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# WHEN VOTERS LIKE TO BE RIGHT: AN ANALYSIS OF THE CONDORCET JURY THEOREM WITH MIXED MOTIVES<sup>\*</sup>

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### Abstract

We study the aggregation of private information through voting in committees where agents are rewarded based on both the correctness of the committee decision (instrumental payoffs) and the correctness of their vote (expressive payoffs). Surprisingly, we find that even when expressive payoffs are perfectly aligned with instrumental payoffs, expressive payoffs can prevent committees from aggregating private information, suggesting that committees will make better decisions if agents are *not* held individually responsible for the correctness of their vote. We show that this finding holds in situations with heterogeneous expressive payoffs and reputation payoffs that depend on the aggregate profile of votes.

**JEL-codes:** D71, D72

**Key words:** Information aggregation, Voting, Expressive payoffs, Condorcet Jury Theorem.

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# 1 Introduction

The Condorcet jury model shows that when a body of voters hold diffuse private information that is collectively more accurate than public information, a decision taken by a majority vote will optimally reflect the private information of its members (Feddersen and Pesendorfer, 1997). A key assumption of the Condorcet jury model is that voters only receive payoffs related to the correctness of the committee decision (instrumental payoffs). In many real-world cases, however, voters may also receive payoffs related to the correctness of their individual vote (expressive payoffs). For an example, consider Senator Hillary Clinton's vote supporting military intervention in Iraq, which she and many others viewed as incorrect ex post:

I thought I had acted in good faith and made the best decision I could with the information I had. And I wasn't alone in getting it wrong. But I still got it wrong. Plain and simple. (Clinton, 2014)

Despite making the best decision given the *available information*, Hillary Clinton faced individual consequences based on the perceived correctness of her vote: many pundits identified Hillary Clinton's vote for the war as a key factor in her loss in the 2008 presidential primary to Barak Obama, who had opposed the war (Byler, 2019).<sup>1</sup>

Both political and expert committees often face voting decisions under a high degree of uncertainty regarding which outcome will be viewed as ex post correct. Based on voting data from FDA committees approving new drug applications Newham and Midjord (2020) estimate that the probability that a committee member's private information is incorrect ranges from 17.5-27.5%. For politicians, who are often not experts but are forced to consider specific questions ranging from the impact of fiscal policy on inflation to the appropriateness of public health measures to address the COVID pandemic, the ex ante uncertainty as to the ex post correct choice may be even higher. Unsurprisingly, this uncertainty can lead to incorrect choices: e.g. several state legislatures in the US have recently implemented bans on mask mandates in schools, only to see a resurgence of COVID rates among school-age children. Quoting Governor

<sup>&</sup>lt;sup>1</sup>Mixed motives, i.e. both expressive and instrumental payoffs, are arguably present in non-political committees as well: members of a board of directors may benefit professionally from voting to support a successful venture relative to directors who voted against the measure; and independent of the committee decision, professors may receive intrinsic payoffs from voting to hire candidates whose job-market papers are subsequently published in top-5 journals.

Asa Huchinson on Arkansas' ban on mask mandates: ""In hindsight, I wish that it had not become law." Fischels (2021)

While simple intuition would suggest that rewarding agents for getting their individual vote right may have a positive impact on the quality of committee decisions in these situations, we show that expressive payoffs—even when perfectly aligned with instrumental payoffs—destroy the key property of the pivotal-voter equilibrium of the Condorcet jury model that generally enables committees to aggregate the private information of its members. That is, individual incentives to vote correctly may prevent agents from voting in a way that reflects their private information, resulting in a decision that is no more informative than public information. In this sense, our research suggests society may be better served if politicians are *not* held individually responsible for the correctness of their vote.

First, to see why private information is aggregated in the canonical pivotal-voter model, note that instrumental payoffs depend only on the collective action (the committee decision). Therefore, instrumental payoffs incentivize agents to condition their voting choice on the event that their vote impacts the collective action (i.e. their vote is pivotal), and it is always equilibrium behavior for agents to at least partially reveal their private information through their vote. The reason that agents vote informatively in equilibrium is that conditional upon being pivotal, the agents infer information about the signals of the other agents and the updated equilibrium beliefs make voting according to one's private signal optimal. It follows that when agents only have instrumental incentives, the decisions of the committee will reflect the aggregate profile of private information (Feddersen and Pesendorfer, 1997).

In contrast to instrumental payoffs, expressive payoffs depend on individual action (the vote). Accordingly, a payoff for getting their vote right gives agents an incentive to vote for the option that, based on their interim beliefs, maximizes the probability of being individually correct, independent of whether their vote has an impact on the collective action.<sup>2</sup> This implies that when the precision of the private information is relatively low, agents maximize their chance of being individually correct by voting according to the public information, effectively disregarding their private information. For example, if the public prior in Arkansas was that

 $<sup>^{2}</sup>$ More precisely, expressive payoffs give agents an incentive to vote for the option that maximizes their expected expressive payoffs given their individual beliefs.

the COVID pandemic was largely over, state senators may have had an individual incentive to vote for a mask mandate ban even if they had a private signal that a COVID resurgence was likely. It follows that even when instrumental and expressive payoffs are perfectly aligned, they do not necessarily incentivize the same behavior: when the precision of the private information is relatively low, in equilibrium agents maximize their instrumental payoffs by voting according to their private information, but maximize their expressive payoffs by voting according to the public information. In large committees, where the incentive to vote expressively overwhelms the vanishingly small instrumental incentive associated with being pivotal, this tension between instrumental and expressive payoffs leads to a failure of information aggregation.

This shows that a key assumption behind the positive information aggregation result of the pivotal voter model is that agents are only motivated by the correctness of the committee decision. That is, if voters face payoffs based on the correctness of their vote, regardless of whether they are small or perfectly aligned with instrumental payoffs, then agents' voting choices in large committees will depend only on the agents' interim beliefs. Perversely, this implies that the ability of the committee decision to reflect the private information of its members will be harmed. Moreover, the harm from expressive payoffs is not marginal—when information aggregation fails in large committees it fails completely and the committee decision is no more informative than the public information.

We also show that the failure of information aggregation is not just a large committee prediction: expressive payoffs systematically bias decisions for committees of all sizes and for reasonable parameter values information aggregation can fail completely for committees of sizes less than thirty. Additionally, the result holds with any distribution of heterogeneous expressive payoffs (and in asymmetric equilibria), and for "sophisticated" models of reputation where the size of the expressive payoffs depends on the number of other agents who are correct.

For a more detailed illustration of our main finding, consider a simple example. Assume a large committee takes a decision between options a and b, by a majority vote. If the state is  $\alpha$  ( $\beta$ ) then a (b) is socially optimal; i.e. if the committee matches the decision to the state, then agents receive an instrumental payoff of 100, and if not then they receive an instrumental payoff of 0. The state of the world, however, is unknown and the ex-ante probability that the state is  $\alpha$  is equal to 4/5. Additionally, each committee member receives an i.i.d. private signal

of a or b, where the probability that their signal is a (b) given a state of  $\alpha$  ( $\beta$ ) is equal to 3/4.<sup>3</sup>

When agents only receive instrumental payoffs, the pivotal voter reasoning results in an equilibrium in mixed strategies where agents with a signal of a vote a, while agents with a signal of b mix between voting for a and b. Note that it is not an equilibrium for all agents with a signal of b to vote their signal: in this case, when an agent with a signal of b is pivotal they know that 50% of other agents received a signal of b and 50% a signal of a, and the conditional probability that the state is  $\alpha$  is 4/7. Since this is greater than 1/2, the pivotal-voter logic implies that agents with a signal of b would have a best response of voting for a. However, if agents with a signal of b mix between voting for a and b, then when an agent is pivotal they expect that more than 50% of other agents received a signal of b. Therefore, an equilibrium exists where agents with a signal of b mix, but reveal enough private information in equilibrium so that as the committee grows, a law of large numbers applies and the committee makes the correct decision almost certainly.

Next, in addition to instrumental payoffs, assume committee members also receive an expressive payoff of 1 if their vote matches the state, and zero otherwise. First, note that under the mixed strategies outlined above, the addition of the expressive payoff does not impact the willingness of agents with a signal of b to vote for b when they are pivotal. However, to maximize their expected expressive payoffs, agents should vote for the option that is most likely given the public information and their private signal. In our example, this implies that voting for a maximizes the expressive payoffs of agents of both signal types, since the conditional probability that the state is  $\alpha$  given a signal of b is 4/7. Therefore, while for small committees, an equilibrium persists where agents with a signal of b still vote for b with a positive, but smaller, probability than with instrumental payoffs only, in large committees the unique equilibrium is for all agents to vote for a regardless of their private information.

 $<sup>^{3}</sup>$ We also use these parameters in a more detailed numerical example in Section 3.1, and these parameters are reasonable given the empirical evidence: for FDA experts, Newham and Midjord (2020) estimate the average probability of an incorrect signal of up to 27.5% and prior probabilities that the drug is "good" of 44% to 84% depending upon the drug category; for Supreme Court Justices, Iaryczower and Shum (2012) estimate average individual probabilities of an incorrect signal of up to 35% and average common prior probabilities in favor of the plaintiff from 33% to 61% depending on the area of law. Since these estimates are based on a continuous signal, there is not a perfect translation to our model with discrete signals. However, given the dispersion of individual cases about these average estimates, this implies that expressive payoffs distort voting behavior for a significant proportion of committee decisions.

# Literature Review

Our primary contribution is to the literature that highlights the limitations of aggregating private information through voting. Information aggregation in large committees has been shown to fail due to, among others, the decision rule (Feddersen and Pesendorfer, 1998), uncertainty regarding the signal structure (Mandler, 2012), a failure of preference monotonicity (Bhattacharya, 2013), and uncertainty regarding the size of the population (Ekmekci and Lauermann, 2019). Additionally, the literature has shown that expressive voting can result in a failure of information aggregation when voters are bribed (Dal Bó, 2007), have a preference for winning (Callander, 2007, 2008), moral payoffs (Feddersen, Gailmard and Sandroni, 2009), and partial expressive motives (Morgan and Várdy, 2012). One common theme of the articles relating to expressive motives is that the failure of information aggregation stems from the fact that expressive payoffs are independent of the state—if all agents are bribed to vote for a as in Dal Bó (2007), then information about the state is irrelevant to maximizing expressive payoffs. Moreover, no matter how small the bribe is, it is the dominant incentive in a large committee since the probability of influencing the committee decision is vanishingly small. Therefore, in large committees agents will not condition their vote on information about the state and information aggregation will fail. In contrast, our paper shows that information aggregation can fail with mixed motives even when expressive and instrumental payoffs are perfectly aligned and agents take information about the state into account when maximizing expressive payoffs.

There is one special case where information aggregation can be achieved despite a misalignment of expressive and instrumental payoffs. In a previous paper (Midjord, Rodríguez Barraquer and Valasek, 2017), we show that when a committee faces an individual "disesteem payoff" from, say, individually voting to approve a drug that is shown to have severe side effects, information aggregation can be achieved if the expressive payoff only realizes when the committee also incorrectly approves the drug for general use.<sup>4</sup> However, the key assumption that allows for information aggregation in this setting is that there are no expressive payoffs for one of the two committee decisions—as is highlighted in our current paper, information aggregation

 $<sup>^{4}</sup>$ In a related paper, Breitmoser and Valasek (2017) show that the socially optimal outcome can be achieved with expressive payoffs if committee members have access to cheap talk prior to voting *and* decisions are taken via a unanimity rule. The present paper takes a different approach, and characterizes the set of expressive payoffs that result in information aggregation in large committees, where pre-vote cheap talk may not be feasible.

will fail for some information structures whenever expressive payoffs realize for both committee decisions.<sup>5</sup>

Our paper also relates to the literature on reputation payoffs in committees, which considers the Holmstrom (1999) model of career concerns applied to a committee setting with private information. This literature primarily considers the question of the impact of transparency on information aggregation; see Fehrler and Hughes (2018) for a review. In this literature, agents have an incentive to vote in a way that maximizes a principal's ex-post belief that the agent is of high ability, and it has been shown in various settings that reputation concerns can cause committee members to: unite behind voting for the same option (Visser and Swank, 2007), vote against the ex ante most likely option (Levy, 2007), or distort pre-vote communication (Gersbach and Hahn, 2008, Fehrler and Hughes, 2018). Most of these articles analyze situations in which agents do not have instrumental incentives (Gersbach and Hahn, 2008, Levy, 2007, and Fehrler and Hughes, 2018), which inhibits the comparability to the classical Condorcet Jury model.<sup>6</sup> Perhaps most related to our paper in the reputation literature is a paper by Mattozzi and Nakaguma (2019) that considers agents with instrumental payoffs, reputation concerns and partisan bias. Their paper shows that partial information aggregation can be achieved in large committees in this setting when agents receive a signal that is either perfectly informative or uninformative. In contrast, we show that given any expressive payoffs tied to the correctness of the vote, information aggregation fails completely when agents' signals are of low precision relative to public information.

Lastly, our paper also shows that there is a link between expressive incentives in large committees and the concept of "naive voting" as discussed in the canonical article by Feddersen and Pesendorfer (1997).<sup>7</sup> Feddersen and Pesendorfer (1997) define naive voting as agents ignoring

 $<sup>^{5}</sup>$ Midjord, Rodríguez Barraquer and Valasek (2017) also showed that information aggregation fails in this setting if there exists a small but non-vanishing probability that even an arbitrarily large committee voting informatively is incorrect.

 $<sup>^{6}</sup>$ Visser and Swank (2007) consider agents with both instrumental and reputation payoffs. However, in their model the principle never observes the correctness of the committee decision or the individual votes and therefore reputation payoffs do not depend on the correctness of the vote.

<sup>&</sup>lt;sup>7</sup>There is also an extensive literature on the Condorcet Jury Theorem that preceded Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997) and considers information aggregation with non-strategic or "naive" voters. However, the basic assumption of this literature is that voters each have a fixed probability of voting for the correct state  $p_i$ —i.e. voters are "naive" in the sense that there is no model of voter behavior based on information and preferences. The question addressed by this literature is under what assumptions on the distributions of  $p_i$  there is information aggregation; e.g., heterogeneous  $p_i$  (Shapley and Grofman, 1984), multiple policy dimensions (Peyton, 1988) and correlated errors (Ladha, 1992).

strategic considerations and voting as if their own vote determined the committee outcome. That is, ignoring the fact that their vote only matters when pivotal, naive voters vote to maximize expected instrumental payoffs given only their own interim beliefs. In our paper, we show that expressive motives can cause even strategic agents to act as if they are naive—in large committees, agents vote to maximize expected expressive payoffs based on their own interim beliefs. Our paper therefore highlights that it is not just the assumption of strategic voting that is key in the pivotal voter model, rather it is also that agents are only motivated by collective instrumental payoffs: if agents have any individual incentives to vote correctly—no matter how small—then large committees of strategic agents will do no better than naive agents.

# 2 Model

We consider a model with n agents, indexed by  $i \in \{1, ..., n\}$ , who vote for option a or option b. The outcome of the vote,  $x \in \{a, b\}$ , is determined by simple majority.<sup>8</sup> Let  $v_i \in \{a, b\}$  denote the vote cast by voter i. There are two states of the world  $\omega \in \{\alpha, \beta\}$  where the commonly known prior probability of state  $\alpha$  is  $\Pr(\alpha) = \pi \in (0, 1)$ . Each voter receives an imperfect private signal,  $s_i \in \{a, b\}$ , about the state of the world. The signals are i.i.d. conditional on  $\omega$ and  $\Pr(a|\alpha) = \Pr(b|\beta) = \varepsilon$ , where  $\varepsilon \in (\frac{1}{2}, 1)$ . Hence, signal a (b) is associated with the state  $\alpha$  ( $\beta$ ) and  $\varepsilon$  is the signal precision. We sometimes refer to the pair ( $\pi, \varepsilon$ ) as the *information structure* of the voting game, since ( $\pi, \varepsilon$ ) determines the structure of interim beliefs.

Voters receive both instrumental and expressive payoffs. We represent voter *i*'s payoff by the following expression, where the first term captures instrumental-values (we denote  $(x, \omega) =$  $(a, \alpha)$  or  $(x, \omega) = (b, \beta)$  by  $x = \omega$ ), and the second two terms capture expressive-values:

$$U_{i}(k_{i}, v_{i}, x, \omega) = 1 (x = \omega) + 1 (v_{i} = a, \omega = \alpha) k_{i}(a, \alpha) + 1 (v_{i} = b, \omega = \beta) k_{i}(b, \beta)$$

First, note that all voters receive an instrumental payoff of 1 if the majority decision matches the state (i.e.  $(x, \omega) = (a, \alpha)$  or  $(x, \omega) = (b, \beta)$ ), and a payoff of zero otherwise (our results extend to more general instrumental payoffs). Second, voters receive expressive payoffs that depend on their individual vote and the state. This gives four possible expressive-payoff realizations for each voter. However, we normalize two of these payoffs to zero and let voter *i*'s

<sup>&</sup>lt;sup>8</sup>Assume without loss of generality that option b wins in case of a tie.

expressive payoffs be represented by a vector  $k_i = [k_i(a, \alpha), k_i(b, \beta)] \in M = [-1, 1] \times [-1, 1].$ <sup>9</sup> That is, an expressive payoff from voting *a* when the state is  $\alpha$  and an expressive payoff from voting *b* when the state is  $\beta$ .

For expositional clarity, we first analyze the homogeneous model where all agents face the same expressive payoffs. However, our main specification allows for heterogeneity, where  $k_i$  is independently drawn from some joint probability distribution F with density f. In this case, the vector  $k_i$  is privately observed by voter i and we refer to  $k_i$  as voter i's type. We assume that F has full support on M and  $\int_X f(k_i)dk_i > 0$  for all nonempty open subsets X of M.

A strategy for voter *i* is a function that maps voter *i*'s signal and payoff-type into a probability of voting for  $a, \sigma_i : \{a, b\} \times M \to [0, 1]$ . A full strategy profile is denoted by  $\sigma$  and we denote by  $\{\sigma^n\}$  a sequence of strategy profiles for increasing *n*. Our equilibrium notion is Bayesian Nash. Since  $U_i(k_i, v_i, x, \omega)$  is continuous in  $k_i$  we are guaranteed the existence of a Bayesian Nash equilibrium for any n.<sup>10</sup> In our initial analysis of homogeneous expressive payoffs we confine ourselves to symmetric Bayesian Nash equilibria. Our general results, however, are with respect to both symmetric and asymmetric equilibria.

As our performance benchmark, we follow Morgan and Várdy (2012) and define the socially optimal committee decision as the decision that aggregates agents' private information and matches the decision to the state of the world; we therefore use the term "information aggregation" and socially optimal interchangeably.

# 3 Analysis

To build intuition for our main results, we first expand on the simple example introduced in the introduction and focus on the case in which all voters receive a positive expressive payoff for an ex-post "correct vote:"  $k_i(a, \alpha) = k_{\alpha} > 0$  and  $k_i(b, \beta) = k_{\beta} > 0$ . Note that, in contrast to our example in the introduction, we do allow the size of the expressive payoff for getting the individual vote right to depend on the realized state of the world. We begin by formally

<sup>&</sup>lt;sup>9</sup>We allow expressive payoffs to take positive and negative values in order to study all potential impacts of payoffs that condition on the correctness of the individual vote, and we focus on expressive payoffs that are small relative to instrumental payoffs so that both instrumental and expressive concerns matter for the voting decision. However, this assumption is without loss of generality in the sense that our results hold for any other closed rectangle other than  $[-1, 1] \times [-1, 1]$ .

 $<sup>^{10}</sup>$ See e.g. Theorem 3.1. in Balder (1988).

establishing the fact that for large committees, expressive payoffs will drive behavior since instrumental payoffs are only associated with the unlikely pivotal event.

Let  $\Phi_{s_i}^n$  denote the relative expected utility that voter *i* receives from voting *a* given  $s_i$ ,  $k_{\alpha}$ ,  $k_{\beta}$  and the strategy of all others. In the following expression, we present a simplified equation for  $\Phi_{s_i}^n$ —letting  $piv_i$  indicate the event that voter *i* is pivotal for the outcome we get:

$$\Phi_{s_i}^n(\sigma_{-i}) = Pr(\alpha|s_i)[Pr(piv_i|\alpha) + k_\alpha] - Pr(\beta|s_i)[Pr(piv_i|\beta) + k_\beta]$$
(1)

Throughout the equilibrium analysis we follow the rule that  $\Phi_{s_i}^n(\sigma_{-i}) > 0$  implies that *i* will vote for *a* and  $\Phi_{s_i}^n(\sigma_{-i}) < 0$  implies *i* will vote for *b*.

Since all probability terms involving  $piv_i$  converge uniformly to 0 as  $n \to \infty$ ,<sup>11</sup> Expression 1 suggests that as n becomes large, the relative expected utility of voting a,  $\Phi_{s_i}^n(\sigma_{-i})$ , becomes insensitive to the strategies played by other agents. That is, we define  $\Phi_{s_i}$  as follows:

$$\Phi_{s_i} = \Pr(\alpha|s_i)k_\alpha - \Pr(\beta|s_i)k_\beta.$$
<sup>(2)</sup>

Loosely,  $\Phi_{s_i}$  can be thought of as the limiting expression of agent *i*'s relative payoff of voting for option a as  $n \to \infty$ . Note that  $\Phi_{s_i}$  is only a function of the expressive payoffs, the individual signal and the information structure, and is not a function of the strategies of the other agents or the instrumental payoffs—essentially, in large committees expressive payoffs turn the voting decision into an individual-choice problem. Additionally, other than the knife-edge case where  $Pr(\alpha|s_i)k_{\alpha} = Pr(\beta|s_i)k_{\beta}$  for  $s_i = a$  or  $s_i = b$ , Expression 2 suggests that agents will have a strict incentive to vote for either *a* or *b*.

We formalize this logic in the following proposition (we address voter behavior in the special cases where  $Pr(\alpha|s_i)k_{\alpha} = Pr(\beta|s_i)k_{\beta}$  in the appendix).

PROPOSITION 1 (Approximation of voter behavior in large committees). For all payoff vectors  $(k_{\alpha}, k_{\beta})$  and information structures  $\{\pi, \epsilon\}$  such that  $\Pr(\alpha|s_i)k_{\alpha} \neq \Pr(\beta|s_i)k_{\beta}$ : For any sequence of equilibria,  $\{\sigma^n\}$ , there exists n' such that for all n > n':

- 1.  $\sigma^n(a) = 1$ ,  $\sigma^n(b) = 1$  if  $\Phi_a > 0$ ,  $\Phi_b > 0$ .
- 2.  $\sigma^n(a) = 1$ ,  $\sigma^n(b) = 0$  if  $\Phi_a > 0$ ,  $\Phi_b < 0$ .

<sup>&</sup>lt;sup>11</sup>Given a symmetric strategy profile the probability of exactly  $\lfloor n/2 \rfloor$  votes for b out of n-1 follows a binomial distribution. By applying Stirling's formula it follows that there is an upper bound on the pivotal probability that converges to zero as  $n \to \infty$ .

3. 
$$\sigma^n(a) = 0, \ \sigma^n(b) = 0 \ if \ \Phi_a < 0, \ \Phi_b < 0.$$

The intuition for Proposition 1 follows from the logic introduced above—in large committees, voting behavior is driven by the expressive payoffs  $(k_{\alpha}, k_{\beta})$ .

While Proposition 1 characterizes the behavior of individual agents, it also reveals when expressive payoffs for getting the vote right cause information aggregation to fail (we give a full characterization result below). That is, for the committee decision to reflect the private information of the agents, it must be the case that agents *vote* according to their private information—either all agents set  $v_i = s_i$ , or agents place a higher weight on voting for the same option as their private signal. However, Proposition 1 shows that with expressive payoffs, agents only vote according to their private signal in large committees when  $\Phi_a > 0$  and  $\Phi_b < 0$ . Therefore, information aggregation will fail if either:

$$\Phi_a = \Pr(\alpha|a)k_\alpha - \Pr(\beta|a)k_\beta > 0, \text{ or}$$
(3)

$$\Phi_b = \Pr(\alpha|b)k_\alpha - \Pr(\beta|b)k_\beta < 0.$$
(4)

On the other hand, if neither of these conditions are satisfied, then a large committee will select the correct option with arbitrary precision.

To see how these conditions depend on the information structure, we substitute for  $Pr(\omega|b)$ in the above expressions, which shows that information aggregation will fail if either:

$$\frac{k_{\alpha}}{k_{\beta}} > \frac{(1-\pi)(1-\varepsilon)}{\pi\varepsilon}, \text{ or } \frac{k_{\alpha}}{k_{\beta}} < \frac{(1-\pi)\varepsilon}{\pi(1-\varepsilon)}.$$

Or, put differently:

$$\left[\frac{\pi}{(1-\pi)}\right]\frac{k_{\alpha}}{k_{\beta}} \notin \left(\frac{1-\varepsilon}{\varepsilon}, \frac{\varepsilon}{1-\varepsilon}\right).$$
(5)

First, note that the interval  $\left(\frac{1-\varepsilon}{\varepsilon}, \frac{\varepsilon}{1-\varepsilon}\right)$  includes 1 for all values of  $\varepsilon$ . Therefore, a necessary condition for information aggregation to fail is that the ratio of the expressive payoffs,  $k_{\alpha}/k_{\beta}$ , differs from the ratio of the prior probabilities of the two states,  $(1-\pi)/\pi$ . Put differently, when state  $\alpha$  is more likely,  $(1-\pi)/\pi < 1/2$ , then the relative payoff of voting correctly in state  $\beta$  must be higher for information aggregation to be assured.

Second, note that the size of the interval  $\left(\frac{1-\varepsilon}{\varepsilon}, \frac{\varepsilon}{1-\varepsilon}\right)$  depends on the precision of the private signal. If the signal precision is low—i.e.  $\varepsilon$  is low—then information aggregation will fail even

if  $k_{\alpha}/k_{\beta}$  and  $(1-\pi)/\pi$  only differ by a small amount. Conversely, if the signal precision is high, then information aggregation will not fail for a large imbalance between  $k_{\alpha}/k_{\beta}$  and  $(1-\pi)/\pi$ . Together, these two insights suggest the following proposition:

PROPOSITION 2 (Failure of information aggregation). Given any set of expressive payoffs  $(k_{\alpha}, k_{\beta})$  the committee decision is non-responsive to the profile of private signals for some information structures  $\{\pi, \varepsilon\}$ : That is, for any  $\pi$  such that  $k_{\alpha}/k_{\beta} \neq (1 - \pi)/\pi$ , there exists  $\varepsilon'$  such that information aggregation fails for all  $\varepsilon < \varepsilon'$ .

Proposition 2 highlights that in large committees the balance of expressive payoffs relative to the prior is more important than the alignment of instrumental and expressive payoffs. Returning to our running example of  $k_{\alpha} = k_{\beta}$ , as long as the likelihood of the two states of the world are not equal then information aggregation will fail for some signal precisions. Unlike the collective payoffs for getting the committee decision right, individual expressive payoffs for getting the vote right cannot incentivize agents to vote according to their private signals for all signal structures.

Next we formally link voting behavior to the committee outcome to fully characterize committee behavior. First, we introduce the following notation for the conditional probability that the committee selects a when the state is  $\alpha$  ( $\beta$ ): let  $Z_{\alpha}^{n} \equiv Pr(x = a | \sigma^{n}, \alpha)$  and  $Z_{\beta}^{n} \equiv Pr(x = a | \sigma^{n}, \beta)$ . The pair  $(Z_{\alpha}^{n}, Z_{\beta}^{n})$  is denoted by  $Z^{n}$  and thus  $Z^{n} = (1, 0)$  indicates that the committee achieves the socially optimal decision. Additionally, we define a *limit outcome* as a pair of conditional decision probabilities  $(Z_{\alpha}, Z_{\beta})$  that are consistent with the limiting values of a sequence of strategies  $\sigma^{n}$  that are best responses.

PROPOSITION 3 (Committee behavior in large committees). For any sequence of equilibria as  $n \to \infty, Z^n = (Z^n_{\alpha}, Z^n_{\beta})$  converges to:

- 1. (1,1) if  $\Phi_a > 0$ ,  $\Phi_b > 0$ .
- 2. (1,0) if  $\Phi_a \ge 0$ ,  $\Phi_b \le 0$ .
- 3. (0,0) if  $\Phi_a < 0$ ,  $\Phi_b < 0$ .

Note that when the inequalities in Proposition 3 are strict, the results follow directly from the characterization of voter behavior in Proposition 1. In the cases where one or both of  $\Phi_a$ ,  $\Phi_b$  equal 0 there is a unique non-babbling equilibrium for all sufficiently large n, and the associated sequence of conditional acceptance probabilities,  $Z^n$ , converges to (1,0) as  $n \to \infty$ . A full discussion of these cases are given in the appendix before the proof of Proposition 3.

Lastly, note that Proposition 3 shows that when information aggregation breaks down, it breaks down completely (bang-bang result) and the committee simply selects either option a or b regardless of the private information of the agents. Moreover, as we will demonstrate in the following subsection, this complete break-down of information aggregation is not only a limit prediction, but occurs at a finite n.

### **3.1** Behavior of small committees

Next we analyze the impact of expressive payoffs for small committees. In particular, we focus on the case of expressive payoffs that are aligned with instrumental payoffs, and information structures such that information aggregation fails for large committees. That is,  $k_{\alpha} = k_{\beta} > 0$ and  $\Phi_a > 0$ ,  $\Phi_b > 0$  (the case of  $\Phi_a < 0$ ,  $\Phi_b < 0$  is analogous). As a benchmark, we consider the unique informative symmetric equilibrium for the model with no expressive payoffs ( $k_{\alpha} = k_{\beta} =$ 0), which we denote by  $\sigma^{n*}(s_i)$ . As shown by McLennan (1998),  $\sigma^{n*}(s_i)$  is equivalent to the optimal symmetric voting strategy (i.e. the symmetric strategy that maximizes instrumental payoffs) given the aggregate profile of private information.

We begin by noting that the failure of information aggregation is not only a limit prediction, but a prediction for committees of large, but finite size.

COROLLARY 1. For any sequence of equilibria,  $(\sigma^n)$ , if  $\Phi_a > 0$ ,  $\Phi_b > 0$  there exists n' such that for all n > n',  $Z^n = (Z^n_{\alpha}, Z^n_{\beta})$  equals (1, 1).

Corollary 1 follows directly from Proposition 1 and shows that when Z = (1, 1), then  $Z^n = (1, 1)$  for all committees above a certain size. Moreover, as seen in the numerical example below, this complete failure of information aggregation can occur for relatively low values of n (n = 23 in the example).

EXAMPLE 1. Consider the case in which the payoffs for voting correctly are half of the instrumental payoff, that is,  $k_{\alpha} = k_{\beta} = 0.5$ . Furthermore, suppose that ex-ante, agents believe that  $\alpha$ 



Figure 1: This figure shows the voting behavior (we only show  $\sigma^n(b)$  since  $\sigma^n(a) = 1$  for all equilibria) and the corresponding probabilities that the committee decides *a* conditional on the two states of the world  $(Z^n_{\alpha}, Z^n_{\beta})$  for Example 1 for (1) the optimal symmetric voting strategy (green, solid line), and (2) informative equilibria with expressive payoffs (blue, dashed and red, dotted lines). There are two such equilibria for all small enough *n* in this example.

is four times as likely as  $\beta$  ( $\pi = 0.8$ ) and that  $\varepsilon = 0.75$ . These values are consistent with the estimates of Newham and Midjord (2020) based on voting data from FDA advisory committees where experts vote on questions concerning the approval of new drug applications. According to those estimates, the range for the probability that a committee member gets an incorrect signal is 17.5% and 27.5%<sup>12</sup> and the estimated prior probabilities that the drug is "good" range from 44% to 84%. In this example there are two non-babbling equilibria for all sufficiently small n. Both of these of the form  $\sigma^n(a) = 1$  and  $\sigma^n(b) \in (0,1]$  as is the unique non-babbling equilibrium in the case in which there are no expressive payoffs and all other parameters are as above. Figure 1 summarizes information aggregation in small committees for this example.<sup>13</sup> Finally, it is worth noting that the value of n at which the complete collapse of information aggregation takes place (n = 23) in this example is within the range of sizes of FDA advisory committees.

Figure 1 also illustrates several regularities of committee outcomes when expressive payoffs and information structures satisfy the conditions detailed above. First, note that the committee decision may partially reflect the private information for small committees, in the sense that

 $<sup>^{12}</sup>$ The model of Newham and Midjord (2020) is based on a continuous signal so there is not a perfect translation between of the error probability in the context of our discrete signals and their setting. For the purpose of these calculations we label a signal in their setting as erroneous if it leads to updated beliefs that are further away from the true state of the world.

<sup>&</sup>lt;sup>13</sup>In the appendix (Proposition 9) we show that as long as  $k_{\alpha} < 0$  and  $k_{\beta} < 0$ , all non-babbling equilibria must be either of the form  $\sigma^n(a) = 1$  and  $\sigma^n(b) \in [0, 1)$ , as is the case in this example, or of the form  $\sigma^n(a) \in (0, 1]$ and  $\sigma^n(b) = 0$ .

 $\sigma^n(b) \in (0,1)$  for small *n*. However, relative to the symmetric strategy that maximizes instrumental payoffs, the committee decision is biased towards the option that maximizes expressive payoffs (*a*) for committees of all sizes. This finding is formalized in the following proposition:

PROPOSITION 4 (Bias in small committees). For  $k_{\alpha} = k_{\beta} > 0$  and  $\Phi_a > 0$ ,  $\Phi_b > 0$ , in any symmetric equilibrium  $\sigma^n(s_i)$ ,  $\sigma^n(b) > \sigma^{n*}(b)$  and  $\sigma^n(a) = \sigma^{n*}(a) = 1$ .

Second, Figure 1 illustrates that as the committee size increases, there is a point at which  $\sigma^n(b)$  jumps to 1 and partial information aggregation suddenly fails. As shown by the following proposition this is a general feature of the model, and the size of the jump of  $\sigma^n(b)$  can be bounded from below as described.

PROPOSITION 5. For  $k_{\alpha} = k_{\beta} > 0$  and  $\Phi_a > 0$ ,  $\Phi_b > 0$ , if an equilibrium exists with  $\sigma^n(b) < 1$  then:

$$1 - \sigma^n(b) \ge \frac{|\Phi_b|}{2}.$$

Loosely, Proposition 5 follows from the fact that along the sequence of mixed strategy equilibria, the net instrumental payoff from voting for a must equal the net expected expressive payoff from voting for a,  $\Phi_b$ . This can only be the case if the probability of being pivotal is sufficiently high, which defines a lower bound for  $1 - \sigma(b)$  (given that  $\sigma(a) = 1$ ). It follows that as n increases and the probability of being pivotal decreases, the expected instrumental payoff of voting for a conditional upon being pivotal must increase. However, past a point the probability of being pivotal also decreases with  $1 - \sigma(b)$ , resulting in a sudden disappearance of the non-babbling mixed strategy equilibrium.

# 4 General Analysis and Extensions

In this section we analyze the general model that (1) allows for any conditional expressive payoffs that depend on the individual vote and the state, and (2) considers any continuous distribution of heterogeneous expressive payoffs. That is, we consider individual expressive payoffs  $k_i =$  $[k_i(a, \alpha), k_i(b, \beta)]$  drawn from a distribution F with a full support of  $M = [-1, 1] \times [-1, 1]$ . We begin by establishing that, as with homogeneous expressive payoffs, in equilibrium the pivotal probability converges to zero as n increases for any distribution of expressive payoffs. LEMMA 1 (Uniform convergence in the set of equilibrium strategies). For any sequence of equilibria  $\{\sigma^n\}$  of the game  $(F, \pi, \varepsilon, n)$  the pivotal probability converges uniformly to zero as n goes to infinity.

As above, Lemma 1 allows us to appeal to the relative expected expressive payoff of voting for a,  $\Phi_{s_i}$ , to characterize voter behavior in large committees.

$$\Phi_{s_i} = Pr(\alpha|s_i)k_i(a,\alpha) - Pr(\beta|s_i)k_i(b,\beta)$$
(6)

However, in contrast to the case of homogeneous expressive payoffs, Expression 6 only details individual voting behavior, since the value of  $\Phi_{s_i}$  depends on the individual values of  $k_i(v_i, \omega)$ .

To characterize aggregate voting behavior in large committees, we use  $\Phi_{s_i}$  to divide agents into four categories based on their payoff type  $k_i$ : partisan voters (a/b) who vote for one option for either signal, informative voters who set their vote equal to their private signal, and "antiinformative" voters who vote opposite of their private signal. Note that these categories depend on both the individual expressive payoffs and the information structure  $(\pi, \varepsilon)$ . That is, given a distribution F this partition of the set of expressive payoffs is not general, but will vary depending on the information structure of the game.

Formally, given  $(F, \pi, \varepsilon)$  take:

$$S_{v_i(a),v_i(b)} = \begin{cases} S_{a,b} = \{(k_i(a,\alpha), k_i(b,\beta)) \in M : \Phi_a > 0, \Phi_b < 0\}, \text{ (Informative)} \\ \\ S_{a,a} = \{(k_i(a,\alpha), k_i(b,\beta)) \in M : \Phi_a > 0, \Phi_b > 0\}, \text{ (partisan } a) \\ \\ \\ S_{b,b} = \{(k_i(a,\alpha), k_i(b,\beta)) \in M : \Phi_a < 0, \Phi_b < 0\}, \text{ (partisan } b) \\ \\ \\ \\ \\ S_{b,a} = \{(k_i(a,\alpha), k_i(b,\beta)) \in M : \Phi_a < 0, \Phi_b > 0\}. \text{ (anti-informative)} \end{cases}$$

Where  $\{v_i(a), v_i(b)\} \equiv \{v_i(s_i)\}$  corresponds to the voting behavior of an agent with  $k_i(v_i, \omega) \in S_{v_i(s_i)}$  in a large committee given, respectively, a signal of a, b. Henceforth we will use "type" to refer to an agent's classification into one of the four categories *informative*, *partisan a*, *partisan b*, *anti-informative*, while we continue use "payoff-type" to refer to an agent's expressive payoffs  $k_i$ .

We establish the correspondence between the classification of the types of the agents and voting behavior in the following corollary. COROLLARY 2 (Voter Behavior). Given any sequence of equilibria and any given  $k_i \in S_{v_i(a),v_i(b)}$ there exists n' such that for all n > n',  $v_i = v_i(s_i)$ .

Corollary 2 follows Lemma 1 and Proposition 1, and the intuition for the result is the same as above—as the probability of being pivotal diminishes, agents have an incentive to vote to maximize their expressive payoffs. Note, however, that Corollary 2 only applies to each payofftype individually, and we cannot use the result to establish that there exist an n' large enough so that all agents vote according to their type since agents with payoff-types "close" to the closure of the sets in the partition of the payoff types only face marginal incentives to vote to maximize their expressive payoffs. Therefore, we still require a result that ties the aggregate profile of types to committee outcomes.

To establish this result we first introduce the following notation to refer to the probability that an agent has a given type:

$$\mathbf{S}_{v_i(s_i)}^F = \int_{S_{v_i(s_i)}} f(k_i) dk_i$$

Loosely, given a large committee we can think of this as the size of the set of agents. Given this notation, we introduce our first main result:

THEOREM 1 (Socially Optimal Committee Decisions). Existence: Given a sequence of games  $\{(F, \pi, \varepsilon, n)\}$ , there exists a sequence of equilibria  $\{\sigma^n\}$  with  $\lim_{n\to\infty} Z^n = (1, 0)$  if the following condition holds:

$$\mathbf{S}_{a,b}^{F} > \mathbf{S}_{b,a}^{F} + \frac{|\mathbf{S}_{a,a}^{F} - \mathbf{S}_{b,b}^{F}|}{1 - 2(1 - \varepsilon)}$$

$$\tag{7}$$

Uniqueness: If Condition 7 is met, then any sequence of equilibria has  $\lim_{n\to\infty} Z^n = (1,0)$ .

To show existence, we first establish that for large committees, the proportion of agents of each type is approximated by  $\mathbf{S}_{v_i(s_i)}^F$  (see the formal proof in the appendix for details). Next, note that for information to be aggregated, it must be the case that the set of informative voters are 'pivotal' for the committee decision. That is, since only informative voters vote according to their private signal, the set of informative voters must be large enough to swing the election to *a* in state  $\alpha$  and *b* in state  $\beta$ . This occurs in state  $\alpha$  if:

$$\varepsilon \mathbf{S}_{a,b}^{F} + (1-\varepsilon)\mathbf{S}_{b,a}^{F} + \mathbf{S}_{a,a}^{F} > (1-\varepsilon)\mathbf{S}_{a,b}^{F} + \varepsilon \mathbf{S}_{b,a}^{F} + \mathbf{S}_{b,b}^{F}, \tag{8}$$

And in state  $\beta$  if:

$$\varepsilon \mathbf{S}_{a,b}^{F} + (1-\varepsilon)\mathbf{S}_{b,a}^{F} + \mathbf{S}_{b,b}^{F} > (1-\varepsilon)\mathbf{S}_{a,b}^{F} + \varepsilon \mathbf{S}_{b,a}^{F} + \mathbf{S}_{a,a}^{F}, \tag{9}$$

Simplifying and combining Expressions 8 and 9 gives Condition 7 in Theorem 1.

Intuitively, Condition 7 can be broken down into two parts. First, the size of the set of informative agents must be greater than the set of agents who are incentivized to vote against their signal ( $\mathbf{S}_{a,b}^{F} > \mathbf{S}_{b,a}^{F}$ ). Second, the proportion of informative agents who receive the correct signal (given by  $\varepsilon$ ) must outweigh any aggregate bias introduced by the partian voters  $(|\mathbf{S}_{a,a}^{F} - \mathbf{S}_{b,b}^{F}|)$ .

Additionally, Theorem 1 shows that when Condition 7 is fulfilled then any sequence of equilibria leads to the instrumentally optimal outcome. This is due to the fact that agents have a strict incentive to vote to maximize their individual expressive payoffs. To this end, our model with mixed motives strengthens the Condoret Jury Theorem when the Condition 7 holds, as the expressive payoffs rule out babbling equilibria that otherwise feature in models with pure instrumental values.

Next, we characterize how information aggregation fails when Condition 7 is not satisfied.

COROLLARY **3** (Failure of Information Aggregation). Given a sequence of games  $\{(F, \pi, \varepsilon, n)\}$ , if the following condition holds:

$$\mathbf{S}_{a,b}^{F} < \mathbf{S}_{b,a}^{F} + \frac{|\mathbf{S}_{a,a}^{F} - \mathbf{S}_{b,b}^{F}|}{1 - 2(1 - \varepsilon)},\tag{10}$$

then any sequence of equilibria has  $\lim_{n\to\infty} Z^n \in \{(1,1), (0,0), (0,1)\}.$ 

That is, when Condition 7 is not satisfied, then informative agents are not pivotal for the committee decision. Instead, the set of pivotal voters are either partian or anti-informative, which implies that the committee will either select the same option for all signal profiles, or select option a (b) in state  $\beta$  ( $\alpha$ ). Also, note that by Corollary 2, all agents play pure strategies for large n. Therefore, even with heterogeneous  $k_i$ , for large committees there are no equilibria where the committee partially aggregates information: when information aggregation fails, it fails completely.

Lastly, we consider the impact of the information structure  $(\pi, \varepsilon)$  on committee behavior, fixing the distribution of expressive payoffs F. Of particular interest is how the size of the set of informative voters,  $\mathbf{S}_{a,b}^F$ , depends on the precision of the private signal  $\varepsilon$ . First, note that the set of  $k_i$  that support informative types,  $S_{a,b}$ , is equivalent to the set of expressive payoffs that support informative voting in the homogeneous model. That is,  $k_i \in S_{a,b}$  iff:

$$\left[\frac{\pi}{(1-\pi)}\right]\frac{k_i(a,\alpha)}{k_i(b,\beta)} \in \left(\frac{1-\varepsilon}{\varepsilon}, \frac{\varepsilon}{1-\varepsilon}\right).$$
(11)

This demonstrates that, analogous to the homogeneous case,  $S_{a,b}$  and hence  $\mathbf{S}_{a,b}^F$  is increasing in  $\varepsilon$ . That is, as the precision of the private signal decreases, fewer payoff-types maximize their expected payoffs by voting informatively.

Moreover, the precision of the private signal has an additional effect with heterogeneous expressive payoffs. That is, it is not only the size of  $\mathbf{S}_{a,b}^F$  that matters for information aggregation the relative number of informative agents that vote correctly ( $\varepsilon \mathbf{S}_{a,b}^F - (1 - \varepsilon) \mathbf{S}_{a,b}^F$ ) also matters, since the relative number of correct votes must outweigh any partian bias ( $|\mathbf{S}_{a,a}^F - \mathbf{S}_{b,b}^F|$ ). Since the relative number of informative voters that vote correctly is increasing in  $\varepsilon$ , this condition is less likely to be satisfied as the signal precision decreases. This combined logic leads to our second main result:

THEOREM 2 (General Failure of the CJT with Mixed Motives). For any F the committee decision is non-responsive to the profile of private signals for some information structures  $(\pi, \varepsilon)$ : that is, for a set of  $\pi \in (0,1)$ ,<sup>14</sup> there exists  $\bar{\varepsilon} \in (\frac{1}{2}, 1)$  such that for all  $\varepsilon < \bar{\varepsilon}$  every sequence of equilibria has  $\lim_{n\to\infty} Z^n \in \{(0,0), (1,1)\}$ .

Theorem 2 shows that Proposition 2 extends to the general case, and that in contrast to the case with instrumental payoffs, for any distribution of expressive payoffs information aggregation fails in large committees for some information sets.

### 4.1 Reputational motives

Above, we consider expressive payoffs that condition only on the correctness of the agents individual vote. Naturally, however, the reputational payoff of voting correctly or incorrectly may also depend on the number of other agents who voted correctly/incorrectly. For example, in certain situations, it may be advantageous to be among the minority of agents who are

<sup>&</sup>lt;sup>14</sup>The required conditions on  $\pi$  are that  $\pi \neq 1/2$  and that  $\int_S f(k_i) dk_i \neq \frac{1}{2}$  where  $S = \{(k_i \in M : \pi k_i(a, \alpha) > (1 - \pi)k_i(b, \beta)\}$  is the set of types who optimally votes for a under both signals given zero pivotal probability and  $\varepsilon$  set to one half (i.e. uninformative signals).

correct, analogous to the reputation payoffs that have been received by the small number of experts who correctly predicted the 2008 financial crisis. In other situations, however, it may be more important to not be in the minority of agents who are incorrect, as illustrated in the opening quote by Hillary Clinton, where she qualifies her own error with the fact that "...I wasn't alone in getting it wrong."

In this subsection we consider reputation payoffs that are a function of the number of other agents who voted correctly. Note that in contrast with the expressive payoffs analyzed in our main specification, the expected reputational payoffs are a function of the strategies of the other agents, implying that voting is a strategic choice even in large committees. As we will show, however, our main results are robust to reputational considerations.<sup>15</sup>

In what follows, let  $\delta_{\alpha}$  ( $\delta_{\beta}$ ) indicate the proportion of votes for a when the state is  $\alpha$  ( $\beta$ ). Moreover, given some strategy profile  $\sigma$ , let  $\bar{\delta}_{\alpha}$  ( $\bar{\delta}_{\beta}$ ) indicate the expected value of  $\delta_{\alpha}$  ( $\delta_{\beta}$ ). In this subsection, we consider expressive payoffs that are a function of the proportion of agents who vote for a. In particular, we assume that  $\delta_{\omega}$  has a linear impact on expressive payoffs—as above, agents draw  $k_i$  according to a continuous distribution F with full support on  $M = [0, 1]^2$ , and agents' expressive payoffs takes one of the following forms:

$$k_i(\omega, v_i, \delta_{\omega}) = \begin{cases} (1) & (k_i(a, \alpha)(1 - \delta_{\alpha}), k_i(b, \beta)\delta_{\beta}) \text{ or} \\ \\ (2) & (k_i(a, \alpha)\delta_{\alpha}, k_i(b, \beta)(1 - \delta_{\beta})). \end{cases}$$

That is, in Case 1 agents face an increasing reward for being right when others are wrong, and in Case 2 there is an decreasing cost of being wrong when others are also wrong.

Next, in this subsection we also let the instrumental payoffs be heterogeneous across agents. This assumption simplifies the proof on the uniform convergence of the pivotal probability to zero along equilibrium sequences, but otherwise has no substantive impact on the model our results hold for *any* distribution of instrumental payoffs, including a distribution where an arbitrarily high portion of the probability mass is distributed arbitrarily close to an instrumental payoff of 1/0. Let  $w_i(a, \alpha)$  ( $w_i(b, \beta)$ ) be agent *i*'s instrumental payoff when the committee

<sup>&</sup>lt;sup>15</sup>As we mention in the literature review, there is also a literature that applies the Holmstrom (1999) model of career concerns to committees. In such models, low-information types have an incentive to mimic highinformation types, which can result in coordination or anti-coordination problems. While we do not formally analyze a model of career concerns, note that our main results apply straightforwardly—if  $\pi \neq 1/2$  and the precision of the signal received by the high-information type is low enough, then both high-information and low-information types will condition their vote on the public information rather than their private information.

decides for option a (b) and the state is  $\alpha$  ( $\beta$ ). Agents draw  $w_i(x, \omega)$  according to a continuous distribution F'' with full support on  $M = [0, 1]^2$ , and take F' to be the joint distribution of expressive and instrumental payoffs.

Proposition 6 extends our Theorem 2 to situations where either the benefit of being right or the cost of being wrong depends on the number of other agents who are right/wrong.

PROPOSITION 6 (General failure of the CJT with reputational motives). For any F' the committee decision is non-responsive to the profile of private signals for some information structures  $(\pi, \varepsilon)$ : that is, for a set of  $\pi \in (0, 1)$ , there exists  $\overline{\varepsilon} \in (\frac{1}{2}, 1)$  such that for all  $\varepsilon < \overline{\varepsilon}$  every sequence of equilibria has  $\lim_{n\to\infty} Z^n \in \{(0,0), (1,1)\}$ .

In Case 2, there is an incentive for conformity as agents have a disincentive for being wrong in a minority, which reinforces the incentive to vote for the option that maximizes expressive payoffs and Theorem 2 extends. In Case 1 the result is perhaps more surprising since there is an incentive for anti-conformity. While this does increase the incentive to vote opposite to the majority, this effect is never large enough to push the committee decision away from selecting one of the two options with certainty, since the size of the effect diminishes as the expected margin of victory decreases.

### 4.2 Optimal decision rules

Above we consider voting under a simple majority rule given a fixed distribution of expressive payoffs and show that for some information structures, a committee will not make optimal decisions. In this subsection, we will demonstrate that this problem can be mitigated by selecting the optimal decision rule. That is, we consider the set of q-rules, with  $q \in (0, 1)$ , where the outcome of the vote is a if  $|\{i|v_i = a\}|/n > q$  and b otherwise. We show that if the type distribution and information structure is such that the informative types outweigh the anti-informative types, then there always exists a set of q-rules such that a version of Condition 7 of Theorem 2 is met, and large committees approach the socially optimal decision.

PROPOSITION 7 (Optimal decision rule). If  $\mathbf{S}_{a,b}^F > \mathbf{S}_{b,a}^F$  there exists a non-empty convex set of *q*-rules for which every sequence of equilibria has  $\lim_{n\to\infty} Z^n = (1,0)$ .

The intuition behind Proposition 7 is that by appropriately increasing or decreasing q it is possible to ensure that an informative type is the pivotal voter, effectively neutralizing the impact of the two partisan groups. While Proposition 7 does not provide a general solution to the problem of information aggregation with expressive payoffs, it shows that for a given information structure and distribution of expressive payoffs, a decision rule can be tailored to the collective choice to ensure that the decisions of large committees reflect the private information of its members.

# 5 Conclusion

In this paper, we extend the literature of voting under mixed motives in two important directions: first, we allow for mixed motives that depend on the correctness of the individual vote, examine how they operate in committees of small size and show that they can lead to complete failure of information aggregation in large committees. Second we show that this finding is robust to substantial variations of our baseline model including arbitrary distributions of privately known heterogeneous payoffs and expressive payoffs that depend on the observed voting profile as might be the case when they arise from reputational concerns.

Our paper also further highlights the frailty of the canonical pivotal voter model/Condorcet Jury Theorem (Feddersen and Pesendorfer, 1997) by demonstrating that the assumption that agents only derive payoffs based on the collective outcome is key for committee decisions to consistently reflect agents' private information.

From a practical standpoint, our results illustrate that, surprisingly, holding committee members accountable for the correctness of their individual vote can distort committee outcomes and result in overly conservative decisions, in the sense that the committee is biased against options that are perceived as less likely ex ante. We end by noting that expressive payoffs might also serve a useful purpose in the sense of providing incentives to turnout or to acquire private information. Therefore, for future research, it may be important to explore the interaction effect of expressive payoffs on voting decisions and other relevant voter decisions, such as the decision of whether or not to vote.

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# A Appendix

### A.1 Proofs for Section 3

### Proof of Proposition 1:

Consider the case in which  $\Phi_a > 0$  and  $\Phi_b < 0$ . The other cases in the proposition are addressed analogously. Given that  $Pr(piv_i|\alpha)$  and  $Pr(piv_i|\beta)$  converge uniformly to 0 (uniformly with respect to  $\sigma$ ), then for any h > 0 there exists  $n^*$  such that for all  $n > n^*$ , and  $\sigma$ ,  $|\Phi_a^n - \Phi_a| < h$ and  $|\Phi_b^n - \Phi_b| < h$ . It follows that for all large enough  $n, \sigma^n(a) = 1, \sigma^n(b) = 0$  is an equilibrium of the game. This argument also shows that for sufficiently large n the only equilibrium of the game involves  $\sigma^n(a) = 1$  and  $\sigma^n(b) = 0$ . We complete the sequence for  $n \leq n^*$ , by selecting for each game in the sequence an arbitrary symmetric Bayesian Nash equilibrium. Lemma 2 below shows that such an equilibrium always exists. ■

LEMMA 2. Given any payoff vector  $(k_{\alpha}, k_{\beta}) > (0, 0)$ , the game associated to a committee with n members has a symmetric Bayesian Nash equilibrium.

### Proof of Lemma 2:

Notice that given  $Pr(\alpha|a) > Pr(\alpha|b)$  and  $Pr(\beta|a) < Pr(\beta|b)$ , we have that for all n, and  $\sigma_{-i}$ ,  $\Phi_a^n(\sigma_{-i}) > \Phi_b^n(\sigma_{-i})$ . It follows that there are only four possible kinds of symmetric Bayesian Nash equilibria: (1)  $\sigma(a) = \sigma(b) = 0$  if and only if  $\Phi_a^n(\sigma) < 0$  and  $\Phi_b^n(\sigma) < 0$ , <sup>16</sup> (2)  $0 < \sigma(a) \le 1$ ,  $\sigma(b) = 0$  if  $\Phi_a^n(\sigma) = 0$  and  $\Phi_b^n(\sigma) < 0$ , (3)  $\sigma(a) = 1, 0 \le \sigma(b) \le 1$  if  $\Phi_a^n(\sigma) > 0$  and  $\Phi_b^n(\sigma) = 0$ , (4)  $\sigma(a) = 1, \sigma(b) = 1$  if  $\Phi_a^n(\sigma) > 0$  and  $\Phi_b^n(\sigma) > 0$ .

Case (I)  $\Phi_b^n(0,0) \leq 0$ : If  $\Phi_a^n(0,0) \leq 0$  then  $\sigma(a) = \sigma(b) = 0$  is an equilibrium and we are done. Else, consider  $\Phi_a^n(1,0)$ .

Case (Ia)  $\Phi_a^n(1,0) \leq 0$ : Then, by continuity, there must exist  $0 < \sigma(a) < 1$  such that  $\Phi_a^n(\sigma(a),0) = 0$ . Furthermore, it must be the case that  $\Phi_b^n(\sigma(a),0) < 0$  and thus  $(\sigma(a),0)$  is an equilibrium.

Case (Ib)  $\Phi_a^n(1,0) > 0$ : If  $\Phi_b^n(1,0) \le 0$ , then (1,0) is an equilibrium. Else, if  $\Phi_b^n(1,0) > 0$ , then consider  $\Phi_b^n(1,1)$ . If  $\Phi_b^n(1,1) \ge 1$ , then  $\Phi_a^n(1,1) > 1$  and (1,1) is an equilibrium. If  $\Phi_b^n(1,1) < 0$  then by continuity there must exist  $\sigma(b)$  such that  $\Phi_b^n(1,\sigma(b)) = 0$ . It follows that  $\Phi_a^n(1,\sigma(b)) > 0$  and  $(1,\sigma(b))$  is an equilibrium.

Case (II)  $\Phi_b^n(0,0) > 0$ : If  $\Phi_b^n(1,0) > 0$  then consider  $\Phi_b^n(1,1)$ . If  $\Phi_b^n(1,1) \ge 0$  then  $\Phi_a^n(1,1) > 0$  and (1,1) is an equilibrium. If  $\Phi_b^n(1,0) \le 0$ , then consider  $\Phi_a^n(1,0)$ .

Case (IIa) If  $\Phi_a^n(1,0) \ge 0$ , then (1,0) is an equilibrium.

Case (IIb) If  $\Phi_a^n(1,0) < 0$  then by continuity there must exist  $\sigma(a)$  such that  $\Phi_a^n(\sigma(a),0) = 0$ . It follows that  $\Phi_b^n(\sigma(a),0) < 0$  and that  $(\sigma(a),0)$  is an equilibrium.

<sup>&</sup>lt;sup>16</sup>With a slight abuse of notation, by  $\Phi_{s_i}^n(\sigma)$ , we mean evaluating the best response function by setting  $\sigma_j = \sigma$  for all  $j \neq i$ .

### Proof of Proposition 2:

Notice that for any  $(k_{\alpha}, k_{\beta})$  and  $\pi$  such that  $k_{\alpha}/k_{\beta} \neq (1 - \pi)/\pi$ , there exists  $\varepsilon'$  such that for all  $\varepsilon < \varepsilon'$ 

$$\frac{k_{\alpha}}{k_{\beta}} \notin \left[\frac{Pr(\beta|s_i=a)}{Pr(\alpha|s_i=a)}, \frac{Pr(\beta|s_i=b)}{Pr(\alpha|s_i=b)}\right]$$

then either (1)  $\Phi_a < 0$  and  $\Phi_b < 0$  or (2)  $\Phi_a > 0$  and  $\Phi_b > 0$ . Suppose without loss of generality that we are in case (1). Then, because  $Pr(piv_i|\alpha)$  and  $Pr(piv_i|\beta)$  converge uniformly to 0, it follows that along any sequence of equilibria, it must be the case that  $\sigma^n(a) = 0$  and  $\sigma^n(b) = 0$  for all large enough n. It follows from the law of large numbers and the fact that  $\varepsilon > \frac{1}{2}$  that  $Z^n_{\alpha} \to 0$  and  $Z^n_{\beta} \to 0$  as  $n \to \infty$ .

Analysis of special cases:  $\Phi_a = 0$  or  $\Phi_b = 0$ . Here we address the special cases of  $\Phi_a = 0$  or  $\Phi_b = 0$ . We first cover the intuition, followed by the formal results.

When  $\Phi_a = 0$  and  $\Phi_b = 0$  it follows that for all n the inequalities determining an agent's best response are exactly the ones which he/she would face in the absence of mixed motives. As discussed in Feddersen and Pesendorfer, 1997 when this is the case, the unique sequence of non-babbling mixed strategy equilibria leads to perfect information aggregation in the limit as  $n \to \infty$ . That is,  $Z^n = (Z^n_{\alpha}, Z^n_{\beta})$  converges to (1, 0) as  $n \to \infty$ . Along the sequence, equilibria can take one of two forms: (1)  $\sigma(b) = 0$  and  $\sigma(a) \in (0, 1]$  or (2)  $\sigma(b) = \in [0, 1)$  and  $\sigma(a) = 1$ , depending on the payoff and information structures.

When only  $\Phi_a = 0$  or  $\Phi_b = 0$ , the inequality  $\Phi_{s_i} \neq 0$  establishes the unique best response of agents for all sufficiently large n, conditional on observing  $s_i$ ; either  $\sigma(s_i) = 1$  or  $\sigma(s_i) = 0$ . For all n, the expression  $\Phi_{s'_i}^n$  associated to the other signal  $s'_i$  is identical to the one in the model with purely instrumental payoffs. Consider the case in which  $\Phi_a = 0$  and  $\Phi_b < 0$ . In the absence of expressive payoffs having a signal revealing pure strategy equilibrium requires that each signal makes its corresponding state more likely than the alternative according to agents' interim beliefs. Specifically we require  $Pr(\beta|a) \leq Pr(\alpha|a)$  and  $Pr(\beta|b) \geq Pr(\alpha|b)$ . Given that  $\Phi_b < 0$ , the second inequality no longer needs to hold, and then as long as signal a makes the state  $\alpha$  more likely, we have a signal revealing pure strategy equilibrium for all sufficiently large *n*. When even upon observing signal  $a, \beta$  is more likely than  $\alpha$ , then equilibria are of the form  $\sigma^n(b) = 0$  and  $\sigma^n(a) > 0$  and identical to those of the case without expressive payoffs. It follows that we obtain perfect information aggregation in the limit as  $n \to \infty$ . This intuition is formalized in the following proposition.

PROPOSITION 8. (1) When either  $\Phi_a = 0$  and  $\Phi_b = 0$ , then as  $n \to \infty$ ,  $Z^n = (Z^n_{\alpha}, Z^n_{\beta})$ converges to (1,0), in the unique sequence of non-babbling equilibria. Furthermore:

(2) When  $\Phi_a = 0$  and  $\Phi_b < 0$  there exists an  $n^*$  such that for all  $n > n^*$ ,  $\sigma^n(a) = 1$  and  $\sigma^n(b) = 0$  is an equilibrium if and only if  $Pr(\beta|a) \leq Pr(\alpha|a)$ .

(3) When  $\Phi_a > 0$  and  $\Phi_b = 0$  there exists an  $n^*$  such that for all  $n > n^*$ ,  $\sigma^n(a) = 1$  and  $\sigma^n(b) = 0$  is an equilibrium if and only if  $Pr(\beta|b) \ge Pr(\alpha|b)$ .

### Proof of Proposition 8:

(1) Suppose that  $\Phi_a = 0$  and  $\Phi_b = 0$ . It then follows that  $\Phi_a^n$  and  $\Phi_b^n$  are just as in the absence of expressive payoffs and the conditions for the aggregation of information as  $n \to \infty$  are just as in the canonical model.

(2) Suppose that  $\Phi_b < 0$  and  $\Phi_a = 0$ . Then it follows that for sufficiently large n, in any equilibrium it must be the case that  $\sigma^n(b) = 0$ . In what follows we focus on these large enough values of n. Further, note that  $\Phi_a^n$  will be exactly as in the absence of expressive payoffs.

It follows that there will be a pure strategy equilibrium in which agents vote according to their signal if and only if:

$$\frac{Pr(piv_{i}|\alpha)}{Pr(piv_{i}|\beta)} \geq \frac{Pr(\beta|a)}{Pr(\alpha|a)} \equiv h(a)$$

$$\Leftrightarrow \frac{\binom{n-1}{\lfloor n/2 \rfloor} \mu_{\alpha}^{\lfloor n/2 \rfloor} (1-\mu_{\alpha})^{n-1-\lfloor n/2 \rfloor}}{\binom{n-1}{\lfloor n/2 \rfloor} \mu_{\beta}^{\lfloor n/2 \rfloor} (1-\mu_{\beta})^{n-1-\lfloor n/2 \rfloor}} \geq h(a) \text{ where } \mu_{\alpha} = \sigma^{n}(a)\varepsilon \text{ and } \mu_{\beta} = \sigma^{n}(a)(1-\varepsilon)$$

$$\Leftrightarrow \left(\frac{\varepsilon}{1-\varepsilon}\right)^{\lfloor n/2 \rfloor} \left(\frac{1-\varepsilon}{\varepsilon}\right)^{n-1-\lfloor n/2 \rfloor} \geq h(a), \text{ letting } \sigma^{n}(a) = 1$$

The inequality above is equivalent to  $\frac{\varepsilon}{1-\varepsilon} \ge h(a)$  for all even n and  $1 \ge h(a)$  for all odd n. It follows that we will have a signal revealing pure strategy equilibrium for all sufficiently large n if and only if  $1 \ge h(a)$ . That is, if and only if  $\frac{Pr(\beta|a)}{Pr(\alpha|a)} \ge 1$  as stated in the proposition.

We will have a non-babbling mixed strategy equilibrium with  $\sigma(a) \in (0, 1)$  if:

$$\frac{\binom{n-1}{\lfloor n/2 \rfloor} \mu_{\alpha}^{\lfloor n/2 \rfloor} (1-\mu_{\alpha})^{n-1-\lfloor n/2 \rfloor}}{\binom{n-1}{\lfloor n/2 \rfloor} \mu_{\beta}^{\lfloor n/2 \rfloor} (1-\mu_{\beta})^{n-1-\lfloor n/2 \rfloor}} = h(a) \text{ where } \mu_{\alpha} = \sigma^{n}(a)\varepsilon \text{ and } \mu_{\beta} = \sigma^{n}(a)(1-\varepsilon)$$

$$\Leftrightarrow \frac{1-\sigma^{n}(a)\varepsilon}{1-(1-\varepsilon)\sigma^{n}(a)} = h(a)^{1/(n-1-\lfloor n/2 \rfloor)} \left(\frac{1-\varepsilon}{\varepsilon}\right)^{\frac{\lfloor n/2 \rfloor}{n-1-\lfloor n/2 \rfloor}} \equiv f(n)$$

$$\Leftrightarrow \sigma^{n}(a) = \frac{1-f(n)}{\varepsilon-(1-\varepsilon)f(n)}$$

Noting that f(n) converges to  $\frac{1-\varepsilon}{\varepsilon} < 1$  it follows that for all sufficiently large  $n, \sigma^n(a) > 0$ . Moreover  $\sigma^n(a) \le 1$  if and only if  $f(n) \ge \frac{1-\varepsilon}{\varepsilon}$  which will be the case as long as  $h(a) \ge 1$ .

Notice that given that evaluating at  $\sigma^n(b) = 0$ ,  $\sigma^n(a)$  as above it will be the case that,

$$\frac{Pr(piv_i|\alpha)}{Pr(piv_i|\beta)} = \frac{Pr(\beta|a)}{Pr(\alpha|a)} < \frac{Pr(\beta|b)}{Pr(\alpha|b)},$$

the resulting sequence of non-babbling equilibria will also be a sequence of non-babbling equilibria in the canonical game, without expressive payoffs. We know that in the limit as  $n \to \infty$  such a sequence converges to perfect information aggregation.

The proof of the case in which  $\Phi_a > 0$  and  $\Phi_b = 0$  is analogous to the above.

### Proof of Proposition 3:

Proposition 3 follows directly from voting behavior detailed in Proposition 1 and the characterization of the special cases in Proposition 8 above.

### A.2 Proofs for Section 3.1

We first introduce the following Proposition that characterizes the shape of non-babbling equilibria of the game when  $k_{\alpha} > 0$  and  $k_{\beta} > 0$ .

PROPOSITION 9. For any information and payoff structures such that  $(k_{\alpha}, k_{\beta}) > (0, 0)$  equilibria are of one of two forms:

(1)  $\sigma(b) = 0$  and  $\sigma(a) \in [0, 1]$  or,

(2)  $\sigma(b) \in [0, 1]$  and  $\sigma(a) = 1$ 

Proof of Proposition 9:

Note that,  $\Phi_{s_i}^n = \Phi_{s_i} + Pr(piv_i|\alpha)Pr(\alpha|s_i) - Pr(piv_i|\beta)Pr(\beta|s_i).$ 

The proposition follows from the fact that since  $k_{\alpha} > 0$  and  $k_{\beta} > 0$  it is the case that  $\Phi_a^n > \Phi_b^n$  regardless of  $\sigma^n$ .

### Proof of Proposition 4:

First, we show that equilibria must take the form  $\sigma^n(b) \in (0,1]$ ,  $\sigma^n(a) = 1$ . Assume an equilibrium exists with  $\sigma^n(b) = 0$  and  $\sigma^n(a) \in [0,1]$ . If follows that  $\Pr(\alpha | piv_i, b) \ge \Pr(\alpha | b)$  and  $\Pr(\beta | piv_i, b) \le \Pr(\beta | b)$ . However, rewriting the expression for  $\Phi^n_a$  gives:

$$\Phi_b^n = (\Pr(\alpha | piv_i, b) - \Pr(\beta | piv_i, b)) \Pr(piv_i | b) + \Phi_b$$

Which implies that

$$\Phi_b^n > (\Pr(\alpha|b) - \Pr(\beta|a)) \Pr(piv_i|b) + \Phi_b > 0.$$

This contradicts the assumption that  $\sigma^n(a) = 0$  for any  $k_{\alpha} = k_{\beta} \ge 0$  since  $\Phi_b > 0$  and  $\Pr(\alpha|b) < \Pr(\beta|b)$  (given that  $k_{\alpha} = k_{\beta}$  and  $\Phi_b > 0$ ). By Proposition 9 this implies that equilibria must take the form  $\sigma^n(b) \in (0, 1], \sigma^n(a) = 1$ .

Next, by contradiction, assume that  $\sigma^n(b) \leq \sigma^{n*}(b)$ . Note that  $\sigma^{n*}(b)$  solves:

$$\Pr(\alpha|piv_i, b) - \Pr(\beta|piv_i, b) = 0$$

While  $\sigma^n(b)$  solves:

$$\left(\Pr(\alpha|piv_i, b) - \Pr(\beta|piv_i, b)\right) \Pr(piv_i|b) + \Phi_b = 0$$

This implies that:

$$(\Pr(\alpha|piv_i, b) - \Pr(\beta|piv_i, b)) + \frac{\Phi_b}{\Pr(piv_i|b)} = 0$$

However:

$$(\Pr(\alpha|piv_i, b, \sigma) - \Pr(\beta|piv_i, b, \sigma) \ge (\Pr(\alpha|piv_i, b, \sigma^*) - \Pr(\beta|piv_i, b, \sigma^*) = 0$$

since the LHS term is decreasing in  $\sigma^n(b)$  (given  $\sigma^n(a) = 1$ ) and  $\frac{\Phi_b}{\Pr(piv_i|b)} > 0$ , which is a contradiction.

Proof of Proposition 5:

Any such equilibrium requires:

$$\Pr(piv_i|b) \left( \Pr(\beta|piv_i, b) - \Pr(\alpha|piv_i, b) \right) \ge \Phi_b$$
  
$$\Rightarrow \Pr(piv_i|b) \ge \Phi_b$$

$$\Rightarrow Pr(piv_i|\alpha)Pr(\alpha|b) + Pr(piv_i|\omega = \beta)Pr(\beta|b) = Pr(piv_i|b) \ge \Phi_b$$
$$\Rightarrow 1 - \sigma^n(b) \ge max\{Pr(piv_i|\alpha), Pr(piv_i|\beta)\} \ge \frac{\Phi_b}{2}$$

Where the inequality on the left follows from the fact that for each  $\omega$ ,  $1 - \sigma^n(b) \ge 1 - \mu_{\omega} \ge Pr(piv_i|\omega)$ . This is the case because  $1 - \mu_{\alpha} = (1 - \varepsilon)(1 - \sigma^n(b))$  and  $1 - \mu_{\beta} = \varepsilon(1 - \sigma^n(b))$ .

### A.3 Proofs for Section 4

Proof of Lemma 1: Suppose  $\{\sigma^n\}$  is a sequence of equilibria. Note from expression (1) that given any signal precision and  $Pr(piv_i|\omega)$  there is a positive measure of types such that  $\Phi_{s_i}^n$  is strictly positive (negative) for both signal realizations. Hence, any player votes for option awith a probability that is bounded away from 0 and 1 and the bound is independent of n.

Conditional on the state, we can consider the distribution of the number of votes by agents other than *i* for option *a*, given  $\sigma^n$ , as a sum of independent Bernoulli trials (Poisson binomial distribution) with success probabilities  $p_j$  belonging to a closed set [l, 1-l] bounded away from 0 and 1. The probability of *i* being pivotal is linear on each  $p_j$ . Specifically it is a sum of  $\binom{n-1}{\lfloor n/2 \rfloor}$  products in each of which  $p_j$  appears as one of the multipliers as such or as  $(1 - p_j)$ . Therefore the probability of *i* being pivotal achieves a maximum at the extreme points of the set  $[l, 1 - l]^{n-1}$ . Choosing the number of  $p_j$ 's equal to *l* (denoted by |l|), with the remaining equal to (1 - l) (denoted by |(1 - l)| equalling (n - 1) - |l|), to maximize the probability of *i* being pivotal implies that  $|l| = |(1 - l)| \pm 1$ . This is true due to the following (by contradiction):

Suppose otherwise and the pivotal probability is maximized with |l| > |(1 - l)| + 1 (the following argument works analogously if |l| < |(1 - l)| - 1. First, note that the mode of a Poisson binomial distribution differs from the mean by at most 1 (Darroch, 1317-1321) and the mean is given by the sum of the  $p_j$ 's. Let our |l| be denoted by X and then our |(1 - l)| equals (n-1)-X. Next, consider the case with |l| = X - 1 and |(1-l)| = (n-1)-X (i.e. one trial less and |l| decreased by one). We know that the mode (even in special cases with two consecutive modes) of the Poisson Binomial distribution with |l| = X - 1 and |(1 - l)| = (n - 1) - X is strictly less than  $\lfloor n/2 \rfloor$ . Hence, given |l| = X - 1 and |(1 - l)| = (n - 1) - X, the probability of  $\lfloor n/2 \rfloor$  successes (denoted W) is strictly less than the probability of  $\lfloor n/2 \rfloor - 1$  successes (denoted Z). We can now express the pivotal probability (i.e.  $\lfloor n/2 \rfloor$  successes) as:  $W(1 - l) + Z \times l$ .

If instead we have |l| = X - 1 and |(1 - l)| = (n - 1) - X + 1 the pivotal probability is given by:  $W \times l + Z(1-l)$ . Since l < (1-l) and W < Z we have a contradiction since the pivotal probability is strictly greater for |l| = X - 1 and |(1 - l)| = (n - 1) - X + 1 than for |l| = Xand |(1-l)| = (n-1) - X.

Having showed that to maximize the pivotal probability we must have  $|l| = |(1 - l)| \pm 1$  we can apply Central Limit Theorem (also noting that the Lyapunov condition is satisfied). Given any sequence of equilibria we can establish an upper bound on the pivotal probability along this sequence. Consider some n' and maximize the pivotal probability with regards to |l| and |(1-l)|. Then take n = n' + 2 and maximize the pivotal probability with regards to |l| and |(1-l)|, which implies that both |l| and |(1-l)| is greater, or equal to, |l| and |(1-l)| for n', whereby we can fix |l| and |(1-l)| for n' and add two independent binary r.v.'s to reach |l| and |(1-l)| for n = n'+2. Same for n = (n'+2)+2 and so on. Same procedure if we instead start form n = n' + 1. Hence, we can apply the Central Limit Theorem and have an upper bound on the pivotal probability that converges uniformly to zero (the normal distribution).

### Proof of Theorem 1:

(the "if" part) Since  $U_i$  is continuous in  $k_i$  and the set of strategies available to each type is compact we know that a Bayesian Nash equilibrium exists for each n (see e.g. Theorem 3.1.) in Balder (1988)). We will now show that for any such sequence of equilibria  $\{\sigma^{n'}\}$  it must be the case that  $\lim_{n\to\infty} Z^n = (1,0)$  if condition 7 is satisfied (equivalent to conditions 8 and 9) jointly).

Suppose conditions 8 and 9 are satisfied and pick  $w \in (0,1)$  small enough so that the left-hand-side of condition (8) is greater than  $\frac{1}{2} + w$  and the right-hand-side of condition (9) is smaller than  $\frac{1}{2} - w$ .<sup>17</sup> Consider the exhaustion by compact sets of the open set  $S_{y,z}$  as an increasing sequence of compact sets  $S_{y,z}^j$  such that  $S_{y,z}^j \subset S_{y,z}^{j+1}$  with the limit of the sequence being  $S_{y,z}$ , where  $y, z \in \{a, b\}$ .<sup>18</sup>

Notice that because  $S_{y,z}^j \subseteq S_{y,z}$ , the inequalities characterizing each of the sets  $S_{y,z}$  hold  $\overline{ \begin{array}{l} & \overset{17}{} \text{Note that 8 is equivalent to } \varepsilon \mathbf{S}_{a,b}^{F} + (1-\varepsilon) \mathbf{S}_{b,a}^{F} + \mathbf{S}_{a,a}^{F} > \frac{1}{2}, \text{ because } \mathbf{S}_{a,b}^{F} + \mathbf{S}_{b,a}^{F} + \mathbf{S}_{b,a}^{F} + \mathbf{S}_{b,b}^{F} = 1 \text{ and similarly } \\ 9 \text{ is equivalent to } (1-\varepsilon) \mathbf{S}_{a,b}^{F} + \varepsilon \mathbf{S}_{b,a}^{F} + \mathbf{S}_{a,a}^{F} < \frac{1}{2}. \\ \overset{18}{} \text{That is, } \bigcup_{j} S_{y,z}^{j} = S_{y,z}. \end{array}$ 

within  $S_{y,z}^j$ . Let  $\{\sigma^{n'}\}$  be a sequence of equilibrium strategies. Notice that  $\Phi_{s_i}$  is continuous in  $k_i$ . Hence, the maximum and minimum of  $\Phi_{s_i}(k_i \in S_{y,z}^j)$  are always attained for  $k_i$  in  $S_{y,z}^j$ , because  $S_{y,z}^j$  are compact.<sup>19</sup>

It follows that if we fix j, there exists  $h_{y,z} > 0$  such that the inequalities defining set  $S_{y,z}$ hold with a margin of  $h_{y,z}$ . Given the uniform convergence of the probability of being pivotal (within the collection sequences of equilibrium strategies) to 0 it follows that there exists  $n_{x,y}^{j}$ such that for all  $n \ge n_{x,y}^{j}$  the difference between  $\Phi_{s_i}^{n}(\sigma_{-i}^{n'})$  and  $\Phi_{s_i}(\sigma_{-i}^{n'})$  is at most  $h_{z,y}/2$  for all  $k_i$  in  $S_{y,z}^{j}$ . Letting  $n^{j} = max\{n_{a,a}^{j}, n_{a,b}^{j}, n_{b,a}^{j}, n_{b,b}^{j}\}$  it follows that whenever  $n > n_{j}$  the inequalities defining the set  $S_{y,z}$  given in terms of  $\Phi_a$  and  $\Phi_b$  also hold for  $\Phi_{s_i}^{n}(\sigma_{-i}^{n'})$ . It must therefore be the case that for all such  $n, i \in \{1, ..., n\}, v_i(s_i = a) = y$  and  $v_i(s_i = b) = z$ whenever  $k_i \in S_{y,z}^{n}$ 

We end by noting, that because the limit of the sequence  $S_{y,z}^j$  is  $S_{y,z}$  there exists large enough  $j^*$  such that for  $j' > j^*$ 

$$\int_{S_{a,a}^{j'}} f(k_i) dk_i + \int_{S_{a,b}^{j'}} f(k_i) dk_i \varepsilon + \int_{S_{b,a}^{j'}} f(k_i) dk_i (1-\varepsilon) > \frac{1}{2} + \frac{w}{2}$$

$$\int_{S_{a,a}^{j'}} f(k_i) dk_i + \int_{S_{a,b}^{j'}} f(k_i) dk_i (1-\varepsilon) + \int_{S_{b,a}^{j'}} f(k_i) dk_i \varepsilon < \frac{1}{2} - \frac{u}{2}$$

But then conditional on  $\alpha$  the expected share of votes for option a for all  $n > n^{j'}$  given  $\sigma^{n'}$ must be greater than  $\frac{1}{2} + \frac{w}{2}$  and conditional on  $\beta$  must be smaller than  $\frac{1}{2} - \frac{w}{2}$ . By the law of large numbers,  $Z^n(\sigma^{n'}) \to (1,0)$  as  $n \to \infty$ . This completes the "if" part.

(the "only if" part) Suppose condition 8 does not hold in a strict sense. Pick  $w \in (0, 1)$ small enough that the left-hand-side of condition 8 is greater than  $\frac{1}{2} + w$ . Consider again the exhaustion by compact sets of the open set  $S_{y,z}$  as an increasing sequence of compact sets  $S_{y,z}^{j}$ such that  $S_{y,z}^{j} \subset S_{y,z}^{j+1}$  with the limit of the sequence being  $S_{y,z}$ , where  $y, z \in \{a, b\}$ .

Then, there exists large enough j, call it j', such that

$$\int_{S_{a,a}^{j'}} f(k_i) dk_i + \int_{S_{a,b}^{j'}} f(k_i) dk_i (1-\varepsilon) + \int_{S_{b,a}^{j'}} f(k_i) dk_i \varepsilon > \frac{1}{2} + \frac{w}{2}$$

As shown in the "if" part above optimal behavior for large enough n always entails  $v_i(s_i =$ 

<sup>&</sup>lt;sup>19</sup>In fact, the maximum and the minimum must be attained in the boundary of the set.

a) = y and  $v_i(s_i = b) = z$  whenever  $k_i \in S_{y,z}^{n'}$ . Thereby, in any equilibrium for large enough n we must have that the expected fraction of a votes conditional on  $\beta$  is greater than  $\frac{1}{2} + \frac{w}{2}$  and, by the law of large numbers,  $Z_{\beta}^n \to 1$  as  $n \to \infty$ . Similarly if condition 9 is violated in a strict sense, whereby for any equilibrium sequence we get  $Z_{\alpha}^n \to 0$  as  $n \to \infty$ .

#### Proof of Theorem 2:

Consider any pair  $(F,\pi)$ , where  $\int_S f(k_i)dk_i \neq \frac{1}{2}$ , with  $S = \{(k_i \in M : \pi k_i(a,\alpha) > (1 - \pi)k_i(b,\beta)\}$ , and  $\pi \in (0,1)$ . Note that if it is not the case that  $\int_S f(k_i)dk_i > \frac{1}{2}$  then it must be that  $\int_{S'} f(k_i)dk_i > \frac{1}{2}$  where  $S' = \{k_i \in M : \pi k_i(a,\alpha) < (1 - \pi)k_i(b,\beta)\}$ , the set of types for whom always voting for option b is optimal given a zero probability of being pivotal and  $\varepsilon$  set to one half.

First suppose that  $\int_{S} f(k_i)dk_i > \frac{1}{2}$ . Pick w > 0 small enough so that  $\int_{S} f(k_i)dk_i > \frac{1}{2} + w$ . By inspection of Expression 6 we see that both  $\Phi_a$  and  $\Phi_b$  converge uniformly to  $\pi k_i(a, \alpha) - (1 - \pi)k_i(b, \beta)$  as  $\varepsilon \to 1/2$ . Hence, the term  $\int_{S_{a,a}} f(k_i)dk_i$  converges uniformly to  $\int_{S} f(k_i)dk_i$  as  $\varepsilon \to 1/2$ .

Considering the exhaustion by compact sets of the open set  $S_{a,a}$  as an increasing sequence of compact sets  $S_{a,a}^{j}$  such that  $S_{a,a}^{j} \subset S_{a,a}^{j+1}$  with the limit of the sequence being  $S_{a,a}$  there exists large enough j, call it j', and  $\varepsilon$  close enough to 1/2, call it  $\overline{\varepsilon}$ , such that  $\int_{S_{a,a}^{j'}} f(k_i) dk_i > \frac{1}{2} + \frac{w}{2}$ . As shown in the "if" part of the proof of Theorem 1 we know that for any voter i and all large enough n, optimal behavior for types  $k_i \in S_{a,a}^{j'}$ , given  $\varepsilon \leq \overline{\varepsilon}$ , entail voting for option aunder both signals regardless of the strategies of others and thus for all large enough n the expected share of votes for a conditional on  $\alpha$  or  $\beta$  exceeds  $\frac{1}{2} + \frac{w}{2}$ . Hence, given any pair  $(F, \pi)$  such that  $\int_S f(k_i) dk_i > \frac{1}{2}$  there exists  $\overline{\varepsilon}$  such that for all  $\varepsilon \leq \overline{\varepsilon}$  any sequence of equilibria has  $\lim_{n\to\infty} Z^n = (1, 1)$ . By analogous reasoning, if  $(F, \pi)$  is such that  $\int_S f(k_i) dk_i < \frac{1}{2}$  there exists  $\overline{\varepsilon}$  such that for any  $\varepsilon \leq \overline{\varepsilon}$  all sequences of equilibria have  $\lim_{n\to\infty} Z^n = (0, 0)$ . In the non-generic case where  $(F, \pi)$  is such that  $\int_S f(k_i) dk_i = \frac{1}{2}$  our methodology does not allow us to solve the game asymptotically.

### A.4 Proofs Section 4.1

Proof of Proposition 6: (Case 1) First, note that Lemma 1 applies whereby the pivotal probability along any equilibrium sequence  $\{\sigma^n\}$  converges uniformly to zero as  $n \to \infty$ . In equilibrium, given any  $(F', \pi, \varepsilon, n)$ , there is a positive measure of types such that  $\Phi_{s_i}^n$  is strictly positive (negative) for both signal realizations. Hence, any player votes for option a with a probability that is bounded away from 0 and 1. To check this, suppose otherwise and thus either  $\bar{\delta}_{\alpha}, \bar{\delta}_{\beta}$  is close to zero or  $\bar{\delta}_{\alpha}, \bar{\delta}_{\beta}$  is close to one.<sup>20</sup> Then, we have a contradiction for any given  $(F', \pi, \varepsilon, n)$ and for any  $Pr(piv_i|\alpha), Pr(piv_i|\beta)$  as  $\Phi_b^n > 0$  for a positive measure of types, given  $\bar{\delta}_{\alpha}, \bar{\delta}_{\beta}$  close to zero, and  $\Phi_a^n < 0$  for a positive measure of types given  $\bar{\delta}_{\alpha}, \bar{\delta}_{\beta}$  close to one. We can then apply Lemma 1 whereby the pivotal probability along any equilibrium sequence  $\{\sigma^n\}$  converges uniformly to zero as  $n \to \infty$ .

Next, consider the set of types  $S' = \{t_i \in M' : \pi k_i(a, \alpha) - (1 - \pi)k_i(b, \beta) > 0\}$  and suppose  $\int_{S'} f'(t_i)dt_i > 1/2$ . The set S' contains types optimally voting for a given uninformative signals, zero pivotal probability, and  $\bar{\delta}_{\alpha}, \bar{\delta}_{\beta}$  set to one half. Pick  $w \in (0, 1)$  so that  $\int_{S'} f'(t_i)dt_i > 1/2 + w$ . Then there exists  $\delta'_{\beta} \in (\frac{1}{2}, 1)$  such that for all  $\delta''_{\beta} \in [0, \delta'_{\beta}]$  we have  $\int_{S''} f'(t_i)dt_i > 1/2 + \frac{w}{2}$ , with  $S'' = \{t_i \in M' : \pi k_i(a, \alpha)(1 - \delta''_{\beta}) - (1 - \pi)k_i(b, \beta)\delta''_{\beta} > 0\}$ . Since  $Pr(\alpha|b)$  is continuous in  $\varepsilon$  and  $Pr(\alpha|b) \to \pi$  as  $\varepsilon \to \frac{1}{2}$  and  $Pr(\beta|b)$  is continuous in  $\varepsilon$  and  $Pr(\beta|b) \to (1 - \pi)$  as  $\varepsilon \to \frac{1}{2}$  there exists  $\bar{\varepsilon} \in (\frac{1}{2}, 1)$  such that for all  $\varepsilon \leq \bar{\varepsilon}$  and for all  $\delta''_{\beta} \in [0, \delta'_{\beta}]$  we have  $\int_{S_{a,a}} f'(t_i)dt_i > 1/2 + \frac{w}{4}$ , where  $S_{a,a} = \{t_i \in M' : Pr(\alpha|b)k_i(a, \alpha)(1 - \delta''_{\beta}) - Pr(\beta|b)k_i(b, \beta)\delta''_{\beta} > 0\}$ . Note that in equilibrium  $\bar{\delta}_{\alpha} > \bar{\delta}_{\beta}$  as  $\Phi^n_a > \Phi^n_b$ . Taken together (including Lemma 1) we have that for any  $\varepsilon \leq \bar{\varepsilon}$  and sequence of equilibria  $\{\sigma^n\}$  and some  $z \in (0, 1)$  we must have  $\bar{\delta}_{\beta} > \frac{1}{2} + z$  for all large enough n and by the law of large numbers we have  $\lim_{n\to\infty} Z^n = (1, 1)$ . Note that at least one equilibrium exists for any n (see e.g. Balder (1988), Theorem 3.1.) and if, for all large enough n and  $\varepsilon \leq \bar{\varepsilon}$ , we would have  $\bar{\delta}_{\beta} < \frac{1}{2} + z$  we run into a contradiction since, as shown above, the expected fraction of a-votes in state  $\beta$  is larger, and bounded away from, one half.

Analogously if we suppose  $\int_{S'} f'(t_i) dt_i < 1/2$  then for any  $\varepsilon \leq \tilde{\varepsilon}$  and sequence of equilibria  $\{\sigma^n\}$  we have  $\lim_{n\to\infty} Z^n = (0,0)$ . Hence, for all  $\varepsilon \leq \min\{\bar{\varepsilon}, \tilde{\varepsilon}\}$  we have the desired result.

(Case 2) First, we show that for any convergent sub-sequence of equilibria  $\{\sigma^l\}$ , of any

<sup>&</sup>lt;sup>20</sup>Note that in equilibrium no type votes for a when signal b and for b when signal a as  $w_i(a, \alpha), w_i(b, \beta), k_i(a, \alpha), k_i(b, \beta) > 0$  whereby  $\Phi_a^n > \Phi_b^n$ .

sequence of equilibria  $\{\sigma^n\}$ , with  $\lim_{l\to\infty} Z^l \notin \{(0,0), (1,1)\}$  the pivotal probability converges uniformly to zero. Consider any sequence of equilibria  $\{\sigma^n\}$  and some small  $\epsilon > 0$ . For any n where  $\bar{\delta}_{\alpha}, \bar{\delta}_{\beta} > 0 + \epsilon$  or  $\bar{\delta}_{\alpha}, \bar{\delta}_{\beta} < 1 - \epsilon$  it follows that for any given  $(F', \pi, \varepsilon)$  and  $Pr(piv_i|\alpha), Pr(piv_i|\beta)$  there is a positive measure of types for which  $\Phi_b^n > 0$  and a positive measure of types for which  $\Phi_a^n < 0$ . Consider any sub-sequence,  $\{\sigma^l\}$ , of  $\{\sigma^n\}$  such that for all l we have  $\bar{\delta}_{\alpha}, \bar{\delta}_{\beta}$  bounded away from zero or  $\bar{\delta}_{\alpha}, \bar{\delta}_{\beta}$  bounded away from 1. It follows from the proof of Lemma 1 that the pivotal probability along such sub-sequence converges uniformly to zero.

Next, consider the set of types  $S' = \{t_i \in M' : \pi k_i(a, \alpha) - (1 - \pi)k_i(b, \beta) > 0\}$  and suppose  $\int_{S'} f'(t_i)dt_i > \frac{1}{2}$ . Note that S' contains types optimally voting for a given uninformative signals, zero pivotal probability, and  $\bar{\delta}_{\alpha}, \bar{\delta}_{\beta}$  set to one half. Pick  $w \in (0, 1)$  so that  $\int_{S'} f'(t_i)dt_i > \frac{1}{2} + w$ . Then there exists  $m \in (0, 1)$  such that for all  $\delta'_{\beta} \in [\frac{1}{2} - m, \frac{1}{2} + m]$  we have  $\int_{S''} f'(t_i)dt_i > \frac{1}{2} + \frac{w}{2}$  for  $S'' = \{t_i \in M' : \pi k_i(a, \alpha)\delta'_{\beta} - (1 - \pi)k_i(b, \beta)(1 - \delta'_{\beta}) > 0\}$ . Since  $Pr(\alpha|b)$  is continuous in  $\varepsilon$  and  $Pr(\alpha|b) \to \pi$  as  $\varepsilon \to \frac{1}{2}$  and  $Pr(\beta|b)$  is continuous in  $\varepsilon$  and  $Pr(\beta|b) \to (1 - \pi)$  as  $\varepsilon \to \frac{1}{2}$  there exists  $\bar{\varepsilon} \in (\frac{1}{2}, 1)$  such that for all  $\varepsilon \leq \bar{\varepsilon}$  and for all  $\delta'_{\beta} \in [\frac{1}{2} - m, \frac{1}{2} + m]$  we have  $\int_{S_{a,a}} f'(t_i)dt_i > \frac{1}{2} + \frac{w}{4}$  for  $S_{a,a} = \{t_i \in M' : Pr(\alpha|b)k_i(a, \alpha)\delta'_{\beta} - Pr(\beta|b)k_i(b, \beta)(1 - \delta'_{\beta}) > 0\}$ . Letting  $\varepsilon \leq \bar{\varepsilon}$ : (i) Consider any sub-sequence  $\{\sigma^l\}$ , of any sequence of equilibria  $\{\sigma^n\}$ , such that for each l we have  $\bar{\delta}_{\beta} \in [\frac{1}{2} - m, \frac{1}{2} + m]$ . Since  $\int_{S_{a,a}} f'(t_i)dt_i > \frac{1}{2} + \frac{w}{4}$  and noting that in equilibrium  $\bar{\delta}_{\alpha} > \bar{\delta}_{\beta}$  as  $\Phi^n_a > \Phi^n_b$  and that Lemma 1 applies, there exists l' such that for all  $l \geq l'$  we have  $\int_{S'_{a,a}} f'(t_i)dt_i > \frac{1}{2} + \frac{w}{8}$  for  $S'_{a,a} = \{t_i \in M' : Pr(\alpha|b)k_i(a, \alpha)\bar{\delta}_{\alpha} - Pr(\beta|b)k_i(b, \beta)(1 - \bar{\delta}_{\beta}) > 0\}$  and by the law of large numbers  $\lim_{l\to\infty} Z^l = (1, 1)$ .

(ii) For any sub-sequence  $\{\sigma^l\}$ , of any  $\{\sigma^n\}$ , such that for each l we have  $\bar{\delta}_{\beta} > \frac{1}{2} + m$  then surely  $\lim_{l\to\infty} Z^l = (1,1)$  by the law of large numbers and since  $\bar{\delta}_{\alpha} > \bar{\delta}_{\beta}$  in equilibrium.

(iii) Lastly, consider any sub-sequence  $\{\sigma^l\}$ , of any sequence of equilibria  $\{\sigma^n\}$ , such that for each l we have  $\bar{\delta}_{\beta} < \frac{1}{2} - m$ . Note that as  $\varepsilon \to \frac{1}{2}$  then  $\Phi^n_a \to \Phi^n_b$  and therefore  $\bar{\delta}_{\beta} \to \bar{\delta}_{\alpha}$ . Hence, there exists  $\bar{\varepsilon}' \leq \bar{\varepsilon}$  such that for all  $\varepsilon \leq \bar{\varepsilon}'$  we have that  $\bar{\delta}_{\alpha} < \frac{1}{2} - \frac{m}{2}$  and thus  $\lim_{l\to\infty} Z^l = (0,0)$ by the law of large numbers.

We then have that for all  $\varepsilon \leq \overline{\varepsilon}'$  every convergent sub-sequence of equilibria have  $\lim_{n\to\infty} Z^n \in \{(0,0), (1,1)\}$ . Note that at least one equilibrium exists for each n (see e.g. Balder (1988), Theorem 3.1.).

Analogously if we suppose  $\int_{S'} f'(t_i) dt_i < 1/2$  then every convergent sub-sequence of equilibria have  $\lim_{n\to\infty} Z^n \in \{(0,0), (1,1)\}$  for all  $\varepsilon \leq \overline{\varepsilon}''$ . Hence, for all  $\varepsilon \leq \min\{\overline{\varepsilon}', \overline{\varepsilon}''\}$  we have the desired result.

## A.5 Proofs Section 4.2

Proof of Proposition 7: Suppose  $\mathbf{S}_{a,b}^F > \mathbf{S}_{b,a}^F$ . Let  $A = \mathbf{S}_{a,a}^F + \varepsilon \mathbf{S}_{a,b}^F + (1-\varepsilon)\mathbf{S}_{b,a}^F$  and  $B = \mathbf{S}_{a,a}^F + (1-\varepsilon)\mathbf{S}_{a,b}^F + \varepsilon \mathbf{S}_{b,a}^F$ . Note that A > B and pick  $w \in (0,1)$  such that A > B + w. For any sequence of equilibria,  $\{\sigma^n\}$ , there exists n such that for all  $n \ge n'$  the expected share of votes for a in state  $\alpha$  is within the interval  $(A - \frac{w}{4}, A + \frac{w}{4})$  and the expected share of votes for a in state  $\beta$  is within the interval  $(B - \frac{w}{4}, B + \frac{w}{4})$ . This follows from Lemma 1 and the same arguments as in the proof of Theorem 1. By the law of large numbers we have that for any  $q \in (B + \frac{w}{3}, A - \frac{w}{3})$  every sequence of equilibria has  $\lim_{n\to\infty} Z^n = (1, 0)$ .