Speculation, Sentiment, and Interest Rates

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ABSTRACT

We compare the implications of speculation versus hedging channels for bond markets in heterogeneous agents economies. Treasuries command a significant risk premium when optimistic agents speculate by leveraging their positions using bonds. Disagreement drives a wedge between marginal agent vs. econometrician beliefs (sentiment). When speculative demands dominate, the interaction between belief heterogeneity and sentiment helps rationalize several puzzling characteristics of Treasury markets. Empirically, we test model predictions and find that larger disagreement (i) lowers the risk-free rate, (ii) raises the slope of the yield curve; and (iii) with positive sentiment increases bond risk premia and makes its dynamics counter-cyclical.

Keywords: Fixed income, Bond Risk Premia, Heterogeneous Agents, Speculation.

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Daily trading volume in U.S. Treasury bonds is about ten times daily U.S. GDP; in 2018, daily Treasury trading volume was \$882 billion (\$408 billion in cash Treasury notes and bond and \$474 billion of notional face value in Treasury futures). The extent of this daily turnover is large compared to the total notional stock of all Treasury Bonds, which is \$16 trillion (all maturities).[1] Regulators have often commented that this amount of trading is unlikely to be exclusively due to hedging demands and strategic portfolio allocation decisions. For this reason, the Commodity Exchange Act (CEA) has placed the Commodity and Futures Trading Commission (CFTC) in the position to actively monitor the amount of speculation in futures markets and impose agents specific limits on the size of speculative positions.

Surprisingly, however, little is known about the bond pricing implications of speculation. At the same time, a vast literature documents how traditional term structure models assuming homogeneous agents find it difficult to reproduce several properties of bond returns. For instance, it is known that benchmark consumption-based models can produce either pro-cyclical real short term interest rates (as in the data) or upward sloping real term structure, but not both at the same time. Indeed, when short rates are pro-cyclical bonds can be used to hedge aggregate consumption shocks so should earn a negative risk premium, which makes it difficult to match the observed sign of bond risk premia.[2] Since a significant component of trading in bond markets is done by institutional investors acting as agents, a natural question to ask is whether speculation can help us understand how bond markets function.

We introduce multiple agents who disagree about the consumption process in an otherwise standard economy that borrows characteristics from the long-run risk literature. The real growth rate contains a highly persistent, low volatility, time-varying conditional mean component. Hansen, Heaton, and Li (2008) argue that if long run risks exist, then agents within these models must face significant measurement challenges in quantifying the long-run risk-return trade-off.[3] We embed this observation within the model by allowing agents to deviate from full information rational expectations by holding subjective models for the long-run growth rate of the economy.[4] This

[1] Source: `https://www.cmegroup.com/education/featured-reports/interest-rate-futures-liquidity-update.html` and `www.sifma.org/research/statistics.aspx`

[2] In the context of long-run risk models, see, for instance, the discussion in Beeler and Campbell (2009). For a survey of this literature see Duffee (2012) and references therein. For studies of the real term structure properties see Pflueger and Viceira (2013) or Ermolov (2018).

[3] Hansen, Heaton, and Li (2008) write that *'the same statistical challenges that plague econometricians presumably also plague market participants'*.

[4] Disagreement about the state of the economy is often discussed in the financial press. For example, to quote a recent article, 'Whether rates will be high or low a few years from now has very little to do with what the Fed does this week. It has quite a lot to do with what happens to forces deep inside the economy that are poorly understood

induces agents to demand different portfolios of state contingent claims, which generates trade in equilibrium, and gives rise to a set of implications.

The first implication is that speculative trading among agents with different beliefs leads to an endogenous ex-post redistribution of wealth toward those agents whose models displayed better accuracy in the past. Suppose that an initial set of agents were optimistic. A negative shock to aggregate endowment induces a wealth redistribution away from these agents. Thus, because of their own trading activity, agents's beliefs become an endogenous source of individual specific consumption risk which affects agents' marginal valuations.

The second effect is due to variation in relative wealth. Indeed, in the previous example, the ex-post redistribution of wealth reduces the economic weight of the optimist and shifts the marginal agent's beliefs towards those who were relatively pessimistic. Thus, depending on the distribution of wealth, the beliefs of the marginal agent differ from those in a homogeneous belief economy. We refer to this bias as market "sentiment".

The previous two effects are amplified when risk aversion is low (risk tolerance is high), since this increases the willingness of agents to speculate on their beliefs. Moreover, the magnitude of these effects depends on the interaction between speculative demand (risk aversion) and the distribution of wealth-weighted beliefs (sentiment). When risk aversion is low and optimists have a sufficient share of the wealth, heterogeneous belief economies are characterised by an upward sloping yield curve, pro-cyclical short-term interest rates, and realistic positive bond risk premia. This is interesting given that these properties cannot be easily generated in an equivalent economy with homogeneous investors without introducing ad-hoc inflation dynamics. Finally, when optimists also have lower risk aversion than pessimists, this effect is amplified even further.

The final prediction we explore relates to risk premia. In a heterogeneous beliefs equilibrium, bond risk premia are equal to: (risk aversion $\times$ aggregate consumption volatility - sentiment) $\times$ interest rate volatility. When risk aversion is small (or volatility of aggregate consumption is small) but sentiment is large and positive (when optimists have a larger wealth share) bond risk premia are dominated by the product of sentiment and interest rate volatility. Depending on the sign of sentiment, the price of risk can switch sign and become negative. Thus, since interest rates are procyclical, long-term bonds are unattractive to the optimists since they prefer assets that are positively correlated with the state of the economy. As a consequence, in equilibrium long-term

and extremely hard to forecast.' The same article highlights the substantial disagreement among economists about long-term growth and interest rates. New York Times 15/12/2015.

bond prices drop to support larger (positive) expected excess returns to compensate their relative unattractiveness.

We estimate the model by Simulated Method of Moments (SMM) using nominal U.S. Treasury zero-coupon bond data for maturities between 3 months and 5 years over the sample period 1962.1 - 2019.1. The set of moment conditions includes both properties of the consumption dynamics and of bond yields. To isolate the specific contribution of the speculative channel, we assume inflation to follow a non-distortionary process that does not affect consumption growth. Thus, in the economy we study, bond risk premia are driven by real consumption growth risks.

We use the over-identifying set of moment restrictions from the structural model to run a specification test based on the asymptotic chi-square statistic proposed by Lee and Ingram (1991), which is the SMM analogue of the Hansen (1982) J_T-statistic. The null hypothesis that beliefs are irrelevant for bond prices is strongly rejected at a *p-value* less than 1%. The estimated parameters measuring the extent of heterogeneity in beliefs are both economically and statistically significant. At the estimated parameter values the model can reproduce an upward sloping term structure, procyclical short term interest rates and positive bond risk premia. Moreover, estimates of the parameters that capture disagreement (the long-run growth rate of the economy and the instantaneous correlation between consumption and growth rate shocks) are statistically significantly different from zero. Finally, the estimated value of the risk aversion parameter γ is lower than 1, which suggests that the speculative channel is important to generate the properties of the yield curve observed in the data.

In the second half of the paper, we tease out and empirically study joint predictions of speculative channel versus hedging channels. We construct a data set on the distribution of expectations from professional forecasters about real GDP growth and inflation. This dataset is unique in that it records dis-aggregated forecaster specific projections from a panel of financial institutions, spans the period January 1988 to January 2020, and it is based on a large and stable cross-section. Disagreement is obtained from the cross-sectional dispersion in 1-quarter ahead out GDP growth rates. To proxy for sentiment, we borrow from the index of Huang, Jiang, Tu, and Zhou (2015), which develops and updates the index of Baker and Wurgler (2006) by separating the components of sentiment which relate to expected returns from noise.[5] The following results emerge.

First, we find strong evidence that nominal short-term interest rates are negatively related

[5] Huang, Jiang, Tu, and Zhou (2015) and Baker and Wurgler (2006) sentiment indices are based on 5 individual sentiment proxies variables: The value-weighted dividend premium, first-day returns on IPOs, IPO volume, closed-end fund discount, equity share in new issues. Our results are quantitatively similar regardless of which index is employed.

to disagreement. Changes in disagreement about real economic growth are negatively correlated with changes in short term interest rates, with a correlation coefficient of -25%. Moreover, after controlling for consensus expectations about real growth and inflation, the slope coefficient of a regression of changes in the 3 month yield on changes in disagreement about GDP growth is negative and statistically significant at the 5% confidence level and the R^2 range between 16% and 25%. The link is also economically significant as the dynamics of disagreement on GDP can explain about a one fifth of standard deviation in 3-month changes, which is quantitatively similar to the effect coming from expected inflation or GDP growth. This result is consistent with the prediction that the marginal investor is risk tolerant.

Second, we find a strong link between real disagreement and the slope of the term structure. Increases in real disagreement load positively on nominal forward spreads (forward rates minus the one-year yield). The slope coefficient of a regression that controls for consensus expectations about inflation and economic growth yields slope coefficients on real disagreement that are economically large and significant at the 1% confidence level for all forward maturities. Moreover, when we examine the link between disagreement and real yields, we find that, consistent with the results for forward rates on nominal bonds, the relationship between real forward spreads and real disagreement continues to be statistically significant at the 1% confidence level for all maturities. Again, the sign of the slope coefficients are consistent with models in which the speculative channel plays a significant role ($\gamma < 1$). The R^2 are quite large, ranging between 13% (nominal forward spread) and 42% (real forward spread).

The third set of results relates to bond risk premia implications and are cast in the context of predictability regressions. Real disagreement predicts one year holding period excess bond returns at large degrees of statistical confidence after controlling for disagreement about inflation. Economically, a 1-standard deviation increase to real disagreement implies a 0.20-standard deviation increase in expected excess returns on 5-year bonds. When we interact disagreement and sentiment, we find that both variables contribute significantly to explain the dynamics of 1 year ahead bond excess returns. In periods of optimism, larger disagreement implies larger expected excess returns. The opposite holds when sentiment is negative. This highlights the non linearity of the speculative channel. Depending on the level of the interaction between optimism and disagreement, bond risk premia can switch sign. Consistent with the data, they are larger at the end of recessions when disagreement is largest. The R^2 is larger for 2-year bonds (21%) than for 5-year bonds (12%).

To summarize, while the potential importance of speculation is discussed by the theoretical literature studying principal-agents models in which agents (financial institutions, hedge funds, and proprietary traders) have limited liability and convex incentives, little is known about speculation in bond markets. We show through the lens of a model with heterogeneous risk tolerant agents, and by testing a series of empirical predictions arising from the model, that speculation can be an important determinant of interest rates.

Related Literature: Our paper contributes to the literature that studies asset pricing implications of heterogeneity. Early contributions recognised that speculative motivates can generate trade in equilibrium when investors *agree to disagree* (Harrison and Kreps (1978) and Harris and Raviv (1993)).[6] Specific to bond markets, the importance of heterogeneity is well recognised. Xiong and Yan (2010) show that when log-utility agents disagree bond prices are given by a linear wealth weighted average of fictitious homogeneous economy prices. Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018) derive a prediction that inflation disagreement is positively related to real yields. In contrast, Hong, Sraer, and Yu (2014) study a model and provide empirical evidence that when heterogeneous investors are subject to short sale constraints inflation disagreement lowers long term yields. More recently, Barillas and Nimark (2017) show that belief aggregation, due to heterogeneous private information, can lead to a speculative component in bonds dynamics that is orthogonal to traditional components of the yield curve.

Our paper investigates a different channel that is closer to David (2008) who is the first to study the importance of risk tolerance and speculation for the equity risk premium puzzle. We show that, in contrast to conventional wisdom, bonds are risky in economies where optimistic agents speculate through leveraged positions in bonds. This observation relates to the literature on risk bearing capacity in financial markets. When financial institutions play a role in capital markets, their incentives structure may affect asset prices. For example, when delegated managers earn convex performance-based incentives or do not fully bear the consequences of their decisions, risk shifting may emerge such that agents become risk tolerant and, in some cases, risk seeking (Carpenter (2000), Panageas and Westerfield (2009), Buraschi, Kosowski, and Sritrakul (2014)). Complimenting these studies, this paper shows that speculation among heterogeneous risk tolerant agents plays a role in resolving bond market puzzles.

We also contribute to the large literature studying the role of sentiment in asset prices.[7] Theo-

[6]Additional contributions have focused on heterogeneity in labour income shocks Constantinides and Duffie (1996), beliefs (Scheinkman and Xiong (2003), Basak (2005), Buraschi and Jiltsov (2006), Dumas, Kurshev, and Uppal (2009), Chen, Joslin, and Tran (2012), Atmaz and Basak (2017)), preferences (Wang (1996) Chan and Kogan (2002), and Bhamra and Uppal (2014)), and frictions (Gallmeyer and Hollifield (2008) and Chabakauri (2015)).

[7]For a recent survey of the literature see Zhou (2017).

retically, De Long, Shleifer, Summers, and Waldmann (1990) investigate equilibrium asset prices in an economy populated by both irrational noise traders with erroneous but stochastic beliefs (sentiment) and risk averse rational arbitrageurs. They show that the unpredictability of noise traders' beliefs creates a risk in the price of the asset that limits the desire of rational arbitrageurs to trade against them. The sentiment risk of these noise traders increases asset risk premia. Thereafter, several studies explored the dynamics of investor sentiment and its market impact in the context of equity markets. We show that the sentiment index of Huang, Jiang, Tu, and Zhou (2015), which is based on the index of Baker and Wurgler (2006), once interacted with disagreement, is also important in understanding bond market dynamics.

1. Institutional Oversight of Speculative Demand

Since the 1936 *Commodity Exchange Act* (CEA) the American Congress has enacted a series of legislations that focus on the regulation of speculation. Following passage of the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010, the role of the Commodity Futures Trading Commission (CFTC) has been enhanced and the CFTC can impose limits on the size of speculative positions in futures. Accordingly, central clearing houses must classify and report trading activity according to registered users.[8] Indeed, as part of its market surveillance program, the CFTC compiles specific information on the identity of all participants on futures exchanges, which becomes part of the 'CFTC Commitment of Traders Report'. We use this data to investigate the potential significance of the speculative motives in bond markets.

The Treasury futures market constitutes a large proportion of trade across fixed income markets. Baker, McPhail, and Tuckman (2018) combines transactions data in the cash securities market from TRACE with transactions data in the futures market at the CFTC and find that DV01 risk-adjusted volume across all days is distributed 44% in futures and 56% in cash. The CFTC Commission classifies each trader in the following categories: A Producer is an entity that predominantly engages in the production, processing, packing or handling of a physical commodity and uses the futures markets to manage or hedge risks associated with those activities. A Dealer/Intermediary is an (sell-side) entity that uses futures markets to manage or hedge the risk associated with its flow business. They tend to have matched books or offset their risk across markets and clients. The rest of the market comprises the buy-side, which is divided into three separate categories. Asset

[8]Section 4a(a) of the CEA provides that "for the purpose of diminishing, eliminating, or preventing unreasonable or unwarranted price fluctuations, the Commission (CFTC) may impose limits on the amount of speculative trading that may be done or speculative positions that may be held in contracts for future delivery."

Manager includes mutual funds, insurance companies, endowments, and pension funds. Leveraged funds typically include hedge funds and various types of money managers, including registered commodity trading advisors (CTAs); registered commodity pool operators (CPOs) or unregistered funds identified by CFTC. These traders are engaged in managing and conducting proprietary trading on behalf of speculative clients. Other Reportables include corporate treasuries, mortgage originators, and trade unions.

We use disaggregated data provided by the CFTC to construct separate time series for long and short net positions measured in terms of number of contracts for each category of users.[9] We conservatively assume that 'Asset Managers' and 'Other Reportables' portfolio decisions are not driven by speculative motives but by hedging, wealth shocks, and passive portfolio rebalancing. Thus, we limit our proxy of speculators to the category of 'Leveraged Funds' . Thus, let $\mathcal{R}_t = L_t/(AM_t + O_t)$ be the ratio of the positions of Leveraged funds (L_t) to Asset Managers (AM_t) plus Other Reportables (O_t). This ratio captures the relative proportion of the open interest of traders whose trades are likely motivated by their beliefs versus those motivated by other reasons. The net position of Dealers is, by virtue of their mandate, small.

We combine CFTC data futures and options report, which converts positions in the options market to futures on a delta-equivalent basis. Figure 1 documents the time series of $\mathcal{R}_t$ distinguishing between Long (blue line) and Short (red line) positions since June 2006.[10]

[INSERT FIGURE 1 HERE]

The average ratio is 0.90 (1.27) for the Long (Short) for the 5 years and 1.40 (1.70) for the 2 years Treasury future. In both cases we see extended periods of time when the speculative positions are large and the time-series displays significant variation. The maximum value of the ratio is 2.93 (5.39) for the Long (Short) positions for the 5 years Treasury bond future and 4.56 (7.52) for the 2 year Treasury bond future.

In terms of raw quantities we observe a large long and short speculative positions. For example, on December 18th 2012, Leveraged Money traders were holding 612,000 short contracts in 2-Year Treasury futures versus 78,000 held by Asset Managers. For the 5-year Treasury futures, on December 31st 2018 Leveraged Money held 1,932,000 short contracts versus 461,000 short contacts by Asset Managers, a ratio of more than four to one.

[9] `www.cftc.gov/MarketReports/CommitmentsofTraders/index.htm`

[10] By construction, the long and short positions do not net out to zero due to the existence of other agents with non zero positions at CFTC, such as Dealer/Intermediary and Producers.

In the following, we investigate the term structure implications of speculation in the context of a heterogeneous beliefs models. In the context of the model developed below, the red (blue) line can be interpreted as a proxy for the positions of agents A (agent B), due to their optimism (pessimism) about economic growth and higher (lower) interest rates.

2. Theory

1. Fundamentals

We study an endowment economy where a single consumption good and the nominal price level evolve according to

$$\begin{aligned} dC_t/C_t &= g_t dt + \sigma_c dW_t^c, \\ dQ_t/Q_t &= q dt + \sigma_q dW_t^q, \\ dg_t &= \kappa_g \big(\theta - g_t\big) dt + \sigma_g dW_t^g, \end{aligned} \tag{1}$$

where $g(t)$ is the time-varying growth rate component in consumption, q is a constant inflation rate, and correlated shocks are given by $\langle dW_t^c, dW_t^g \rangle = \rho_{c,g} dt$, $\langle dW_t^q, dW_t^g \rangle = \rho_{q,g} dt$ and $\langle dW_t^c, dW_t^q \rangle = 0$. Now, consider two agents, each representing their own class, that have common information sets and 'agree to disagree' about how to process information. Mathematically, agents have different filtered probability spaces $\{\Omega, \mathcal{F}_t, \mathcal{P}(\Theta^i)\}$, where the parameters that determine subjective measures $\mathcal{P}^i$ are contained in Θ^i. Since C_t is common and observable, consistent perceptions require that subjective innovations are related by

$$dW_t^{c,p} = dW_t^{c,o} + \sigma_c^{-1} \left(g_t^o - g_t^p\right) dt = dW_t^{c,o} + \Psi_t dt \tag{2}$$

which defines the standardised 'real disagreement' process Ψ_t. Model disagreement leads to different empirical likelihoods for the two agents. The difference in likelihood is summarized by the Radon-Nikodym derivative whose solution is given by $\eta_t = \eta_0 \exp\left(-1/2 \int_0^t (\Psi_t)^2 ds - \int_0^t \Psi_s dW_s^{c,o}\right)$. Intuitively, the Radon-Nikodym derivative encodes the difference in beliefs between agents by assigning a higher (lower) weight for states of nature which they deem more likely (unlikely). Intuitively, if agents p (pessimist) believes consumption is likely to be smaller tomorrow than agent o (optimist) does, then η_T will be larger in down states.

In equilibrium, agents trade to the point that ex-ante expected marginal utility of consumption equate. Thus, a frictionless equilibrium requires that for $\forall u > t$

$$E_t^o\big[U'(C_u^o)\big] = E_t^o\big[\eta_u U'(C_u^p)\big] \tag{3}$$

so that innovations in η_t necessarily imply a different allocation of state-contingent consumption C_t^o and C_t^p between the two agents. One readily sees that the Radon-Nikodym derivative must also equal the ratio of agents marginal utilities: $\eta_t = \frac{U'(C_t^o)}{U'(C_t^p)}$.

2. Learning and disagreement

In the literature, the Radon-Nikodym derivative η_t is either assumed as an exogenous process or obtained as the outcome of an optimal learning problem. We assume Bayesian agents who learn from identical information sets that include realisations of consumption and the price level, which is correlated with stochastic growth: $\mathcal{F}_t = \{C_\tau, Q_\tau\}_{\tau=0}^t$. However, consistent with Das, Kuhnen, and Nagel (2017), we assume that agents update their beliefs about the states using different models, which are indexed by two sets of parameters Θ^i. Denote agent i's conditional forecast $\widehat{g}_t^i = E_t^i\big[g_t|\mathcal{F}_t, \Theta^i\big]$ and posterior variance $\nu_t^i = E_t^i\big[(\widehat{g}_t^i - g_t)^2|\mathcal{F}_t, \Theta^i\big]$, where Θ^i is a set of subjective model parameters.

The first source of belief heterogeneity is about correlation: agents disagree about the (i) correlation between shocks to consumption levels and future consumption growth rates ($\rho_{c,g}^i$); and (ii) shocks to consumption levels and inflation ($\rho_{q,g}^i$). These parameters play an important role for term structure properties in homogeneous agent economies since they determine the extent to which bonds are risky bets or hedging instruments against consumption and inflation shocks. Thus, disagreement about these correlation is equivalent to disagreement about the instantaneous hedging properties of bonds. The second heterogeneous parameter is the long-run consumption growth rate ($\theta^o \neq \theta^p$). Indeed, a significant stream of the empirical asset pricing literature argues about the existence of significant challenges in measuring the long-run properties of the economy.[11]

Since state dynamics are conditionally Gaussian, standard Kalman filtering methods can be applied to the system of fundamentals given by equation 1. The optimal posterior mean and variance of each agent i, conditional on θ^i, satisfy the following conditions (see online appendix for

[11] Hansen, Heaton, and Li (2008) argue that econometricians face severe measurement challenges when quantifying the long-run components of the economy. For a related discussion see Pastor and Stambaugh (2000) who study the statistical properties of predictive systems when the predictors are autocorrelated but κ_g is not known. Chen, Joslin, and Tran (2012) argue that difficult to measure parameters of the economy, such as the likelihood of rare disasters, are a natural source of disagreement.

details):

$$d\widehat{g}_t^i = \kappa_g(\theta^i - \widehat{g}_t^i)dt + \underbrace{\left(\frac{\nu^{*,i} + \rho_{c,g}^i \sigma_c \sigma_g}{\sigma_c}\right)}_{\sigma_{c,g}^i} d\widehat{W}_t^{c,i} + \underbrace{\rho_{q,g}^i \sigma_g}_{\sigma_{q,g}^i} d\widehat{W}_t^{q,i}, \tag{4}$$

$$\nu^{*,i} = \sigma_c \sqrt{\kappa_g^2 \sigma_c^2 + 2\kappa_g \rho_{c,g}^i \sigma_c \sigma_g + \sigma_g^2(1 - (\rho_{q,g}^i)^2)} - \kappa_g \sigma_c^2 - \rho_{c,g}^i \sigma_c \sigma_g. \tag{5}$$

Using these conditions, standardised disagreement, $\Psi_t = \sigma_c^{-1}(\widehat{g}_t^o - \widehat{g}_t^p)$, under the measure of agent a satisfies:

$$\begin{aligned} d\Psi_t^g &= \frac{\kappa\sigma_c + \sigma_{c,g}^p}{\sigma_c}\left(\left(\frac{\kappa(\theta^o - \theta^p)}{\kappa\sigma_c + \sigma_{c,g}^p}\right) - \Psi_t^g\right)dt + \left(\frac{\sigma_{c,g}^o - \sigma_{c,g}^p}{\sigma_c}\right)dW_t^{c,o} + \left(\frac{\sigma_{q,g}^o - \sigma_{q,g}^p}{\sigma_c}\right)dW_t^q \\ &= \kappa_\Psi\left(\theta_\Psi - \Psi_t^g\right)dt + \sigma_{c,\Psi} dW_t^{c,o} + \sigma_{q,\Psi} dW_t^q. \end{aligned} \tag{6}$$

Two notable implications emerge. First, disagreement about θ and ρ have significantly different effects on the dynamics of Ψ_t^g. When $\theta^o \neq \theta^p$, the disagreement process has a *non-zero* long run mean both conditionally and unconditionally. In this case, the disagreement process does not revert to zero in steady state. Moreover, conditional disagreement can take both positive and negative values: growth rate optimists can become growth rate pessimists and vice-versa, depending on the realization of signals.

When $\rho^o \neq \rho^p$, disagreement is stochastic. This implies even if today agents agree on g_t (i.e. $g_t^o = g_t^p$), they are aware that, almost surely, they will disagree tomorrow even if they will observe the same public signals. Indeed, different correlations parameters ρ will induce different posterior distributions. This implies that Ψ_t^g is stochastic and an endogenous source of time variation in the investment opportunity set of each agent. Therefore, this source of disagreement has distinct implications on bond volatility and risk premia.

3. General Equilibrium and Bond Prices

The dynamic properties of asset prices depend on the characteristics of the stochastic discount factor of the representative agent. In complete markets, Basak (2000) extends Cuoco and He (1994) approach to show how the competitive equilibrium solution can be obtained from the solution of a central planner problem.[12] Indeed, a representative investor utility function can be constructed

[12] Constantinides (1982) extends Negishi (1960)'s results and proves the existence of a representative agent with heterogeneous preferences and endowments but with homogeneous beliefs. In an incomplete market setting with

from a (stochastic) weighted average of each individual utilities.

In a homogeneous agent economy, complete markets ensure that the stochastic discount factor depends only on aggregate consumption (C_t). With belief heterogeneity, the stochastic discount factor (Λ_t) is also driven by the path of agent specific consumption (c_t^i):

$$\Lambda_t = \underbrace{\left[e^{-\delta t} C_t^{-\gamma}\right]}_{H_t} \times \left(1 + \eta_t^{1/\gamma}\right)^{\gamma} \tag{7}$$

$$c_t^o = \frac{C_t}{1 + \eta_t^{1/\gamma}} \quad , \quad c_t^p = C_t \frac{\eta_t^{1/\gamma}}{1 + \eta_t^{1/\gamma}} \quad , \quad \eta_t = \left(\frac{\omega_t^p}{\omega_t^o}\right)^{\gamma} \tag{8}$$

where $\omega_t^i = c_t^i / C_t$ is investor's i consumption share. Since beliefs induce agents to trade, they affect ex-post consumption allocations $\frac{\omega_t^p}{\omega_t^o}$ thus becoming a source of variation is asset prices both via the quantity and price of risk channels. The short term real interest rate r_t and prices of a nominal bonds $B^{\$}(t,T)$ with maturity at time T are related to the properties of Λ_t:

$$\begin{aligned} r_t dt &= -E_t^i \left[\frac{d\Lambda_t}{\Lambda_t}\right] \\ B^{\$}(t,T) &= E_t^i \left[e^{-\delta\tau} \frac{\Lambda_T}{\Lambda_t} \frac{Q_T}{Q_t}\right]. \end{aligned}$$

In what follows, we derive closed-form solutions for the yield curve. Initially, to keep notation simple, we assume that agents have homogeneous CRRA risk aversion γ. Later, we extend the model to allow also for heterogeneity in risk aversion and study under what conditions our main result is amplified or attenuated.

3.1. Short Term Interest Rates

Applying Itô's lemma to equation 7, when agents have the same risk aversion, the equilibrium real risk free rate is given by[13]

$$r_t = \underbrace{\delta - \frac{1}{2}\gamma(\gamma+1)\sigma_c^2}_{\text{Lucas-Tree Term}} + \gamma \underbrace{(\omega_t^o \widehat{g}_t^o + \omega_t^p \widehat{g}_t^p)}_{\text{Consensus Bias}} + \underbrace{\frac{\gamma - 1}{2\gamma} \omega_t^o \omega_t^p (\Psi_t^g)^2}_{\text{Speculative Demand}} \tag{9}$$

homogeneous agents Cuoco and He (1994) show a representative agent can be constructed from a social welfare function with stochastic weights. Basak (2000) discuss the aggregation properties in economies with heterogeneous beliefs but complete markets. He shows that a representative can be constructed from a stochastic weighted average of individuals marginal utilities.

[13]The nominal risk free rate is given by $r_t^{\$} = r_t + q - \sigma_q^2 - \gamma\sigma_c\sigma_q\rho_{cq}$.

When disagreement is zero the short term interest rate is given by the Lucas solution. In the heterogeneous case, the short term interest rates includes two new terms. The first is $[\omega_t^o \widehat{g}^o + \omega_t^p \widehat{g}_t^p]$ and includes an income effect. When $\eta_t \neq 1$, this term differs from the consensus belief $\frac{1}{2}\widehat{g}_t^o + \frac{1}{2}\widehat{g}_t^p$. Speculative activity undertaken in the past affects agents' relative wealth today and this term biases the short rate towards the belief of the agent who has been relatively more successful at forecasting in the past.[14] If the path of the economy were such that the distribution of wealth was shifted towards pessimists (optimists) bond prices will be inflated (deflated) with respect to their homogeneous counterparts. The third term is due to speculative demand, which is proportional to the level of disagreement. When $\gamma = 1$, this last term is zero. For $\gamma \neq 1$ the impact of the speculative demand on the short rate is largest when $\omega_t^o = \omega_t^p = 1/2$.

To visualise the interaction of risk aversion, expectations, and relative wealth, consider the sensitivity of the short rate with respect to the state vector $[\widehat{g}_t^o,\ \eta_t,\ \Psi_t^g]$:

$$\frac{\partial r}{\partial \widehat{g}^o} = \gamma = \frac{1}{EIS} \tag{10}$$

$$\frac{\partial r}{\partial \Psi^g} = -\gamma\sigma_c \left(\frac{\eta_t^{1/\gamma}}{1+\eta_t^{1/\gamma}} \right) + \left(\frac{\gamma-1}{\gamma} \right) \frac{\eta_t^{1/\gamma}}{(1+\eta_t^{1/\gamma})^2} \Psi_t^g \tag{11}$$

$$\frac{\partial r}{\partial \eta} = \sigma_c \left(\frac{\eta_t^{1/\gamma}}{(1+\eta_t^{1/\gamma})^2} \right) \Psi_t^g - \frac{1}{2} \frac{\eta_t^{\frac{1-\gamma}{\gamma}}}{(1+\eta_t^{1/\gamma})^3} \left(\eta_t^{\frac{1}{\gamma}} - 1 \right) \left(\frac{1}{\gamma} \right) \left(\frac{\gamma-1}{\gamma} \right) (\Psi_t^g)^2 \tag{12}$$

Figure 2 summarizes the results. The first panel shows that when $\gamma < 1$ the substitution effect dominates and short term interest rates are unambiguously negatively related to disagreement, $\partial r/\partial\Psi^g < 0$. Moreover, the larger the level of disagreement Ψ_t^g, the larger is the reduction of the short-term rate due to an increase in Ψ_t^g. This is due to the second term in equation (11). The reason is that changes in Ψ_t^g also affect the investment opportunity set by increasing speculative opportunities between agents. The sign of the effect depends on whether γ is greater or smaller than 1. Considering the relative wealth of agents a

$$d\omega_t^o = \underbrace{\frac{\gamma-1}{2\gamma}\omega_t^o\omega_t^p(\Psi_t^g)^2}_{\text{Speculative Demand}} \left(\frac{(\gamma-1)+2\gamma\omega_t^p}{\gamma(\gamma-1)} \right) dt + \frac{1}{\gamma}\omega_t^o\omega_t^p(\Psi^g)_t^2 d\widehat{W}_t^{c,o}. \tag{13}$$

we see that for $\gamma > 1$ the wealth effect dominates: speculation raises the drift of planned consumption, which is fixed today; thus, interest rates must rise to clear the market. When $\gamma < 1$

[14] Jouini and Napp (2006) also construct a consensus investor whose stochastic discount factor contains an aggregation bias.

the substitution effect dominates: speculation increases expected returns raising the price of current consumption relative to future consumption, lowering the drift of planned consumption; thus, interest rates must fall.

The risk-free rate also depends explicitly on η_t. The sign of the effect depends on the interaction between risk aversion and the distribution of wealth. A negative (positive) endowment shock $d\widehat{W}_t^{c,o}$ decreases (increases) the wealth of agents o and increases (reduces) η_t, since in equilibrium $\eta_t = (\omega_t^p/\omega_t^o)^\gamma$.

When the economy is dominated by optimists ($\omega_t^o = 0.75$, so that $\eta_t < 1$) with $\gamma < 1$ the term $(\eta_t^{\frac{1}{\gamma}} - 1)\frac{\gamma-1}{\gamma}$ is positive so that for a sufficiently small value of γ the second term in equation (12) dominates and interest rates decrease (increase): $\partial r/\partial\eta > 0$. The economic intuition is simple. When risk aversion is very low, the speculative channel plays an important role since the optimist wants to leverage his position in risky assets by short-selling bonds (borrowing). This leverage is provided by the pessimists, agent p. When a negative shock reduces the relative wealth weight of the optimist ($d\omega_t^o < 0$), interest rates drop for two reasons. First, wealth-weighted aggregate beliefs shift toward those of the pessimist. Second, optimists deleverage their positions by purchasing short-term bonds. As a consequence, the short-term interest rate drops more significantly than in a homogeneous Lucas economy. This effect is further amplified when disagreement Ψ_t is large. Figure 2 summarizes this effect showing that $\partial r/\partial\eta < 0$ ($\omega_t^o = 0.75$ and $\gamma < 1$). When γ is large, this effect is not present since the speculative demand changes sign for $\gamma > 1$. Notice that when the economy is dominated by pessimists ($\omega_t^o = 0.25$ and $\eta_t > 1$), if $\gamma < 1$ the term $(\eta_t^{\frac{1}{\gamma}} - 1)\frac{\gamma-1}{\gamma}$ is negative and a similar argument implies that $\partial r/\partial\eta > 0$. Thus, negative (positive) aggregate consumption shock increases (reduces) η_t and increases (reduces) interest rates.

[Insert figure 2 about here]

3.2. Bond Prices and the Yield Curve

Solving for bond prices require solving for the conditional expectation of the product of two terms. The first emerges in the traditional homogeneous case; the second one arises because of the impact of disagreement on the ex-post redistribution of wealth.

Define $X_t = \log C_t$, $Y_t = \log Q_t$, and $Z_t = \log \eta_t$. For integer risk aversion, one could binomial expand $(1 + e^{\frac{1}{\gamma}Z_T})^\gamma$ resulting in a sum of exponential functions that can be solved in closed form. However, since we want to study the implications of trade amongst a set of agents, such as intermediaries and hedge funds, who are risk tolerant ($\gamma < 1$) we cannot follow this approach.[15] For arbitrary risk aversion, one cannot directly take the transform of the SDF directly because it is not a tempered distribution.[16] However, expanding around the integer case leaves a residual component that can be transformed after applying an appropriate damping factor.[17] The final result is a sum of characteristic functions of the form

$$\phi(\tau; u) = E_t^i \left(e^{u_1 X_T + u_2 Y_T + u_3 Z_T} \right). \tag{14}$$

Following Cheng and Scaillet (2007) we conjecture a solution that is exponentially affine in the extended state vector $V_t = \left(X_t, Y_t, Z_t, \widehat{g}_t^o, \Psi_t^g, (\Psi_t^g)^2 \right)$ from which we can recover the characteristic function in terms of a set of separable ordinary differential equations.

Theorem 1 (Bond Prices)**.** *The term structure of bond prices is a weighted sum of exponentially affine functions that depend on growth rate dynamics, differences in beliefs, and the distribution of wealth.*

$$B^{\$}(\tau) = e^{-\delta\tau}(1 + \eta_t^{1/\gamma})^{-\gamma} \sum_{j=0}^{\lfloor\gamma\rfloor+1} \binom{\lfloor\gamma\rfloor + 1}{j} \frac{\gamma}{\pi} \int_0^{+\infty} Re \left[\eta_t^{u_3} \phi(\tau; u) \frac{\Gamma[g_1]\Gamma[g_2]}{\Gamma[g_1 + g_2]} \right] dk \tag{15}$$

where $\Gamma[\cdot]$ is the (complex) gamma function, $\lfloor\cdot\rfloor$ is the floor operator, and

$$\alpha = \lfloor\gamma\rfloor + 1 - \gamma \quad , \quad g_1 = \alpha/2 - i\gamma k \quad , \quad g_2 = \alpha/2 + i\gamma k$$
$$u_1 = -\gamma \quad , \quad u_3 = (2j - \alpha)/2\gamma - ik,$$

$\phi(\tau; u) = e^{\alpha(\tau;u)+\beta(\tau;u)'v}$ and $\{\alpha(\tau), \beta(\tau)\}$ are functions of time and the structural parameters of the economy.

3. Data

We use three different datasets:

Bond Data: For Treasury bonds data, we use the nominal zero-coupon bond yields dataset

[15] The solution given by Xiong and Yan (2010) only applies to the case of log utility investors. Dumas, Kurshev, and Uppal (2009) derive solutions for the optimal holdings of risky assets for γ integers using binomial expansions of the Fourier transform. These methods do not allow for an analytical solution for the $\gamma < 1$ case, which is the focus of this paper.

[16] See Chen and Joslin (2012) for a discussion of the class of functions that admit transforms.

[17] The online appendix reports details of the steps in these calculations.

of Gürkaynak, Sack, and Wright (2006) (GSW) for maturities between 3 months and 5 years for the sample period 1962.1 — 2020.1. U.S inflation protected Treasuries were first issued in 1997 which adjust to the all urban consumer price index with a 3-month lag. In the early years of issue this market suffered significant liquidity problems (see, for example Roll (2004)) and our sample for real yields focuses on the period 2002.01 - 2020.1. We obtain zero coupon TIPS estimated by GSW and discussed in Gürkaynak, Sack, and Wright (2010). For further details on the estimation of real and nominal zero coupon yields, and for links to the datasets see `www.federalreserve.gov/data/yield-curve-models.htm`

We denote the date t log price of a n-year discount bond as $p_t^{(n)}$ and the continuously compounded yield is defined as $y_t^{(n)} = -\frac{1}{n} p_t^{(n)}$. The date-$t$ 1-year forward rate for the year from $t+n$ and $t+n+1$ is $F_t^{n,n+1} = p_t^{(n)} - p_t^{(n+1)}$. The one year log holding period return is the realised return on an n-year maturity bond bought at date t and sold as an $(n-1)$-year maturity bond at date $t+12$:

$$r_{t,t+12}^{(n)} = p_{t+12}^{(n-1)} - p_t^{(n)}. \tag{16}$$

Excess holding period returns are denoted by:

$$rx_{t,t+12}^{(n)} = r_{t,t+12}^{(n)} - y_t^{(1)}. \tag{17}$$

Macro Data: Inflation is computed from the year-on-year growth rate in the Consumer Price Index for All Urban Consumers All Items [CPIAUCSL] obtained from the Federal Reserve Economic Data Set. Consumption data is from the U.S. Bureau of Economic Analysis (BEA) from which we compute annual real per capital consumption growth on non-durables by combining NIPA Tables 2.3.3, 2.8.6, and 7.1. The sample period for macro data is 1962.1 — 2020.1.

Survey Data: to construct proxies of disagreement and sentiment, we use professional survey data from BlueChip Financial Forecasts Indicators (BCFF). This is a monthly publication that provides an extensive panel data on expectations by agents who are working at institutions active in financial markets. At the start of this project, digital copies of BCFF were available only since 2001. Thus, we obtained the complete BCFF paper archive directly from Wolters Kluwer and proceeded to digitize all the data. The digitization process required inputting around 750,000 entries of named forecasts plus quality control checking and was completed in a joint venture with the Federal Reserve Board. The resulting dataset represents an extensive and unique dataset to investigate the role of formation of expectations about the compensation for bearing interest rate risk. Each month, BlueChip carry out surveys of professional economists from leading financial institutions and service companies regarding all maturities of the yield curve and economic fundamentals and are asked to give point forecasts at quarterly horizons out to 5-quarters ahead (6 from January 1997). While exact timings of the surveys are not published, the survey is usually conducted between the $25th$ and $27th$ of the month and mailed to subscribers within the first 5 days of the subsequent month, thus our empirical analysis is unaffected by biases induced by staleness or overlapping observations between returns and responses. The sample period for BCFF is 1988.1 — 2020.1.

4. Estimation

We estimate the model via SMM which is analogous to the generalized method of moments (GMM) estimator, but allows us to estimate the parameters even if the latent variables disagreement and wealth are not directly observable. Moreover, SMM avoids difficulties of computing analytical moment conditions which, in the context of our model, include multiple integrals over nonlinear functions. To save space, a detailed discussed of the moment conditions and numerical details of the estimation are reported in the Online Appendix. For a textbook treatment of the subject we refer the reader to Singleton (2009). The data used in estimation is discussed in section 3 above and spans the sample period 1962.1 — 2020.1

1. Macro Dynamics

The dynamics of the consumption process depend on the parameter vector $\beta_0 = [\theta, \sigma_c, \kappa_g, \sigma_g]$. We follow Bansal and Yaron (2004) and assume all macro shocks are orthogonal under the objective measure and a set of six moments conditions for time-aggregated annual consumption growth. The vector of moments include: (i) mean consumption growth; (ii) consumption volatility; (iii) AR(1); (iv) AR(2); (v) AR(5); (vi) AR(10). With $p = 6$ moments and $q = 4$ parameters the system is over identified with two degrees of freedom.

[INSERT TABLE 1 HERE]

Table 1, panel A, summarizes the results. The estimated parameters $[\theta, \sigma_c, \kappa_g, \sigma_g]$ are equal to $[2.0, 1.03, 0.09, 0.43]$, implying that average expectations about long-term growth rate are equal to 2% and consumption volatility is slightly above 1%. The estimation error implies a confidence interval for θ equal to $[1.79, 2.22]$. Panel B shows the difference between the six empirical moments and their model implied values. At the estimated parameter values, the J_T statistics testing the over-identifying restrictions generates a p-value of 0.47, so that we cannot reject the null hypothesis that the model is correctly specified at these parameter values. The estimates for the mean ($\bar{\pi}$) and volatility of inflation (σ_q) are set equal to their empirical counter parts of 3.81% and 2.6%, respectively.

2. Term Structure Estimation

In a second step, we take the parameters of the macro economy as given and estimate the remaining parameters using information from the panel of nominal yields: $[y^{3m}, y^{12m}, y^{24m}, y^{36m}, y^{48m}, y^{60m}]$. Keeping the parameter set to be estimated small and focusing on the effect of heterogeneity, we set the rate of time preference $\delta = 1\%$. We then estimate $\beta_0 = [\gamma, \theta^o, \theta^p, \rho^o_{c,g}, \rho^p_{c,g}, \rho^o_{q,g}, \rho^p_{q,g}]$ assuming that agents disagree symmetrically about consumption growth rates and react in equal but opposite directions to shocks. Specifically, we parameterise

$$\theta_o = \theta + \Delta_\theta/2 \quad \text{and} \quad \theta_p = \theta - \Delta_\theta/2 \tag{18}$$

$$\rho^o_{c,g} = \Delta_\rho \quad \text{and} \quad \rho^p_{c,g} = -\Delta_\rho \tag{19}$$

$$\rho^o_{q,g} = \Delta_\rho \quad \text{and} \quad \rho^p_{q,g} = -\Delta_\rho. \tag{20}$$

This reduces the parameter set to be estimated to $\beta_0 = [\gamma, \Delta_\theta, \Delta_\rho]$. The vector of moment conditions includes the (i) mean, (ii) volatility; (iii) skewness; (iv) kurtosis of monthly changes of y^{3m}, (v) AR(1) coefficient of the y^{3m}; and (vi) - (X) mean of monthly levels of $[y^{12m}, y^{24m}, y^{36m}, y^{48m}, y^{60m}]$ The model is over-identified with $p - q = 10 - 3 = 7$ degrees of freedom.

[INSERT TABLE 2 AND FIGURE 3 HERE]

The estimated values of $[\gamma, \Delta_\theta, \Delta_\rho]$ are $[0.62, 0.45, 0.29]$. The coefficient of risk aversion is 0.62, which suggests that, while agents are risk averse, the low level of risk aversion can give rise to speculative effects in asset prices. Moreover, both disagreement parameters are statistically and economically different than zero: Δ_θ is equal to 0.45 with a 95% confidence level of $[0.25, 0.65]$ and Δ_ρ is equal to 0.29 with a 95% confidence level of $[0.16, 0.40]$. This suggests that both dimensions of disagreement are needed to reproduce the moment conditions. Moreover, the estimate of Δ_θ implies that the long-term growth rate disagreement parameters, $[\theta_o, \theta_p] = [1.775; 2.225]$ are broadly consistent with the confidence interval of the parameter $\theta = [1.79, 2.22]$.

The model-implied average 3 month yield is 4.93%, versus 4.90% in the data. The unconditional yield curve implied by the model at the SMM estimated parameter values is upward sloping and the five year yield implied by the model is 5.58%, versus 5.78% in the data. The model can also reproduce both the autocorrelation of the 3 month yield, which is 0.95 versus 0.98 in the data, and the kurtosis of bond yield changes, which is 15.78 versus 15.86 in the data.

Figure 3 summarizes the model-implied shape of the unconditional yield curve and compare it with the empirical one. While the model produces a slightly flatter yield curve, the results is very accurate and well within the 95% confidence region, which is highlighted by the red shaded area. The model finds it more difficult to exactly match the unconditional volatility of changes in the 3 month bond and it produces somewhat wide confidence bounds for the skewness in short-term bond yield changes $[-0.80, 0.83]$, versus a point estimate of -0.74 in the data. We use the chi-square statistics $J_T = T(1 + 1/\tau)\ G_T^\top\ \widehat{W}^\star\ G_T$, proposed by Lee and Ingram (1991), to test the the overidentifying restrictions of the model at the estimated values summarized in table 2. The J_T statistics has seven degrees of freedom and generates a p-value equal to 0.49. Even in this case, we cannot reject the model at traditional confidence levels.

To investigate the role played by the interaction of risk aversion and disagreement, Figure 3 compares the yield curves implied by two different levels of risk aversion: $\gamma = 0.62$ (SMM estimate) and $\gamma = 2$ around the SMM estimated parameters, namely $[\theta, \sigma_c, \kappa_g, \sigma_g] = [2.0, 1.03, 0.09, 0.43]$ and $[\Delta_\theta, \Delta_\rho] = [0.45, 0.29]$. We find that when $\gamma = 2$ the yield curve shifts upward to unrealistic levels (around 10%) and the slope of the yield curve turns negative. At this level of risk aversion one can immediately notice the emergence of the well-known risk-free rate puzzle and belief heterogeneity is not sufficient to reproduce, at the same time, both the risk free rate and the bond term premia. However, when $\gamma = 0.62$ the model with disagreement can relax this tension without the cost of creating excessive bond volatility.

[INSERT TABLE 2 AND FIGURES 3 HERE]

At the SMM estimated parameter values ($\gamma = 0.62$), Figure 2, top panel, shows that the sensitivity of the short term interest rate with respect to disagreement is negative: the larger the disagreement, the lower the short term interest rate.

5. Discussion and Predictions

1. Level and Slope

Figure 4 plots term structures for an economy corresponding to the SMM estimated parameters reported in table 2. Panel A investigates the properties of the yield curve in an economy in which the initial distribution of wealth is skewed towards pessimist agents, namely $\omega_o = 0.40$. We consider two scenarios depending on the initial level of disagreement about growth rates g_t. The first relates to a low level of disagreement when $g_t^o - g_t^p = 0.45\%$ (red line); the second relates to a large level of disagreement when $g_t^o - g_t^p = 0.90\%$ (green line). We compare these two scenarios to the shape of the yield curve emerging in an economy with no disagreement on future growth rates $g_t^o - g_t^p = 0\%$ (blue line). One can immediately notice that in absence of disagreement one obtains the well know result that the yield curve is downward sloping and the level of three month interest rates is above 6%, which is substantially larger than the 4.90% observed in the data.

When disagreement increases, the yield curve steepens and the short term rate drops. For a level of disagreement such that $g_t^o - g_t^p = 0.45\%$, the short term rate is around 5% and the spread between the long and short term rate is about 50 basis points. For levels of disagreement above average, namely $g_t^o - g_t^p = 0.90\%$, the model-implied short term rate drops very significantly while long term rates remain elevated, thus inducing a sharp steepening of the term structure.

[INSERT FIGURE 4 HERE]

2. Risk Premia and Quantities of Risk

Bond volatility is monotonically increasing in disagreement, becoming an important factor driving the quantity of risk. The economic mechanism is rather simple but different from traditional channels. When agents agree to disagree, they engage in beliefs-based trading. The larger the amount of trade, the larger the ex-post volatility of individual consumption, which implies a large volatility for discount bonds. The diffusion component of $d\omega_t^o$ makes this explicit:

$$d\omega_t^o - E_t^o[d\omega_t^o] = \frac{1}{\gamma}\omega_t^o\omega_t^p\Psi_t^g d\widehat{W}_t^{c,o} \tag{21}$$

Individual agent consumption volatility is higher for lower levels of risk aversion. Since agents are forward looking, equilibrium bond prices must discount larger individual consumption risks, thus requiring larger risk premia. Bond risk premia are given by:

$$\mu_t^\tau - r_t = \underbrace{\Big[\gamma\sigma_c + \frac{1}{\sigma_c}\Big[g_t - (\omega_t^o \widehat{g}_t^o + \omega_t^p \widehat{g}^p)\Big]\Big]}_{\text{PoR}} \times \underbrace{\sigma_{b,c}^\tau(t)}_{\text{QoR}} \tag{22}$$

$$= \Big[\gamma\sigma_c - \mathcal{S}_t\Big]\sigma_{b,c}^\tau(t), \tag{23}$$

where μ_t^τ is the objective drift of a maturity τ bond. Equation (22) shows that bond risk premia are equal to the quantity of risk ($\sigma_{b,c}^\tau$) multiplied by the sum: price of risk arising in a homogeneous Lucas economy ($\gamma\sigma_c$) plus $\mathcal{S}_t = \sigma_c^{-1}\sum\omega_t^i(\widehat{g}^i - g_t)$ which is an ex-ante measure of bias in the economy that we refer to as *'Sentiment'*.

[INSERT FIGURE 5 AND 6]

Figure 5 summarizes how different levels of ω_t^o and Ψ_t^g interact to affect equilibrium bond risk premia. Two key results emerge. First, for intermediate values of $\omega_o \simeq 0.5$ (i.e. small sentiment, $\mathcal{S}_t \simeq 0$) bond risk premia are small and similar to those arising in a homogeneous Lucas economy. Second, bond risk premia are non-linear and depend on the interaction between $\mathcal{S}_t$ and Ψ_t^g. When $\mathcal{S}_t > 0$ (i.e. when ω_o is large and optimists are the wealthiest), bond risk premia are positive but bond risk premia can turn negative when $\mathcal{S}_t < 0$ (when ω_o is small and pessimists are the wealthiest).

Under the objective measure, the price of risk switches sign depending on the relative value of $\mathcal{S}_t$ with respect to $\gamma\sigma_c$. This is an interesting property since reduced form evidence (for example, Duffee (2002) or Cochrane and Piazzesi (2005)) demonstrates that expected bond returns do indeed take both positive and negative values.[18]

States of the world when bond risk premia significantly deviate from $\gamma\sigma_c$ are when (i) sentiment $\mathcal{S}_t$ is large, and / or (ii) disagreement is large which induces trade leading to an increase in agent specific consumption volatility. When these conditions are met, bond risk premia can be large even if $\gamma\sigma_c$ is small. This is juxtapose to the CRRA representative agent paradigm which requires unrealistically large levels of risk aversion to reproduce bond risk premia comparable to the data. Indeed, different than traditional Lucas endowment economies, heterogeneity in beliefs makes agents exposed to risks originating from their own belief motivated trades, above and beyond the volatility of fundamentals. The smaller the agents risk aversion, the larger the trading and the greater the

[18] While both habit and long risk economies can generate positive or negative bond risk premia, for a given parameter set, the sign is bounded by zero.

amount of individual consumption risks. As a consequence, the price of interest rate risk can be large even if the volatility of aggregate endowment is small.

The quantity of risk depends on the sensitivity of bond return to shocks to the three state variables $[\widehat{g}^o, \eta, \Psi^g]$:

$$\left[\sigma_{b,c}^{\tau}\ ,\ \sigma_{b,q}^{\tau}\right] = \frac{1}{B^{\$}(\tau)}\left[\frac{\partial B^{\$}(\tau)}{\partial \widehat{g}^o}\ ,\ \frac{\partial B^{\$}(\tau)}{\partial \eta}\ ,\ \frac{\partial B^{\$}(\tau)}{\partial \Psi^g}\right]\begin{bmatrix}\sigma_{c,g} & \sigma_{q,g} \\ -\Psi_t^g & 0 \\ \sigma_{c,\Psi^g} & \sigma_{q,\Psi^g}\end{bmatrix}. \tag{24}$$

Figure 6 plots the components of equation (24) as a function of the relative wealth ω_o of agent o. It shows that the quantity of risk can change sign depending on whether the economy is dominated by optimists or pessimists. When both sentiment $\mathcal{S}_t$ and disagreement Ψ_t^g are large, bond prices become very sensitive to consumption shocks. Moreover, the dominant component of the quantity of risk is played by the dependence of bond returns on η_t, which is given by $-\Psi_t^g \frac{\partial B^{\$}(\tau)/\partial \eta}{B^{\$}(\tau)}$ (see yellow line in Figure 6, first Panel). For large disagreement (i.e. $\hat{g}_t^o - \hat{g}_t^p = 0.90\%$) and large positive sentiment $\mathcal{S}_t$ (i.e. large ω_t^o), $-\Psi_t^g \frac{\partial B^{\$}(\tau)/\partial \eta}{B^{\$}(\tau)}$ is largely negative. Then, for $\mathcal{S}_t > \gamma\sigma_c$, PoR < 0 so that bond risk premia are positive and large in magnitude.

Even if bonds were hedges against aggregate consumption shocks in equivalent homogeneous economies, long term bonds can command a time-varying positive bond risk premium in the heterogeneous agent equilibrium. The economic intuition for this effect is that agents are exposed to individual consumption risks due to their speculative trades. Optimists (agent o) want to leverage their positions in risky assets by borrowing from the pessimists (agent p) by shorting bonds. Negative consumption shocks reduce optimists relative wealth weight ($d\omega_t^o < 0$), resulting in long term bonds prices to appreciate for two reasons. First, aggregate beliefs shift toward those of the pessimist. Thus, interest rates drop. Second, the wealth shock forces optimists to deleverage their positions by covering their short bond holdings. Long-term bond prices must increase more significantly than in a homogeneous Lucas economy. As a consequence, the speculative channel makes long term bonds risky for optimists. To summarize, significant positive bond risk premia can arise in speculative heterogeneous agent economies and this premium can be time-varying due to the interaction of disagreement with positive sentiment $\mathcal{S}_t > 0$.

6. Empirics

1. *Disagreement and Sentiment*

To study the evolution over time of the cross-sectional properties of beliefs, we employ a proxy for real belief disagreement, denoted Ψ_t^g, from the the cross-sectional mean-absolute-deviation in 1-quarter ahead forecasts for real GDP from BlueChip Financial Forecasts (see section 3). Moreover, as pointed out in the literature review, existing studies hav shown the potential importance of disagreement about inflation in bond markets and so in what follows we use the same approach to control for inflation disagreement on CPI growth, which we label Ψ_t^π. Controlling for consensus expectations we compute the cross-sectional mean 1-quarter ahead forecasts for real GDP and CPI growth, denoted as $E_t[g]$ and $E_t[\pi]$, respectively. All variables are sampled quarterly at the beginning of January, April, July, October.

Panel (a) of figure 7 displays time-series plots for the real disagreement proxy Ψ_t^g, the three month yield y_t^3, and the forward spread defined as the difference between the nominal 1-year forward rate 4-years from date t, and the 1-year nominal rate at time t. There are three NBER economic recessions in our sample period (1991, 2002, and 2009) where we observe sharp contractions in expected growth, a drop in the short-term interest rate, and an increase in forward spreads. In 1991 and 2002 real growth bottomed out at -1% while in 2009 GDP growth reached -3.2%. Average real disagreement is 0.49 and its standard deviation is 0.22. Disagreement on the real economy has a significant business cycle component. In all previous three recessions Ψ_t^g is low before recessions and increasing during the recession and the unconditional correlation between $E_t[g]$ and in Ψ_t^g is -56%. This is notable since financial commentaries are known to report large disagreement about the state of the economy especially during recessions. Panel (b) of 7 compares the disagreement on GDP with that on inflation. The correlation between changes in Ψ_t^π and changes in Ψ_t^g is 13% and the time-series properties of Ψ_t^g versus Ψ_t^π suggest the two disagreement factors reveal rather different information. During the 2001 recession, for instance, most of the disagreement related to future real economic growth; during the 2008-2009 crisis, however, disagreement about inflation was more than double real disagreement.

When belief formation deviates from full information rational expectations, agents' subjective beliefs about the future value of economic quantities, such as asset values or other economic variables, may deviate from the expectation implied by the true data generating process. The literature refers to this deviation as economic sentiment. Empirically, however, measuring sentiment is difficult and many proxies have been proposed by the literature. The most cited sentiment index is that of Baker and Wurgler (2006) which is based on 5 individual sentiment proxies variables: the value-weighted dividend premium, first-day returns on IPOs, IPO volume, closed-end fund discount, equity share in new issues. Extending and updating the index of Baker and Wurgler (2006), Huang, Jiang, Tu, and Zhou (2015) propose a modified index more closely aligned with expected returns.[19] The intuition for the index makes it a natural candidate to proxy for sentiment, $\mathcal{S}_t$, given by equation 22: when sentiment is high (low) investors tend to make or form overly optimistic (pessimistic) choices or beliefs.

Panel (c) of 7 compares the time series of sentiment $\mathcal{S}_t$ and Ψ_t^g. Sentiment was low during the late 1980s, increased after the 1991 recession, in the run up to the collapse of LTCM and then reached a peak during the Internet bubble in the late 1990s. Sentiment fell to a trough during the during the mid-2000's and then rose again in the run up to the 2008/2009 subprime crisis and then falls again in the second half of 2009 and remains low until the end of our sample. We note that while disagreement and sentiment are positively correlated (19%) they capture rather different properties of beliefs. Indeed, sentiment tends to spike in the run up to recessions after which sentiment drops, while disagreement tends to be peak during and towards the end of recessions. Moreover, at times the variables move in opposite directions. Panel (d) summarizes the dynamics of consensus beliefs on GDP and CPI.

Summary statistics for all left and hand variables in the regressions that follow are displayed in tables 3, 4 and 5.

[INSERT FIGURE 7 AND TABLES 3, 4 AND 5 HERE.]

2. Short Rates

The first prediction of the model links shocks to disagreement to the dynamics of short term interest rates. A knife edge case says that the sign of the relationship depends the level of γ. We test this hypothesis by regressing the 1-quarter change in the $3, 6, 9$ and 12-month interest rate on changes in disagreement, controlling for changes in consensus beliefs. The sample period for the regression is January 1988 - January 2020.[20]

[19] Two versions of this index are available which are either orthogonalised, with respect to a set of macro variables, or not. The index is available for download here: `https://dashanhuang.weebly.com/`. In what follows, we proxy for sentiment using non-orthogonalized version but our findings are similar regardless of which version is used. Results available on request.

[20] Excluding the financial crisis, our results get stronger as the Federal Reserve effectively pegged the target rate at the zero lower bound so there is a post 2009 period with little variation in short term interest rates.

Figure 7, panel (a), shows the joint dynamics of the time series of Ψ_t^g and the three month yield. The correlation between changes in Ψ_t^g and changes in the three month yield is -0.25 over the 1988:1-2020:1 sample period. The three periods of the 1990, 2000, and 2008 crisis are particular striking. Indeed, during these periods as Ψ_t^g spiked the three month yield dropped.

To avoid potential issues related to the persistence in interest rates, we test the first null hypothesis of the model by running the following regression in differences:

$$\Delta y_t^{(n)} = const + \Delta\beta_1 E_t[\pi] + \beta_2 \Delta E_t[g] + \beta_3 \Delta\Psi_t^g + \beta_4 \Delta\Psi_t^\pi + \varepsilon_t^{(n)}. \tag{25}$$

Table 6 reports the point estimates and standard errors which are computed from the block bootstrap procedure of Politis and Romano (1992) using a block bootstrap with a block length of 4 and compute from 10000 replications, which preserves the serial dependence and heteroskedasticity in the underlying data.

[INSERT TABLE 6 HERE.]

As predicted by a model with $\gamma < 1$, the slope coefficient β_3 is negative and statistically significant at the 5% level for all maturities. The speculative channel is also economically significant. The value of β_3 for the three month yield regression is -0.65 so that, on average, a one standard deviation change in $\Delta\Psi_t^g$ is associated with a $0.65 \times 0.15 = 10$ basis point (a 0.20-standard deviation) drop in the three month yield. Economically, this is larger than the effect coming from expected inflation but smaller than the effect coming from expected GDP. Given that the range of $\Delta\Psi_t^g$ has been between -0.73 and 0.37, in some periods changes in disagreement have contributed to a change in short term rates between -24 and 47 basis points. The R^2 ranges of the regression ranges between 16% and 25%, for the 3 month and 12 months yield respectively, which is surprisingly large given that the regression is run in differences.

After controlling for expected inflation and GDP growth, the slope coefficient on real disagreement is consistently negative, as predicted by the speculative channel in heterogeneous economies with $\gamma < 1$ (see Figure 2). It is notable to observe that the existence of a knife edge case for $\gamma = 1$ is a robust feature of the model since it does not depend on the values of other parameters. On the other hand, disagreement about inflation is positive and significant for 3,6 and 9 month maturities and insignificant for 12-month yield changes. Finally, as expected, the slope coefficients of the consensus expected inflation and GDP are positive for all maturities.

3. Term Structure Slopes

The second prediction of the model describes how differences in beliefs affect the slope of the term structure. This implication is important since it relates to the joint properties of short term interest rates and bond risk premia, which many models find it difficult to explain. To investigate this link, we run contemporaneous regression of the forward-spot spread, considered a direct measure of term premia (Fama and Bliss (1987)), on consensus expectations and disagreement

$$FS_t^{n,n+1} = const + \beta_2 E_t[\pi] + \beta_3 E_t[g] + \beta_4 \Psi_t^g + \beta_5 \Psi_t^\pi + \varepsilon_t^{(n)}, \tag{26}$$

where $FS_t^{n,n+1} = F_t^{n,n+1} - y_t^1$ is the spread between the nominal 1-year forward rate n-years from date t and the 1-year nominal rate.

A test of the model is equivalent to $H_0 : \beta_4 > 0$, namely that real disagreement increases the nominal forward spread. The results are summarized in Table 7, which reports point estimates and standard errors computed using a block bootstrap, as above. We find robust evidence that the slope coefficient on Ψ_t^g is positive for all maturities. It ranges between 1.17 for the $FS_t^{1,2}$ forward spread and 2.57 for the $FS_t^{4,5}$. The estimates are significant at the 1% confidence levels for all maturities and the R^2 ranges between 13% and 14%. A one standard deviation positive change in Ψ_t^g translates in a $2.57 \times 0.23 = 59$ basis point (a 0.45-standard deviation) increase in the $FS_t^{4,5}$ spread, which is economically significant.

Expected fundamentals are largely insignificant, except for expected inflation for the longer maturity spreads, and disagreement on inflation is never significant. Figure 7, panel (a), summarizes the joint dynamics of Ψ_t^g and the forward spread. The positive correlation between the two time series is very apparent and, once again, it becomes particularly striking during the 1990, 2000, and 2008 crisis when both forward spreads and disagreement widened.

It is well known that the U.S. nominal term structure slopes upward and is strongly time-varying. In the sample period 1988.1 — 2020.1 (2002.1 — 2020.1) the 4-year nominal forward spot spread averaged 1.6% (1.9%) with a standard deviation of 1.3% (1.4%). Less well known is that the real curve is also upward sloping and strongly time-varying. In the sample 2002.1 — 2020.1 the 4-year real forward spot spread averaged 1.2% with a standard deviation of 1.1%. This point is important since extant equilibrium term structure models face well known difficulties generating upward sloping real yields. Since the mechanism in our model is inherently real and there is no inflation risk premium, we investigate if disagreement can also explain variation in the slope of the real yield curve. We run the following regression:

$$\text{REAL: } FS_t^{n,n+1} = const + \beta_2 E_t[\pi] + \beta_3 E_t[g] + \beta_4 \Psi_t^g + \beta_5 \Psi_t^\pi + \varepsilon_t^{(n)}, \tag{27}$$

where REAL: $FS_t^{n,n+1} = F_t^{n,n+1} - y_t^2$ is the spread between the 1-year real forward rate n-years from date t and the 2-year real rate. We compute spread with respect to the 2-year real rate since shorter dated TIPS are relatively illiquid. The results are summarized in Table 8. Standard errors are based on a block bootstrap.

Controlling for expecting fundamentals, we again find a strong positive link between disagreement on GDP growth and slope of the real yield curve. The slope coefficient on the short-term and long-term real forward spread are 2.96 and 3.47, respectively. They are both significant at the 1% confidence level and the R^2 of these regressions range between 40% and 42%. The results are also economically significant as a one standard deviation increase in disagreement translates, on average, to a $3.47 \times 0.20 = 70$ basis point (a 0.60-standard deviation) increase in the real $FS_t^{4,5}$ forward spread.

[INSERT TABLE 7 AND 8 HERE.]

4. Bond Risk Premia

The third set of predictions relates to bond risk premia. Models with heterogeneous beliefs suggest the existence of a common component driving both forward spreads and expected bond returns. This common factor depends on both the time-series and cross-sectional properties of the beliefs structure. First, we investigate the relative importance of real and inflation disagreement in explaining bond risk premia by estimating predictability regressions in the spirit of Fama and Bliss (1987)

$$rx_{t,t+12}^{(n)} = const + b_1 \Psi_t^g + b_3 \Psi_t^\pi + \varepsilon_{t+12}^{(n)}. \tag{28}$$

The specification uses forecasts on 1-year excess holding period returns conditional on our time series of quarterly disagreement. Thus, we sample annual future returns at the quarterly frequency and project on beginning-of-quarter disagreement. Table 9 reports point estimates and the standard errors computed from a block bootstrap, as above. We find that the slope coefficient on Ψ_t^g is positive and significant at the 5% confidence level for all bond maturities. The slope coefficient of Ψ_t^π, on the other hand, is not statistically significant and all of the predictable variation in bond risk premia is coming from real disagreement. Indeed, the $\overline{R}^2$'s in the multivariate regression, which range between 6% and 11%, are almost identical to the $\overline{R}^2$'s in an unreported univariate regression that only includes Ψ_t^g. In terms of economic significance, a one standard deviation increase in real disagreement correspond, on average, to an increase of $4.97 \times 0.23 = 1.14\%$ (0.25 standard deviations) in expected excess returns on five-year bonds. This suggests that disagreement is an important economic driver in the dynamics of bond excess returns.

Next, we examine the prediction that the impact of disagreement is amplified in states of large sentiment as discussed in the theory section above. Specifically, we consider the impact of sentiment using the following predictability regression

$$rx_{t,t+12}^{(n)} = const + b(\mathcal{S}_t)\Psi_t^g + \varepsilon_{t+12}^{(n)}. \tag{29}$$

If we assume that $b(\mathcal{S}_t) = b_1 + b_2\mathcal{S}_t$, the relationship can be estimated as follows

$$rx_{t,t+12}^{(n)} = const + b_1\Psi_t^g + b_2\Psi_t^g\mathcal{S}_t + \varepsilon_{t+12}^{(n)}. \tag{30}$$

The results are summarized in Table 10. The interaction between sentiment and disagreement $\Psi_t^g \times \mathcal{S}_t$ yields an interesting result. The slope coefficient b_2 is positive and statistically significant at the 5% (10%) level for maturities of 2 and 3 (4 and 5)-years. The regressions $\overline{R}^2$ are large and range between 21% and 12% (two and five years bonds, respectively). These results are, once again, consistent with a speculative model with $\gamma < 1$, in which bond expected excess returns are large and positive when relative wealth is skewed toward agents holding optimistic beliefs (i.e. positive and large $\mathcal{S}_t$) in states with large disagreement. These are states in which optimists want to hold leveraged positions in risky assets and finance their portfolio exposure by short selling bonds. If a negative aggregate shock occurs, there is a wealth redistribution that shifts wealth-weighted beliefs toward the pessimists'. Interest rates fall, bond prices rise, and optimists are exposed to significant individual consumption risk. Consistent with this scenario, equilibrium bond risk premia need to be large to compensate the optimists' specific ex-post risk associated to their belief motivated trades: the product of the PoR and QoR is positive and large, thus implying that $b_2 > 0$.

[INSERT TABLE 9 AND 10 HERE]

7. The Role of Risk Aversion

Summarising, we examined three predictions from the theory sections, documenting consistent evidence of a speculative channel driving, at the same time, the short term rate, the slope of the term structure, and bond excess returns. The signs of the slope coefficients are all jointly consistent with economies in which the marginal trader is affected by financial institutions with small risk aversion ($\gamma < 1$). This motivates two questions: Is it realistic to expect agents active in bond markets to behave as if they have low risk aversion? Are the result robust in presence of heterogeneity in both risk aversion and beliefs?

1. Convex Incentives and Endogenous Risk Shifting

A large literature highlights the potential importance of convex incentives in the financial intermediation industry on capital markets. Indeed, when delegated managers earn option-like incentives agents become risk tolerant and, in some cases, risk seeking. Carpenter (2000) studies the dynamic investment problem of a risk averse manager compensated with a call option on the assets he controls. She shows that as the asset value drops, the optimal leverage of the manager increases and a conflict of interest emerges between the manager and his clients. The compensation of delegated managers can be convex even in absence of explicit options incentives. One example is when the downside risk of a trader's compensation is protected by limited liability. A second example is that of mutual fund managers. Sirri and Tufano (2002) and Chevalier and Ellison (1997) find an asymmetric relation between performance and subsequent new money flows. As winners receive a larger share of new flows, this creates a convex endogeneous compensation function that affects ex-ante the willingness of delegated manager to behave as if they had a lower effective risk aversion.[21] Our paper relates to this literature since it shows that speculation in the presence of a set of risk tolerant agents can play a key role in resolving bond market puzzles.

2. Bond Price Implications of Heterogeneity in Risk Version

Let us study the conditions under which the previous results are robust to other forms of heterogeneity, such as differences in risk aversion. The derivation here draws upon Bhamra and Uppal (2014) who provide analytical solutions for an economy with catching up with the Joneses utility

[21] Additional work stressing studying risk shifting in delegated portfolio management include Hodder and Jackwerth (2007), Basak, Pavlova, and Shapiro (2007), Panageas and Westerfield (2009), and Buraschi, Kosowski, and Sritrakul (2014).

functions where agents differ in their beliefs and their preference parameters for time discount, risk aversion, and sensitivity to habit. We apply their results to the CRRA case. A detailed derivation is provided in the Online Appendix.

Suppose that agents are heterogeneous with respect to both risk aversion γ^i and their subjective probability measure $\mathbb{P}^i$. Combining the first order condition of the individual agent problems with the first-order condition from the central planner's problem and clearing the consumption market gives the consumption sharing rule between the two agents:

$$\omega_t^o \eta_t^{\frac{1}{\gamma^o}} = (\omega_t^p)^{\frac{\gamma^p}{\gamma^o}} C_t^{\frac{\gamma^p - \gamma^o}{\gamma^p}}. \tag{31}$$

Let us assume, with no loss of generality, that $\gamma^p = m\gamma^o$. Substituting and after some algebra, one can obtain that the relative wealth share must satisfy:

$$\frac{(\omega_t^p)^m}{\omega_t^o} = \left(\eta_t C_t^{\gamma^o(1-m)}\right)^{\frac{1}{\gamma^o}}. \tag{32}$$

It is easy to observe that when $m = 1$, namely when there is no heterogeneity in preferences, the sharing rule becomes identical to equation (8). However, when $\gamma^o \neq \gamma^p$, the sharing rule is non-linear and explicitly depends on the extent of heterogeneity m.

2.1. Risk Free Rate

Non-linearity in the sharing rule has implications for equilibrium bond prices beyond differences in belief as the dynamics of short term interest rate now depend both on the aggregate relative risk aversion, $\mathbf{R}_t \equiv \left[\frac{\omega_t^o}{\gamma^o} + \frac{\omega_t^p}{\gamma^p}\right]^{-1}$, and aggregate prudence, $\mathbf{P}_t \equiv (1+\gamma^o)\omega_t^o(\frac{\mathbf{R}_t}{\gamma^o})^2 + (1+\gamma^p)\omega_t^p(\frac{\mathbf{R}_t}{\gamma^p})^2$. When $\gamma^o \neq \gamma^p$, the short-term interest rate is equal to:

$$r_t = \delta + \mathbf{R}_t(\omega_t^o g_t^o + \omega_t^p g_t^p) - \frac{1}{2}\mathbf{R}_t \mathbf{P}_t \sigma_c^2 \tag{33}$$

$$+ \underbrace{\frac{1}{2}\omega_t^o \omega_t^p \mathbf{R}_t^2 \left(1 - \frac{\mathbf{R}_t}{\gamma^o \gamma^p}\right)\Psi_t^2}_{Term\ 1} - \underbrace{\omega_t^o \omega_t^p \mathbf{R}_t^3 \left(\frac{1}{\gamma^o} - \frac{1}{\gamma^p}\right)\sigma_c \Psi_t}_{Term\ 2}. \tag{34}$$

In the special case $\gamma^o = \gamma^p$, the solution reduces to the homogeneous economy one reported in equation (9). With heterogeneity in risk aversion, the impact of disagreement on short term interest rates is driven by two additional terms. The first term ("Term 1") is quadratic in disagreement; the second term ("Term 2") is linear in disagreement and is proportional to the extent of heterogeneity in preferences.

However, when $\gamma^o \neq \gamma^p$ new terms may either reinforce or reduce the negative effect of Ψ_t^2 on r_t. It can be immediately noticed that a sufficient condition for the effect to be enhanced, i.e. interest rates to drop when disagreement increases, is: (i) $\frac{\mathbf{R}_t}{\gamma^o\gamma^p} > 1$ and (ii) $\gamma^o < \gamma^p$. This occurs when the optimist agent o, is less risk averse than the pessimist and $\gamma^p\omega_t^o + \gamma^o\omega_t^p < 1$, namely when the relative wealth of the optimist is below a maximum threshold which depends on the level of risk aversion of agent p, i.e. $\omega_t^o < \frac{(1-\gamma^o)}{(\gamma^p-\gamma^o)}$. Obviously, this is a sufficient condition and interest rates can still be decreasing in disagreement Ψ_t even if it is violated, as long as the linear term dominates the quadratic one. This can occur, for instance, if $\gamma^o < \gamma^p$ as long as $\mathbf{R}_t \cong \gamma^o\gamma^p$ (which occurs when the weighted average risk aversion is one). On the other hand, the negative effect on short term interest rates is attenuated when the risk aversion of the optimist is higher than the pessimist, namely $\gamma^o > \gamma^p$. This can potentially give rise to significant non-linearities in the dynamics of short-term interest rates.

It is notable that an important literature argues that financial intermediaries are, at the same time, both less risk averse than the average investor, due to their convex incentives, and more optimistic on average. In states of the world when this is true, one can expect that $\gamma^o < \gamma^p$. In support of this argument, a stream of the literature documents that banks "sell-side" analysts (acting on behalf of brokers) forecasts and recommendations are, on average, excessively optimistic (See Kato, Skinner, and Kunimura (2009) and Cho, Hah, and Kim (2011)). An example which is often cited is the degree of economic optimism of banks in the run up to the 2008 crisis (see, for example, Agrawal and Chen (2012)).

8. Conclusion

In this paper, we study bond market implications in economies where agents agree to disagree about economic fundamentals. In these economies interest rates and risk premia are affected both by fundamental risk and by the individual consumption risk generated by agents' beliefs themselves.

When optimistic traders leverage long positions by shorting bonds, they must take into account the risk of a price rise if wealth weights of pessimists' beliefs are to increase (either because of unexpected positive shocks or because of shifts in beliefs) and he has to buy back the stock. The risk of a further change of the wealth-weighted average opinion is an additional channel affecting prices ex-ante and risk premia. Lower levels of risk aversion amplify this effect since it increases trade and ex-post individual consumption risk in equilibrium.

Our paper shows that this effect is significant to help explaining several empirical properties of bonds prices. Both an estimation and reduced form empirical tests of the model show that the distribution of beliefs affects both short terms interest rates and the slope of the term structure. Moreover, bond risk premia are nonlinear in sentiment and disagreement. In states of large positive sentiment, greater disagreement induces large bond risk premia and the effect is magnified for low risk aversion.

The main predictions of the model are consistent with the data, and show that speculation, instead of being a challenge to traditional equilibrium models, can help to explain at least three key empirical regularities in bond markets which are otherwise difficult to be reconciled with single agent models in which the main asset pricing channel is driven by hedging demands.

These results raise several interesting additional questions. First, in our model we take risk aversion as an exogenous parameter. However, propensity to bare risk may be endogenous and may depend on a variety of characteristics, such as incentives, corporate governance, market structure. It would be interesting to study structural economies in which Treasury bond prices may depend on some of these features directly. Second, since the economy we study generates implications for equilibrium volumes of trade, it would be interesting to study the extent to which bond trading volumes are correlated to disagreement, as opposed to shocks to fundamentals. We leave these questions to future research.

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9. Appendix C: Tables

Table 1. Consumption Estimation

This table reports empirical and simulated method of moments estimates for the consumption dynamics reported in equation 1 in the main body of the paper. Consumption data is from the U.S. Bureau of Economic Analysis (BEA) from which we compute annual real per capital consumption growth on non-durables by combining NIPA Tables 2.3.3, 2.8.6, and 7.1. Model implied moments are reported for annual time aggregated consumption growth rates. Panel (a) reports parameter estimates and Panel (b) reports estimated moments. MD is the empirical moment while MS is the estimated moment. Upper and Lower are the 95% confidence intervals for moment estimates. The sample period is 1961.1 - 2019.1.

(a) Parameters

	$\widehat{\beta}$	Lower	Upper
θ	2.00	1.79	2.22
σ_c	1.03	0.82	1.23
κ	0.09	0.06	0.12
σ_g	0.43	0.21	0.66

(b) Moments

	MD	$\widehat{MS}$	Lower	Upper
mean	1.94	1.89	1.75	2.02
std	1.25	1.16	1.12	1.20
AR(1)	0.52	0.46	0.43	0.49
AR(2)	0.20	0.26	0.22	0.30
AR(5)	0.09	0.09	0.06	0.13
AR(10)	0.02	-0.05	-0.08	-0.02

Table 2. Term Structure Estimation

This table reports empirical and simulated method of moments estimates for the term structural of nominal interest rates. Zero coupon Treasury bonds data is from the Gürkaynak, Sack, and Wright (2006) (GSW) dataset for maturities between 3 months and 5 years. Moment estimates are for continuously compounded log yields. Panel (a) reports parameter estimates and Panel (b) reports estimated moments. MD is the empirical moment while MS is the estimated moment. Upper and Lower are the 95% confidence intervals for moment estimates. The sample period is 1961.1 - 2019.1.

(a) Parameters

	$\widehat{\beta}$	lower	upper
γ	0.62	0.36	0.88
Δ_θ	0.45	0.25	0.65
Δ_ρ	0.29	0.16	0.40

(b) Moments

	MD	$\widehat{MS}$	lower	upper
y^{3m} mean	4.90	4.93	4.56	5.30
$\Delta\ y^{3m}$ std	0.59	0.20	0.09	0.31
$\Delta\ y^{3m}$ skew	-0.74	0.01	-0.80	0.83
$\Delta\ y^{3m}$ kurt	15.86	15.78	15.69	15.87
y^{3m} AR(1)	0.98	0.95	0.92	0.98
y^{12m} mean	5.09	5.12	4.96	5.29
y^{24m} mean	5.31	5.31	5.13	5.48
y^{36m} mean	5.49	5.43	5.24	5.62
y^{48m} mean	5.65	5.52	5.32	5.71
y^{60m} mean	5.78	5.58	5.39	5.78

Table 3. Summary Statistics: Full Sample Differences

Table reports summary statistics for the left and right hand variables used in our regression analysis. From left to right across the column heading these are: expected 1-quarter ahead inflation and GDP growth consensus expectations, interquartile range in 1-quarter ahead inflation ahead GDP growth (disagreement), and 3,6, 9 and 12 month nominal yields. Data is quarterly from 1988.1 - 2020.1.

	$E[\pi]$	$E[g]$	Ψ^{π}	Ψ^{g}	$y^{(3)}$	$y^{(6)}$	$y^{(9)}$	$y^{(12)}$
Mean	-0.02	0.01	0.00	-0.01	-0.03	-0.03	-0.04	-0.04
std	0.66	0.91	0.18	0.15	0.48	0.50	0.52	0.53
min	-3.40	-3.20	-0.75	-0.73	-2.14	-1.99	-1.86	-1.75
max	3.10	3.50	0.81	0.37	0.97	1.19	1.35	1.34
Skew	-0.55	-0.06	0.38	-0.54	-1.32	-1.01	-0.79	-0.70
Kurt	13.30	6.67	7.94	7.15	6.32	5.20	4.55	4.14
AR(1)	-0.20	0.00	-0.34	-0.17	0.34	0.28	0.21	0.16

Table 4. Summary Statistics: Full Sample Levels

Table reports summary statistics for the left and right hand variables used in our regression analysis. From left to right across the column heading these are: expected 1-quarter ahead inflation and GDP growth consensus expectations, interquartile range in 1-quarter ahead inflation ahead GDP growth (disagreement), Sentiment, 3 and 12 month nominal yields, 2 and 5-year forward spreads, 2 and 5 year annual excess bond returns. Data is quarterly from 1988.1 - 2020.1.

	$E[\pi]$	$E[g]$	Ψ^{π}	Ψ^{g}	$\mathcal{S}$	$y^{(3)}$	$y^{(12)}$	$FS^{1,2}$	$FS^{4,5}$	rx^{2}	rx^{5}
Mean	2.58	2.36	0.49	0.54	-0.32	3.23	3.45	0.94	1.65	0.65	2.33
std	0.99	1.19	0.22	0.23	0.72	2.51	2.62	0.86	1.31	1.26	4.28
min	-1.30	-3.20	0.23	0.24	-1.14	0.07	0.12	-0.64	-0.60	-2.52	-8.49
max	7.00	4.60	1.72	1.35	2.37	9.22	9.66	3.02	4.09	3.46	12.46
Skew	0.84	-1.89	2.18	1.30	2.11	0.32	0.27	0.24	0.16	0.27	-0.01
Kurt	7.39	8.36	10.22	4.49	7.74	1.96	1.90	2.33	1.84	2.65	2.55
AR(1)	0.77	0.71	0.67	0.78	0.93	0.98	0.98	0.88	0.92	0.80	0.71

Table 5. Summary Statistics: Real Yields

Table reports summary statistics for the left and right hand variables used in our regression analysis. From left to right across the column heading these are: expected 1-quarter ahead inflation and GDP growth consensus expectations, interquartile range in 1-quarter ahead inflation ahead GDP growth (disagreement), and 2, 3, and 4 year real forward spreads where REAL: $FS_t^{n,n+1} = F_t^{n,n+1} - y_t^2$. Data is quarterly from 2002.1 - 2020.1.

	$E[\pi]$	$E[g]$	Ψ^{π}	Ψ^{g}	$REAL:$ $FS^{2,3}$	$REAL:$ $FS^{3,4}$	$REAL:$ $FS^{4,5}$
Mean	2.07	2.43	0.56	0.46	0.59	0.90	1.15
std	0.61	1.23	0.24	0.20	0.98	1.08	1.12
min	-1.30	-3.20	0.25	0.24	-2.47	-2.36	-2.24
max	3.90	4.30	1.72	1.35	4.70	4.57	4.15
Skew	-2.05	-2.16	2.23	1.85	1.16	0.58	0.14
Kurt	15.20	9.74	9.79	7.33	7.77	4.93	3.58
AR(1)	0.38	0.76	0.58	0.76	0.62	0.69	0.72

Table 6. Short Term Interest Rate Regressions

This table reports the results from a regression of *changes* in maturity τ nominal short term interest rates on *changes* in consensus inflation ($E[\pi]$) and GDP ($E_t[g]$) expectations, real (Ψ_t^g) and inflation (Ψ_t^π) disagreement proxies

$$\Delta y_t^{(n)} = const + \Delta\beta_1 E_t[\pi] + \beta_2 \Delta E_t[g] + \beta_3 \Delta\Psi_t^g + \beta_4 \Delta\Psi_t^\pi + \varepsilon_t^{(n)}$$

Standard errors are reported in parentheses below the point estimates and are computed using a block bootstrap with a block length of 4 and computed from 10000 replications. Superscripts $*$, $**$ and $***$ denote statistical significance at 90%, 95% and 99%, respectively. Data is quarterly from 1988.1 - 2020.1.

	const	$\Delta E[\pi]$	$\Delta E[g]$	$\Delta\Psi^g$	$\Delta\Psi^\pi$	$\overline{R}^2(\%)$
$\Delta y^{(3)}$	-0.04	0.12^{*}	0.19^{***}	-0.65^{*}	0.31^{**}	16.20
	(0.06)	(0.06)	(0.06)	(0.32)	(0.17)	
$\Delta y^{(6)}$	-0.04	0.16^{***}	0.20^{***}	-0.64^{**}	0.37^{**}	21.31
	(0.06)	(0.06)	(0.06)	(0.32)	(0.16)	
$\Delta y^{(9)}$	-0.04	0.19^{***}	0.21^{***}	-0.62^{**}	0.39^{**}	23.64
	(0.05)	(0.06)	(0.06)	(0.31)	(0.17)	
$\Delta y^{(12)}$	-0.04	0.21^{***}	0.21^{***}	-0.59^{**}	0.40^{**}	24.96
	(0.05)	(0.06)	(0.06)	(0.30)	(0.17)	

Table 7. Nominal Forward Spread Regressions

This table reports the results from a regression of forward spreads (forward rates minus the one year yield) on consensus inflation ($E[\pi]$) and GDP ($E_t[g]$) expectations, real (Ψ_t^g) and inflation (Ψ_t^π) disagreement proxies:

$$FS_t^{n,n+1} = const + \beta_2 E_t[\pi] + \beta_3 E_t[g] + \beta_4 \Psi_t^g + \beta_5 \Psi_t^\pi + \varepsilon_t^{(n)}$$

where $FS_t^{n,n+1} = F_t^{n,n+1} - y_t^1$. Standard errors are reported in parentheses below the point estimates and are computed using a block bootstrap with a block length of 4 and computed from 10000 replications. Superscripts $*$, $**$ and $***$ denote statistical significance at 90%, 95% and 99%, respectively. Data is quarterly from 1988.1 - 2020.1.

	const	$E[\pi]$	$E[g]$	Ψ^g	Ψ^π	$\overline{R}^2(\%)$
$FS^{1,2}$	0.10	-0.12^{*}	0.12^{*}	1.17^{***}	-0.43	13.64
	(0.37)	(0.08)	(0.07)	(0.36)	(0.33)	
$FS^{2,3}$	0.50	-0.30^{**}	0.18	1.85^{***}	-0.46	12.63
	(0.63)	(0.13)	(0.12)	(0.57)	(0.56)	
$FS^{3,4}$	0.87	-0.45^{***}	0.22	2.28^{***}	-0.33	13.26
	(0.81)	(0.17)	(0.16)	(0.71)	(0.72)	
$FS^{4,5}$	1.22	-0.57^{***}	0.24	2.57^{***}	-0.17	14.48
	(0.95)	(0.20)	(0.18)	(0.82)	(0.84)	

Table 8. Real Forward Spread Regressions

This table reports the results from a regression of real forward spreads (real forward rates minus the two real year yield) on consensus inflation ($E[\pi]$) and GDP ($E_t[g]$) expectations, real (Ψ_t^g) and inflation (Ψ_t^π) disagreement proxies:

$$\text{REAL: } FS_t^{n,n+1} = const + \beta_2 E_t[\pi] + \beta_3 E_t[g] + \beta_4 \Psi_t^g + \beta_5 \Psi_t^\pi + \varepsilon_t^{(n)}$$

where REAL: $FS_t^{n,n+1} = F_t^{n,n+1} - y_t^2$. Standard errors are reported in parentheses below the point estimates and are computed using a block bootstrap with a block length of 4 and computed from 10000 replications. Superscripts $*$, $**$ and $***$ denote statistical significance at 90%, 95% and 99%, respectively. Data is quarterly from 2002.1 - 2020.1.

	const	$E[\pi]$	$E[g]$	Ψ^g	Ψ^π	$\overline{R}^2(\%)$
$FS^{2,3}$	-1.74	0.15	0.45^{***}	2.96^{***}	-0.80	40.39
	(0.75)	(0.17)	(0.14)	(0.78)	(0.55)	
$FS^{3,4}$	-1.62	0.06	0.54^{***}	3.39^{***}	-0.86	41.94
	(0.89)	(0.20)	(0.16)	(0.79)	(0.67)	
$FS^{4,5}$	-1.35	0.00	0.57^{***}	3.47^{***}	-0.89	41.11
	(1.01)	(0.24)	(0.16)	(0.77)	(0.76)	

Table 9. Bond Predictability Regressions 1

This table reports estimates from the return forecasting specification

$$rx_t^n = a + b_1 \Psi_t^g + b_3 \Psi_t^\pi + \epsilon_{t,t+4}^n.$$

where 1-year holding period excess returns (rx_t^n) are projected on real (Ψ_t^g) and inflation (Ψ_t^π) disagreement proxies. Standard errors are reported in parentheses below the point estimates and are computed using a block bootstrap with a block length of 4 and computed from 10000 replications. Superscripts $*$, $**$ and $***$ denote statistical significance at 90%, 95% and 99%, respectively. Data is quarterly from 1988.1 - 2020.1.

	const	Ψ^g	Ψ^π	$\overline{R}^2$ (%)
rx^2	-0.44	1.92**	0.16	11.26
	(0.46)	(0.86)	(0.65)	
rx^3	-0.60	3.32**	0.28	9.20
	(0.88)	(1.53)	(1.27)	
rx^4	-0.48	4.29**	0.18	7.20
	(1.26)	(2.03)	(1.83)	
rx^5	-0.13	4.97**	-0.16	5.45
	(1.59)	(2.42)	(2.35)	

Table 10. Bond Predictability Regressions 2

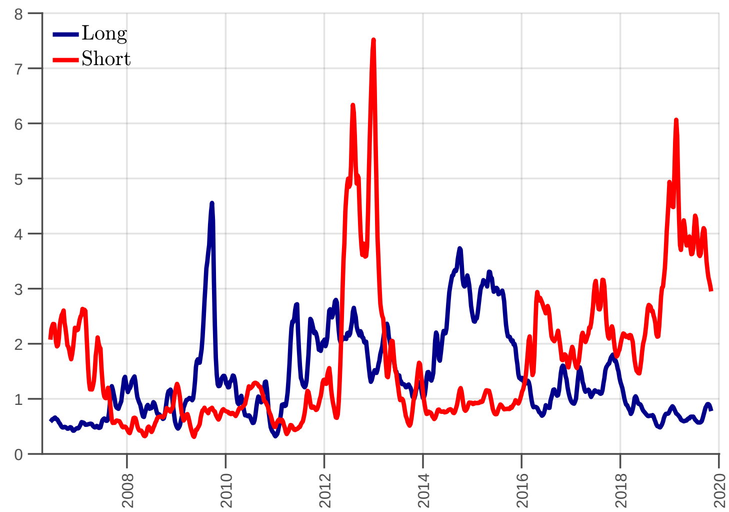
This table reports estimates from the return forecasting specification

$$rx_t^n = const + b_1 \Psi_t^g + b_2 \Psi_t^g \mathcal{S} + \varepsilon_{t+4}^{(n)}$$

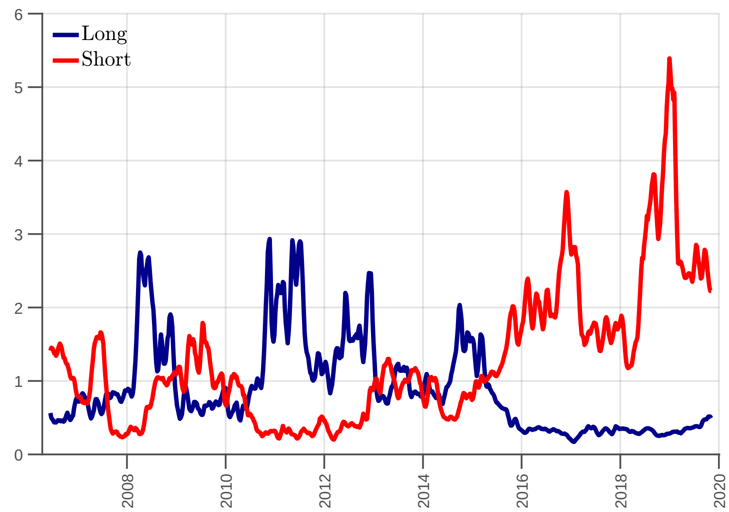
where 1-year holding period excess returns (rx_t^n) are projected on real disagreement (Ψ_t^g) and the interaction between real disagreement and sentiment ($\Psi^g \times \mathcal{S}$). Standard errors are reported in parentheses below the point estimates and are computed using a block bootstrap with a block length of 4 and computed from 10000 replications. Superscripts $*$, $**$ and $***$ denote statistical significance at 90%, 95% and 99%, respectively. Data is quarterly from 1988.1 - 2020.1.

	const	Ψ^g	$\Psi^g \times \mathcal{S}$	$\overline{R}^2$ (%)
rx^2	-0.24	1.89**	0.75**	20.77
	(0.47)	(0.87)	(0.37)	
rx^2	-0.26	3.27**	1.33**	17.57
	(0.82)	(1.47)	(0.66)	
rx^2	-0.09	4.18**	1.73*	14.40
	(1.09)	(1.90)	(0.94)	
rx^2	0.20	4.73**	2.03*	11.66
	(1.31)	(2.23)	(1.20)	

10. Figures

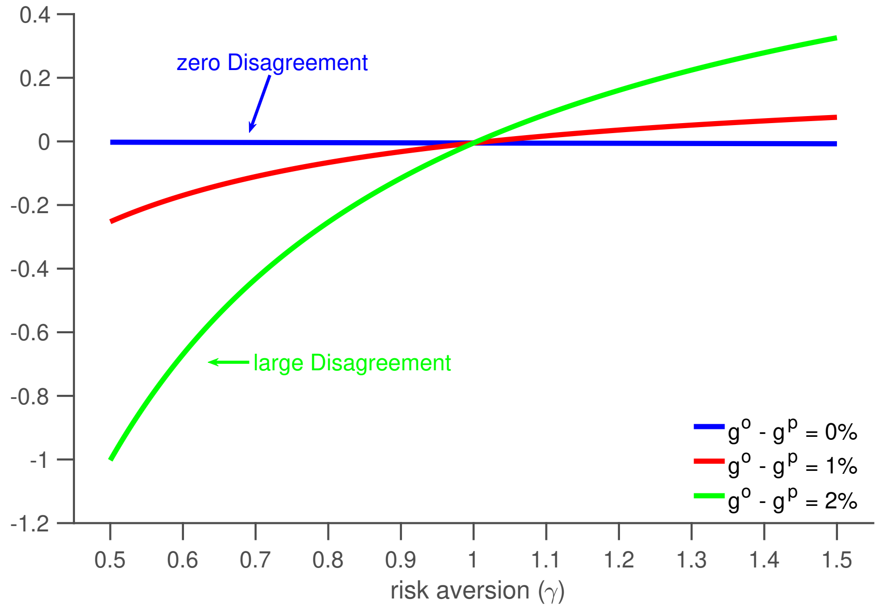


(a) 2-year

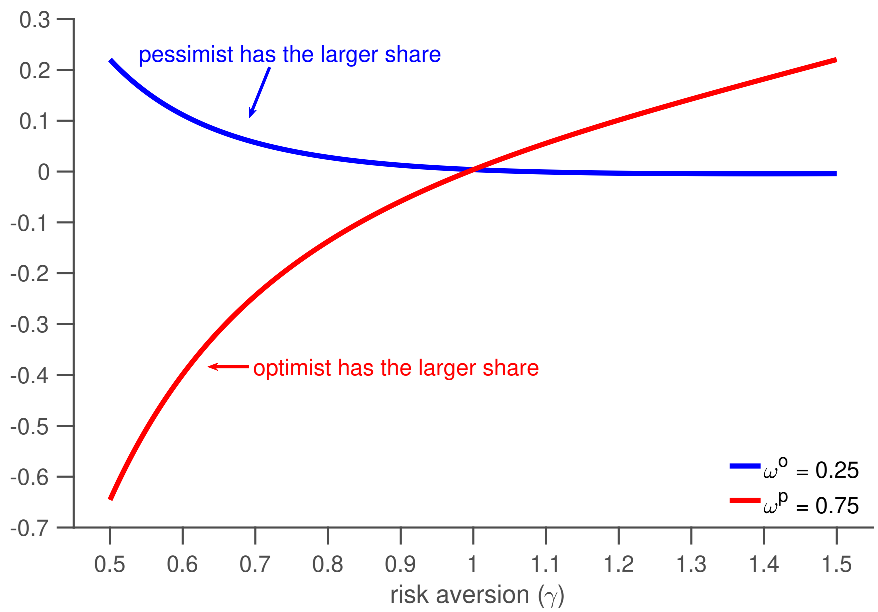


(b) 5-year

Figure 1. Speculative Demand - Treasury Bond Future
The two plots use disaggregated data provided by the CFTC to construct separate time series for long and short net positions measured in terms of number of contracts for each category of users. Let $R_t = L_t/(AM_t + O_t)$ be the ratio of the positions of "Leveraged funds" (L_t) and "Asset Managers" (AM_t) plus "Other Reportables" (O_t). The top panel plots R_t for the 2-year Treasury bond contract (blue=Long; red=Short); the bottom panel plots R_t for the 5-year Treasury bond contract.



(a)



(b)

Figure 2. Short Rate Sensitivities: differences in belief
The top panel plots the sensitivity of the short rate with respect to disagreement as a function of risk aversion for disagreements of zero, one, and two percent. The bottom panel plots the sensitivity of the short rate with respect to the Radon-Nikodym derivative $\eta_t = \left(\frac{\omega_t^p}{\omega_t^o}\right)^\gamma$ for optimistic ($\omega^o = 0.75$) and pessimistic ($\omega^o = 0.25$) economies for $g_t^o - g_t^p = 2\%$.

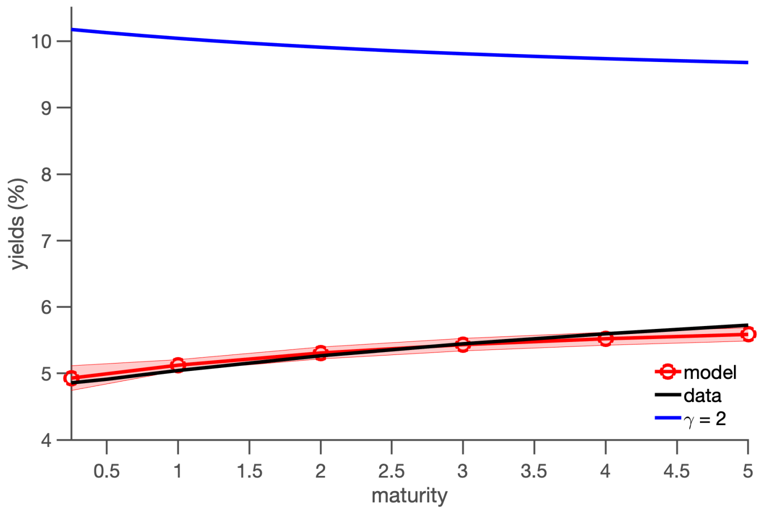


Figure 3. Term Structure Estimation

This figure displays the model implied and empirical estimates for the average levels of $\{y^{3m}, y^{12m}, y^{24m}, y^{36m}, y^{48m}, y^{60m}\}$ continuously compounded log zero coupon yields. The red line and shaded red region displays the point estimates and 95% confidence intervals corresponding to the parameter and moments reported in table 2. The black line displays empirical counterparts and the blue line plots a counter factual that takes the estimated model parameters but replaces the estimated risk aversion with $\gamma = 2.00$. The sample period is 1962.1 - 2019.1

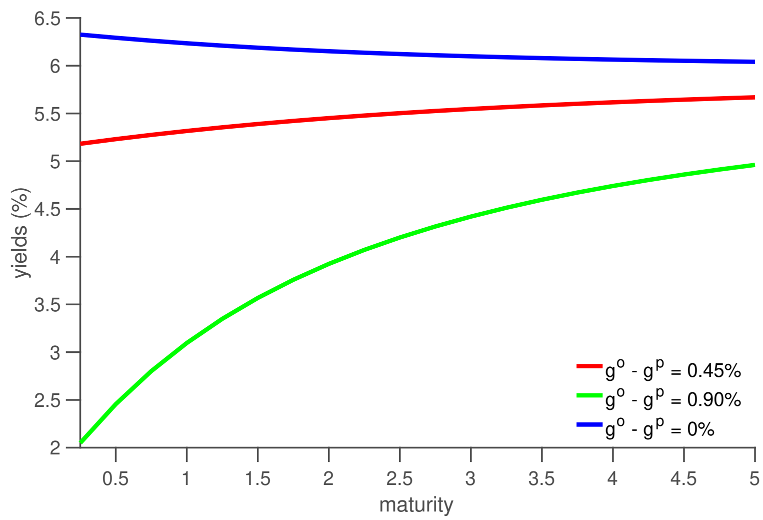
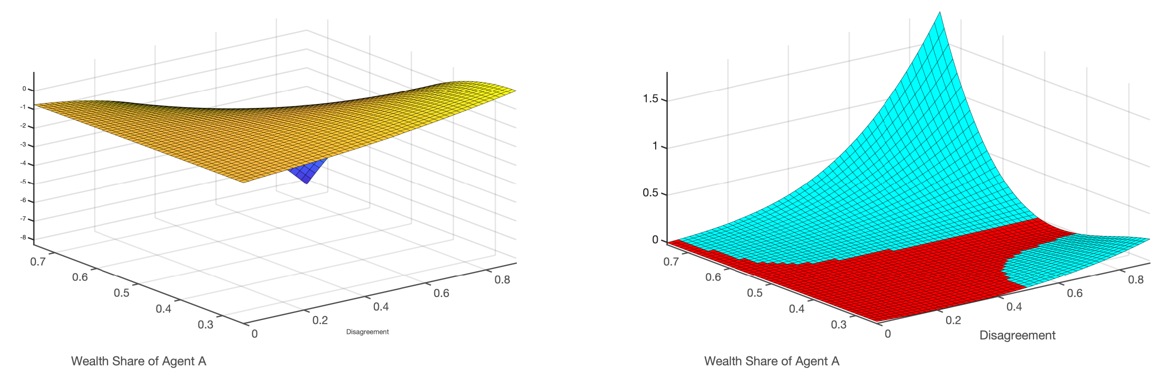


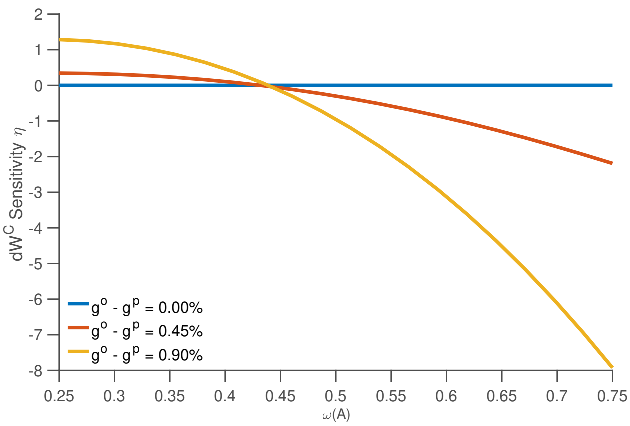
Figure 4. Term Structure Implications

Figure displays the term structure of interest rates for an economy populated by agents risk aversions: $\gamma = 0.62$ and where the distribution of wealth is skewed towards the pessimist agents: $\omega^o = 0.40$. Term structures are plotted for steady state disagreement equal to $g^o - g^p = 0.45\%$ (red line), a large disagreement state $g^o - g^p = 0.90\%$ (green line), and an economy where agents agree on future growth rates $g^o - g^p = 0.00\%$ (blue line).

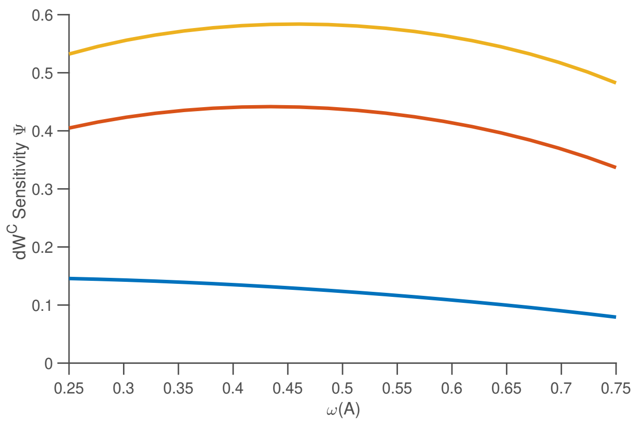


(a) $\sigma^5_{b,c}$ **(b)** $\mathrm{E}\left[rx^5\right]$

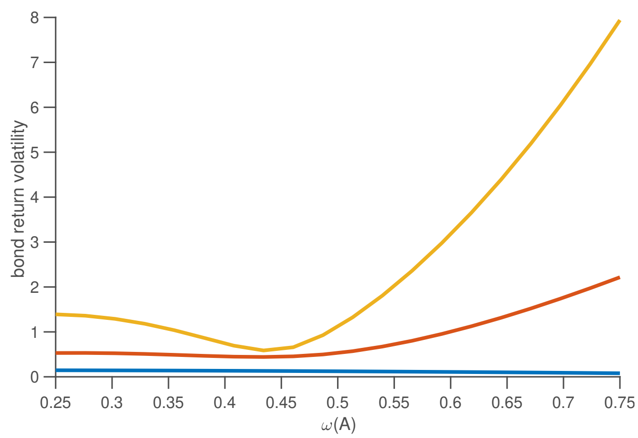
Figure 5. Quantities of Risk and Expected Bond Returns.
The left panels plot quantities of risk ($\sigma^\tau_{b,c}$), i.e, bond sensitivities to aggregate consumption shocks dW_c for maturity $\tau = 5$ year bonds. The right panels plot instantaneous excess returns given by the product of quantities of risk and prices of risk. Both panels are for an economy populated by agents with risk aversion $\gamma = 0.62$.



(a)



(b)



(c)

Figure 6. Bond Sensitivities and Volatility.
The sensitivity of bond returns to dW_c shocks is given by the sum of three terms:

$$\sigma_{b,c}^{\tau} = \frac{1}{B^{\tau}} \left[\frac{\partial B^{\tau}}{\partial \widehat{g}^{a}} \; \sigma_{c,g} + \frac{\partial B^{\tau}}{\partial \eta} \; (-\Psi^{g}) + \frac{\partial B^{\tau}}{\partial \Psi^{g}} \sigma_{c,\Psi^{g}} \right]$$

For the a bond with maturity $\tau = 5$ years and for $\gamma = 0.62$, panel (a) plot the component of this gradient due to η (the middle term) while panel (b) plots the component of this gradient due to Ψ^g (the last term). Sensitivities are plotted as a function of the relative wealth of agent a (along the x-axis) and for three different levels of disagreement (Ψ^g).

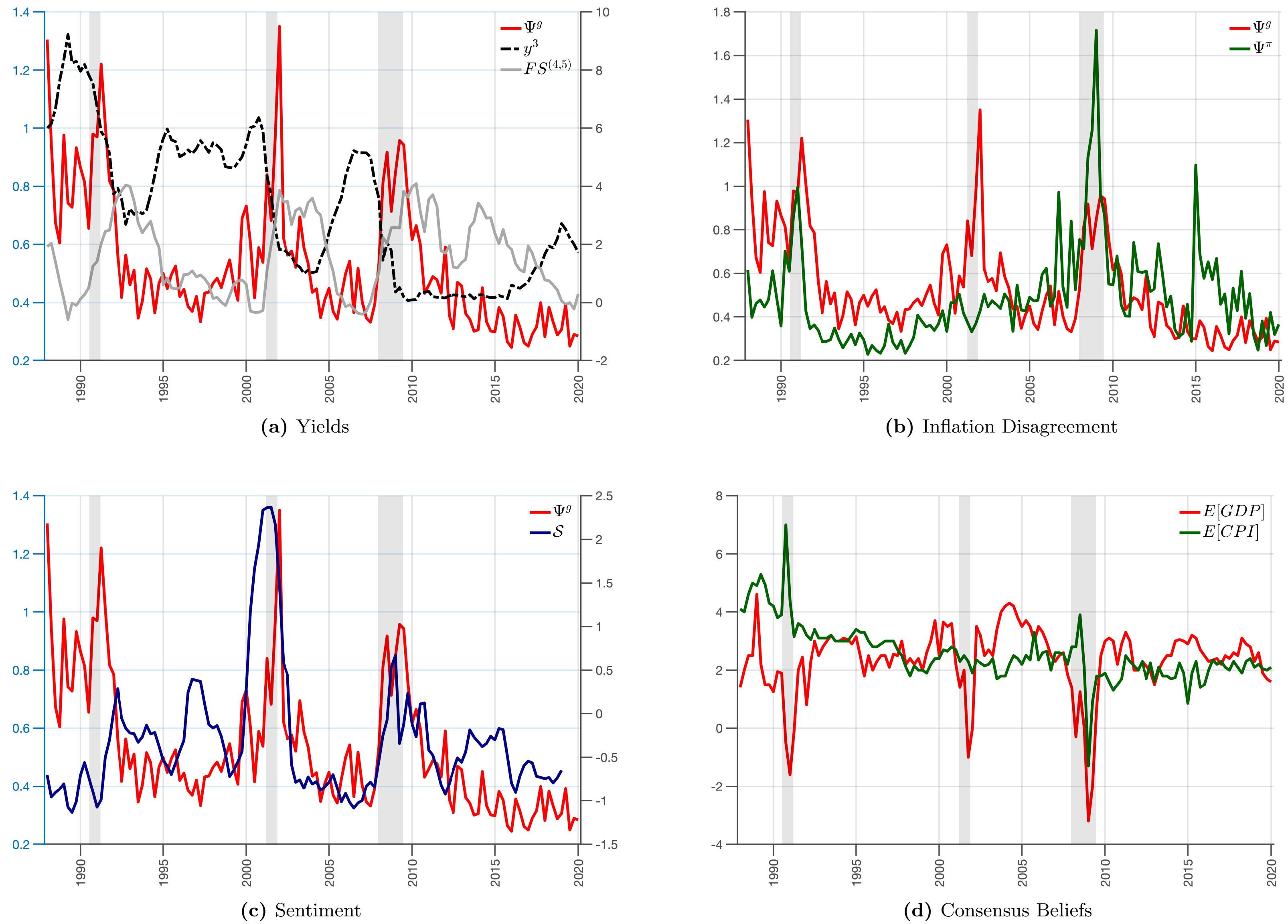


Figure 7. Beliefs and Yields

Panel (a) displays time-series plots for 1-quarter ahead GDP and CPI growth rate expectations. Panel (b) displays our proxy for real disagreement proxy (Ψ^g) and inflation disagreement proxy (Ψ^π). Sample period for the belief proxies is 1988.1 — 2020.1 Panel (c) plots the sentiment index of Huang, Jiang, Tu, and Zhou (2015) alongside Ψ^g. Sample period for sentiment is 1988.1 — 2018.12.