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QALYs, DALYs, and HALYs: a unifying framework for the evaluation of population health*

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Abstract

We provide a unifying framework for the evaluation of population health. We formalize several axioms for social preferences over distributions of health. We show that a specific combination of those axioms characterizes a large class of *population health evaluation functions* combining concerns for quality of life, quantity of life and health shortfalls. We refer to the class as (unweighted) aggregations of *health-adjusted life years* (HALYs). Two focal (and somewhat polar) members of this family are the (unweighted) aggregations of *quality-adjusted life years* (QALYs), and of *disability-adjusted life years* (DALYs). We also provide new characterization results for these focal members that enable us to scrutinize their normative foundations and shed new light on their similarities and differences.

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Keywords: *population health, QALYs, DALYs, HYEs, axioms.*

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1 Introduction

The ability to assess the effect on a population of specific health interventions is crucial for decisions of priority and financing in the health care sector. Social scientists, health services researchers, and operations researchers alike have long been concerned with developing appealing quantitative measures to evaluate the health of a population.[1] Mortality indicators (such as life expectancy) were typically used first. Although they are still of great importance nowadays, there has been a growing consensus to combine them with morbidity indicators. By now, it is widely accepted that the benefit a patient derives from a particular health care intervention can be defined according to two natural dimensions: quality of life and quantity of life. Pliskin et al., (1980) developed the so-called *quality-adjusted life years* (in short, QALYs), which offer a straightforward procedure to combine the two natural dimensions. It is arguably the most widely accepted methodology in the economic evaluation of health care nowadays, and a reference standard in cost-effectiveness analysis (e.g., Gold et al., 1996). Nevertheless, alternative health outcome measures are also popular. A special emphasis goes to the so-called *disability-adjusted life years* (in short, DALYs), primarily a measure of disease burden, which arose in the early 1990s (e.g., World Bank, 1993) as a result of an effort to quantify the global burden of premature death, disease, and injury and to make recommendations that would improve health, particularly in developing nations. DALYs have been extensibly studied ever since (e.g., Murray, 1994; Murray and Acharya, 1997; Anand and Hanson, 1998). As with QALYs, they have also been systematically used in applied work (e.g., Murray et al., 2012; Murray et al., 2015; Kyu et al., 2018) and remain extremely popular for a wide variety of cases as of today (e.g., Briggs and Vassall, 2021; Giannino et al., 2021; Chapman et al., 2022; Xiong et al., 2022). QALYs and DALYs have usually been confronted to each other (e.g., Sassi, 2006; Martinez et al., 2019; Feng et al., 2020). Nevertheless, both measures can actually be seen as (admittedly, polar) instances of *Health Adjusted Life Years* (HALYs), an umbrella term for a family of measures endorsing concerns for health attainments as well as health shortfalls (e.g., Gold et al., 2002).[2]

[1]Early instances are Fanshel and Bush (1970) and Torrance (1976). For comprehensive surveys, the reader is referred to Dolan (2000), Gold et al., (2002) or Murray et al., (2002) among others.

[2]The notion of acceptable health as a reference point in health priority setting is, for instance, explored by Wouters et al., (2015, 2017).

We provide in this paper normative foundations for HALY-based measures. Normative foundations of population health measures are crucial to guide public authority choices among them. Nevertheless, the literature has not paid sufficient attention to them. To wit, although they have been established for basic families of QALY-based measures of population health, this has not been the case for measures involving a health loss. In this paper, we aim to fill that gap upon introducing a framework broad enough to allow for the analysis of measures based on both approaches. In doing so, we are also able to investigate the normative principles underlying models in which the health of a population is either measured in terms of (health) gains, losses or both.

In our model, we assume that society has preferences over distributions of (average) health states and lifetime spans in a population, and we determine specific combinations of normative principles (axioms) that characterize different measures for the evaluation of population health, dubbed *population health evaluation functions* (in short, PHEFs). More precisely, we assume that the distribution of health in a population is defined by a collection of triplets, each indicating the status that an agent of the population achieves in the health dimension (quality of life), the time dimension (quantity of life), as well as the (individual) reference lifetime. The framework that we set up thus allows us to approach the problem from both a health asset view and a health gap view. As a result, we are able to axiomatize population health evaluation functions concerned with the loss of life and/or the accumulation of disability or ill health, as well as those concerned with the health gains.

Our approach builds upon the framework introduced in Hougaard et al., (2013). Therein, a number of population health evaluation functions, such as the (time linear) QALY and HYE (acronym for *Healthy Years Equivalent*) population health evaluation functions, concerned with the accumulation of health, are characterized axiomatically. Our generalization of that framework allows us to characterize not only those population health evaluation functions but also others taking a 'health gap' approach, including the (time linear) DALY population health evaluation function. Furthermore, by characterizing the two types of population health evaluation functions side by side, we are able to highlight the similarities and differences between the two approaches. The similarities and differences between the QALY and DALY measures (with a special emphasis on whether there is an impact from using the latter) have previously been discussed (e.g., Sassi, 2006; Airoldi and Morton, 2009; Morton, 2010). Our results add new insights to this discussion. We present general classes of population health evaluation functions that are able to encompass both approaches into one. In particular, we present a one-parameter family of population health evaluation functions that contain the time linear QALY and time linear DALY as special cases, thereby compromising among them.

Our first result is precisely a characterization of this family by the combination of a pack of basic structural axioms (dubbed "COMMON") with two additional axioms. One (*lifetime invariance at common health*) states that the planner should be indifferent between adding (the same amount of) lifetime to one agent or another, provided both experience the same health status (although probably different lifetimes) and the gap between lifetime and reference age is kept fixed. Another (*reference age invariance at common health*) states that the evaluation of a population health distribution in which the reference age of an individual is increased by a certain amount does not depend on whether this is done for one individual or another, as long as the two individuals have the same health state.

We also show that adding just one axiom (out of a pair of dual independence axioms) to those listed above allows us to characterize the focal elements within the above-mentioned family that correspond to the time linear QALY and time linear DALY population health evaluation functions. This shows that both (polar) measures actually share a solid ground.

Finally, we provide additional results characterizing more general families of population health evaluation functions encompassing the HALY-based measures.

Our paper obviously lies within the literature on health economics dealing with the normative foundations of health measures. As such, it is connected to the sizable literature on decision theory making use of multiattribute utility functions, pioneered by Debreu (1960), Fishburn (1965), Raiffa (1968) and Keeney (1974), among others. We should, nevertheless, stress that we do not presuppose in our analysis the existence of an individual utility function to evaluate health.[3]

A novelty of our formal analysis is to augment the concept of population health distributions to include (individual) reference lifetimes, beyond the standard dimensions of quality of life and

[3]This is in line with Hougaard et al., (2013) and Moreno-Ternero and Østerdal (2017). There exist earlier contributions within the health economics literature providing normative foundations for QALY-based measures of population health, but they presume the existence of individual health-related utility (QALY) measures (e.g., Bleichrodt, 1995, 1997) or individual preference relations over quality and quantity of life (e.g., Østerdal, 2005).

quantity of life.[4] This opens the door to explore possible connections between evaluations on attainment and shortfall health. The discussion on attainment and shortfall inequality (e.g., Sen, 1992) has been particularly lively within health economics (e.g., Erreygers, 2009; Lambert and Zheng, 2011). In measuring inequality of a bounded variable such as health status, one can focus on attainments or shortfalls. Both look at the same situation, but from a different point of view. Thus, they can move in opposite directions. This would be in line with the psychology literature, where it is well known that losses loom larger than gains (e.g., Kahneman and Tversky, 1979). Nevertheless, the health economics literature has focussed on the requirement that both are measured *consistently*, which leads towards strong consequences. In the context of our paper, this would translate, for instance, into *consistent* evaluations of health care programs when assessed via the QALYs they generate and the DALYs they generate.

We conclude this introduction mentioning that we align with the tradition of axiomatic work in economics that can be traced back to the 1950's. The axiomatic method has not been frequently used for health measurement within health economics. This is in contrast with other fields, which have witnessed in the last decades numerous applications of the method to the evaluation of a variety of concepts. These applications range from classical ones such as conflict resolution (e.g., Gupta and Livne, 1988), taxation (e.g., Young, 1988), income inequality (e.g., Bossert, 1990), or polarization (e.g., Esteban and Ray, 1994) to somewhat unconventional ones treated recently, such as resilience (e.g., Asheim et al., 2020), broadcasting problems (e.g., Bergantiños and Moreno-Ternero, 2020), individual productivity (e.g., Flores-Szwagrzak and Treibich, 2020) or financial networks (e.g., Csóka and Herings, 2021).

The rest of the paper proceeds as follows. In Section 2, we introduce the framework, some basic instances of population health evaluation functions, as well as a list of seven 'COMMON' axioms that will be used for all the results in the paper. In Section 3, we provide characterizations for the three focal classes of population health evaluation functions mentioned above. Section 4 provides additional characterization results for more general families. Section 5 concludes. All proofs have been deferred to an Appendix.

[4]As such, our move is reminiscent of the so-called *baseline* rationing (e.g., Hougaard et al., 2012), which enriches the standard claim problems (e.g., O'Neill, 1982) to account for additional references in the allocation process. Similarly, Ju et al., (2021) recently augmented the standard model for the allocation of greenhouse gas emissions to account for historical emissions, which can also play the role of references in the allocation process.

2 The preliminaries

Imagine a policy maker who has to compare distributions of health for a population of fixed size $n \geq 3$. Let us identify the population (society) with the set $N = \{1, ..., n\}$. The health of each individual in the population is described by a triplet $h_i = (a_i, t_i, r_i)$, where $a_i \in A$ is a health state, $t_i \in T = [0, +\infty)$ is the number of years lived, and $r_i \in T$, such that $r_i > t_i$, is a reference age.[5] The set of possible health states, A, is defined generally enough to encompass all possible health states for everybody in the population. We emphasize that A is an abstract set without any particular mathematical structure. A population health distribution (or, simply, a health profile) $h = [h_1, \ldots, h_n] = [(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)]$ specifies the health triplet of each individual in society.[6] Denote the set of all possible health profiles by H. Even though we do not impose a specific mathematical structure on the set A, we assume that it contains a specific element, a_*, referred to as *perfect health*, which is univocally identified as a "superior" state by all agents in the population.

The policy maker's preferences (or social preferences) over health profiles are expressed by a preference relation $\succsim$, to be read as "at least as preferred as". As usual, $\succ$ denotes strict preference and $\sim$ denotes indifference. Assume the relation $\succsim$ is a weak order, i.e., it is complete (for each health profiles h, h', either $h \succsim h'$, or $h' \succsim h$, or both) and transitive (if $h \succsim h'$ and $h' \succsim h''$ then $h \succsim h''$).

2.1 Population health evaluation functions

A *population health evaluation function* (PHEF) is a real-valued function $P : H \to \mathbb{R}$. We say that P represents $\succsim$ if it holds that, for each pair $h, h' \in H$, $P(h) \geq P(h')$ if and only if $h \succsim h'$. Note that if P represents $\succsim$ then any strictly increasing transformation of P would also do so.

The following population health evaluation function, which we call *(aggregated) time-linear QALY*, evaluates population health distributions by means of the unweighted aggregation of individual QALYs in society, or, in other words, by the weighted (through health levels) aggregate

[5]We can think of r_i as the aspirational number of life years for individual i, i.e., a target or an expectation under the best possible conditions.

[6]For ease of exposition, we establish the notational convention that $h_S \equiv (h_i)_{i \in S}$, for each $S \subset N$.

time span the distribution yields. Formally,

$$P^q[h_1, \ldots, h_n] = P^q[(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] = \sum_{i=1}^{n} q(a_i)t_i, \tag{1}$$

where $q : A \to [0, 1]$ is a function satisfying $0 \leq q(a_i) \leq q(a_*) = 1$, for each $a_i \in A$.

The *(aggregated) time-linear DALY* population health evaluation function evaluates population health distributions by means of the unweighted aggregation of individual gaps from reference age and QALYs in society. Formally,

$$P^d[h_1, \ldots, h_n] = P^d[(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] = \sum_{i=1}^{n} (q(a_i)t_i - r_i), \tag{2}$$

where $q : A \to [0, 1]$ is a function satisfying $0 \leq q(a_i) \leq q(a_*) = 1$, for each $a_i \in A$.

Equivalently, (2) can be expressed as follows.

$$P^d[h_1, \ldots, h_n] = P^d[(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] = -\sum_{i=1}^{n} (q(a_*) - q(a_i))t_i - \sum_{i=1}^{n} (r_i - t_i),$$

where $q : A \to [0, 1]$ is a function satisfying $0 \leq q(a_i) \leq q(a_*) = 1$, for each $a_i \in A$. That is, the *(aggregated) time-linear DALY* population health evaluation function evaluates population health distributions by means of the reverse unweighted aggregation of individual quality losses (with respect to perfect health) and lifetime gaps (with respect to reference age). That is, the lower individual quality losses and reference-lifetimes gaps, the higher the value achieved by the *(aggregated) time-linear DALY* population health evaluation function.

The previous two population health evaluation functions are instances of the class introduced next. This class encompasses various interpretations of the importance of reference age and health shortfalls, while capturing both quality and quantity of time. That is why we refer to the class as Health Adjusted Life Years (HALY) population health evaluation functions.[7] Formally,

$$P^h[h_1, \ldots, h_n] = P^h[(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] = \sum_{i=1}^{n} (q(a_i)t_i - \alpha r_i), \tag{3}$$

where $q : A \to [0, 1]$ is a function satisfying $0 \leq q(a_i) \leq q(a_*) = 1$ for each $a_i \in A$.

At the risk of stressing the obvious, note that when $\alpha = 0$ at (3) we obtain the QALY population health evaluation function, whereas when $\alpha = 1$ at (3) we obtain the DALY population health evaluation function.

[7] As mentioned above, this has already been used as an umbrella term for QALY and DALY-like measures of population health (e.g., Gold et al., 2002).

2.2 COMMON axioms

We now present a set of seven (common) axioms that can be considered as basic axioms for social preferences in the current context.[8] They are all satisfied by the population health evaluation functions introduced above.

The first one, *anonymity*, states that a permutation of agents should not matter for the evaluation of the population health. Formally, let Π^N denote the class of bijections from N into itself. Then,

ANON: $h \sim h_\pi$ for each $h \in H$, and each $\pi \in \Pi^N$.

The second axiom, *separability*, says that if the distribution of health in a population changes only for a subgroup of agents, the relative evaluation of the two corresponding distributions should only depend on that subgroup. Formally,

SEP: $\left[h_S, h_{N\setminus S}\right] \succsim \left[h'_S, h_{N\setminus S}\right] \Leftrightarrow \left[h_S, h'_{N\setminus S}\right] \succsim \left[h'_S, h'_{N\setminus S}\right]$, for each $S \subseteq N$, and each pair $h, h' \in H$.

Continuity indicates that, for fixed distributions of health states, small changes in lifetimes and/or references should not lead to large changes in the evaluation of the population health distribution. Formally,

CONT: Let $h, h' \in H$, and $h^{(k)}$ be a sequence in H such that, for each $i \in N$, $h_i^{(k)} = (a_i, t_i^{(k)}, r_i^{(k)}) \to (a_i, t_i, r_i) = h_i$. If $h^{(k)} \succsim h'$ for each k then $h \succsim h'$, and if $h' \succsim h^{(k)}$ for each k then $h' \succsim h$.

The next pair of axioms refers to the focal state of perfect health (a_*).

Perfect health superiority says that replacing an agent's health status by perfect health cannot hurt the evaluation of the population health. Formally,

PHS: $[(a_*, t_i, r_i), h_{N\setminus\{i\}}] \succsim h$, for each $h = [h_1, \ldots, h_n] \in H$ and each $i \in N$.

Lifetime monotonicity at perfect health says that if an agent is at perfect health, then a higher lifetime for that agent (keeping the reference age fixed) is strictly better. Formally,

LMPH: Let $h \in H$ and $i \in N$ such that $t_i > t'_i$. Then $[(a_*, t_i, r_i), h_{N\setminus\{i\}}] \succ [(a_*, t'_i, r_i), h_{N\setminus\{i\}}]$.

[8] They all extend to this general setting the axioms in Hougaard et al., (2013).

The last two basic axioms we consider deal with the other focal situation of zero lifetime.

Positive lifetime desirability states that moving an agent from zero lifetime to positive lifetime (for a given health state and reference) is a societal improvement. Formally,

PLD: $h \succsim [h_{N\setminus\{i\}}, (a_i, 0, r_i)]$, for each $h = [h_1, \ldots, h_n] \in H$ and $i \in N$.

The *social zero condition* says that the health state of an agent with zero lifetime is irrelevant for the evaluation of the health distribution.[9] Formally,

ZERO: For each $h \in H$ and each $i \in N$ such that $t_i = 0$, and each $a_i' \in A$, $h \sim [h_{N\setminus\{i\}}, (a_i', 0, r_i)]$.

In what follows, we refer to the set of axioms introduced above as the *common structural axioms* (in short, **COMMON**).

3 Normative foundations for HALYs, QALYs and DALYs

Our first result says that the class of HALY population health evaluation functions (P^h) introduced above is characterized by the combination of the common structural axioms described above plus the following two specific axioms. First, *lifetime invariance at common health*, which says that the planner should be indifferent between adding (the same amount of) lifetime to one agent or another, provided both experience the same health status (although probably different lifetimes) and the gap between lifetime and reference age is kept fixed. Second, *reference age invariance at common health*, which says that the evaluation of a population health distribution in which the reference age of an individual is increased by a certain amount does not depend on whether this is done for one individual or another, as long as the two individuals have the same health state. Formally,

LICH: For each $h \in H$, each $c > 0$, and each pair $i, j \in N$, such that $a_i = a_j = a$,

$$\left[(a, t_i + c, r_i + c), (a, t_j, r_j), h_{N\setminus\{i,j\}}\right] \sim \left[(a, t_i, r_i), (a, t_j + c, r_j + c), h_{N\setminus\{i,j\}}\right].$$

RICH: For each $h \in H$, each $c > 0$, and each pair $i, j \in N$, with $a_i = a_j = a$,

$$[(a, t_i, r_i + c), (a, t_j, r_j), h_{N\setminus\{i,j\}}] \sim [(a, t_i, r_i), (a, t_j, r_j + c), h_{N\setminus\{i,j\}}].$$

[9] Miyamoto et al., (1988) introduced the counterpart individual version of this axiom.

Theorem 1 *The following statements are equivalent:*

1. $\succsim$ *is represented by a PHEF satisfying* (3).

2. $\succsim$ *satisfies COMMON, LICH, and RICH.*

We now consider another specific axiom. *Independence of reference age at perfect health* says that when an individual lives in perfect health, the reference age (and therefore the gap between lifetime and reference age) is irrelevant for the evaluation of population health. Formally,

IRPH: For each $h \in H$, each $i \in N$, and each pair $r'_i \neq r_i$,

$$[(a_*, t_i, r_i), h_{N \setminus \{i\}}] \sim [(a_*, t_i, r'_i), h_{N \setminus \{i\}}].$$

As the next result states, if we add *independence of reference age at perfect health* to the axioms at Theorem 1, we characterize the QALY population health evaluation function (P^q).[10]

Theorem 2 *The following statements are equivalent:*

1. $\succsim$ *is represented by a PHEF satisfying* (1).

2. $\succsim$ *satisfies COMMON, LICH, RICH and IRPH.*

We also consider a last axiom. *Gap invariance at perfect health* says that when an individual enjoys perfect health, only the gap between lifetime and reference age matters, not lifetime per se. Formally,

GIPH: For each $h \in H$ and each $i \in N$,

$$[(a_*, 0, r_i), h_{N \setminus \{i\}}] \sim [(a_*, t_i, r_i + t_i), h_{N \setminus \{i\}}].$$

If we now add *gap invariance at perfect health*, instead of *independence of reference age at perfect health*, to the axioms at Theorem 1, we characterize the DALY population health evaluation function (P^d). In other words, our next result says that P^d is characterized by the combination of the common structural axioms, *lifetime invariance at common health*, *reference age invariance at common health* and *gap invariance at perfect health*.

Theorem 3 *The following statements are equivalent:*

[10] This result can be seen as an extension of Theorem 2 in Hougaard et al., (2013) to our setting.

1. $\succsim$ *is represented by a PHEF satisfying* (2).

2. $\succsim$ *satisfies COMMON, LICH, RICH and GIPH.*

A simple inspection of the statements of the previous two theorems, allows us to infer that the characterizations of two allegedly polar procedures for the evaluation of population health, such as (time-linear) QALYs and DALYs, actually share many axioms: the so-called COMMON axioms (anonymity, separability, continuity, perfect health superiority, lifetime monotonicity at perfect health, positive lifetime desirability, and the social zero condition), as well as *lifetime invariance at common health* and *reference age invariance at common health.* They only differ in two: the time-linear QALY requires *independence of reference age at perfect health*, whereas the time-linear DALY requires *gap invariance at perfect health.*

The common ground of DALYs and QALYs is further revealed by means of the characterization we provide at Theorem 1 of a class of population health evaluation functions, which encompasses both the time-linear QALY and DALY representations as specific cases. Such a characterization is obtained by just adding two axioms to the list of COMMON axioms mentioned above: *lifetime invariance at common health* and *reference age invariance at common health.*

One might argue that the class of HALY population health evaluation functions characterized above is too large and it would make sense to consider only its members arising from convex combinations of its most focal members (the DALY and QALY population health evaluation functions). This is equivalent to restricting the parameter α at (3) to the range $[0, 1]$. It turns out that such a sub-class can be characterized adding two natural axioms to those listed at Theorem 1. The first axiom states that if an individual enjoys perfect health with a certain lifetime t equal to its reference lifetime, then increasing t cannot hurt the population evaluation. The second axiom states that decreasing the lifetime gap for an individual enjoying perfect health (by means of her reference) cannot hurt the population evaluation.

More precisely, *joint monotonicity at perfect health* says that if an individual enjoys perfect health with a certain lifetime equal to its reference lifetime, then increasing them equally cannot hurt the population evaluation. *Gap monotonicity at perfect health* says that decreasing the reference while keeping the lifetime fixed, for an individual enjoying perfect health (by means of her reference) cannot hurt the population evaluation. Formally,

JMPH: For each $h \in H$, each $c > 0$, and each $i \in N$, such that $a_i = a_*$, and $t_i = r_i$,

$$\left[(a_*, t_i + c, r_i + c), h_{N\setminus\{i\}}\right] \succsim \left[(a_*, t_i, r_i), h_{N\setminus\{i\}}\right].$$

GMPH: For each $h \in H$, each $c > 0$, and each $i \in N$, such that $a_i = a_*$,

$$\left[(a_*, t_i, r_i), h_{N\setminus\{i\}}\right] \succsim \left[(a_*, t_i, r_i + c), h_{N\setminus\{i\}}\right].$$

Corollary 1 *The following statements are equivalent:*

1. $\succsim$ *is represented by a PHEF satisfying* (3) *with* $\alpha \in [0, 1]$.
2. $\succsim$ *satisfies COMMON, LICH, RICH, JMPH and GMPH.*

4 Going beyond time linearity

In this section, we obtain normative foundations for more general classes of population health evaluation functions that include HALYs. They all rely on the so-called *healthy years equivalent*, formally introduced next.

Let $f : A \times T^2 \to \mathbb{R}_+$ be a continuous function with respect to its second and third variable, such that

- $0 \leq f(a_i, t_i, r_i) \leq t_i$, for each $(a_i, t_i, r_i) \in A \times T^2$,
- $h \sim [(a_*, f(a_i, t_i, r_i), r_i)_{i\in N}]$, for each $h = [h_1, \ldots, h_n] = [(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] \in H$.

We refer to f as the *Healthy Years Equivalent* (HYE) function.

One might find it plausible to generalize the class of HALY population health evaluation functions to account for (separable) functional forms relying on the gap between *healthy years equivalent* (instead of QALYs) and reference lifetimes. It turns out that such a family could be characterized by just adding one of the axioms introduced in the previous section (*gap invariance at perfect health*) to the list of COMMON structural axioms.

Let $P^l : H \to \mathbb{R}$ be such that, for each $h = [h_1, \ldots, h_n] = [(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] \in H$,

$$P^l[h_1, \ldots, h_n] = P^l[(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] = \sum_{i=1}^{n} g(f(a_i, t_i, r_i) - r_i), \tag{4}$$

where $g : \mathbb{R}_- \to \mathbb{R}$ is a strictly increasing continuous function, and f is the HYE function.

Theorem 4 *The following statements are equivalent:*

1. $\succsim$ *is represented by a PHEF satisfying* (4).

2. $\succsim$ *satisfies COMMON and GIPH.*

If instead of *gap invariance at perfect health*, the only axiom we add to the list of COMMON structural axioms is *independence of reference age at perfect health*, then we characterize a class of (separable) population health evaluation functions conveying a transformation of *healthy years equivalent*. Formally, let $P^f : H \to \mathbb{R}$ be such that, for each $h = [h_1, \ldots, h_n] = [(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] \in H$,

$$P^f[h_1, \ldots, h_n] = P^f[(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] = \sum_{i=1}^{n} g(f(a_i, t_i, r_i)), \tag{5}$$

where $g : \mathbb{R}_+ \to \mathbb{R}$ is a strictly increasing continuous function, and f is the HYE function.

Theorem 5 *The following statements are equivalent:*

1. $\succsim$ *is represented by a PHEF satisfying* (5).

2. $\succsim$ *satisfies COMMON and IRPH.*

Finally, we provide a characterization for the most general (separable) functional form, relying on the *healthy years equivalent* and reference lifetimes.[11] Formally, let $P^g : H \to \mathbb{R}$ be such that, for each $h = [h_1, \ldots, h_n] = [(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] \in H$,

$$P^g[h_1, \ldots, h_n] = P^g[(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] = \sum_{i=1}^{n} g(f(a_i, t_i, r_i), r_i), \tag{6}$$

where $g : \mathbb{R}_+^2 \to \mathbb{R}$ is a continuous function that is strictly increasing in its first variable, and f is the HYE function. We refer to P^g as a *generalized HALY* PHEF.

Theorem 6 *The following statements are equivalent:*

1. $\succsim$ *is represented by a PHEF satisfying* (6).

2. $\succsim$ *satisfies COMMON.*

[11] The result can be seen as an extension of Theorem 1 in Hougaard et al., (2013) to our setting.

5 Conclusion

We have presented in this paper an axiomatic approach to the evaluation of population health, when individuals' statuses are described in the health dimension (quality of life), the time dimension (quantity of life), as well as the (individual) reference lifetime. We have characterized two focal representations of social preferences over population health distributions; namely, the time-linear QALY and DALY representations. Those representations, which are widely used in applied work for the economic evaluation of health care programs, were initially considered as polars. Some of their differences, as well as similar aspects, were addressed in the literature (e.g., Sassi, 2006; Airoldi and Morton, 2009; Morton, 2010). We have seen in this paper that the two representations (QALYs and DALYs) actually share a solid common ground, being both focal elements of a class of population health evaluation functions (dubbed HALY), which we have also characterized. Typically, QALYs and DALYs will not provide *consistent* evaluations of health care programs. But if they do, this will also be the case with members of the HALY family compromising between them.

We have also provided further insights, obtaining additional characterizations of more general HALY-related population health evaluation functions. We note that all population health evaluation functions we characterize impose unweighted aggregation across individuals, an aspect usually criticized in the health economics literature by its lack of concern for distributive justice (e.g., Wagstaff, 1991). That is why more general population health evaluation functions, such as the ones mentioned at the end of the previous section, may be of interest to capture such a concern. A natural course of action is the so-called Bergsonian approach, which can be traced back to Bergson (e.g., Burk, 1936).[12] In a health economics context, power functions of QALYs were introduced, at an individual level, by Pliskin et al., (1980). The concept was also studied by Wagstaff (1991), Williams (1997), Østerdal (2005) and Hougaard et al., (2013), among others. Nevertheless, power functions of DALYs (or other population health evaluation functions involving reference lifetimes, such as those captured by the HALY family characterized in this paper) have not been proposed yet in the literature. This issue is left for further research.

[12] See also Moulin (1988, Chapter 2) for further details.

6 Appendix. Proofs of the results

Proof of Theorem 6

We start with this proof, as it will be instrumental for the remaining ones.

Suppose first that $\succsim$ is represented by a PHEF satisfying (6). We start by noticing that, from inspection of (6), it follows immediately that ANON and SEP hold. CONT holds because f and g are continuous functions themselves. As $0 \leq f(a_i, t_i, r_i) \leq t_i$, it follows that $f(a_i, 0, r_i) = 0$, implying ZERO. Thus, as $f(a_i, 0, r_i) = 0 \leq f(a_i, t_i, r_i)$, this in turn implies PLD. Furthermore, as $h \sim [(a_*, f(a_i, t_i, r_i), r_i)_{i\in N}]$, it follows that if $h_i = [(a_*, t_i, r_i)], h'_i = [(a_*, t'_i, r_i)]$, and $t_i > t'_i$, then $f(a_*, t_i, r_i) = t_i > t'_i = f(a_i, t'_i, r_i)$, which implies that $[h_i, h_{N\setminus\{i\}}] \succ [h'_i, h_{N\setminus\{i\}}]$, so LMPH holds. Finally, as $h \sim [(a_*, f(a_i, t_i, r_i), r_i)_{i\in N}]$, and $f(a_i, t_i, r_i) \leq t_i$, it follows from LMPH that $[(a_*, t_i, r_i)_{i\in N}] \succsim [(a_*, f(a_i, t_i, r_i), r_i)_{i\in N}] \sim h$. Thus, PHS holds.

Conversely, suppose $\succsim$ satisfies COMMON. We start by showing that there exists a function $f : A \times T^2 \to \mathbb{R}$ such that f is continuous with respect to its second and third variable and such that for each $h = [h_1, \ldots, h_n] = [(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i, r_i), r_i)_{i\in N}],$$

where $0 \leq f(a_i, t_i, r_i) \leq t_i$ for each $(a_i, t_i, r_i) \in A \times T^2$. Note that this part of the proof follows along similar lines to the proofs of existence of HYEs in Østerdal (2005) and Hougaard et al., (2013) for individual or social health profiles, respectively, without reference age.

First, we prove that for each $h \in H$ and each $i \in N$, there exists $t^*_i \in T$, such that

$$h \sim [(a_*, t^*_i, r_i), h_{N\setminus\{i\}}].$$

If $t_i = 0$, then it follows from ZERO that $t^*_i = t_i = 0$. Therefore, let $t_i > 0$. By contradiction, assume that t^*_i does not exist. Then, $T = A \cup B$, where

$$A = \{s \in T | h \succ [(a_*, s, r_i), h_{N\setminus\{i\}}]\},$$

$$B = \{s \in T | [(a_*, s, r_i), h_{N\setminus\{i\}}] \succ h\}.$$

We show first that both A and B are non-empty sets. By PHS, $[(a_*, t_i, r_i), h_{N\setminus\{i\}}] \succsim h$, implying that either $t^*_i = t_i$ (a contradiction), or $t_i \in B$. By PLD and ZERO, it follows that either $t^*_i = 0$ (a contradiction), or $0 \in A$.

Now, by CONT, A and B are open sets. Thus, as $A \cap B = \emptyset$, it follows that T is not a connected set, which is a contradiction. Therefore, t_i^* exists, and due to LMPH, it is uniquely determined. By SEP, we can determine each t_i^* separately. Therefore, let $f_i : A \times T^2 \to \mathbb{R}$ be such that $f_i(a_i, t_i, r_i) = t_i^*$ for each $i \in N$. By ANON, $f_i() = f_j() = f()$ for each pair $i, j \in N$. By CONT, f is continuous with respect to its second and third variables. And, as $t_i \in B$ and $0 \in A$, $0 \leq f(a_i, t_i, r_i) \leq t_i$, so the range of f is a connected subset of $\mathbb{R}$. Furthermore,

$$h \sim [(a_*, f(a_i, t_i, r_i), r_i)_{i \in N}].$$

Thus, using the notation $h^* = [(a_*, f(a_i, t_i, r_i), r_i)_{i \in N}]$, there exists an induced social preference relation $\succsim^*$ such that $h \succsim h'$ if and only if $h^* \succsim^* h'^*$. By CONT and SEP, it follows that $\succsim^*$ is continuous on its domain and satisfies separability across individuals. It then follows by application of Theorem 3 in Debreu (1960) that there exists a continuous function $g : \mathbb{R}_+^2 \to \mathbb{R}$, such that

$$h^* \succsim h'^* \Leftrightarrow \sum_{i=1}^{n} g(f(a_i, t_i, r_i), r_i) \geq \sum_{i=1}^{n} g(f(a_i', t_i', r_i'), r_i').$$

Moreover, by LMPH, g is strictly increasing in its first variable.

Proof of Theorem 1

We focus on the non-trivial implication. Suppose $\succsim$ satisfies COMMON, LICH and RICH. Then, by Theorem 6, $\succsim$ is represented by a generalized HALY PHEF. That is,

$$P[h_1, \ldots, h_n] = \sum_{i=1}^{n} g(f(a_i, t_i, r_i), r_i),$$

where $g : \mathbb{R}_+^2 \to \mathbb{R}$ is a continuous function, strictly increasing in its first variable, and $f : A \times T^2 \to \mathbb{R}_+$ is the HYE function.

Let $\varphi : A \times T^2 \to \mathbb{R}$ be such that $\varphi(a_i, t_i, r_i) = g(f(a_i, t_i, r_i), r_i)$, for each $(a_i, t_i, r_i) \in A \times T^2$. Assume, without loss of generality, that $\varphi(\overline{a}, 0, 0) = 0$ for some $\overline{a} \in A$. Let $a \in A$. By iterated application of LICH and RICH and the transitivity of $\succsim$, as well as ZERO,

$$\begin{aligned}\sum_{i=1}^{n}\varphi(a,t_i,r_i) &\overset{[\text{LICH}]}{=}\varphi(a,\sum_{i=1}^{n}t_i,r_i+\sum_{j\neq i}t_j)+(n-1)\varphi(a,0,r_j-t_j)\\ &\overset{[\text{RICH}]}{=}\varphi(a,\sum_{i=1}^{n}t_i,\sum_{i=1}^{n}r_i)+(n-1)\varphi(a,0,0)\\ &\overset{[\text{ZERO}]}{=}\varphi(a,\sum_{i=1}^{n}t_i,\sum_{i=1}^{n}r_i)+(n-1)\varphi(\overline{a},0,0)\\ &=\varphi(a,\sum_{i=1}^{n}t_i,\sum_{i=1}^{n}r_i).\end{aligned}\tag{7}$$

Let $i,j\in N$. It then follows from the above, as well as LICH, RICH and ZERO, that

$$\begin{aligned}&\varphi(a,t_i,r_i)+\varphi(a,t_j,r_j)\\ \overset{[\text{LICH}]}{=}&\varphi(a,t_i+t_j,r_i+t_j)+\varphi(a,0,r_j-t_j)\\ \overset{[\text{RICH}]}{=}&\varphi(a,t_i+t_j,t_i+t_j)+\varphi(a,0,r_i-t_i+r_j-t_j)\\ \overset{[(7)]}{=}&\varphi(a,t_i,t_i)+\varphi(a,t_j,t_j)+\varphi(a,0,r_i-t_i)+\varphi(a,0,r_j-t_j)\\ \overset{[\text{ZERO}]}{=}&\varphi(a,t_i,t_i)+\varphi(a_*,0,r_i-t_i)+\varphi(a,t_j,t_j)+\varphi(a_*,0,r_j-t_j).\end{aligned}$$

Therefore, $\varphi(a,t_i,r_i)$ can be decomposed as follows:

$$\varphi(a,t_i,r_i)=\varphi(a,t_i,t_i)+\varphi(a_*,0,r_i-t_i).\tag{8}$$

Next, define the function $\phi: A\times T^2\to\mathbb{R}$ such that $\phi(a_i,t_i)=\varphi(a,t_i,t_i)$, for each $(a_i,t_i)\in A\times T$. Let $a\in A$. Then, $\sum_{i=1}^{n}\phi(a,t_i)=\phi(a,\sum_{i=1}^{n}t_i)$, for each $t_i\in T$. In particular, $\phi(a,t_1+t_2)=\phi(a,t_1)+\phi(a,t_2)$ for each pair $t_1,t_2\in T$, which is precisely one of Cauchy's canonical functional equations. As $\phi(a,\cdot)$ is a continuous function, it follows that the unique solutions to such an equation are the linear functions (e.g., Aczel, 2006; page 34). More precisely, there exists a function $\hat{q}: A\to\mathbb{R}$ such that

$$\varphi(a,t,t)=\phi(a,t)=\hat{q}(a)t,$$

for each $a\in A$, and each $t\in T$. It follows from PHS that $\hat{q}(a_*)\geq\hat{q}(a)$.

Let $h_i=(a_*,0,r_i)\in A\times T^2$. From (7), it follows that

$$\sum_{i=1}^{n}\varphi(a_*,0,r_i)=\varphi(a_*,0,\sum_{i=1}^{n}r_i).$$

Next, define the function $\psi : T \to \mathbb{R}$ such that $\psi(r_i) = \varphi(a_*, 0, r_i)$, for each $r_i \in T$. Then, $\psi(r_1) + \psi(r_2) = \psi(r_1 + r_2)$, for each pair $r_1, r_2 \in T$. As before, as ψ is a continuous function, it follows that the unique solutions to such an equation are the linear functions. Therefore, there exists $\beta \in \mathbb{R}$ such that

$$(a_*, 0, r_i) = \psi(r_i) = \beta r_i,$$

for each $r_i \in T$.

Thus, by (8),

$$\varphi(a, t_i, r_i) = \hat{q}(a)t_i + \beta(r_i - t_i) = (\hat{q}(a) - \beta)t_i + \beta r_i,$$

for each $a \in A$ and each $(t_i, r_i) \in T^2$ such that $r_i \geq t_i$, where $\hat{q} : A \to \mathbb{R}$ is a function satisfying $\hat{q}(a) \leq \hat{q}(a_*)$, for each $a \in A$. To conclude, let $\alpha = -\beta$ and $q : A \to \mathbb{R}$ be such that $q(a) = \frac{\hat{q}(a)-\beta}{\hat{q}(a_*)-\beta}$, for each $a \in A$. By PLD and LMPH, it follows that $1 = q(a_*) \geq q(a) \geq 0$, for each $a \in A$. Then, we may write:

$$\varphi(a, t_i, r_i) = q(a)t_i - \alpha r_i,$$

where $\alpha \in \mathbb{R}$, and $0 \leq q(a) \leq q(a_*) = 1$, for each $a \in A$, as desired. ■

Proof of Theorem 2

We focus on the non-trivial implication. Suppose $\succsim$ satisfies COMMON, LICH, RICH and IRPH. Then, by Theorem 1, $\succsim$ is represented by a HALY PHEF, i.e.,

$$P[h_1, \ldots, h_n] = P[(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] = \sum_{i=1}^{n} (q(a_i)t_i - \alpha r_i).$$

By IRPH,

$$[(a_*, t_i, r_i), h_{N\setminus\{i\}}] \sim [(a_*, t_i, r_i'), h_{N\setminus\{i\}}],$$

for each $i \in N$, and each pair $r_i' \neq r_i$. Equivalently,

$$\begin{aligned} P[(a_*, t_i, r_i), h_{N\setminus\{i\}}] &= q(a_*)t_i - \alpha r_i + \sum_{j\neq i}(q(a_j)t_j - \alpha r_j) \\ &= q(a_*)t_i - \alpha r_i' + \sum_{j\neq i}(q(a_j)t_j - \alpha r_j) = P[(a_*, t_i, r_i'), h_{N\setminus\{i\}}], \end{aligned}$$

from where it follows that $\alpha = 0$. ■

Proof of Theorem 3

We focus on the non-trivial implication. Suppose $\succsim$ satisfies COMMON, LICH, RICH and GIPH. Then, by Theorem 1, $\succsim$ is represented by a HALY PHEF, i.e.,

$$P[h_1, \ldots, h_n] = P[(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] = \sum_{i=1}^{n}(q(a_i)t_i - \alpha r_i).$$

By GIPH,

$$[(a_*, 0, r_i), h_{N\setminus\{i\}}] \sim [(a_*, t_i, r_i + t_i), h_{N\setminus\{i\}}],$$

for each $i \in N$. Equivalently,

$$\begin{aligned} P[(a_*, 0, r_i), h_{N\setminus\{i\}}] &= -\alpha r_i + \sum_{j\neq i}(q(a_j)t_j - \alpha r_j) \\ &= t_i - \alpha(r_i + t_i) + \sum_{j\neq i}(q(a_j)t_j - \alpha r_j) = P[(a_*, t_i, r_i + t_i), h_{N\setminus\{i\}}], \end{aligned}$$

from where it follows that $\alpha = 1$. ■

Proof of Corollary 1

We focus on the non-trivial implication. Suppose $\succsim$ satisfies COMMON, LICH, RICH, JMPH and GMPH. Then, by Theorem 1, $\succsim$ is represented by a HALY PHEF, i.e.,

$$P[h_1, \ldots, h_n] = P[(a_1, t_1, r_1), \ldots, (a_n, t_n, r_n)] = \sum_{i=1}^{n}(q(a_i)t_i - \alpha r_i).$$

By GMPH, for each $h \in H$, each $i \in N$, and each $c > 0$,

$$\left[(a_*, t_i, r_i), h_{N\setminus\{i\}}\right] \succsim \left[(a_*, t_i, r_i + c), h_{N\setminus\{i\}}\right].$$

Equivalently,

$$\begin{aligned} P\left[(a_*, t_i, r_i), h_{N\setminus\{i\}}\right] &= t_i - \alpha r_i + \sum_{j\neq i}(q(a_j)t_j - \alpha r_j) \\ &\geq t_i - \alpha(r_i + c) + \sum_{j\neq i}(q(a_j)t_j - \alpha r_j) = P\left[(a_*, t_i, r_i + c), h_{N\setminus\{i\}}\right], \end{aligned}$$

from where it follows that $\alpha \geq 0$. By JMPH, for each $h \in H$, each $c > 0$, and each $i \in N$, such that $a_i = a_*$, and $t_i = r_i$,

$$\left[(a_*, t_i + c, r_i + c), h_{N\setminus\{i\}}\right] \succsim \left[(a_*, t_i, r_i), h_{N\setminus\{i\}}\right].$$

Equivalently,

$$P\left[(a_*, t_i + c, r_i + c), h_{N\setminus\{i\}}\right] = t_i + c - \alpha(r_i + c) + \sum_{j\neq i}(q(a_j)t_j - \alpha r_j)$$
$$\geq t_i - \alpha r_i + \sum_{j\neq i}(q(a_j)t_j - \alpha r_j) = P\left[(a_*, t_i, r_i), h_{N\setminus\{i\}}\right],$$

from where it follows that $\alpha \leq 1$. ■

Proof of Theorem 4

We focus on the non-trivial implication. Suppose $\succsim$ satisfies COMMON and GIPH. Then, by Theorem 6, $\succsim$ is represented by a generalized HALY PHEF. In particular, for each $h \in H$, $h \sim h^* \equiv [(a_*, f(a_i, t_i, r_i), r_i))_{i\in N}]$. Thus, there exists an induced social preference relation $\succsim^*$ such that $h \succsim h'$ if and only if $h^* \succsim^* h'^*$. By CONT and SEP, it follows that $\succsim^*$ is continuous on its domain and satisfies separability across individuals. It then follows by application of Theorem 3 in Debreu (1960) that there exists a continuous function $\hat{g} : \mathbb{R}^2_+ \to \mathbb{R}$, such that

$$h^* \succsim h'^* \Leftrightarrow \sum_{i=1}^{n} \hat{g}(f(a_i, t_i, r_i), r_i) \geq \sum_{i=1}^{n} \hat{g}(f(a'_i, t'_i, r'_i), r'_i).$$

By GIPH, $[(a_*, f(a_i, t_i, r_i), r_i))_{i\in N}] \sim [(a_*, 0, r_i - f(a_i, t_i, r_i))_{i\in N}]$. Thus, $h \sim [(a_*, 0, r_i - f(a_i, t_i, r_i))_{i\in N}]$, for each $h \in H$. Let $g : \mathbb{R}_- \to \mathbb{R}$ be such that $g(x) = \hat{g}(0, -x)$, for each $x \in \mathbb{R}_-$. By construction and LMPH (part of COMMON), g is continuous and strictly increasing. Altogether,

$$h \succsim h' \Leftrightarrow \sum_{i=1}^{n} g(f(a_i, t_i, r_i) - r_i) \geq \sum_{i=1}^{n} g(f(a'_i, t'_i, r'_i) - r'_i),$$

as desired. ■

Proof of Theorem 5

We focus on the non-trivial implication. Suppose $\succsim$ satisfies COMMON and IRPH. Then, as in the previous proof, there exists a continuous function $\hat{g} : \mathbb{R}^2_+ \to \mathbb{R}$, such that

$$h^* \succsim h'^* \Leftrightarrow \sum_{i=1}^{n} \hat{g}(f(a_i, t_i, r_i), r_i) \geq \sum_{i=1}^{n} \hat{g}(f(a'_i, t'_i, r'_i), r'_i).$$

By IRPH, $\hat{g}(f(a_i, t_i, r_i), r_i) = \hat{g}(f(a_i, t_i, r_i), r_i')$ for all $r_i' \neq r_i$. Let $g : \mathbb{R}_+ \to \mathbb{R}$ be the corresponding univariate function. Then, g is continuous and, by LMPH, strictly increasing. Furthermore,

$$h \succsim h' \Leftrightarrow \sum_{i=1}^{n} g(f(a_i, t_i, r_i)) \geq \sum_{i=1}^{n} g(f(a_i', t_i', r_i')),$$

as desired. ■

References

[1] Aczel, J., 2006. Lectures on functional equations and their applications. Dover.

[2] Airoldi, M., Morton, A., 2009. Adjusting life for quality or disability: stylistic difference or substantial dispute? Health Economics 18, 1237-1247.

[3] Anand, S., Hanson, K., 1998. DALYs: efficiency versus equity. World Development 26, 307-310.

[4] Asheim, G., Bossert, W., D'Ambrosio, C., Vögele, C., 2020. The measurement of resilience. Journal of Economic Theory 189, 105104.

[5] Bergantiños, G., Moreno-Ternero, J.D., 2020. Sharing the revenues from broadcasting sport events. Management Science 66, 2417-2431.

[6] Bleichrodt, H., 1995. QALYs and HYEs: Under what conditions are they equivalent? Journal of Health Economics 14, 17-37.

[7] Bleichrodt, H., 1997. Health utility indices and equity considerations. Journal of Health Economics 16, 65-91.

[8] Bossert, W., 1990. An axiomatization of the single-series Ginis. Journal of Economic Theory 50, 83-92.

[9] Briggs A., Vassall A., 2021. Count the cost of disability caused by COVID-19. Nature 593, 502-505.

[10] Burk, A., 1936. Real income, expenditure proportionality, and Frisch's new methods of measuring marginal utility. Review of Economic Studies 4, 33-52.

[11] Chapman, L.A., Shukla, P., Rodríguez-Barraquer, I., Shete, P.B., León, T.M., Bibbins-Domingo, K., Rutherford, G.W., Schechter, R., Lo, N.C., 2022. Risk factor targeting for vaccine prioritization during the COVID-19 pandemic. Scientific Reports 12, 1-12.

[12] Csóka, P., Herings, P.J.J., 2021. An axiomatization of the proportional rule in financial networks. Management Science 67, 2799-2812.

[13] Debreu, G., 1960, Topological methods in cardinal utility theory, in K. Arrow, S. Karlin and P. Suppes, (Eds.), Mathematical methods in the social sciences. Stanford University Press.

[14] Dolan, P., 2000. The measurement of health-related quality of life, in Culyer, A.J., Newhouse, J.P. (Eds.), Handbook of Health Economics. Elsevier Science, North-Holland, Amsterdam.

[15] Erreygers, G., 2009. Can a single indicator measure both attainment and shortfall inequality? Journal of Health Economics 28, 885-893.

[16] Esteban, J.-M., Ray, D., 1994. On the measurement of polarization. Econometrica 62, 819-851.

[17] Fanshel, L. Bush, J., 1970. A health-status index and its application to health-services outcomes, Operations Research 18, 1021-1066.

[18] Feng, X., Kim, D.D., Cohen, J.T., Neumann, P.J., Ollendorf, D.A., 2020. Using QALYs versus DALYs to measure cost-effectiveness: how much does it matter? International Journal of Technology Assessment in Health Care 36, 96-103.

[19] Fishburn, P., 1965. Independence in utility theory with whole product sets, Operations Research 13, 28-45.

[20] Flores-Szwagrzak, K., Treibich, R., 2020. Teamwork and individual productivity. Management Science 66, 2523-2544.

[21] Gianino, M.M., Savatteri, A., Politano, G., Nurchis, M.C., Pascucci, D., Damiani, G., 2021. Burden of COVID-19: Disability-Adjusted Life Years (DALYs) across 16 European Countries. European Review for Medical and Pharmacological Sciences 25, 5529-5541.

[22] Gold, M., Siegel, J., Russell, L., Weinstein, M., 1996, Cost-effectiveness in health and hedicine, Oxford University Press, New York.

[23] Gold, M.R., Stevenson, D., Fryback, D.G., 2002. HALYs and QALYs and DALYs, oh my: similarities and differences in summary measures of population health. Annual Review of Public Health 23, 115-134.

[24] Gupta, S. and Livne, Z.A., 1988. Resolving a conflict situation with a reference outcome: an axiomatic model. Management Science 34, 1303-1314.

[25] Hougaard, J.L., Moreno-Ternero, J.D., Østerdal, L.P., 2012. A unifying framework for the problem of adjudicating conflicting claims. Journal of Mathematical Economics 48, 107-114.

[26] Hougaard, J., Moreno-Ternero, J.D., Østerdal, L.P., 2013. A new axiomatic approach to the evaluation of population health. Journal of Health Economics 32, 515-523.

[27] Ju, B-G., Kim, M., Kim, S., Moreno-Ternero, J.D., 2021. Fair international protocols for abatement of GHG emissions. Energy Economics 94, 105091.

[28] Kahneman D, Tversky A., 1979. Prospect theory: an analysis of decision making under risk. Econometrica 47, 263-291.

[29] Keeney, R., 1974. Multiplicative utility functions, Operations Research 22, 22-34.

[30] Kyu, H.H., Abate, D., Abate, K.H., Abay, S.M., Abbafati, C., Abbasi, N., Abbastabar, H., Abd-Allah, F., Abdela, J., Abdelalim, A., Abdollahpour, I., 2018. Global, regional, and national disability-adjusted life-years (DALYs) for 359 diseases and injuries and healthy life expectancy (HALE) for 195 countries and territories, 1990–2017: a systematic analysis for the Global Burden of Disease Study 2017. The Lancet 392, 1859-1922.

[31] Lambert, P., Zheng, B., 2011. On the consistent measurement of attainment and shortfall inequality. Journal of Health Economics 30, 214-219.

[32] Martinez, R., Soliz, P., Caixeta, R., Ordunez, P., 2019. Reflection on modern methods: years of life lost due to premature mortality-a versatile and comprehensive measure for monitoring non-communicable disease mortality. International Journal of Epidemiology 48, 1367-1376.

[33] Miyamoto, J., Wakker, P., Bleichrodt, H., Peters, H., 1998. The zero-condition: A simplifying assumption in QALY measurement and multiattribute utility, Management Science 44, 839-849.

[34] Moreno-Ternero, J.D., Østerdal, L.P., 2017. A normative foundation for equity-sensitive health evaluation: The role of relative comparisons of health gains. Journal of Public Economic Theory 19, 1009-1025.

[35] Morton, A., 2010. Bridging the gap: health equality and the gap framing of health. Health Economics 19, 1497-1501.

[36] Moulin, H., 1988. Axioms of cooperative decision making. Cambridge University Press.

[37] Murray C., 1994. Quantifying the burden of disease: the technical basis for disability-adjusted life years. Bulletin of the World Health Organization 72, 429-45.

[38] Murray C., Acharya A., 1997. Understanding DALYs (disability- adjusted life years). Journal of Health Economics 16, 703–30.

[39] Murray C., Salomon J., Mathers C., 2002. A critical examination of summary measures of population health, in Murray C., Salomon J., Mathers C., Lopez A., (Eds.) Summary Measures of Population Health: Concepts, Ethics, Measurement and Application. World Health Organization.

[40] Murray, C.J., Vos, T., Lozano, R., Naghavi, M., Flaxman, A.D., Michaud, C., Ezzati, M., Shibuya, K., Salomon, J.A., Abdalla, S. and Aboyans, V., 2012. Disability-adjusted life years (DALYs) for 291 diseases and injuries in 21 regions, 1990–2010: a systematic analysis for the Global Burden of Disease Study 2010. The Lancet 380, 2197-2223.

[41] Murray, C.J., Barber, R.M., Foreman, K.J., Ozgoren, A.A., Abd-Allah, F., Abera, S.F., Aboyans, V., Abraham, J.P., Abubakar, I., Abu-Raddad, L.J., Abu-Rmeileh, N.M., 2015. Global, regional, and national disability-adjusted life years (DALYs) for 306 diseases and injuries and healthy life expectancy (HALE) for 188 countries, 1990–2013: quantifying the epidemiological transition. The Lancet 386, 2145-2191.

[42] O'Neill, B., 1982. A problem of rights arbitration from the Talmud, *Mathematical Social Sciences* 2, 345-371.

[43] Pliskin, J., Shepard, D., Weinstein, M., 1980. Utility functions for life years and health status. Operations Research 28, 206-224.

[44] Raiffa, H., 1968. Decision Analysis, Addison-Wesley, Reading, Mass.

[45] Sassi, F., 2006. Calculating QALYs, comparing QALY and DALY calculations. Health Policy and Planning 21, 402-408.

[46] Sen, A., 1992. Inequality reexamined. Clarendon Press, Oxford.

[47] Torrance, G., 1976. Health status index models: A unified mathematical view. Management Science 22, 990-1001.

[48] Wagstaff A., 1991. QALYs and the equity-efficiency trade-off. Journal of Health Economics 10, 21-41.

[49] Williams, A., 1997. Intergenerational equity: an exploration of the 'fair innings' argument. Health Economics 6, 117-132.

[50] World Bank, 1993. World Development Report 1993: Investing in health. New York: Oxford University Press.

[51] Wouters, S., Van Exel, N.J.A., Rohde, K.I.M., Brouwer, W.B.F., 2015. Are all health gains equally important? An exploration of acceptable health as a reference point in health care priority setting. Health and quality of life outcomes 13, 1-10.

[52] Wouters, S., van Exel, N.J.A., Rohde, K.I.M., Vromen, J.J., Brouwer, W.B.F., 2017. Acceptable health and priority weighting: discussing a reference-level approach using sufficientarian reasoning. Social Science and Medicine, 181, pp.158-167.

[53] Xiong, P., Liu, M., Liu, B., Hall, B.J., 2022. Trends in the incidence and DALYs of anxiety disorders at the global, regional, and national levels: estimates from the Global Burden of Disease Study 2019. Journal of Affective Disorders 297, 83-93.

[54] Østerdal L.P., 2005. Axioms for health care resource allocation. Journal of Health Economics 24, 679-702.