieuwerburgh (2021) finding that buyout funds generate cash flows that appear to be risk-free in part of their harvesting period.

5.3 Venture Capital

Table 7 reports GPME estimations for venture capital funds. As a starting point, we estimate a log-utility GPME of 0.14 and statistically indistinguishable from zero. For comparison, Korteweg and Nagel (2016) find a marginally positive log-utility GPME of 0.05 for venture capital funds. The higher GPME in our sample might come from a larger number of funds in pre-1998 vintages. Historically, those vintages have high risk-adjusted performance for venture capital.

[11] Grey bars represent the risk-neutral present value of average cash flows at each horizon. Instead, the J-curve is typically plotted as the average cash flows at each horizon without discounting. Nonetheless, the two quantities are close, especially at horizons shorter than 10 years.

Considering the GPME decomposition for log-utility, venture capital has a risk-neutral value of 0.48 and a risk adjustment of -0.35. Compared to buyout, this risk adjustment is considerably larger (-0.35 vs. -0.09). The log-utility model has a constant risk price of $\gamma = 1$ across samples. Differences in risk adjustment are consequently entirely due to different covariance between cash flows and market returns. Thus, higher risk adjustment for venture capital suggests higher market exposure of venture capital's cash flows relative to buyout. Larger risk adjustment for venture capital is consistent with Driessen et al. (2012), who estimates a market beta of 2.4 for venture capital and 1.3 for buyout, and with Ang et al. (2018), estimating a market beta 1.8 for venture capital and 1.2 for buyout.

Venture Capital for CAPM and Long-Term Investors

In Table 7, the GPME estimate for the CAPM investor is -0.15 with single intercept and 0.07 with multiple intercepts. Both methods indicate that the CAPM implies lower GPMEs relative to log-utility. This difference is consistent with the findings of Korteweg and Nagel (2016) in a smaller sample. The single intercept and multiple intercepts methods disagree on the GPME sign and magnitude, however, applying multiple intercepts result in marginally positive GPME for venture capital.

Differences in the GPME between the single intercept and multiple intercept models can arise from differences in the SDF intercepts, a_h, but also from differences in the estimated risk price, γ. Table 7 shows that CAPM's risk price is 2.93 with single intercept and 2.03 with multiple intercepts. We show in Section 6.1 that differences in γ do not entirely explain the GPME difference between the two methods, as the GPME of the CAPM investor with multiple intercepts remains higher relative to single intercept even assuming the same risk price of 2.93 for both models.

For the long-term investor, we observe qualitatively similar differences between the single and multiple intercepts specifications. Using a single intercept, we estimate negative GPMEs of -0.19 for LT (DP) and -0.08 for LT (Full). With multiple intercepts, we estimate positive GPMEs of 0.13 for LT (DP) and 0.15 for LT (Full). In Section 6.1, we also show that this difference is not entirely explained by lower risk prices with multiple intercepts.

Comparing GPMEs between the CAPM and long-term investors, we find that the long-term investor assigns higher value to venture capital, relative to the CAPM investor, with multiple intercepts. GPME estimates with multiple intercepts for LT (DP) and LT (Full) are almost double the GPME estimate for the CAPM. Considering the single intercept method, we find more stable performance across investors for venture capital relative to buyout. To investigate this result, we plot the implied discounting for venture capital funds.

For venture capital, Figure 4 plots the cross-sectional average SDF as a function of the cash flow horizon. As we do for buyout, the figure distinguishes between multiple intercepts and the three models with a single intercept. The figure confirms that multiple intercepts imply a more stable time discounting across horizons. The single intercept method implies qualitatively similar time discounting between buyout and venture capital. With venture capital, however, time discounting of the single intercept method does not vary as widely across the three models.

Components of Venture Capital Performance across Horizons

Following the discussion of the results for buyout funds, we also decompose GPMEs by horizon for venture capital. Figure 5 plots the decomposition for CAPM and LT (DP) models with multiple intercepts. We focus on the decomposition of the risk adjustment.

In the figure, risk adjustment varies similarly with horizon across the two models. For both models, risk adjustment is marginally positive from year zero to two. This result suggests that contributions tend to be slightly pro-cyclical, which is similar to buyout funds although quantitatively smaller. Starting from year 3, risk adjustment turns negative, and it is most important between year 4 and 7. This result contrasts with buyout, where risk adjustment tends to be small especially in year 6 and 7. The distribution of venture capital risk adjustment suggests markedly different risk across horizons relative to buyout.

5.4 Generalist

Our analysis of generalist funds is similar to that of buyout and venture capital. Here we provide an overview of the results.

Table 8 shows the results of the GPME estimations for generalist funds. With log-utility, we estimate a statistically significant GPME of 0.16, which is lower relative to buyout and marginally higher relative to venture capital. For CAPM and long-term investors, we estimate consistently positive GPME with single intercept and multiple intercepts methods. GPME estimates are higher and more stable across estimations with multiple intercepts relative to the single intercept method.

Figure 6 shows differences in time discounting across methods by plotting the average SDF by horizon implied by the different specifications. Qualitatively, the figure shows results similar to buyout and venture capital. With multiple intercepts, time discounting is positive, constant across investors, and stable across horizons.

Figure 7 plots the risk-neutral value and the risk adjustment components by horizon. From year zero to two, the figure shows risk adjustment similar to the other fund categories, consistent with contributions hedging some risk for PE investors. From year 3 onwards, risk adjustment turns negative. Compared to the other categories, the risk adjustment of generalist funds can be attributed more homogeneously to cash flows received from year 4 to 10.

6 Robustness

In this section, we focus exclusively on the method with multiple intercepts and examine the robustness of our results along two dimensions. First, we study the sensitivity of the GPME as we exogenously change risk aversion, γ. Second, we estimate GPMEs for investors whose portfolio weight in the market is either $\omega = 50\%$ or 200%, as opposed to 100% in Section 5.

6.1 Risk Aversion

Our GPME decomposition does not provide clear guidance on how to estimate risk prices for proper benchmarking of PE cash flows. As described in Section 2, our method identifies risk prices by constraining the SDF to price risky benchmark returns at a 10-year horizon. This is a heuristic approach based on the typical horizon of PE funds, and we study the sensitivity to this heuristic by changing risk prices exogenously. Specifically, we change risk aversion, γ, since risk prices are primarily determined by this parameter for our investors.

As we change risk aversion exogenously, we do not need to run new estimations. Instead, the resulting GPME with multiple intercepts is computed as follows:

$$\text{GPME}(\gamma) = \sum_{h=1}^{H} \left(\frac{1}{N} \sum_{i=1}^{N} \frac{1}{R^{\text{f}}_{t,t+h}} \right) \left(\overline{C}_h + A_h(\gamma) \right) \tag{17}$$

To obtain this expression, we rewrite the GPME decomposition (5) using time discounting restrictions: $\frac{1}{N}\sum_{i=1}^{N} M_{t,t+h} = \frac{1}{N}\sum_{i=1}^{N} 1/R^{\text{f}}_{t,t+h}$. We use this notation to highlight that A_h is the only term of the GPME affected by risk prices. Further, A_h is the only term affected by the SDF, but it does not depend on intercept parameters.[12]

Using expression (17), we compute GPMEs for risk aversion between 1 and 12 for each type of investor in each fund category. Figure 8 plots the resulting GPMEs as a function of γ. For each category, the three lines correspond to different investors. The solid line represents the CAPM, the dotted line represents LT (DP), and the dash-dotted line represents LT (Full). Further, there are

[12]To see why A_h does not depend on intercept parameters, recall that $A_h = \frac{1}{N}\sum_i (M_{t,t+h}/\overline{M}_h - 1)(C_{i,t+h} - \overline{C}_h)$ with $\overline{M}_h = \frac{1}{N}\sum_i M_{t,t+h}$. The SDF enters A_h only through its normalized form, $M_{t,t+h}/\overline{M}_h$, and intercepts cancel out because of the normalization.

two circles over each line. The black circle represents the combination of GPME and γ estimated for that investor with multiple intercepts in Table 5, Table 7, or Table 8, depending on the category. For comparison, the white circle corresponds to γ estimated with single intercept and GPME computed with expression (17).

The top-left panel of Figure 8 shows results for buyout funds. For the CAPM investor, the GPME ranges from 0.5 to 0.1, it is monotonically decreasing in risk aversion, and it remains positive even at risk aversion of 12. For long-term investors, the LT (DP) and LT (Full) models imply approximately the same GPME across all levels of risk aversion. For these investors, the GPME ranges from 0.5 to 0.3, and it is non-monotonic in risk aversion. Overall, we find a robustly positive performance of buyout funds and only moderate sensitivity to risk prices.

The top-right panel of Figure 8 shows results for venture capital. As opposed to buyout, we find high sensitivity of venture capital's GPME to risk prices. This sensitivity is highest for the CAPM investor, whose GPME estimate goes from 0.35 to -0.25 as risk aversion increases. For long-term investors, the GPME displays marginally lower sensitivity, ranging from 0.35 to -0.15. Across all investors, we find that venture capital's GPME is the most sensitive to risk aversion in the range of risk aversion between 1 and 3, which contains the three point estimates of γ with multiple intercepts from Table 7.

In the bottom panel of Figure 8, we report sensitivity results for generalist funds. The GPME of generalist funds display only moderate sensitivity to risk prices, similarly to buyout, and it remains positive for all investors at all levels of risk aversion between 1 and 12. The GPME ranges from 0.35 to 0.05 for the CAPM investor, and from 0.35 to 0.15 for long-term investors.

Across investors and fund categories, we find a tendency for the GPME to decrease in risk aversion, especially for risk aversion between 1 and 5, which is typically the most relevant range. For buyout and generalist funds, we find quantitatively modest GPME sensitivity to risk prices, and the GPME remains positive across a wide range of risk aversion. For venture capital, instead, we find high sensitivity of the GPME to risk prices, with positive GPME for risk aversion below 2 and negative GPME for risk aversion above 3. Because of the high sensitivity, venture capital seems the most challenging category to evaluate.

6.2 Investor Leverage

Another way to compare CAPM and long-term investors is by looking at the effect of the investor's leverage on the GPME. A natural measure of leverage, in our model, is the portfolio weight in the market, ω. While we assume $\omega = 100\%$ in previous sections, we now consider two different values representing a conservative investor with low leverage ($\omega = 50\%$) and an aggressive investor with high leverage ($\omega = 200\%$).

The SDF expression (12) presented in Section 3 suggests two considerations about leverage. First, CAPM investors with different leverage assign the same GPMEs to cash flows. For those investors, ω enters the SDF only through the product $\omega\gamma$ determining the market risk price, and it is a redundant parameter. For CAPM investors with higher leverage, our GPME estimation will mechanically result in proportionally lower risk aversion. Second, long-term investors with different leverage can assign different GPMEs to cash flows. For long-term investors, ω affects the importance of the market risk price, $\omega\gamma$, relative to the DR news risk price, $\omega(\gamma - 1)$. Since $\gamma > \gamma - 1$, DR news risk is less important for aggressive investors with large ω, and provided that DR news matters when evaluating PE cash flows, leverage can affect GPMEs.

In Table 9, we report GPME estimations similar to the multiple intercepts part of Table 5, Table 7, and Table 8, except that we do not assume $\omega = 100\%$. Instead, we assume $\omega = 50\%$ in the first panel and $\omega = 200\%$ in the second panel. For CAPM investors, the table confirms that leverage has no effect on GPMEs, and higher leverage is mechanically offset by lower risk aversion. For long-term investors, we find some differences in GPMEs across leverage. The conservative long-term investor assigns a GPME of 0.35 to buyout, 0.25 to generalist, and in the 0.15-0.20 range to venture capital. The aggressive long-term investor, instead, assigns a GPME of 0.33 to buyout, 0.22 to generalist, and 0.07 to venture capital. Thus, the conservative investor assigns greater value to PE across all three fund categories. By construction, these differences are entirely due to different risk adjustments. Quantitatively, however, the GPME differences across leverage appear largely negligible, at least for the case of buyout and generalist funds.

Overall, we estimate that all three fund categories provide positive value to both CAPM investors and long-term investors across a wide range of leverage levels.

7 Conclusion

PE funds are illiquid investments whose true return is unobservable. Since investment returns are unobservable risk and performance cannot be estimated with standard approaches. Consequently, the literature has developed methods to evaluate these investments by discounting fund cash flows with SDFs. In this paper, we show that existing SDF methods for the valuation of PE funds result in unrealistic time discounting, which can generate implausible performance estimates. We propose a modified method and compare it to existing ones.

Theoretically, our approach is based on a standard asset pricing decomposition of PE performance in a risk-neutral part and a risk adjustment. We fix the risk-neutral part by constraining the SDF such that the subjective term structure of interest rates is determined by market data. By construction, the risk-neutral part does not vary as we add or remove risk factors from the SDF, so we can meaningfully measure the economic cost of PE risk and compare it across models. Empirically, we evaluate our approach against existing methods and find that our approach results in more robust PE performance across SDFs and lower variation of performance across funds.

We apply our method to measure PE performance and risk adjustment for two types of investors: a CAPM investor and a long-term investor distinguishing between permanent and transitory wealth shocks. We discount net-of-fees cash flows of 1866 PE funds started in the US between 1978 and 2009 and divided into three categories: buyout, venture capital, and generalist. We find largely negligible differences between the two investors, especially for buyout and generalist funds. Overall, we find positive risk-adjusted performance for buyout, generalist, and venture capital funds. For venture capital, however, large risk exposure makes performance estimates particularly sensitive to estimated risk prices.

Our performance decomposition does not provide clear guidance on how to estimate risk prices for proper benchmarking of PE cash flows. Therefore we rely on heuristic SDF restrictions and study the sensitivity of performance estimates with respect to risk prices. The open issue on risk price estimation, among others, is left to future research.

Table 1
Summary Statistics of VAR Variables

This table reports summary statistics of variables entering the full VAR model. Variables are computed quarterly from 1950 to 2018 for a total of 276 observations. The expression $r^{\mathrm{m}} - r^{\mathrm{f}}$ indicates the excess log-return on the S&P 500, DP is the logarithm of dividend yield on the S&P 500, VS is the difference in the log book-to-market ratio of small-value and small-growth stocks, CS is the yield difference between BAA and AAA rated corporate bonds, and $TERM$ is the yield difference between treasuries with 10-year and 3-month maturity. Panel A reports descriptive statistics. Panel B reports correlations between contemporaneous and lagged variables.

Panel A: Descriptive Statistics

	Mean	Std	Min	Max	Autocorr.
$r^{\mathrm{m}}_t - r^{\mathrm{f}}_t$	0.016	0.078	−0.311	0.192	0.099
DP_t	−3.536	0.424	−4.497	−2.624	0.982
VS_t	1.575	0.157	1.280	2.111	0.890
CS_t	0.010	0.004	0.003	0.034	0.878
$TERM_t$	0.017	0.014	−0.035	0.045	0.841

Panel B: Correlations

	$r^{\mathrm{m}}_t - r^{\mathrm{f}}_t$	DP_t	VS_t	CS_t	$TERM_t$
$r^{\mathrm{m}}_{t-1} - r^{\mathrm{f}}_{t-1}$	0.099	−0.069	−0.017	−0.220	0.060
DP_{t-1}	0.130	0.982	−0.439	0.159	−0.221
VS_{t-1}	−0.099	−0.429	0.890	0.030	0.326
CS_{t-1}	0.022	0.144	0.060	0.878	0.321
$TERM_{t-1}$	0.103	−0.273	0.283	0.171	0.841

Table 2
VAR Estimation

This table reports the results of two VAR estimations. In Panel A, the VAR includes only the excess log-return and the logarithm of dividend yield on the S&P 500. In Panel B, the VAR includes also the value spread, credit spread, and term premium as defined in Table 1 and in the main text. Variables are computed quarterly from 1950 to 2018 for a total of 276 observations. OLS Standard errors are reported in parenthesis, and the symbols ***, **, and * indicate significance at 1%, 5%, and 10%.

Panel A: DP only							
	Constant	$r^{\mathrm{m}}_t - r^{\mathrm{f}}_t$	DP_t	VS_t	CS_t	$TERM_t$	R^2
$r^{\mathrm{m}}_{t+1} - r^{\mathrm{f}}_{t+1}$	0.102***	0.107*	0.025**				0.028
	(0.039)	(0.060)	(0.011)				
DP_{t+1}	−0.084**	−0.100	0.977***				0.965
	(0.040)	(0.062)	(0.011)				

Panel B: Full VAR							
	Constant	$r^{\mathrm{m}}_t - r^{\mathrm{f}}_t$	DP_t	VS_t	CS_t	$TERM_t$	R^2
$r^{\mathrm{m}}_{t+1} - r^{\mathrm{f}}_{t+1}$	0.162***	0.082	0.027**	−0.040	−0.622	0.929**	0.051
	(0.055)	(0.060)	(0.012)	(0.034)	(1.118)	(0.373)	
DP_{t+1}	−0.101*	−0.085	0.975***	0.019	−0.705	−0.780**	0.966
	(0.056)	(0.062)	(0.013)	(0.035)	(1.153)	(0.385)	
VS_{t+1}	0.101**	0.064	−0.027**	0.868***	1.840*	−0.240	0.798
	(0.051)	(0.056)	(0.012)	(0.031)	(1.039)	(0.347)	
CS_{t+1}	0.001	−0.009***	0.000	0.000	0.875***	−0.011	0.798
	(0.001)	(0.002)	(0.000)	(0.001)	(0.029)	(0.010)	
$TERM_{t+1}$	−0.011**	−0.004	0.000	0.006*	0.371***	0.789***	0.725
	(0.005)	(0.006)	(0.001)	(0.003)	(0.106)	(0.036)	

Table 3
Discount Rate News

This table decomposes point estimates and variance of N_t^{DR} from the two VAR estimations of Table 2. In Panel A, we report estimates of the vector λ such that $N_t^{\mathrm{DR}} = \lambda\varepsilon_t$ where ε_t is the VAR error term. The vector is $\lambda = \rho e'\Theta(I - \rho\Theta)^{-1}$, where Θ is the matrix of VAR coefficients, I is the identity matrix, e is a column vector with 1 as first element and 0 elsewhere, and $\rho = 0.95^{1/4}$. In parentheses, we report standard errors calculated with the delta method. Statistical significance is computed using the normal distribution, and the symbols ***, **, and * indicate significance at 1%, 5%, and 10%. In Panel B, we decompose the variance of N_t^{DR} expressed as $\lambda\Sigma_\varepsilon\lambda'$. We report the vector of variance components $\lambda \circ \lambda\Sigma_\varepsilon$, where $\circ$ is the element-wise product. We also report components as percentages of the total variance.

Panel A: Long-Run Coefficients

	DP only	Full VAR
	λ	λ
$r^{\mathrm{m}} - r^{\mathrm{f}}$	0.039	0.046
	(0.027)	(0.041)
DP	0.713***	0.782***
	(0.112)	(0.139)
VS		−0.110
		(0.097)
CS		−4.869
		(3.843)
$TERM$		1.977***
		(0.747)

Panel B: Variance Decomposition

	DP only		Full VAR	
	$\lambda \circ \lambda\Sigma_\varepsilon$	%	$\lambda \circ \lambda\Sigma_\varepsilon$	%
$r^{\mathrm{m}} - r^{\mathrm{f}}$ shock	−0.00016	−5.4	−0.00019	−5.4
DP shock	0.00303	105.4	0.00343	99.2
VS shock			0.00011	3.0
CS shock			−0.00002	−0.4
$TERM$ shock			0.00013	3.6
$\mathrm{Var}(N_t^{\mathrm{DR}})$	0.00287	100.0	0.00345	100.0

Table 4
Summary Statistics of Funds Data

This table reports summary statistics of funds data from Burgiss. In Panel A, *#Funds* is the sample size, and for each fund, Fund Size is total commitment, Effective Years counts the years between the first and last available cash flow, #Cash flows/Fund is the number of cash flows, TVPI is the ratio of distributions over contributions. Furthermore, #Unresolved Funds counts funds that are not fully liquidated in sample, and NAV/Distributions is the ratio of their residual NAV over total distributions. Panel B reports mean and median TVPI for different vintage years. For confidentiality, we do not report figures computed with 4 funds or less.

Panel A: Descriptive Statistics

	Buyout			Venture Capital			Generalist		
	Mean	Median	St.dev.	Mean	Median	St.dev.	Mean	Median	St.dev.
#Funds	652			971			243		
Fund Size ($M)	1099.10	421.00	2118.19	222.10	126.00	282.75	557.77	225.00	1290.66
Effective Years	14.16	13.75	3.13	15.52	15.25	3.37	14.42	13.50	3.37
#Cash flows/Fund	35.83	36.00	10.45	27.60	27.00	9.62	32.99	33.00	10.53
TVPI	1.83	1.68	1.15	2.08	1.39	3.07	1.80	1.64	1.02
#Unresolved Funds	311			353			88		
NAV/Distributions	0.10	0.06	0.11	0.14	0.10	0.13	0.12	0.07	0.13

Panel B: TVPI by Vintage Year

	Buyout			Venture Capital			Generalist		
Vintage	# Funds	Mean	Median	# Funds	Mean	Median	# Funds	Mean	Median
1978-91	57	3.09	2.22	238	2.12	1.77	15	2.76	2.55
1992	8	1.97	1.64	17	3.19	1.76	5	3.12	2.62
1993	7	1.68	1.72	20	5.35	3.15	7	2.30	1.90
1994	18	1.73	1.49	16	6.15	4.50	8	2.60	2.18
1995	26	1.63	1.53	27	5.69	2.72	6	3.08	2.37
1996	17	1.64	1.70	18	6.68	3.31	8	2.00	1.43
1997	26	1.24	1.23	47	3.48	1.94	17	1.49	1.28
1998	40	1.45	1.45	53	1.97	1.18	17	1.48	1.39
1999	34	1.45	1.55	94	0.86	0.72	20	1.29	1.09
2000	50	1.79	1.68	119	0.96	0.85	26	1.43	1.41
2001	31	1.90	1.94	60	1.26	1.12	6	2.04	2.20
2002	21	1.89	1.85	21	1.08	1.11	7	1.73	1.63
2003	22	2.07	1.82	21	1.38	1.10	4	* * *	* * *
2004	38	1.77	1.63	34	1.49	0.91	10	1.66	1.74
2005	57	1.64	1.52	52	1.63	1.31	18	1.84	1.53
2006	61	1.67	1.64	54	1.56	1.47	27	1.53	1.39
2007	66	1.77	1.70	45	2.21	1.90	22	1.58	1.68
2008	55	1.72	1.69	28	2.06	1.71	13	1.96	1.81
2009	18	2.06	2.05	7	1.92	2.01	7	1.71	1.59

Table 5
Buyout Performance

For $N = 652$ buyout funds in our sample, we estimate expected GPME by summing discounted cash flows of each fund and averaging the result across funds. Cash flows are discounted with the following SDF:

$$M_{t,t+h} = \exp\left(a_h h - \omega\gamma\, r^{\mathrm{m}}_{t,t+h} - \omega(\gamma-1)N^{\mathrm{DR}}_{t,t+h}\right)$$

In this table, the investor has stock allocation $\omega = 100\%$, and each column corresponds to different restrictions on the SDF. With Log-Utility, $a_h = 0$ and $\gamma = 1$ as in the PME of Kaplan and Schoar (2005). With Single Intercept, $a_h = a$ for all h, and the two parameters (a and γ) are estimated following Korteweg and Nagel (2016) so that the SDF prices artificial funds invested in the S&P 500 and in quarterly T-bills. With Multiple Intercepts, we estimate γ and one a_h for every h such that the SDF prices T-Bills investments at every horizon and stock investments at horizon $h = 40$ quarters. CAPM corresponds to $N^{\mathrm{DR}}_{t,t+h} = 0$, while LT(DP) and LT(Full) use $N^{\mathrm{DR}}_{t,t+h}$ from the VAR estimations of Table 2. For each specification, we report point estimates of GPME and γ. In parentheses, we report GPME standard errors that account for error dependence between overlapping funds and ignore parameters uncertainty. In brackets, we report p-values for a J-test of GPME $= 0$. In the last two rows of the table, we divide the GPME in two components, and we let $\overline{M}_h = \frac{1}{N}\sum_i M_{t,t+h}$ be the average SDF at each horizon, $\overline{C}_h = \frac{1}{N}\sum_i C_{i,t+h}$ be the average cash flow, and $A_h = \frac{1}{N}\sum_i (M_{t,t+h}/\overline{M}_h - 1)(C_{i,t+h} - \overline{C}_h)$ be a risk adjustment.

		Single Intercept			Multiple Intercepts		
	Log-Utility	CAPM	LT (DP)	LT (Full)	CAPM	LT (DP)	LT (Full)
GPME	0.203	0.279	0.803	0.405	0.298	0.350	0.346
	(0.027)	(0.159)	(0.706)	(0.285)	(0.113)	(0.118)	(0.114)
	[0.000]	[0.079]	[0.255]	[0.155]	[0.008]	[0.003]	[0.002]
γ	1.00	3.37	10.30	6.72	3.16	3.98	4.03
Components of GPME							
$\sum_h \overline{M}_h \overline{C}_h$	0.293	0.792	2.502	1.139	0.608	0.608	0.608
$\sum_h \overline{M}_h A_h$	−0.090	−0.513	−1.699	−0.735	−0.310	−0.258	−0.261

Table 6
GPME Distributions

The table reports mean, standard deviation, and selected percentiles of the GPME distribution for buyout, venture capital, and generalist funds across different models. The models are estimated in Table 5, 7, and 8.

		Single Intercept			Multiple Intercepts		
	Log-Utility	CAPM	LT (DP)	LT (Full)	CAPM	LT (DP)	LT (Full)
			Buyout ($N = 652$)				
Mean	0.20	0.28	0.80	0.41	0.30	0.35	0.35
St.Dev.	0.64	1.42	9.25	2.99	0.98	1.05	1.04
Min	−1.25	−4.15	−15.39	−5.86	−2.21	−1.60	−1.66
p10	−0.36	−0.83	−4.30	−1.65	−0.47	−0.48	−0.47
p25	−0.12	−0.38	−0.85	−0.56	−0.20	−0.23	−0.20
p50	0.13	−0.04	−0.17	−0.14	0.02	0.03	0.04
p75	0.44	0.59	0.61	0.69	0.51	0.61	0.58
p90	0.82	1.72	4.06	2.76	1.35	1.56	1.51
Max	10.89	13.98	165.94	42.62	8.78	8.32	8.66
			Venture Capital ($N = 971$)				
Mean	0.14	−0.15	−0.19	−0.08	0.07	0.13	0.15
St.Dev.	1.40	1.18	1.43	1.68	1.07	1.23	1.32
Min	−1.14	−2.57	−3.21	−2.84	−1.50	−1.36	−1.40
p10	−0.67	−0.95	−1.12	−1.02	−0.66	−0.65	−0.65
p25	−0.44	−0.63	−0.68	−0.63	−0.42	−0.41	−0.41
p50	−0.19	−0.33	−0.37	−0.34	−0.16	−0.16	−0.15
p75	0.23	0.04	−0.04	0.03	0.26	0.30	0.31
p90	0.88	0.72	0.73	0.90	0.87	0.91	0.98
Max	15.13	16.47	20.42	24.55	15.51	17.31	17.46
			Generalist ($N = 243$)				
Mean	0.16	0.13	0.05	0.15	0.21	0.24	0.25
St.Dev.	0.52	1.01	2.28	1.58	0.75	0.79	0.79
Min	−1.15	−2.72	−6.43	−3.75	−1.50	−1.47	−1.51
p10	−0.39	−0.72	−1.95	−1.08	−0.41	−0.46	−0.42
p25	−0.17	−0.44	−0.69	−0.51	−0.24	−0.24	−0.23
p50	0.06	−0.12	−0.24	−0.17	0.00	0.03	0.06
p75	0.44	0.42	0.34	0.40	0.46	0.51	0.53
p90	0.79	1.19	2.38	2.07	1.01	1.11	1.10
Max	2.63	6.11	11.52	8.94	4.15	4.01	4.20

Table 7
Venture Capital Performance

We estimate expected GPME for $N = 971$ venture capital funds in our sample. The construction of this table follows the description of Table 5 using the sample of venture capital funds.

		Single Intercept			Multiple Intercepts		
	Log-Utility	CAPM	LT (DP)	LT (Full)	CAPM	LT (DP)	LT (Full)
GPME	0.135	−0.150	−0.188	−0.077	0.073	0.126	0.151
	(0.084)	(0.067)	(0.081)	(0.101)	(0.065)	(0.078)	(0.084)
	[0.109]	[0.026]	[0.020]	[0.447]	[0.261]	[0.108]	[0.072]
γ	1.00	2.93	4.68	4.19	2.03	2.37	2.40
Components of GPME							
$\sum_h \overline{M}_h \overline{C}_h$	0.484	0.910	0.903	0.892	0.725	0.725	0.725
$\sum_h \overline{M}_h A_h$	−0.349	−1.060	−1.091	−0.969	−0.652	−0.599	−0.574

Table 8
Generalist Performance

We estimate expected GPME for $N = 243$ generalist funds in our sample. The construction of this table follows the description of Table 5 using the sample of generalist funds.

		Single Intercept			Multiple Intercepts		
	Log-Utility	CAPM	LT (DP)	LT (Full)	CAPM	LT (DP)	LT (Full)
GPME	0.156	0.127	0.046	0.153	0.213	0.241	0.249
	(0.023)	(0.094)	(0.173)	(0.129)	(0.069)	(0.074)	(0.071)
	[0.000]	[0.177]	[0.791]	[0.234]	[0.002]	[0.001]	[0.000]
γ	1.00	3.05	7.49	5.43	2.53	3.14	3.19
Components of GPME							
$\sum_h \overline{M}_h \overline{C}_h$	0.318	0.703	1.043	0.832	0.563	0.563	0.563
$\sum_h \overline{M}_h A_h$	-0.162	-0.576	-0.998	-0.679	-0.350	-0.321	-0.313

Table 9
PE Performance and Investor's Leverage

We estimate expected GPME for of a conservative investor with 50% of wealth in stocks ($\omega = 50\%$) and an aggressive investor with 200% of wealth in stocks ($\omega = 200\%$). This table reports the results separately for buyout, venture capital, and generalist funds. The estimation follows the description of Table 5 limited to the case with Multiple Intercepts.

	Buyout			Venture Capital			Generalist		
	CAPM	LT (DP)	LT (Full)	CAPM	LT (DP)	LT (Full)	CAPM	LT (DP)	LT (Full)
Conservative Investor ($\omega = 50\%$)									
GPME	0.298	0.358	0.354	0.073	0.155	0.198	0.212	0.250	0.262
	(0.113)	(0.119)	(0.114)	(0.065)	(0.086)	(0.096)	(0.069)	(0.076)	(0.071)
	[0.008]	[0.003]	[0.002]	[0.262]	[0.071]	[0.040]	[0.002]	[0.001]	[0.000]
γ	6.32	8.29	8.41	4.08	5.07	5.17	5.07	6.66	6.78
Components of GPME									
$\sum_h \overline{M}_h \overline{C}_h$	0.608	0.608	0.608	0.725	0.725	0.725	0.563	0.563	0.563
$\sum_h \overline{M}_h A_h$	−0.310	−0.250	−0.254	−0.652	−0.570	−0.527	−0.350	−0.313	−0.301
Aggressive Investor ($\omega = 200\%$)									
GPME	0.298	0.329	0.327	0.073	0.075	0.075	0.212	0.223	0.225
	(0.113)	(0.116)	(0.114)	(0.065)	(0.065)	(0.066)	(0.069)	(0.071)	(0.069)
	[0.008]	[0.004]	[0.004]	[0.262]	[0.253]	[0.250]	[0.002]	[0.002]	[0.001]
γ	1.58	1.81	1.83	1.02	1.02	1.03	1.27	1.38	1.39
Components of GPME									
$\sum_h \overline{M}_h \overline{C}_h$	0.608	0.608	0.608	0.725	0.725	0.725	0.563	0.563	0.563
$\sum_h \overline{M}_h A_h$	−0.310	−0.279	−0.281	−0.652	−0.650	−0.649	−0.350	−0.339	−0.337

Figure 1
Aggregate Cash Flows

This figure plots aggregate (normalized) contributions, distributions, and net cash flows for the three fund categories. Buyout corresponds to the top-left, venture capital is in the top-right, and generalist at the bottom. The blue area represents distributions, the red area represents contributions, and the solid line represents net cash flows. The grey shaded areas correspond to NBER recessions.

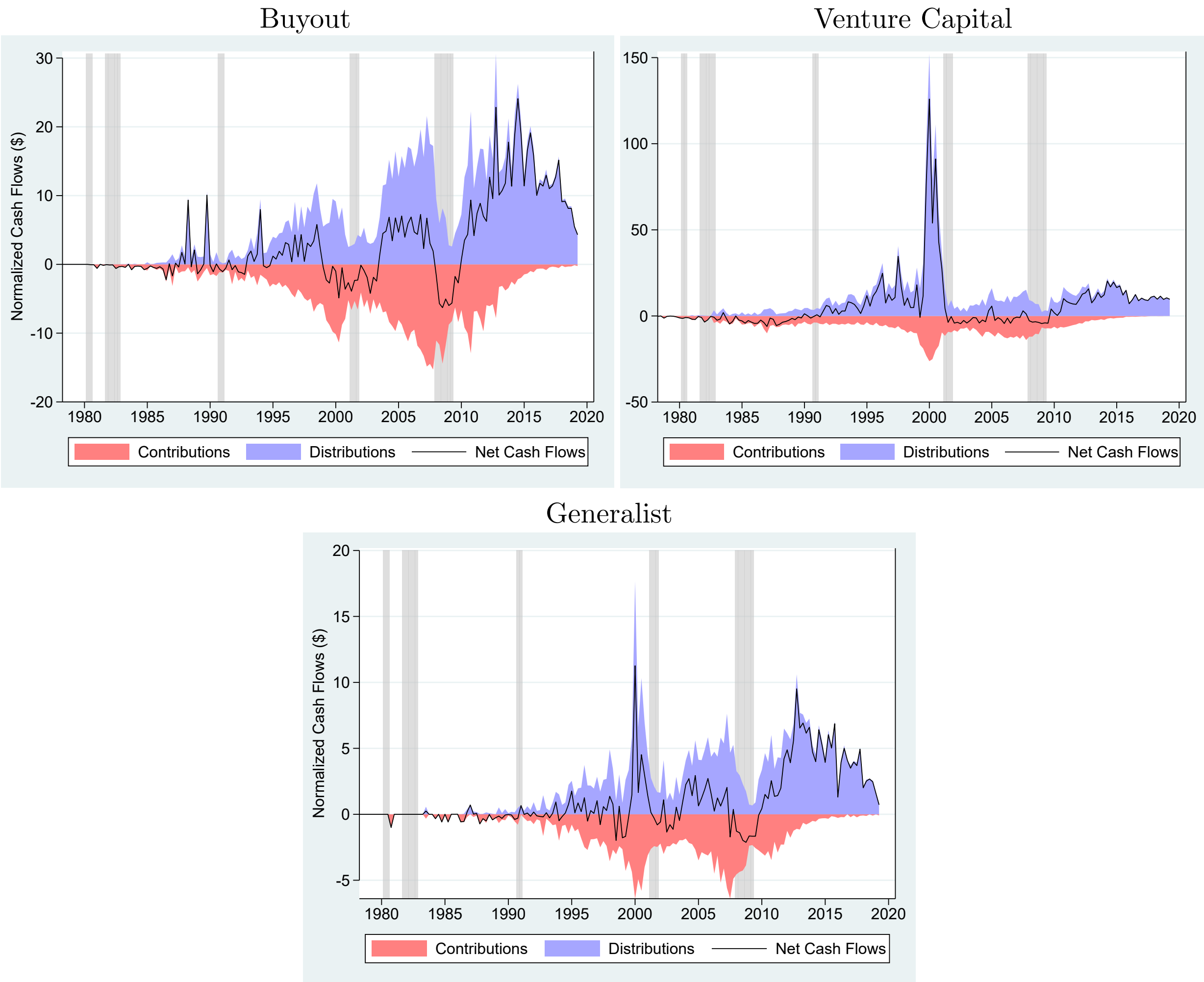


Figure 2
Time Discounting for Buyout

This figure plots the average multi-period SDF, $\overline{M}_h = \frac{1}{N}\sum_i M_{t,t+h}$, across buyout funds every quarter. We consider different SDFs resulting from the estimations of Table 5.

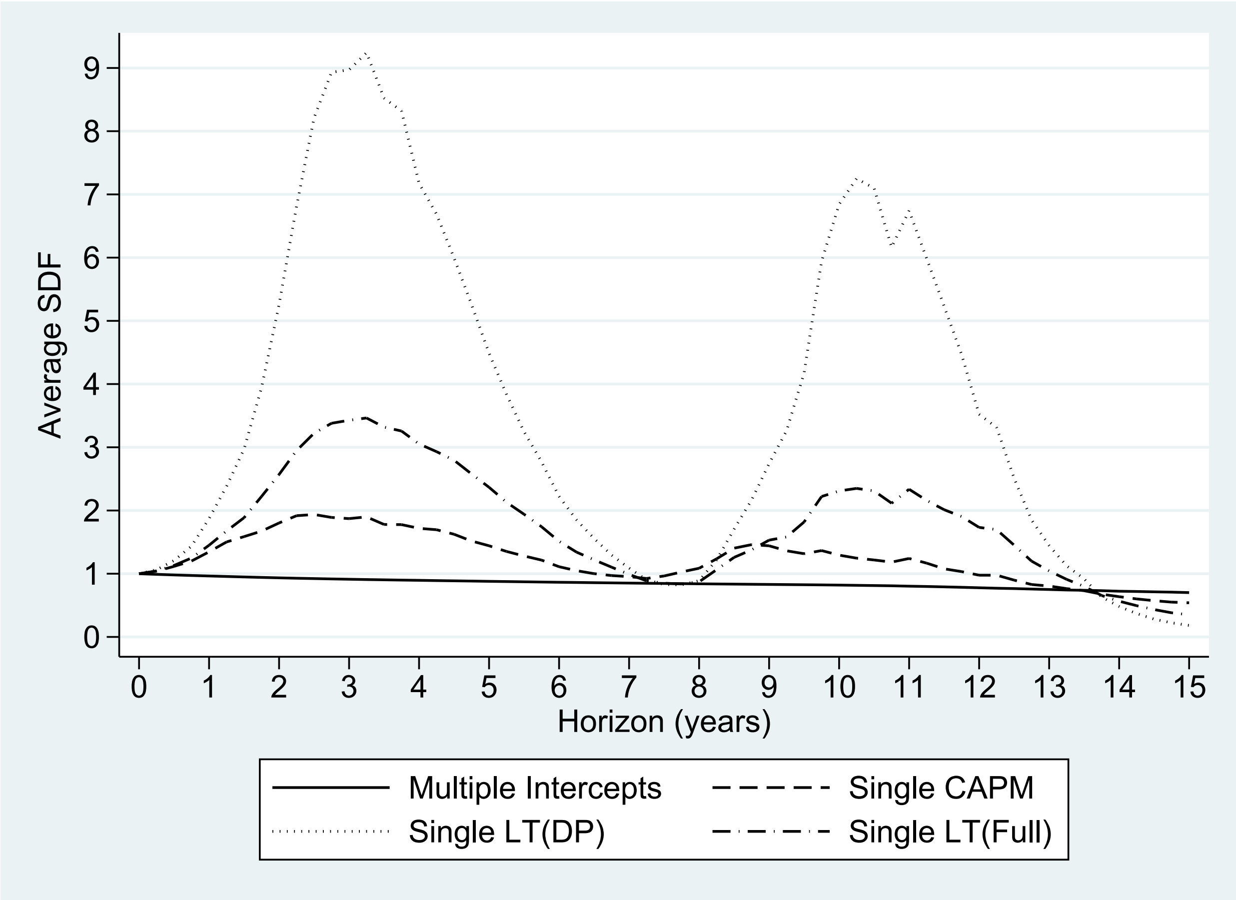


Figure 3
GPME Decomposition for Buyout

In this figure, we decompose GPMEs estimated in Table 5 for the CAPM and LT(DP) models with multiple intercepts. Discounted Value of Avg. Cash Flow is $\overline{C}_0$ in year 0, $\sum_{h=1}^{4} \overline{M}_h \overline{C}_h$ in year 1, $\sum_{h=4y-3}^{4y} \overline{M}_h \overline{C}_h$ in year y between 2 and 14, and $\sum_{h=57}^{H} \overline{M}_h \overline{C}_h$ in year 15. By construction, Discounted Value of Avg. Cash Flow is the same across the two models. Discounted Value of Risk is 0 in year 0, $\sum_{h=1}^{4} \overline{M}_h A_h$ in year 1, $\sum_{h=4y-3}^{4y} \overline{M}_h A_h$ in year y between 2 and 14, and $\sum_{h=57}^{H} \overline{M}_h A_h$ in year 15.

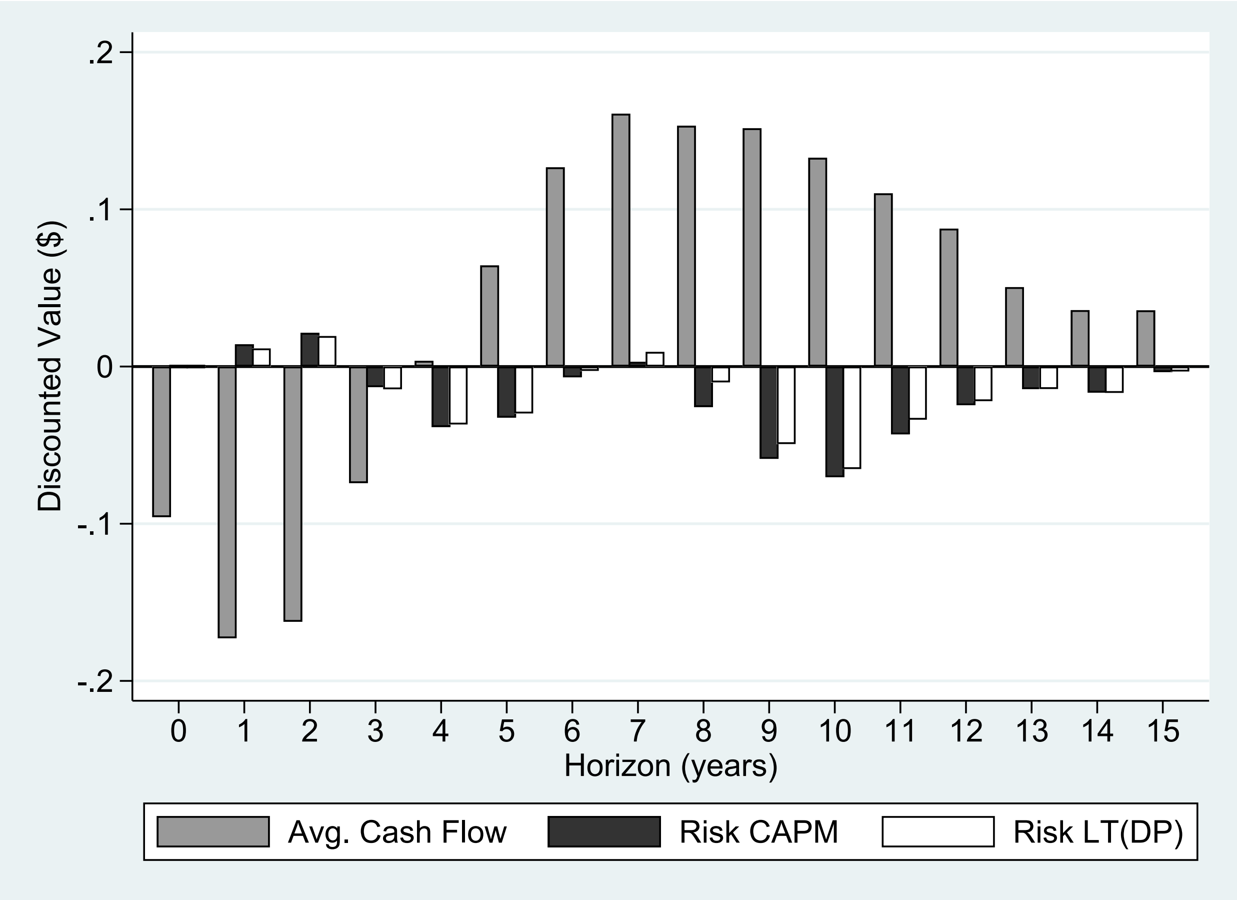


Figure 4
Time Discounting for Venture Capital

This figure plots the average multi-period SDF, $\overline{M}_h = \frac{1}{N}\sum_i M_{t,t+h}$, across venture capital funds every quarter. We consider different SDFs resulting from the estimations of Table 7.

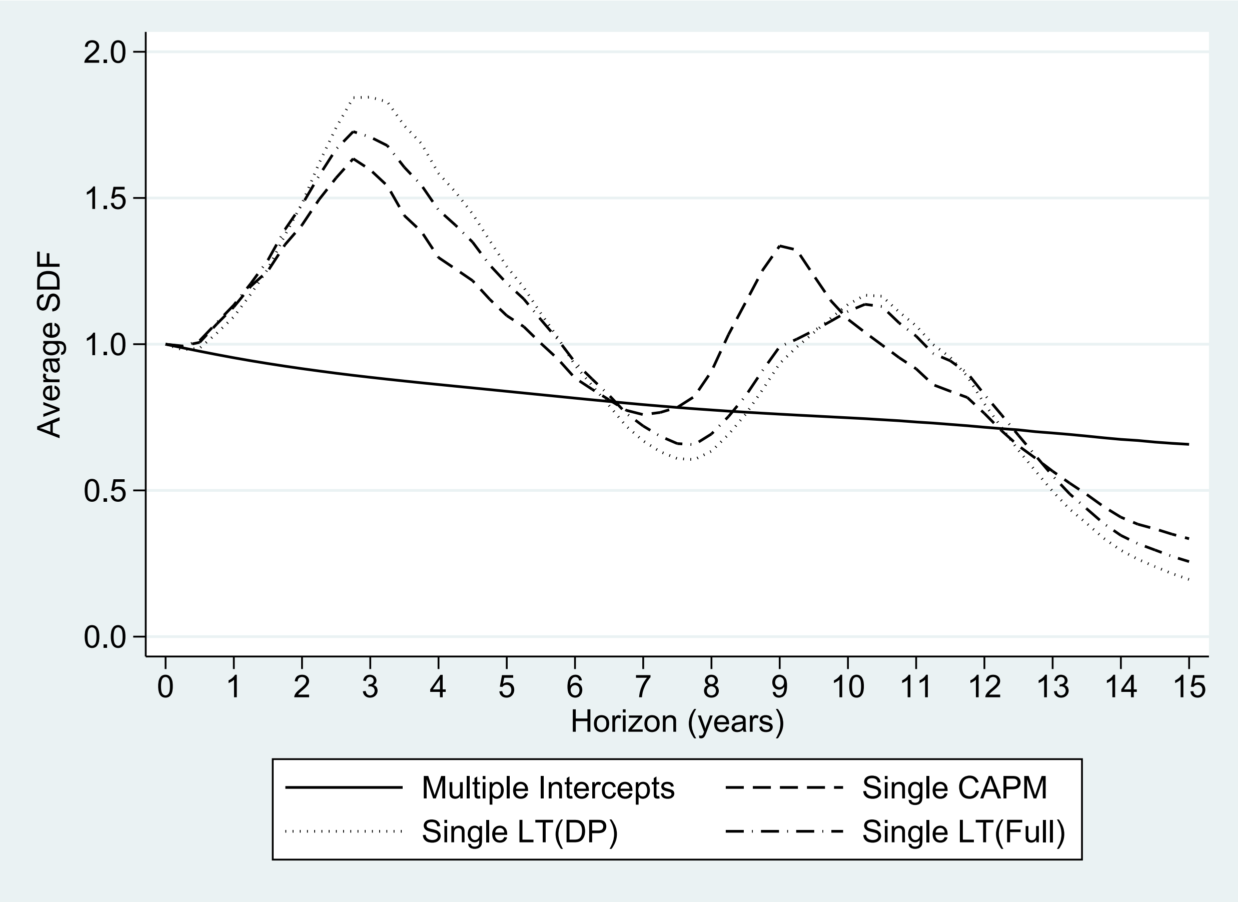


Figure 5
GPME Decomposition for Venture Capital

In this figure, we decompose GPMEs estimated in Table 7 for the Multiple Intercepts CAPM and LT(DP) models. The plot is constructed as in Figure 3.

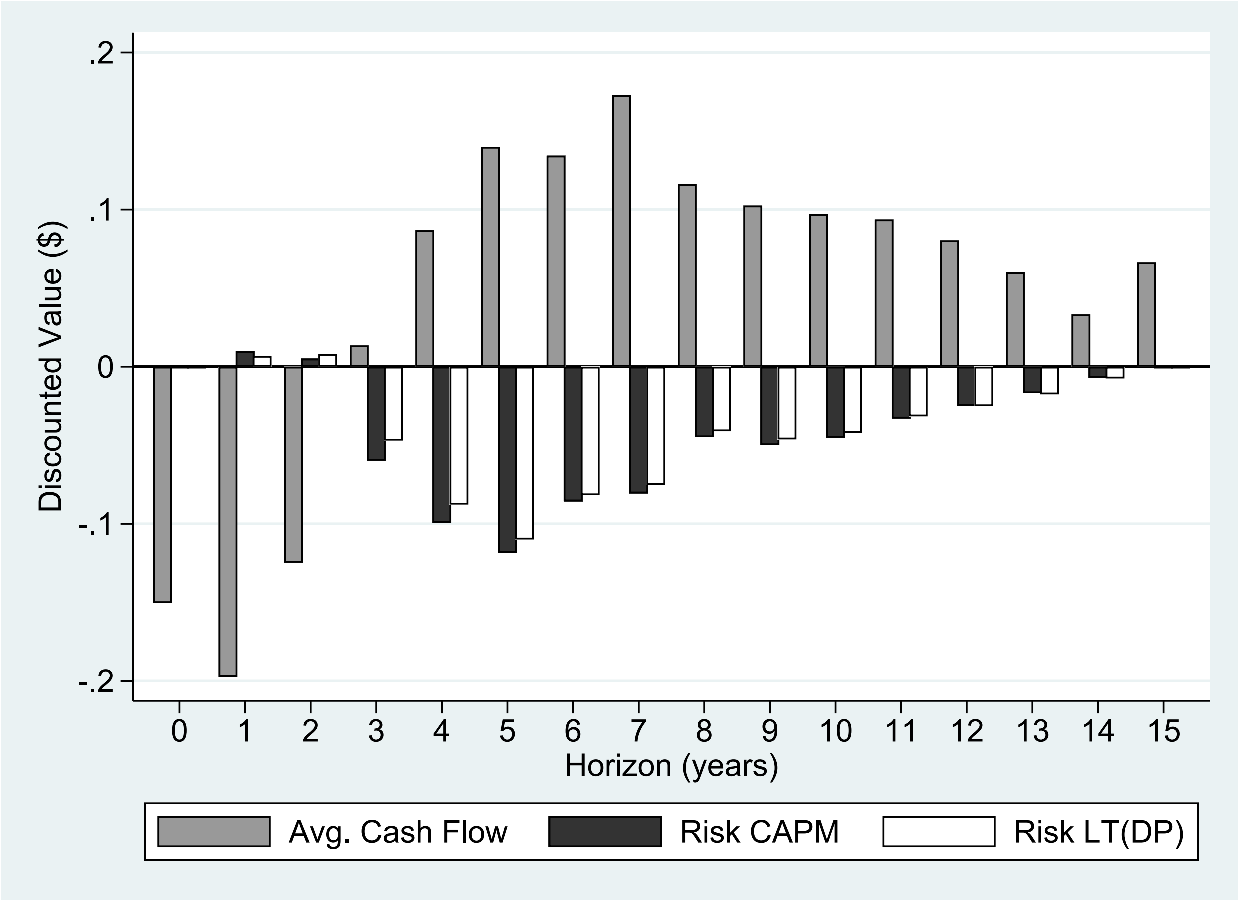


Figure 6
Time Discounting for Generalist

This figure plots the average multi-period SDF, $\overline{M}_h = \frac{1}{N}\sum_i M_{t,t+h}$, across generalist funds every quarter. We consider different SDFs resulting from the estimations of Table 8.

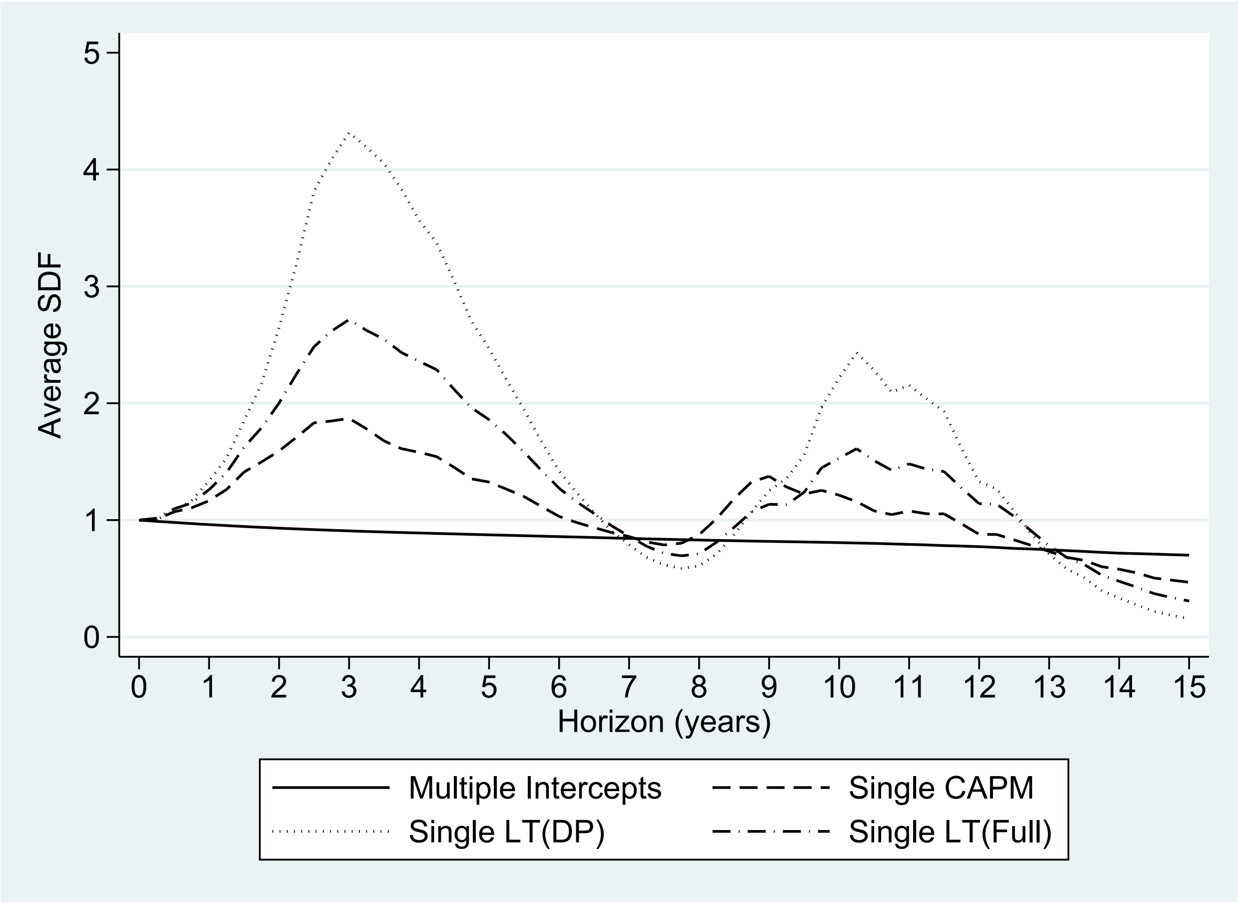


Figure 7
GPME Decomposition for Generalist

In this figure, we decompose GPMEs estimated in Table 8 for the Multiple Intercepts CAPM and LT(DP) models. The plot is constructed as in Figure 3.

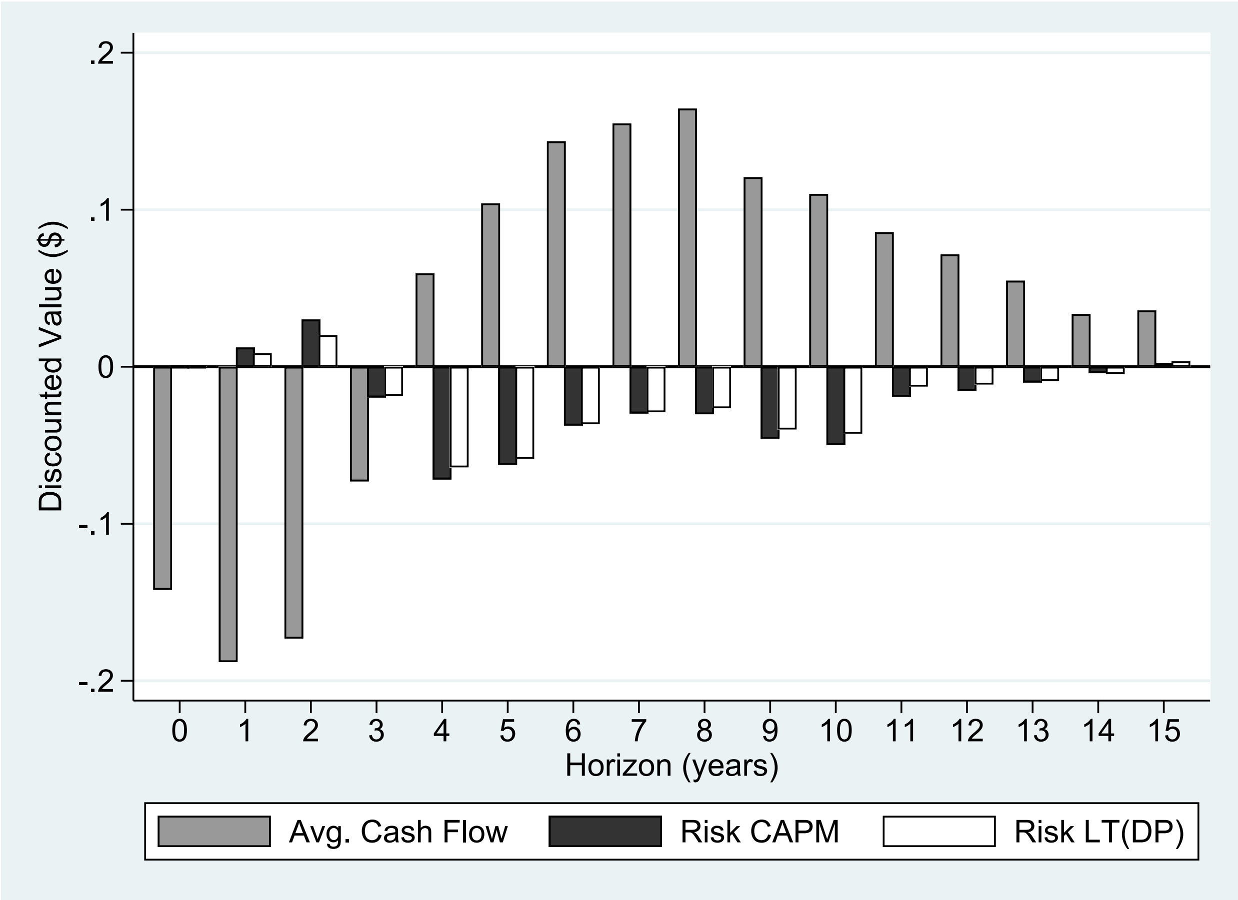
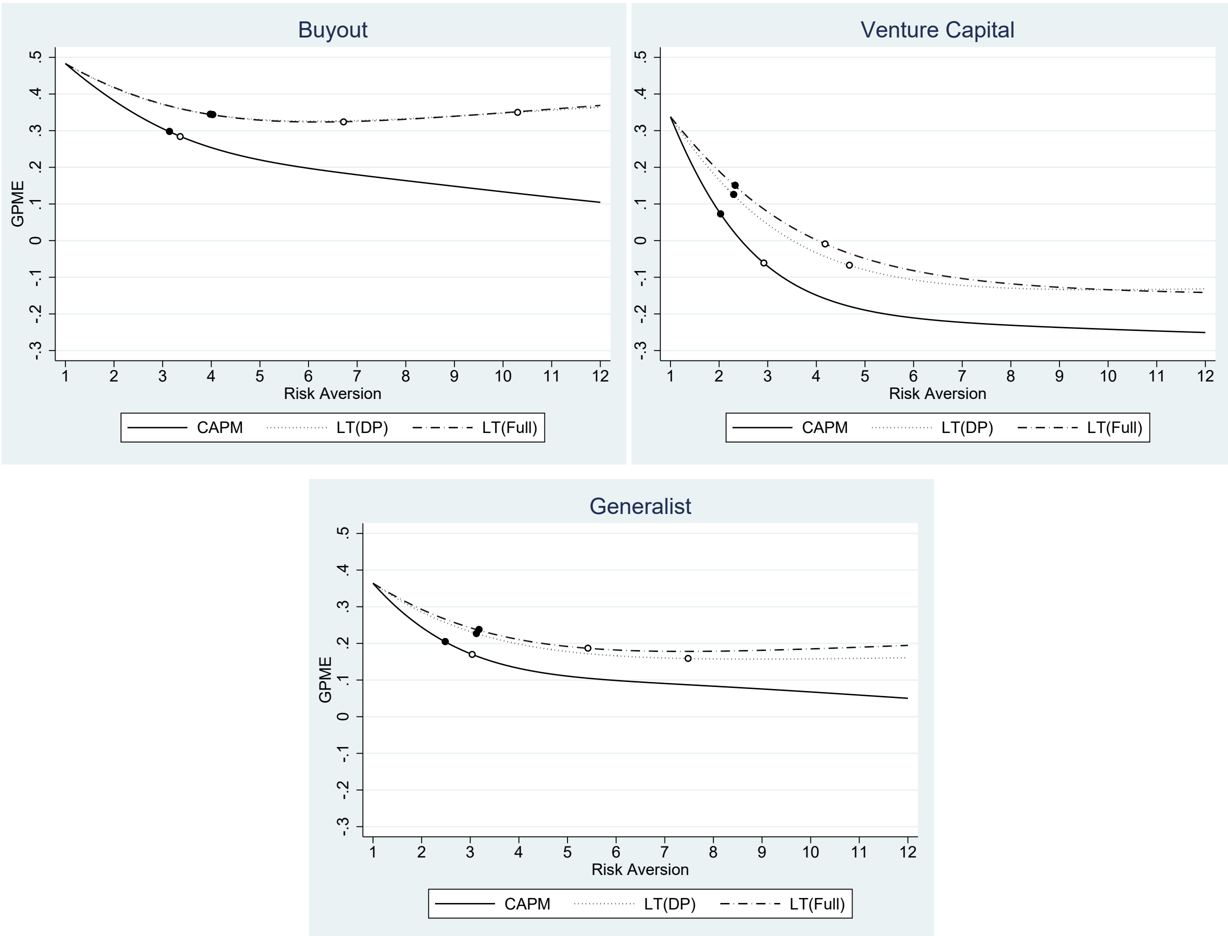


Figure 8
GPME Sensitivity to Risk Aversion Estimates

In this figure, we plot GPMEs for models with multiple intercepts using exogenous values of γ between 1 and 12. For each category and for each model, we compute GPMEs as $\sum_h(\frac{1}{N}\sum_i 1/R^{\mathrm{f}}_{t,t+h})\ (\overline{C}_h + A_h)$, where $A_h = \frac{1}{N}\sum_i(M_{t,t+h}/\overline{M}_h - 1)(C_{i,t+h} - \overline{C}_h)$. The term $M_{t,t+h}/\overline{M}_h$ is computed as $\exp(-\gamma r^{\mathrm{m}}_{t,t+h})/\frac{1}{N}\sum_i \exp(-\gamma r^{\mathrm{m}}_{t,t+h})$ for CAPM and similarly with an additional risk factor for LT(DP) and LT(Full). On each line, the black circle indicates the GPME estimate using γ from the corresponding model in Table 5 for buyout, Table 7 for venture capital, and Table 8 for generalist. The white circle indicates the GPME estimate using γ from the single intercept version of the model.



Appendix

A Theoretical Stochastic Discount Factor

In this section, we connect the long-term SDF from the main text to theory by deriving its theoretical version. We use the setup of Campbell (1993) extending his results to price payoffs received several periods in the future.

The investor has infinite-horizon Epstein-Zin preferences over consumption, and these preferences correspond to the following general form of SDF (Epstein and Zin, 1989):

$$M_{t,t+1}^{\text{theory}} = \exp\left(\theta \log \delta - \frac{\theta}{\psi}(c_{t+1} - c_t) - (1-\theta)\, r_{t+1}^{\mathrm{W}}\right) \tag{A.1}$$

In this expression, c_t is the natural logarithm of consumption, and r_{t+1}^{W} is the log-return on wealth. Greek letters indicate parameters with δ being the subjective discount factor, ψ being the elasticity of intertemporal substitution, and $\theta = \frac{1-\gamma}{1-1/\psi}$, which depends on ψ and relative risk aversion γ.

The budget constraint of the investor can be written as follows:

$$W_{t+1} = (W_t - C_t) R_{t+1}^{\mathrm{W}} \tag{A.2}$$

Wealth is W_t, while $C_t = \exp(c_t)$ is consumption, and $R_{t+1}^{\mathrm{W}} = \exp(r_{t+1}^{\mathrm{W}})$ is the return on wealth. Following Campbell (1993), we represent the investor's budget constraint with the following log-linear approximation:

$$w_{t+1} - w_t = r_{t+1}^{\mathrm{W}} + k + \left(1 - \frac{1}{\rho}\right)(c_t - w_t) \tag{A.3}$$

In this expression, w_t is the logarithm of wealth, k and ρ are approximation constants.

A.1 Consumption Growth

Assuming that second and higher moments of r_{t+1}^{W} are constant over time, consumption can be expressed as a function of returns. For that, we use two equations derived in Campbell (1993) under the same set of assumptions. The first equation connects expected consumption growth to expected return on wealth:

$$\mathrm{E}_t\left[c_{t+1}-c_t\right]=\psi\mathrm{E}_t\left[r_{t+1}^{\mathrm{W}}\right]+\underbrace{\psi\log(\delta)+\frac{1}{2}\frac{1-\gamma}{1-\psi}\mathrm{Var}_t\left(c_{t+1}-c_t-\psi r_{t+1}^{\mathrm{W}}\right)}_{\text{constant}} \tag{A.4}$$

The second equation connects the unexpected component of consumption growth to the unexpected component of r_{t+1}^{W} and to discount rate (DR) news about investor's wealth:

$$c_{t+1}-c_t-\mathrm{E}_t\left[c_{t+1}-c_t\right]=r_{t+1}^{\mathrm{W}}-\mathrm{E}_t\left[r_{t+1}^{\mathrm{W}}\right]+(1-\psi)\underbrace{\left(\mathrm{E}_{t+1}-\mathrm{E}_t\right)\sum_{j=1}^{\infty}\rho^j r_{t+1}^{\mathrm{W}}}_{N_{t+1}^{\mathrm{DR,W}}} \tag{A.5}$$

Combining (A.4) with (A.5), we get the following expression for the logarithm of consumption growth:

$$c_{t+1}-c_t=r_{t+1}^{\mathrm{W}}+(1-\psi)N_{t+1}^{\mathrm{DR,W}}-(1-\psi)\mathrm{E}_t\left[r_{t+1}^{\mathrm{W}}\right]+\text{constant} \tag{A.6}$$

A.2 Return on Wealth

DR news on wealth, $N_{t+1}^{\mathrm{DR,W}}$, can be expressed in terms of DR news on underlying assets. For that, we specify a simple investment strategy for the investor. His portfolio is a constant combination of two assets. One asset is risk-free, while the other is a market index of public equities. The return on the risk-free asset is constant and its logarithm is denoted r^{f}. The return on the market is risky and its logarithm is denoted r_t^{m}. Expected log-return on the market can vary over time, while its variance and higher conditional moments remain constant.

Following Campbell and Viceira (1999), log-returns on individual assets determine log-return on wealth through the following approximate relation:

$$r_{t+1}^{\mathrm{W}}=\omega\left(r_{t+1}^{\mathrm{m}}-r^{\mathrm{f}}\right)+r^{\mathrm{f}}+\frac{1}{2}\omega(1-\omega)\mathrm{var}\left(r_{t+1}^{\mathrm{m}}\right) \tag{A.7}$$

In this expression, ω is the constant portfolio weight in the market. Under our assumptions, this approximation implies:

$$N_{t+1}^{\mathrm{DR,W}}=\omega N_{t+1}^{\mathrm{DR}} \tag{A.8}$$

where N_{t+1}^{DR} is one-period DR news on the market as defined in the main text.

A.3 Theoretical Long-Term SDF

To obtain the theoretical form of the long-term SDF, we substitute (A.6) and (A.7) inside the SDF expression (A.1). As a result, we obtain the one-period theoretical version of the long-term SDF:

$$M_{t+1}^{\text{theory}} = \exp\left(a_t - \omega\gamma\, r_{t+1}^{\text{m}} - \omega(\gamma-1)\, N_{t+1}^{\text{DR}}\right) \tag{A.9}$$

with $a_t = \omega(\gamma-1)\mathrm{E}_t\left[r_{\text{m},t+1}\right] + \text{constant}$.

Two Periods

For the two-period version, the product $M_{t+1}^{\text{theory}} \times M_{t+2}^{\text{theory}}$ can be written as follows:

$$M_{t,t+2}^{\text{theory}} = \exp\left(a_{2,t} - \omega\gamma\, r_{t,t+2}^{\text{m}} - \omega(\gamma-1) N_{t,t+2}^{\text{DR}}\right) \tag{A.10}$$

This expression contains the following objects:

$$a_{2,t} = \omega(\gamma-1)\left(\mathrm{E}_t[r_{t+1}^{\text{m}}] + \mathrm{E}_t[r_{t+2}^{\text{m}}]\right) + 2\cdot\text{constant} \tag{A.11}$$

$$\begin{aligned} N_{t,t+2}^{\text{DR}} &= (\mathrm{E}_{t+2} - \mathrm{E}_t)\sum_{j=1}^{\infty}\rho^j r_{t+2+j}^{\text{m}} - (1-\rho)(\mathrm{E}_{t+1} - \mathrm{E}_t) r_{t+2}^{\text{m}} \\ &\approx (\mathrm{E}_{t+2} - \mathrm{E}_t)\sum_{j=1}^{\infty}\rho^j r_{t+2+j}^{\text{m}} \end{aligned} \tag{A.12}$$

and the approximation of $N_{t,t+2}^{\text{DR}}$ is accurate for ρ close 1.

Many Periods

For the multi-period version, the product $M_{t+1}^{\text{theory}} \times M_{t+2}^{\text{theory}} \times \cdots \times M_{t+h}^{\text{theory}}$ can be written as:

$$M_{t,t+h}^{\text{theory}} = \exp\left(a_{h,t} - \omega\gamma\, r_{t,t+h}^{\text{m}} - \omega(\gamma-1) N_{t,t+h}^{\text{DR}}\right) \tag{A.13}$$

where

$$a_{h,t} = \omega(\gamma-1)\sum_{s=1}^{h}\mathrm{E}_t[r_{t+s}^{\text{m}}] + h\cdot\text{constant} \tag{A.14}$$

Furthermore, multi-period DR news is written as follows:

$$
\begin{aligned}
N^{\mathrm{DR}}_{t,t+h} &= \sum_{s=1}^{h} N^{\mathrm{DR}}_{t+s} - \sum_{s=1}^{h-1}\left[(\mathrm{E}_{t+s}-\mathrm{E}_{t+s-1})\sum_{j=1}^{h-s} r^{\mathrm{m}}_{t+s+j}\right] && \text{(A.15)}\\
&= \sum_{s=1}^{h}\left[(\mathrm{E}_{t+s}-\mathrm{E}_{t+s-1})\sum_{j=1}^{\infty}\rho^{j} r^{\mathrm{m}}_{t+s+j}\right] - \sum_{s=1}^{h-1}\left[(\mathrm{E}_{t+s}-\mathrm{E}_{t+s-1})\sum_{j=1}^{h-s} r^{\mathrm{m}}_{t+s+j}\right]\\
&= \sum_{s=1}^{h}\left[(\mathrm{E}_{t+s}-\mathrm{E}_{t+s-1})\sum_{j=h-s+1}^{\infty}\rho^{j} r^{\mathrm{m}}_{t+s+j}\right] - \sum_{s=1}^{h-1}\left[(\mathrm{E}_{t+s}-\mathrm{E}_{t+s-1})\sum_{j=1}^{h-s}(1-\rho^{j}) r^{\mathrm{m}}_{t+s+j}\right]\\
&\approx \sum_{s=1}^{h}\left[(\mathrm{E}_{t+s}-\mathrm{E}_{t+s-1})\sum_{j=h-s+1}^{\infty}\rho^{j} r^{\mathrm{m}}_{t+s+j}\right]\\
&= \sum_{s=1}^{h}\left[(\mathrm{E}_{t+s}-\mathrm{E}_{t+s-1})\sum_{j=1}^{\infty}\rho^{j} r^{\mathrm{m}}_{t+h+j}\right]\\
&= (\mathrm{E}_{t+h}-\mathrm{E}_{t})\sum_{j=1}^{\infty}\rho^{j} r^{\mathrm{m}}_{t+h+j} && \text{(A.16)}
\end{aligned}
$$

The approximation is accurate for ρ close to 1.

Bibliography

Andonov, A. (2020). Delegated Investment Management in Alternative Assets. *Working Paper*.

Ang, A., B. Chen, W. N. Goetzmann, and L. Phalippou (2018). Estimating Private Equity Returns from Limited Partner Cash Flows. *The Journal of Finance 73*(4), 1751–1783.

Ang, A., D. Papanikolaou, and M. M. Westerfield (2014). Portfolio Choice with Illiquid Assets. *Management Science 60*(11), 2737–2761.

Aït-Sahalia, Y. and M. W. Brandt (2001). Variable Selection for Portfolio Choice. *The Journal of Finance 56*(4), 1297–1351.

Axelson, U., T. Jenkinson, P. Strömberg, and M. S. Weisbach (2013). Borrow Cheap, Buy High? The Determinants of Leverage and Pricing in Buyouts. *The Journal of Finance 68*(6), 2223–2267.

Bain & Company (2021). Global Private Equity Report. Technical Report.

Bansal, R. and A. Yaron (2004). Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles. *The Journal of Finance 59*(4), 1481–1509.

Black, F. (1972). Capital Market Equilibrium with Restricted Borrowing. *The Journal of Business 45*(3), 444–55.

Boyer, B., T. D. Nadauld, K. P. Vorkink, and M. S. Weisbach (2021). Discount Rate Risk in Private Equity: Evidence from Secondary Market Transactions. *Working Paper*.

Brandt, M. W. (1999). Estimating Portfolio and Consumption Choice: A Conditional Euler Equations Approach. *The Journal of Finance 54*(5), 1609–1645.

Brandt, M. W. and P. Santa-Clara (2006). Dynamic Portfolio Selection by Augmenting the Asset Space. *The Journal of Finance 61*(5), 2187–2217.

Brandt, M. W., P. Santa-Clara, and R. Valkanov (2009). Parametric Portfolio Policies: Exploiting Characteristics in the Cross-Section of Equity Returns. *The Review of Financial Studies 22*(9), 3411–3447.

Britten-Jones, M. (1999). The Sampling Error in Estimates of Mean-Variance Efficient Portfolio Weights. *The Journal of Finance 54*(2), 655–671.

Brown, G., R. Harris, W. Hu, T. Jenkinson, S. N. Kaplan, and D. T. Robinson (2021). Can Investors Time Their Exposure to Private Equity? *Journal of Financial Economics 139*(2), 561–577.

Brown, G., W. Hu, and B.-K. Kuhn (2021). Private Investments in Diversified Portfolios. *Working Paper*.

Brown, G. W., E. Ghysels, and O. Gredil (2020). Nowcasting Net Asset Values: The Case of Private Equity. *Working Paper*.

Brown, G. W., O. R. Gredil, and S. N. Kaplan (2019). Do Private Equity Funds Manipulate Reported Returns? *Journal of Financial Economics 132*(2), 267–297.

Brown, G. W., R. S. Harris, T. Jenkinson, S. N. Kaplan, and D. T. Robinson (2015). What Do Different Commercial Data Sets Tell Us About Private Equity Performance? *Working Paper*.

Campbell, J. Y. (1991). A Variance Decomposition for Stock Returns. *The Economic Journal 101*(405), 157–179.

Campbell, J. Y. (1993). Intertemporal Asset Pricing without Consumption Data. *American Economic Review 83*(3), 487–512.

Campbell, J. Y. (1996). Understanding Risk and Return. *Journal of Political Economy 104*(2), 298–345.

Campbell, J. Y. and J. H. Cochrane (1999). By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. *Journal of Political Economy 107*(2), 205–251.

Campbell, J. Y. and R. J. Shiller (1988). The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors. *The Review of Financial Studies 1*(3), 195–228.

Campbell, J. Y. and L. M. Viceira (1999). Consumption and Portfolio Decisions when Expected Returns are Time Varying. *The Quarterly Journal of Economics 114*(2), 433–495.

Campbell, J. Y. and L. M. Viceira (2002). *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*. Oxford University Press.

Campbell, J. Y. and T. Vuolteenaho (2004). Bad Beta, Good Beta. *The American Economic Review 94*(5), 1249–1275.

Cavagnaro, D. R., B. A. Sensoy, Y. Wang, and M. S. Weisbach (2019). Measuring Institutional Investors' Skill at Making Private Equity Investments. *The Journal of Finance 74*(6), 3089–3134.

Cochrane, J. (1996). A Cross-Sectional Test of an Investment-Based Asset Pricing Model. *Journal of Political Economy 104*(3), 572–621.

Cochrane, J. H. (2005). The Risk and Return of Venture Capital. *Journal of Financial Economics 75*(1), 3 – 52.

Cochrane, J. H. (2011). Discount Rates. *The Journal of Finance 66*(4), 1047–1108.

Cochrane, J. H. (2017, 03). Macro-Finance. *Review of Finance 21*(3), 945–985.

Da Rin, M. and L. Phalippou (2017). The Importance of Size in Private Equity: Evidence from a Survey of Limited Partners. *Journal of Financial Intermediation 31*, 64–76.

Driessen, J., T.-C. Lin, and L. Phalippou (2012). A New Method to Estimate Risk and Return of Nontraded Assets from Cash Flows: The Case of Private Equity Funds. *Journal of Financial and Quantitative Analysis 47*(3), 511–535.

Epstein, L. G. and S. E. Zin (1989). Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica 57*(4), 937–969.

Fama, E. F. and K. R. French (1989). Business Conditions and Expected Returns on Stocks and Bonds. *Journal of Financial Economics 25*(1), 23 – 49.

Farnsworth, H., W. Ferson, D. Jackson, and S. Todd (2002). Performance Evaluation with Stochastic Discount Factors. *The Journal of Business 75*(3), 473–583.

Ferson, W. E. and C. R. Harvey (1991). The Variation of Economic Risk Premiums. *Journal of Political Economy 99*(2), 385.

Ferson, W. E. and C. R. Harvey (1999). Conditioning Variables and the Cross Section of Stock Returns. *The Journal of Finance 54*(4), 1325–1360.

Ferson, W. E. and R. W. Schadt (1996). Measuring Fund Strategy and Performance in Changing Economic Conditions. *Journal of Finance 51*(2), 425–61.

Franzoni, F., E. Nowak, and L. Phalippou (2012). Private Equity Performance and Liquidity Risk. *The Journal of Finance 67*(6), 2341–2373.

Gao, C. and I. Martin (2019). Volatility, Valuation Ratios, and Bubbles: An Empirical Measure of Market Sentiment. *Working Paper*.

Giommetti, N. and M. Sorensen (2021). Optimal Allocation to Private Equity. *Working Paper*.

Gompers, P. A. and J. Lerner (1997). Risk and Reward in Private Equity Investments: The Challenge of Performance Assessment. *The Journal of Private Equity 1*(2), 5–12.

Gredil, O., B. E. Griffiths, and R. Stucke (2014). Benchmarking Private Equity: The Direct Alpha Method. *Working Paper*.

Gredil, O., Y. Liu, and B. Sensoy (2021). Diversifying Private Equity. *Working Paper*.

Gredil, O., M. Sørensen, and W. Waller (2020). Evaluating Private Equity Performance using Stochastic Discount Factors. *Working Paper*.

Gupta, A. and S. Van Nieuwerburgh (2021). Valuing Private Equity Investments Strip by Strip. *Journal of Finance, forthcoming*.

Harris, R. S., T. Jenkinson, and S. N. Kaplan (2014). Private Equity Performance: What Do We Know? *The Journal of Finance 69*(5), 1851–1882.

Harris, R. S., T. Jenkinson, S. N. Kaplan, and R. Stucke (2018). Financial Intermediation in Private Equity: How Well Do Funds of Funds Perform? *Journal of Financial Economics 129*(2), 287–305.

Harris, R. S., T. Jenkinson, S. N. Kaplan, and R. Stucke (2020). Has Persistence Persisted in Private Equity? Evidence from Buyout and Venture Capital Funds. *Working Paper*.

Harvard (2020). Financial Report. Annual report, Harvard University.

Jagannathan, R. and Z. Wang (1996). The Conditional CAPM and the Cross-Section of Expected Returns. *The Journal of Finance 51*(1), 3–53.

Jeffers, J., T. Lyu, and K. Posenau (2021). The Risk and Return of Impact Investing Funds. *Working Paper*.

Jenkinson, T., M. Sousa, and R. Stucke (2013). How Fair are the Valuations of Private Equity Funds? *Working Paper*.

Kaplan, S. N. and A. Schoar (2005). Private Equity Performance: Returns, Persistence, and Capital Flows. *The Journal of Finance 60*(4), 1791–1823.

Keim, D. B. and R. F. Stambaugh (1986). Predicting Returns in the Stock and Bond Markets. *Journal of Financial Economics 17*(2), 357 – 390.

Kelly, B. and S. Pruitt (2013). Market Expectations in the Cross-Section of Present Values. *The Journal of Finance 68*(5), 1721–1756.

Kelly, B. and S. Pruitt (2015). The Three-Pass Regression Filter: A New Approach to Forecasting Using Many Predictors. *Journal of Econometrics 186*(2), 294–316.

Korteweg, A. (2019). Risk Adjustment in Private Equity Returns. *Annual Review of Financial Economics 11*(1), 131–152.

Korteweg, A. and S. Nagel (2016). Risk-Adjusting the Returns to Venture Capital. *The Journal of Finance 71*(3), 1437–1470.

Lamont, O. (1998). Earnings and Expected Returns. *The Journal of Finance 53*(5), 1563–1587.

Lerner, J., A. Schoar, and W. Wongsunwai (2007). Smart Institutions, Foolish Choices: The Limited Partner Performance Puzzle. *The Journal of Finance 62*(2), 731–764.

Lettau, M. and S. Ludvigson (2001). Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia Are Time-Varying. *Journal of Political Economy 109*(6), 1238–1287.

Lettau, M. and H. Uhlig (2002). The Sharpe Ratio and Preferences: A Parametric Approach. *Macroeconomic Dynamics 6*(2), 242–265.

Li, H., Y. Xu, and X. Zhang (2016). Hedge Fund Performance Evaluation under the Stochastic Discount Factor Framework. *The Journal of Financial and Quantitative Analysis 51*(1), 231–257.

Ljungqvist, A. and M. Richardson (2003). The Cash Flow, Return and Risk Characteristics of Private Equity. *Working Paper*.

Long, A. M. and C. J. Nickels (1996). A Private Investment Benchmark. *Working Paper*.

Lustig, H. and S. Van Nieuwerburgh (2006). The Returns on Human Capital: Good News on Wall Street is Bad News on Main Street. *The Review of Financial Studies 21*(5), 2097–2137.

Lynch, A. W. and J. A. Wachter (2013). Using Samples of Unequal Length in Generalized Method of Moments Estimation. *The Journal of Financial and Quantitative Analysis 48*(1), 277–307.

Martin, I. (2016). What is the Expected Return on the Market? *The Quarterly Journal of Economics 132*(1), 367–433.

McKinsey & Company (2020). McKinsey Global Private Markets Review. Technical Report.

Phalippou, L. (2014). Performance of Buyout Funds Revisited? *Review of Finance 18*(1), 189–218.

Phalippou, L. and O. Gottschalg (2009). The Performance of Private Equity Funds. *The Review of Financial Studies 22*(4), 1747–1776.

Preqin (2020). Preqin Global Private Equity and Venture Capital Report. Technical Report.

Pástor, L. and R. F. Stambaugh (2003). Liquidity Risk and Expected Stock Returns. *Journal of Political Economy 111*(3), 642 – 685.

Robinson, D. T. and B. A. Sensoy (2016). Cyclicality, Performance Measurement, and Cash Flow Liquidity in Private Equity. *Journal of Financial Economics 122*, 521–543.

Rouvinez, C. (2003). Private Equity Benchmarking with PME+. *Private Equity International*.

Rubinstein, M. (1976). The Strong Case for the Generalized Logarithmic Utility Model as the Premier Model of Financial Markets. *The Journal of Finance 31*(2), 551–571.

Sorensen, M. and R. Jagannathan (2015). The Public Market Equivalent and Private Equity Performance. *Financial Analysts Journal 71*(4), 43–50.

Stafford, E. (2020). Replicating Private Equity with Value Investing, Homemade Leverage and Hold-to-Maturity Accounting. *Working Paper*.

Welch, I. and A. Goyal (2008). A Comprehensive Look at the Empirical Performance of Equity Premium Prediction. *The Review of Financial Studies 21*(4), 1455–1508.

TITLER I PH.D.SERIEN:

2004

1. Martin Grieger
Internet-based Electronic Marketplaces and Supply Chain Management

2. Thomas Basbøll
LIKENESS
A Philosophical Investigation

3. Morten Knudsen
Beslutningens vaklen
En systemteoretisk analyse of moderniseringen af et amtskommunalt sundhedsvæsen 1980-2000

4. Lars Bo Jeppesen
Organizing Consumer Innovation
A product development strategy that is based on online communities and allows some firms to benefit from a distributed process of innovation by consumers

5. Barbara Dragsted
SEGMENTATION IN TRANSLATION AND TRANSLATION MEMORY SYSTEMS
An empirical investigation of cognitive segmentation and effects of integrating a TM system into the translation process

6. Jeanet Hardis
Sociale partnerskaber
Et socialkonstruktivistisk casestudie af partnerskabsaktørers virkelighedsopfattelse mellem identitet og legitimitet

7. Henriette Hallberg Thygesen
System Dynamics in Action

8. Carsten Mejer Plath
Strategisk Økonomistyring

9. Annemette Kjærgaard
Knowledge Management as Internal Corporate Venturing
– a Field Study of the Rise and Fall of a Bottom-Up Process

10. Knut Arne Hovdal
De profesjonelle i endring
Norsk ph.d., ej til salg gennem Samfundslitteratur

11. Søren Jeppesen
Environmental Practices and Greening Strategies in Small Manufacturing Enterprises in South Africa
– A Critical Realist Approach

12. Lars Frode Frederiksen
Industriel forskningsledelse
– på sporet af mønstre og samarbejde i danske forskningsintensive virksomheder

13. Martin Jes Iversen
The Governance of GN Great Nordic
– in an age of strategic and structural transitions 1939-1988

14. Lars Pynt Andersen
The Rhetorical Strategies of Danish TV Advertising
A study of the first fifteen years with special emphasis on genre and irony

15. Jakob Rasmussen
Business Perspectives on E-learning

16. Sof Thrane
The Social and Economic Dynamics of Networks
– a Weberian Analysis of Three Formalised Horizontal Networks

17. Lene Nielsen
Engaging Personas and Narrative Scenarios – a study on how a user-centered approach influenced the perception of the design process in the e-business group at AstraZeneca

18. S.J Valstad
Organisationsidentitet
Norsk ph.d., ej til salg gennem Samfundslitteratur

19. Thomas Lyse Hansen
Six Essays on Pricing and Weather risk in Energy Markets

20. Sabine Madsen
Emerging Methods – An Interpretive Study of ISD Methods in Practice

21. Evis Sinani
The Impact of Foreign Direct Investment on Efficiency, Productivity Growth and Trade: An Empirical Investigation

22. Bent Meier Sørensen
Making Events Work Or, How to Multiply Your Crisis

23. Pernille Schnoor
Brand Ethos
Om troværdige brand- og virksomhedsidentiteter i et retorisk og diskursteoretisk perspektiv

24. Sidsel Fabech
Von welchem Österreich ist hier die Rede?
Diskursive forhandlinger og magtkampe mellem rivaliserende nationale identitetskonstruktioner i østrigske pressediskurser

25. Klavs Odgaard Christensen
Sprogpolitik og identitetsdannelse i flersprogede forbundsstater
Et komparativt studie af Schweiz og Canada

26. Dana B. Minbaeva
Human Resource Practices and Knowledge Transfer in Multinational Corporations

27. Holger Højlund
Markedets politiske fornuft
Et studie af velfærdens organisering i perioden 1990-2003

28. Christine Mølgaard Frandsen
A.s erfaring
Om mellemværendets praktik i en transformation af mennesket og subjektiviteten

29. Sine Nørholm Just
The Constitution of Meaning – A Meaningful Constitution?
Legitimacy, identity, and public opinion in the debate on the future of Europe

2005

1. Claus J. Varnes
Managing product innovation through rules – The role of formal and structured methods in product development

2. Helle Hedegaard Hein
Mellem konflikt og konsensus – Dialogudvikling på hospitalsklinikker

3. Axel Rosenø
Customer Value Driven Product Innovation – A Study of Market Learning in New Product Development

4. Søren Buhl Pedersen
Making space
An outline of place branding

5. Camilla Funck Ellehave
Differences that Matter
An analysis of practices of gender and organizing in contemporary workplaces

6. Rigmor Madeleine Lond
Styring af kommunale forvaltninger

7. Mette Aagaard Andreassen
Supply Chain versus Supply Chain
Benchmarking as a Means to Managing Supply Chains

8. Caroline Aggestam-Pontoppidan
From an idea to a standard
The UN and the global governance of accountants' competence

9. Norsk ph.d.

10. Vivienne Heng Ker-ni
An Experimental Field Study on the

Effectiveness of Grocer Media Advertising
Measuring Ad Recall and Recognition, Purchase Intentions and Short-Term Sales

11. Allan Mortensen
Essays on the Pricing of Corporate Bonds and Credit Derivatives

12. Remo Stefano Chiari
Figure che fanno conoscere
Itinerario sull'idea del valore cognitivo e espressivo della metafora e di altri tropi da Aristotele e da Vico fino al cognitivismo contemporaneo

13. Anders McIlquham-Schmidt
Strategic Planning and Corporate Performance
An integrative research review and a meta-analysis of the strategic planning and corporate performance literature from 1956 to 2003

14. Jens Geersbro
The TDF – PMI Case
Making Sense of the Dynamics of Business Relationships and Networks

15 Mette Andersen
Corporate Social Responsibility in Global Supply Chains
Understanding the uniqueness of firm behaviour

16. Eva Boxenbaum
Institutional Genesis: Micro – Dynamic Foundations of Institutional Change

17. Peter Lund-Thomsen
Capacity Development, Environmental Justice NGOs, and Governance: The Case of South Africa

18. Signe Jarlov
Konstruktioner af offentlig ledelse

19. Lars Stæhr Jensen
Vocabulary Knowledge and Listening Comprehension in English as a Foreign Language
An empirical study employing data elicited from Danish EFL learners

20. Christian Nielsen
Essays on Business Reporting
Production and consumption of strategic information in the market for information

21. Marianne Thejls Fischer
Egos and Ethics of Management Consultants

22. Annie Bekke Kjær
Performance management i Proces-innovation
– belyst i et social-konstruktivistisk perspektiv

23. Suzanne Dee Pedersen
GENTAGELSENS METAMORFOSE
Om organisering af den kreative gøren i den kunstneriske arbejdspraksis

24. Benedikte Dorte Rosenbrink
Revenue Management
Økonomiske, konkurrencemæssige & organisatoriske konsekvenser

25. Thomas Riise Johansen
Written Accounts and Verbal Accounts
The Danish Case of Accounting and Accountability to Employees

26. Ann Fogelgren-Pedersen
The Mobile Internet: Pioneering Users' Adoption Decisions

27. Birgitte Rasmussen
Ledelse i fællesskab – de tillidsvalgtes fornyende rolle

28. Gitte Thit Nielsen
Remerger
– skabende ledelseskræfter i fusion og opkøb

29. Carmine Gioia
A MICROECONOMETRIC ANALYSIS OF MERGERS AND ACQUISITIONS

30. Ole Hinz
Den effektive forandringsleder: pilot, pædagog eller politiker?
Et studie i arbejdslederes meningstilskrivninger i forbindelse med vellykket gennemførelse af ledelsesinitierede forandringsprojekter

31. Kjell-Åge Gotvassli
Et praksisbasert perspektiv på dynamiske
læringsnettverk i toppidretten
Norsk ph.d., ej til salg gennem Samfundslitteratur

32. Henriette Langstrup Nielsen
Linking Healthcare
An inquiry into the changing performances of web-based technology for asthma monitoring

33. Karin Tweddell Levinsen
Virtuel Uddannelsespraksis
Master i IKT og Læring – et casestudie i hvordan proaktiv proceshåndtering kan forbedre praksis i virtuelle læringsmiljøer

34. Anika Liversage
Finding a Path
Labour Market Life Stories of Immigrant Professionals

35. Kasper Elmquist Jørgensen
Studier i samspillet mellem stat og erhvervsliv i Danmark under 1. verdenskrig

36. Finn Janning
A DIFFERENT STORY
Seduction, Conquest and Discovery

37. Patricia Ann Plackett
Strategic Management of the Radical Innovation Process
Leveraging Social Capital for Market Uncertainty Management

2006

1. Christian Vintergaard
Early Phases of Corporate Venturing

2. Niels Rom-Poulsen
Essays in Computational Finance

3. Tina Brandt Husman
Organisational Capabilities, Competitive Advantage & Project-Based Organisations
The Case of Advertising and Creative Good Production

4. Mette Rosenkrands Johansen
Practice at the top
– how top managers mobilise and use non-financial performance measures

5. Eva Parum
Corporate governance som strategisk kommunikations- og ledelsesværktøj

6. Susan Aagaard Petersen
Culture's Influence on Performance Management: The Case of a Danish Company in China

7. Thomas Nicolai Pedersen
The Discursive Constitution of Organizational Governance – Between unity and differentiation
The Case of the governance of environmental risks by World Bank environmental staff

8. Cynthia Selin
Volatile Visions: Transactons in Anticipatory Knowledge

9. Jesper Banghøj
Financial Accounting Information and Compensation in Danish Companies

10. Mikkel Lucas Overby
Strategic Alliances in Emerging High-Tech Markets: What's the Difference and does it Matter?

11. Tine Aage
External Information Acquisition of Industrial Districts and the Impact of Different Knowledge Creation Dimensions

A case study of the Fashion and Design Branch of the Industrial District of Montebelluna, NE Italy

12. Mikkel Flyverbom
Making the Global Information Society Governable
On the Governmentality of Multi-Stakeholder Networks

13. Anette Grønning
Personen bag
Tilstedevær i e-mail som inter-aktionsform mellem kunde og med-arbejder i dansk forsikringskontekst

14. Jørn Helder
One Company – One Language?
The NN-case

15. Lars Bjerregaard Mikkelsen
Differing perceptions of customer value
Development and application of a tool for mapping perceptions of customer value at both ends of customer-supplier dyads in industrial markets

16. Lise Granerud
Exploring Learning
Technological learning within small manufacturers in South Africa

17. Esben Rahbek Pedersen
Between Hopes and Realities: Reflections on the Promises and Practices of Corporate Social Responsibility (CSR)

18. Ramona Samson
The Cultural Integration Model and European Transformation.
The Case of Romania

2007

1. Jakob Vestergaard
Discipline in The Global Economy
Panopticism and the Post-Washington Consensus

2. Heidi Lund Hansen
Spaces for learning and working
A qualitative study of change of work, management, vehicles of power and social practices in open offices

3. Sudhanshu Rai
Exploring the internal dynamics of software development teams during user analysis
A tension enabled Institutionalization Model; "Where process becomes the objective"

4. Norsk ph.d.
Ej til salg gennem Samfundslitteratur

5. Serden Ozcan
EXPLORING HETEROGENEITY IN ORGANIZATIONAL ACTIONS AND OUTCOMES
A Behavioural Perspective

6. Kim Sundtoft Hald
Inter-organizational Performance Measurement and Management in Action
– An Ethnography on the Construction of Management, Identity and Relationships

7. Tobias Lindeberg
Evaluative Technologies
Quality and the Multiplicity of Performance

8. Merete Wedell-Wedellsborg
Den globale soldat
Identitetsdannelse og identitetsledelse i multinationale militære organisationer

9. Lars Frederiksen
Open Innovation Business Models
Innovation in firm-hosted online user communities and inter-firm project ventures in the music industry
– A collection of essays

10. Jonas Gabrielsen
Retorisk toposlære – fra statisk 'sted' til persuasiv aktivitet

11. Christian Moldt-Jørgensen
Fra meningsløs til meningsfuld evaluering.
Anvendelsen af studentertilfredshedsmålinger på de korte og mellemlange videregående uddannelser set fra et psykodynamisk systemperspektiv

12. Ping Gao
Extending the application of actor-network theory
Cases of innovation in the telecommunications industry

13. Peter Mejlby
Frihed og fængsel, en del af den samme drøm?
Et phronetisk baseret casestudie af frigørelsens og kontrollens sameksistens i værdibaseret ledelse!

14. Kristina Birch
Statistical Modelling in Marketing

15. Signe Poulsen
Sense and sensibility:
The language of emotional appeals in insurance marketing

16. Anders Bjerre Trolle
Essays on derivatives pricing and dynamic asset allocation

17. Peter Feldhütter
Empirical Studies of Bond and Credit Markets

18. Jens Henrik Eggert Christensen
Default and Recovery Risk Modeling and Estimation

19. Maria Theresa Larsen
Academic Enterprise: A New Mission for Universities or a Contradiction in Terms?
Four papers on the long-term implications of increasing industry involvement and commercialization in academia

20. Morten Wellendorf
Postimplementering af teknologi i den offentlige forvaltning
Analyser af en organisations kontinuerlige arbejde med informationsteknologi

21. Ekaterina Mhaanna
Concept Relations for Terminological Process Analysis

22. Stefan Ring Thorbjørnsen
Forsvaret i forandring
Et studie i officerers kapabiliteter under påvirkning af omverdenens forandringspres mod øget styring og læring

23. Christa Breum Amhøj
Det selvskabte medlemskab om managementstaten, dens styringsteknologier og indbyggere

24. Karoline Bromose
Between Technological Turbulence and Operational Stability
– An empirical case study of corporate venturing in TDC

25. Susanne Justesen
Navigating the Paradoxes of Diversity in Innovation Practice
– A Longitudinal study of six very different innovation processes – in practice

26. Luise Noring Henler
Conceptualising successful supply chain partnerships
– Viewing supply chain partnerships from an organisational culture perspective

27. Mark Mau
Kampen om telefonen
Det danske telefonvæsen under den tyske besættelse 1940-45

28. Jakob Halskov
The semiautomatic expansion of existing terminological ontologies using knowledge patterns discovered

on the WWW – an implementation and evaluation

29. Gergana Koleva
European Policy Instruments Beyond Networks and Structure: The Innovative Medicines Initiative

30. Christian Geisler Asmussen
Global Strategy and International Diversity: A Double-Edged Sword?

31. Christina Holm-Petersen
Stolthed og fordom
Kultur- og identitetsarbejde ved skabelsen af en ny sengeafdeling gennem fusion

32. Hans Peter Olsen
Hybrid Governance of Standardized States
Causes and Contours of the Global Regulation of Government Auditing

33. Lars Bøge Sørensen
Risk Management in the Supply Chain

34. Peter Aagaard
Det unikkes dynamikker
De institutionelle mulighedsbetingelser bag den individuelle udforskning i professionelt og frivilligt arbejde

35. Yun Mi Antorini
Brand Community Innovation
An Intrinsic Case Study of the Adult Fans of LEGO Community

36. Joachim Lynggaard Boll
Labor Related Corporate Social Performance in Denmark
Organizational and Institutional Perspectives

2008

1. Frederik Christian Vinten
Essays on Private Equity

2. Jesper Clement
Visual Influence of Packaging Design on In-Store Buying Decisions

3. Marius Brostrøm Kousgaard
Tid til kvalitetsmåling?
– Studier af indrulleringsprocesser i forbindelse med introduktionen af kliniske kvalitetsdatabaser i speciallægepraksissektoren

4. Irene Skovgaard Smith
Management Consulting in Action
Value creation and ambiguity in client-consultant relations

5. Anders Rom
Management accounting and integrated information systems
How to exploit the potential for management accounting of information technology

6. Marina Candi
Aesthetic Design as an Element of Service Innovation in New Technology-based Firms

7. Morten Schnack
Teknologi og tværfaglighed
– en analyse af diskussionen omkring indførelse af EPJ på en hospitalsafdeling

8. Helene Balslev Clausen
Juntos pero no revueltos – un estudio sobre emigrantes norteamericanos en un pueblo mexicano

9. Lise Justesen
Kunsten at skrive revisionsrapporter.
En beretning om forvaltningsrevisionens beretninger

10. Michael E. Hansen
The politics of corporate responsibility: CSR and the governance of child labor and core labor rights in the 1990s

11. Anne Roepstorff
Holdning for handling – en etnologisk undersøgelse af Virksomheders Sociale Ansvar/CSR

12. Claus Bajlum
Essays on Credit Risk and Credit Derivatives

13. Anders Bojesen
The Performative Power of Competence – an Inquiry into Subjectivity and Social Technologies at Work

14. Satu Reijonen
Green and Fragile
A Study on Markets and the Natural Environment

15. Ilduara Busta
Corporate Governance in Banking
A European Study

16. Kristian Anders Hvass
A Boolean Analysis Predicting Industry Change: Innovation, Imitation & Business Models
The Winning Hybrid: A case study of isomorphism in the airline industry

17. Trine Paludan
De uvidende og de udviklingsparate
Identitet som mulighed og restriktion blandt fabriksarbejdere på det aftayloriserede fabriksgulv

18. Kristian Jakobsen
Foreign market entry in transition economies: Entry timing and mode choice

19. Jakob Elming
Syntactic reordering in statistical machine translation

20. Lars Brømsøe Termansen
Regional Computable General Equilibrium Models for Denmark
Three papers laying the foundation for regional CGE models with agglomeration characteristics

21. Mia Reinholt
The Motivational Foundations of Knowledge Sharing

22. Frederikke Krogh-Meibom
The Co-Evolution of Institutions and Technology
– A Neo-Institutional Understanding of Change Processes within the Business Press – the Case Study of Financial Times

23. Peter D. Ørberg Jensen
OFFSHORING OF ADVANCED AND HIGH-VALUE TECHNICAL SERVICES: ANTECEDENTS, PROCESS DYNAMICS AND FIRMLEVEL IMPACTS

24. Pham Thi Song Hanh
Functional Upgrading, Relational Capability and Export Performance of Vietnamese Wood Furniture Producers

25. Mads Vangkilde
Why wait?
An Exploration of first-mover advantages among Danish e-grocers through a resource perspective

26. Hubert Buch-Hansen
Rethinking the History of European Level Merger Control
A Critical Political Economy Perspective

2009

1. Vivian Lindhardsen
From Independent Ratings to Communal Ratings: A Study of CWA Raters' Decision-Making Behaviours

2. Guðrið Weihe
Public-Private Partnerships: Meaning and Practice

3. Chris Nøkkentved
Enabling Supply Networks with Collaborative Information Infrastructures
An Empirical Investigation of Business Model Innovation in Supplier Relationship Management

4. Sara Louise Muhr
Wound, Interrupted – On the Vulnerability of Diversity Management

5. Christine Sestoft
Forbrugeradfærd i et Stats- og Livsformsteoretisk perspektiv

6. Michael Pedersen
Tune in, Breakdown, and Reboot: On the production of the stress-fit self-managing employee

7. Salla Lutz
Position and Reposition in Networks – Exemplified by the Transformation of the Danish Pine Furniture Manufacturers

8. Jens Forssbæck
Essays on market discipline in commercial and central banking

9. Tine Murphy
Sense from Silence – A Basis for Organised Action
How do Sensemaking Processes with Minimal Sharing Relate to the Reproduction of Organised Action?

10. Sara Malou Strandvad
Inspirations for a new sociology of art: A sociomaterial study of development processes in the Danish film industry

11. Nicolaas Mouton
On the evolution of social scientific metaphors:
A cognitive-historical enquiry into the divergent trajectories of the idea that collective entities – states and societies, cities and corporations – are biological organisms.

12. Lars Andreas Knutsen
Mobile Data Services:
Shaping of user engagements

13. Nikolaos Theodoros Korfiatis
Information Exchange and Behavior
A Multi-method Inquiry on Online Communities

14. Jens Albæk
Forestillinger om kvalitet og tværfaglighed på sygehuse
– skabelse af forestillinger i læge- og plejegrupperne angående relevans af nye idéer om kvalitetsudvikling gennem tolkningsprocesser

15. Maja Lotz
The Business of Co-Creation – and the Co-Creation of Business

16. Gitte P. Jakobsen
Narrative Construction of Leader Identity in a Leader Development Program Context

17. Dorte Hermansen
"Living the brand" som en brandorienteret dialogisk praxis:
Om udvikling af medarbejdernes brandorienterede dømmekraft

18. Aseem Kinra
Supply Chain (logistics) Environmental Complexity

19. Michael Nørager
How to manage SMEs through the transformation from non innovative to innovative?

20. Kristin Wallevik
Corporate Governance in Family Firms
The Norwegian Maritime Sector

21. Bo Hansen Hansen
Beyond the Process
Enriching Software Process Improvement with Knowledge Management

22. Annemette Skot-Hansen
Franske adjektivisk afledte adverbier, der tager præpositionssyntagmer indledt med præpositionen à som argumenter
En valensgrammatisk undersøgelse

23. Line Gry Knudsen
Collaborative R&D Capabilities
In Search of Micro-Foundations

24. Christian Scheuer
Employers meet employees
Essays on sorting and globalization

25. Rasmus Johnsen
The Great Health of Melancholy
A Study of the Pathologies of Performativity

26. Ha Thi Van Pham
Internationalization, Competitiveness Enhancement and Export Performance of Emerging Market Firms:
Evidence from Vietnam

27. Henriette Balieu
Kontrolbegrebets betydning for kausativalternationen i spansk
En kognitiv-typologisk analyse

2010

1. Yen Tran
Organizing Innovationin Turbulent Fashion Market
Four papers on how fashion firms create and appropriate innovation value

2. Anders Raastrup Kristensen
Metaphysical Labour
Flexibility, Performance and Commitment in Work-Life Management

3. Margrét Sigrún Sigurdardottir
Dependently independent
Co-existence of institutional logics in the recorded music industry

4. Ásta Dis Óladóttir
Internationalization from a small domestic base:
An empirical analysis of Economics and Management

5. Christine Secher
E-deltagelse i praksis – politikernes og forvaltningens medkonstruktion og konsekvenserne heraf

6. Marianne Stang Våland
What we talk about when we talk about space:
End User Participation between Processes of Organizational and Architectural Design

7. Rex Degnegaard
Strategic Change Management
Change Management Challenges in the Danish Police Reform

8. Ulrik Schultz Brix
Værdi i rekruttering – den sikre beslutning
En pragmatisk analyse af perception og synliggørelse af værdi i rekrutterings- og udvælgelsesarbejdet

9. Jan Ole Similä
Kontraktsledelse
Relasjonen mellom virksomhetsledelse og kontraktshåndtering, belyst via fire norske virksomheter

10. Susanne Boch Waldorff
Emerging Organizations: In between local translation, institutional logics and discourse

11. Brian Kane
Performance Talk
Next Generation Management of Organizational Performance

12. Lars Ohnemus
Brand Thrust: Strategic Branding and Shareholder Value
An Empirical Reconciliation of two Critical Concepts

13. Jesper Schlamovitz
Håndtering af usikkerhed i film- og byggeprojekter

14. Tommy Moesby-Jensen
Det faktiske livs forbindtlighed
Førsokratisk informeret, ny-aristotelisk ἦθος-tænkning hos Martin Heidegger

15. Christian Fich
Two Nations Divided by Common Values
French National Habitus and the Rejection of American Power

16. Peter Beyer
Processer, sammenhængskraft og fleksibilitet
Et empirisk casestudie af omstillingsforløb i fire virksomheder

17. Adam Buchhorn
Markets of Good Intentions
Constructing and Organizing Biogas Markets Amid Fragility and Controversy

18. Cecilie K. Moesby-Jensen
Social læring og fælles praksis
Et mixed method studie, der belyser læringskonsekvenser af et lederkursus for et praksisfællesskab af offentlige mellemledere

19. Heidi Boye
Fødevarer og sundhed i senmodernismen
– En indsigt i hyggefænomenet og de relaterede fødevarepraksisser

20. Kristine Munkgård Pedersen
Flygtige forbindelser og midlertidige mobiliseringer
Om kulturel produktion på Roskilde Festival

21. Oliver Jacob Weber
Causes of Intercompany Harmony in Business Markets – An Empirical Investigation from a Dyad Perspective

22. Susanne Ekman
Authority and Autonomy
Paradoxes of Modern Knowledge Work

23. Anette Frey Larsen
Kvalitetsledelse på danske hospitaler
– Ledelsernes indflydelse på introduktion og vedligeholdelse af kvalitetsstrategier i det danske sundhedsvæsen

24. Toyoko Sato
Performativity and Discourse: Japanese Advertisements on the Aesthetic Education of Desire

25. Kenneth Brinch Jensen
Identifying the Last Planner System
Lean management in the construction industry

26. Javier Busquets
Orchestrating Network Behavior for Innovation

27. Luke Patey
The Power of Resistance: India's National Oil Company and International Activism in Sudan

28. Mette Vedel
Value Creation in Triadic Business Relationships. Interaction, Interconnection and Position

29. Kristian Tørning
Knowledge Management Systems in Practice – A Work Place Study

30. Qingxin Shi
An Empirical Study of Thinking Aloud Usability Testing from a Cultural Perspective

31. Tanja Juul Christiansen
Corporate blogging: Medarbejderes kommunikative handlekraft

32. Malgorzata Ciesielska
Hybrid Organisations.
A study of the Open Source – business setting

33. Jens Dick-Nielsen
Three Essays on Corporate Bond Market Liquidity

34. Sabrina Speiermann
Modstandens Politik
Kampagnestyring i Velfærdsstaten.
En diskussion af trafikkampagners styringspotentiale

35. Julie Uldam
Fickle Commitment. Fostering political engagement in 'the flighty world of online activism'

36. Annegrete Juul Nielsen
Traveling technologies and transformations in health care

37. Athur Mühlen-Schulte
Organising Development Power and Organisational Reform in the United Nations Development Programme

38. Louise Rygaard Jonas
Branding på butiksgulvet Et case-studie af kultur- og identitets-arbejdet i Kvickly

2011

1. Stefan Fraenkel
Key Success Factors for Sales Force Readiness during New Product Launch A Study of Product Launches in the Swedish Pharmaceutical Industry

2. Christian Plesner Rossing
International Transfer Pricing in Theory and Practice

3. Tobias Dam Hede
Samtalekunst og ledelsesdisciplin – en analyse af coachingsdiskursens genealogi og governmentality

4. Kim Pettersson
Essays on Audit Quality, Auditor Choice, and Equity Valuation

5. Henrik Merkelsen
The expert-lay controversy in risk research and management. Effects of institutional distances. Studies of risk definitions, perceptions, management and communication

6. Simon S. Torp
Employee Stock Ownership: Effect on Strategic Management and Performance

7. Mie Harder
Internal Antecedents of Management Innovation

8. Ole Helby Petersen
Public-Private Partnerships: Policy and Regulation – With Comparative and Multi-level Case Studies from Denmark and Ireland

9. Morten Krogh Petersen
'Good' Outcomes. Handling Multiplicity in Government Communication

10. Kristian Tangsgaard Hvelplund
Allocation of cognitive resources in translation - an eye-tracking and key-logging study

11. Moshe Yonatany
The Internationalization Process of Digital Service Providers

12. Anne Vestergaard
Distance and Suffering Humanitarian Discourse in the age of Mediatization

13. Thorsten Mikkelsen
Personligsheds indflydelse på forretningsrelationer

14. Jane Thostrup Jagd
Hvorfor fortsætter fusionsbølgen udover "the tipping point"? – en empirisk analyse af information og kognitioner om fusioner

15. Gregory Gimpel
Value-driven Adoption and Consumption of Technology: Understanding Technology Decision Making

16. Thomas Stengade Sønderskov
Den nye mulighed Social innovation i en forretningsmæssig kontekst

17. Jeppe Christoffersen
Donor supported strategic alliances in developing countries

18. Vibeke Vad Baunsgaard
Dominant Ideological Modes of Rationality: Cross functional

integration in the process of product innovation

19. Throstur Olaf Sigurjonsson
Governance Failure and Icelands's Financial Collapse

20. Allan Sall Tang Andersen
Essays on the modeling of risks in interest-rate and inflation markets

21. Heidi Tscherning
Mobile Devices in Social Contexts

22. Birgitte Gorm Hansen
Adapting in the Knowledge Economy
Lateral Strategies for Scientists and Those Who Study Them

23. Kristina Vaarst Andersen
Optimal Levels of Embeddedness
The Contingent Value of Networked Collaboration

24. Justine Grønbæk Pors
Noisy Management
A History of Danish School Governing from 1970-2010

25. Stefan Linder
Micro-foundations of Strategic Entrepreneurship
Essays on Autonomous Strategic Action

26. Xin Li
Toward an Integrative Framework of National Competitiveness
An application to China

27. Rune Thorbjørn Clausen
Værdifuld arkitektur
Et eksplorativt studie af bygningers rolle i virksomheders værdiskabelse

28. Monica Viken
Markedsundersøkelser som bevis i varemerke- og markedsføringsrett

29. Christian Wymann
Tattooing
The Economic and Artistic Constitution of a Social Phenomenon

30. Sanne Frandsen
Productive Incoherence
A Case Study of Branding and Identity Struggles in a Low-Prestige Organization

31. Mads Stenbo Nielsen
Essays on Correlation Modelling

32. Ivan Häuser
Følelse og sprog
Etablering af en ekspressiv kategori, eksemplificeret på russisk

33. Sebastian Schwenen
Security of Supply in Electricity Markets

2012

1. Peter Holm Andreasen
The Dynamics of Procurement Management
- A Complexity Approach

2. Martin Haulrich
Data-Driven Bitext Dependency Parsing and Alignment

3. Line Kirkegaard
Konsulenten i den anden nat
En undersøgelse af det intense arbejdsliv

4. Tonny Stenheim
Decision usefulness of goodwill under IFRS

5. Morten Lind Larsen
Produktivitet, vækst og velfærd
Industrirådet og efterkrigstidens Danmark 1945 - 1958

6. Petter Berg
Cartel Damages and Cost Asymmetries

7. Lynn Kahle
Experiential Discourse in Marketing
A methodical inquiry into practice and theory

8. Anne Roelsgaard Obling
Management of Emotions in Accelerated Medical Relationships

9. Thomas Frandsen
Managing Modularity of Service Processes Architecture

10. Carina Christine Skovmøller
CSR som noget særligt
Et casestudie om styring og meningsskabelse i relation til CSR ud fra en intern optik

11. Michael Tell
Fradragsbeskæring af selskabers finansieringsudgifter
En skatteretlig analyse af SEL §§ 11, 11B og 11C

12. Morten Holm
Customer Profitability Measurement Models
Their Merits and Sophistication across Contexts

13. Katja Joo Dyppel
Beskatning af derivater
En analyse af dansk skatteret

14. Esben Anton Schultz
Essays in Labor Economics
Evidence from Danish Micro Data

15. Carina Risvig Hansen
"Contracts not covered, or not fully covered, by the Public Sector Directive"

16. Anja Svejgaard Pors
Iværksættelse af kommunikation
- patientfigurer i hospitalets strategiske kommunikation

17. Frans Bévort
Making sense of management with logics
An ethnographic study of accountants who become managers

18. René Kallestrup
The Dynamics of Bank and Sovereign Credit Risk

19. Brett Crawford
Revisiting the Phenomenon of Interests in Organizational Institutionalism
The Case of U.S. Chambers of Commerce

20. Mario Daniele Amore
Essays on Empirical Corporate Finance

21. Arne Stjernholm Madsen
The evolution of innovation strategy
Studied in the context of medical device activities at the pharmaceutical company Novo Nordisk A/S in the period 1980-2008

22. Jacob Holm Hansen
Is Social Integration Necessary for Corporate Branding?
A study of corporate branding strategies at Novo Nordisk

23. Stuart Webber
Corporate Profit Shifting and the Multinational Enterprise

24. Helene Ratner
Promises of Reflexivity
Managing and Researching Inclusive Schools

25. Therese Strand
The Owners and the Power: Insights from Annual General Meetings

26. Robert Gavin Strand
In Praise of Corporate Social Responsibility Bureaucracy

27. Nina Sormunen
Auditor's going-concern reporting
Reporting decision and content of the report

28. John Bang Mathiasen
Learning within a product development working practice:
- an understanding anchored in pragmatism

29. Philip Holst Riis
Understanding Role-Oriented Enterprise Systems: From Vendors to Customers

30. Marie Lisa Dacanay
Social Enterprises and the Poor
Enhancing Social Entrepreneurship and Stakeholder Theory

31. Fumiko Kano Glückstad
Bridging Remote Cultures: Cross-lingual concept mapping based on the information receiver's prior-knowledge

32. Henrik Barslund Fosse
Empirical Essays in International Trade

33. Peter Alexander Albrecht
Foundational hybridity and its reproduction
Security sector reform in Sierra Leone

34. Maja Rosenstock
CSR - hvor svært kan det være?
Kulturanalytisk casestudie om udfordringer og dilemmaer med at forankre Coops CSR-strategi

35. Jeanette Rasmussen
Tweens, medier og forbrug
Et studie af 10-12 årige danske børns brug af internettet, opfattelse og forståelse af markedsføring og forbrug

36. Ib Tunby Gulbrandsen
'This page is not intended for a US Audience'
A five-act spectacle on online communication, collaboration & organization.

37. Kasper Aalling Teilmann
Interactive Approaches to Rural Development

38. Mette Mogensen
The Organization(s) of Well-being and Productivity
(Re)assembling work in the Danish Post

39. Søren Friis Møller
From Disinterestedness to Engagement
Towards Relational Leadership In the Cultural Sector

40. Nico Peter Berhausen
Management Control, Innovation and Strategic Objectives – Interactions and Convergence in Product Development Networks

41. Balder Onarheim
Creativity under Constraints
Creativity as Balancing 'Constrainedness'

42. Haoyong Zhou
Essays on Family Firms

43. Elisabeth Naima Mikkelsen
Making sense of organisational conflict
An empirical study of enacted sense-making in everyday conflict at work

2013

1. Jacob Lyngsie
Entrepreneurship in an Organizational Context

2. Signe Groth-Brodersen
Fra ledelse til selvet
En socialpsykologisk analyse af forholdet imellem selvledelse, ledelse og stress i det moderne arbejdsliv

3. Nis Høyrup Christensen
Shaping Markets: A Neoinstitutional Analysis of the Emerging Organizational Field of Renewable Energy in China

4. Christian Edelvold Berg
As a matter of size
THE IMPORTANCE OF CRITICAL MASS AND THE CONSEQUENCES OF SCARCITY FOR TELEVISION MARKETS

5. Christine D. Isakson
Coworker Influence and Labor Mobility
Essays on Turnover, Entrepreneurship and Location Choice in the Danish Maritime Industry

6. Niels Joseph Jerne Lennon
Accounting Qualities in Practice
Rhizomatic stories of representational faithfulness, decision making and control

7. Shannon O'Donnell
Making Ensemble Possible
How special groups organize for collaborative creativity in conditions of spatial variability and distance

8. Robert W. D. Veitch
Access Decisions in a Partly-Digital World
Comparing Digital Piracy and Legal Modes for Film and Music

9. Marie Mathiesen
Making Strategy Work
An Organizational Ethnography

10. Arisa Shollo
The role of business intelligence in organizational decision-making

11. Mia Kaspersen
The construction of social and environmental reporting

12. Marcus Møller Larsen
The organizational design of offshoring

13. Mette Ohm Rørdam
EU Law on Food Naming
The prohibition against misleading names in an internal market context

14. Hans Peter Rasmussen
GIV EN GED!
Kan giver-idealtyper forklare støtte til velgørenhed og understøtte relationsopbygning?

15. Ruben Schachtenhaufen
Fonetisk reduktion i dansk

16. Peter Koerver Schmidt
Dansk CFC-beskatning
I et internationalt og komparativt perspektiv

17. Morten Froholdt
Strategi i den offentlige sektor
En kortlægning af styringsmæssig kontekst, strategisk tilgang, samt anvendte redskaber og teknologier for udvalgte danske statslige styrelser

18. Annette Camilla Sjørup
Cognitive effort in metaphor translation
An eye-tracking and key-logging study

19. Tamara Stucchi
The Internationalization of Emerging Market Firms:
A Context-Specific Study

20. Thomas Lopdrup-Hjorth
"Let's Go Outside":
The Value of Co-Creation

21. Ana Alačovska
Genre and Autonomy in Cultural Production
The case of travel guidebook production

22. Marius Gudmand-Høyer
Stemningssindssygdommenes historie i det 19. århundrede
Omtydningen af melankolien og manien som bipolære stemningslidelser i dansk sammenhæng under hensyn til dannelsen af det moderne følelseslivs relative autonomi.
En problematiserings- og erfarings-analytisk undersøgelse

23. Lichen Alex Yu
Fabricating an S&OP Process
Circulating References and Matters of Concern

24. Esben Alfort
The Expression of a Need
Understanding search

25. Trine Pallesen
Assembling Markets for Wind Power
An Inquiry into the Making of Market Devices

26. Anders Koed Madsen
Web-Visions
Repurposing digital traces to organize social attention

27. Lærke Højgaard Christiansen
BREWING ORGANIZATIONAL RESPONSES TO INSTITUTIONAL LOGICS

28. Tommy Kjær Lassen
EGENTLIG SELVLEDELSE
En ledelsesfilosofisk afhandling om selvledelsens paradoksale dynamik og eksistentielle engagement

29. Morten Rossing
Local Adaption and Meaning Creation in Performance Appraisal

30. Søren Obed Madsen
Lederen som oversætter
Et oversættelsesteoretisk perspektiv på strategisk arbejde

31. Thomas Høgenhaven
Open Government Communities
Does Design Affect Participation?

32. Kirstine Zinck Pedersen
Failsafe Organizing?
A Pragmatic Stance on Patient Safety

33. Anne Petersen
Hverdagslogikker i psykiatrisk arbejde
En institutionsetnografisk undersøgelse af hverdagen i psykiatriske organisationer

34. Didde Maria Humle
Fortællinger om arbejde

35. Mark Holst-Mikkelsen
Strategieksekvering i praksis
– barrierer og muligheder!

36. Malek Maalouf
Sustaining lean
Strategies for dealing with organizational paradoxes

37. Nicolaj Tofte Brenneche
Systemic Innovation In The Making
The Social Productivity of Cartographic Crisis and Transitions in the Case of SEEIT

38. Morten Gylling
The Structure of Discourse
A Corpus-Based Cross-Linguistic Study

39. Binzhang YANG
Urban Green Spaces for Quality Life
- Case Study: the landscape architecture for people in Copenhagen

40. Michael Friis Pedersen
Finance and Organization:
The Implications for Whole Farm Risk Management

41. Even Fallan
Issues on supply and demand for environmental accounting information

42. Ather Nawaz
Website user experience
A cross-cultural study of the relation between users´ cognitive style, context of use, and information architecture of local websites

43. Karin Beukel
The Determinants for Creating Valuable Inventions

44. Arjan Markus
External Knowledge Sourcing and Firm Innovation
Essays on the Micro-Foundations of Firms' Search for Innovation

2014

1. Solon Moreira
Four Essays on Technology Licensing and Firm Innovation

2. Karin Strzeletz Ivertsen
Partnership Drift in Innovation Processes
A study of the Think City electric car development

3. Kathrine Hoffmann Pii
Responsibility Flows in Patient-centred Prevention

4. Jane Bjørn Vedel
Managing Strategic Research
An empirical analysis of science-industry collaboration in a pharmaceutical company

5. Martin Gylling
Processuel strategi i organisationer
Monografi om dobbeltheden i tænkning af strategi, dels som vidensfelt i organisationsteori, dels som kunstnerisk tilgang til at skabe i erhvervsmæssig innovation

6. Linne Marie Lauesen
Corporate Social Responsibility in the Water Sector: How Material Practices and their Symbolic and Physical Meanings Form a Colonising Logic

7. Maggie Qiuzhu Mei
LEARNING TO INNOVATE: The role of ambidexterity, standard, and decision process

8. Inger Høedt-Rasmussen
Developing Identity for Lawyers Towards Sustainable Lawyering

9. Sebastian Fux
Essays on Return Predictability and Term Structure Modelling

10. Thorbjørn N. M. Lund-Poulsen
Essays on Value Based Management

11. Oana Brindusa Albu
Transparency in Organizing: A Performative Approach

12. Lena Olaison
Entrepreneurship at the limits

13. Hanne Sørum
DRESSED FOR WEB SUCCESS? An Empirical Study of Website Quality in the Public Sector

14. Lasse Folke Henriksen
Knowing networks How experts shape transnational governance

15. Maria Halbinger
Entrepreneurial Individuals Empirical Investigations into Entrepreneurial Activities of Hackers and Makers

16. Robert Spliid
Kapitalfondenes metoder og kompetencer

17. Christiane Stelling
Public-private partnerships & the need, development and management of trusting A processual and embedded exploration

18. Marta Gasparin
Management of design as a translation process

19. Kåre Moberg
Assessing the Impact of Entrepreneurship Education From ABC to PhD

20. Alexander Cole
Distant neighbors Collective learning beyond the cluster

21. Martin Møller Boje Rasmussen
Is Competitiveness a Question of Being Alike? How the United Kingdom, Germany and Denmark Came to Compete through their Knowledge Regimes from 1993 to 2007

22. Anders Ravn Sørensen
Studies in central bank legitimacy, currency and national identity Four cases from Danish monetary history

23. Nina Bellak
Can Language be Managed in International Business? Insights into Language Choice from a Case Study of Danish and Austrian Multinational Corporations (MNCs)

24. Rikke Kristine Nielsen
Global Mindset as Managerial Meta-competence and Organizational Capability: Boundary-crossing Leadership Cooperation in the MNC The Case of 'Group Mindset' in Solar A/S.

25. Rasmus Koss Hartmann
User Innovation inside government Towards a critically performative foundation for inquiry

26. Kristian Gylling Olesen
Flertydig og emergerende ledelse i folkeskolen
Et aktør-netværksteoretisk ledelses-studie af politiske evalueringsreformers betydning for ledelse i den danske folkeskole

27. Troels Riis Larsen
Kampen om Danmarks omdømme 1945-2010
Omdømmearbejde og omdømmepolitik

28. Klaus Majgaard
Jagten på autenticitet i offentlig styring

29. Ming Hua Li
Institutional Transition and Organizational Diversity: Differentiated internationalization strategies of emerging market state-owned enterprises

30. Sofie Blinkenberg Federspiel
IT, organisation og digitalisering: Institutionelt arbejde i den kommunale digitaliseringsproces

31. Elvi Weinreich
Hvilke offentlige ledere er der brug for når velfærdstænkningen flytter sig – er Diplomuddannelsens lederprofil svaret?

32. Ellen Mølgaard Korsager
Self-conception and image of context in the growth of the firm – A Penrosian History of Fiberline Composites

33. Else Skjold
The Daily Selection

34. Marie Louise Conradsen
The Cancer Centre That Never Was
The Organisation of Danish Cancer Research 1949-1992

35. Virgilio Failla
Three Essays on the Dynamics of Entrepreneurs in the Labor Market

36. Nicky Nedergaard
Brand-Based Innovation
Relational Perspectives on Brand Logics and Design Innovation Strategies and Implementation

37. Mads Gjedsted Nielsen
Essays in Real Estate Finance

38. Kristin Martina Brandl
Process Perspectives on Service Offshoring

39. Mia Rosa Koss Hartmann
In the gray zone
With police in making space for creativity

40. Karen Ingerslev
Healthcare Innovation under The Microscope
Framing Boundaries of Wicked Problems

41. Tim Neerup Themsen
Risk Management in large Danish public capital investment programmes

2015

1. Jakob Ion Wille
Film som design
Design af levende billeder i film og tv-serier

2. Christiane Mossin
Interzones of Law and Metaphysics
Hierarchies, Logics and Foundations of Social Order seen through the Prism of EU Social Rights

3. Thomas Tøth
TRUSTWORTHINESS: ENABLING GLOBAL COLLABORATION
An Ethnographic Study of Trust, Distance, Control, Culture and Boundary Spanning within Offshore Outsourcing of IT Services

4. Steven Højlund
Evaluation Use in Evaluation Systems – The Case of the European Commission

5. Julia Kirch Kirkegaard
AMBIGUOUS WINDS OF CHANGE – OR FIGHTING AGAINST WINDMILLS IN CHINESE WIND POWER
A CONSTRUCTIVIST INQUIRY INTO CHINA'S PRAGMATICS OF GREEN MARKETISATION MAPPING CONTROVERSIES OVER A POTENTIAL TURN TO QUALITY IN CHINESE WIND POWER

6. Michelle Carol Antero
A Multi-case Analysis of the Development of Enterprise Resource Planning Systems (ERP) Business Practices

Morten Friis-Olivarius
The Associative Nature of Creativity

7. Mathew Abraham
New Cooperativism:
A study of emerging producer organisations in India

8. Stine Hedegaard
Sustainability-Focused Identity: Identity work performed to manage, negotiate and resolve barriers and tensions that arise in the process of constructing or ganizational identity in a sustainability context

9. Cecilie Glerup
Organizing Science in Society – the conduct and justification of resposible research

10. Allan Salling Pedersen
Implementering af ITIL® IT-governance - når best practice konflikter med kulturen Løsning af implementerings-problemer gennem anvendelse af kendte CSF i et aktionsforskningsforløb.

11. Nihat Misir
A Real Options Approach to Determining Power Prices

12. Mamdouh Medhat
MEASURING AND PRICING THE RISK OF CORPORATE FAILURES

13. Rina Hansen
Toward a Digital Strategy for Omnichannel Retailing

14. Eva Pallesen
In the rhythm of welfare creation
A relational processual investigation moving beyond the conceptual horizon of welfare management

15. Gouya Harirchi
In Search of Opportunities: Three Essays on Global Linkages for Innovation

16. Lotte Holck
Embedded Diversity: A critical ethnographic study of the structural tensions of organizing diversity

17. Jose Daniel Balarezo
Learning through Scenario Planning

18. Louise Pram Nielsen
Knowledge dissemination based on terminological ontologies. Using eye tracking to further user interface design.

19. Sofie Dam
PUBLIC-PRIVATE PARTNERSHIPS FOR INNOVATION AND SUSTAINABILITY TRANSFORMATION
An embedded, comparative case study of municipal waste management in England and Denmark

20. Ulrik Hartmyer Christiansen
Follwoing the Content of Reported Risk Across the Organization

21. Guro Refsum Sanden
Language strategies in multinational corporations. A cross-sector study of financial service companies and manufacturing companies.

22. Linn Gevoll
Designing performance management for operational level
- A closer look on the role of design choices in framing coordination and motivation

23. Frederik Larsen
Objects and Social Actions
– on Second-hand Valuation Practices

24. Thorhildur Hansdottir Jetzek
The Sustainable Value of Open Government Data
Uncovering the Generative Mechanisms of Open Data through a Mixed Methods Approach

25. Gustav Toppenberg
Innovation-based M&A
– Technological-Integration Challenges – The Case of Digital-Technology Companies

26. Mie Plotnikof
Challenges of Collaborative Governance
An Organizational Discourse Study of Public Managers' Struggles with Collaboration across the Daycare Area

27. Christian Garmann Johnsen
Who Are the Post-Bureaucrats?
A Philosophical Examination of the Creative Manager, the Authentic Leader and the Entrepreneur

28. Jacob Brogaard-Kay
Constituting Performance Management
A field study of a pharmaceutical company

29. Rasmus Ploug Jenle
Engineering Markets for Control: Integrating Wind Power into the Danish Electricity System

30. Morten Lindholst
Complex Business Negotiation: Understanding Preparation and Planning

31. Morten Grynings
TRUST AND TRANSPARENCY FROM AN ALIGNMENT PERSPECTIVE

32. Peter Andreas Norn
Byregimer og styringsevne: Politisk lederskab af store byudviklingsprojekter

33. Milan Miric
Essays on Competition, Innovation and Firm Strategy in Digital Markets

34. Sanne K. Hjordrup
The Value of Talent Management
Rethinking practice, problems and possibilities

35. Johanna Sax
Strategic Risk Management
– Analyzing Antecedents and Contingencies for Value Creation

36. Pernille Rydén
Strategic Cognition of Social Media

37. Mimmi Sjöklint
The Measurable Me
- The Influence of Self-tracking on the User Experience

38. Juan Ignacio Staricco
Towards a Fair Global Economic Regime? A critical assessment of Fair Trade through the examination of the Argentinean wine industry

39. Marie Henriette Madsen
Emerging and temporary connections in Quality work

40. Yangfeng CAO
Toward a Process Framework of Business Model Innovation in the Global Context
Entrepreneurship-Enabled Dynamic Capability of Medium-Sized Multinational Enterprises

41. Carsten Scheibye
Enactment of the Organizational Cost Structure in Value Chain Configuration
A Contribution to Strategic Cost Management

2016

1. Signe Sofie Dyrby
Enterprise Social Media at Work

2. Dorte Boesby Dahl
The making of the public parking attendant
Dirt, aesthetics and inclusion in public service work

3. Verena Girschik
Realizing Corporate Responsibility Positioning and Framing in Nascent Institutional Change

4. Anders Ørding Olsen
IN SEARCH OF SOLUTIONS
Inertia, Knowledge Sources and Diversity in Collaborative Problem-solving

5. Pernille Steen Pedersen
Udkast til et nyt copingbegreb
En kvalifikation af ledelsesmuligheder for at forebygge sygefravær ved psykiske problemer.

6. Kerli Kant Hvass
Weaving a Path from Waste to Value: Exploring fashion industry business models and the circular economy

7. Kasper Lindskow
Exploring Digital News Publishing Business Models – a production network approach

8. Mikkel Mouritz Marfelt
The chameleon workforce:
Assembling and negotiating the content of a workforce

9. Marianne Bertelsen
Aesthetic encounters
Rethinking autonomy, space & time in today's world of art

10. Louise Hauberg Wilhelmsen
EU PERSPECTIVES ON INTERNATIONAL COMMERCIAL ARBITRATION

11. Abid Hussain
On the Design, Development and Use of the Social Data Analytics Tool (SODATO): Design Propositions, Patterns, and Principles for Big Social Data Analytics

12. Mark Bruun
Essays on Earnings Predictability

13. Tor Bøe-Lillegraven
BUSINESS PARADOXES, BLACK BOXES, AND BIG DATA: BEYOND ORGANIZATIONAL AMBIDEXTERITY

14. Hadis Khonsary-Atighi
ECONOMIC DETERMINANTS OF DOMESTIC INVESTMENT IN AN OIL-BASED ECONOMY: THE CASE OF IRAN (1965-2010)

15. Maj Lervad Grasten
Rule of Law or Rule by Lawyers?
On the Politics of Translation in Global Governance

16. Lene Granzau Juel-Jacobsen
SUPERMARKEDETS MODUS OPERANDI – en hverdagssociologisk undersøgelse af forholdet mellem rum og handlen og understøtte relationsopbygning?

17. Christine Thalsgård Henriques
In search of entrepreneurial learning – Towards a relational perspective on incubating practices?

18. Patrick Bennett
Essays in Education, Crime, and Job Displacement

19. Søren Korsgaard
Payments and Central Bank Policy

20. Marie Kruse Skibsted
Empirical Essays in Economics of Education and Labor

21. Elizabeth Benedict Christensen
The Constantly Contingent Sense of Belonging of the 1.5 Generation Undocumented Youth
An Everyday Perspective

22. Lasse J. Jessen
Essays on Discounting Behavior and Gambling Behavior

23. Kalle Johannes Rose
Når stifterviljen dør…
Et retsøkonomisk bidrag til 200 års juridisk konflikt om ejendomsretten

24. Andreas Søeborg Kirkedal
Danish Stød and Automatic Speech Recognition

25. Ida Lunde Jørgensen
Institutions and Legitimations in Finance for the Arts

26. Olga Rykov Ibsen
An empirical cross-linguistic study of directives: A semiotic approach to the sentence forms chosen by British, Danish and Russian speakers in native and ELF contexts

27. Desi Volker
Understanding Interest Rate Volatility

28. Angeli Elizabeth Weller
Practice at the Boundaries of Business Ethics & Corporate Social Responsibility

29. Ida Danneskiold-Samsøe
Levende læring i kunstneriske organisationer
En undersøgelse af læringsprocesser mellem projekt og organisation på Aarhus Teater

30. Leif Christensen
Quality of information – The role of internal controls and materiality

31. Olga Zarzecka
Tie Content in Professional Networks

32. Henrik Mahncke
De store gaver
- Filantropiens gensidighedsrelationer i teori og praksis

33. Carsten Lund Pedersen
Using the Collective Wisdom of Frontline Employees in Strategic Issue Management

34. Yun Liu
Essays on Market Design

35. Denitsa Hazarbassanova Blagoeva
The Internationalisation of Service Firms

36. Manya Jaura Lind
Capability development in an off-shoring context: How, why and by whom

37. Luis R. Boscán F.
Essays on the Design of Contracts and Markets for Power System Flexibility

38. Andreas Philipp Distel
Capabilities for Strategic Adaptation: Micro-Foundations, Organizational Conditions, and Performance Implications

39. Lavinia Bleoca
The Usefulness of Innovation and Intellectual Capital in Business Performance: The Financial Effects of Knowledge Management vs. Disclosure

40. Henrik Jensen
Economic Organization and Imperfect Managerial Knowledge: A Study of the Role of Managerial Meta-Knowledge in the Management of Distributed Knowledge

41. Stine Mosekjær
The Understanding of English Emotion Words by Chinese and Japanese Speakers of English as a Lingua Franca An Empirical Study

42. Hallur Tor Sigurdarson
The Ministry of Desire - Anxiety and entrepreneurship in a bureaucracy

43. Kätlin Pulk
Making Time While Being in Time
A study of the temporality of organizational processes

44. Valeria Giacomin
Contextualizing the cluster Palm oil in Southeast Asia in global perspective (1880s–1970s)

45. Jeanette Willert
Managers' use of multiple Management Control Systems: The role and interplay of management control systems and company performance

46. Mads Vestergaard Jensen
Financial Frictions: Implications for Early Option Exercise and Realized Volatility

47. Mikael Reimer Jensen
Interbank Markets and Frictions

48. Benjamin Faigen
Essays on Employee Ownership

49. Adela Michea
Enacting Business Models An Ethnographic Study of an Emerging Business Model Innovation within the Frame of a Manufacturing Company.

50. Iben Sandal Stjerne
Transcending organization in temporary systems Aesthetics' organizing work and employment in Creative Industries

51. Simon Krogh
Anticipating Organizational Change

52. Sarah Netter
Exploring the Sharing Economy

53. Lene Tolstrup Christensen
State-owned enterprises as institutional market actors in the marketization of public service provision: A comparative case study of Danish and Swedish passenger rail 1990–2015

54. Kyoung(Kay) Sun Park
Three Essays on Financial Economics

2017

1. Mari Bjerck
Apparel at work. Work uniforms and women in male-dominated manual occupations.

2. Christoph H. Flöthmann
Who Manages Our Supply Chains? Backgrounds, Competencies and Contributions of Human Resources in Supply Chain Management

3. Aleksandra Anna Rzeźnik
Essays in Empirical Asset Pricing

4. Claes Bäckman
Essays on Housing Markets

5. Kirsti Reitan Andersen
Stabilizing Sustainability in the Textile and Fashion Industry

6. Kira Hoffmann
Cost Behavior: An Empirical Analysis of Determinants and Consequences of Asymmetries

7. Tobin Hanspal
Essays in Household Finance

8. Nina Lange
Correlation in Energy Markets

9. Anjum Fayyaz
Donor Interventions and SME Networking in Industrial Clusters in Punjab Province, Pakistan

10. Magnus Paulsen Hansen
Trying the unemployed. Justification and critique, emancipation and coercion towards the 'active society'. A study of contemporary reforms in France and Denmark

11. Sameer Azizi
Corporate Social Responsibility in Afghanistan – a critical case study of the mobile telecommunications industry

12. Malene Myhre
The internationalization of small and medium-sized enterprises: A qualitative study

13. Thomas Presskorn-Thygesen
The Significance of Normativity – Studies in Post-Kantian Philosophy and Social Theory

14. Federico Clementi
Essays on multinational production and international trade

15. Lara Anne Hale
Experimental Standards in Sustainability Transitions: Insights from the Building Sector

16. Richard Pucci
Accounting for Financial Instruments in an Uncertain World Controversies in IFRS in the Aftermath of the 2008 Financial Crisis

17. Sarah Maria Denta
Kommunale offentlige private partnerskaber Regulering I skyggen af Farumsagen

18. Christian Östlund
Design for e-training

19. Amalie Martinus Hauge
Organizing Valuations – a pragmatic inquiry

20. Tim Holst Celik
Tension-filled Governance? Exploring the Emergence, Consolidation and Reconfiguration of Legitimatory and Fiscal State-crafting

21. Christian Bason
Leading Public Design: How managers engage with design to transform public governance

22. Davide Tomio
Essays on Arbitrage and Market Liquidity

23. Simone Stæhr
Financial Analysts' Forecasts Behavioral Aspects and the Impact of Personal Characteristics

24. Mikkel Godt Gregersen
Management Control, Intrinsic Motivation and Creativity – How Can They Coexist

25. Kristjan Johannes Suse Jespersen
Advancing the Payments for Ecosystem Service Discourse Through Institutional Theory

26. Kristian Bondo Hansen
Crowds and Speculation: A study of crowd phenomena in the U.S. financial markets 1890 to 1940

27. Lars Balslev
Actors and practices – An institutional study on management accounting change in Air Greenland

28. Sven Klingler
Essays on Asset Pricing with Financial Frictions

29. Klement Ahrensbach Rasmussen
Business Model Innovation The Role of Organizational Design

30. Giulio Zichella
Entrepreneurial Cognition. Three essays on entrepreneurial behavior and cognition under risk and uncertainty

31. Richard Ledborg Hansen
En forkærlighed til det eksisterende – mellemlederens oplevelse af forandringsmodstand i organisatoriske forandringer

32. Vilhelm Stefan Holsting
Militært chefvirke: Kritik og retfærdiggørelse mellem politik og profession

33. Thomas Jensen
Shipping Information Pipeline: An information infrastructure to improve international containerized shipping

34. Dzmitry Bartalevich
Do economic theories inform policy? Analysis of the influence of the Chicago School on European Union competition policy

35. Kristian Roed Nielsen
Crowdfunding for Sustainability: A study on the potential of reward-based crowdfunding in supporting sustainable entrepreneurship

36. Emil Husted
There is always an alternative: A study of control and commitment in political organization

37. Anders Ludvig Sevelsted
Interpreting Bonds and Boundaries of Obligation. A genealogy of the emergence and development of Protestant voluntary social work in Denmark as shown through the cases of the Copenhagen Home Mission and the Blue Cross (1850 – 1950)

38. Niklas Kohl
Essays on Stock Issuance

39. Maya Christiane Flensborg Jensen
BOUNDARIES OF PROFESSIONALIZATION AT WORK An ethnography-inspired study of care workers' dilemmas at the margin

40. Andreas Kamstrup
Crowdsourcing and the Architectural Competition as Organisational Technologies

41. Louise Lyngfeldt Gorm Hansen
Triggering Earthquakes in Science, Politics and Chinese Hydropower - A Controversy Study

2018

1. Vishv Priya Kohli
Combatting Falsifi cation and Counterfeiting of Medicinal Products in the E uropean Union – A Legal Analysis

2. Helle Haurum
Customer Engagement Behavior in the context of Continuous Service Relationships

3. Nis Grünberg
The Party -state order: Essays on China's political organization and political economic institutions

4. Jesper Christensen
A Behavioral Theory of Human Capital Integration

5. Poula Marie Helth
Learning in practice

6. Rasmus Vendler Toft-Kehler
Entrepreneurship as a career? An investigation of the relationship between entrepreneurial experience and entrepreneurial outcome

7. Szymon Furtak
Sensing the Future: Designing sensor-based predictive information systems for forecasting spare part demand for diesel engines

8. Mette Brehm Johansen
Organizing patient involvement. An ethnographic study

9. Iwona Sulinska
Complexities of Social Capital in Boards of Directors

10. Cecilie Fanøe Petersen
Award of public contracts as a means to conferring State aid: A legal analysis of the interface between public procurement law and State aid law

11. Ahmad Ahmad Barirani
Three Experimental Studies on Entrepreneurship

12. Carsten Allerslev Olsen
Financial Reporting Enforcement: Impact and Consequences

13. Irene Christensen
New product fumbles – Organizing for the Ramp-up process

14. Jacob Taarup-Esbensen
Managing communities – Mining MNEs' community risk management practices

15. Lester Allan Lasrado
Set-Theoretic approach to maturity models

16. Mia B. Münster
Intention vs. Perception of Designed Atmospheres in Fashion Stores

17. Anne Sluhan
Non-Financial Dimensions of Family Firm Ownership: How Socioemotional Wealth and Familiness Influence Internationalization

18. Henrik Yde Andersen
Essays on Debt and Pensions

19. Fabian Heinrich Müller
Valuation Reversed – When Valuators are Valuated. An Analysis of the Perception of and Reaction to Reviewers in Fine-Dining

20. Martin Jarmatz
Organizing for Pricing

21. Niels Joachim Christfort Gormsen
Essays on Empirical Asset Pricing

22. Diego Zunino
Socio-Cognitive Perspectives in Business Venturing

23. Benjamin Asmussen
Networks and Faces between Copenhagen and Canton, 1730-1840

24. Dalia Bagdziunaite
Brains at Brand Touchpoints A Consumer Neuroscience Study of Information Processing of Brand Advertisements and the Store Environment in Compulsive Buying

25. Erol Kazan
Towards a Disruptive Digital Platform Model

26. Andreas Bang Nielsen
Essays on Foreign Exchange and Credit Risk

27. Anne Krebs
Accountable, Operable Knowledge Toward Value Representations of Individual Knowledge in Accounting

28. Matilde Fogh Kirkegaard
A firm- and demand-side perspective on behavioral strategy for value creation: Insights from the hearing aid industry

29. Agnieszka Nowinska
SHIPS AND RELATION-SHIPS Tie formation in the sector of shipping intermediaries in shipping

30. Stine Evald Bentsen
The Comprehension of English Texts by Native Speakers of English and Japanese, Chinese and Russian Speakers of English as a Lingua Franca. An Empirical Study.

31. Stine Louise Daetz
Essays on Financial Frictions in Lending Markets

32. *Christian Skov Jensen*
Essays on Asset Pricing

33. Anders Kryger
Aligning future employee action and corporate strategy in a resource-scarce environment

34. Maitane Elorriaga-Rubio
The behavioral foundations of strategic decision-making: A contextual perspective

35. Roddy Walker
Leadership Development as Organisational Rehabilitation: Shaping Middle-Managers as Double Agents

36. Jinsun Bae
Producing Garments for Global Markets Corporate social responsibility (CSR) in Myanmar's export garment industry 2011–2015

37. Queralt Prat-i-Pubill
Axiological knowledge in a knowledge driven world. Considerations for organizations.

38. Pia Mølgaard
Essays on Corporate Loans and Credit Risk

39. Marzia Aricò
Service Design as a Transformative Force: Introduction and Adoption in an Organizational Context

40. Christian Dyrlund Wåhlin-Jacobsen
Constructing change initiatives in workplace voice activities Studies from a social interaction perspective

41. Peter Kalum Schou
Institutional Logics in Entrepreneurial Ventures: How Competing Logics arise and shape organizational processes and outcomes during scale-up

42. Per Henriksen
Enterprise Risk Management Rationaler og paradokser i en moderne ledelsesteknologi

43. Maximilian Schellmann
The Politics of Organizing Refugee Camps

44. Jacob Halvas Bjerre
Excluding the Jews: The Aryanization of Danish-German Trade and German Anti-Jewish Policy in Denmark 1937-1943

45. Ida Schrøder
Hybridising accounting and caring: A symmetrical study of how costs and needs are connected in Danish child protection work

46. Katrine Kunst
Electronic Word of Behavior: Transforming digital traces of consumer behaviors into communicative content in product design

47. Viktor Avlonitis
Essays on the role of modularity in management: Towards a unified perspective of modular and integral design

48. Anne Sofie Fischer
Negotiating Spaces of Everyday Politics: -An ethnographic study of organizing for social transformation for women in urban poverty, Delhi, India

2019

1. Shihan Du
ESSAYS IN EMPIRICAL STUDIES BASED ON ADMINISTRATIVE LABOUR MARKET DATA

2. Mart Laatsit
Policy learning in innovation policy: A comparative analysis of European Union member states

3. Peter J. Wynne
Proactively Building Capabilities for the Post-Acquisition Integration of Information Systems

4. Kalina S. Staykova
Generative Mechanisms for Digital Platform Ecosystem Evolution

5. Ieva Linkeviciute
Essays on the Demand-Side Management in Electricity Markets

6. Jonatan Echebarria Fernández
Jurisdiction and Arbitration Agreements in Contracts for the Carriage of Goods by Sea – Limitations on Party Autonomy

7. Louise Thorn Bøttkjær
Votes for sale. Essays on clientelism in new democracies.

8. Ditte Vilstrup Holm
The Poetics of Participation: the organizing of participation in contemporary art

9. Philip Rosenbaum
Essays in Labor Markets – Gender, Fertility and Education

10. Mia Olsen
Mobile Betalinger - Succesfaktorer og Adfærdsmæssige Konsekvenser

11. Adrián Luis Mérida Gutiérrez
Entrepreneurial Careers: Determinants, Trajectories, and Outcomes

12. Frederik Regli
Essays on Crude Oil Tanker Markets

13. Cancan Wang
Becoming Adaptive through Social Media: Transforming Governance and Organizational Form in Collaborative E-government

14. Lena Lindbjerg Sperling
Economic and Cultural Development: Empirical Studies of Micro-level Data

15. Xia Zhang
Obligation, face and facework: An empirical study of the communicative act of cancellation of an obligation by Chinese, Danish and British business professionals in both L1 and ELF contexts

16. Stefan Kirkegaard Sløk-Madsen
Entrepreneurial Judgment and Commercialization

17. Erin Leitheiser
The Comparative Dynamics of Private Governance
The case of the Bangladesh Ready-Made Garment Industry

18. Lone Christensen
STRATEGIIMPLEMENTERING: STYRINGSBESTRÆBELSER, IDENTITET OG AFFEKT

19. Thomas Kjær Poulsen
Essays on Asset Pricing with Financial Frictions

20. Maria Lundberg
Trust and self-trust in leadership identity constructions: A qualitative exploration of narrative ecology in the discursive aftermath of heroic discourse

21. Tina Joanes
Sufficiency for sustainability
Determinants and strategies for reducing clothing consumption

22. Benjamin Johannes Flesch
Social Set Visualizer (SoSeVi): Design, Development and Evaluation of a Visual Analytics Tool for Computational Set Analysis of Big Social Data

23. Henriette Sophia Groskopff Tvede Schleimann
Creating innovation through collaboration – Partnering in the maritime sector

24. Kristian Steensen Nielsen
The Role of Self-Regulation in Environmental Behavior Change

25. Lydia L. Jørgensen
Moving Organizational Atmospheres

26. Theodor Lucian Vladasel
Embracing Heterogeneity: Essays in Entrepreneurship and Human Capital

27. Seidi Suurmets
Contextual Effects in Consumer Research: An Investigation of Consumer Information Processing and Behavior via the Applicati on of Eye-tracking Methodology

28. Marie Sundby Palle Nickelsen
Reformer mellem integritet og innovation: Reform af reformens form i den danske centraladministration fra 1920 til 2019

29. Vibeke Kristine Scheller
The temporal organizing of same-day discharge: A tempography of a Cardiac Day Unit

30. Qian Sun
Adopting Artificial Intelligence in Healthcare in the Digital Age: Perceived Challenges, Frame Incongruence, and Social Power

31. Dorthe Thorning Mejlhede
Artful change agency and organizing for innovation – the case of a Nordic fintech cooperative

32. Benjamin Christoffersen
Corporate Default Models:
Empirical Evidence and Methodical Contributions

33. Filipe Antonio Bonito Vieira
Essays on Pensions and Fiscal Sustainability

34. Morten Nicklas Bigler Jensen
Earnings Management in Private Firms:
An Empirical Analysis of Determinants and Consequences of Earnings Management in Private Firms

2020

1. Christian Hendriksen
Inside the Blue Box: Explaining industry influence in the International Maritime Organization

2. Vasileios Kosmas
Environmental and social issues in global supply chains:
Emission reduction in the maritime transport industry and maritime search and rescue operational response to migration

3. Thorben Peter Simonsen
The spatial organization of psychiatric practice: A situated inquiry into 'healing architecture'

4. Signe Bruskin
The infinite storm: An ethnographic study of organizational change in a bank

5. Rasmus Corlin Christensen
Politics and Professionals: Transnational Struggles to Change International Taxation

6. Robert Lorenz Törmer
The Architectural Enablement of a Digital Platform Strategy

7. Anna Kirkebæk Johansson Gosovic
Ethics as Practice: An ethnographic study of business ethics in a multinational biopharmaceutical company

8. Frank Meier
Making up leaders in leadership development

9. Kai Basner
Servitization at work: On proliferation and containment

10. Anestis Keremis
Anti-corruption in action: How is anti-corruption practiced in multinational companies?

11. Marie Larsen Ryberg
Governing Interdisciolinarity: Stakes and translations of interdisciplinarity in Danish high school education.

12. Jannick Friis Christensen
Queering organisation(s): Norm-critical orientations to organising and researching diversity

13. Thorsteinn Sigurdur Sveinsson
Essays on Macroeconomic Implications of Demographic Change

14. Catherine Casler
Reconstruction in strategy and organization: For a pragmatic stance

15. Luisa Murphy
Revisiting the standard organization of multi-stakeholder initiatives (MSIs): The case of a meta-MSI in Southeast Asia

16. Friedrich Bergmann
Essays on International Trade

17. Nicholas Haagensen
European Legal Networks in Crisis: The Legal Construction of Economic Policy

18. Charlotte Biil
Samskabelse med en sommerfugle-model: Hybrid ret i forbindelse med et partnerskabsprojekt mellem 100 selvejende daginstitutioner, deres paraplyorganisation, tre kommuner og CBS

19. Andreas Dimmelmeier
The Role of Economic Ideas in Sustainable Finance: From Paradigms to Policy

20. Maibrith Kempka Jensen
Ledelse og autoritet i interaktion - En interaktionsbaseret undersøgelse af autoritet i ledelse i praksis

21. Thomas Burø
LAND OF LIGHT: Assembling the Ecology of Culture in Odsherred 2000-2018

22. Prins Marcus Valiant Lantz
Timely Emotion: The Rhetorical Framing of Strategic Decision Making

23. Thorbjørn Vittenhof Fejerskov
Fra værdi til invitationer - offentlig værdiskabelse gennem affekt, potentialitet og begivenhed

24. Lea Acre Foverskov
Demographic Change and Employment: Path dependencies and institutional logics in the European Commission

25. Anirudh Agrawal
A Doctoral Dissertation

26. Julie Marx
Households in the housing market

27. Hadar Gafni
Alternative Digital Methods of Providing Entrepreneurial Finance

28. Mathilde Hjerrild Carlsen
Ledelse af engagementer: En undersøgelse af samarbejde mellem folkeskoler og virksomheder i Danmark

29. Suen Wang
Essays on the Gendered Origins and Implications of Social Policies in the Developing World

30. Stine Hald Larsen
The Story of the Relative: A Systems-Theoretical Analysis of the Role of the Relative in Danish Eldercare Policy from 1930 to 2020

31. Christian Casper Hofma
Immersive technologies and organizational routines: When head-mounted displays meet organizational routines

32. Jonathan Feddersen
The temporal emergence of social relations: An event-based perspective of organising

33. Nageswaran Vaidyanathan
ENRICHING RETAIL CUSTOMER EXPERIENCE USING AUGMENTED REALITY

2021

1. Vanya Rusinova
The Determinants of Firms' Engagement in Corporate Social Responsibility: Evidence from Natural Experiments

2. Lívia Lopes Barakat
Knowledge management mechanisms at MNCs:
The enhancing effect of absorptive capacity and its effects on performance and innovation

3. Søren Bundgaard Brøgger
Essays on Modern Derivatives Markets

4. Martin Friis Nielsen
Consuming Memory: Towards a conceptualization of social media platforms as organizational technologies of consumption

05. Fei Liu
Emergent Technology Use in Consumer Decision Journeys: A Process-as-Propensity Approach

06. Jakob Rømer Barfod
Ledelse i militære højrisikoteams

07. Elham Shafiei Gol
Creative Crowdwork Arrangements

08. Árni Jóhan Petersen
Collective Imaginary as (Residual) Fantasy: A Case Study of the Faroese Oil Bonanza

09. Søren Bering
"Manufacturing, Forward Integration and Governance Strategy"

10. Lars Oehler
Technological Change and the Decomposition of Innovation:
Choices and Consequences for Latecomer Firm Upgrading: The Case of China's Wind Energy Sector

11. Lise Dahl Arvedsen
Leadership in interaction in a virtual context:
A study of the role of leadership processes in a complex context, and how such processes are accomplished in practice

12. Jacob Emil Jeppesen
Essays on Knowledge networks, scientific impact and new knowledge adoption

13. Kasper Ingeman Beck
Essays on Chinese State-Owned Enterprises:
Reform, Corporate Governance and Subnational Diversity

14. Sönnich Dahl Sönnichsen
Exploring the interface between public demand and private supply for implementation of circular economy principles

15. Benjamin Knox
Essays on Financial Markets and Monetary Policy

16. Anita Eskesen
Essays on Utility Regulation: Evaluating Negotiation-Based Approaches inthe Context of Danish Utility Regulation

17. Agnes Guenther
Essays on Firm Strategy and Human Capital

18. Sophie Marie Cappelen
Walking on Eggshells: The balancing act of temporal work in a setting of culinary change

19. Manar Saleh Alnamlah
About Gender Gaps in Entrepreneurial Finance

20. Kirsten Tangaa Nielsen
Essays on the Value of CEOs and Directors

21. Renée Ridgway
Re:search - the Personalised Subject vs. the Anonymous User

22. Codrina Ana Maria Lauth
IMPACT Industrial Hackathons: Findings from a longitudinal case study on short-term vs long-term IMPACT implementations from industrial hackathons within Grundfos

23. Wolf-Hendrik Uhlbach
Scientist Mobility: Essays on knowledge production and innovation

24. Tomaz Sedej
Blockchain technology and inter-organizational relationships

25. Lasse Bundgaard
Public Private Innovation Partnerships: Creating Public Value & Scaling Up Sustainable City Solutions

26. Dimitra Makri Andersen
Walking through Temporal Walls: Rethinking NGO Organizing for Sustainability through a Temporal Lens on NGO-Business Partnerships

27. Louise Fjord Kjærsgaard
Allocation of the Right to Tax Income from Digital Products and Services: A legal analysis of international tax treaty law

28. Sara Dahlman
Marginal alternativity: Organizing for sustainable investing

29. Henrik Gundelach
Performance determinants: An Investigation of the Relationship between Resources, Experience and Performance in Challenging Business Environments

30. Tom Wraight
Confronting the Developmental State: American Trade Policy in the Neoliberal Era

31. Mathias Fjællegaard Jensen
Essays on Gender and Skills in the Labour Market

32. Daniel Lundgaard
Using Social Media to Discuss Global Challenges: Case Studies of the Climate Change Debate on Twitter

33. Jonas Sveistrup Søgaard
Designs for Accounting Information Systems using Distributed Ledger Technology

34. Sarosh Asad
CEO narcissism and board composition: Implications for firm strategy and performance

35. Johann Ole Willers
Experts and Markets in Cybersecurity On Definitional Power and the Organization of Cyber Risks

36. Alexander Kronies
Opportunities and Risks in Alternative Investments

37. Niels Fuglsang
The Politics of Economic Models: An inquiry into the possibilities and limits concerning the rise of macroeconomic forecasting models and what this means for policymaking

38. David Howoldt
Policy Instruments and Policy Mixes for Innovation: Analysing Their Relation to Grand Challenges, Entrepreneurship and Innovation Capability with Natural Language Processing and Latent Variable Methods

2022

01. Ditte Thøgersen
Managing Public Innovation on the Frontline

02. Rasmus Jørgensen
Essays on Empirical Asset Pricing and Private Equity

TITLER I ATV PH.D.-SERIEN

1992

1. Niels Kornum
Servicesamkørsel – organisation, økonomi og planlægningsmetode

1995

2. Verner Worm
Nordiske virksomheder i Kina
Kulturspecifikke interaktionsrelationer ved nordiske virksomhedsetableringer i Kina

1999

3. Mogens Bjerre
Key Account Management of Complex Strategic Relationships
An Empirical Study of the Fast Moving Consumer Goods Industry

2000

4. Lotte Darsø
Innovation in the Making
Interaction Research with heterogeneous Groups of Knowledge Workers creating new Knowledge and new Leads

2001

5. Peter Hobolt Jensen
Managing Strategic Design Identities
The case of the Lego Developer Network

2002

6. Peter Lohmann
The Deleuzian Other of Organizational Change – Moving Perspectives of the Human

7. Anne Marie Jess Hansen
To lead from a distance: The dynamic interplay between strategy and strategizing – A case study of the strategic management process

2003

8. Lotte Henriksen
Videndeling
– om organisatoriske og ledelsesmæssige udfordringer ved videndeling i praksis

9. Niels Christian Nickelsen
Arrangements of Knowing: Coordinating Procedures Tools and Bodies in Industrial Production – a case study of the collective making of new products

2005

10. Carsten Ørts Hansen
Konstruktion af ledelsesteknologier og effektivitet

TITLER I DBA PH.D.-SERIEN

2007

1. Peter Kastrup-Misir
Endeavoring to Understand Market Orientation – and the concomitant co-mutation of the researched, the re searcher, the research itself and the truth

2009

1. Torkild Leo Thellefsen
Fundamental Signs and Significance effects
A Semeiotic outline of Fundamental Signs, Significance-effects, Knowledge Profiling and their use in Knowledge Organization and Branding

2. Daniel Ronzani
When Bits Learn to Walk Don't Make Them Trip. Technological Innovation and the Role of Regulation by Law in Information Systems Research: the Case of Radio Frequency Identification (RFID)

2010

1. Alexander Carnera
Magten over livet og livet som magt
Studier i den biopolitiske ambivalens