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Dynamic Global Currency Hedging *

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Abstract

This paper proposes a model for *discrete-time* currency hedging based on *continuous-time* movements in portfolio and foreign exchange rate returns. The vector of optimal currency exposures is given by the negative realized regression coefficients from a one-period conditional expectation of the intra-period quadratic covariation matrix for portfolio and exchange rate returns. Empirical results from an extensive hedging exercise for equity investments illustrate that currency exposures exhibit important time variation, leading to substantial volatility reductions when hedging, without sacrificing returns. A risk-averse investor is willing to pay several hundred annual basis points to switch from existing hedging methods to the proposed dynamic strategies.

Keywords: Currency Hedging, Foreign Exchange Rates, High-frequency Data, Mean-Variance Analysis, Quadratic Covariation, Realized Currency Beta.

JEL classification: C14, C32, C58, G11, G15

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"Currency hedging is the hottest thing in investing right now."

Article headline, Business Insider UK, March, 2015.

1 Introduction

The potential benefits from international diversification have been recognized in the academic finance literature since the seminal work by Grubel (1968) and Levy & Sarnat (1970). Many empirical studies, however, find little, if any, statistically significant diversification benefits from investments across developed countries in more recent times, unless carried out using specific investment styles, e.g., by leveraging size, value, and momentum anomalies.[1] A possible explanation for these somewhat discouraging results (seen from the perspective of an investor) is the continuing integration of international financial markets, resulting in higher correlation among international assets and, thereby, reducing the potential for harvesting diversification benefits, see, e.g., Longin & Solnik (1995), Bekaert, Hodrick & Zhang (2009), and Christoffersen, Errunza, Jacobs & Langlois (2012).

Most of the aforementioned studies, however, neglect an important component of international investments: The currency exposure implicit in the international equity portfolio holdings. In other words, international investments in a given foreign country are exposed to exchange rate movements, and investors need to decide whether, and how, to hedge this additional source of risk. In practice, investment professionals often choose to hedge a certain fraction of their currency exposure, popular choices being the half-hedge and the full hedge. Some studies have analyzed hedging strategies that go beyond simple rule-of-thumb guides. In particular, Glen & Jorion (1993), de Roon, Nijman & Werker (2003), Campbell, de Medeiros & Viceira (2010), Schmittmann (2010), Kroencke, Schindler & Schrimpf (2014), and Opie & Dark (2015) analyze diversification benefits from optimal hedging strategies based on the theory originally proposed by Anderson & Danthine (1981), albeit with mixed empirical results.[2] Whereas they all reject leaving international investments unhedged, the first two studies find no significant evidence that a static, optimal, volatility-minimizing hedging strategy provides diversification benefits beyond what can be achieved by fully hedging international equity investments. However, when implementing a pseudo-dynamic hedging strategy according to which the optimal currency exposure to a given foreign country depends on the level of its interest rates relative to those in the domestic country, thus mimicking some form of carry trade, they find significant gains over full hedging. Campbell et al. (2010) find that a static volatility-minimizing hedging strategy significantly reduces the risk of international equity investments, compared to the gains from full hedging, and a similar pseudo-dynamic hedging strategy provides additional diversification benefits. However, the latter are economically modest, judging by their Sharpe ratios (SRs), and often statistically insignificant. Furthermore, their subsample analysis suggests that optimal currency exposures are quite sensitive to

[1] This includes the mean-variance analyses in, among others, Britten-Jones (1999), Errunza, Hogan & Hung (1999), Eun, Huang & Lai (2008), Eun, Lai, de Roon & Zhang (2010), and Fama & French (2012).

[2] Optimal in this setting is to be understood in a mean-variance sense, i.e., as the solution to a quadratic optimization problem for an investor seeking to maximize her risk-return tradeoff.

the specific sample under consideration. Similar results are obtained by Schmittmann (2010) and Opie & Dark (2015) from different countries' perspectives and across various horizons, corroborating the conclusions. Finally, Kroencke et al. (2014) take a deeper look into the diversification benefits from using traditional currency investment styles such as carry trade, momentum, and value strategies in said framework, thus promoting the pseudo-dynamic aspect of the optimal hedging strategies. They find significant diversification benefits, in particular when including foreign exchange rates for countries outside of the G10. However, the diversification benefits from their two-step procedure stem from the speculative asset allocation in the second step, not from the hedging itself, for which their results resemble those of Campbell et al. (2010). Thus, they reflect the profitability of the three currency investment styles over the last 30-40 years.

In the present paper, we reconsider the first step, i.e., we focus on enhancing the diversification benefits from volatility-minimizing hedging strategies, conditionally on a given portfolio, not on speculative currency investments.[3] To set the stage, we highlight two important aspects of previous approaches to currency hedging that demand further attention. First, all aforementioned studies of optimal currency exposure rely on the theoretical results from Anderson & Danthine (1981), who assume that asset prices are observed at the same frequency as that at which the investor rebalances her portfolio, that is, the frequency at which hedging decisions are made. This implies, for example, that if an investor rebalances her portfolio at a monthly frequency, then movements in asset prices occur at the monthly frequency, as well. Hence, this approach neglects all information from asset price movements occurring at higher frequencies, e.g., daily or intra-daily. Second, the hedging strategies are often studied in their "static", or unconditional, form, suggesting that optimal, volatility-minimizing currency exposures should be constant, often over a time span of 30-40 years, and estimated using full sample information. When such hedging strategies are given a time-varying flavor, it is by conditioning on variables related to currency investment styles, such as past interest rate differentials. The latter approach is labelled "pseudo-dynamic" for two reasons: (1) All intertemporal movements in the optimal currency exposures are determined by slowly varying conditioning variables. Hence, no traditional time series modeling (ARMA, GARCH, or stochastic volatility) is actually performed. (2) The implementation of the hedging strategies is often in-sample, i.e., the functional link to the interest differentials is estimated using full sample information, then used for conditional hedging decisions.[4] Hence, neither the static nor the pseudo-dynamic implementation of the optimal hedging strategies is designed for real-time investment decisions, and they provide inadequate descriptions of the dynamic properties of optimal currency exposures. The main exception to these caveats is Opie & Dark (2015),

[3]A related body of work considers optimal hedging of spot exchange rate risk using equivalent currency futures contracts in *conditional* frameworks resemblant of that developed by Anderson & Danthine (1981), see, for example, Baillie & Bollerslev (1989). However, this problem is distinct from the present setting of strategic utilization of currency exposures to improve the performance of an existing portfolio.

[4]Note that the implementation of the two-step optimal hedging strategies using currency investment styles in Kroencke et al. (2014) does not suffer from the caveat in (2), as the conditioning variables for the investment styles are contemporaneously available when the investor rebalances her portfolio. However, the caveat in (1) still describes their analysis. Similar comments apply to the robustness check in Campbell et al. (2010, Section 6).

who perform an out-of-sample analysis in which they compare rule-of-thumb hedges to a static optimal hedging strategy and a dynamic strategy based on a multivariate VAR-GARCH model, both implemented with rolling windows to estimate the currency exposures. The models are based on daily data, and rebalancing occurs if the changes in exposure generate increases in utility. Similarly to Campbell et al. (2010), they show that the two optimal strategies produce the lowest portfolio volatility, but also display statistically indistinguishable performance. That is, they find no additional improvements from actual dynamic modeling. However, despite addressing the second caveat, their framework, as well as analysis, does not explicitly treat the issue of sampling versus rebalancing frequency, but rather implicitly via a utility function.

This paper directly addresses both caveats by introducing a new economic model for *discrete time* currency hedging that not only allows the assets of interest - the portfolio and foreign currencies - to exhibit *within-period* movements, but actively utilizes additional "infill" information to construct accurate measures of optimal, volatility-minimizing, currency exposures. In particular, the latter are shown to be the negative realized regression coefficients from a one-period conditional expectation of the intra-period quadratic covariation matrix for portfolio and currency returns, labelled the *realized currency betas*. The model, hence, facilitates dynamic hedging strategies, depending exclusively on the dynamic evolution of the ex-post quadratic covariation matrix. This has the strong theoretical implication that interest rate differentials have *no* asymptotic impact on the optimal currency demands for a given international portfolio, in stark contrast with existing hedging theory, e.g., Anderson & Danthine (1981), Glen & Jorion (1993), and Campbell et al. (2010).[5] Moreover, as the proposed strategies do *not* rely on information about local trends in currencies in their construction, they are notably different from traditional currency investment styles, such as carry, momentum, and value trading. From a theoretical perspective, the development of the realized currency beta hedging framework involves establishing new results for optimal currency exposures based on the notion of quadratic covariation measures and infill asymptotic limits. From a practical perspective, the theory suggests that an investor should sample as frequently as possible within fixed time intervals between portfolio rebalances to construct accurate estimates the quadratic covariation matrix and model its dynamics. Hence, this paper proposes to implement the new hedging strategies using modern, yet simple, nonparametric techniques to accurately measure and dynamically model historical quadratic covariation matrices, imposing only few parametric restrictions on the underlying processes.

The new dynamic hedging strategies are analyzed in an extensive empirical exercise, covering different international equity portfolios, as well as a balanced fixed income-equity portfolio, and different rebalancing horizons, sampling frequencies, and currency investment universes (sets of foreign currencies available for hedging purposes). This produces several new and striking results that may be summarized as follows: (i) The optimal currency exposures display substantial time-variation, which is tied to important economic events such as the 2008-2009 global financial crisis, the European sovereign

[5] Even the dynamic VAR-GARCH implementation of currency hedging strategies in Opie & Dark (2015) depends implicitly on interests rate differentials via the conditional mean specification for currency excess returns.

debt crisis, and the "bloody Christmas" of 2018 global stock sell-off. The Swiss Franc and Japanese Yen are, on average, the most important hedging currencies, and the Canadian Dollar and Euro the main funding currencies. (*ii*) The proposed dynamic hedging strategies produce statistically significant, as well as economically substantial, volatility reductions for *all* baseline portfolios, compared to fully hedging currency exposure, as well as to existing, static and pseudo-dynamic, approaches to optimal hedging. (*iii*) These volatility reductions come without sacrificing returns, especially when implemented using intra-daily data to estimate the quadratic covariation between assets, thereby delivering Sharpe ratios that are 61% larger than key benchmarks. (*iv*) The estimated economic gains to the new hedging strategies are equally substantial, being 120-465 annual basis points after transaction costs over full hedging – depending on baseline portfolio and investor risk-aversion – and 120-520 annual basis points over existing hedging procedures. (*v*) The quality of the input quadratic covariation measure seems to be more important for designing profitable realized currency beta hedging strategies than the dynamic model specification or an expansion of the set of hedging currencies, with the former being the second most important feature. (*vi*) The currency overlay behind the dynamic realized currency beta investment strategy is negatively correlated with the FX carry trade, and only modestly correlated with momentum and value investments. Interestingly, the empirical analysis strongly suggests that the unwinding of carry trades, at least partially, fund the strong performance of the proposed dynamic hedging strategy during the global financial crisis of 2008-2009.

All of the findings (*i*)-(*vi*) are new to the literature on global currency hedging. In particular, the empirical hedging results go well beyond those in existing studies, such as Glen & Jorion (1993), Campbell et al. (2010) and Opie & Dark (2015), by not only showing how dynamic hedging strategies can be designed to obtain better volatility *and* risk-return trade-offs than full hedging and static optimal procedures, but also by estimating the economic gains from such strategies to a risk-averse investor, documenting important time-variation in optimal currency exposures, showing how this links to key economic events, and by providing results that speak to the relative importance of currency universe, dynamic model, and sampling frequency. Moreover, this paper is the first to leverage intra-daily data to construct currency hedging strategies, and this feature is paramount for designing procedures that deliver significant volatility reductions and superior economic performance.

The finding that dynamic hedging strategies based on intra-daily rather than daily data improves portfolio performance is consistent with Fleming, Kirby & Ostdiek (2001, 2003), who study dynamic asset allocation between S&P 500, Treasury bond, and gold futures.[6] However, in addition to the present analysis being one of hedging rather than asset allocation, the elicitation of gains from intra-daily data in the international investments and currency trading case is more challenging than in their single-country analysis, due to assets being traded on different exchanges with only partially overlap-

[6]It is also consistent with Andersen, Bollerslev, Diebold & Labys (2003), who consider VaR estimation using 30-minute returns on two currencies, and with Chiriac & Voev (2011), and Varneskov & Voev (2013), who carry out mean-variance analyses using intra-daily data on DJIA stocks. However, none of these studies considers the interaction between equity investments and currency exposures, nor do they use intra-daily data to design currency hedging strategies. Finally, it is consistent with Christoffersen & Diebold (2000), who show that volatility forecastability is important for risk management, see also Andersen, Bollerslev, Christoffersen & Diebold (2013) for a comprehensive review.

ping trading hours. Our results demonstrating that dynamic rather than static modeling of exchange rate covariances leads to economic gains for a risk-averse investor are also consistent with Della Corte, Sarno & Tsiakas (2009), who analyze asset allocation between fixed income and currencies by applying different univariate dynamic models to monthly data. In contrast with these studies, the problem of currency hedging can be viewed as a constrained, or conditional, asset allocation exercise in which currency exposures are selected for a given baseline portfolio, unlike their unconstrained approaches. Hence, it mirrors the problem faced by many institutional investors where one investment team takes a strategy, or portfolio, of another as given, and carries out hedging to improve its performance.

Even though individual currencies have traditionally been viewed as poor investment vehicles with low return and high volatility, there has been a recent surge of academic papers in a separate strand of the literature on exchange rate modeling, showing that systematic application of traditional currency trading, in particular carry trade, momentum, and value investments, may be highly profitable, even on a risk-adjusted basis, see, for example, the recent contributions by Lustig & Verdelhan (2007), Brunnermeier, Nagel & Pedersen (2009), Burnside, Eichenbaum, Kleshchelski & Rebelo (2011), Lustig, Roussanov & Verdelhan (2011), Menkhoff, Sarno, Schmeling & Schrimpf (2012*a*, 2012*b*), Moskowitz, Ooi & Pedersen (2012), Asness, Moskowitz & Pedersen (2013), and many references therein. The dynamic hedging strategies proposed in this paper similarly constitute systematic trading opportunities in currencies. However, they are designed with the specific purpose of improving the performance of an already existing baseline portfolio. Moreover, as the realized currency betas are asymptotically invariant to changes in interest rate differentials and only use information about the covariance between foreign exchange rate and portfolio returns in their construction, that is, no information about local trends in the former, they are notably different from traditional investment styles. In fact, their favorable correlation properties, noted in (*vi*), suggest not only that the proposed dynamic strategies may provide a hedge for carry trade, but also that there may be intriguing opportunities to combine the four different methods in designing tactical foreign currency exchange rate trading.

The outline of the paper is as follows. Section 2 introduces the new economic model and the assumptions, then derives the theoretical foundation for the proposed dynamic currency hedging strategies. Section 3 discusses the non-parametric implementation procedure. Section 4 introduces the data and provides empirical evidence of time variation in the optimal currency exposures. The risk-return performance and economic benefits from implementing different hedging strategies are examined in Section 5. Section 6 relates the returns to the dynamic hedging strategies to those from traditional currency investment styles, and Section 7 concludes. The Online Appendix provides additional theory, proofs of the theoretical results, various robustness checks, and implementation details.

2 The Dynamic Modeling Framework

This section introduces a multi-period model for *discrete-time hedging* based on *continuous-time* within-period movements in the underlying portfolio and foreign exchange rate returns. The model is intended to capture the decision problem of an investor who rebalances, or re-hedges, her portfolio in fixed time intervals, but observes both portfolio and exchange rate movements within each interval. Optimal currency exposures are established using *infill* asymptotic theory for a general class of continuous-time price processes. The discrete time framework follows along the lines of Anderson & Danthine (1981) and Campbell et al. (2010). However, as shown below, allowing for the continuous-time within-period movements in the processes of interest not only generalizes the framework considerably, it also simplifies the optimal, volatility-minimizing, hedging decision.

2.1 Discrete Time Decision Making

Suppose that at each discrete point in time $t = 1, 2, \dots, T$, an investor holds a position $w_{c,t}$ in country c's equities, $c = 0, \dots, C$, from time t until $t + 1$, when the holding pays a gross continuously compounded return of $R_{c,t+1}$.[7] For simplicity, let $c = 0$ index the home country, which is assumed to be the US, and let $S_{c,t+1}$ be the corresponding time $t + 1$ spot exchange rate quoted in USD per foreign currency unit. In this setting, the US investor earns an unhedged return $R^u_{c,t+1} = R_{c,t+1}S_{c,t+1}/S_{c,t}$ on her country c investment. To hedge the latter against currency risk, the investor buys a holding of the one-period forward exchange rate $F_{c,t}$, equivalently measured in USD per foreign currency unit, at time t in country c. Let $\theta_{c,t}$ be the dollar value of this holding per USD invested in the equity portfolio. Thus, the investor gets to exchange $\theta_{c,t}/S_{c,t}$ units of $R_{c,t+1}w_{c,t}/S_{c,t}$ back into USD at the exchange rate $F_{c,t}$, and the remaining $(R_{c,t+1}w_{c,t}/S_{c,t} - \theta_{c,t}/S_{c,t})$ units of foreign currency at the spot exchange rate $S_{c,t+1}$. This suggests writing the hedged portfolio return as

$$R^h_{t+1} = \sum_{c=0}^{C} w_{c,t}R^u_{c,t+1} + \sum_{c=0}^{C} \theta_{c,t}\frac{F_{c,t}}{S_{c,t}} - \sum_{c=0}^{C} \theta_{c,t}\frac{S_{c,t+1}}{S_{c,t}}. \tag{1}$$

Notice that the choice of domestic hedge ratio, $\theta_{0,t}$, is arbitrary, since $S_{0,t} = F_{0,t} = 1$, for all t. Hence, for ease of exposition, the hedge ratios are normalized to sum up to one, implying that

$$\sum_{c=0}^{C} w_{c,t} = 1, \qquad \theta_{0,t} = 1 - \sum_{c=1}^{C} \theta_{c,t}, \tag{2}$$

for all t. Maintaining an assumption of absence of arbitrage throughout, it follows by covered interest rate parity (CIP) that $F_{c,t}/S_{c,t} = (1+I_{0,t})/(1+I_{c,t})$, where $I_{c,t}$ denotes the nominal short-term risk-free interest rate. This identity may be inserted in (1).

The form of the portfolio return in (1) also allows for speculative positions in exchange rates if, for example, the currency demand $\theta_{c,t}$ is driven by, e.g., a model for local trends in $S_{c,t}$, regardless of its correlation with the portfolio return. Hence, to avoid confusion going forward, the label "hedging"

[7] The exposition is laid out for equities. This may without loss of generality, however, be changed to other assets held in foreign countries, such as corporate bonds, commodities, derivates, etc., as long as the assumptions on the assets, as outlined below, are satisfied. A balanced bond-equity portfolio is examined in the empirical analysis.

in this paper signifies that currency demands are determined with the explicit objective of reducing the risk of the portfolio return, thus seeking currencies with favorable correlation properties. In other words, currency *hedgers* and *speculators* are distinguished according to whether they emphasize correlation properties or local trends, respectively, when selecting foreign exchange rate exposure.

2.2 Intra-period Dynamics

Suppose that the processes of interest – equities, currencies, and bonds – are defined on a filtered probability space, $(\Omega, \mathcal{F}, (\mathcal{F}_{t,\tau}), \mathbb{P})$, where $\tau \in [t, t+1]$ is the within-period time indicator. In the absence of arbitrage, prices are assumed to follow semimartingales, e.g., Back (1991). Hence, denote by $P_{c,\tau}$ the price of the equity holdings in country c, measured in local currency, and $B_{c,\tau}$ the price of a country c-denominated riskless bond. Then, for $c = 0, \dots, C$, the system of equity, currency, and bond prices obeys

$$dP_{c,\tau}/P_{c,\tau} = \mu_{c,\tau} d\tau + \sigma_{c,\tau} dW_{c,\tau}, \tag{3}$$

$$dS_{c,\tau}/S_{c,\tau} = \alpha_{c,\tau} d\tau + \varphi_{c,\tau} dY_{c,\tau}, \tag{4}$$

$$dB_{c,\tau}/B_{c,\tau} = \lambda_{c,\tau} d\tau, \tag{5}$$

in which $(\mu_{c,\tau}, \alpha_{c,\tau})$ and $(\sigma_{c,\tau}, \varphi_{c,\tau})$ capture the within-period drift and stochastic volatility of equity and currency returns, $W_{c,\tau}$ and $Y_{c,\tau}$ are standard Brownian motions adapted to $(\mathcal{F}_{t,\tau})$, and $\lambda_{c,\tau}$ models the instantaneous return from holding a short-term riskless bond.[8] Moreover, for $c \neq k$, we define the quadratic covariations $d[W_c, W_k]_\tau = \sigma_{c,k,\tau} d\tau$ and $d[Y_c, Y_k]_\tau = \varphi_{c,k,\tau} d\tau$. Finally, let $d[W_c, Y_k]_\tau = \psi_{c,k,\tau} d\tau$ for all c, k. The theoretical analysis, then, necessitates additional structure on the system:

Assumption 1. *For all $c, k \in \{0, \dots, C\}$, the components of* (3)-(5) *satisfy:*

(a) $\mu_{c,\tau}$, $\alpha_{c,\tau}$, and $\lambda_{c,\tau}$ are $\mathcal{F}_{t,\tau}$-predictable and locally bounded;

(b) $\sigma_{c,\tau}$ and $\varphi_{c,\tau}$ are $\mathcal{F}_{t,\tau}$-adapted, locally bounded, càdlàg, and strictly greater than zero;

(c) $\sigma_{c,k,\tau}$, $\varphi_{c,k,\tau}$, and $\psi_{c,k,\tau}$ are $\mathcal{F}_{t,\tau}$-adapted, locally bounded, and càdlàg.

The present setting generalizes the previously developed framework for deriving optimal currency exposure in Anderson & Danthine (1981), Glen & Jorion (1993), de Roon et al. (2003), and Campbell et al. (2010), by allowing for stochastic drift and volatility, as well as intra-period movements in equites and currencies. The latter are assumed to belong to a general class of continuous Brownian semimartingales, which, again, is commonly used in the literature on high-frequency volatility and covariance estimation, since it nests many continuous-time models in financial economics.[9] For example, the class accommodates the widely documented presence of leverage effects, i.e., non-zero correlation

[8] The time t subscript is dropped for notational simplicity when describing the intra-period price system (3)-(5), since the representation is valid for all intervals, with $\tau \in [t, t+1]$, $t = 1, \dots, T$.

[9] See, e.g., Andersen & Benzoni (2012) and Andersen et al. (2006, 2013) for reviews.

between innovations in the price process and the stochastic volatility process. The modeling system implies that in a given time interval, $[t, t+1]$, between the rebalancing times of the portfolio of equities, currencies, and bonds, asset prices are allowed to evolve according to intra-period trajectories, which will be important for determining the investor's optimal, volatility-minimizing, currency position.

Remark 1. *While it is convenient to work with locally riskless bonds, it is important to note that the analytical results below are not contingent on a diffusive component being absent in* (5). *All our results are asymptotically invariant to replacing the latter with* $dB_{c,\tau}/B_{c,\tau} = \lambda_{c,\tau}d\tau + \varsigma_{c,\tau,\epsilon}dZ_{c,\tau}$, *where* $dZ_{c,\tau}$ *is a standard Brownian motion, which may have non-trivial quadratic covariation with* $dW_{k,\tau}$ *and* $dY_{k,\tau}$, *and* $\varsigma_{c,\tau,\epsilon} = \varsigma_{c,\tau} \times (d\tau)^{\epsilon}$ *captures stochastic volatility, with* $\epsilon > 0$ *and* $\varsigma_{c,\tau}$ *satisfying conditions similar to Assumption 1(b). This models the volatilities of short-term bonds as an order of magnitude smaller than the corresponding volatilities for equity and currency prices. For* $\epsilon \to \infty$, *(5) is recovered.*

Remark 2. *The vector price system* (3)-(5) *may be extended to include jumps. This only leads to minor changes in the interpretation of the results. The role of jumps is discussed in Section A of the Online Appendix, where the theoretical results, provided below, are also extended.*

Before deriving the optimal currency exposures, it is important to characterize the path of the hedged portfolio return at each time $\tau \in [t, t+1]$. Hence, with V_τ denoting the value of the hedged portfolio at time τ, the evolution of its instantaneous return may be described using (3)-(5),

$$\frac{dV_\tau}{V_\tau} = \sum_{c=0}^{C} w_{c,t}\frac{d(P_{c,\tau}S_{c,\tau})}{P_{c,\tau}S_{c,\tau}} + \sum_{c=0}^{C}\theta_{c,t}\frac{d(B_{0,\tau}/B_{c,\tau})}{B_{0,\tau}/B_{c,\tau}} - \sum_{c=0}^{C}\theta_{c,t}\frac{dS_{c,\tau}}{S_{c,\tau}}. \tag{6}$$

As in Campbell et al. (2010), it simplifies the problem to work in logarithms and use matrix notation. Hence, let $r^h_{t+1} = \ln(R^h_{t+1})$ and $x_{c,\tau} = \ln(X_{c,\tau})$ for $X \in \{P, S, V\}$. Similarly, let $\boldsymbol{w}_t = (w_{0,t}, \ldots, w_{C,t})'$ be the $(C+1)\times 1$ vector of portfolio weights, $\boldsymbol{\Theta}_t = (\theta_{0,t}, \ldots, \theta_{C,t})'$ the corresponding $(C+1)$-dimensional vector of currency hedging positions, $\boldsymbol{x}_\tau = (x_{0,\tau}, \ldots, x_{C,\tau})'$ for $x = (p, s, \lambda)'$, and $\boldsymbol{\lambda}_{0,\tau} = \boldsymbol{\iota}\lambda_{0,\tau}$, with $\boldsymbol{\iota}$ a $(C+1)\times 1$ vector of ones. Furthermore, to explicitly capture the fact that an investor can alter her currency exposure by lending and borrowing (going long or short in bonds or forward contracts), define $\boldsymbol{\beta}_t \equiv (\beta_{0,t}, \ldots, \beta_{C,t})' = \boldsymbol{w}_t - \boldsymbol{\Theta}_t$ as the net exposures to the foreign currencies. For example, $\beta_{c,t} = 0$ corresponds to having a fully hedged position in country c's equities, and $\beta_{c,t} = w_{c,t}$ to leaving the exposure completely unhedged. In general, $\beta_{c,t} > 0$ implies that the investor demands exposure to currency c and, equivalently, she wants to be underexposed if $\beta_{c,t} < 0$. Note that (2) implies $\boldsymbol{\beta}_t'\boldsymbol{\iota} = 0$, that is, the dynamic currency hedging portfolio is a zero investment, meaning that all long positions in currencies are financed by shorting bonds in funding currencies, similarly as in traditional currency investment styles. Finally, a regularity condition is imposed on the elements of the vectors $\boldsymbol{w}_t$ and $\boldsymbol{\Theta}_t$ to simplify the further theoretical analysis.

Assumption 2. *For all* $t = 1, \ldots, T$, $\sup_{c=0,\ldots,C}|w_{c,t}| + \sup_{c=0,\ldots,C}|\theta_{c,t}| < \infty$.

Assumption 2 innocuously states that both the equity portfolio weight in and currency exposure to country c, $c = 0, \ldots, C$, must be finite.[10] The following proposition provides a representation result for the *within-period* currency hedged log-returns, dv_τ.[11]

Proposition 1. *Suppose the representation* (6) *and Assumptions 1-2 hold. Then*

$$dv_\tau = \boldsymbol{w}_t'(d\boldsymbol{p}_\tau + \boldsymbol{\lambda}_{0,\tau}d\tau - \boldsymbol{\lambda}_\tau d\tau) + \boldsymbol{\beta}_t'(d\boldsymbol{s}_\tau - \boldsymbol{\lambda}_{0,\tau}d\tau + \boldsymbol{\lambda}_\tau d\tau) + \Sigma_\tau^h d\tau + o_p(d\tau),$$

where Σ_τ^h is $\mathcal{F}_\tau$-adapted, locally bounded, and càdlàg.

Proposition 1, similarly to the representation in Campbell et al. (2010, Equation (1)), provides a decomposition of the hedged log-return into three components; the first is the instantaneous excess return on a *fully* hedged portfolio; the second term represents the instantaneous excess return on currencies, which depends on the selected exposure, $\boldsymbol{\beta}_t$; and the last term is a Jensen's inequality correction. However, unlike in the corresponding framework in Anderson & Danthine (1981) and Campbell et al. (2010), the instantaneous log-return on the hedged portfolio is allowed to evolve stochastically in the interval $\tau \in [t, t+1]$, implying that the one-period log-return may be written

$$r_{t+1}^h = \int_t^{t+1} dv_\tau, \qquad t = 1, \ldots, T, \tag{7}$$

thus formally providing a link between their framework and ours. Equation (7) suggests that the one-period log-return on a hedged portfolio may be interpreted as the sum of returns at a higher frequency. For an investor with a monthly investment horizon, this could, e.g., be a sum of daily log-returns.

2.3 Optimal Dynamic Currency Exposure

The optimal dynamic selection of currency exposure requires the choice of an appropriate objective function. Usually, in portfolio selection problems, this involves choosing the portfolio weights to minimize portfolio variance subject to certain constraints. Similarly to the one-period log-return (7), which is measured by cumulating returns at higher frequency, a measure of its variance must also reflect the stochastic intra-period movements in dv_τ. In this setting, quadratic variation (QV) offers such a variability measure, see, e.g., Andersen, Bollerslev & Diebold (2010). Formally, suppose the intra-period hedged log-return dv_τ is observed on a discrete partitioning τ_i of the time interval, $t = \tau_0 < \tau_1 < \cdots < \tau_n = t+1$. The QV of r_{t+1}^h is, then, defined as

$$[dv_\tau]_{t+1} \equiv \operatorname*{plim}_{n\to\infty} \sum_{i=1}^n (v_{\tau_i} - v_{\tau_{i-1}})^2 = \lim_{h\to 0} \int_t^{t+1} \mathbb{E}\left[\mathcal{M}(dv_\tau)^2 | \mathcal{F}_{t,\tau-h}\right] d\tau, \tag{8}$$

[10] Strictly speaking, the condition $\sup_{c=0,\ldots,C} |\theta_{c,t}| < \infty$ should be shown endogenously in the dynamic model, since $\theta_{c,t}$ will depend on the components of the intra-period price system (3)-(5). However, by assuming it from the outset, rather than showing it endogenously, the proofs of Propositions 1-4 may be shortened considerably, without loss of intuition.

[11] We will be using the nomenclature $o_p(d\tau)$ to describe higher-order terms of the form $(d\tau)^2$, $d\tau \times dW_{c,\tau}$, $d\tau \times dY_{c,\tau}$, etc., which have no asymptotic impact in the further analysis below.

with $\mathcal{M}(\cdot)$ isolating the martingale component of dv_τ, for $\sup_i\{\tau_{i+1} - \tau_i\} \to 0$ as $n \to \infty$, see, e.g., Jacod & Shiryaev (2003).[12] QV captures the entire realized ex-post variation of the hedged log-returns, and its use will simplify the computations of the optimal currency exposures via the next result.

Proposition 2. *Suppose the conditions of Proposition 1 hold. Then, as $n \to \infty$,*

$$[dv_\tau]_{t+1} = \left[\boldsymbol{w}_t' d\boldsymbol{p}_\tau + \boldsymbol{\beta}_t' d\boldsymbol{s}_\tau\right]_{t+1}.$$

Proposition 2 shows that the QV of the hedged log-return depends *only* on the QVs of the fully hedged log-return and the total currency exposure return, as well as on their quadratic covariation (QC). Hence, there is no impact from movements in nominal short-term risk-free interest rate differentials nor from the Jensen's inequality induced term, $\Sigma_\tau^h d\tau$. This distinct advantage of the proposed *within-period* model for equities, currencies, and bonds is due to the fact that drift components have no asymptotic impact on QV in the *infill* asymptotic limit. As a result, Proposition 2 provides a variance measure that contrasts starkly with the corresponding long-span variance measure used for the development of the existing currency hedging theory by Anderson & Danthine (1981), and applied in Glen & Jorion (1993), de Roon et al. (2003), and Campbell et al. (2010), and which depends on period-by-period movements in short-term interest rate differentials.

Since the vector of dynamic net currency exposures, $\boldsymbol{\beta}_t$, represents a zero-investment portfolio, it suffices to determine the $C \times 1$ vector of foreign currency exposures $\tilde{\boldsymbol{\beta}}_t = (\beta_{1,t}, \ldots, \beta_{C,t})'$, which spans the unique elements of $\boldsymbol{\beta}_t$. Formally, and consistently with our distinction between foreign exchange hedgers and speculators, exposures are selected to minimize the one-period conditional quadratic variation of the hedged log-return, that is, as

$$\tilde{\boldsymbol{\beta}}_t^* = \underset{\boldsymbol{\beta}_t|\boldsymbol{w}_t}{\operatorname{argmin}}\, \mathcal{L}_t(\boldsymbol{\beta}_t, \boldsymbol{w}_t), \qquad \mathcal{L}_t(\boldsymbol{\beta}_t, \boldsymbol{w}_t) = \frac{1}{2}\mathbb{E}_t\left[[dv_\tau]_{t+1}\right]. \tag{9}$$

Before stating the optimality result, let $\tilde{\boldsymbol{s}}_\tau = (s_{1,\tau}, \ldots, s_{C,\tau})'$ denote the vector of currencies corresponding to the unique exposures $\tilde{\boldsymbol{\beta}}_t$. Then, the following proposition solves (9).

Proposition 3. *Suppose the conditions of Proposition 2 hold, and that $\mathbb{E}_t\left[[d\tilde{\boldsymbol{s}}_\tau]_{t+1}\right]$ is positive definite for all $t = 1, \ldots, T$. Then the limiting, unique, optimal currency exposures are determined by*

$$\tilde{\boldsymbol{\beta}}_t^* = -\mathbb{E}_t\left[[d\tilde{\boldsymbol{s}}_\tau]_{t+1}\right]^{-1} \mathbb{E}_t\left[[d\tilde{\boldsymbol{s}}_\tau, \boldsymbol{w}_t' d\boldsymbol{p}_\tau]_{t+1}\right].$$

Proposition 3 demonstrates that the vector of optimal currency exposures is the negative vector of realized regression coefficients from an implicit projection of the fully hedged log-return on the vector of foreign exchange rate innovations, which is embedded in the one-period conditional expectation of the QC matrix. This former is labelled the *realized currency beta*, in analogy with the market

[12] The quadratic covariation between two appropriately dimensioned vector processes $\boldsymbol{x}_{\tau_i}$ and $\boldsymbol{y}_{\tau_i}$, for a similar partition of the sample $\tau_i \in [t, t+1]$, $i = 0, \ldots, n$, is analogously defined as $[\boldsymbol{x}, \boldsymbol{y}]_{t+1}$, that is, as the probability limit of a sum of outer products of their increments as the distance between observations tends to zero.

exposure measured by the CAPM beta. However, it is stressed that while the market beta reflects the uncertainty of a given asset in terms of its sensitivity to market movements, the realized currency beta reflects the hedging potential from having active, and systematic, currency exposure in a *given* equity portfolio and is, as a result, *not* a deep characteristic of a currency.

In addition, Proposition 3 suggests that realized currency betas may be computed dynamically using only *within-period* equity and foreign exchange rate data by first obtaining a time series of their QC estimates, and then specifying an appropriate dynamic model for these, to obtain one-step-ahead conditional expectations. This is a highly desirable property, since it implies that the optimal currency exposures are not only asymptotically invariant to short-term interest rate differentials, but also to the validity of CIP, which is otherwise used to substitute out forward rates with interest rate differentials in (1) and (6).[13] Importantly, this invariance separates the optimal currency exposures from the popular carry trade investments, which are designed with long positions in baskets of currencies with high short-term interest rates and short in baskets of currencies with low interest rates, resting on the failure of the uncovered interest rate parity. The resulting realized currency beta hedging strategies may, thus, be viewed as an alternative to traditional currency investments, such as carry, momentum, and value trading strategies, which rely solely on the modeling of local trends, rather than cross-asset covariances. Empirical comparisons are made in Section 6 below.

Finally, focusing solely on volatility reduction in the objective function (9) has two additional advantages. First, it mirrors the problem faced by many institutional investors, where one investment team takes a trading strategy as given (here, an equity portfolio), then executes a hedging procedure to improve its risk profile. Second, from an econometric perspective, Engle & Colacito (2006) show that the economic value of time-varying covariances can only be consistently measured in a mean-variance setting by the minimum variance portfolio. Hence, the objective function (9) facilitates consistent evaluation of the proposed intra-period model for currency hedging.

Remark 3. *Although the exposition is given from the perspective of a US investor, it is important to note that the realized currency betas are dynamically invariant to base currency. This implies that, e.g., a UK investor with the same equity portfolio will be choosing identical optimal currency exposures. This invariance result is formally shown in the Online Appendix.*

3 Estimating Optimal Currency Exposures

Dynamic implementation of the proposed realized currency beta hedging strategy requires both estimation of the latent QC matrix over each discrete time interval between portfolio rebalances, and subsequent dynamic modeling of the covariance matrices. Hence, two different non-parametric approaches

[13] While Akram, Rime & Sarno (2008) find that CIP holds approximately at daily or lower frequencies, Du, Tepper & Verdelhan (2018) find persistent deviations from the no-arbitrage condition. Although realized currency betas are invariant to CIP, the latter will have an impact on whether to apply forward rates or interest rate differentials to evaluate the return performance of optimal hedging strategies ex-post. In the empirical analysis below, the investor is assumed to trade FX forwards, and forward rates are applied, to avoid concerns about CIP violations.

to QC estimation, which may be applied to data sampled at different frequencies, are discussed first. Second, a simple filtering procedure for the construction of one-period-ahead conditional expectations of the quadratic covariation matrix is then detailed.

3.1 Measuring Quadratic Covariation

Suppose that the vector $\boldsymbol{x}_{\tau_i} = (\boldsymbol{w}_t' d\boldsymbol{p}_{\tau_i}, d\tilde{\boldsymbol{s}}_{\tau_i}')'$ is observed at the $n+1$ discrete time points from the portfolio rebalancing at t to the next, that is, at $\tau_i \in [t, t+1]$, $i = 0, \ldots, n$, then the realized covariance (RC) estimator, introduced by Andersen et al. (2003) and Barndorff-Nielsen & Shephard (2004), represents the empirical approximation to the computations (8). Formally, the estimator is defined as

$$RC(\boldsymbol{x}) = \sum_{i=1}^{n} \Delta\boldsymbol{x}_{\tau_i} \Delta\boldsymbol{x}_{\tau_i}', \tag{10}$$

where $\Delta = 1 - L$ is the usual first difference operator. Under mild conditions on the vector price system in (3)-(5), $RC(\boldsymbol{x}) \xrightarrow{\mathbb{P}} [\Delta\boldsymbol{x}]_{t+1}$ for $\sup_i\{\tau_{i+1} - \tau_i\} \to 0$ as $n \to \infty$. Its associated central limit theory demonstrates that convergence occurs at the optimal rate, $n^{1/2}$, to a mixed Gaussian distribution. Implicit in these statements, however, is that the individual entries in $\boldsymbol{x}_{\tau_i}$ are observed synchronously and without measurement errors. This approximation may not be too damaging if the rebalancing horizon is, for example, weekly or monthly, and the intra-period observations are recorded daily or even intra-daily at sufficiently sparse intervals.[14] If the data are sampled intra-daily at higher frequencies, on the other hand, market microstructure (MMS) effects and non-synchronicity related errors drive a wedge between the observed equity prices and exchange rates and their theoretical counterparts, leading the individual entries of standard covariance matrix estimators such as RC to diverge. Hence, if the data are available intra-daily at frequencies higher than the conventional five minute rule-of-thumb, it is pertinent to use an estimator that actively mitigates the impact from these measurement errors while maintaining good efficiency properties. A class of estimators fitting these requirements is the flat-top realized kernels, proposed by Varneskov (2016, 2017).

The notion of measurement errors may be quantified as follows: Let the observable, synchronized, intra-daily observations follow an additive noise model of the form $\boldsymbol{y}_{\tau_i} = \boldsymbol{x}_{\tau_i} + \boldsymbol{u}_{\tau_i}$, where $\boldsymbol{u}_{\tau_i}$ summarizes the effects from an array of market imperfections, including synchronization errors, and is referred to as MMS noise.[15] Next, let $\Gamma_h(\boldsymbol{y}) = \sum_{i=|h|+1}^{n} \Delta\boldsymbol{y}_{\tau_i} \Delta\boldsymbol{y}_{\tau_{i-|h|}}'$ for $h \geq 0$ and $\Gamma_h(\boldsymbol{y}) = \Gamma_{-h}(\boldsymbol{y})'$ for $h < 0$ be the realized autocovariance of $\boldsymbol{y}$ for given lag h. The class of flat-top realized kernels is designed to eliminate the noise-induced bias and variance of the realized covariance estimator by

[14] It is generally not recommended to sample much more sparsely than daily since the asymptotic approximation of negligible drift, or local trends, may be poor at such frequencies. If the series display non-negligible drift, this obviously needs to be taken into account when computing the quadratic covariation estimates.

[15] Besides synchronization errors, the MMS noise captures both exogenous effects, such as bid-ask bounce movements, and endogenous effects, such as asymmetric information and strategic learning among market participants.

weighting higher-order realized autocovariances appropriately as

$$RK^*(\boldsymbol{y}) = RC(\boldsymbol{y}) + \sum_{h=1}^{n-1} k(h/H) \left\{ \Gamma_h(\boldsymbol{y}) + \Gamma_{-h}(\boldsymbol{y}) \right\}, \tag{11}$$

for a bandwidth parameter $H = an^{1/2}$, $a > 0$, and, in particular, a non-stochastic kernel function, $k(\cdot)$, designed as

$$k(z) = \mathbf{1}_{\{|z| \leq k\}} + \lambda(|z| - k)\mathbf{1}_{\{|z|>k\}}, \tag{12}$$

with $k = H^{-\xi}$, $\xi \in (0, 1)$, a shrinking function of the bandwidth H, and $\lambda(\cdot)$ a second-order smooth kernel function, satisfying some mild regularity conditions, an example being the Parzen kernel. The properties of these HAC-style estimators depend crucially on the kernel function, and by selecting $k(z)$ as in (12), the resulting class of flat-top realized kernels achieves optimal asymptotic properties in this setting, such as consistency, asymptotic unbiasedness, and mixed Gaussianity at the optimal rate of convergence, $n^{1/4}$, under mild assumptions on the MMS noise and (possibly random) sampling times.[16] If optimally designed, the estimator is, in addition, efficient in a Cramér-Rao sense. As a result, it performs well in finite samples, even for sparse observations available at 1 to 5-minute frequencies. Implementation details are provided in the Online Appendix.

When intra-daily observations are only available for a certain part of a day, the trading window, and there is no recorded trading during weekends, holidays, etc., the estimates from the flat-top realized kernel may be supplemented with the squared close-to-open return since the preceding (trading) day. This approach essentially combines the estimates from $RC(\boldsymbol{x})$ and $RK^*(\boldsymbol{y})$.

3.2 A Simple Filtering Approach to Covariance Modeling

A number of different procedures to construct one-step conditional expectations of the QC matrix have been proposed in the literature. However, rather than searching for the best covariance model, the aim of this paper is to provide a baseline approach for dynamic implementation of the realized currency betas, which is simple, of non-parametric flavor, and easy to implement for QC estimates with different degrees of measurement errors, such that it can accommodate within-period sampling at both daily and intra-daily frequencies. In particular, the procedure that is introduced here adapts the rolling window estimator proposed by Foster & Nelson (1996) and Andreou & Ghysels (2002) in the univariate case, and extended to the multivariate case in Fleming, Kirby & Ostdiek (2001), to the present setting. To this end, let $\boldsymbol{\Sigma}_t$ and $\hat{\boldsymbol{\Sigma}}_t$ be short-hand notation for the latent conditional quadratic covariation matrix and a generic estimator of this, respectively. Then, the use of rolling window estimators implies the relation $\boldsymbol{\Sigma}_t = \sum_{i=1}^{\infty} \boldsymbol{\varpi}_{t-i} \odot \hat{\boldsymbol{\Sigma}}_{t-i}$, where $\boldsymbol{\varpi}_{t-i}$ is a symmetric $(C + 1) \times (C + 1)$ matrix of weights, and $\odot$ is the Hadamard product. As proved by Foster & Nelson (1996, Theorem 5) under weak assumptions, the mean-squared error optimal covariance weights are given by $\boldsymbol{\varpi}_{t-i} = \gamma \exp(-\gamma i)\iota\iota'$, such that $\boldsymbol{\Sigma}_t = \exp(-\gamma)\boldsymbol{\Sigma}_{t-1} + \gamma \exp(-\gamma)\hat{\boldsymbol{\Sigma}}_{t-1}$. The resulting covariance estimate is, however, generally

[16] The presence of additive MMS noise slows down the best attainable rate of convergence from $n^{1/2}$ to $n^{1/4}$.

downward biased since $(1+\gamma)\exp(-\gamma) < 1$ for $\gamma > 0$. Hence, a new bias-corrected version is introduced,

$$\boldsymbol{\Sigma}_t = (1 - (1+\gamma)\exp(-\gamma))\bar{\boldsymbol{\Sigma}}_{t-1}. + \exp(-\gamma)\boldsymbol{\Sigma}_{t-1} + \gamma\exp(-\gamma)\hat{\boldsymbol{\Sigma}}_{t-1}, \tag{13}$$

where $\bar{\boldsymbol{\Sigma}}_{t-1}$ is the prevailing mean of $\hat{\boldsymbol{\Sigma}}_{t-1}$.[17] In other words, this version of the rolling window covariance estimator may be thought of as an exponentially weighted multivariate GARCH model for the time series of QC estimates, $\hat{\boldsymbol{\Sigma}}_t$, whose rate of decay is determined by a single parameter γ. Despite being parsimoniously parametrized, this approach allows for persistent time-variation in $\boldsymbol{\Sigma}_t$, while implicitly reducing the impact of measurement errors in $\hat{\boldsymbol{\Sigma}}_t$. Despite its simplistic structure, Varneskov & Voev (2013) show that the forecasting performance of (13), without the bias-correction, is insignificantly different from that of more sophisticated multivariate Cholesky decomposed HAR and ARFIMA models when evaluated using a global minimum variance criterion for a portfolio of ten stocks and various realized covariance measures.[18] As a robustness check, however, the empirical analysis of currency hedging strategies, presented in the next sections, has been carried out using a multivariate HAR model in place of the MGARCH model. The results are very similar, albeit with slightly worse risk-return properties, and are provided in Section F of the Online Appendix.

4 Data, Implementation, and Summary Statistics

This section introduces the data, which consist of daily and intra-daily observations covering the time span from January 2000 through December 2019. Furthermore, it provides details on the construction of the QC estimates and forecasts, as well as the computation of the realized currency betas. Moreover, bid-ask spread data are used to estimate transaction costs for the evaluation of the dynamic hedging strategies. Finally, novel evidence of time variation in optimal currency exposures is presented.

4.1 Data Collection and Construction

The empirical analysis is performed for a US investor who holds either an equity portfolio or a balanced portfolio with fixed income and equity, and may use (a subset of) the G10 currencies to hedge her foreign currency exchange rate exposure. In particular, two sets of currencies are considered. The first includes the very liquid Canadian Dollar (CAD), Swiss Franc (CHF), Euro (EUR), Great Britain Pound (GBP), and Japanese Yen (JPY). The second set further allows active investments in the Australian Dollar (AUD), the Norwegian Krona (NOK), the New Zealand Dollar (NZD), and the Swedish Krona (SEK). These sets are labeled G06 and G10, respectively. For each exchange rate, the last *daily* spot price (bid, ask, trade) quoted on Bloomberg during NYSE trading hours is obtained, along with

[17] Specifically, the present approach differs from the procedure in Fleming, Kirby & Ostdiek (2003) by replacing the outer product of returns, or the realized covariance estimator, with a generic quadratic covariation estimator, similarly to the study in Varneskov & Voev (2013). Moreover, it differs from the latter by introducing a bias-correction.

[18] Using a statistical loss function, on the other hand, Varneskov & Voev (2013) find statistically significant gains from using multivariate Cholesky decomposed HAR and ARFIMA models over the multivariate GARCH model.

the corresponding one-month forward points, to construct forward prices, as well as estimates of the transaction costs associated with FX trading. Summary statistics for forward returns, spot returns, and implied returns on interest rate differentials are provided in Table 1, along with estimates of transaction costs in basis points per annum (BP). Note that *all* return series are log-transformed.[19]

Table 1 illustrates that currencies such as the CHF and JPY have lower returns on interest rates than the USD, and vice versa for the AUD and NZD. Moreover, the spot return fails to offset the interest rate differentials, consistent with the former two being funding currencies for carry trades, and the latter being high-yielding currencies in the G10 universe, see, e.g., Lustig et al. (2011). For all individual currencies, the excess returns are modest compared to volatility, generating annualized Sharpe Ratios (SRs) of no higher than 0.3. However, an extensive literature (see the introduction for references) has documented that systematic FX trading can be very profitable, on a risk-adjusted basis. The CHF exhibits extreme kurtosis relative to the remaining currencies, caused by the Swiss National Bank's abandonment of the EUR peg on January 15, 2015. The estimated transaction costs, computed as $10,000(\texttt{ask} - \texttt{bid})/((\texttt{ask} + \texttt{bid})/2)$ for the forward prices, shows that, on average, it is cheaper to trade G06 currencies than G10. Following convention in the literature, e.g. Lyons (2001), the cost of trading is fixed at half the average spread, that is, at 3BP for the G06 currencies and 6BP for the G10 set. The results, however, are robust to increasing these numbers.

Three different baseline portfolios are considered. The first is the S&P 500, whose currency exposure is determined implicitly through the international investments and cash flows of its constituents. The second is an equally weighted portfolio in the DAX, FTSE 100, and S&P 500. All equity index investments are carried out via futures contracts. The third portfolio has a 60% weight on the S&P 500 and 40% on 10-year Treasury bond futures. This balanced, risk-parity-style, benchmark has been popularized by mutual funds, such as Vanguard, and is a prominent benchmark for passive investors. As such, the setup resembles the one in Campbell et al. (2010), who consider currency hedging for the S&P 500 and equal weighted (EW) international equity investments, i.e., the first two portfolios. The main differences are that they use either monthly or quarterly observations, while we collect *daily* data on the futures contracts. Moreover, they include the AUD in their set of (seven) hedging currencies, whereas we consider both the G06 and G10 sets of currencies. Finally, the investment horizons considered here are weekly and monthly, rather than monthly and quarterly.

In addition to the daily futures data for the baseline portfolios and FX forwards, we collect intra-daily, one-minute data for the baseline portfolios and spot prices of the G06 currencies. The intra-daily data are generally non-synchronous, and *refresh time sampling* is used to align observations, see, e.g., Varneskov (2016, Definition 1). Then, from either the daily or intra-daily data, estimates of QC is constructed using RC or flat-top realized kernels, respectively.[20] Hence, our final data set consists

[19]Since all summary statistics are computed directly from log-returns, there is no need to consider the Jensen's inequality correction in Proposition 1. The latter is only important when transferring log-returns back into gross return form. Moreover, as seen by the general identity from Itô's lemma, $R_t = r_t + [r]_t/2$, the use of log-returns leads to a conservative assessment of the benefits from applying the proposed dynamic hedging strategies.

[20]Details on the cleaning of the intra-daily data as well as its characteristics, e.g., number of synchronized observations, noise-to-signal ratios, etc., are provided in Section E of the Online Appendix.

of either weekly or monthly observations of returns on the baseline portfolios and FX forwards, QC estimates for G06 and G10 constructed from daily data, as well as QC estimates for G06 constructed from intra-daily data. This facilitates assessments of whether the new dynamic realized currency beta hedging theory generates improved portfolio performance, whether there are differences across baseline portfolios, or across different sets of hedging currencies, G06 versus G10, and whether there are additional gains from leveraging intra-daily data to increase precision of the QC estimates.

4.2 Implementing Realized Currency Betas

The empirical analysis features several benchmark implementations of the dynamic realized currency beta hedging strategies. First, to extrapolate expectations of the QC matrix, the dynamic MGARCH specification in (13) is estimated using standard multivariate Gaussian maximum likelihood techniques in conjunction with either the RC estimates from (10) for the G06 currency set in place of $\hat{\mathbf{\Sigma}}_t$, the corresponding for the G10 set, or the flat-top realized kernel estimates from (11)-(12) for the G06 set. Specifically, the smoothing parameter γ is estimated recursively using an expanding window of observations and a two-year initialization period. Given this, a QC forecast is generated from (13) and used to compute the realized currency betas from Proposition 3. These three adaptive dynamic hedging (ADH) methods are labeled ADH-06, ADH-10, and ADH-HF to indicate the FX set for daily data-based QC estimates and high-frequency data-based estimates, respectively. In addition, a standard MGARCH model is included as a benchmark in the same G06 framework. This model is nested in the setting (13) by replacing $\hat{\mathbf{\Sigma}}_t$ with the outer product of returns (weekly or monthly, according to rebalancing frequency) and is labeled ADH-SM for Standard MGARCH.

These dynamic strategies are compared to a fully hedged baseline portfolio to examine if active currency hedging adds economic value to the existing investments. Moreover, a *real-time* version of the strategy in Campbell et al. (2010) is implemented, using the negative slope from a regression of the portfolio returns on excess FX returns. This is included for both the G06 and G10 sets of currencies, labeled CMV-06 and CMV-10, respectively, and utilizes an expanding window of either weekly or monthly observations, depending on rebalancing frequency. Finally, to examine the relative importance of the dynamic model for intra-day-based QC estimates, a further benchmark is added, in which the QC expectations are based on the average flat-top realized kernel estimates using an expanding window up to and including ten years, and subsequently a rolling ten year window. This simple procedure circumvents the forecasting step (13) and is dubbed the ROL-HF estimator.

As a preliminary gauge of the similarities and differences between strategies, the average currency exposures are provided in Table 2 for each benchmark portfolio. Some interesting discrepancies between average exposures obtained from the various hedging procedures appear. First, for the S&P 500 portfolio, the CMV-06 strategy takes large long positions in the CHF and USD, funded by short positions in CAD and EUR. The ADH-HF procedure holds JPY rather than USD, and the ADH-06 strategy has approximately equal (long) exposures to the JPY and USD. This is consistent with the CHF, JPY and USD being considered as safe haven currencies during episodes of financial turmoil.

Second, the absolute magnitude of the average FX exposures vary, with exposures in CMV-06 larger than those in ADH-06, which in turn are larger than the corresponding in ADH-HF. Third, when the FX set is extended from G06 to G10 for the CMV and ADH strategies, this generates a reduction in the short positions in CAD and EUR (whose exposure actually changes sign), in favor of shorting the AUD and SEK. Moreover, the CMV procedure indicates positive exposure to NOK. Fourth, whereas the average exposures are similar in magnitude and sign for the S&P 500 and EW portfolios, the optimal exposures for the balanced portfolio are significantly reduced. This is consistent with the latter having lower volatility (cf. Table 3 below), implying that the hedging strategies must match a lower volatility target.

Figure 1 depicts the optimal exposures for the ADH-HF procedure applied to the three baseline portfolios. While this underscores the propensity of the hedging strategy to go long CHF and JPY, and funding this by shorting the EUR and CAD, there are interesting dynamic patterns. First, the exposures to the CHF and JPY are much larger during the global financial crisis of 2008-2009 than during the years leading up to and following it. Hence, the strategy takes on larger "safe haven bets" during financial turmoil. Second, the status of the CHF as a hedging currency is dramatically affected by the Swiss National Banks's abandonment of its EUR peg during January 2015, and while the optimal exposure remains positive afterwards, it leaves the hedging demand much smaller. The optimal exposure to the JPY, on the other hand, remains large after the financial crisis, with further spikes occurring in early 2016, when fears of a slowdown of China's economy lead to global stock sell-offs, and during late 2018, when, again, stocks exhibit massive sell-offs over Christmas. Third, the optimal exposure to the USD is flat both before 2010 and after 2015. However, the model suggests to hold USD during the European sovereign debt crisis. Finally, Figure 1 also illustrates that is optimal to hold more than a $\pm 100\%$ exposure to certain currencies over short portions of the sample period. This may not be feasible for all institutional investors. Hence, as a robustness check, we will also examine the performance of a modified version of the ADH-HF strategy, in which, if the optimal exposure is larger than ± 1, it is fixed at ± 1. This constrained version is labeled the CON-HF strategy.

5 Benefits from Dynamic Global Currency Hedging

This section demonstrates that beside displaying interesting time-variation closely tied to important economic events (cf. Figure 1), the estimated dynamic realized currency betas give rise to hedging strategies that provide economic benefits to an investor beyond what is achieved by either fully hedged equity investments or existing optimal hedging strategies from Campbell et al. (2010), which ignore within-period variation in the economic system. Specifically, the gains from dynamic currency hedging are illustrated from three different perspectives. First, standard risk-return results are provided. Second, the statistical significance of the volatility reductions are formally tested. Third, the economic benefits to a risk-averse investor are assessed. The results are described for monthly returns. The corresponding results for weekly returns are very similar and available in Section F of the Online

Appendix, together with robustness checks using the multivariate HAR model.

5.1 Risk-Return Benefits

As an initial gauge of the benefits to dynamic currency hedging, Table 3 reports the annualized mean return, standard deviation, and Sharpe ratio, along with skewness and kurtosis, for an equity investor who implements one of the eight hedging strategies, for each of the three baseline portfolios. There are several striking findings, which are described in detail for the S&P 500 portfolio and subsequently generalized to the remaining two portfolios. First, all strategies except ADH-SM provide non-trivial volatility reductions relative to full hedging. Second, the dynamic ADH-06 and ADH-HF strategies generate larger volatility reductions than the corresponding CMV-06 procedure, with the HF version performing best. Specifically, the ADH-HF strategy delivers a 100BP improvement in volatility over CMV-06, in addition to average returns that are 150BP higher. Consequently, the SR of ADH-HF is 61% higher than those for the fully and CMV-06 hedged portfolios, and 29% higher than for ADH-06. Third, while the CMV-06 procedure delivers substantial volatility reductions, these come at a cost of returns, thus failing to improve the SR relative to the fully hedged portfolio. This is consistent with the empirical findings in Campbell et al. (2010), despite the different setup. Fourth, when comparing equivalent strategies using either the G06 or G10 sets of currencies, the latter is seen to generate the largest volatility reductions, but these are very costly, and the strategies deliver worse overall risk-return performance. From Table 2, this follows since these strategies, on average, substitute a large part of the short position in the CAD to the high-yielding AUD, which is very expensive.

The results in Table 3 demonstrate that a better risk-return trade-off can be achieved by the proposed realized currency beta framework. Moreover, they show that obtaining precise high-frequency data-based estimates of QC deliver the best results, judging by the SR, and that daily data-based QC estimates also provide economic value over standard benchmarks. They further illustrate that the selection of currency universe is important for overall performance, and that active trading in high-yielding currencies can be expensive. Interestingly, the results also indicate that reasonable performance can be achieved by the simple ROL-HF procedure, which has higher SR than ADH-06, suggesting that the input covariance measure is relatively more important than the dynamic model. Finally, and not surprisingly, given Figure 1, the results for ADF-HF and CON-HF are almost identical. The few and small violations of ± 1 exposure have little impact on overall performance.

In sum, the risk-return benefits from applying the realized currency beta hedging strategies, especially those leveraging intra-daily data, appear substantial. Importantly, they are also not confined to the S&P 500 portfolio. The remaining portions of Table 3 show that the same performance pattern appears for the EW and balanced portfolios. Specifically, the relative volatility and SR gains are similar in magnitude for ADH-HF, thus delivering superior risk-return performance. The next two subsections test the significance of the gains and assess them from the perspective of a risk-averse economic agent.

5.2 Significance Testing of Volatility Reductions

This subsection elaborates on the risk-return results by testing whether the volatility reductions achieved by the dynamic hedging strategies are statistically significant. Specifically, the best set of hedging models is determined by their ability to reduce volatility of the baseline portfolio. To this end, the model confidence set (MCS) approach of Hansen, Lunde & Nason (2011) is applied to identify the models in a manner which is robust against multiple testing biases. The testing procedure requires the selection of an ex-post QC proxy and is implemented using both the RC and the flat-top realized kernel estimates. Since the hedging strategies using the G10 universe consistently exhibit lower SRs than those actively trading G06 currencies, and since flat-top realized kernel estimates are only available for the latter, this exercise will focus on the G06 universe. Moreover, the MCS is configured with the `T-max` statistic and a 10% significance level (see Hansen et al. (2011) for details).

Results are collected in Table 4. The table shows the average ex-post volatility for each of the QC measures, and indicates (by an asterisk) whether a given strategy belongs to the MCS. The message is clear. The ADH-06 and ADH-HF strategies consistently deliver the largest volatility reductions, significantly outperforming the fully hedged portfolio, as well as the CMV-06, ADH-SM, and ROL-HF strategies, for all three baseline portfolios. The ADH-HF strategy achieves the most significant volatility reductions when the flat-top realized kernel estimates are used as QC proxy, while the ADH-06 is also included in the set of best performing strategies when the RC estimate is used to calculate ex-post volatility. In fact, the latter is the best strategy for the EW portfolio in this case, although its average volatility edge over ADH-HF is small. As for the risk-return results in Table 3, there is no significant difference between ADH-HF and CON-HF.

The volatility reductions from applying the dynamic ADH-HF strategy rather than fully hedging currency exposure or applying the existing CMV-06 approach to currency hedging are not only economically meaningful, for the S&P 500 portfolio of the order 250BP and 100BP for the two strategies (cf. Table 3), they are also statistically significant, based on the results in Table 4. Moreover, whereas ADH-HF achieves the highest SRs in Table 3 for all baseline portfolios, the results in Table 4 show that similar volatility reductions can be achieved by ADH-06. The advantage in terms of Sharpe ratio from using intra-daily data in the implementation, thus, stems mainly from enhancing the returns to the hedging strategy.

5.3 Economic Benefits from Dynamic Currency Hedging

The economic benefits are assessed via three different measures. First, using the SRs in Table 3, the gain is quantified as the number of BP an investor is better off at a 10% volatility level, that is, by

$$\varpi_{b,s} = (\mathtt{SR}_b - \mathtt{SR}_s) \times 10 \times 100, \tag{14}$$

where $\mathtt{SR}_b$ and $\mathtt{SR}_s$ are the SRs of a benchmark strategy b and an alternative strategy s, respectively. However, as emphasized by Han (2006) and Della Corte et al. (2009), the SR may underestimate the performance of dynamic portfolio strategies. In particular, as the SR is computed using the full sample realized portfolio return and standard deviation, it may not adequately describe the *conditional risk* faced by an investor at each point in time. Hence, following Fleming et al. (2001), the economic value of a benchmark hedging strategy b relative to the alternative s is also assessed by determining the fee that may be subtracted from the hedged portfolio return corresponding to the benchmark each period, while still leaving average utility unchanged, compared to that achieved by investing according to the alternative hedging strategy. In other words, this fee equals the amount a risk-averse investor is willing to pay in order to switch from the alternative strategy s to the hedging benchmark b. Formally, as in Della Corte et al. (2009), let $Z_t^b = 1 + r_t^b$ and $Z_t^s = 1 + r_t^s$ be the payoffs to the benchmark and the alternative hedging strategy, respectively, then the switching fee $\Phi_{b,s}$ solves

$$\sum_{t=T_1+1}^{T} \left\{ (Z_t^b - \Phi_{b,s}) - \frac{\delta}{2(1+\delta)} \left(Z_t^b - \Phi_{b,s} \right)^2 \right\} = \sum_{t=T_1+1}^{T} \left\{ Z_t^s - \frac{\delta}{2(1+\delta)} \left(Z_t^s \right)^2 \right\}. \tag{15}$$

Specifically, (15) equates the average realized period-by-period utility across the benchmark and alternative hedging strategies for an investor with quadratic preferences and relative risk-aversion indexed by the parameter δ. In the empirical application, $\delta \in \{3, 8\}$ is fixed (see Fleming et al. (2001) and Della Corte et al. (2009) for detailed discussions of this preference specification).[21]

Finally, while both $\varpi_{b,s}$ and $\Phi_{b,s}$ speak to the (conditional) properties of means and volatilities for returns to the currency hedging strategies, it is worth examining the robustness to higher moments. For example, from Table 3, kurtosis and negative skewness are reduced by an expansion of the currency universe, for otherwise identical stategies. Thus, the hedging strategies are further evaluated using a measure capturing features of the whole return distribution, specifically, the Omega ratio statistic introduced by Bernardo & Ledoit (2000) and Keating & Shadwick (2002), and studied extensively by Caporin, Costola, Jannin & Maillet (2018). The ratio is defined as

$$\Omega_s(\varrho) = \frac{\mathbb{E}[r_t^s - \varrho | r_t^s > \varrho]}{\mathbb{E}[\varrho - r_t^s | r_t^s \leq \varrho]} \times \frac{1 - F_s(\varrho)}{F_s(\varrho)}, \tag{16}$$

with the conditional expectation taken with respect to a threshold ϱ, and $F_s(\varrho)$ the cumulative distribution function. Hence, the statistic quantifies the ratio of favorable and unfavorable outcomes with respect to a given threshold, where $\varrho \in \{0, 0.1\}$ is selected, following Caporin et al. (2018). As above, define $\Omega_{b,s}(\varrho) = \Omega_s(\varrho)/\Omega_b(\varrho)$ to assess the omega ratio relative to a benchmark strategy.

Table 5 reports the estimates of $\varpi_{b,s}$, $\Phi_{b,s}$, and $\Omega_{b,s}(\varrho)$ using ADH-HF as the benchmark strategy for all three baseline portfolios. Interestingly, when considering the results for gains in SR, $\varpi_{b,s}$, the ADH-HF strategy delivers 210-250 BP improvements over full hedging, 200-350 BP over the existing CMV procedures, and 150-190 BP over ADH-06. These numbers are substantial, clearly illustrating the value of using intra-daily data and the proposed economic model that actively utilizes within-

[21] Fleming et al. (2001, 2003) fix $\delta \in \{1, 10\}$ and Della Corte et al. (2009) let $\delta \in \{2, 6\}$. A higher value of δ, such as 10, implies that the investor is willing to pay a higher fee for strategies that reduce volatility.

period information to carry out dynamic currency hedging. The differences are smaller relative to the ROL-HF strategy, 70-100 BP, further underscoring the importance of QC measure quality relative to dynamic model specification. The ADH-HF and CON-HF strategies are economically identical.

When turning to the corresponding estimates of switching fees, $\Phi_{b,s}$, thus speaking to the relative conditional risk of the hedging strategies, the qualitative rankings are identical.[22] However, there are differences between the absolute magnitudes of the $\varpi_{b,s}$ and $\Phi_{b,s}$ estimates. The results for the S&P 500 and EW equity portfolios indicate even larger return differences than before. For the balanced portfolio, on the other hand, the estimated switching fees are generally lower. However, since these fees are always greater than the 96 BP reported for the ADH-06 strategy, they remain economically large. The exception is the ROL-HF strategy, for which the relative gain is 35-60 BP, again demonstrating the value arising from using the precise flat-top realized kernel estimates of QC. The smaller switching fees for the balanced portfolio may readily be explained by the latter having substantially lower volatility than both equity portfolios. Hence, a risk-averse investor places a smaller premium on further volatility reductions, despite the $\varpi_{b,s}$ measure indicating that the risk-return gains are large.

Finally, the $\Omega_{b,s}(\varrho)$ estimates show that these conclusions remain robust when allowing higher-order return moments to impact the economic evaluation. The ADH-HF strategy, and its constrained version, consistently outperform other approaches to currency hedging. In sum, the economic benefits achieved by the dynamic realized currency beta hedging strategies, especially when implemented using intra-daily data, are economically large, and volatility reductions statistically significant.

6 RCB and Traditional Currency Investment Styles

The previous section demonstrates that the investor can achieve substantial gains in baseline portfolio performance by supplementing the latter with a tactical foreign exchange rate overlay based on the proposed realized currency beta (RCB) hedging procedure, especially, the dynamic ADH-HF approach. To synthesize and further explore these findings, this section relates the performance of the zero net currency portfolio from the ADH-HF strategy to traditional currency investment styles, in particular, carry, momentum, and value strategies. Following, among others, Lustig et al. (2011), Menkhoff, Sarno, Schmeling & Schrimpf (2012*a*, 2012*b*), and Asness et al. (2013), the carry trade is constructed by sorting on interest rate differentials, momentum on three month excess currency returns, and the value trade by betting on mean-reversion against five-year average returns. Each of these strategies is implemented with standard rank-based weights and using the G10 set of currencies.

Table 6 reports summary statistics of the FX strategies, as well as correlations between them, with the currency overlay in the ADH-HF approach being designed to hedge the S&P 500 portfolio.[23] Moreover, their respective cumulative returns are depicted in Figure 2. In line with prior work, the

[22] Note that the switching fee estimates in Table 5 are new to the currency hedging literature and, thus, provide further perspectives on the economic value a risk-averse investor receives from implementing also the existing optimal hedging procedures in Glen & Jorion (1993), de Roon et al. (2003), and Campbell et al. (2010).

[23] The results are very similar when applying the other two baseline portfolios and are, thus, omitted for brevity.

carry and momentum investment strategies are very profitable until the 2008-2009 global financial crisis, when, specifically, carry trades exhibit a massive drawdown during the fall of 2008, culminating with a 12.4% loss in October. The strategy subsequently recovers, but has only delivered modest returns since 2010. This observed tail behavior of carry trades is consistent with prior findings in the literature, documenting that the strategy is exposed to "crash risk", e.g, Brunnermeier et al. (2009), Burnside et al. (2011), and Menkhoff, Sarno, Schmeling & Schrimpf (2012*a*). Momentum, on the other hand, acts as a hedge during the financial crisis, delivering a positive return of 11.5% in October 2008, but has performed abysmally since 2012. Finally, the value trading strategy similarly performed well during the financial crises, but has been largely flat since 2010.

When traditional FX investment styles are compared to the ADH-HF overlay, the latter is also seen to provide protection during the financial crisis, returning 13.5% in October 2008. Interestingly, when comparing its performance to carry trades, keeping in mind the time-varying exposures of ADH-HF from Figure 1, the long positions in traditional carry funding currencies, such as CHF and JPY, suggest that the strong performance of ADH-HF is, at least partially, funded by carry traders unwinding their positions. In addition, the hedging strategy provides protection during other episodes of financial turmoil, such as those surrounding the downgrade of Greece's sovereign debt to junk bond status during the European sovereign debt crisis in April-May 2010, the Brexit vote in June 2016, and the "bloody Christmas" equity sell-off in 2018, delivering returns of 6.4%, 5.0% and 4.5%, respectively, during these episodes. In contrast, momentum and value strategies return $-(1.4, 5.9, 2.7)\%$ and $(0.6, 1.5, 2.2)\%$ over the same time intervals. Hence, as intended, the ADH-HF overlay provides a robust hedge for an equity portfolio when protection is needed the most, and it performs better than existing FX strategies during such episodes. Moreover, the hedging strategy has lost less, on average, during bull market periods than it has gained during sell-offs, thus providing a positive return on average, albeit not as high as carry. Finally, the correlation between carry and the ADH-HF overlay is strongly negative, at -0.62 (cf. Table 6), suggesting that a profitable trading strategy could be constructed by combining the two. However, asset allocation among currency investment styles is beyond the scope of the present paper.

7 Conclusion

This paper proposes a model for *discrete-time* currency hedging based on *continuous-time* movements in portfolio and foreign exchange rate returns. The vector of optimal currency exposures is shown to be given by the negative realized regression coefficients computed from a one-period conditional expectation of the intra-period quadratic covariation matrix for portfolio and foreign exchange rate returns, labeled the *realized currency betas*. The theoretical model, hence, facilitates the design of dynamic hedging strategies that depend exclusively on the evolution of the intra-period quadratic covariation matrix. This implies that interest differentials have *no* asymptotic impact on optimal currency hedging demands, and that an investor should sample observations as frequently as possible in fixed time intervals between portfolio rebalances to improve the accuracy of the quadratic covariation estimates. Both implications contrast with prior theoretical results in the extant currency hedging literature, which assume that assets are observed at the same frequency as that at which the portfolio is being rebalanced. Moreover, since the proposed strategies only use information about the covariance between exchange rate and portfolio returns, not about local trends in the former, they are notably different from traditional currency investment styles, such as carry, momentum, and value.

The realized currency beta hedging strategies are implemented using modern, yet simple, non-parametric techniques to accurately measure the historical quadratic covariation between assets and, subsequently, capture their dynamic evolution. Methodologically, this procedure addresses two general caveats in the literature. First, there has been a lack of dynamic modeling when computing optimal currency exposures, except when tied to slowly varying conditioning variables, such as past interest rate differentials. Second, previous work has been plagued by the use of forward-looking information when estimating optimal exposures, thus providing investors with the benefit of hindsight. Addressing both caveats is important for accurate assessments of intertemporal currency hedging demands and real-time investment decisions.

In an extensive empirical analysis, the use of the new hedging strategies, based on realized currency betas, produces novel results: (*i*) The optimal currency exposures display substantial time-variation, which is tied to important economic events, such as the 2008-2009 global financial crisis, the European sovereign debt crisis, and the "bloody Christmas" 2018 global stock sell-off. (*ii*) The proposed dynamic hedging strategies produce statistically significant, as well as economically substantial, volatility reductions for international equity portfolios and a balanced fixed income-equity portfolio, compared to either fully hedging currency exposure or applying existing approaches to (static) optimal hedging. (*iii*) These volatility reductions come without sacrificing returns, especially when implemented using intra-daily data, delivering Sharpe ratios 61% larger than key benchmarks. (*iv*) The estimated economic gains to the new hedging strategies are substantial, at 120-465 annual basis points over full hedging – depending on baseline portfolio and investor risk-aversion – and 120-520 basis points over existing static approaches. (*v*) The quality of the input quadratic covariation measure seems to be more important for designing profitable realized currency beta hedging strategies than the dynamic model specification or an expansion of the hedging universe beyond the G06 currencies. (*vi*) The currency overlay behind the dynamic realized currency beta investment strategy is negatively correlated with the FX carry trade, and only modestly correlated with momentum and value investments. Interestingly, the empirical analysis suggests that carry traders, at least partially, fund the strong performance of the proposed dynamic strategy during global financial crisis of 2008-2009.

Summary Statistics for Excess FX Returns									
	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
mean(IR)	2.16	0.15	-1.61	-0.55	0.44	-2.05	0.91	2.51	-0.34
mean(SR)	0.31	0.52	2.32	0.44	-1.01	-0.33	-0.52	1.20	-0.54
mean(ER)	2.47	0.67	0.71	-0.12	-0.58	-2.37	0.39	3.72	-0.88
Std. Dev.	12.43	8.75	11.12	9.60	9.10	9.75	11.75	12.65	11.75
Skewness	-0.34	-0.10	3.73	0.05	-0.76	0.07	-0.15	-0.30	-0.04
Kurtosis	12.66	6.01	120.73	4.64	13.71	7.11	5.62	5.82	5.56
SR	0.20	0.08	0.06	-0.01	-0.06	-0.24	0.03	0.29	-0.07
mean(BA)	6.92	4.45	6.47	3.08	4.22	3.78	15.13	12.76	11.58
Std. Dev.	6.60	3.90	7.84	3.20	5.67	5.00	17.33	11.61	10.83
Q05	1.52	1.05	1.21	0.82	0.68	1.04	3.67	3.99	4.08
Q95	17.95	9.81	19.31	9.45	12.24	9.66	41.47	30.62	29.79

Table 1: Excess FX Returns. This table presents annualized summary statistics for daily excess FX returns, in logarithms, from January 2000 through December 2019. Moreover, the mean excess return (ER) is decomposed into interest rate (IR) differentials and spot FX returns (SR). The bottom part of the table provides the mean and standard deviation as well as the 5% and 95% quantiles of estimated bid-ask (BA) spreads in basis poins (BP). Specifically, the latter are computed as $10,000(\texttt{ask} - \texttt{bid})/((\texttt{ask} + \texttt{bid})/2)$ for the forward prices.

Average Exposures for Currency Hedging Strategies										
	CAD	CHF	EUR	GBP	JPY	AUD	NZD	SEK	NOK	USD
S&P 500										
CMV-06	-0.96	0.83	-0.69	0.09	-0.08	-	-	-	-	0.81
CMV-10	-0.45	0.70	0.34	0.17	-0.04	-0.40	-0.03	-1.03	0.22	0.53
ADH-SM	-0.99	0.53	-0.46	0.01	0.06	-	-	-	-	0.86
ADH-06	-0.69	0.61	-0.49	-0.06	0.34	-	-	-	-	0.29
ADH-10	-0.42	0.54	0.05	0.04	0.32	-0.27	-0.12	-0.32	-0.05	0.23
ADH-HF	-0.43	0.30	-0.26	-0.03	0.39	-	-	-	-	0.04
ROL-HF	-0.42	0.43	-0.35	-0.01	0.36	-	-	-	-	-0.01
EW										
CMV-06	-0.99	1.04	-0.91	0.30	0.07	-	-	-	-	0.48
CMV-10	-0.48	0.93	0.14	0.39	0.11	-0.39	-0.05	-1.00	0.17	0.20
ADH-SM	-1.39	0.76	-0.58	0.13	0.12	-	-	-	-	0.96
ADH-06	-0.63	0.59	-0.44	0.01	0.30	-	-	-	-	0.17
ADH-10	-0.36	0.52	0.13	0.10	0.29	-0.30	-0.04	-0.33	-0.11	0.10
ADH-HF	-0.48	0.37	-0.32	0.01	0.44	-	-	-	-	-0.02
ROL-HF	-0.47	0.46	-0.38	0.03	0.44	-	-	-	-	-0.08
Balanced										
CMV-06	-0.54	0.45	-0.44	0.09	-0.12	-	-	-	-	0.56
CMV-10	-0.26	0.38	0.13	0.13	-0.10	-0.21	-0.01	-0.60	0.14	0.40
ADH-SM	-0.57	0.27	-0.28	0.04	-0.03	-	-	-	-	0.56
ADH-06	-0.39	0.32	-0.27	-0.04	0.13	-	-	-	-	0.25
ADH-10	-0.23	0.28	0.03	0.02	0.12	-0.14	-0.09	-0.18	-0.03	0.21
ADH-HF	-0.24	0.15	-0.15	-0.02	0.17	-	-	-	-	0.10
ROL-HF	-0.23	0.22	-0.20	-0.01	0.16	-	-	-	-	0.07

Table 2: Average Exposures. This table presents the average exposures for seven different approaches to currency hedging and the three different baseline portfolios using a monthly rebalancing frequency. The baseline portfolios are the S&P 500, an equal weighted (EW) basket of DAX, FTSE 100, and S&P 500 futures contracts, and a balanced portfolio with 60% S&P 500 and 40% 10-year US Treasury bond futures. The different hedging strategies are detailed in Section 4.2. The sample spans January 2000 through December 2019. The dynamic covariance models are estimated using an expanding window with a two-year initialization period.

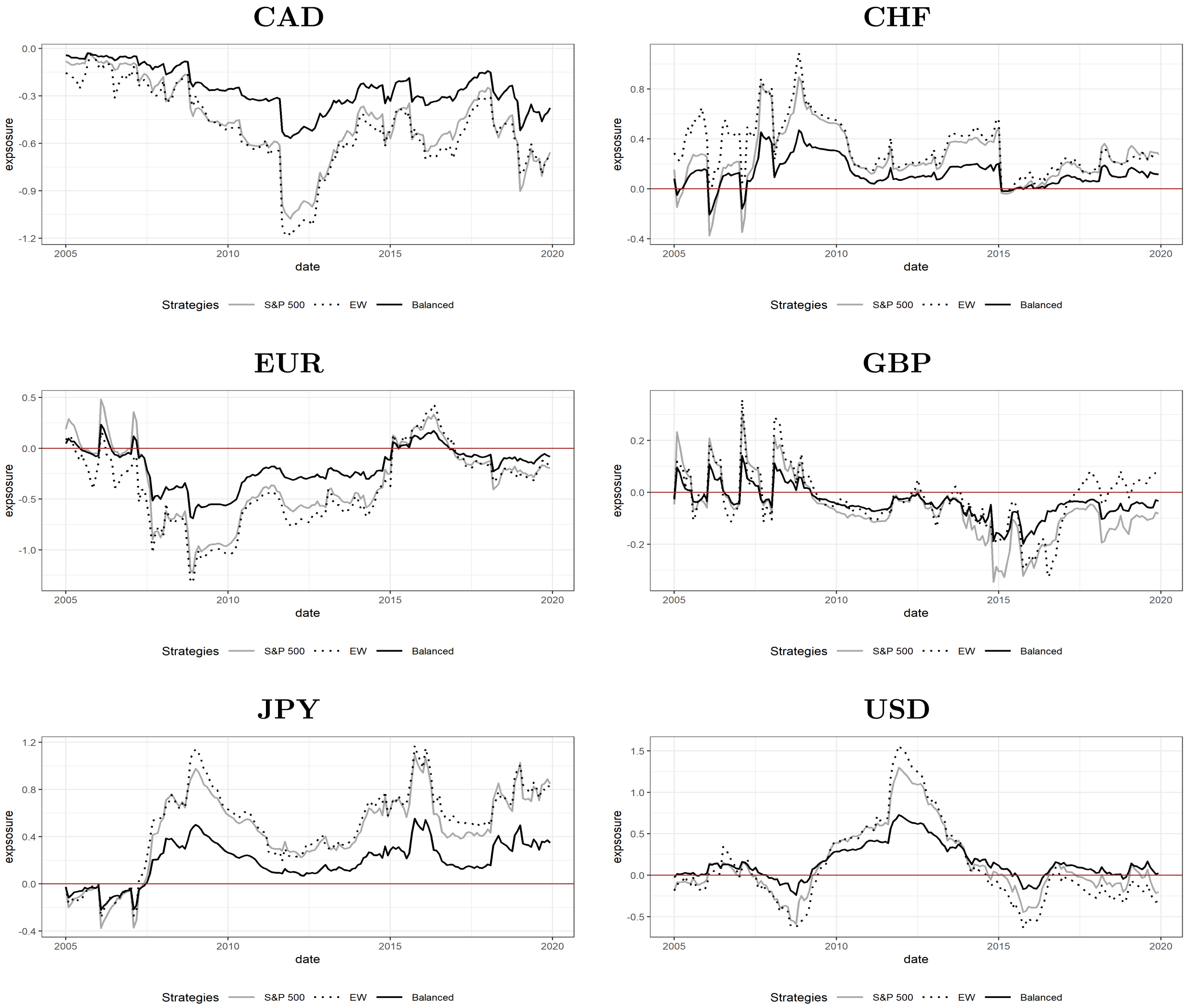

Figure 1: Exposure plots. This figure depicts the optimal currency exposures to the CAD, CHF, EUR, GBP, JPY, and USD, respectively, computed using the ADH-HF procedure for three different baseline portfolios and a monthly rebalancing frequency. The exposures are depicted for January 2005 through December 2019.

Risk-Return Performance for Currency Hedging Strategies									
	full	CMV-06	CMV-10	ADH-SM	ADH-06	ADH-10	ADH-HF	ROL-HF	CON-HF
S&P 500									
mean	5.93	5.31	4.39	2.71	6.34	4.14	7.87	7.24	8.00
StdDev	14.29	12.79	12.47	15.49	12.22	11.58	11.76	12.85	11.79
Skewness	-0.92	-0.78	-0.37	-1.54	-0.74	-0.43	-0.81	-0.81	-0.82
Kurtosis	5.23	4.89	3.77	9.42	6.73	5.97	6.33	4.66	6.29
SR	0.41	0.41	0.35	0.17	0.52	0.36	0.67	0.56	0.68
EW									
mean	4.40	3.41	2.10	1.29	4.02	1.81	6.73	6.01	6.90
StDev	14.96	14.02	13.80	16.59	12.74	12.23	13.27	13.99	13.33
Skewness	-0.93	-0.54	-0.43	-0.16	-0.45	-0.65	-0.58	-0.57	-0.55
Kurtosis	5.24	6.09	5.33	4.18	7.35	6.31	7.77	5.87	7.58
SR	0.29	0.24	0.15	0.08	0.32	0.15	0.51	0.43	0.52
Balanced									
mean	5.03	4.89	4.38	2.92	5.05	3.71	5.96	5.73	5.96
StDev	8.15	7.37	7.20	8.59	7.13	6.65	6.88	7.49	6.88
Skewness	-1.00	-0.39	-0.19	-1.52	-0.40	-0.13	-0.73	-0.80	-0.73
Kurtosis	6.55	4.13	3.79	8.60	6.61	6.52	6.38	5.35	6.38
SR	0.62	0.66	0.61	0.34	0.71	0.56	0.87	0.76	0.87

Table 3: Risk-return performance. This table presents annualized risk-return performance for nine different approaches to currency hedging and three different baseline portfolios using a monthly rebalancing frequency. The baseline portfolios are the S&P 500, an equal weighted (EW) basket of DAX, FTSE 100, and S&P 500 futures contracts, and a balanced portfolio with 60% S&P 500 and 40% 10-year US Treasury bond futures. The different hedging strategies are detailed in Section 4.2. The sample spans January 2000 through December 2019. The dynamic covariance models are estimated using an expanding window with a two-year initialization period.

Volatility Reductions for Currency Hedging Strategies							
	full	CMV-06	ADH-SM	ADH-06	ADH-HF	ROL-HF	CON-HF
S&P 500							
FTRK	15.47	15.51	16.26	13.72	13.29⋆	14.34	13.30⋆
RC	15.37	13.85	14.61	12.35⋆	12.51⋆	13.69	12.52⋆
EW							
FTRK	16.02	15.71	17.65	13.69	13.49⋆	14.75	13.50⋆
RC	14.83	13.75	15.55	12.28⋆	12.43	13.52	12.45
Balanced							
FTRK	8.70	9.02	9.42	7.99	7.74⋆	8.23	7.74⋆
RC	8.75	8.28	8.60	7.41⋆	7.46⋆	8.03	7.46⋆

Table 4: Volatility reductions. This table presents the average, annualized, ex-post volatility for seven approaches to currency hedging and three baseline portfolios using a monthly rebalancing frequency. The ex-post QC measures are constructed using either realized covariance (RC) based on daily data or the flat-top realized kernel (FTRK) based on intra-daily data. The baseline portfolios are the S&P 500, an equal weighted (EW) basket of DAX, FTSE 100, and S&P 500 futures contracts, and a balanced portfolio with 60% S&P 500 and 40% 10-year US Treasury bond futures. The hedging strategies are detailed in Section 4.2. An asterisk (⋆) signifies that the hedging strategy belongs to the 10% model confidence set of Hansen et al. (2011). The sample spans January 2000 through December 2019. The dynamic covariance models are estimated using an expanding window with a two-year initialization period.

Economic Benefits of Currency Hedging Strategies									
	full	CMV-06	CMV-10	ADH-SM	ADH-06	ADH-10	ADH-HF	ROL-HF	CON-HF
S&P 500									
$\varpi_{b,s}$	255.06	254.74	318.02	494.84	151.06	311.96	0.00	106.49	-8.72
$\Phi_{b,s}(3)$	294.68	294.97	374.87	669.15	170.37	366.85	0.00	104.57	-11.31
$\Phi_{b,s}(8)$	465.80	360.71	419.66	923.56	199.82	356.25	0.00	175.52	-9.22
$\Omega_{b,s}(0)$	0.82	0.82	0.78	0.69	0.90	0.79	1.00	0.92	1.01
$\Omega_{b,s}(0.1)$	0.84	0.82	0.78	0.70	0.90	0.79	1.00	0.93	1.01
EW									
$\varpi_{b,s}$	213.25	264.07	355.11	429.66	191.55	358.77	0.00	77.41	-10.20
$\Phi_{b,s}(3)$	305.27	363.10	484.79	692.49	250.14	451.80	0.00	101.56	-14.15
$\Phi_{b,s}(8)$	427.19	415.37	520.96	936.57	214.06	384.15	0.00	152.88	-10.30
$\Omega_{b,s}(0)$	0.83	0.80	0.74	0.70	0.85	0.74	1.00	0.93	1.01
$\Omega_{b,s}(0.1)$	0.84	0.80	0.74	0.71	0.85	0.74	1.00	0.93	1.01
Balanced									
$\varpi_{b,s}$	248.43	203.09	258.29	526.20	158.09	308.95	0.00	101.99	0.00
$\Phi_{b,s}(3)$	121.73	118.13	165.19	343.91	96.47	220.81	0.00	36.97	0.00
$\Phi_{b,s}(8)$	170.97	136.07	176.83	410.97	105.57	212.79	0.00	59.87	0.00
$\Omega_{b,s}(0)$	0.84	0.85	0.82	0.68	0.91	0.80	1.00	0.93	1.00
$\Omega_{b,s}(0.1)$	0.86	0.86	0.82	0.69	0.90	0.79	1.00	0.94	1.00

Table 5: Economic gains. This table presents estimates of the economic gains arising from applying the ADH-HF strategy in place of eight alternative currency hedging strategies. The gains are computed for three baseline portfolios and a monthly rebalancing frequency. Specifically, as described in Section 5.3, $\varpi_{b,s}$ quantifies the SR difference, $\Phi_{b,s}(\delta)$ the switching fee for a risk-averse investor with quadratic utility function and risk-aversion parameter $\delta \in \{3, 8\}$, and $\Omega_{b,s}(\varrho)$ the relative Omega ratio for $\varrho \in \{0, 0.1\}$. The return differences $\varpi_{b,s}$ and $\Phi_{b,s}(\delta)$ are quoted in annualized basis points (BP), and an Omega ratio less than one indicates that ADH-HF achieves a higher value. The baseline portfolios are the S&P 500, an equal weighted (EW) basket of DAX, FTSE 100, and S&P 500 futures contracts, and a balanced portfolio with 60% S&P 500 and 40% 10-year US Treasury bond futures. The different hedging strategies are detailed in Section 4.2. The sample spans January 2000 through December 2019. The dynamic covariance models are estimated using an expanding window with a two-year initialization period.

	Summary Statistics					Correlation			
	mean	Std. Dev.	Skewness	Kurtosis	SR	RCB	MOM	CARRY	VALUE
RCB	1.95	9.15	1.35	7.97	0.21	1.00	0.21	-0.62	0.24
MOM	-1.24	7.39	0.52	6.23	-0.17	0.21	1.00	-0.10	-0.03
CARRY	3.38	8.01	-0.88	6.86	0.42	-0.62	-0.10	1.00	-0.60
VALUE	0.34	7.30	1.42	9.28	0.05	0.24	-0.03	-0.60	1.00

Risk-Return Performance for FX Trading Strategies

Table 6: FX trading strategies. This table presents annualized risk-return performance for four different FX trading strategies using a monthly rebalancing frequency. These are carry, momentum, and value investments based on the G10 currency set, as well as the FX overlay of the ADH-HF realized currency beta (RCB) hedging strategy for the S&P 500 portfolio. Moreover, correlations between the strategies are reported. The sample spans January 2000 through December 2019. The dynamic covariance model is estimated using an expanding window with a two-year initialization period.

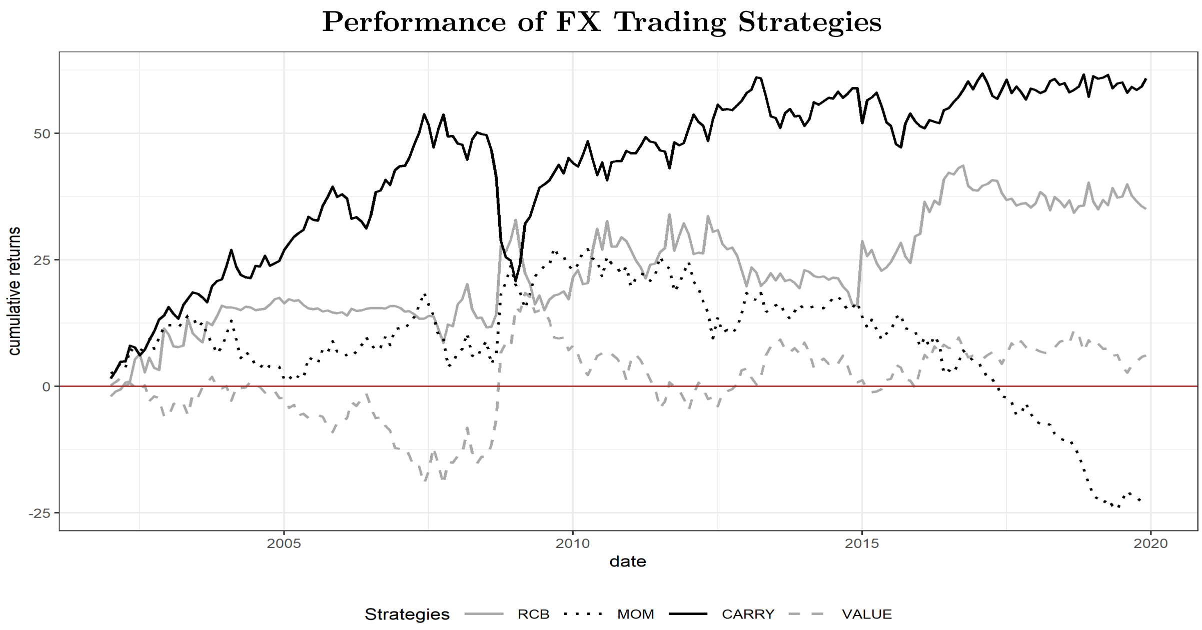

Figure 2: Exposure plots. This figure depicts cumulative (log-)returns to four different FX trading strategies using a monthly rebalancing frequency. These are carry, momentum, and value investments based on the G10 currency set, as well as the FX overlay of the ADH-HF realized currency beta (RCB) hedging strategy for the S&P 500 portfolio. The sample spans January 2000 through December 2019. The dynamic covariance model is estimated using an expanding window with a two-year initialization period.

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