Cryptocurrencies in optimized portfolios: a portfolio theory approach

The portfolio performance-enhancing capabilities of cryptocurrencies

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Abstract

Since cryptocurrencies entered the investment scene, they have attracted substantial interest from investors, both retail and professional, governments and corporations. The foundation of this interest is twofold; first, cryptocurrencies differ from traditional investment assets in the way that they are not based on any tangible asset nor are associated with any government or monetary authority. The second foundation is that cryptocurrencies have since their surfacing had a monumentally high price appreciation and returns compared to other investment assets. However, the high returns have been combined with extreme volatility compared to other asset classes. These characteristics build the foundation of the research performed in this analysis which purpose is to add to the literature on cryptocurrencies as an investment asset by investigating the effect of adding a professionally computed index of cryptocurrencies to optimized portfolios of five asset classes and empirically testing whether portfolio performance can be enhanced when including the index in an out-of-sample setting.

A large theoretical foundation of portfolio theory exists, mostly built on Markowitz (1952), which relies on two parameters: expected return and variances. This analysis is performed across six portfolio construction methods as well as three levels of risk aversion, to make the results as robust and applicable as possible. The period studied is from March 2016 to December 2021 and performance is measured in terms of annualized excess returns, annualized Sharpe Ratio, and annualized Sortino Ratio of the portfolios with and without an investment in cryptocurrencies. Five optimization techniques are employed: The Global Minimum Variance portfolio, the Tangency portfolio, the Optimal portfolio, the Bayes-Stein Optimal portfolio, the Risk Parity portfolio, as well as the Equally Weighted portfolio.

The results show strong evidence for cryptocurrencies´ ability to enhance portfolio performance as all performance metrics considered increase when including an investment in cryptocurrencies across all portfolios constructed. On average, annualized excess returns increase by 69.87%, the annualized Sharpe Ratio increases by 31.44%, and the annualized Sortino Ratio increases by 56.73% across all portfolios. While these results point to a clear enhancement of portfolio performance, it is recommended that the study is revisited when more data is available for the returns of cryptocurrencies.

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1 Introduction

Ever since the establishment of financial markets, investors have sought ways to achieve returns on their investments. The way investors achieve returns differs significantly among them; some prefer investing in riskier assets in the hope that it will yield a significant return, while others seek investments in safer assets, not wanting to take on a high risk, but as a result, are more likely to realize a lower return. Financial markets have evolved significantly from their establishment, believed to have been in the 17th century (Atack & Neal, 2009). Not only the instruments traded have changed, but also the way they are traded.

The changes and forward developments in financial markets and intermediaries have been exponential in the last decades. The central theme of the late 20th century was the deregulation of financial markets, setting the scene for unprecedented growth in market activity and ultimately the global financial crash of 2008. For example, subprime lending, collateralized debt obligations (CDO), and collateralized loan obligations (CLO) were all instruments that were central to the global financial crash and were issued by banks for the first time in 2005 (Cresci, 2005). This illustrates how rapidly the environment in financial markets can change with developments and progression. Another development that has substantially affected financial markets in recent years is the rise of electronic trading. Gone are the days of outcries in a trading pit. Today, computers trade complex instruments in a matter of seconds, screening the universe of thousands of financial instruments contemporaneously (Taylor, 2018).

The securities that are traded in the market have also evolved. In the beginning, financial markets mainly traded government bonds. The equity market expanded rapidly in the 19th century with globalization of markets as capital flowed without restrictions between countries. Commodities, derivatives, and other more complex instruments soon followed (Taylor, 2018). Then, in the last years a new asset class surfaced, an asset class based on exotic technology and ideas, cryptocurrencies.

In 2008 Bitcoin was introduced in a whitepaper by the unknown individual or a group, Satoshi Nakamoto. Nakamoto (2008) sought to develop a digital currency that was not based on any tangible asset and was not associated with any government or monetary authority. The size and popularity of Bitcoin has increased greatly since its introduction and thousands of additional cryptocurrencies have been developed.

Bitcoin and other cryptocurrencies, commonly referred to as altcoins, have gained rapid growth and media coverage in recent years. Many investors have been interested in this alternative financial asset that could provide high returns, especially given the financial and economic conditions, the “new normal”, after the financial crisis of 2007-2008 and the aftermath of the 2008-2012 global recession. Under the “new normal” era investors desire new alternative assets that yield excess returns, which potentially supports and reinforces upward pressure on Bitcoin price (Hong, 2017). Bitcoin was initially developed as an alternative currency; however, it is difficult to overlook the amount of Bitcoin and altcoins used as investments. Academics argue that majority of Bitcoins is held by investors and since the supply of Bitcoin is predetermined it is more likely that the asset will be used as an investment asset than as a medium of exchange (Baur et al., 2015).

The way investors invest in financial instruments has also evolved alongside the financial market’s development. The idea of minimizing risks while achieving acceptable returns has been around for a long time but was theorized in the twentieth century. Traditional portfolio theory focused on analysing individual securities, while the perspective of a portfolio of securities was largely ignored (Lekovic, 2021). Modern portfolio theory was born with the publishing of the ground-breaking paper by Markowitz, titled Portfolio Selection, in 1952. The article was the first to mathematically optimize the relationship between expected return and the risk associated with that return in a portfolio setting. Central to Markowitz’s theory is that investors structure their portfolios based on the first and second moments of probability distribution: means (expectation) and variances. Further, the risk associated with each security is not the most important, rather the covariance between the securities and their contribution to the risk of the portfolio (Lekovic, 2021).

Since cryptocurrencies entered the investment scene, they have attracted interest from investors, individuals, corporations, and governments. Cryptocurrencies share features with both commodities and fiat currencies; however, fiat money is usually used as a medium of exchange. Further, both types can be used as a store of value and thus an investment (Baur et al., 2015). The literature on cryptocurrencies as an investment asset has rapidly grown alongside the increased interest and coverage. In the literature, cryptocurrencies are frequently compared to gold in terms of their hedging and safe haven potential (Dyhrberg, 2016; Klein et al., 2018; Shahzad et al., 2019). Numerous studies investigate the ability of Bitcoin to act as hedge and safe haven assets for example Bouri et al. (2017a), Bouri et al. (2017b), Demir et al.

(2018), Wu et al. (2019) and the potential of a wider range of cryptocurrencies to act as a hedge or a safe haven like Wang et al. (2019), Bouri et al. (2020b) and Colon et al. (2021), as historically the correlation between cryptocurrencies and other asset classes has been low or negative (Aslandis, Bariviera & Martinez, 2019).

Due to Bitcoin's characteristics, high returns and volatility, academics have begun investigating the effect of adding Bitcoin to optimized portfolios, for example Wu and Pandey (2014), Briere, Oosterlinck and Szafarz (2015), Eisl, Gasser and Weinmayer (2015), Gangwal (2017), Agarwal et al. (2018), Kajtazi and Moro (2019), Platanikis and Urquhart (2020), and Bakry et al. (2021). These studies are unanimous regarding the ability of Bitcoin to improve risk adjusted excess returns, looking at the Sharpe Ratio, Sortino Ratio, or the Risk-Return Ratio.

The development in financial markets, the growth, and media exposure of cryptocurrencies in recent years, the attention the asset class has gained in the literature and in practice as an investment asset, cryptocurrencies´ characteristics, their very high return and volatility are the focal considerations in the problem identification in this thesis. The young literature on cryptocurrencies as an alternative investment in optimized portfolios has mainly focused on adding Bitcoin to portfolios of different assets. For that reason, the effect of adding a wider range of cryptocurrencies to optimized portfolios will be analysed in this thesis. Investigating the findings of the existing literature and further extending it this analysis aims to find evidence for or against the following research question:

Does an investment in cryptocurrencies enhance the performance of optimized portfolios?

Three hypotheses are proposed with the aim to provide an answer to the research question. This analysis takes a different approach by including a professionally computed cryptocurrency index, the Royalton CRIX Index, that represents the total market return characteristics, using the portfolio optimization techniques to find evidence for or against the asset´s ability to enhance the performance of portfolios. The research combines the latest developments in financial instruments with older theory, generating insights into whether cryptocurrencies are a feasible investment choice to have in one's portfolio.

The period under investigation in this analysis is the 1st of March 2016 to the 31st of December 2021. The analysis mainly follows Markowitz's portfolio framework and more recent theoretical developments, using a back-test of optimized portfolios. Five optimization techniques are used and an Equally Weighted portfolio, as well as three risk aversion coefficients. Two portfolios are constructed for each approach, a portfolio with an investment in cryptocurrencies and a portfolio without an investment in cryptocurrencies, making it 20 portfolios in total, 10 with cryptocurrencies and 10 without cryptocurrencies. The study is out of sample; only data that was available to the investor at the time of portfolio construction or rebalancing is used. Using an out of sample method to construct the portfolios yields realistic and attainable portfolio weights. In an in-sample analysis, one uses data and information unavailable to the investor at the time of portfolio construction. The performance of the portfolios constructed with and without an investment in cryptocurrencies is compared using standard portfolio performance measures.

1.1 Delimitations

The analysis is delimited in some aspects to enable an in-depth analysis of the topic. First, a professionally computed index, the Royalton CRIX Index, is used as an approximation for an investment in cryptocurrencies. There are thousands of cryptocurrencies available to invest in, however the CRIX Index tracks the total market index of cryptocurrencies, consisting of a minimum of five liquid cryptocurrencies, which at the time of writing are nine in total. The index relies on a dynamic approach which determines the optimal number of cryptocurrencies quarterly to ensure that it represents the total market return characteristics (Royalton CRIX Crypto Index, 2022). Therefore, this index is deemed sufficient to represent the cryptocurrency market much like the S&P500 index is often thought of as representative of the equity market. However, results cannot be fully generalized for all cryptocurrencies.

Second, transaction costs are ignored throughout the analysis. That means that returns can be overstated, as usually there are some transaction costs associated with investing in a financial instrument. The transaction costs are generally higher for illiquid securities, but in this analysis, the securities used to construct the portfolios are very liquid and have minimal transaction costs as they are all indices. By the same token, it is assumed that investors can observe security prices at the open on the first day of each month and trade on them contemporaneously. This can lead to overstated results, but from the perspective of comparing the performance of optimal portfolios, it does not change the results as the same goes for both portfolios.

Third, the analysis is performed on a pre-specified time horizon, March 2016 to December 2021. This is due to the availability of the CRIX Index values. However, during the period financial markets were far from calm. Asset prices, in particular equities fell sharply during the outbreak of the COVID-19 pandemic in early 2020, as well as experiencing a volatile period in 2018, before periods of high positive returns in both cases. The time horizon includes 36 months of active portfolio rebalancing, as the first three years, 2016, 2017, and 2018 are only used to estimate initial portfolios. However, results are not necessarily applicable to all periods.

Lastly, the field of portfolio theory is vast. The chosen methods are among the most widely used in academia and practice. The underlying theory is based on Markowitz's portfolio theory but also considers some developments made to the theory. These methods are thus deemed a good representation of portfolio theory in the research.

In the remainder of this thesis, cryptocurrencies are considered an investment asset instead of medium of exchange.

1.2 Research Structure and Approach

The research approach conducted in this thesis is of deductive nature. A deductive research approach revolves around developing hypotheses based on previous theories and researchers' views before applying a research strategy to test the hypotheses. The support for or against the hypotheses based on the research conducted leads to an answer to the research question proposed in section 1, Introduction. The methodology applied in the research has deep theoretical roots within portfolio theory. This allows for rigorous analysis of the subject, as well as strengthening the hypotheses, as they are answered with numerical facts and thus leave less room for biases.

The first section of this thesis, Introduction, sets the scene for the analysis and the motivation for the research performed in this thesis. Section 2 further lays the foundation of the analysis by giving insights into the theory of asset allocation and diversification, followed by a detailed introduction to cryptocurrencies. Section 3 gives a comprehensive overview of the existing literature on cryptocurrencies, emphasizing cryptocurrencies as an investment to enhance portfolio performance. Results of past literature are discussed and compared. Section 4 outlines the hypotheses to be tested to find evidence for or against the research question. Section 5 describes the methodological approach in detail as well as describing the data used in the research. Section 6 presents the empirical results of the analysis. Thereupon, in section 7, a detailed discussion is set forward about the empirical findings, and they are compared to results of previous literature. Section 8 concludes the research with a short overview of the main findings and possibilities for further research. A visualization of the thesis outline can be seen in figure 1.

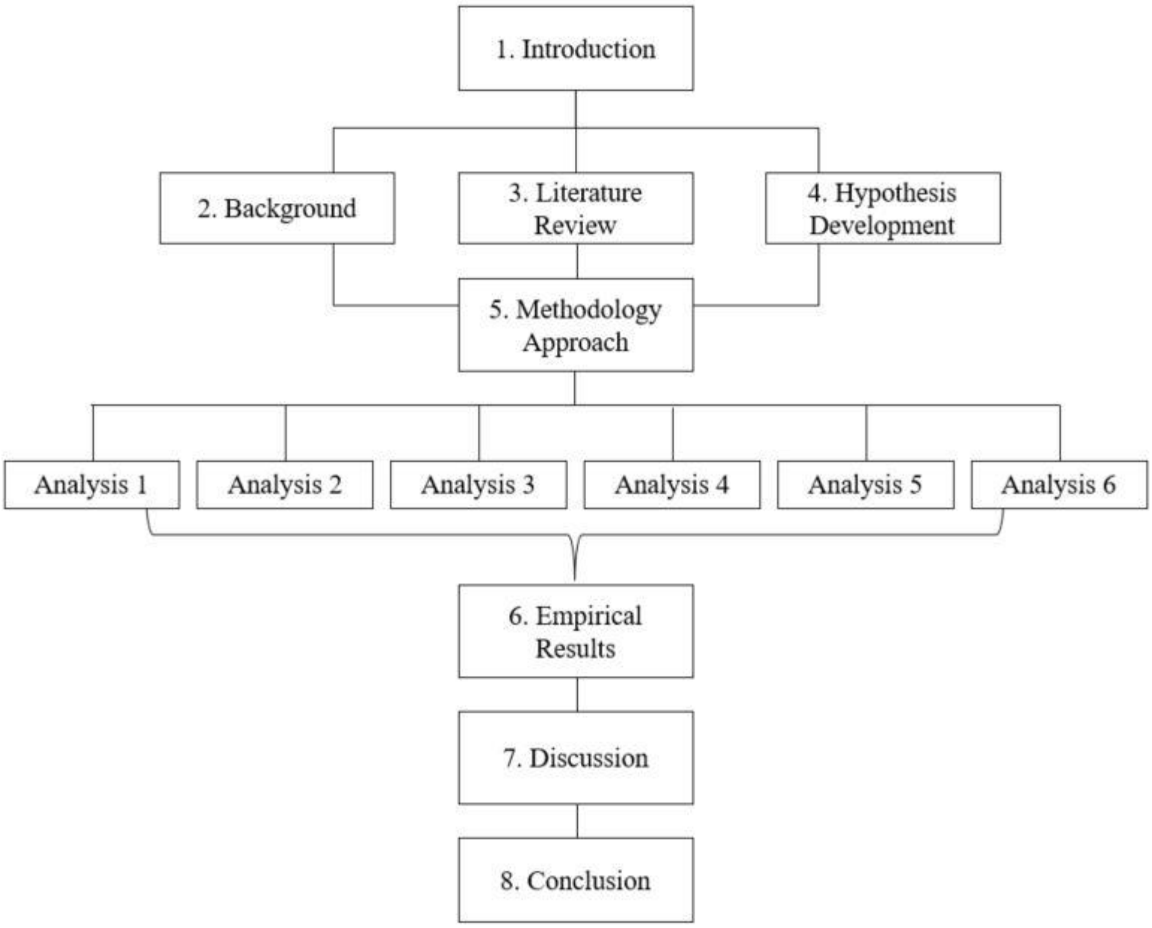

Figure 1: Thesis outline

2 Background

2.1 Asset allocation and Diversification

2.1.1 Asset Allocation

Investment professionals, individual investors, and others who invest in the financial market take different approaches to invest their money. Investors have different views on risk, the degree of risk aversion, and different opinions on what asset classes are favorable or unfavorable at a given time. According to Brinson, Hood, and Beebower (1986) portfolio design consists of four steps. The first step is deciding what asset classes an investor wants to include in a portfolio and what asset classes to exclude. Then, the investor needs to decide on a benchmark weight, what proportion of his portfolio is invested in each asset class on average over the long term. Third, to alter the long-term average weights to capture returns from short-term fluctuation in the market. Fourth, select securities within an asset class to capture returns that are more significant relative to that asset class as a whole (Brinson et al., 1986).

Having a long-term average weight for asset classes is called strategic asset allocation. The long-term average is decided with the investor's goals in mind; for example, a pension fund has a larger long–term average weight on bonds compared to more risky funds such as mutual funds. The methods used to decide the long-term weights of asset classes in a portfolio are risk-based asset allocation, passive asset allocation, and liquidity-based asset allocation. Historically, passive asset allocation has been a popular choice among individual and value investors, investing in the market in broad asset classes but not rebalancing or trading other than buying new securities when listed (Pedersen, 2015)

When investors have opinions about the short-term properties of a given asset class, the strategic asset allocation weights can change to attempt to capture protentional return in a market. Such deviations from the long-term weights are called tactical asset allocation. An example could be a hedge fund that usually is market neutral. Still, analysts discover short-term opportunities in the equity market, and the fund decides to take more long than short positions for a short time before returning to the long-term weights (Pedersen, 2015).

2.1.2 Diversification

Generally, the level of risk in a portfolio depends on whether securities have isolated risks or if they have common risks. The former can be minimized with diversification, but the latter often remains. Unsystematic risk which is specific to a company or an industry can, for example, be bad news coming out about the company or industry, worse than expected earnings, or an external shock to demand for the products of an industry (Berk, DeMarzo & Harford, 2015).

On the other hand, systematic risk concerns the whole economy and thus is not company or sector specific. An example of systematic risk is the monetary policy or overall economic movements. Systematic risk can not be diversified away with diversification, as it affects all assets (Berk, DeMarzo & Harford, 2015).

Combining many assets classes and securities from different sectors and issuers creates diversification. On average, unsystematic risk averages out; for example, bad news for one company can be good news for another company or industry. A standard method to measure this effect is to look at historical correlations of securities. Preferably, an investor would want to hold a portfolio of uncorrelated securities. Holding a portfolio of uncorrelated securities means that it is more likely that more unsystematic risk can be diversified away, as opposed to when holding correlated securities, such as investing in equities of many airline companies. In that case, news about movements in oil prices would affect them all – leaving industry-specific risk in the portfolio (Berk, DeMarzo & Harford, 2015).

Figure 2 shows how the risk of a portfolio, measured in standard deviation, decreases when investing in many different equities, and shows how the systematic risk remains constant.

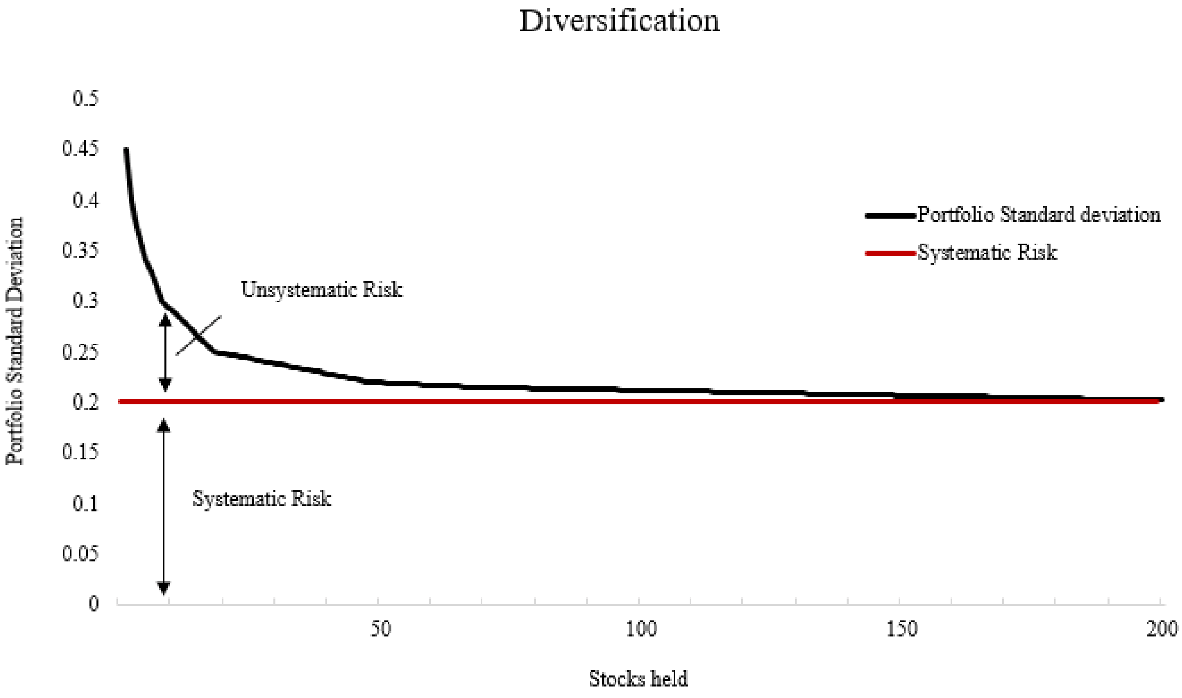

Figure 2: Hypothetical diversification

2.2 Cryptocurrencies

2.2.1 Introduction to Cryptocurrencies

Although the concept of electronic currency dates back to the late 1980s, Bitcoin, launched in 2009 is the first successful decentralized cryptocurrency (Farell, 2015). In 2008, the anonymous individual or group Satoshi Nakamoto published a whitepaper titled "Bitcoin: A Peer-to-Peer Electronic Cash System," which introduces a solution to overcome the weaknesses of the traditional trust-based model, where financial institutions serve as a trusted third party to process electronic payments. The weaknesses of the trust-based model are inherent in high transaction costs in terms of mediation, completely non-reversible transactions are not feasible, small casual transactions are limited, and a certain amount of fraud is taken as given. These problems could be solved using physical currency, however, with digital currency, there is a danger that someone can spend the same coin more than once. Thus, Nakamoto sought to develop a digital currency that removed any trusted central authority completely and replace it with cryptographic proof. This system, based on cryptographic proof, would solve the double-spending problem using a peer-to-peer distributed timestamp server to generate a computational confirmation of the chronological order of transactions (Nakamoto, 2008). Specifically, the Bitcoin software encrypts each transaction, the sender and the receiver are identified by a string of numbers, but a public record of every coin's movement is published across the entire network. Buyers and sellers remain anonymous, but everyone can see that a coin has moved from one party to another (Davis, 2011). The system keeps records of ownership and transaction timestamps, eliminating the possibility of digital copying and thus, double spending (Farell, 2015).

Bitcoin is fundamentally a chain of digital signatures where "each owner transfers the coin to the next by digitally signing a hash of the previous transaction and the public key of the next owner and adding these to the end of the coin" so that the ownership can be programmed into the coin (Nakamoto, 2008). These lines of computer code are stored in a program called "wallet" on personal hard drives and/or via online wallets (Farell, 2015). A transaction is only complete and added to the blockchain when a certain amount of computational power is used (Nakamoto, 2008). At this point, the transaction is considered complete, and the ownership of the coin has been transferred, without the risk of double spending, because the entire network is informed of which wallet the coin is located (Farell, 2015).

The system is protected from cyber attackers as long as the majority of the computational power is controlled by honest nodes (miners). The honest chain will grow faster and outpace the attacker. Miners are incentivized to mine and stay honest. The Bitcoin protocol offers rewards in two forms: the miner gains ownership of the mined coin, and in the long run, when the number of coins in circulation has reached 21 million, the incentive transitions to transaction fees and is completely inflation free. Additionally, this incentivizes minors to support the network and provides a way to create more coins, since there is no central authority to issue them. Under the unlikely circumstances that an attacker can assemble more computational power than all the honest minors, he ought to find it more profitable to play by the rules, such rules that favour him with more coins (Nakamoto, 2008).

Blockchain is the core technology of many cryptocurrencies and it first appeared in Nakamoto's paper when Bitcoin took the blockchain technology to the extreme. After that, many cryptocurrencies based on blockchain have been developed (Zhang & Lee, 2020). Unlike traditional currencies, cryptocurrencies are not issued or backed by any central authority and their value is not based on any tangible asset like commodities, nor to any economy like conventional currencies (Corbet et al., 2019; Plantankis et al., 2020; Tzoucanas et al., 2020). There is no legal entity responsible for the activities, and therefore cryptocurrencies fall out of the traditional regulation of fiat currencies (Gangwal, 2017).

2.2.2 Historical Development of Cryptocurrencies

Following Nakamoto's paper in 2008, Bitcoin was introduced to the public on January 3rd, 2009. Until February 2011, it traded for less than 1 US dollar (Farell, 2015). The size and popularity of this peer-to-peer system has increased extremely since its creation and thousands of additional cryptocurrencies have been developed (Today's Cryptocurrency Prices by Market Cap, 2022). The second cryptocurrency, Namecoin emerged two years later in April 2011 (Hileman & Rauchs, 2017). In the fall of 2011, Litecoin was released, which soon became the cryptocurrency with the highest market capitalization after Bitcoin until it was overtaken by Ripple in the fall of 2014. The idea with Litecoin was that it would be more appropriate for day-to-day transactions, modifying Bitcoin's protocol and increasing transaction speed. Ripple, created in 2013, introduced a different model to that used by Bitcoin and had the second-highest market capitalization in April 2015 (Farell, 2015). What these cryptocurrencies have in common is the public ledger (blockchain) that is shared between network participants and the use of native tokens as a way to incentivize participants for running the network in the absence of a central authority. Most altcoins use technology identical to the Bitcoin technology and simply feature different parameter values, for example different block time, currency supply and issuance scheme (Hileman & Rauchs, 2017).

At the moment of writing, the cryptocurrency industry consists of over 9,000 coins with varying user bases and trading volumes. Bitcoin currently has the highest market capitalization, and Ethereum, Tether, BNB, USD Coin, and XRP follow (Today's Cryptocurrency Prices by Market Cap, 2022). Because of its high volatility, the market capitalization of the cryptocurrency industry changes drastically but is estimated at the time of writing to be just over 2.0 trillion (Global Cryptocurrency Charts, 2022). For the last year, the total cryptocurrency market capitalization has fluctuated considerably as illustrated in Figure 3.

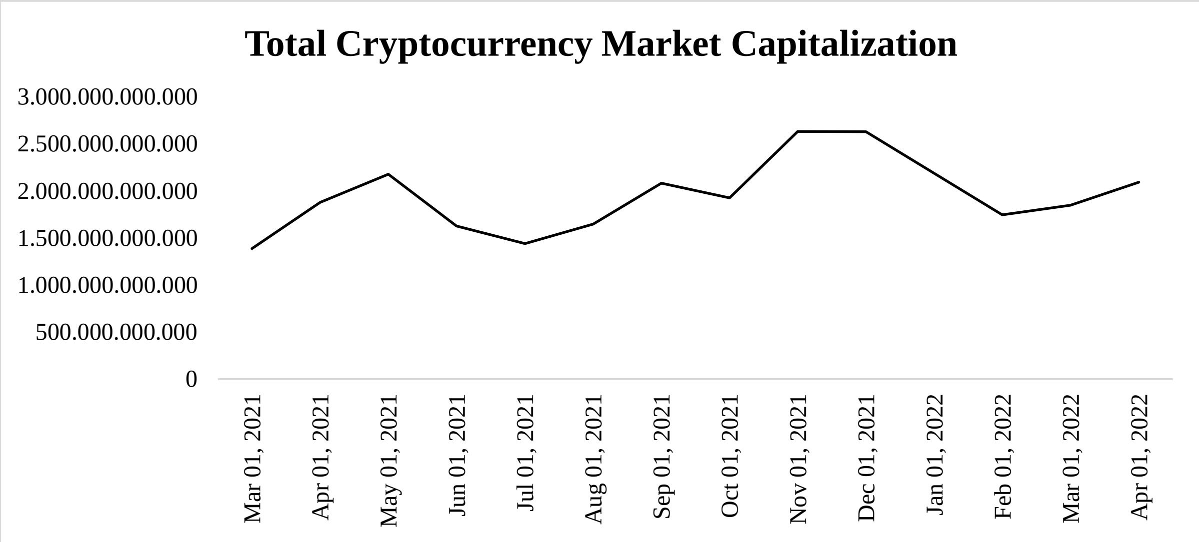

Figure 3: Total Cryptocurrency Market Capitalization (Global Cryptocurrency Charts, 2022)

2.2.3 Cryptocurrency as a Medium of Exchange or an Investment Asset

Bitcoin is originally developed as an online peer-to-peer payment system in which users can transact directly without a financial intermediary. This definition suggests that it is mainly used as an alternative currency. However, academics have contrasting views about the usage of Bitcoin which implies that it does not meet its original purpose. Some argue that merchants are incentivized to accept it as a form of payment due to lower fees than imposed by credit card processors while others argue that it is not used broadly in retail transactions (Hong, 2017). Baur et al. (2015) argue that Bitcoin is mainly used as an investment and that majority of Bitcoins are held by investors with the aim of capturing returns through price appreciation. Although Nakamoto (2008) defined Bitcoin as an alternative currency and a peer-to-peer electronic cash system, it is difficult to overlook the amount of Bitcoin used as an investment. Already in late 2013, both individuals and institutional investors started to invest in Bitcoins. Many investors have been interested in this alternative financial asset that could provide higher returns, especially given the financial and economic conditions, the “new normal”, after the financial crisis of 2007-2008 and the aftermath of the 2008-2012 global recession. Under the “new normal” era investors desire new alternative assets that yield excess returns, which potentially supports and reinforces upward pressure on Bitcoin price (Hong, 2017). Bitcoin shares features with both commodities such as gold and fiat currencies such as the US dollar. Whilst commodities have other use than being a medium of exchange, fiat money is usually used as a medium of exchange. Additionally, both types can be used as a store of value and thus as an investment. Since the supply of Bitcoins is predetermined and is expected to increase until 2040 and remain at that level, potential deflationary effects may emerge. These effects make it more likely that Bitcoins will be used as an investment asset than as a medium of exchange (Baur et al., 2015). As stated in section 1.1, Delimitations, cryptocurrencies are considered an investment asset in this thesis.

2.2.4 Bitcoin, Ethereum and BNB

The four largest cryptocurrencies by market capitalization are Bitcoin, Ethereum, Tether, and BNB coin. In this section Bitcoin, Ethereum, and BNB will be elaborated upon. Tether is a stable coin and will be discussed further in the following section.

Each day, these cryptocurrencies are traded at numerous exchanges around the world, where the top exchanges are for example Binance, OKX, and Coinflex. Bitcoin is the largest cryptocurrency by market capitalization, equal to $829 billion at the time of writing. The current supply of Bitcoin is 18.95 million, but the supply of Bitcoin is predetermined to be capped at 21 million (Bitcoin, 2022). Bitcoin's price reached its all-time high in November 2021 when the price went over $68,000 (Bitcoin, 2022; Kollewe, 2021). Ethereum was first described in a whitepaper by Vitalik Buterin in 2013 and is the second-largest cryptocurrency by market capitalization, currently $366 billion. Therefore, Bitcoin is almost 2.3 times larger than Ethereum in terms of market capitalization. Ethereum has an unlimited supply, and its current supply is 119 million coins (Ethereum, 2022). BNB coin is a dedicated utility token of the Binance exchange. Binance was launched in 2017 and is currently the biggest cryptocurrency exchange globally based on daily trading volume. BNB coin's market capitalization is $70 billion, and its circulating supply is 163 million, but the supply limit is 165 million (BNB, 2022). Figure 4 illustrates the log prices of Bitcoin, Ethereum and BNB coin. The Bitcoin log price starts in October 2014, the Ethereum log price in August 2015 and BNB log price in July 2017. Log prices for all assets are illustrated until April 2022.

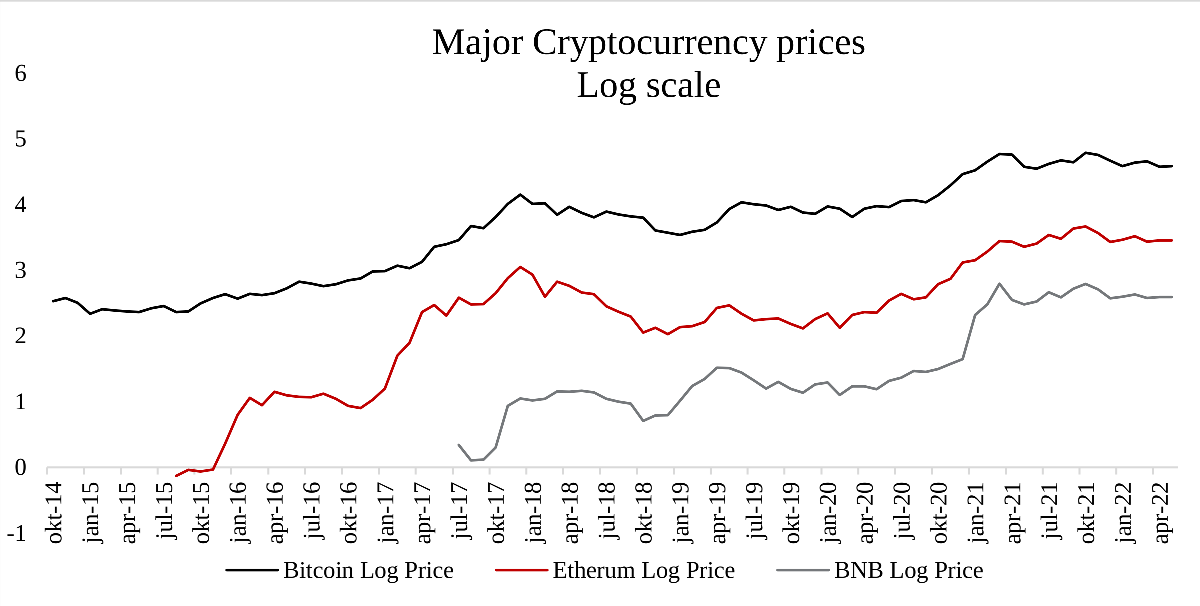

Figure 4: Major Cryptocurrency Prices on a Log Scale

2.2.5 Stable Coins

Despite the fascinating technology behind Bitcoin, there are some challenges, including its extreme price volatility. Stable coins are crypto assets that are developed with the aim of minimizing price volatility by including a stability mechanism. As previously mentioned, the supply of Bitcoins is predetermined and converges to 21 million units, thereafter no additional coins are mined. This aggregate supply schedule meets a constantly changing aggregate demand driven by changing expectations. This results in considerable price volatility. On the contrary, all central banks in developed countries seek to stabilize the value of their currencies by providing “elastic currency” to mitigate the price fluctuations and adjusting the aggregate supply of money to a changing aggregate demand. This mechanism is absent in the current Bitcoin protocol, and therefore it displays much higher short-term price fluctuations than many government-backed fiat currencies (Berentsen & Schär, 2018; Berentsen & Schär, 2019).

Stable coins track traditional fiat currencies, like the US dollar, the Euro, or the Japanese yen, and are commonly used by crypto investors who aim to avoid the severe volatility of other cryptocurrencies (Frankenfield, 2022). They can be pegged to other assets as well, like gold or a basket of goods. The first stable coin was developed in 2014. The market capitalization of all stable coins was approximately $2.7 billion at the beginning of 2019, which was 2.2% of the total market capitalization of all crypto assets (Berentsen & Schär, 2019). Tether is a blockchain-based cryptocurrency whose tokens in circulation are backed by a corresponding amount of U.S. dollars, making it a stable coin with a price pegged to USD $1.00 (Frankenfield, 2022). At the time of this paper, Tether is the largest stable coin token by market capitalization, with a market capitalization of around $78 billion, and the third-largest cryptocurrency by market capitalization (Tether, 2022). As can be seen on figure 5, Tether's price is fairly stable around a price of 1.

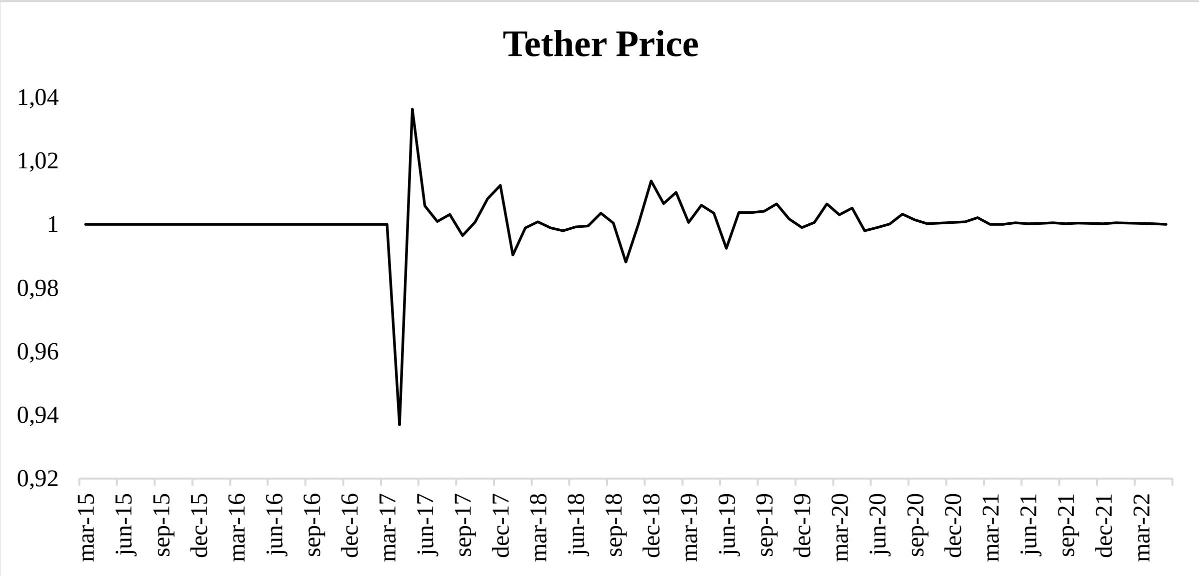

Figure 5: Tether Price

2.2.6 The Value of Cryptocurrencies

Speculation on how cryptocurrencies derive their value has been a frequent topic in recent years, and academics have conflicting views on the matter. Several factors have been proposed to predict cryptocurrency returns and drivers of cryptocurrency prices, specifically network and production factors (Liu & Tsyvinski, 2021). As suggested by the literature, Liu & Tsyvisnki (2021) show that cryptocurrency returns strongly respond to the cryptocurrency network factors.

Cryptocurrencies serve as a membership to a platform where a pool of users uses a decentralized platform by using blockchain technology. Therefore, the network effect of user adoption can play a central role in the valuation of cryptocurrencies (Pagnotta & Buraschi, 2018; Biais et al., 2020; Sockin & Xiong, 2020; Cong, Li & Wang, 2021). These factors measure the network effect of user adoption and are essential drivers of cryptocurrency prices

(Liu & Tsyvisnki, 2021). The transactions made within the platform are difficult to make outside of it. Therefore, the platform´s value lies with its design in filling the users´ transaction needs and in its potential to pool together a considerable number of users with the need to transact with each other. Users can make transactions with one another if they belong to the platform. With this, the network effect comes into play, which indicates that each user´s desire to join the platform grows with the number of other users. If more users join the platform, each user benefits more from joining the platform and is therefore willing to adopt a lower participation threshold and pay a higher price (Sockin & Xiong, 2020).

According to several papers, the costs of mining are essential for the infrastructure and security of cryptocurrency (Abadi & Brunnermeier, 2018; Sockin & Xiong, 2019; Cong, He & Li, 2021; (Liu & Tsyvisnki, 2021). To mine cryptocurrencies, two things are required; electricity and computer power or production factors (Liu & Tsyvinski, 2020). Some argue that the prices of cryptocurrencies are closely associated with the marginal cost of mining (Shickin & Xiong, 2019). However, Liu & Tsyvinski (2020) showed that cryptocurrency market returns are not significantly exposed to most production factors, using proxies for electricity cost and computer power.

The nature of cryptocurrencies is debated in the literature. Some argue that the prices of cryptocurrencies are linked to that of fiat currencies (Liu & Tsyvisnki, 2021). Nonetheless, Liu & Tsyvisnki (2021) argue that there is no evidence of systematic currency exposures in cryptocurrencies. It has further been proposed that cryptocurrencies are "digital gold" and serve the purpose of a traditional precious metal commodity. If cryptocurrency investors hold this assumption, it can be expected that the returns of cryptocurrencies will move in the same manner as the returns of traditional precious metal commodities. However, Liu & Tsyvisnki (2021) found no evidence of systematic precious metal commodity exposures in cryptocurrencies.

3 Literature Review

Research on cryptocurrencies as an investment asset is relatively young compared to other aspects of financial literature, as cryptocurrencies entered the scene in the first decade of the 21st century. However, the characteristics of cryptocurrencies, particularly the high returns and high volatility compared to other asset classes, as well as extensive coverage of cryptocurrencies has attracted the attention of academics, growing the literature. Majority of existing literature on cryptocurrencies as an investment asset revolves around two principles; adding cryptocurrencies to optimized portfolios and investigating the effect it has on portfolio performance, and research on the hedging and safe haven properties of cryptocurrencies due to the correlation of cryptocurrencies with other assets. The intrinsic value of cryptocurrencies, the pricing dynamics of cryptocurrencies and cryptocurrencies as an investment asset or a medium of exchange is another strand of literature. These aspects are discussed in Section 2, Background. As Bitcoin is the largest and most recognized cryptocurrency, most of the existing literature considers Bitcoin.

This section is split into two parts, where two themes of literature is discussed. First, literature on cryptocurrencies and its effect on portfolio performance is presented, where the majority of research focuses on adding Bitcoin to optimized portfolios. Second, literature on cryptocurrencies to serve as a hedge or a safe haven asset is presented.

These perspectives, the high return and high volatility of cryptocurrencies, as well as the asset´s potential to serve as a hedge and a safe haven, give interesting insights into the role of cryptocurrencies within a portfolio. The two concepts are interlinked, as a good hedging asset should by definition improve the performance of a portfolio.

Cryptocurrencies are, as previously mentioned, a young phenomenon. Because of that fact, the literature surrounding them is at times conflicting, and sensitive to what time periods are investigated. In this literature review, the goal is to shed light on the existing literature relevant for this analysis, comparing the results and methods of researchers to this day. The section also lays the ground for the research of this thesis, both in terms of methodology and in terms of research ideas and approach to the research.

To ensure the quality of the reviewed literature and to conduct it in the most prudent manner, most of the articles were either published in the major financial and economics journals, meaning that they had to obtain external approval from third parties, or are peer reviewed and often cited in others work. Emphasis is placed on ensuring adequate quality of the literature underlying the topic of the research, as it is fundamental in producing quality research.

3.1 Cryptocurrencies and Portfolio Performance

This section gives an overview of the literature on adding cryptocurrencies to optimized portfolios, especially looking at the risk-return profile of portfolios and comparing portfolios with an investment in cryptocurrencies and without an investment in cryptocurrencies. Most of the studies use optimization techniques to optimize the risk-return profile, first presented by Markowitz (1952). Markowitz's portfolio theory is centred around the risk-return profile of portfolios, constructing portfolios that maximize returns for a level of volatility, as well as minimizing volatility for a given level of return. The risk-return profile can be measured with the Sharpe Ratio (Sharpe, 1966). Using the Sharpe Ratio to compare optimized portfolios allows for a direct comparison of the performance of the portfolios on a risk-adjusted basis.

Two reasons generally attract the attention of researchers to the investment characteristics of cryptocurrencies; cryptocurrencies usually do not have an association with any authority or monetary policy, and to a greater extent in a portfolio construction context; they have shown extraordinary returns paired with exceptionally high volatility since their inception. Due to these characteristics of cryptocurrencies, academics have begun investigating the effect of adding cryptocurrencies to optimized portfolios, where the majority of studies focus on Bitcoin (Wu and Pandey, 2014; Briere, Oosterlinck and Szafarz, 2015; Eisl, Gasser and Weinmayer, 2015; Gangwal 2017; Agarwal et al, 2018; Kajtazi and Moro, 2019; Platanikis and Urquhart, 2020; Bakry et al., 2021).

According to Briére, Oosterlinck, and Szafarz (2015) their article is the first to explore the investment characteristics of Bitcoin in a portfolio context. They mention that up until that point, the literature on Bitcoin had mainly focused on whether Bitcoin was, in fact, a credible investment but not on its ability to diversify a portfolio. The authors take the perspective of a U.S. investor who holds a diversified portfolio consisting of worldwide equities, bonds, currencies as well as alternative investments in commodities and real estate, using weekly returns of these assets and Bitcoin between 2010 and 2013. They mention that Bitcoin showed a high average monthly return of 7.8% over the period, with a high monthly volatility of

24.43%. They also emphasize the low correlation of Bitcoin and the other asset classes over the period.

Employing a spanning test to inspect the effect on mean-variance characteristics by adding Bitcoin to portfolios, Briére, Oosterlinck and Szafarz (2015) conclude that adding Bitcoin to the portfolios results in superior mean-variance trade-offs compared to the portfolios excluding Bitcoin. They also construct mean-variance optimal portfolios with and without Bitcoin. Although Bitcoin is too volatile to be included in the least risky portfolio, results show that the risk-adjusted return increases significantly for all portfolios when adding Bitcoin, with a much steeper efficient frontier compared to the portfolio where Bitcoin is excluded. The downside risk adjusted return, or the Sortino Ratio also increases with the investment in Bitcoin. However, the authors mention that the results could be affected by the early-stage behaviour of Bitcoin and the fact that the analysis is conducted in a Bitcoin bull market.

Platanikis & Urquhart (2020) extend the literature by constructing eight different portfolios based on various portfolio optimization theories. According to them, their analysis is the first to examine the out-of-sample benefits of adding Bitcoin to a portfolio. They further note that the classic Markowitz mean-variance optimal portfolios are often unstable as they are sensitive to estimation risk and show poor out-of-sample performance. To address this problem, they employ techniques to make the portfolios less sensitive to estimation risk, namely the techniques of Bayes-Stein and Black-Litterman and the classic Markowitz mean-variance portfolios with varying constraints and estimation windows.

An out-of-sample analysis is employed to analyse whether adding Bitcoin to a traditional stock-bond portfolio carries diversification benefits from an U.S. investor´s perspective. To make the estimates of inputs in the optimization procedures more robust they employ both an expanding estimation window as well as a rolling estimation window, up until the day of rebalancing, which happens on a weekly basis, using weekly return data from October 2011 to June 2018.

The results of the analysis show that including Bitcoin in a traditional stock-bond portfolio increases returns for all methods and all levels of risk aversion. Further, the inclusion of Bitcoin increases the standard deviation of all portfolios. However, the increase in return of the portfolios including Bitcoin compared to the portfolios without it outweighs the increase in standard deviation when including an investment in Bitcoin, resulting in higher risk-adjusted measures for all portfolios. The mean Sharpe Ratio for portfolios without Bitcoin is 1.03, while the mean Sharpe Ratio for portfolios including Bitcoin is 1.64. Both the Sortino Ratio and the Omega Ratio increase when including Bitcoin in the portfolios.

The authors note that even though they find significant evidence for the benefits of including Bitcoin in optimized portfolios, the volatility of Bitcoin means that results cannot be generalized to apply to all future periods, as historical estimates can be sub-par estimates of future performance for very volatile assets.

Kajtazi and Moro (2019) investigate the effects of adding Bitcoin to optimal portfolios built with the mean-CVaR approach or mean Conditional Value at Risk, constructing portfolios from the standpoints of US, European and Chinese investors. They construct four different portfolios for each standpoint; the Equally Weighted portfolio (naïve diversification), a long-only portfolio, a semi-constrained portfolio, and a semi-constrained portfolio where Bitcoin cannot be shorted. The two authors differentiate their study from the existing literature by not constructing the optimal portfolios using Markowitz's approach but using an in-sample mean-CVaR approach. CVaR or Expected Shortfall is calculated as the average loss beyond the given VaR threshold and can be used to optimize portfolios. Daily data from 2012 to 2019 is used to construct the portfolios, both without rebalancing and with semi-annual rebalancing.

The results of the study show that the addition of Bitcoin to mean-CVaR portfolios increases both the return and volatility of the portfolios. Comparable to the results of Platanikis and Urquhart (2020) and Briére, Oosterlinck, and Szafarz (2015), the increase in return outweighs the increase in volatility, resulting in a higher risk-adjusted return for all portfolios.

Kajtazi and Moro (2019) further note that most of the benefits of including Bitcoin in a portfolio stems from the sharp increase in the price of Bitcoin in 2013. Afterwards, the price shows speculative behaviour. The authors also note that Bitcoin and other cryptocurrencies are assets that need to be specially handled, pointing out the high volatility and the short time span of available data.

Aggarwal et al. (2018) study the effect of adding Bitcoin to a diversified portfolio of eight indices across six different asset classes; fixed income, commodities, real estate, equities, gold, and alternative investments, by taking the perspective of an Indian investor. The methods used to construct optimal portfolios are the Equally Weighted portfolio, long only portfolio and a constrained portfolio. Log return series of monthly data for all asset classes are obtained from

August 2010 to December 2016. The risk-return ratios are compared for the portfolios with and without an investment in Bitcoin. The results show that for the long only and Equally Weighted portfolios, the portfolio performance improves significantly in terms of risk adjusted returns when adding an investment in Bitcoin.

Similar to Aggarwal et al. (2018), Gangwal (2017) investigates the effect on risk adjusted returns of adding Bitcoin to a diversified portfolio of equities, bonds, crude oil, real estate, gold, MXEF and Baltic Index by taking the perspective of an international investor. Seven optimal portfolios are constructed. The period under investigation is from July 2010 to August 2016 and daily data is used. The results show that including Bitcoin in the different portfolios over the period increased the Sharpe Ratio in all cases. Like Platanikis and Urquhart (2020), Gangwal (2017) notes that Bitcoin's high returns outweigh its high volatility, resulting in a higher Sharpe Ratio.

Wu and Pandey (2014) analyse two components of Bitcoin, its possible role as a currency, as well as analysing the usefulness of Bitcoin as an investment asset. The first part of the analysis, whether Bitcoin is useful as a currency, yields a negative result as the return pattern of Bitcoin significantly deviates from the pattern of other major currencies, as well as gold, and suggests that Bitcoin should be regarded as a very illiquid financial asset. The second part of the analysis is more relevant to this paper. Wu and Pandey (2014) construct portfolios of currencies, equities, real estate, commodities, and the VIX index. They then decide the optimal portfolio by simulating 1,000 trials of random weights of the components of the portfolios, both with and without Bitcoin, observing weekly prices over the period between July 2010 to December 2013. The portfolios selected as optimal portfolios are the ones that optimize the examined measure each time. The measures Wu and Pandey use in their simulations are minimized total variance, minimized downside variance, maximizing the Sharpe Ratio, maximizing the Sortino Ratio, maximizing the Omega Ratio, and lastly, using a Black-Litterman portfolio construction technique incorporating pessimistic return views on Bitcoins.

The results of Wu and Pandey (2014) show that an inclusion of Bitcoin increases portfolio returns in all portfolios, as well as decreasing the probability of incurring a loss compared to the portfolios that excluded Bitcoin. Even in the Black-Litterman portfolio, where biased views of negative returns of Bitcoin and underperformance compared to other asset classes are incorporated, the algorithm continues to place positive allocations to Bitcoin. Like in most of the other literature, Wu and Pandey note that even if the research shows improved performance with Bitcoin included in the portfolios, investors need to treat it with care because of the speculative nature of prices and the high volatility of Bitcoin.

Eisl, Gasser and Weinmayer (2018) investigate the diversification effect of adding Bitcoin to portfolios based on the mean-CVaR approach, taking the view of an U.S. investor, and constructing well diversified portfolios including a broad range of asset classes. The period studied is from July 2010 to April 2015. Four different portfolios are constructed; the Equally Weighted portfolio (naïve diversification), unconstrained portfolio where no weight related constraints are applied, long-only portfolio where short selling is not possible and -100%/+100% portfolio which allows for asset weights to shift between +100% and -100%. Each portfolio is constructed with and without an investment in Bitcoin.

Several indices for equity, fixed income, commodity, money market, real estate and alternative investment opportunities are included in the study. A back-testing technique is then used to calculate monthly out-of-sample portfolio returns based on the optimal weights of each asset, given the maximized risk-return ratio. Further, a 12-month rolling horizon to estimate portfolio weights is applied throughout the investment period, and the out-of-sample expected monthly returns and CVaRs are calculated. Then, the effect of adding Bitcoin to a portfolio is explored by comparing the risk-return ratios of the optimal portfolios, eight portfolios in total, four of which include Bitcoin. The risk return ratio is used as a performance indicator to evaluate the risk-return efficiency of the portfolios in the same way as the Sharpe Ratio (Eisl, Gasser and Weinmayer, 2018).

The results indicate that Bitcoin should be included in optimal portfolios, further concluding that an investment in Bitcoin increases the risk of a portfolio, but this additional risk is offset by high returns resulting in better risk-return ratios (Eisl, Gasser and Weinmayer, 2018).

Bakry et al. (2021) investigate the diversification effect of adding Bitcoin to optimized portfolios. Several constraining optimization frameworks are employed with and without an investment in Bitcoin, with the aim of maximizing the Sharpe Ratio by optimizing the weights allocated to each asset class. Weekly data from August 2011 to May 2021 is used. The frameworks used to evaluate the performance are for example the Equally Weighted portfolio, the Risk Parity portfolio, semi-constrained portfolio, and unconstrained portfolio. The results show that for almost all the portfolio optimization techniques the performance measures improve considerably when adding Bitcoin to the portfolios.

Literature also exists on portfolio optimization using only cryptocurrencies as investable assets (Platanikis, Sutcliffe, and Urquhart, 2018; Braunies and Mestel, 2019). Platanikis Sutcliffe and Urquhart (2018) use four of the most widely traded cryptocurrencies to construct optimized portfolios based on Markowitz´s mean-variance framework and compare them to a 1/N Equally Weighted portfolio of the same cryptocurrencies in an in-sample test. The results of their analysis show that very little evidence is found for the benefits of optimizing a portfolio of cryptocurrencies over equally allocating capital. They further mention that no or only marginal improvement or even a decline is found in the Sharpe and Omega Ratios when comparing the Equally Weighted and optimized portfolios. They note that the optimal diversification of portfolios only including cryptocurrencies is likely to be offset by estimation errors in the models' inputs.

Similarly, Braunies and Mestel (2019) employ the Markowitz mean-variance framework to optimize portfolios consisting of 500 cryptocurrencies between January 2015 and December 2017, comparing it to a benchmark Equally Weighted portfolio using an out-of-sample test. Monthly rebalancing with a rolling estimation window of input parameters is used. Results are comparable to the results of Platanikis, Sutcliffe and Urquhart (2018). Expectedly, the optimized portfolios have lower volatility compared to single cryptocurrencies, but the optimized portfolios have a lower Sharpe Ratio than the 1/N portfolios in 75% of cases.

Taking a different approach to the analysis of the diversification abilities of cryptocurrencies, Corbet et al. (2018) and Guesmi et al. (2019) look at the statistical relationship between cryptocurrencies and various other financial assets.

Corbet et al. (2018) examine the returns and volatility transmission among three cryptocurrencies and other financial assets, employing a generalized variance decomposition methodology and investigating volatility spill over as well as unconditional connectedness between the cryptocurrencies and other financial assets, using data from 2013 to 2017. Their results show that the cryptocurrencies were conditionally disconnected from the mainstream financial assets, thus potentially offering investors diversification benefits when included in a portfolio.

Guesmi et al. (2019) also examine the conditional cross effect and volatility spill-over from Bitcoin to other financial assets. They use a multivariate DCC-GARCH to model the volatility spill-over. Their results point to Bitcoin as a very good tool for diversification in portfolios, crediting its high average return and low correlation with other financial assets, resulting in lower volatility when included in a portfolio of gold, oil, and equities, as well as lowering the volatility of a portfolio constructed with Bitcoin and emerging markets equities.

The properties of cryptocurrencies – their relatively high returns and high volatilities, as well as their rising coverage and attention within the financial markets scheme has made them an interesting subject of literature with regards to portfolio construction in the recent years; this is common when new asset classes emerge on the scene, as investors and researchers constantly seek new ways to further optimize portfolios (Ma et al., 2020).

The results of the literature reviewed seem coherent in the context of adding Bitcoin into a well-diversified portfolio of assets. Wu and Pandey (2014), Briere, Oosterlinck and Szafarz (2015), Eisl, Gasser and Weinmayer (2015), Gangwal (2017), Agarwal et al. (2018), Kajtazi and Moro (2019), Platanikis and Urquhart (2020), and Bakry et al. (2021) all find significant evidence of the ability of Bitcoin in particular to improve risk-adjusted performance of optimized portfolios, all employing different optimization techniques and data periods. Further analysis, for example by Matkovskyy et al. (2021) shows that optimized probabilistic utility portfolios of the ten worst-performing stocks in the SP100, SP400, and SP600 indices and the ten largest cryptocurrencies enhanced returns to a degree where the portfolio had a better return than a portfolio of the top ten best-performing stocks in the indices. However, the studies by Platanikis, Sutcliffe and Urquhart (2018) and Braunies and Mestel (2019) show that optimizing portfolios that solely consisted of cryptocurrencies did not show an improvement in the risk-adjusted performance of the portfolios, as the 1/N portfolio consistently outperformed the optimized portfolios. Lastly, Corbet et al. (2018) and Guesmi et al. (2019) conclude that statistical properties of cryptocurrencies are favorable for diversification in portfolios.

3.2 Cryptocurrencies as a Hedge and a Safe Haven

Research on cryptocurrencies has continued to consider whether Bitcoin and other crypto assets possess safe haven and hedging properties, particularly due to the correlation of cryptocurrencies with other assets. This has laid the foundation of analyses investigating the ability of cryptocurrencies to act as hedge and safe haven assets. This section of literature centers around this topic. Investors seek to limit their exposure to losses during times of market turbulence by investing in assets that serve as safe havens. Additionally, they aim to reduce the risk of adverse price movements in an asset by entering a hedged position (Bodie et al, 2014). The prospect theory developed by Kahneman & Tversky in 1979, introduced the idea of investor behaviour, related to decision making under risk. The theory assumes that losses and gains are valued differently, and therefore investors make decisions based on perceived gains rather than perceived losses, also known as loss aversion (Kahneman & Tversky, 1979). During periods of market uncertainty, the term safe haven often refers to the assets suggested to investors to "park their money" (Li & Lucey, 2017). Various assets have been investigated in terms of their safe haven and hedging potential including gold (Baur & Lucey, 2010; Baur & McDermott, 2010; Coudert & Raymond, 2011; Ciner et al., 2013; Flavin et al., 2014; Bechmann et al, 2015; Bredin et al., 2015), other precious metals like silver, platinum and palladium (Hiller et al., 2006; Lucey & Li, 2015; Li & Lucey, 2017), commodities (Henriksen, 2018), and long-horizon treasury bonds (Flavin et al., 2014).

The characteristics of cryptocurrencies have led to a body of research to investigate their hedging and safe haven potential. In the literature, cryptocurrencies such as Bitcoin are frequently compared to gold in terms of the hedging and safe haven properties of the assets (Dyhrberg, 2016; Klein et al., 2018; Shahzad et al., 2019). The two assets share characteristics related to their high price volatility and their finite supply (Dyhrberg, 2016), however Bitcoin is considerably more volatile than gold (Smales, 2019). Dyhrberg (2016) finds that Bitcoin possesses some of the same hedging capabilities as gold and can therefore be incorporated into the assortment of tools available to hedge market specific risk. More specifically, using asymmetric GARCH methodology, the findings show that Bitcoin can be used as a hedge against stocks in the Financial Times Stock Exchange and that it can be used as a hedge against the US dollar in the short term. In contrast to Dyhrnberg's (2016) findings, Klein et al. (2018) argue that the two assets are extremely different in terms of their hedging capabilities. The results indicate that Bitcoin is positively correlated with downward markets, whereas gold behaves the opposite way and therefore plays an important role in times of market distress. Additionally, no evidence is found for Bitcoin's stable hedging capabilities. Contributing to this discussion, Shahzad et al. (2019) investigate Bitcoin's ability to serve as a safe-haven for stock market investments during market distress and compare it to the properties of gold and the general commodity index. The period under investigation is from July 2010 to February

2018 and focuses on several stock market indices, including those of China, the US, and other developed and emerging economies. The research concludes that Bitcoin, gold, and the commodity index are all considered weak safe haven assets in some cases, the results reveal that the safe haven abilities of the aforementioned assets are time-varying and differ across the stock market indices considered.

Recently, a growing number of academics have investigated Bitcoin‘s properties to act as a hedge and a safe haven in times of extreme economic and political uncertainty (Bouri et al., 2017a; Bouri et al., 2017b; Demir et al., 2018; Wu et al., 2019).

Demir et al. (2018) analyse the prediction of the economic policy uncertainty (EPU) index on daily Bitcoin returns, using the Bayesian Graphical Structural Vector Autoregressive model as well as the Ordinary Least Squares and the Quantile-on-Quantile regression estimations. The authors find that the EPU has a predictive power of Bitcoin returns, which are fundamentally negatively associated with the EPU. These findings indicate that Bitcoin can serve as a hedging tool against uncertainty.

Bouri et al. (2017a) come to a similar conclusion, where they examine whether Bitcoin could hedge global uncertainty. The authors use Volatility Indexes (VIX‘s) of 14 developed and developing equity markets to measure Bitcoin‘s hedging potential. Furthermore, a quantile regression is employed after decomposing Bitcoin‘s returns into various investment horizons, and given evidence of heavy-tails. The empirical results reveal that Bitcoin reacts positively to uncertainty at both higher quantiles and shorter frequency movements of Bitcoin returns, therefore it acts as a hedge against uncertainty. Additionally, the authors use the same method as Demir et al. (2018), namely a quantile-on-quantile regression and find that hedging is realized at shorter investment horizons, and at both upper and lower ends of Bitcoin returns and global uncertainty (Bouri et al., 2017a).

Bouri et al. (2017b) find that Bitcoin serves as a poor hedge and is merely suitable for diversification purposes, using a dynamic conditional correlation model to investigate Bitcoin‘s potential to serve as a hedge and a safe haven for major world stock indices, oil, gold, bonds, the general commodity index, and the US dollar index. The data spans the period from July 2011 to December 2015. Nonetheless, the results indicate that Bitcoin can serve as a strong safe haven against weekly extreme down movements in Asian stocks.

Wu et al. (2019) examine whether gold or Bitcoin could serve as a safe haven against the EPU index. The properties are calculated via a GARCH model and quantile regression with dummy variables. The findings indicate no evidence of Bitcoin nor gold to serve as a strong safe haven or a hedge for EPU at the average condition. However, while gold maintains stability with smaller hedge and safe haven coefficients, Bitcoin is more responsive to EPU shocks. Additionally, both Bitcoin and gold can serve as a weak safe-haven and weak hedge against EPU during extreme bearish and bullish markets, which two can be considered for portfolio diversification during the normal market.

It is apparent that the majority of the existing literature focuses solely on Bitcoin as a hedge and a safe haven. However, research on the hedging and safe haven properties of a variety of cryptocurrencies has recently been growing. Wang et al. (2019) included 973 cryptocurrencies and 30 international indices in a study on the hedge and safe haven properties of cryptocurrencies. The cryptocurrencies included in the study all have a trading period of more than ten months. The study investigates whether cryptocurrencies can serve as a hedge or a safe haven for the major international stock and volatility indices in both developed and developing markets. The findings indicate that cryptocurrencies do not serve as a hedge for most of the international indices. The safe haven properties were more evident in developed markets. Additionally, the authors employ a rolling window analysis to detect the dynamic safe haven properties of the cryptocurrency composite indices. The results show that the safe haven property for the developed markets decreases in 2017 and increases in 2018 (Wang et al., 2019).

Colon et al. (2021) study the hedging capabilities of 25 cryptocurrencies. The cryptocurrencies are selected based on the ranking on the market capitalization as of August 2019 (including Bitcoin, Ethereum, XRP, Litecoin, Bitcoin Cash, Binance Coin, Stellar, Monero, Dash, Nero, Nem, and more). The findings show that the cryptocurrency market reacts differently to uncertainty, depending on the type of uncertainty. Furthermore, the cryptocurrency market serves as a strong hedge against geopolitical risks in most cases, but it is considered a weak hedge and a safe haven against economic policy uncertainty during a bull market.

Bouri et al. (2020b) use daily data from eight different cryptocurrencies to investigate the hedging and safe haven properties of these cryptocurrencies against down movements in the S&P 500 and its 10 equity sectors. The data comprises daily prices of Bitcoin, Ripple, Litecoin, Ethereum, Stellar, Dash, Nem, and Monero. Daily closing prices of the S&P 500 index and its

10 sector indices is further included in the study. Their findings indicate that the interaction between cryptocurrencies and equities is multifaceted and is commonly mixed. More specifically, Bitcoin, Stellar, Ripple, and to a lesser extent, Monero and Litecoin share somewhat the same safe haven properties against US market-wide and sector level equity indices. However, for Ethereum, Nem, and Dash, the results indicate that the hedging and safe haven properties generally differ across these three cryptocurrencies and US equity sectors.

In another 2020 study, different types of cryptocurrencies are studied, namely Bitcoin, Ethereum, Litecoin, Ripple, and Stellar. Additionally, four MSCI equity indices are included of USA, Europe, Japan, and Asia-Pacific excluding Japan, all series are in US dollar. The period under investigation starts in August 2015 and ends in July 2018. A conditional correlation approach is used combined with regression analysis and the results indicate that not only Bitcoin, but other leading cryptocurrencies can serve as effective diversifiers and hedges. The strongest diversification and hedging potential were against Asian-Pacific and Japanese equities. Further analysis, involving both in-sample and out-of-sample hedging effectiveness, shows the diversification advantages of adding cryptocurrencies to an equity portfolio (Bouri et al., 2020a).

Despite the conflicting views on using Bitcoin, and other cryptocurrencies as a hedge or a safe haven, the results of the aforementioned studies indicate that cryptocurrencies do pose characteristics that make them a favorable hedge and a safe haven asset. Cryptocurrencies are frequently compared to gold in terms of their hedging and safe haven capabilities and research has shown that the two assets do possess some of the same capabilities in this regard (Dyhrber, 2016) although Klein et al. (2018) argue that Bitcoin and Gold are dissimilar in their hedging capabilities. In terms of uncertainty, both Demir et al. (2018) and Bouri et al. (2017) find that Bitcoin serves as a hedge against uncertainty, and Wu et al. (2019) find evidence that Bitcoin responds to EPU shocks. However, Colon et al. (2021) argue that the type of uncertainty matters. Although the majority of previous research has been on Bitcoin, studies on other cryptocurrencies have shown that a variety of those assets can serve as a hedge and a safe haven, particularly against equity indices (Bouri et al., 2020a; Bouri et al., 2020b).

4 Hypotheses Development

This section introduces the research hypotheses investigated in this analysis. First, the potential shortcomings of the academic literature reviewed is outlined, before introducing the hypotheses and the arguments for the hypotheses. Finally, the contribution to the academic literature is presented.

This section puts forward three main hypotheses, which purpose is to either find support or fail to find support for the research question of the analysis:

Does an investment in cryptocurrencies enhance the performance of optimized portfolios?

4.1 Shortcomings of Academic Literature

The objective of this thesis is to analyze the effect of adding cryptocurrencies to optimized portfolios, and therefore shortcomings in relation to understanding that effect are addressed. The literature presented in section 3.1 gives a comprehensive overview of the existing literature on adding cryptocurrencies to optimized portfolios. However, the lion's share of previous research on the matter solely focuses on Bitcoin. These studies include for example Wu and Pandey (2014), Briere, Oosterlinck and Szafarz (2015), Eisl, Gasser and Weinmayer (2015), Gangwal (2017), Agarwal et al. (2018), Kajtazi and Moro (2019), Platanikis and Urquhart (2020), and Bakry et al. (2021) which analyze the effect of adding one cryptocurrency to an optimized portfolio as compared to a wider range of cryptocurrencies. Additionally, these studies examine the effect of adding Bitcoin to a portfolio of indices representing various asset classes, therefore including only one cryptocurrency in optimized portfolios of indices.

Although Bitcoin is the cryptocurrency with the longest price history, the highest market capitalization and most recognition, altcoins are gaining more coverage and growth in trading volume, in fact the cryptocurrency with the highest trading volume at the time of writing is Tether (Monthly Volume Rankings, 2022). The CRIX index considers both market capitalization and trading volume in its construction.

The previously mentioned studies all find evidence that adding Bitcoin to diversified portfolios improves their risk adjusted performance. The analyses by Briére, Oosterlinck, and Szafarz, (2015) Kajtazi and Moro, (2019) include price data from Bitcoin´s earliest days and both studies conclude that the benefits of adding Bitcoin to a portfolio might stem from the sharp increase in the price of Bitcoin in 2013 as well as Bitcoin´s early-stage behavior.

4.2 Research Hypotheses

As noted in section 3.1, there are generally two reasons for the attraction of investors interests toward cryptocurrencies and the possible effect of including them as an asset in their portfolios: cryptocurrencies usually do not have an association with any authority or monetary policy, and to a greater extent in a portfolio construction context; they have shown extraordinary returns paired with exceptionally high volatilities since their inception, creating an interesting research prospect of adding cryptocurrencies to well-diversified portfolios (Wu and Pandey, 2014; Briere, Oosterlinck and Szafarz, 2015; Eisl, Gasser and Weinmayer, 2015; Gangwal 2017; Agarwal et al, 2018; Kajtazi and Moro, 2019; Platanikis and Urquhart, 2020; Bakry et al., 2021).

This observation of the return profile of cryptocurrencies lead to the first research hypothesis of the analysis:

H1: An investment in cryptocurrencies increases returns of optimized portfolios.

An inclusion of an asset that has very high returns on average in a portfolio intuitively should increase the average return of the respective portfolio, but the level of increase depends on the weight allocated to the asset. Some of the techniques and methods employed in this study will favour assets with lower volatility to assets with higher volatility, resulting in a low weight allocated to cryptocurrencies. However, the magnitude of average returns of cryptocurrencies should lead to an increase in the average return of portfolios with a positive weight allocation to the asset.

The correlation between cryptocurrencies and other financial assets has during cryptocurrencies' relatively short history been low or negative (Aslandis, Bariviera & Martinez, 2019), sparking the interest of academics to study cryptocurrencies' capability as a safe haven asset and its hedging properties, for example the analyses by Wang et al. (2019), Bouri et al. (2020a), Bouri et al. (2020b), and Colon et al. (2021). What low correlations entail for a portfolio that includes cryptocurrencies, historically very volatile assets, is that not all the volatility of the cryptocurrencies will be absorbed by the portfolio, as covariance is lower with lower correlation and cryptocurrencies might tend to move in opposite directions compared to other assets, creating possible diversification benefits, as well as affecting the risk-return profile of the portfolios. Previous research on the inclusion of cryptocurrencies in portfolios, for example by Wu and Pandey (2014), Briere, Oosterlinck and Szafarz (2015), Eisl, Gasser and Weinmayer (2015), Gangwal (2017), Agarwal et al. (2018), Kajtazi and Moro (2019), Platanikis and Urquhart (2020), and Bakry et al. (2021) all show an increase in the Sharpe Ratio and risk-return ratio of portfolios that included an investment in Bitcoin compared to the same portfolios without Bitcoin. This leads to the second research hypothesis of this analysis:

H2: An investment in cryptocurrencies increases the volatility of optimized portfolios, but the increase in average return offsets the increase in volatility, resulting in a higher Sharpe Ratio.

By the same token, if cryptocurrencies have a low or negative correlation with other assets, as previously mentioned, during times of financial market turbulence and negative returns across other asset classes, downside risk (in this analysis the downside is defined as excess returns below 0%), should be lower compared to a portfolio that does not include cryptocurrencies, as the allocation towards Bitcoin could possibly act as a hedge in times of financial market turbulence, as outlined in section 3.2, and reduce downside variation. If this notion is in fact true, combined with H1, that the average return is higher in portfolios including cryptocurrencies compared to portfolios that don't, there is a dual positive effect on the Sortino Ratio of the portfolio including cryptocurrencies, as the numerator in the ratio, average excess return, is growing, while the denominator, the standard deviation of downside returns, is decreasing, leading to a growth in the ratio. This leads to the third research hypothesis of this analysis:

H3: An investment in cryptocurrencies reduces the downside standard deviation of a portfolio, resulting in a higher Sortino Ratio.

These hypotheses are chosen as relevant for the study as they allow for a direct, quantitative measurement of metrics that are relevant to the research question, taking into account portfolio performance in a wide perspective, both in plain average return terms but also considering the risk-return profile and trade-off of each portfolio.

4.3 Contribution to Academic Literature

This thesis contributes to the existing literature in several ways. Most studies investigating the effect of adding cryptocurrencies to optimized portfolios focus solely on Bitcoin. This analysis takes a different approach by including a wider range of cryptocurrencies, a professionally computed index of cryptocurrencies that represents the total market return characteristics. Combining indices representative of whole asset classes such as the S&P500 index and indices for other asset classes with only one crypto asset can underrepresent the characteristics of the asset class as a whole and therefore a professionally computed index of cryptocurrencies is included in this analysis, providing more robust results. This further provides an internal diversification within cryptocurrencies. Although Bitcoin is a major component of the index throughout the analysis period, the index always includes a minimum of five liquid cryptocurrencies, which at the time of writing are nine in total (Royalton CRIX Crypto Index, 2022). Additionally, the chosen dataset covers the period from the 1st of March 2016 to the 31st of December 2021, including more recent data compared to many of the existing studies presented in the Literature Review.

There are several portfolio optimization techniques that could be used for optimizing the portfolios in this analysis, but the techniques are chosen with the aim of arriving at the most robust and applicable results and representing different perspectives in terms of risk and return. Therefore, five different optimization techniques are computed and an Equally Weighted portfolio as well as considering three levels of risk aversion to evaluate the effect of adding cryptocurrencies to the portfolios of five different asset classes. Each strategy chosen in this analysis may result in different weights being allocated to each asset, which is favorable in providing an extensive argument of the value of adding cryptocurrencies to optimized portfolios. The following section will outline the methodology used in this analysis.

5 Methodology Approach

This section outlines the methodological approach taken to the analysis of this research. The goal of the analysis is to find evidence for or against the hypotheses structured in section 4 and ultimately lay the foundation for answering the research question. The analysis is six-fold, each empirically investigating the hypotheses.

5.1 Data Collection and Description

This section describes the dataset used in this analysis. Table 1 gives an overview of the selected asset classes.

Overview of asset classes

Asset name	Ticker	Currency	Asset class
S&P 500 Index	GSPC	USD	Equity
iShares U.S. Treasury Bond ETF	GOVT	USD	Bond
United States Oil Fund, LP	USO	USD	Oil
iShares iBoxx $ Investment Grade Corporate Bond ETF	LQD	USD	Bond
SPDR Gold Shares	GLD	USD	Gold
Royalton CRIX Crypto Index	CRIX	USD	Cryptocurrency

Table 1: Overview of asset classes

To investigate the effect of adding cryptocurrencies to a portfolio of assets, price data is required for all assets in all our analyses. Price data for GSPC, GOVT, USO, LQD, and GLD was extracted from Yahoo Finance, and the price data for the CRIX index was extracted from Royalton CRIX Index (2020). Monthly prices were collected for all assets as the portfolio is rebalanced monthly. The dataset was chosen with regards to the availability of the CRIX Index values. The period studied is therefore from the 1st of March 2016 to the 31st of December

2021. The one-month risk-free rate was extracted from Kenneth R. French's data library (French, 2022).

5.1.1 GSPC Index

In order to represent the majority of the U.S. equity market, this analysis considers the Standard & Poor's 500 Index, often abbreviated as the S&P500, which is the most influential equity index in the world. The S&P500 index is a market capitalization-weighted index of 500 leading publicly traded U.S. companies. The index was launched in 1957 as the first U.S. market capitalization-weighted equity index. Today it is one of the most commonly followed equity indices worldwide and is considered one of the best indicators of American equities' performance and even the overall global stock market. The index has trillions of dollars benchmarked or indexed to it, representing approximately 80% of available U.S. market capitalization. By investing in an S&P500 Index fund or an Exchange Traded Fund (ETF), an investor gains broad exposure to the stocks in the index (Bloomenthal, 2022).

The median market capitalization of companies in the index is $79.5 billion, the highest being $2.9 trillion. The index includes Apple, Microsoft, Amazon, Alphabet, Tesla, and other large U.S. companies. Therefore, the index significantly reflects the U.S. economy's growth drivers. Using a market capitalization-weighted index, a higher percentage is allocated to companies with the largest market capitalizations (Kenton, 2022). The companies mentioned before are among the largest by index weight and are mainly in the information technology, consumer discretionary, and the communication services sectors (Bloomenthal, 2022).

5.1.2 GOVT ETF

The iShares U.S. Treasury Bond ETF, GOVT, is considered in this analysis as it gives exposure to the entire U.S. Treasury yield curve. The ETF seeks to track the investment results of an index composed of U.S. Treasury bonds (iShares, 2022a). The fund consists of 98.98% AAA-rated, high-quality bonds and 1.02% cash and/or derivatives (iShares, 2021a) and aims to track the investment results of the ICE U.S. Treasury Core Bond Index, which measures the performance of public obligations of the U.S. Treasury (iShares, 2022b). The underlying index is weighted by market capitalization and it includes publicly issued U.S. Treasury securities that have a remaining maturity greater than one year and less than or equal to thirty years. The securities also have $300 million or more of outstanding face value and must be fixed-rate and denominated in U.S. dollars. Additionally, they are updated on the last business day of each month. To achieve the fund's investment objectives, the fund does not try to "beat" the underlying index but instead uses a "passive" or indexing approach (iShares, 2022b).

By investing in the fund, investors gain exposure to U.S. Treasuries ranging from 1-30 year maturities, as well as stability in their portfolio (iShares, 2022a). The weighted average maturity of the fund's assets is 8.44 years (iShares, 2021). Historically, often when stock markets sell-off, government bonds have experienced an increase in value and can therefore diversify an investor's portfolio (iShares, 2021). However, there have also been instances where both equities and bonds depreciate in value, for example in 2022, as of writing both have depreciated YTD.

5.1.3 USO ETF

Investors seek to improve their expected return-risk trade-off by adding commodities to their portfolios (Dorsman et al., 2013). This thesis considers the United States Oil Fund, USO, an exchange-traded security whose shares may be purchased and sold on the NYSE Arca (USCF, 2022). The fund invests primarily in futures contracts for light, sweet crude oil, other types of crude oil, diesel-heating oil, gasoline, natural gas, and other petroleum-based fuels (Yahoo Finance, 2022). The fund's investment objective is for the daily changes in percentage terms of its share's net asset value to reflect the daily changes of the spot price of light sweet crude oil, as measured by the daily changes in percentage terms in the Benchmark Oil Futures Contract. It seeks for the average daily percentage change in USO's net asset value, for any period of 30 successive valuation days, to be within plus/minus 10% of the average daily percentage change in the price of the Benchmark Oil Futures Contract over the same period. Historically, USO has achieved its investment objective by primarily investing in the Benchmark Futures Contract and oil futures contracts for light, sweet crude oil traded on NYMEX and ICE Futures with the same maturity month as the Benchmark Futures Contract (USCF, 2022).

5.1.4 LQD ETF

By investing in iShares iBoxx $ Investment Grade Corporate Bond ETF, investors get access to more than 1000 high quality corporate bonds in a single fund and exposure to a broad range of U.S. investment grade corporate bonds. The fund's investment objective is to track the investment results of an index composed of U.S. dollar-denominated, investment grade corporate bonds (the underlying index).

The underlying index is the Markit iBoxx USD liquid Investment Grade Index, a modified market-value weighted index with a cap in each issuer of 3%. Additionally, the underlying index is designed to broadly represent the U.S. dollar-denominated liquid investment-grade corporate bond market. The weighted average maturity of the fund's securities is 13.77 years. LQD uses a "passive" approach to achieve the fund's investment objectives but does not aim to "beat" the underlying index.

This approach reduces some of the risks of active management, such as poor security selection. The fund typically invests at least 90% of its assets in component securities of the underlying index and at least 95% in investment-grade corporate bonds. Nevertheless, the fund may occasionally invest up to 20% of its assets in certain futures, options and swap contracts, cash and cash equivalents (iShares, 2021b).

5.1.5 GLD ETF

The SPDR Gold Shares (GLD) ETF seeks to reflect the performance of the price of gold bullion and therefore offers investors a relatively secure and cost-efficient way to access the gold market (Morningstar, 2022). The fund was the first ETF to offer investors a cost-effective way to get indirect exposure to one of the world's most popular commodities and is the largest ETF on Wall Street backed by physical gold today. Furthermore, the fund is the most liquid gold ETF on the market. One of the fund's selling points is stability since its performance is not dependent on an underlying management team but solely on the price of gold itself (Gurdus, 2019).

5.1.6 CRIX Index

The Royalton CRIX Crypto Index tracks the total market index of cryptocurrencies using a minimum of five liquid cryptocurrencies (Royalton CRIX Crypto Index, 2022). At the time of writing the number of index components is nine. Professor Wolfgang Karl Härdle and his team saw a need for an index product to emerge as cryptocurrencies became an investable asset (Royalton CRIX Index, 2020). Even though not traded, the index is constructed as an investable index, only consisting of actively traded cryptocurrencies (Trimborn & Härdle, 2018). It relies on a dynamic approach based upon the CRIX Technology Decision Criterion (Royalton CRIX

Index, 2020), which determines the optimal number of cryptocurrencies quarterly to ensure that it represents the total market return characteristics (Royalton CRIX Crypto Index, 2022). Only cryptocurrencies that add value to the index are considered (Trimborn & Härdle, 2018). The constituents are rebalanced monthly and each cryptocurrency in the index is weighted by its market capitalization (Royalton CRIX Index, 2020). However, solely relying on market capitalization raises concerns regarding the dominance of Bitcoin in the index. Therefore, a second weighting scheme was introduced based on trading volume. If altcoins are traded more than Bitcoin, they are allocated a higher weight in the index.

Many indices are constructed with a fixed number of constituents to represent the market which requires trust from investors for the choice of fixed constituents and rules. In the frequently changing cryptocurrency market, where volatility is high and the number of cryptocurrencies changes daily, this challenge is even bigger, and therefore a dynamic approach is more reasonable (Trimborn & Härdle, 2018; Royalton CRIX Index, 2020). The cryptocurrency market has a diversified nature, which makes the inclusion of smaller coins in the index crucial to improve tracking performance. The research of professor Härdle has shown that assigning optimal weights to a selection of constituents helps to reduce the tracking error of a cryptocurrency portfolio, regardless of the fact that the market capitalization of some cryptocurrencies included is much smaller than Bitcoin's (Royalton CRIX Index, 2020)

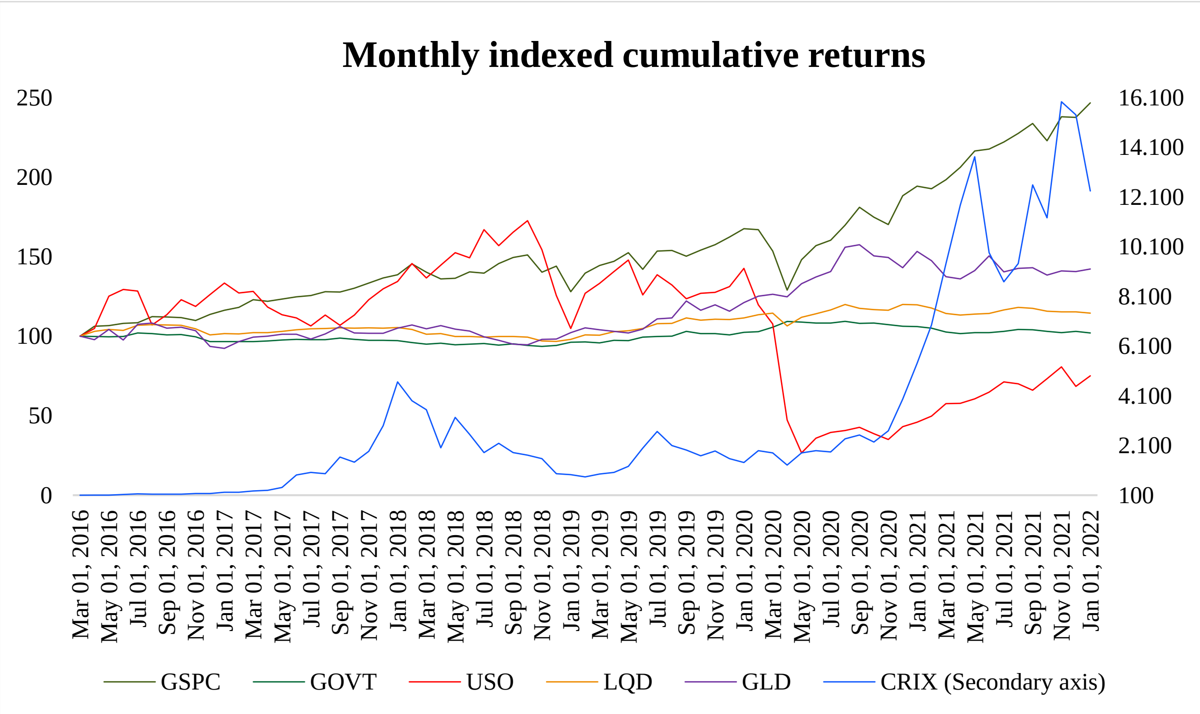

Figure 6: Indexed returns of asset classes

Figure 6 shows the evolution of returns to each asset class used in the analysis. The returns are indexed to 100 on March 1st, 2016. As the CRIX index has extreme returns, the secondary axis applies to that index.

The monthly values of the S&P500 (GSPC) show that the value has an overall upward trend. Note that at the beginning of the year 2020, COVID-19 begins to influence the stock market and between the 1st of March and the 1st of April 2020, the index value drops by approximately 16% amid turbulence in the financial markets. From the 1st of April 2020 to the 1st of January 2022, the index value trend has been upward with short volatile periods.

When analyzing the monthly prices of GOVT over the period studied, it is evident that the price does not fluctuate a lot and is stable over the period. This does not come as a surprise and can also be seen by looking at the monthly standard deviation of 1,16%, presented in table 3, which is the lowest standard deviation of the asset classes studied.

Looking at the monthly prices of USO over the period studied, it is apparent that the price is extremely volatile and fluctuates a lot over the period. This is in line with the standard deviation which is the second-highest (after CRIX) of the assets in the dataset. At the beginning of 2020, the price of USO drops by 81% from the 1st of January to the 1st of May amid uncertainty following the outbreak of COVID-19. In the period after the outbreak of the pandemic in the dataset, demand conditions for oil where unstable as countries moved in and out of lockdowns and travel restrictions, causing volatility in the price of oil. In one month, from the 1st of March to the 1st of April 2020, the price drops by 56%, directly as a consequence of travel restrictions in the world.

When analysing the monthly prices of LQD over the period studied, it shows that the price over the period is relatively stable and does not fluctuate a lot. The standard deviation is the second-lowest one of the assets in the dataset (after GOVT). There is a price drop between the 1st of March and the 1st of April 2020 of approximately 7%, as for the other securities attributable to the outbreak of COVID-19. Other major drawdowns are rare within the period for the ETF.

The monthly prices of GLD over the period studied show that from March 2016 to December 2019 the price is relatively stable. From the 1st of March to the 1st of September 2020, the price increased by approximately 25%, as investors often seek to buy gold in turbulent times in the financial market, as gold as historically been seen as a safe haven asset.

The monthly values of the CRIX index over the period studied show very extreme patterns, the value of the CRIX index rose at a relatively steady pace initially, before the crypto bull-market of 2017 sharply increases the value of the index. Cryptocurrencies experienced large drawdowns in 2018 and the index loses approximately 70% of its value in 2018. The largest movements occur later in the period, as from November 2020 to May 2021 the value of the index increased tremendously. The value increased by approximately 410% over that period. Since then the index has been volatile and has experienced large fluctuations in value. This does not come as a surprise as CRIX has the highest standard deviation in the dataset and cryptocurrencies in general are volatile assets.

5.1.7 Descriptive Return Statistics and Correlations

Tables 2 and 3 present the correlation matrix and descriptive return statistics of the data used in this analysis.

Correlation Matrix

	GSPC	GOVT	USO	LQD	GLD	CRIX
GSPC	1.00	-0.44	0.43	0.48	0.07	0.22
GOVT	-0.44	1.00	-0.42	0.34	0.48	-0.06
USO	0.43	-0.42	1.00	0.19	-0.11	0.04
LQD	0.48	0.34	0.19	1.00	0.49	0.20
GLD	0.07	0.48	-0.11	0.49	1.00	0.11
CRIX	0.22	-0.06	0.04	0.20	0.11	1.00

Table 2: Correlation matrix of assets

Monthly Descriptive Return Statistics

	GSPC	GOVT	USO	LQD	GLD	CRIX
Mean Return	1.40%	0.04%	0.64%	0.21%	0.58%	10.46%
Standard Deviation	4.59%	1.16%	13.13%	1.82%	4.03%	28.21%
Minimum	-16.01%	-3.00%	-56.15%	-7.07%	-9.52%	-43.09%
Maximum	14.85%	3.38%	34.52%	5.02%	10.78%	117.47%
Median	1.94%	-0.06%	3.99%	0.10%	0.45%	6.45%
Skewness	-0.81	0.67	-1.50	-0.65	0.22	0.93
Excess Kurtosis	3.53	1.39	5.81	3.14	0.25	1.84
Observations	70	70	70	70	70	70

Table 3: Descriptive return statistics

Table 2 reports the correlation matrix between the variables considered in the analysis. The CRIX Index has relatively low correlations with all the asset classes, indicating possible diversification benefits. The highest correlation between CRIX and the other asset classes is with S&P500 of 0.22 and it has the lowest correlation with GOVT of -0.06. The correlation between S&P500 and GOVT is negative, whereas S&P500 is positively correlated with all the other asset classes, with a relatively high correlation with LQD and USO.

Additionally, GOVT and GLD have a fairly high positive correlation. GOVT is also positively correlated with LQD whereas it is negatively correlated with the other assets, including CRIX, suggesting that there might be diversification benefits when adding CRIX to the portfolio. LQD is positively correlated with all asset classes. The highest positive correlation of all asset classes is between GLD and LQD (0.49), and the correlation between GOVT and GLD (0.48) is the same as the correlation between LQD and S&P500 (0.48) which is the second highest one.

Table 3 reports the descriptive statistics of the returns of the securities used in this study. The dataset has 70 observations, which includes each month from March 2016 to December 2021. Over the period studied, the CRIX Index has a significantly higher mean return (10.46%) than all the other asset classes, as well as having the highest median return (6.45%).

The big spread in the minimum and maximum return for CRIX indicates its high volatility, which can also be seen on the standard deviation which is the highest one in the sample. The spread further indicates that the dataset includes extreme values, which influences the mean return. Additionally, CRIX is positively skewed, implying that the mean return is greater than the median return as the size of the right tail is larger than the left tail. It further indicates that over the period studied, majority of returns are lower than the mean return, but the returns higher than the mean are more extreme. The excess kurtosis for CRIX is positive implying that the distribution has heavier tails than the normal distribution, meaning that the probability of large positive and large negative returns is higher than normal probability. To further analyse the distribution of the returns of CRIX, figure 7 shows a histogram of the monthly returns of the index, together with the monthly returns of S&P500.

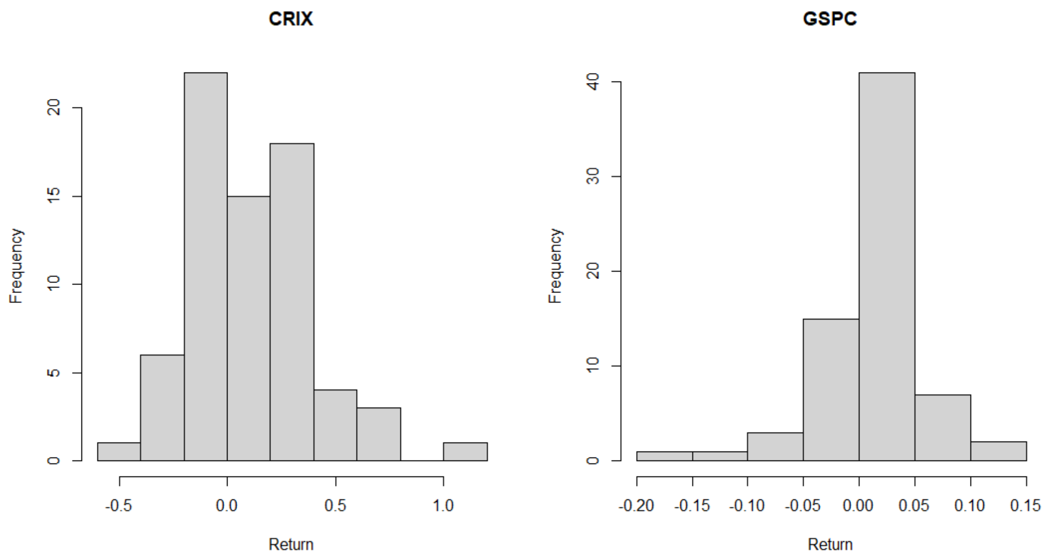

Figure 7: Histograms of monthly returns for GSPC and CRIX

The underlying foundation of most of the analyses in this research, further discussed in upcoming sections, is that returns are normally distributed. The assumption that security returns follow a normal distribution is not perfect, but it is an approximation that is not too far from reality (Munk, 2020). Looking at figure 7, we see that the returns of both the CRIX index and the S&P500 index do partly resemble a normal distribution, although with differing means and standard deviations as covered. While the S&P500 index has a negative skewness, as equity returns tend to have, the CRIX index has positive skewness and surprisingly thin tails, as it has less excess kurtosis compared to the S&P500 index returns. The values on the x-axis are also far more extreme for the CRIX index.

With regards to the other asset classes, S&P500 has the second highest mean return of 1.40%, whereas USO has the second highest median return of 3.99%. GOVT and GLD have positive skewness whereas S&P500, USO, and LQD are negatively skewed, indicating that the majority of returns is higher than the mean return, but the returns lower than the mean are more extreme.

All asset classes have positive kurtosis. USO has the lowest minimum return and second highest maximum return, following CRIX, and subsequently has the second largest standard deviation.

5.2 Notation and Basic Calculations

This section covers basic calculation and the notation used throughout the analysis. To ensure consistency, notation introduced in this section is used for all analyses if not specifically noted in the relevant methodology section.

The monthly return of security i at time t is defined as:

$$r_{i,t} = \left(\frac{P_t}{P_{t-1}}\right) - 1 \tag{1}$$

where P_t is the price of security i at time t and P_{t-1} is the price of security i at time t-1.

The expected return, μ_i of security i at time t is defined as the average monthly return of security i over the period from the beginning of the analysis until each portfolio rebalancing date, T:

$$\mu_{i,T} = \frac{1}{T} * \sum_{s=1}^{T} r_{i,T-s} \tag{2}$$

where T is the number of months from the beginning of the sample until each portfolio rebalancing date, $r_{i,T-s}$ is the return of security i over the T-s month, calculated as in equation (1). Combining the expected returns for all the securities, we obtain the vector of expected returns, which is denoted as **μ.**

The sample covariance of returns at time T of assets i and j is calculated as:

$$cov_{i,j} = \frac{1}{T-1} \sum_{t=1}^{T} \left(r_{i,t} - \bar{r}_i\right) * \left(r_{j,t} - \bar{r}_j\right) \tag{3}$$

where $\bar{r}$ is the mean monthly return of security i, j up until time T, which can be calculated by equation (2). When constructing a portfolio consisting of N assets, the variance-covariance matrix of the returns of the assets is a $N\ x\ N$ matrix. As the variance of single security returns, $Var[r_i]$ is equal to $cov[r_i, r_i]$, the variance covariance matrix of returns can be defined as:

$$\underline{\Sigma}_{ij} = cov[r_i, r_j] \tag{4}$$

even when i = j (Munk, 2020). The return of the portfolios over the holding period is defined as:

$$r_{p,T,T+1} = \sum_{i=1}^{N} \pi_{i,T} * \ r_{i,T,T+1} \tag{5}$$

where $\pi_{i,T}$is the weight of security i in the portfolio after rebalancing at time T and $r_{i,T,T+1}$ is the return of the corresponding security over the time period from T until T+1, where the time period is the following month after rebalancing.

5.2.1 Estimation

This section explains the methodological estimation approach taken through analyses 1-5, in particular how the parameters of models are estimated and the procedure of estimation, explaining an out-of-sample estimation of the models as well as explaining the length of the estimation windows.

5.2.2 Out-Of-Sample Estimation Procedure

Throughout the analysis, an out-of-sample estimation process is used. An out-of-sample estimation process is deemed more relevant, as an in-sample estimation would imply that all the parameters are perfectly forecasted (Platanikis & Urquhart, 2020). It is however well documented that forecasts in returns and variances of returns of financial assets are exposed to estimation errors. The out-of-sample estimation is thus more realistic, as well as making the rebalancing of the portfolios more realistic. An out-of-sample estimation means in its simplest terms that the only data that can be used as an input to the model at any time is data that could have been known to the investor at that time, and the results of this model are applied to new data.

What that entails for this research is at each portfolio rebalancing date, T, the portfolio weights are calculated based on return data available at time T. The portfolio is held for a month and return is calculated based on the weights computed and the following month's returns, when the procedure is repeated, now with a richer dataset of one month, and so on.

5.2.3 Estimation Window

Following Board & Sutcliffe (1994), Tu (2010), Pedersen, Levine & Babu (2021), Platanikis & Urquhart (2020) and others, an expanding estimation window is used to compute the parameters of the models. An advantage of the expanding estimation window is that the parameters of the models should be more stable over time and less sensitive to estimation risk. It is also intuitive in an out-of-sample procedure to use an expanding estimation window.

At the first portfolio formation date, at the beginning of January 2019, the estimation window is 34 months. With the passage of time, more data is used in estimation as more and more data for estimation is available, as at each portfolio rebalancing date one more month of return data is added to the estimation. An alternative method, applied by Bessler & Wolf (2015) would have been to employ a rolling estimation window, however that is deemed inferior to the expanding estimation window for this research as the data available for cryptocurrencies is still limited and the parameters would be more unstable.

5.3 Analysis 1

Analysis 1 investigates the effect of including investment in cryptocurrencies in a The Global Minimum Variance portfolio. The Global Minimum Variance portfolio framework was first developed by Markowitz (1952) and Markowitz (1959). The analysis is conducted by forming portfolios based on two universes of investable securities, first without cryptocurrencies and then with cryptocurrencies. The portfolio weights are then rebalanced monthly based on data available at each portfolio rebalancing point in time. The analysis takes the viewpoint of an U.S. investor diversifying across asset classes.

5.3.1 Theoretical Background of Analysis 1

According to Markowitz (1959), investors have two objectives when investing in securities. First, they seek a high return. While a definition of high returns can vary between investors, investors will always prefer more return to less return. Second, investors want the return to be stable, free of uncertainty to the maximum degree. Like in the case of returns, the degree to which investors wish to eliminate the uncertainty of returns can also vary between them.

However, the methods Markowitz proposes concern investors who prefer stable and predictable returns relative to uncertain returns.

So, what portfolios does a mean-variance optimizing investor seek? The most stable portfolio with the highest expected return could still have considerable uncertainty about the return. Conversely, the portfolio with the lowest uncertainty of expected returns could have a very low expected return. Both portfolios are however mean-variance efficient. Mean-variance efficient portfolios are defined as the portfolios with the lowest uncertainty of a given expected return. As a result, the mean-variance framework of Markowitz only relies on two parameters, expected return and return variance, and in the framework, investors make investment decision solely based on these parameters. In the absence of a risk-free investment, all these efficient portfolios can be graphed to form the efficient frontier of risky assets. (Markowitz, 1959).

If an investor can invest in N risky assets, but no risk-free investment is available, a portfolio vector can be defined as $\boldsymbol{\pi} = (\pi_1, \pi_2, \pi_3, \ldots, \pi_N)^{\boldsymbol{T}}$ where π is defined as the weight of each asset in the portfolio.

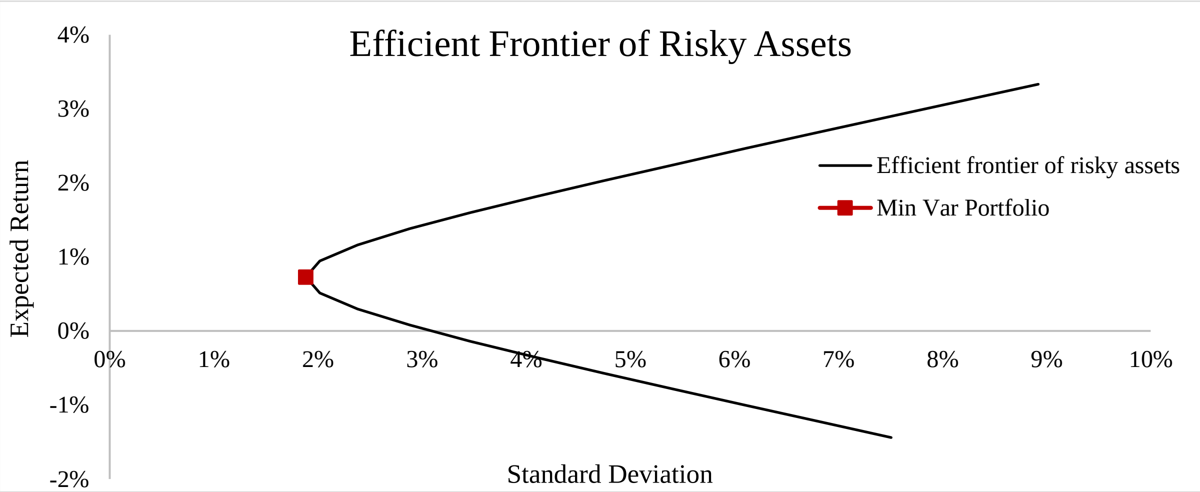

Figure 8: Efficient frontier of risky assets (Authors own make)

Further, the portfolio vector must satisfy: $\boldsymbol{\pi} * \mathbf{1} = \pi_1 + \pi_2 + \pi_3 + \cdots + \pi_N = 1$, indicating that portfolio weights must sum to one. Defining $\boldsymbol{\mu}$ as a vector of expected returns of the securities as defined in equation (2) and defining $\underline{\Sigma}$ as the variance-covariance matrix of returns as defined in equation (4), we can define the expected return, variance and standard deviation of a portfolio $\boldsymbol{\pi}$ as follows (Munk, 2020):

$$\mu(\boldsymbol{\pi}) = \boldsymbol{\pi} * \boldsymbol{\mu} = \sum_{i=1}^{N} \pi_i * \mu_i \tag{6}$$

$$\sigma^2(\boldsymbol{\pi}) = \boldsymbol{\pi} * \underline{\Sigma}\boldsymbol{\pi} = \sum_{i=1}^{N}\sum_{j=1}^{N} \pi_i * \pi_j * \underline{\Sigma}_{ij} \tag{7}$$

$$\sigma(\boldsymbol{\pi}) = \sqrt{\boldsymbol{\pi} * \underline{\Sigma}\boldsymbol{\pi}} \tag{8}$$

A mean-variance efficient portfolio is as a result the solution to the following minimization problem:

$$\min_{\boldsymbol{\pi}} \boldsymbol{\pi} * \underline{\Sigma}\boldsymbol{\pi}$$

$$s.t. \quad \boldsymbol{\pi} * \boldsymbol{\mu} = \bar{\mu}$$

$$\boldsymbol{\pi} * \mathbf{1} = 1$$

where $\bar{\mu}$ is the level of return desirable for the investor. Intuitively, the variance, and as a result the standard deviation is minimized, while constraining the portfolio to achieve a desired return level and the weights must sum to one. If there exists a solution to the minimization problem for the given rate of expected return, the solution portfolio of $\boldsymbol{\pi}$ is mean-variance efficient. The general solution can be written as:

$$\boldsymbol{\pi}(\mu) = \frac{C\bar{\mu} - B}{D}\underline{\Sigma}^{-1}\boldsymbol{\mu} + \frac{A - B\bar{\mu}}{D}\underline{\Sigma}^{-1}\mathbf{1} \tag{9}$$

where $\underline{\Sigma}^{-1}$ is the inverse of the variance-covariance matrix of returns. Calculations in Markowitz (1952) and Markowitz (1959) involve auxiliary constants that change based on the inputs available at a given time, which are defined by A, B, C and D (Munk, 2020):

$$A = \mu^T\underline{\Sigma}^{-1}\boldsymbol{\mu} = \boldsymbol{\mu} * \underline{\Sigma}^{-1}\boldsymbol{\mu} \tag{10}$$

$$B = \mu^T\underline{\Sigma}^{-1}\mathbf{1} = \boldsymbol{\mu} * \underline{\Sigma}^{-1}\mathbf{1} = \mathbf{1} * \underline{\Sigma}^{-1}\boldsymbol{\mu} \tag{11}$$

$$C = 1^T\underline{\Sigma}^{-1}\mathbf{1} = \mathbf{1} * \underline{\Sigma}^{-1}\mathbf{1} \tag{12}$$

$$D = AC - B^2 \tag{13}$$

5.3.2 The Global Minimum Variance Portfolio

In Analysis 1, the Global Minimum Variance portfolio is constructed. The portfolio is rebalanced on a monthly basis, from January 2019 to December 2021. To construct the portfolio, an expanding estimation window is used, as noted in section 5.2.3. Two portfolios are constructed, one where cryptocurrencies are available to invest in and one where cryptocurrencies are not available to invest in. For both portfolios, the performance is measured out-of-sample, with return calculated as in equation (5).

The Global Minimum Variance portfolio is the portfolio of risky assets that has the lowest variance of all possible combinations of portfolios, hence the name "global". As the portfolio with the lowest variance of all portfolios, the Global Minimum Variance is a mean-variance efficient portfolio by definition, as the return of the portfolio can not be achieved with a lower variance (Markowitz, 1959). The Global Minimum Variance portfolio can be seen visually as the red point marked in Figure 8.

The Global Minimum Variance portfolio is the solution to the following minimization problem:

$$\min_{\boldsymbol{\pi}} \boldsymbol{\pi} * \underline{\Sigma} \boldsymbol{\pi}$$

$$s.t. \boldsymbol{\pi} * \mathbf{1} = 1$$

There is no constraint on the expected return, as the aim is only to minimize variance. The solution to the minimization problem is (Munk, 2020):

$$\boldsymbol{\pi}_{min} = \frac{1}{C} \underline{\Sigma}^{-1} \mathbf{1} = \frac{1}{\mathbf{1} * \underline{\Sigma}^{-1} \mathbf{1}} \underline{\Sigma}^{-1} \mathbf{1} \tag{14}$$

this closed form solution is applied in Analysis 1.

The Global Minimum Variance portfolio does not depend on expected return, only variances, so naturally the portfolio will prefer securities with lower standard deviations to securities with higher standard deviations. The portfolio however can also have a relatively large weight on an asset with a high standard deviation, if the asset has low correlation to a lower standard deviation asset, as that potentially creates diversification benefits (Munk, 2020).

5.4 Analysis 2

Analysis 2 investigates the effect of including investment in cryptocurrencies in a Tangency portfolio. The Tangency portfolio is part of the mean-variance framework first developed by Markowitz (1952) and Markowitz (1959). The analysis is conducted by forming portfolios based on two universes of investable securities, first without cryptocurrencies and then with cryptocurrencies. The portfolio weights are then rebalanced monthly based on data available at each portfolio rebalancing point in time. The analysis takes the viewpoint of an U.S. investor diversifying across asset classes.

5.4.1 Theoretical Background of Analysis 2

The Tangency portfolio is the portfolio that maximizes the Sharpe Ratio (Elton & Gruber, 1997). It introduces a mean-variance analysis with both risky assets and a risk-free asset. By investing in a Tangency portfolio, an investor can combine any portfolio of risky assets with an investment in a risk-free asset (Bodie et al., 2014). Figure 9 illustrates the mean-variance frontier of all assets, the mean-variance frontier of risky assets, and the location of the Tangency portfolio. The risk-free asset corresponds to the point $(0, r_f)$ in the figure. The Tangency portfolio corresponds to the point where the line starting at $(0, r_f)$ touches the mean-variance frontier of risky assets (indicated by a triangle). The underlying assumption is that the investor prefers points to the “north west” in Figure 9: a high expected return and a low standard deviation (Munk, 2020).

For combinations with a positive weight on both the risk-free asset and the Tangency portfolio, the straight line between the point $(0, r_f)$ and the point (σ, μ) indicates the (standard deviation, mean) pairs that can be obtained by a combination in the risky asset and the risk-free asset, where the point (σ, μ) corresponds to the portfolio of risky assets (Munk, 2020). The slope of the line is the Sharpe Ratio of the risky portfolio (Bodie et al., 2014). If the weight for the risky portfolio is negative the line starting at $(0, r_f)$ will have a slope equal to minus the Sharpe Ratio. The Sharpe Ratio is positive as long as $\mu > r_f$. It is important whether the risk-free rate r_f is greater than or smaller than the expected return on the Global Minimum Variance portfolio, which is $\mu_{min} = B/C$. Three different scenarios are presented; $\mu_{min} > r_f$, $\mu_{min} < r_f$, and $\mu_{min} = r_f$.

If the risk-free rate is smaller than μ_{min}, $(B > Cr_f)$ the Tangency portfolio is located on the upward sloping branch on the mean-variance efficient frontier of risky assets, where it is located on figure 9 (a). In this scenario, the highest Sharpe Ratio is captured on the tangency line, where the points on the line are acquired by combining a long position in the Tangency portfolio with the risk-free asset. However, if the risk-free rate is larger than μ_{min}, $(B < Cr_f)$ the Tangency portfolio is located on the downward sloping branch of the mean-variance frontier of risky assets, as can be seen on figure 9 (b). The points on this line are captured by combining a short position in the Tangency portfolio with a position of more than 100% in the risk-free asset. The highest Sharpe Ratio in this case is obtained on the upwards sloping straight line (Munk, 2020). In both cases, the mean-variance efficient portfolios of all assets are combinations of the risk-free asset and the Tangency portfolio of risky assets (Munk, 2020).

The case in which the risk-free rate equals the expected return on the Global Minimum Variance portfolio $(B = Cr_f)$, the mean-variance efficient portfolio has 100% invested in the risk-free asset and the Tangency portfolio doesn't exist. However, for this to be obtained the variances, expected returns and covariances must be selected. Therefore, this scenario is not interesting in practice (Munk, 2020).

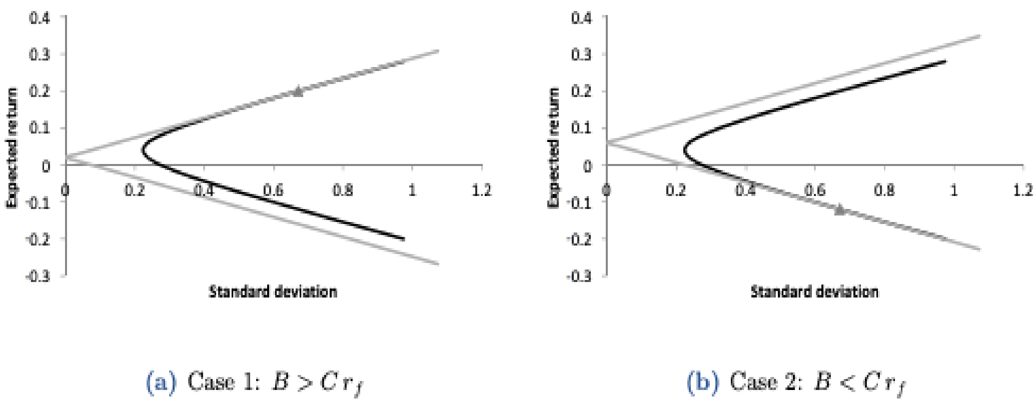

Figure 9: Two cases of efficient frontiers (Munk, 2020)

5.4.2 The Tangency Portfolio

In Analysis 2, the Tangency portfolio is constructed. The portfolio is rebalanced on a monthly basis, from January 2019 to December 2021. To construct the portfolio, an expanding estimation window is used, as noted in section 5.2.3. Two portfolios are constructed, one where cryptocurrencies are available to invest in and one where cryptocurrencies are not available to invest in. For both portfolios, the performance is measured out-of-sample, with return calculated as in equation (5).

Here, $B \neq Cr_f$ is assumed. The portfolio weight vector, $\boldsymbol{\pi}_{tan}$ can be calculated as follows (Munk, 2020):

$$\boldsymbol{\pi}_{tan} = \frac{\underline{\Sigma}^{-1}(\mu - r_f\mathbf{1})}{\mathbf{1} * \underline{\Sigma}^{-1}(\mu - r_f\mathbf{1})} = \frac{1}{B - Cr_f}\underline{\Sigma}^{-1}(\mu - r_f\mathbf{1}) \tag{15}$$

It can be expected, in the case of $\mu_{min} > r_f$, where the Tangency portfolio is the one maximizing the Sharpe Ratio that single assets with greater Sharpe Ratios would weigh higher in the Tangency portfolio. However, diversification comes into play and the Tangency portfolio might allocate large weight to assets with a low Sharpe Ratio if that particular asset has low correlation with assets having higher Sharpe Ratios. This shows the importance of correlations and is important to diversify the risk away (Munk, 2020).

An investor with access to a risk-free asset might nevertheless be interested in the efficient frontier of risky assets. Considering that the Tangency portfolio lies on the efficient frontier of risky assets, the entire frontier of risky assets can be generated by contemplating a number of combinations of the Tangency portfolio and another known frontier portfolio, like the Global Minimum Variance portfolio (Munk, 2020).

Investors´ willingness to take risk varies. If all investors had the same thoughts on the expected returns, variances, and covariances of the risky assets, and agree on the risk-free rate, all of them would have the same idea on the composition of the Tangency portfolio and the location of the mean-variance efficient frontier and would therefore hold the same portfolios (Munk, 2020).

5.5 Analysis 3

Analysis 3 investigates the effect of including investment in cryptocurrencies in an optimally weighted portfolio. The Optimal portfolio is part of the mean-variance framework first developed by Markowitz (1952) and Markowitz (1959). The analysis is conducted by forming portfolios based on two universes of investable securities, first without cryptocurrencies and then with cryptocurrencies. The portfolio weights are then rebalanced monthly based on data available at each portfolio rebalancing point in time. The analysis takes the viewpoint of an U.S. investor diversifying across asset classes.

5.5.1 Theoretical Background of Analysis 3

The underlying assumption of mean-variance portfolio analysis is that investors only look at two parameters when making investment decisions, that is the expectation (expected return) and the variance. However, investors differ in their appetite towards risk. Some investors are willing to take on higher level of risk in the hope of achieving higher returns on their investments, while other investors have less aptitude for risk as they do not want to take the chance of losing their money. This trade-off depends on the risk aversion of investors.

The trade-off of a mean-variance optimizing investor is often characterized in a utility function. If an investor starts with a wealth of W_0 and chooses the portfolio weights as $\boldsymbol{\pi} = (\pi_1, \pi_2, \pi_3, \dots, \pi_N)$, the investors future wealth, say at the end of one period, can be written as $W_1 = W_0 * (\pi_1(1 + r_1) + \pi_2(1 + r_2) + \pi_3(1 + r_3) + \cdots + \pi_N(1 + r_N) + (1 + r_f))$ where $r_1 \dots r_N$ are the returns of each asset invested in, and the last term, $(1 + r_f)$ is the return of a risk-free investment, either money invested at the risk-free rate or borrowed at the risk-free rate (Pedersen, 2015). A utility function, $u(W)$, has values for all the possible outcomes of future period wealth. For an investor, the goal is to maximize his expected utility of all possible portfolios (Munk, 2020).

Utility functions, $u(W)$, are generally increasing, $u'(W) > 0$. The fact that utility functions are increasing refers to the fact that investors generally prefer more wealth to less. Utility functions are also generally concave, $u''(W) < 0$. If the utility function is concave, that entails two things. First, the marginal utility of wealth is decreasing with wealth, an increase in wealth for an investor who has more wealth increases his utility less compared to if he had less wealth, and secondly, the investor is risk averse, as the investor does not take a gamble where the expected profit is zero (Munk, 2020).

The trade-off between an investor´s mean-variance choices can be represented by indifference curves in a (σ, μ) graph. On the same indifference curves, investors are equally well off, they have the same utility at all points. Investors who are mean-variance optimizers will have indifference curves who are increasing, as when standard deviation is increased, the investor will demand a higher expected return on his investment. Further, an indifference curve of a mean-variance optimizing investor is concave, as the investor demands high increase in expected return when standard deviation is already high. This is illustrated in the following figure (Munk, 2020):

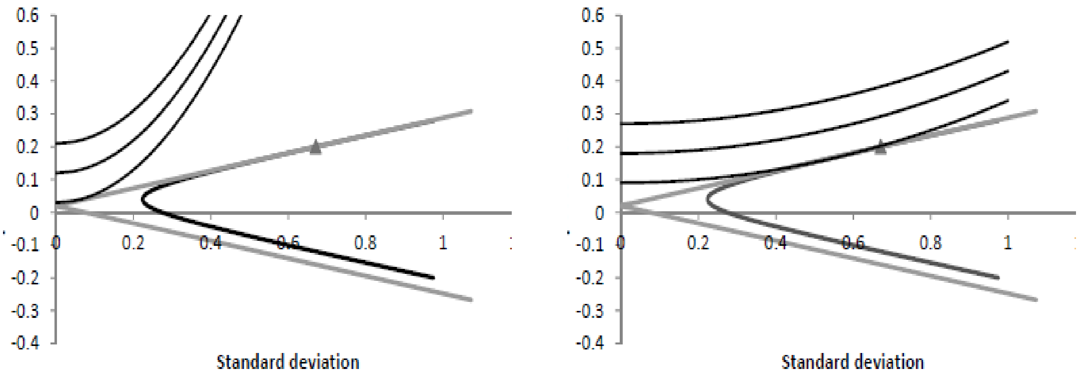

Figure 10: Indifference curves and Optimal Portfolios (Munk, 2020)

The figure shows a mean-variance efficient frontier, both of risky assets and for all assets (as in figures 8 and 9), as well as indifference curves. The left side shows the indifference curves of a risk averse investor, which is characterized by the indifference curves´ steepness. For a relatively low level of standard deviation, to increase the standard deviation, the risk averse investor demands a relatively large increase in expected return. The right side of the figure shows the indifference curves of an investor with relatively lower risk aversion. As the figure shows, the less risk averse investor has flatter indifference curves, reflecting that the investor does not demand a high increase in expected return when standard deviation increases, compared to the risk averse investor (Munk, 2020). The Optimal portfolio for an investor is where an indifference curve is tangent to the upward sloping part of the efficient frontier.

5.5.2 The Optimal Portfolio

In Analysis 3, the Optimal portfolio is constructed. The portfolio is rebalanced on a monthly basis, from January 2019 to December 2021. To construct the portfolio, an expanding estimation window is used, as noted in section 5.2.3. Six portfolios are constructed, three where cryptocurrencies are available to invest in and for risk aversion coefficients of 5, 10 and 15, representing a relatively risk seeking investor, a relatively risk neutral investor and a relatively risk averse investor, and three portfolios where cryptocurrencies are not available to invest in, using the same risk aversion coefficients as before. For all portfolios, the performance is measured out-of-sample, with return calculated as in equation (5).

The objective of the investor is to maximize expected future wealth, but with a penalty for risk. The objective function can thus be written as:

$$\max_{\pi} \ (E[r] - \frac{1}{2} * \ \gamma * Var[r])$$

Where $E[r]$ is the expected return, γ is a positive constant representing the risk aversion coefficient and $Var[r]$ is the variance of the portfolios return. Using portfolio theory notation and vector notation, the objective portfolio optimization function can be re-written as (Pedersen, 2015):

$$\max_{\pi} \ \boldsymbol{\pi} E(R_{\boldsymbol{e}}) - \frac{\boldsymbol{\gamma}}{\mathbf{2}} \boldsymbol{\pi} * \underline{\Sigma} \boldsymbol{\pi}$$

where $E(R_{\boldsymbol{e}})$ is the vector of expected excess return of the portfolio and $\boldsymbol{\pi} * \underline{\Sigma}\boldsymbol{\pi}$ is the variance of the portfolio, as defined before. To solve the optimization problem, the first order conditions are derived and set equal to zero (Pedersen, 2015):

$$0 = E(R_{\boldsymbol{e}}) - \ \boldsymbol{\gamma} \underline{\boldsymbol{\Sigma}} \boldsymbol{\pi}$$

The unconstrained solution to the maximization problem is the Optimal portfolio and can be written as (Pedersen, 2015):

$$\boldsymbol{\pi} * = \ \gamma^{-\mathbf{1}} \underline{\Sigma}^{-1} E(R_{\boldsymbol{e}}) \tag{16}$$

where $\boldsymbol{\pi} *$ is the vector of optimal weights in the portfolio. The solution can also be rewritten as (Munk, 2020):

$$\boldsymbol{\pi} * = \frac{1}{\gamma} \underline{\Sigma}^{-1} (\boldsymbol{\mu} - r_f \mathbf{1}) \tag{17}$$

This solution is not constrained in any way. If the portfolio weights sum up to more than one, an investor would need to borrow, often assumed at the risk-free rate. If the portfolio weights on the other hand sum up to less than one, an investor would invest the difference in the risk-free asset. The unconstrained Optimal portfolio can thus take very large positions in assets with relatively large expected returns, lower variance and low correlations to other assets, taking large short positions in securities that have the opposite characteristics and using borrowing (Munk, 2020). To encounter this, a constraint is introduced, that the weights of the portfolio must sum to one, $\boldsymbol{\pi} * \mathbf{1} = 1$. The optimization problem is then redefined as:

$$\max_{\pi} \left(E[r] - \frac{1}{2} * \gamma * Var[r]\right)$$

$$s.t. \boldsymbol{\pi} * \mathbf{1} = 1$$

The numerical solution of this constrained maximization problem is not covered in this analysis. To generate a solution at each portfolio rebalancing date, Excel's Solver and Visual Basic Analysis tool is used to compute the weights. The objective function is encoded in a cell with inputs from the estimation period, in sample, and maximized by varying the portfolio weights, constraining them to sum to one. That solution portfolio is then held for the following month and repeated over the analysis time period.

5.6 Analysis 4

Analysis 4 investigates the effect of including investment in cryptocurrencies in an optimally weighted portfolio with adjustments to the parameters of the model. The Bayes-Stein Optimal portfolio framework is an extension that tries to encounter the weaknesses of the mean-variance framework first developed by Markowitz (1952) and Markowitz (1959). The analysis is conducted by forming portfolios based on two universes of investable securities, first without cryptocurrencies and then with cryptocurrencies. The portfolio weights are then rebalanced monthly based on data available at each portfolio rebalancing point in time. The analysis takes the viewpoint of an U.S. investor diversifying across asset classes.

5.6.1 Theoretical Background of Analysis 4

Although the Markowitz mean variance optimization framework is a cornerstone of modern portfolio theory, the methods applied in the framework have been subject to criticism and weaknesses have been pointed out. In particular, criticism has been aimed at the fact that the estimates used in the model are exposed to estimation errors (Kolm, Tutuncu, Fabozzi, 2014; Ziemba & Mulvey, 1998; Platanikis & Urquhart, 2020). Estimation errors in the inputs can significantly impact the resulting portfolio weights, resulting in portfolios that are not well diversified and have a poor out-of-sample performance, while often having extreme and non-obtainable weights on some assets or asset classes (Black & Litterman, 1991).

Like earlier mentioned, the mean-variance framework solely rests on two parameters, the expected returns of the securities, the variance of the expected returns. The estimates for these two moments are often computed using historical return time series, but the problem with using historical time series as approximations can be problematic as in practice, they can be unstable, imprecise and subject to estimation errors, and in particular the estimation of the expected returns (Pedersen, 2015).

The performance of mean-variance optimized portfolios in practice has historically not been good, and practitioners consistently find that even the simple 1/N weighted portfolio beats the mean-variance optimized portfolio (Pedersen, Babu & Levine, 2021). The literature on how to best counter the noisy estimators for risk and return to construct a more robust Optimal portfolio than the standard mean-variance framework is vast, but most of the literature centers around a main principle: to shrink the estimates.

Black & Litterman (1991) and (1992) introduced a modified version of the standard mean-variance optimization approach by allowing the investor to incorporate his own views of the returns of assets or asset classes in to the model. The Black-Litterman procedure relies on two sources of information to compute the estimators in the optimizing procedure, the investors subjective views of returns, and a reference portfolio, which is used to compute neutral returns of the securities (Platanikis & Urquhart, 2020). The Black-Litterman procedure also includes a modification of the variance-covariance matrix of returns.

The Bayes-Stein portfolio approach (Jorion, 1986) aims to shrink the estimates used in the portfolio optimization procedure. Instead of using the sample mean as an estimator for expected returns, a shrinkage is applied to shrink the estimates to a common value. Jorion (1986) notes that the sample mean is exposed to taking extreme values that skew the results of portfolio optimization, and that applying the shrinkage to the estimator both reduces estimation risk as well as minimizing the impact of estimation risk on the results of the optimization process (Jorion, 1986). The Bayes-Stein approach is robust and has been widely used in previous research concerning portfolio optimization such as Bessler, Opfer & Wolff (2017) and Platanikis & Sutcliffe (2020).

5.6.2 The Bayes-Stein Optimal Portfolio

In Analysis 4, the Optimal portfolio is constructed, but with a modification of the input parameters based on the Bayes-Stein shrinkage portfolio approach, which is an extension of the standard mean-variance approach by Markowitz. The portfolio is rebalanced on a monthly basis, from January 2019 to December 2021. To construct the portfolio, an expanding estimation window is used, as noted in section 5.2.3. Six portfolios are constructed, three where cryptocurrencies are available to invest in and for a risk aversion coefficient of 5, 10 and 15, representing the relatively risk seeking investor, a relatively risk neutral investor and a relatively risk averse investor, and three portfolios where cryptocurrencies are not available to invest in, using the same risk aversion coefficient as before. For all portfolios, the performance is measured out-of-sample, with return calculated as in equation (5).

The Bayes-Stein column vector of expected returns is a modified version of the previous definitions of expected returns and based on a shrinkage estimation of past returns (Platanikis & Urquhart, 2020). The column vector, μ_{BS}, is computed as:

$$\mu_{BS} = (1-g)\boldsymbol{\mu} + g\mu_G\mathbf{1} \tag{18}$$

where $\boldsymbol{\mu}$ is the original vector of expected returns, μ_G is the return of the Global Minimum Variance Portfolio at the time of rebalancing over the estimation period, and g is shrinkage factor, which is a strictly positive number between 0 and 1, $(0 \leq g \leq 1)$. The shrinkage factor is computed as follows (Jorion, 1986):

$$g = \frac{N+2}{(N+2) + T(\mu - \mu_G\mathbf{1})^{\boldsymbol{T}}\underline{\boldsymbol{\Sigma}}^{-\mathbf{1}}(\mu - \mu_G\mathbf{1})} \tag{19}$$

here, N is the number of assets included in the analysis ant T is the length of the estimation period. As before, $\underline{\Sigma}$ is the variance-covariance matrix of returns. This function of μ_{BS} and g minimizes the loss function that results from the utility loss due to estimation risk. According to Jorion (1986) the modified expected returns vector has uniformly lower risk than the sample mean returns, which is usually used to compute expected returns. The modified Bayes-Stein variance covariance matrix, $\underline{\Sigma}_{BS}$ is computed as (Jorion, 1986):

$$\Sigma_{BS} = \frac{T+\varphi+1}{T+\varphi}\,\underline{\Sigma} + \frac{\varphi}{T(T+\varphi+1)}\,\frac{\mathbf{1}\mathbf{1}^{\boldsymbol{T}}}{\mathbf{1}^{\boldsymbol{T}}\boldsymbol{\Sigma}^{-\mathbf{1}}\mathbf{1}} \tag{20}$$

where T is the length of the estimation period, and φ is defined as (Jorion, 1986):

$$\varphi = \frac{N+2}{(\mu-\mu_G\mathbf{1})^{\boldsymbol{T}}\underline{\boldsymbol{\Sigma}}^{-\mathbf{1}}(\mu-\mu_G\mathbf{1})} \tag{21}$$

The optimization problem is then the same as before, which can be written as:

$$\max_{\pi}\ (E[r]-\frac{1}{2}*\ \gamma*Var[r])$$

$$s.t.\boldsymbol{\pi}*\mathbf{1}=1$$

and rewriting the optimization problem to illustrate the functions of the Bayes-Stein estimators:

$$\max_{\pi}\ (\boldsymbol{\pi}^{\boldsymbol{T}}\mu_{BS}-\frac{\gamma}{2}\ \boldsymbol{\pi}*\underline{\boldsymbol{\Sigma}}_{BS}\boldsymbol{\pi})$$

$$s.t.\boldsymbol{\pi}*\mathbf{1}=1$$

The solution portfolio weights are then held for one month before repeating the procedure and rebalancing the portfolio. To generate a solution at each portfolio rebalancing date, Excel Solver is used to compute the weights. The objective function is encoded in a cell with inputs from the estimation period, in sample, and maximized by varying the portfolio weights, constraining them to sum to one.

5.7 Analysis 5

Analysis 5 investigates the effect of including investment in cryptocurrencies in an Equally Weighted portfolio amongst the asset classes available for investment. The analysis is conducted by forming portfolios based on two universes of investable securities, first without cryptocurrencies and then with cryptocurrencies. The portfolio weights are not rebalanced monthly as in the preceding analyses, as the allocation of capital to the asset classes is constant. The analysis takes the viewpoint of an U.S. investor diversifying across asset classes.

5.7.1 Theoretical Background of Analysis 5

The literature of portfolio theory and optimizing portfolios is vast and a lot of methods are available to an investor at each time. However, because of the methods sensitivity to estimation errors, many of them do not perform well in practice. The weaknesses of traditional optimized portfolios, such as the man-variance optimization framework often leads to it not being used in practice (Pedersen, Babu & Levine, 2021).

Because of the weaknesses of traditional portfolio optimization techniques, various methods have been proposed to counter them, such as the Bayes-Stein shrinkage approach covered in section 5.6. However, for all the research and methods proposed, it has proved difficult for practitioners and researchers to come up with a solution that can beat the Equally Weighted portfolio on a consistent basis (DeMiguel, Garlappi & Uppal, 2009). An Equally Weighted portfolio, or naïve diversification, has shown tendencies to consistently beat the optimized portfolios in practice.

In their research, DeMiguel, Garlappi & Uppal (2009) construct 14 optimized portfolios using both the traditional mean-variance approach as well as using more recent and sophisticated methods. The methods include among others a mean-variance optimized portfolio, a Bayesian diffuse-prior approach optimized portfolio, a Bayes-Stein shrinkage approach optimized portfolio and a combination of the optimized portfolios. The 14 portfolios constructed are tested for 7 different datasets of different asset classes and securities, as well as tested using two lengths of estimation windows.

Over all the portfolios constructed and using three different performance measures, not one portfolio consistently outperformed the Equally Weighted portfolio. The result of DeMiguel, Garlappi & Uppal (2009) indicates that the potential gain from optimally diversifying a portfolio across asset classes and securities is outweighed by estimation errors. Further, by using simulations, DeMiguel, Garlappi & Uppal (2009) estimated that for a portfolio of 25 assets, about 3,000 months of estimation data is required for optimized portfolios, both using the mean-variance framework and extensions of it, to consistently outperform an Equally Weighted portfolio. Similarly, for a portfolio consisting of 50 assets, 6,000 months of estimation data was required to achieve a consistent outperformance of the Equally Weighted portfolio (DeMiguel, Garlappi & Uppal, 2009).

5.7.2 The Equally Weighted Portfolio

In Analysis 5, the Equally Weighted portfolio is constructed. The portfolio is not rebalanced on a monthly basis, like the previous portfolios, but the Equally Weighted portfolio does require trading to keep the weights constant and equal as the value of the assets in the portfolio changes. Two portfolios are constructed, one where cryptocurrencies are available to invest in and one where cryptocurrencies are not available to invest in and the performance of the portfolios is measured and compared.

The weight allocated to each asset is calculated as:

$$\pi_i = \frac{1}{N} \tag{22}$$

where N is the number of assets available for investment. This weight is kept constant over the whole holding period, from 2019 to 2021. By definition, the weights in the portfolio always sum to one. The Equally Weighted portfolio does not rely on any estimation of moments of returns and does not rely on any optimization procedures.

5.8 Analysis 6

Analysis 6 investigates the effect of including an investment in cryptocurrencies in a portfolio based on ideas by Qian (2005), Asness et al. (2011), and Qian (2011) where a portfolio is constructed based on the idea of Risk Parity. In the analysis, two portfolios are constructed, one where cryptocurrencies are available to invest in and one where cryptocurrencies are not available to invest in. The portfolio weights are then rebalanced monthly based on data available at each portfolio rebalancing point in time. The analysis takes the viewpoint of an U.S. investor diversifying across asset classes.

5.8.1 Theoretical Background of Analysis 6

Risk Parity investing first surfaced as an alternative to the classic 60-40 portfolio where 60% of capital is invested in equities and 40% of capital is invested in fixed income. Although such a portfolio is well diversified in Dollar terms, looking at the perspective of an U.S. investor, the portfolio is not well diversified when an investor looks at the risk contribution of each asset to the portfolio. As equities are considerably more volatile than bonds, the return of the portfolio will be significantly directed by the return of the equity component of the portfolio. When seen from this perspective, the 60-40 portfolio in fact does not yield a good diversification (Asness et al., 2011). According to Qian (2005) when a 60-40 equities-bonds portfolio loses over 2% of value, on average 95.6% of the loss is contributed by the equity portion of the portfolio.

Qian (2005) takes an example from a 60-40 portfolio constructed by investing in the Russell 1000 index and the Lehmann Aggregate Bonds Index for the period between 1983-2004. Although the Russell 1000 Index had the highest return in excess of the risk-free rate over the period, 8.3% it had the far highest standard deviation, resulting in a Sharpe Ratio of 0.55, compared to the Sharpe Ratio of 0.80 for the Lehmann Aggregate Bonds Index. A 60-40 portfolio of the two indices resulted in a Sharpe Ratio of 0.67, lower than the Sharpe Ratio of the bond index, an indicator of sub-par diversification.

5.8.2 The Risk Parity Portfolio

In Analysis 6, a Risk Parity portfolio is constructed. The portfolios are constructed by estimating the risk of each asset up until each portfolio rebalancing date. The portfolio is rebalanced on a monthly basis, from January 2019 to December 2021. To construct the portfolio, an expanding estimation window is used, as noted in section 5.2.3. Two portfolios are constructed, one where cryptocurrencies are available to invest in, and one portfolio where cryptocurrencies are not available to invest in. For both portfolios, the performance is measured out-of-sample, with return calculated as in equation (5).

A simple Risk Parity portfolio is constructed following the methodology of Asness et al. (2011) aiming to achieve a risk contribution that is equal among all asset classes. At the end of each month in the sample, the standard deviation of returns of all available asset classes, volatility is measured as:

$$SD_i = \sqrt{\frac{(x_i - \bar{x})^2}{n-1}} = \hat{\sigma}_i \tag{23}$$

The weight of each security, $\pi_{t,i}$, in the portfolio to be held the following month is then computed as (Asness et al., 2011):

$$\pi_{t,i} = k_t * \hat{\sigma}_{t,i}^{-1} \tag{24}$$

here, k_t is a constant number for all securities that indicates the level of leverage of the Risk Parity portfolio. In this analysis, leverage is not considered, so k_t is computed each month for the unlevered portfolio as (Asness et al., 2011):

$$k_t = \frac{1}{\sum_i \hat{\sigma}_{t,i}^{-1}} \tag{25}$$

not using any leverage and computing k_t as in equation 25, leads to monthly weights, $\pi_{t,i}$, of:

$$\pi_{t,i} = \frac{\hat{\sigma}_{t,i}^{-1}}{\sum_i \hat{\sigma}_{t,i}^{-1}} \tag{26}$$

This computation of the monthly weight leads to a simple value-weighted portfolio that takes larger positions in low volatility securities and smaller positions in securities that have higher volatility. By definition, the portfolio weights will sum to one, with long positions only. What this entails for this analysis is that the positions in the more volatile securities, equities and cryptocurrencies will be smaller relatively to some of the analyses discussed earlier, but yields an interesting viewpoint of the effects of including cryptocurrencies in a portfolio where risk is directly controlled for.

5.9 Performance Measures

In order to evaluate if cryptocurrencies affect portfolio performance, several performance measures will be computed. The performance measures will be used to compare two corresponding portfolios. The first portfolio consists of all assets considered in the analysis, except cryptocurrencies, more specifically: S&P500 (GSPC), GOVT, USO, LQD, and GLD, and a portfolio consisting of all assets, including cryptocurrencies, namely the CRIX index. The performance measures for the two portfolios will be calculated over the same time horizon, from January 2019 to December 2021. The performance measures will be computed using monthly data and will be converted into equivalent annualized units.

5.9.1 Sharpe Ratio

In his paper "Mutual Fund Performance" Sharpe (1966) introduced a measure for the performance of mutual funds and presented the term reward-to-variability ratio to describe it, which has been termed the Sharpe Ratio (Sharpe, 1966). Sharpe has contributed tremendously to modern investment theory and was awarded the 1990 Nobel Prize in Economics (Munk, 2020). Investors attempt to find the portfolios promising the greatest expected return for any given degree of risk. The Sharpe Ratio is used to recognize the return of an investment compared to its risk (Sharpe, 1966). It is defined as the ratio between the risk premium and the standard deviation and is often used to quantify the risk-return tradeoff and shows the reward, in terms of extra expected return, per unit of risk. The Sharpe Ratio can be calculated as (Munk, 2020):

$$SR = \frac{E[r]-rf}{\sigma_r} \tag{27}$$

where the excess return: $E[r] - rf$, which is computed using monthly returns and monthly risk-free rates, can be annualized by multiplying it by $n = 12$ (Pedersen, 2015):

$$E[r]^{annual} = E[r] * n \tag{28}$$

$$rf^{annual} = rf * n \tag{29}$$

note that this measure of annualizing returns does not consider compounding of returns. The annual standard deviation scales with the square root of $n = 12$ (Pedersen, 2015):

$$Std\ [r]^{annual} = Std\ [r] * \sqrt{n} \tag{30}$$

Once each component of the risk measure has been annualized, the overall annualized risk measure can be computed. To compute the annualized Sharpe Ratio, the annualized excess return and annualized standard deviation are used (Pedersen, 2015).

Academics and practitioners argue that the standard deviation, where both the upside and downside deviations are included, is not a relevant measure of risk for various investment situations because they don't capture what is at stake (Sortino and Meer, 1991). According to Sortino and Meer (1991) the downside variance is the superior risk measure for many investment situations.

5.9.2 Sortino Ratio

In financial markets, uncertainty is described in terms of a range of possible returns and their chances of occurring, a probability distribution. The shape of the probability distribution is most often a bell shape, or a normal distribution (Sortino and Meer, 1991), which indicates that the standard deviation is a complete risk measure and therefore the Sharpe Ratio is a complete measure of portfolio performance (Bodie et al., 2014). However, in practice, distributions are frequently not normal (Sortino and Meer, 1991), but the normal distribution is often used as an approximation. Therefore, Frank A. Sortino advocated the use of downside risk in investment decisions, instead of using the total standard deviation of portfolio returns.

The Sortino ratio is computed like the Sharpe Ratio, only the total standard deviation is replaced by the lower partial standard deviation. Realizations below either the expected value or below the risk-free return over the same period are used (Bodie et al., 2014; Munk, 2020). The Sortino ratio is considered to give a better view of a portfolio's risk-adjusted performance since positive volatility is a benefit (Sortino and Meer, 1991). Like the Sharpe Ratio, a higher Sortino ratio is better when comparing two similar investments. The Sortino ratio can be calculated as:

$$SR = \frac{E[r]-rf}{\sigma_d} \tag{31}$$

where σ_d is the standard deviation of the downside returns. In this analysis, negative excess returns, or excess returns below zero will be considered when calculating the lower partial standard deviation.

5.9.3 High-Water mark and Drawdown

A high-water mark is the highest price P_t (or highest cumulative return) a fund or a portfolio has achieved in the past (Pedersen, 2015):

$$HWM_t = max_{s \leq t} P_s \tag{32}$$

As an example, hedge funds often charge performance fees only when their returns are above their HWM (Pedersen, 2015).

The drawdown (DD) is the cumulative loss since losses started and is an important risk measure for hedge funds that can be applied to portfolios. In other words, it is the amount in percentage that has been lost since the peak (the HWM) (Pedersen, 2015). The percentage drawdown since the peak is given by (Pedersen, 2015):

$$DD_t = (HWM_t - P_t)/HWM_t \tag{33}$$

where P_t is the cumulative return at time t. If a fund or a portfolio is at its peak, the drawdown is zero, otherwise it is a positive number (Pedersen, 2015). The drawdown can be measured over a specific time period, and in this analysis, it will be measured over the period from January 2019 to December 2021 and will be calculated each month. It is costly and risky when a fund or a portfolio experiences large drawdowns (Pedersen, 2015). The maximum drawdown (MDD) will be computed as well and can be calculated as (Pedersen, 2015):

(34)

$$MDD_T = max_{t \leq T} DD_t$$

5.10 Methodological Assumptions and Limitations

Cryptocurrencies are a relatively young phenomenon and even younger in a context of an asset class. Other than Bitcoin, few are more than a few years old. Majority of previous research concerning cryptocurrencies as a portfolio component have only considered Bitcoin in a portfolio context, often combining it with indices that represent an asset class in a specific region, such as in Wu and Pandey (2014), Briere, Oosterlinck and Szafarz (2015), Eisl, Gasser and Weinmayer (2015), Gangwal (2017), Agarwal et al. (2018), Kajtazi and Moro (2019), Platanikis and Urquhart (2020), and Bakry et al. (2021).

In this analysis, the objective is to view cryptocurrencies as an investment in a wider scope. Combining indices representative of whole asset classes such as the S&P500 index and other similar indices for other asset classes with only one security of cryptocurrencies can underrepresent the characteristics of the asset class as a whole and therefore it was decided to include an index in the analysis, which also provides an internal diversification within cryptocurrencies. Although Bitcoin is a major component of the index throughout the analysis period, it always includes a minimum of five liquid cryptocurrencies (nine at the time of writing), which makes the return more diversified within the asset class.

This approach does yield a limitation for the analysis, as data availability of cryptocurrency indices is relatively limited compared to other asset classes. It is well documented that portfolio optimization techniques suffer from estimation errors, especially if the estimation period is short. In this analysis, to partly encounter that problem, an expanding estimation window is used instead of other techniques such as rolling estimation windows. Using expanding estimation windows increases the estimation data available with the passage of time and as a result should decrease estimation errors. At the end of the sample, 69 observations are used to estimate parameters which is in line with previous research in the field, but earlier periods do have fewer data points.

Another limitation of the analysis is that transaction costs are not considered, neither are possible scenarios of illiquidity of asset classes taken into account. However, the securities considered in this analysis are mostly ETF’s which usually are not subject to high transaction costs and have high liquidity. It is however noted that transaction costs and periods of less liquidity could affect the results of the analysis.

It is assumed in the analysis that an investor can observe opening prices on the first day of the month and trade on them contemporaneously without delay. In practice, this is difficult as the opening price is decided by the first transaction of the day. It is thus assumed that the investor can conterminously view this opening price and trade it immediately, both buying and selling the securities to rebalance the portfolio. Without very sophisticated software this is hard to do. However, as very large price movements after the opening price are relatively rare, as the opening price includes premarket activity and has already absorbed price jumps, which is why in this analysis it was deemed more accurate than the closing price the previous business day.

The index chosen; the Royalton CRIX Index is not a traded index. The index constituents are however displayed in their exact weights on the index's homepage, making computation of the weights needed to invest in each cryptocurrency for each portfolio straight forward. Although a limitation to the study, the index was deemed a good approximation for cryptocurrencies return as an asset class as it is professionally computed and rebalanced.

The analysis is limited to six asset classes; equities, government issued bonds, corporate issued bonds, oil, gold and cryptocurrencies. Although the securities used in the analysis are representative of the respective asset classes, it is noted that the weights computed in the optimized portfolios are indeed dependent on the specific returns of these securities, and a different composition of securities or addition or removal of asset classes would yield different results. The analyses are also exclusive to an U.S. based investor investing in USD denominated securities, as well as cryptocurrencies. According to Bodie, Kane and Marcus (2018) home country biased research, like this analysis is, is not necessarily generalizable to other countries. The U.S. market is the largest and most liquid financial market in the world and thus it was decided to only use U.S. securities. This limitation is however acknowledged.

6 Empirical Results

Section 6 reports the results of Analyses 1-6, outlined in section 5, Methodology approach. The section is structured in the following way: the results of each analysis with and without an investment in cryptocurrencies are presented and finally the main results of all the analyses are summarized in a table.

6.1 Empirical Results – Analysis 1

This section reports the empirical results of Analysis 1, which examines the effect of enabling an investment in an index of cryptocurrencies in a Global Minimum Variance portfolio in an out-of-sample setting. First, return statistics and performance measures are summarized and discussed, as well as graphical illustrations of portfolio performance over time. Statistics of portfolio weights during the holding period are then considered to illustrate the allocation to each portfolio constituent, with an emphasis towards the allocation to cryptocurrencies.

Table 4 illustrates descriptive statistics of the return of the Global Minimum Variance portfolio with and without an investment in a cryptocurrency index, as well as providing portfolio performance metrics.

Global Minimum Variance Portfolio

	With CC	**Without CC**
Average monthly excess return	0.35%	0.34%
Annualized excess return	4.23%	4.16%
St. Dev of Monthly Excess Returns	0.94%	0.94%
Annualized St. Dev of Excess Returns	3.26%	3.26%
Maximum Return	3.07%	3.07%
Minimum Return	-1.38%	-1.39%
Monthly Sharpe Ratio	0.37	0.36
Annualized Sharpe Ratio	1.29	1.28
Monthly Sortino Ratio	0.97	0.94
Annualized Sortino Ratio	3.35	3.27
Maximum Drawdown	1.77%	1.89%
Number of Losing Months	14	14

Table 4: Return statistics of the Minimum Variance Portfolio

As table 4 shows, the Global Minimum Variance portfolios with and without an investment in cryptocurrencies show positive monthly excess returns on average, as well as a low standard deviation of monthly excess returns, in line with expectations of such a portfolio, as only variances and covariances are considered in its construction. The portfolio without cryptocurrencies yields a slightly lower average monthly excess return compared to the portfolio with cryptocurrencies, however the volatility is the same for both portfolios.

The annualized Sharpe Ratio of the portfolio with cryptocurrencies is a bit higher than the Sharpe Ratio of the portfolio without cryptocurrencies, however the difference is not significant. Both portfolios show a good annualized Sharpe Ratio of 1.29 with cryptocurrencies and 1.28 without cryptocurrencies. Sharpe Ratios greater than 1 are generally considered good. The portfolio with an investment in cryptocurrencies also shows a very high annualized Sortino Ratio of 3.35.

The high Sortino Ratio is largely due to limited losses, with the portfolio delivering excess returns lower than -1% on only three occasions of the 14 negative return yielding months, with the largest single month loss being -1.38%, higher than the largest monthly loss of the portfolio without cryptocurrencies. This results in a low standard deviation of downside returns, increasing the Sortino Ratio. The annualized Sortino Ratio of the portfolio when an investor does not invest in cryptocurrencies is marginally lower compared to the portfolio with an investment in cryptocurrencies. Therefore, the portfolio with an investment in cryptocurrencies performs better in terms of the risk adjusted returns compared to the portfolio without an investment in cryptocurrencies.

The maximum drawdown from the portfolio’s high-water mark including cryptocurrencies is only 1.77%, lower compared to the portfolio without cryptocurrencies. Generally, the portfolios perform like one would expect the Global Minimum Variance portfolio to perform, with low volatility of excess returns and low drawdowns. Figure 11 illustrates the cumulative return of the minimum variance portfolios with and without an investment in cryptocurrencies over the holding period, with January 2019 set as 1.

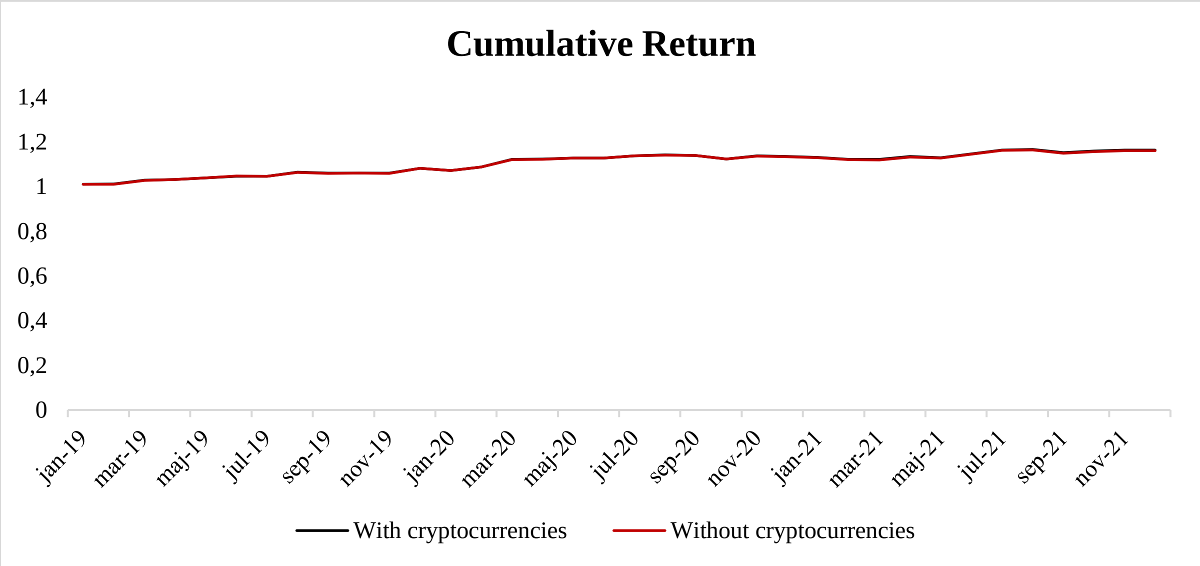

Figure 11: Cumulative Return of the Minimum Variance Portfolio

As figure 11 shows, the Global Minimum Variance portfolio has almost identical return patterns with and without an investment in cryptocurrencies. This stems from the construction of the portfolio, as only variances are taken into account, thus yielding a very low allocation to cryptocurrencies on average. The cumulative return index is relatively flat for both portfolios, with low volatility. In most months, the portfolio has low positive returns and low losses when they occur. The total return over the period from January 2019 to December 2021 is 16.28% for the portfolio with an investment in cryptocurrencies, and 16.02% for the portfolio without an investment in cryptocurrencies.

For both portfolios, the month with the largest return is March 2020, attributable to positive weight in the government bond index and a negative weight in the corporate bond index during a time of high market uncertainty amid the outbreak of COVID-19. For both portfolios, small drawdowns are experienced over the period, with low losses when they happen, resulting in a HWM line which is very similar to the total return index. The period between September 2020 and June 2021 has sustained drawdowns but as mentioned they are insignificant, with a maximum of 1.77% for the portfolio with cryptocurrencies and 1.89% for the portfolio without cryptocurrencies. Graphs showing the high-water mark and drawdowns of the portfolios computed can be seen in Appendix 1. Table 5 shows statistics on weights allocated to the securities in the portfolios over the period.

Portfolio Allocations – Global Minimum Variance Portfolio

With CC						
GMVP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	17.62%	116%	1.84%	-28.69%	-6.82%	0.05%
Std. Dev	1.19%	11.61%	0.30%	13.88%	1.96%	0.08%
Minimum	15.50%	104.45%	0.98%	-51.39%	-9.67%	-0.12%
Maximum	20.49%	134.95%	2.43%	-13.06%	-2.75%	0.16%
Without CC						
GMVP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	17.64%	115.86%	1.84%	-28.57%	-6.78%	0.00%
Std. Dev	1.18%	11.81%	0.31%	14.08%	1.91%	0.00%
Minimum	15.49%	104.12%	0.96%	-51.56%	-9.62%	0.00%
Maximum	20.48%	135.07%	2.43%	-12.68%	-2.79%	0.00%

Table 5: Minimum Variance Portfolio allocations

For both Global Minimum Variance portfolios, the largest weight is allocated to the Government Bond Index fund, GOVT, which is rather foreseeable as GOVT has the lowest volatility of all the available assets, as well as having relatively low correlations with the other assets. The weights are very comparable between the two portfolios, as the weight allocated to cryptocurrencies is very small on average for the portfolio including a cryptocurrency investment.

As shorting is not restricted, LQD and GLD have weights below zero in both portfolios, indicating that the short positions in these securities were used to finance other positions on average. As the Global Minimum Variance portfolio solely relies on variances in its construction, it is unsurprising that the CRIX index has a very low allocation on average, being the most volatile asset of the investable assets. The portfolios also have stable positive allocation to equities, where the portfolio without cryptocurrencies has a slightly higher allocation to the S&P500. As the only input into the computation of the Global Minimum Variance portfolio is variances, the weights tend to stabilize over time, as the estimation period becomes larger.

The Global Minimum Variance portfolio does not reap the benefits nor fall to the drawbacks of investing in cryptocurrencies as the theory does not allocate more than a small fraction of the funds to the cryptocurrency index because of its volatility. However, performance does marginally improve with the small investment in the cryptocurrency index.

6.2 Empirical Results – Analysis 2

This section reports the empirical results of Analysis 2, which examines the effect of enabling an investment in an index of cryptocurrencies in the Tangency portfolio in an out-of-sample setting. First, return statistics and performance measures are summarized and discussed, as well as graphical illustrations of portfolio performance over time. Statistics of portfolio weights during the holding period are then considered to illustrate the allocation to each portfolio constituent, with an emphasis towards the allocation to cryptocurrencies.

Table 6 illustrates descriptive statistics of the return of the Tangency portfolio with and without investment in a cryptocurrency index, as well as providing portfolio performance metrics.

Tangency Portfolio

	With CC	**Without CC**
Average monthly excess return	1.69%	0.98%
Annualized excess return	20.27%	11.73%
St. Dev of Monthly Excess Returns	7.24%	4.33%
Annualized St. Dev of Excess Returns	25.09%	15.01%
Maximum Return	36.04%	14.03%
Minimum Return	-9.72%	-10.16%
Monthly Sharpe Ratio	0.23	0.22
Annualized Sharpe Ratio	0.81	0.78
Monthly Sortino Ratio	0.67	0.43
Annualized Sortino Ratio	2.31	1.50
Maximum Drawdown	16.41%	11.61%
Number of Losing Months	16	14

Table 6: Return statistics of the Tangency Portfolio

The average monthly excess return of the Tangency portfolio with an investment in cryptocurrencies is positive, 1.69%, a high return but the return is volatile. The volatility of returns of the portfolio can be seen in the spread between the maximum and minimum return, a difference of more than 45 percentage points. The computation of the Tangency portfolio takes excess returns into account, therefore sometimes indicating large positions to be held in a month where returns are lower than in the estimation period, making it sensitive to return patterns and the weights relatively unstable over time.

The Tangency portfolio without an investment in cryptocurrencies has a positive average excess return, but considerably lower compared to the portfolio that includes an investment in cryptocurrencies. The standard deviation is also considerably lower, 15.01% on an annual basis compared to 25.09% in the portfolio including cryptocurrencies.

The annualized Sharpe Ratio of the Tangency portfolio including cryptocurrencies is 0.81, which is considered sub-optimal. It is higher than that of the Tangency portfolio without an investment in cryptocurrencies, which has an annualized Sharpe Ratio of 0.78. That indicates that the larger average excess return of the portfolio with cryptocurrencies outweighs the considerably larger standard deviation of returns when computing the Sharpe Ratio.

The annualized Sortino Ratio of the portfolio with cryptocurrencies is 2.31, which is considered a good Sortino Ratio, indicating that the standard deviation of downside returns is considerably lower compared to the standard deviation of all returns, with the minimum and maximum return also pointing towards greater variation in the upside compared to the downside. The portfolio without an investment in cryptocurrencies has a Sortino Ratio of 1.50 on an annual basis, considered a good ratio, but the portfolio including cryptocurrencies also outperforms in terms of Sortino Ratio.

The worst month for the portfolio with cryptocurrencies is in February 2020, when it lost 9.72% of its value, attributable to its positive equity weight in a month were equity prices fell sharply amid the COVID-19 outbreak. The maximum drawdown of the portfolio is 16.41%, at the end of September 2019, at the back of a three-month dive in cryptocurrency prices where the portfolio allocated positive weights to the CRIX index.

The maximum drawdown of the portfolio without an investment in cryptocurrencies occurs in March 2020, after the portfolio lost in two consecutive months during the outbreak of COVID-

19, in which it experienced its worst month of returns of -10.16%. Figure 12 shows the cumulative return index of the Tangency portfolios with and without an investment in cryptocurrencies over the holding period, where January 2019 is set to 1.

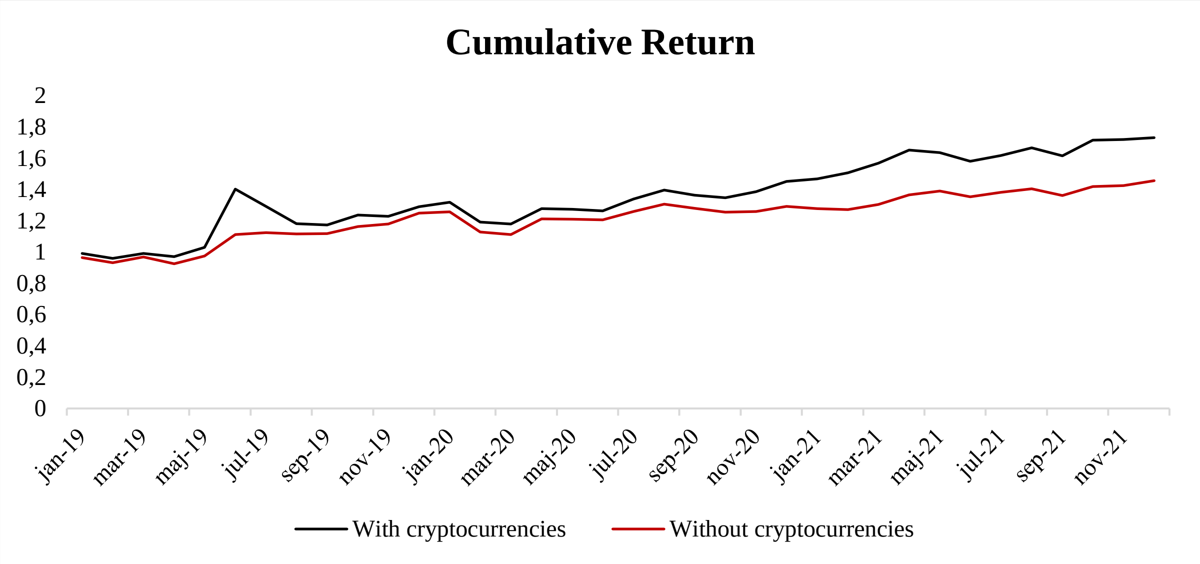

Figure 12: Cumulative Return of the Tangency Portfolio

The returns of both portfolios are volatile over the period, however the return of the portfolio without an investment in cryptocurrencies is less volatile. Both portfolios have an overall upward trending return over the period. For the portfolio with a cryptocurrency investment, high initial returns are followed by drawdowns. The portfolio does show a high total return over the holding period of 73.10%, but an investor would need to be able to cope with periods of considerable losses. The total return of the portfolio without an investment in cryptocurrency over the holding period is 41.02%, considerably lower compared to the portfolio with cryptocurrencies.

The portfolio with cryptocurrencies has a prolonged period of drawdowns in the first half of the holding period. The portfolio reaches a new HWM after June 2019 but falls sharply and does not reach the same cumulative return until in December 2020, a 17 month period of drawdowns. The portfolio performs better in the latter part of the sample, with lower drawdowns over shorter periods when they occur, and the portfolio ends the holding period at a new HWM.

From February 2020 to the beginning of 2021, drawdowns are frequent for the portfolio without an investment in cryptocurrencies. However, the drawdowns are not very large. This pattern can be seen during 2020 and 2021. The maximum drawdown of the Tangency portfolio without cryptocurrencies is 11.61%, lower compared to the Tangency portfolio with cryptocurrencies. Graphs showing the high-water mark and drawdowns of the portfolios computed can be seen in Appendix 2. Table 7 shows statistics on weights allocated to the securities in the portfolios over the period.

Portfolio Allocations – Tangency Portfolio

With CC						
TP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	76.67%	98.70%	-2.11%	-109.88%	27.75%	8.87%
Std. Dev	118.9%	63%	4.1%	137.3%	56.0%	15.7%
Minimum	-168.48%	-161.18%	-17.23%	-596.60%	-81.46%	-18.89%
Maximum	554.97%	172.77%	3.47%	201.84%	247.51%	72.53%
Without CC						
TP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	60.24%	84.97%	-3.52%	-64.88%	23.18%	0.00%
Std. Dev	72.10%	62.60%	3.98%	66.72%	41.45%	0.00%
Minimum	-156.60%	-75.89%	-14.63%	-141.67%	-90.94%	0.00%
Maximum	210.01%	207.12%	3.94%	141.92%	106.02%	0.00%

Table 7: Tangency Portfolio allocations

As table 7 shows, the Tangency portfolio with an investment in cryptocurrencies allocates relatively extreme weights to the securities, especially to S&P500, GOVT and LQD. Like noted earlier, this is because of the sensitivity to the excess return of each security, a few months run of large returns either positive or negative can result in very high weights in both directions, especially during the early months of the portfolio in 2019 when the estimation data is limited. This is, as mentioned earlier in this paper, one of the points of criticism of the Markowitz (1952) framework.

The extremes resulting from lack of estimators is evident in the data, as all maxima and minima occur within the first 14 months of holding the portfolio, and as time passes and the estimators become more stable through more available data, so do the allocations to the securities, where only LQD has a stable allocation of more than -100% after 2019. This is also a key reason for using expanding estimation windows in the analyses.

The CRIX index has considerable allocations throughout the period, with an average allocation of 8.87% and with a standard deviation of 15.7%, entailing that in 24 of the 36 months the allocation is between -6.87% and 24.62%. The portfolio thus has considerable exposure to cryptocurrencies.

What the extreme allocations can entail is volatile return through much higher exposure to single security returns, partly explaining the drawdowns in the first half of the period, when the allocation stabilizes towards the end of the sample, returns also become relatively less volatile.

The Tangency portfolio without cryptocurrencies allocates positive weights to the S&P500, GOVT and GLD on average, while it allocates negative weights to USO and LQD. The portfolio further takes the largest positive weight in GOVT, while taking the largest negative weight in LQD on average. Standard deviation of weights is high for all the securities except for USO, indicating variability in the weights, also illustrated by the spread of the minimum and maximum weight.

Like in the Tangency portfolio with an investment in cryptocurrencies, the weights allocated to the securities are substantially more unstable during the first months of the holding period of the portfolio compared to later periods. All the extreme minima and maxima occur within the first year of holding the portfolio, after which the weights stabilize. The resulting drawdowns in the portfolio are however lower compared to the portfolio with a cryptocurrency investment.

6.3 Empirical Results – Analysis 3

This section reports the empirical results of Analysis 3, which examines the effect of enabling an investment in an index of cryptocurrencies in the Optimal portfolio in an out-of-sample setting. First, return statistics and performance measures are summarized and discussed, as well as graphical illustrations of portfolio performance over time. Statistics of portfolio weights during the holding period is then considered to illustrate the allocation to each portfolio constituent, with an emphasis towards the allocation to cryptocurrencies.

Tables 8-10 illustrate descriptive statistics of the returns of the Optimal portfolios with and without an investment in a cryptocurrency index, for three risk aversion coefficients of $\gamma = 5$, $\gamma = 10$ and $\gamma = 15$, representing a relatively risk seeking investor, a relatively risk neutral investor and a relatively risk averse investor. Additionally, performance metrics are provided for each portfolio. Figures 13-15 illustrate the cumulative returns of the Optimal portfolios with and without an investment in cryptocurrencies, for three risk aversion coefficients.

Optimal Portfolio γ = 5

	With CC	**Without CC**
Average monthly excess return	4.43%	2.84%
Annualized excess return	53.20%	34.02%
St. Dev of Monthly Excess Returns	11.68%	10.42%
Annualized St. Dev of Excess Returns	40.46%	36.09%
Maximum Return	33.91%	35.28%
Minimum Return	-18.63%	-18.04%
Monthly Sharpe Ratio	0.38	0.27
Annualized Sharpe Ratio	1.31	0.94
Monthly Sortino Ratio	0.93	0.56
Annualized Sortino Ratio	3.21	1.95
Maximum Drawdown	28.07%	27.64%
Number of Losing Months	16	13

Table 8: Return statistics of the Optimal Portfolio with γ = 5

The Optimal portfolio of a relatively risk seeking investor with an investment in cryptocurrencies yields a high positive monthly excess return, higher than that of the portfolio without an investment in cryptocurrencies. The standard deviation of monthly excess returns is high for both portfolios, which can also be seen by looking at the spread between the maximum and minimum return, a difference of 52.5 percentage points for the portfolio with cryptocurrencies and 53.3 percentage points for the portfolio without cryptocurrencies.

As cryptocurrencies are the most volatile asset in the dataset, the portfolio with an investment in the asset has higher monthly volatility than the portfolio without an investment in it. However, the high return offsets the increased volatility, and the portfolio with cryptocurrencies yields a higher annualized Sharpe Ratio compared to the portfolio without cryptocurrencies, indicating a better performance in terms of risk adjusted returns. The portfolio further outperforms the portfolio without cryptocurrencies in terms of downside risk adjusted returns, as the portfolio with cryptocurrencies yields a substantially higher Sortino

Ratio on an annual basis compared to the one without cryptocurrencies. Figure 13 shows the cumulative return index of the Optimal portfolios with and without an investment in cryptocurrencies with $\gamma = 5$ over the holding period, where January 2019 is set to 1.

The cumulative return has an overall upward trend for both portfolios. The portfolio with an investment in cryptocurrencies experiences periods of high positive returns and large losses.

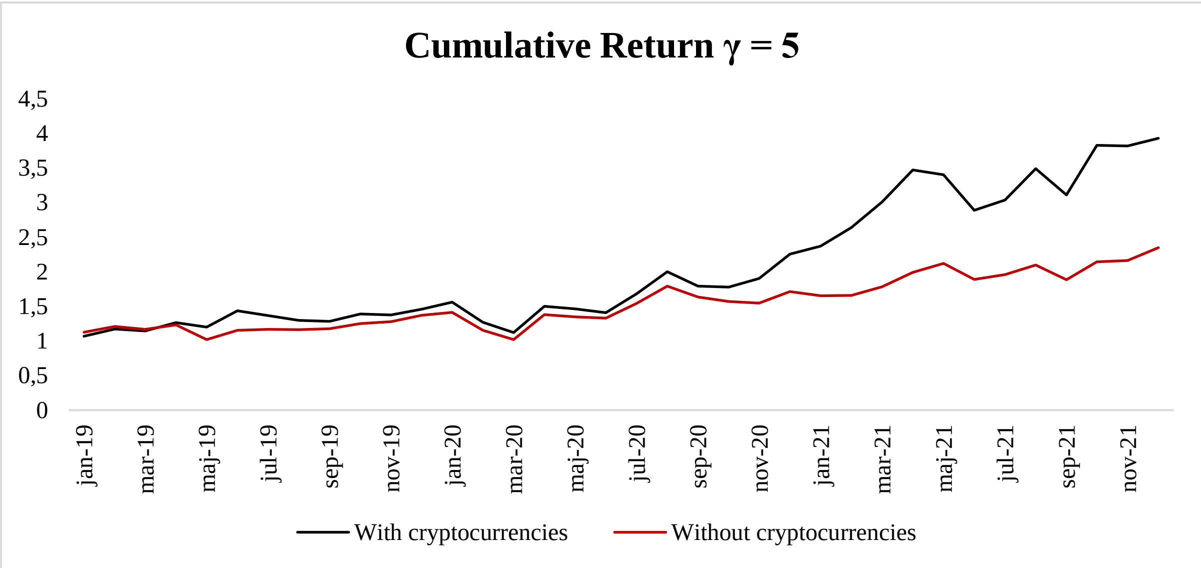

Figure 13: Cumulative Return of the Optimal Portfolio with $\gamma = 5$

Subsequently, there are periods of high volatility in returns. The portfolio shows a high total return over the holding period of 293%, but an investor would need to be able to cope with periods of considerable losses, as can be assumed by a relatively risk seeking investor. Comparing the two portfolios, it is evident that the cumulative return of the portfolio without cryptocurrencies is less volatile over the sample period. However, there are periods of high volatility and significant drawdowns. The portfolio without an investment in cryptocurrencies yields a total return of 135% over the period, substantially lower than the portfolio including cryptocurrencies. The month with the largest return is April 2020 for both portfolios, a return of 33.91% for the portfolio with cryptocurrencies, attributable to high positive weights allocated to securities that yielded high returns over the month, namely S&P500 and the CRIX Index.

The portfolio with cryptocurrencies has periods of significant drawdowns. It reaches a new HWM in January 2020 but yields the lowest return in February of -18.63% and a negative return in March, reaching the maximum drawdown of 28.07% after March 2020. The portfolio performs well between November 2020 and April 2021, with six months of positive returns. The portfolio reaches a new HWM in the last month of the holding period. Further, the portfolio without an investment in cryptocurrencies endured some drawdowns over the period, with the largest drawdown after March 2020 of 27.64%. A graph showing the high-water mark and drawdowns of each portfolio can be seen in Appendix 3.

Optimal Portfolio γ = 10

	With CC	**Without CC**
Average monthly excess return	2.39%	1.59%
Annualized excess return	28.70%	19.09%
St. Dev of Monthly Excess Returns	5.79%	5.18%
Annualized St. Dev of Excess Returns	20.06%	17.94%
Maximum Return	17.03%	17.71%
Minimum Return	-8.82%	-8.31%
Monthly Sharpe Ratio	0.41	0.31
Annualized Sharpe Ratio	1.43	1.06
Monthly Sortino Ratio	1.07	0.67
Annualized Sortino Ratio	3.71	2.31
Maximum Drawdown	12.27%	12.26%
Number of Losing Months	15	13

Table 9: Return statistics of the Optimal Portfolio with γ = 10

The Optimal portfolio of a relatively risk neutral investor with an investment in cryptocurrencies yields a higher excess monthly return compared to the portfolio without an investment in cryptocurrencies. The portfolio further has a higher volatility than the portfolio without cryptocurrencies. Both average monthly excess return and standard deviation of excess return decline with increase in γ compared to the corresponding portfolios.

Again, the portfolio with an investment in cryptocurrencies outperforms the portfolio without cryptocurrencies in terms of risk adjusted returns and downside risk adjusted returns, as the portfolio yields higher Sharpe- and Sortino Ratios on an annual basis. As γ increases, both portfolios yield higher annualized Sharpe- and Sortino Ratios compared to corresponding portfolios with γ = 5.

Figure 14 shows the cumulative return of the Optimal portfolios with and without an investment in cryptocurrencies with γ = 10 over the holding period, where January 2019 is set to 1.

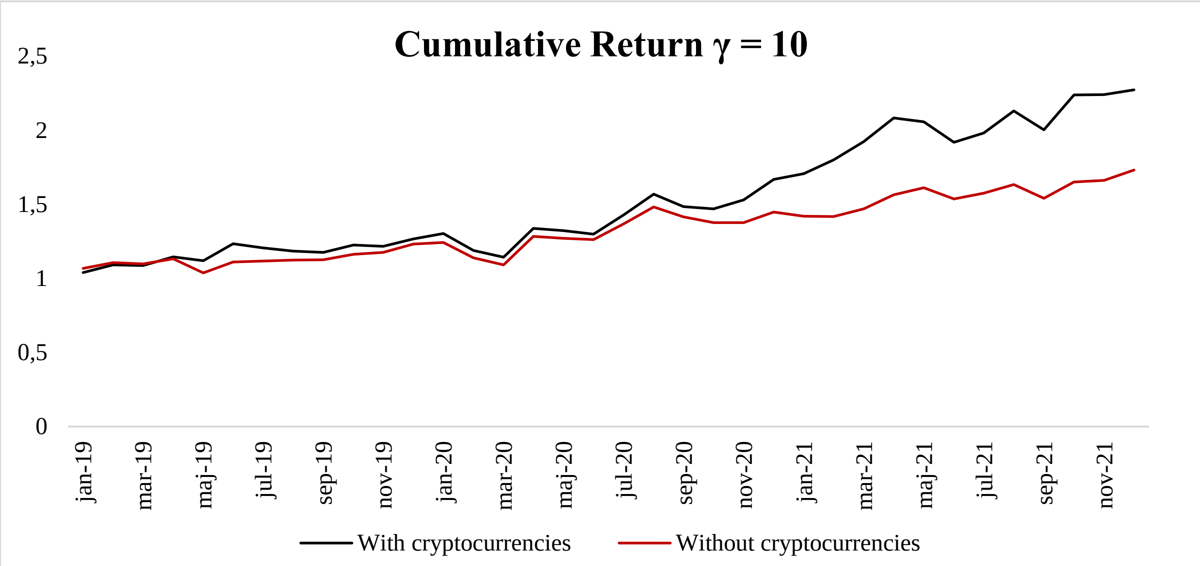

Figure 14: Cumulative Return of the Optimal Portfolio with γ = 10

Comparing the two portfolios with and without an investment in cryptocurrencies, the figure shows that the portfolio without an investment in the asset experiences fluctuations in the cumulative return, however they are less significant compared to the portfolio with an investment in cryptocurrencies.

For the portfolio with cryptocurrencies, there are periods of high positive returns and relatively large losses. The portfolio yields a high total return over the holding period of 127%, but with volatile periods. A lower total return is realized over the period for the portfolio without an investment in cryptocurrencies, or 73%.

For both portfolios, the month with the highest return is April 2020. The portfolio with cryptocurrencies has a return of 17.03% in that month, with a high positive weight in the S&P500 which yielded a high return over the month, and a positive weight in the CRIX index which yielded the highest return over the month. The weights allocated to the riskier securities are however lower than for the relatively risk seeking investor.

The portfolio with an investment in cryptocurrencies reaches a new HWM in January 2020, followed by the month of the largest negative return in February and another loosing month in March, which was additionally the month of the maximum drawdown of 12.27%. In the latter part of the sample, short periods of drawdowns occur.

The portfolio without cryptocurrencies endured some drawdowns over the period with longer periods of drawdowns in the second part of the period, particularly from September 2020 to September 2021. However, more significant drawdowns in terms of magnitude occurred in the first half of the period, with the maximum drawdown after March 2020 of 12.26%. A graph showing the high-water mark and drawdowns of each portfolio can be seen in Appendix 3.

Optimal Portfolio γ = 15

	With CC	**Without CC**
Average monthly excess return	1.71%	1.18%
Annualized excess return	20.52%	14.11%
St. Dev of Monthly Excess Returns	3.86%	3.46%
Annualized St. Dev of Excess Returns	13.36%	11.98%
Maximum Return	11.40%	11.86%
Minimum Return	-5.39%	-5.28%
Monthly Sharpe Ratio	0.44	0.34
Annualized Sharpe Ratio	1.54	1.18
Monthly Sortino Ratio	1.21	0.77
Annualized Sortino Ratio	4.18	2.67
Maximum Drawdown	6.81%	6.81%
Number of Losing Months	14	12

Table 10: Return statistics of the Optimal Portfolio with γ = 15

Table 10 shows the return statistics and performance metrics of the Optimal portfolio with γ =15. Both portfolios show a positive monthly excess return on average for the period. The portfolio with an investment in cryptocurrencies yields a higher monthly excess return on average than the portfolio without an investment in cryptocurrencies, as well as having a slightly higher monthly volatility. Compared to the corresponding portfolios with γ = 5 and γ = 10, the two portfolios with γ = 15 show the lowest average monthly excess return over the period, as well as the lowest standard deviation of monthly excess returns.

The portfolio with cryptocurrencies shows a very high annualized Sortino Ratio of 4.18, substantially higher than that of the portfolio without cryptocurrencies, indicating a very good performance in terms of downside risk adjusted returns. Additionally, the portfolio with cryptocurrencies performs well in terms of risk adjusted returns, as the portfolio shows a high annualized Sharpe ratio of 1.54, also higher than that of the portfolio without an investment in cryptocurrencies. Both the portfolios, with and without cryptocurrencies, show higher annualized Sharpe- and Sortino ratios than the corresponding portfolios with $\gamma = 5$ and $\gamma = 10$. Figure 15 shows the cumulative return of the Optimal portfolios with and without an investment in cryptocurrencies with $\gamma = 15$ over the holding period, where January 2019 is set to 1.

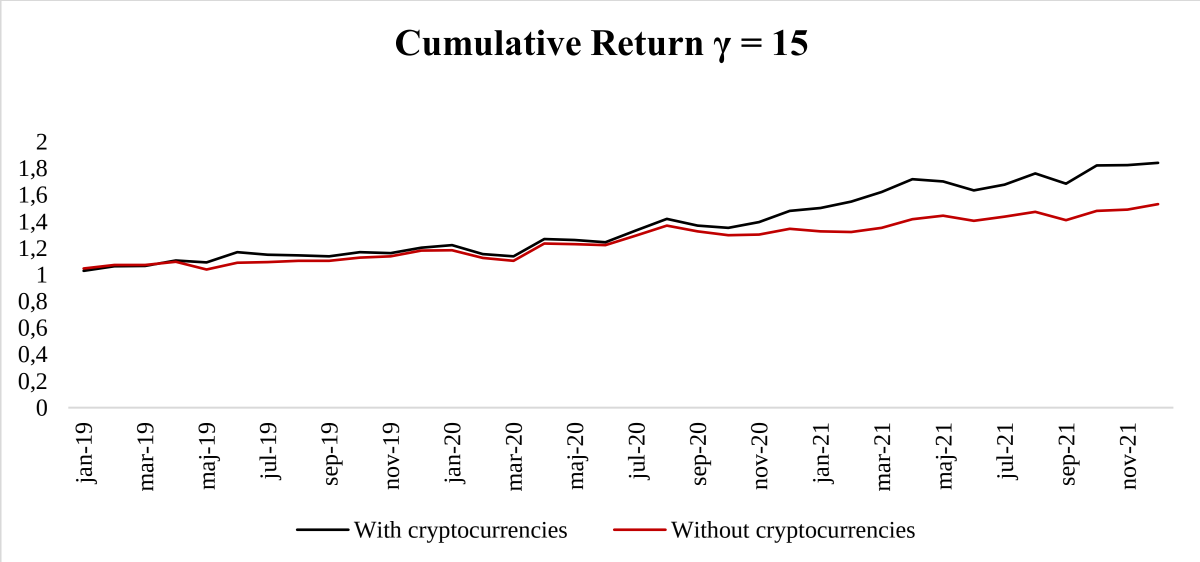

Figure 15: Cumulative Return of the Optimal Portfolio with γ = 15

The cumulative return of both portfolios has an overall upward trend. The portfolios do not experience periods of extreme fluctuations in returns. In the first part of the sample period the returns are very similar for both portfolios, but as the estimation period increases the return for the portfolio with an investment in cryptocurrencies increases compared to the other portfolio. The portfolio with an investment in cryptocurrencies yields a total return of 84% over the period, with a few periods of high positive returns and large losses. The portfolio without cryptocurrencies yields a return total of 53% over the period, lower than the portfolio with cryptocurrencies, and both portfolios have lower total returns compared to the corresponding portfolios with $\gamma = 5$ and $\gamma = 10$.

The month of the maximum return is April 2020 for both portfolios, of 11.40% with cryptocurrencies and 11.86% without cryptocurrencies. Over the period, the portfolio with cryptocurrencies experiences some drawdowns. The period of the largest drawdown is between January and March 2020 with the maximum drawdown of 6.81% occurring after March 2020.

The drawdowns over the period are not significant for the portfolio without cryptocurrencies, with the maximum drawdown of 6.81% after March 2020, the same as for the portfolio with cryptocurrencies. The portfolio ends the period with a new HWM. A graph showing the high-water mark and drawdowns of each of the portfolios computed can be seen in Appendix 3. As the Graphs in Appendix 3 show, the drawdowns decrease with increasing risk aversion.

When the degree of risk aversion decreases, investment in the more risky assets increases. Accordingly, both the average excess return and standard deviation of the portfolios with lower risk aversion coefficients are higher. As has been presented previously, the portfolio with $\gamma = 5$, which can be referred to as the portfolio of a relatively risk seeking investor, yields the highest return but is the most volatile portfolio. Therefore, this portfolio experiences larger drawdowns than the other portfolios with higher risk aversion coefficients. The portfolio with $\gamma = 10$ can be referred to as the portfolio of a relatively risk neutral investor and yields a lower return than the portfolio with $\gamma = 5$ but is less volatile and experiences less extreme drawdowns over the period. Finally, the portfolio with $\gamma = 15$, the portfolio of a relatively risk averse investor yields the lowest return in the sample but has the lowest volatility and experiences the lowest drawdowns. It also has the highest annualized Sharpe Ratio and Sortino Ratio, indicating that it performs best looking at the risk-adjusted performance. Table 11 shows the allocations of securities to the six portfolios.

Portfolio Allocations – The Optimal Portfolio

$\gamma = 5$ with CC						
OP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	183.86%	139.04%	-7.02%	-333.85%	95.56%	22.41%
Std. Dev	20.95%	87.63%	8.01%	85.77%	19.53%	2.39%
Minimum	152.79%	-24.87%	-27.83%	-450.08%	64.57%	18.71%
Maximum	253.83%	247.22%	10.31%	-143.95%	137.66%	27.31%

γ = 5 without CC						
OP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	186.02%	60.73%	-11.73%	-240.58%	105.56%	0.00%
Std. Dev	22.69%	89.89%	8.51%	95.11%	18.42%	0.00%
Minimum	151.51%	-146.56%	-34.49%	-353.48%	74.31%	0.00%
Maximum	242.51%	161.17%	6.46%	-48.07%	148.56%	0.00%
γ = 10 with CC						
OP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	100.57%	126.29%	-2.64%	-179.59%	44.14%	11.22%
Std. Dev	10.28%	40.98%	4.11%	39.73%	9.41%	1.21%
Minimum	85.24%	32.27%	-12.96%	-235.14%	29.08%	9.34%
Maximum	125.67%	176.61%	6.09%	-97.28%	64.49%	13.73%
γ = 10 without CC						
OP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	101.83%	88.30%	-4.95%	-134.57%	49.39%	0.00%
Std. Dev	11.14%	40.03%	4.33%	42.92%	8.97%	0.00%
Minimum	84.54%	-10.71%	-16.30%	-195.25%	35.43%	0.00%
Maximum	129.00%	133.40%	4.17%	-42.83%	69.96%	0.00%
γ = 15 with CC						
OP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	72.77%	123.07%	-1.00%	-129.52%	27.18%	7.50%
Std. Dev	6.83%	23.77%	2.80%	23.23%	6.17%	0.82%
Minimum	62.37%	63.29%	-8.00%	-167.95%	16.88%	6.22%
Maximum	88.94%	153.08%	4.68%	-77.54%	40.10%	9.20%
γ = 15 without CC						
OP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	73.77%	97.49%	-2.68%	-99.24%	30.67%	0.00%
Std. Dev	7.21%	23.28%	2.90%	25.64%	5.80%	0.00%
Minimum	61.90%	34.57%	-10.24%	-142.83%	21.17%	0.00%
Maximum	91.16%	124.14%	3.40%	-41.09%	43.76%	0.00%

Table 11: Optimal Portfolio allocations

As noted, as risk aversion increases, allocation to risky assets such as equities and cryptocurrencies decreases. The highest weight allocated to the CRIX index is for the portfolio with $\gamma = 5$. This is also the highest weight allocated to cryptocurrencies across all the portfolios in all analyses. Additionally, large weights are allocated to the S&P500 for both portfolios, with and without an investment in cryptocurrencies for the risk aversion coefficient $\gamma = 5$, which decreases as γ increases but stays relatively high for all six portfolios. As the Optimal portfolio aims to optimize the mean-variance relationship, the portfolios with an investment in cryptocurrencies allocate a high weight to GOVT on average which is the least volatile asset of the securities. The decrease in allocation to riskier securities, such as the S&P500 index and CRIX index with increase in risk aversion is expected, as a more risk averse investor will naturally allocate more of his capital towards less risky assets, as well as taking less aggressive positions.

For the portfolios without an investment in cryptocurrencies, investment in the government bond index increases as risk aversion increases. For all six portfolios, short positions are taken on average in the oil index, USO and the corporate bond index, LQD. Another observation is that the portfolio weights for the six portfolios are less aggressive to all securities as risk aversion increases. For the portfolios without cryptocurrencies, this is true for all securities other than GOVT.

6.4 Empirical Results – Analysis 4

This section reports the empirical results of Analysis 4, which examines the effect of enabling an investment in an index of cryptocurrencies in the Bayes-Stein Optimal portfolio in an out-of-sample setting. First, return statistics and performance measures are summarized and discussed, as well as graphical illustrations of portfolio performance over time. Statistics of portfolio weights during the holding period is then considered to illustrate the allocation to each portfolio constituent, with an emphasis towards the allocation to cryptocurrencies.

Tables 12-14 illustrate descriptive statistics of the returns of the Bayes-Stein Optimal portfolios with and without an investment in a cryptocurrency index, for three risk aversion coefficients of $\gamma = 5$, $\gamma = 10$ and $\gamma = 15$, representing a relatively risk seeking investor, a relatively risk neutral investor and a relatively risk averse investors. Additionally, performance metrics are provided for each portfolio. Figures 16-18 illustrate the cumulative returns of the Optimal portfolios with and without an investment in cryptocurrencies, for three risk aversion coefficients.

Bayes-Stein Optimal Portfolio γ = 5

	With CC	Without CC
Average monthly excess return	2.53%	1.25%
Annualized excess return	30.41%	14.99%
St. Dev of Monthly Excess Returns	6.53%	4.23%
Annualized St. Dev of Excess Returns	22.61%	14.64%
Maximum Return	16.63%	12.17%
Minimum Return	-10.01%	-7.40%
Monthly Sharpe Ratio	0.39	0.29
Annualized Sharpe Ratio	1.34	1.02
Monthly Sortino Ratio	0.93	0.59
Annualized Sortino Ratio	3.24	2.04
Maximum Drawdown	13.34%	9.05%
Number of Losing Months	15	12

Table 12: Return statistics of the Bayes-Stein Optimal portfolio, γ = 5

Both portfolios have a positive average excess return, but the portfolio with an investment in cryptocurrencies yields a higher average excess return. The portfolio with cryptocurrencies also has a higher standard deviation of excess returns, which is also evident by comparing the spread between the minimum and maximum returns of the portfolios, with a considerably smaller spread in the portfolio without cryptocurrencies, as well as the number of losing months. However, the risk-return profile of the portfolio with an investment in cryptocurrencies is better, as both the annualized Sharpe Ratio, 1.34 compared to 1.02 and the annualized Sortino Ratio, 3.24 compared to 2.04 are higher for the portfolio with cryptocurrencies.

However, with Sharpe Ratios above 1 and Sortino Ratios above 2, both portfolios have a good risk-return profile. Figure 16 shows the cumulative return index of the Bayes-Stein Optimal

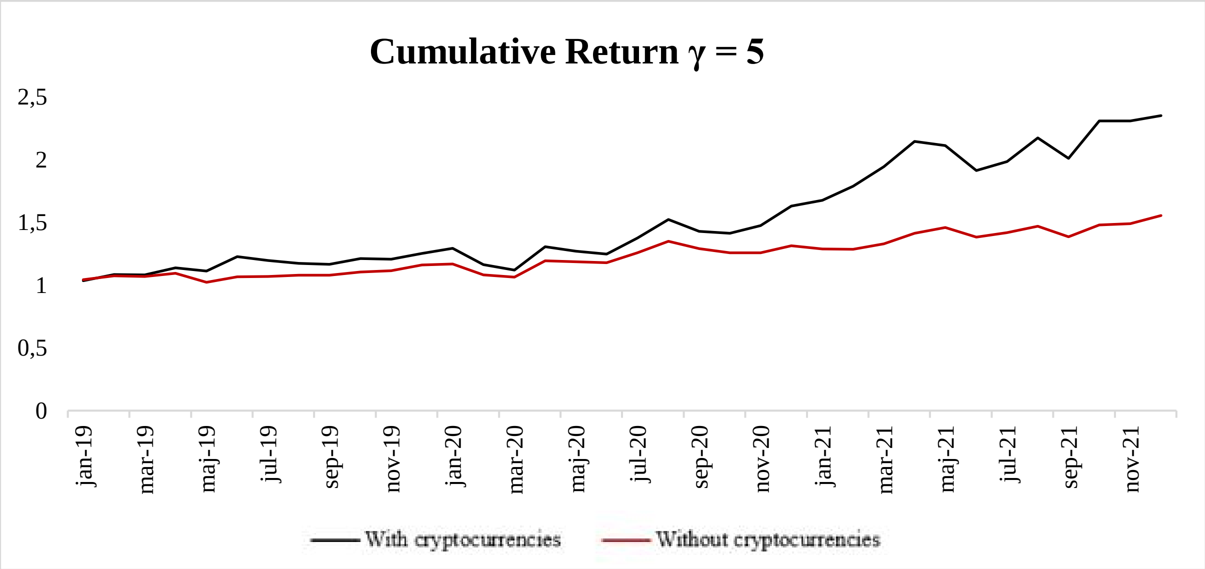

Figure 16: Cumulative Return of the Bayes-Stein Optimal Portfolio with γ = 5

portfolios with and without an investment in cryptocurrencies with γ = 5 over the holding period, where January 2019 is set to 1.

For both portfolios, the return in the first year is stable and without major drawdowns, as well as following a similar pattern. After a period of drawdowns amid the outbreak of COVID-19 in early 2020, the portfolio with cryptocurrencies starts to show very high returns while the portfolio without cryptocurrencies shows positive, but smaller returns.

The portfolio with cryptocurrencies has a high total return of 132.54% over the period. The portfolio endures a period of drawdowns in the beginning of 2020, with a maximum drawdown of 13.34% after March 2020. The second quarter of 2021 was also a period of drawdowns for the portfolio amid price depreciation in the cryptocurrency market, but it quickly rebounded and ended the period at a new HWM.

The portfolio without an investment in cryptocurrencies has a total return of 55.58%, a high return but this return is achieved at a lower Sharpe Ratio than the higher return of the portfolio with cryptocurrencies. The drawdowns are smaller in size in this portfolio however, with the maximum drawdown of 9.05% occurring after October 2020, when the portfolio has a period of sustained drawdowns between September 2020 and April 2021. Towards the end of the period the portfolio shows positive return and ends the period at a new HWM. Table 13 illustrates descriptive return statistics and performance metrics of the Bayes-Stein Optimal portfolio with γ = 10.

Bayes-Stein Optimal Portfolio γ = 10

	With CC	Without CC
Average monthly excess return	1.47%	0.80%
Annualized excess return	17.68%	9.57%
St. Dev of Monthly Excess Returns	3.27%	2.18%
Annualized St. Dev of Excess Returns	11.33%	7.54%
Maximum Return	8.57%	6.16%
Minimum Return	-4.36%	-3.54%
Monthly Sharpe Ratio	0.45	0.37
Annualized Sharpe Ratio	1.56	1.27
Monthly Sortino Ratio	1.16	0.78
Annualized Sortino Ratio	4.02	2.71
Maximum Drawdown	4.91%	4.21%
Number of Losing Months	14	12

Table 13: Return statistics of the Bayes-Stein Optimal portfolio, γ = 10

Both portfolios have a positive average excess return when the risk aversion coefficient is increased to 10. Comparing the portfolios to their corresponding portfolios with γ = 5, it is evident that the average excess return and standard deviation declines with increase in risk aversion. The decreased variation in returns can also be seen in the decreased spread between minimum and maximum returns for both portfolios. Like previously, the portfolio with an investment in cryptocurrencies has a higher average excess return as well as a higher standard deviation compared to the portfolio without cryptocurrencies.

More volatility in the portfolio with cryptocurrencies is again offset by the higher average excess returns, as it yields both a higher Sharpe Ratio and Sortino Ratio, with an annualized Sharpe Ratio of 1.56 compared to 1.27 and an annualized Sortino Ratio of 4.02 compared to 2.71 in the portfolio without cryptocurrencies. The performance metrics of the portfolios all increase with risk aversion when comparing the respective portfolios to their counterpart with γ = 5. Figure 17 shows the cumulative return index of the Bayes-Stein Optimal portfolios with and without an investment in cryptocurrencies with γ = 10 over the holding period, where January 2019 is set to 1.

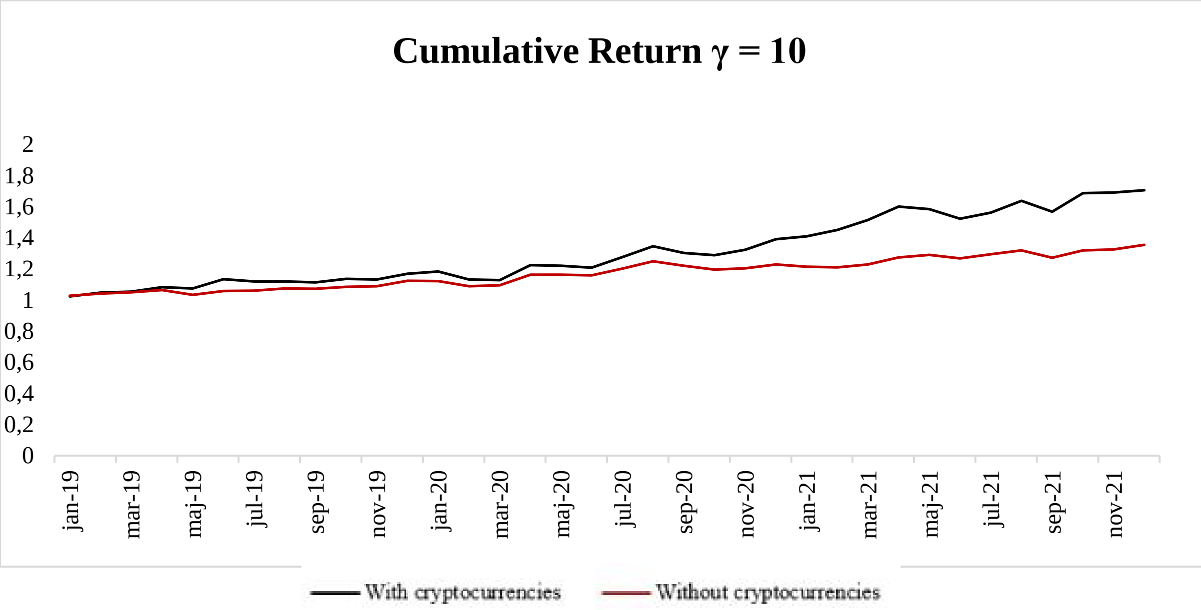

Figure 17: Cumulative Return of the Bayes-Stein Optimal Portfolio with γ = 10

As illustrated in figure 17, the cumulative return patterns are similar to the ones of the portfolios with γ = 5, with the portfolio including cryptocurrencies breaking away in the latter half of the analysis period. The considerably less volatility in the portfolios with γ = 10 is also evident, both in gains and losses. Looking at the portfolio with cryptocurrencies, similar to portfolio with γ = 5, the return is higher in the later years of the holding period, with drawdowns in the latter half of 2019 and the beginning of 2020 and the second quarter of 2021. Over the period, the portfolio has a total return of 70.57%, with a maximum drawdown of 4.91% endured after June 2021, attributable to a short position in LQD which had positive return in that month, and a positive position in CRIX, which lost heavily during the month.

The return of the portfolio without cryptocurrencies, shows similar return patterns as the more risk seeking investor, but with less magnitude of both gains and losses. The portfolio has similar periods of drawdowns, but they are smaller in size, with the maximum drawdown being 4.21%, which occurs after October 2020. Like in the portfolio with γ = 5, the portfolio then shows positive but at time volatile returns after April 2021 and ends the holding period at a new HWM. The total return of the portfolio over the whole period is 35.46%.

Table 14 illustrates return statistics and performance measures of the Bayes-Stein Optimal portfolio with $\gamma = 15$.

Bayes-Stein Optimal Portfolio γ = 15

	With CC	**Without CC**
Average monthly excess return	1.10%	0.65%
Annualized excess return	13.16%	7.84%
St. Dev of Monthly Excess Returns	2.23%	1.56%
Annualized St. Dev of Excess Returns	7.72%	5.40%
Maximum Return	5.70%	4.15%
Minimum Return	-3.34%	-2.79%
Monthly Sharpe Ratio	0.49	0.42
Annualized Sharpe Ratio	1.70	1.45
Monthly Sortino Ratio	1.30	0.94
Annualized Sortino Ratio	4.52	3.24
Maximum Drawdown	3.45%	3.34%
Number of Losing Months	12	11

Table 14: Return statistics of the Bayes-Stein Optimal portfolio, $\gamma = 15$

Both portfolios have a positive average excess return when the risk aversion coefficient is further increased to 15, with a further decrease in both average excess return and standard deviation with the increase in risk aversion. The portfolio with cryptocurrencies once again has the higher average return as well as higher volatility, but also both a higher Sharpe Ratio and Sortino Ratio compared to the portfolio without cryptocurrencies. Both portfolios have high annualized Sharpe Ratios and Sortino Ratios.

The decreased average returns and standard deviations do not come as a surprise as a more risk averse investor will naturally have higher allocations to safer securities in the portfolio compared to the risk seeking investor, who allocates more to riskier securities with higher expected returns.

Performance metrics also increase with risk aversion. The annualized Sharpe Ratio of the portfolio with cryptocurrencies of the relatively risk seeking investor is 1.34, for the risk neutral investor it is 1.56 and for the risk averse investor the Sharpe Ratio is 1.70. Even though the

Sharpe Ratios are all acceptable, this entails that the decrease in return is more than offset by the decrease in volatility of returns, leading to a better risk-return profile of the returns. By the same token, the Sortino Ratios increase with risk aversion, indicating less volatility of downside returns for risk averse investors.

Comparing the average excess return and standard deviation of the Bayes-Stein Optimal portfolios in tables 12-14 to the Optimal Portfolios in tables 8-10, the Bayes-Stein Optimal Portfolio yields lower excess returns on average, but the standard deviation is also significantly lower for all risk aversion coefficients. Further, the Bayes-Stein method yields higher performance metrics for all levels of risk aversion. Figure 18 shows the cumulative return index of the Bayes-Stein Optimal portfolios with and without an investment in cryptocurrencies with γ = 15 over the holding period, where January 2019 is set to 1.

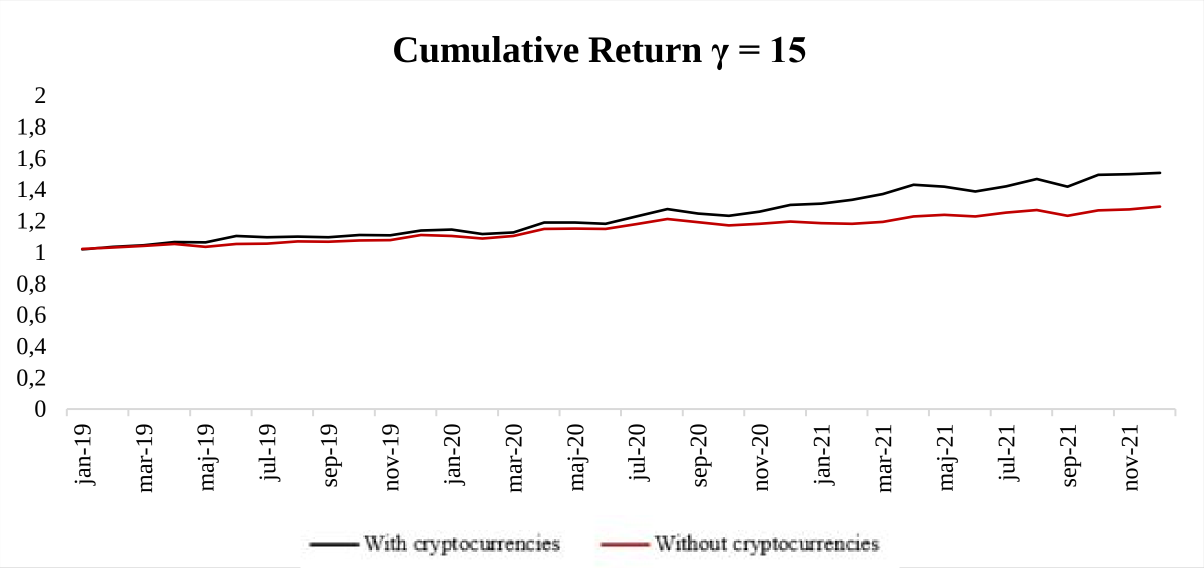

Figure 18: Cumulative Return of the Bayes-Stein Optimal Portfolio with γ = 15

The return pattern of the portfolios shows moderate returns and low volatility, but otherwise a similar pattern to the portfolios with γ = 5 and with γ = 10. The total return of the portfolio with cryptocurrencies over the period is 50.66%, with a high Sharpe and Sortino Ratio, attributable to both low volatility of returns and low volatility of downside returns. Comparing the results of the portfolio with cryptocurrencies and the portfolio without an investment in cryptocurrencies, the return is considerably higher, as well as both the Sharpe Ratio and Sortino Ratio, as can be seen in table 14. The portfolio without cryptocurrencies has a total return of

29.15%. When drawdowns occur, they are relatively small in magnitude in both portfolios, with a maximum of 3.44% in the cryptocurrency portfolio and 3.34% without cryptocurrencies.

Graphs showing the high-water mark and drawdowns of each of the portfolios computed in Analysis 4 can be seen in Appendix 4. As the Graphs in Appendix 4 show, the drawdowns decrease with increase in risk aversion, as they become smaller and cover shorter periods. Table 15 shows the allocations of securities to all portfolios in Analysis 4.

Portfolio Allocations – The Bayes-Stein Optimal Portfolio

γ = 5 with CC						
BSOP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	110.56%	129.37%	-3.40%	-199.25%	50.06%	12.66%
Std. Dev	13.91%	46.43%	4.91%	56.66%	10.62%	2.76%
Minimum	85.64%	20.76%	-20.30%	-303.77%	31.39%	8.60%
Maximum	149.35%	186.78%	6.23%	-95.86%	76.43%	18.17%
γ = 5 without CC						
BSOP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	89.15%	96.00%	-3.87%	-122.17%	40.89%	0.00%
Std. Dev	15.15%	30.29%	3.51%	41.28%	9.72%	0.00%
Minimum	59.19%	1.77%	-10.90%	-198.41%	18.55%	0.00%
Maximum	116.68%	127.67%	3.74%	-41.04%	67.32%	0.00%
γ = 10 with CC						
BSOP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	63.90%	123.45%	-0.67%	-114.78%	21.74%	6.38%
Std. Dev	5.88%	18.44%	2.24%	21.62%	4.67%	1.36%
Minimum	53.07%	73.05%	-5.51%	-160.44%	14.32%	4.25%
Maximum	74.96%	147.01%	4.05%	-66.96%	33.87%	9.16%
γ = 10 without CC						
BSOP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	53.23%	105.94%	-1.01%	-75.07%	16.91%	0.00%
Std. Dev	7.32%	11.52%	1.84%	16.77%	4.30%	0.00%
Minimum	38.83%	63.45%	-4.93%	-107.59%	7.88%	0.00%
Maximum	66.78%	120.80%	2.79%	-39.32%	29.34%	0.00%

γ = 15 with CC						
BSOP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	48.50%	120.88%	0.18%	-85.99%	12.17%	4.26%
Std. Dev	3.90%	9.75%	1.54%	12.04%	2.90%	0.93%
Minimum	41.51%	90.49%	-2.98%	-112.67%	8.29%	2.79%
Maximum	55.77%	134.53%	3.32%	-57.33%	19.82%	6.15%
γ = 15 without CC						
BSOP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	41.49%	109.38%	-0.04%	-59.87%	9.04%	0.00%
Std. Dev	4.72%	6.82%	1.29%	10.17%	2.64%	0.00%
Minimum	32.04%	84.02%	-2.94%	-80.68%	4.32%	0.00%
Maximum	50.14%	120.69%	2.49%	-38.74%	16.68%	0.00%

Table 15: Bayes-Stein Optimal portfolio allocations

As earlier noted, it is evident that as risk aversion grows, the allocations to the riskier assets, such as S&P500 and the CRIX index decreases, with the average allocation to both securities decreasing by more than 50% when looking at the relatively risk seeking investor and going to the most risk averse investor for both cases, with and without an investment in cryptocurrencies. Another observation is that the portfolio weights in general are less aggressive to all securities other than GOVT when risk aversion increases. For all coefficients of risk aversion and both portfolios, the largest weight on average is allocated to GOVT and the smallest weight is allocated to LQD on average.

The Bayes-Stein shrinkage method involves a rather aggressive shrinkage of the vector of expected returns of the securities. Comparing the allocations using the Bayes-Stein method and the classic Optimal portfolio as seen in table 11, it is evident that the weights allocated to the securities in the Bayes-Stein Optimal portfolio are both significantly less aggressive and have a lower variation of the allocation. The shrinkage of the parameters thus seems to do a good job in stabilizing the positions in the portfolio and making them less sensitive to periods of extreme returns in the securities.

Even though the method applies the shrinkage to the parameters, the CRIX index has a considerable allocation throughout the portfolios that include an investment in the index, however the allocation is around 50% higher to the index in the Optimal portfolio as seen in table 11.

6.5 Empirical Results – Analysis 5

This section reports the empirical results of Analysis 5, which examines the effect of enabling an investment in an index of cryptocurrencies in an Equally Weighted portfolio in an out of sample setting. First, return statistics and performance measures are summarized and discussed, as well as graphical illustrations of portfolio performance over time. Statistics of portfolio weights during the holding period is then considered to illustrate the allocation to each portfolio constituents, with an emphasis towards the allocation to cryptocurrencies.

Table 16 illustrates descriptive statistics of the return of the Equally Weighted portfolio with and without an investment in a cryptocurrency index, as well as providing portfolio performance metrics.

Equally Weighted Portfolio

	With CC	**Without CC**
Average monthly excess return	2.30%	0.81%
Annualized excess return	27.58%	9.69%
St. Dev of Monthly Excess Returns	5.60%	4.13%
Annualized St. Dev of Excess Returns	19.39%	14.29%
Maximum Return	13.08%	9.01%
Minimum Return	-17.33%	-15.41%
Monthly Sharpe Ratio	0.41	0.20
Annualized Sharpe Ratio	1.42	0.68
Monthly Sortino Ratio	0.74	0.29
Annualized Sortino Ratio	2.57	1.01
Maximum Drawdown	20.86%	22.29%
Number of Losing Months	12	11

Table 16: Return statistics of the Equally Weighted Portfolio

The Equally Weighted portfolio yields a positive excess return for both portfolios on average, with a higher monthly excess return for the portfolio with cryptocurrencies. The portfolio with cryptocurrencies is fairly volatile with a standard deviation of monthly excess returns of 5.60%, higher than that of the portfolio without a cryptocurrency investment.

Once again, the portfolio with an investment in cryptocurrencies performs better compared to the portfolio without an investment in the asset in terms of risk adjusted returns and downside risk adjusted return, yielding a higher annualized Sharpe- and Sortino Ratios.

As previously mentioned, the literature on portfolio theory and optimizing portfolios is vast and a lot of methods are available to investors. Nevertheless, these methods are sensitive to estimation errors, and do not perform well in practice at times (Pedersen, Babu & Levine, 2021). Several studies show that the Equally Weighted portfolio often outperforms optimized portfolios, for example DeMiguel, Garlappi & Uppal (2009) which show that the Equally Weighted portfolio often yields a higher Sharpe Ratio than the optimized portfolios.

In this analysis, 10 portfolios with an investment in cryptocurrencies are constructed, including the Equally Weighted portfolio. The Equally Weighted portfolio outperforms five of the nine optimized portfolios in terms of the annualized Sharpe Ratio, and therefore has better risk adjusted returns. Additionally, the Equally Weighted portfolio yields a higher average monthly excess return than six of the optimized portfolios

In terms of the portfolios without an investment in cryptocurrencies, the Equally Weighted portfolio shows an annualized Sharpe Ratio of 0.68 which is lower than the annualized Sharpe Ratio of all the nine optimized portfolios without cryptocurrencies. Therefore, the Equally Weighted portfolio performs worse in terms of risk adjusted return than the optimized portfolios without cryptocurrencies. Figure 19 illustrates the cumulative return of the Equally Weighted portfolios with and without an investment in cryptocurrencies over the holding period, with January 2019 set as 1.

The cumulative return of both portfolios has an overall upward trend. The portfolio with an investment in cryptocurrencies has a total return of 120% over the period. However, there are periods of considerable losses. The portfolio without an investment in cryptocurrencies has a flatter cumulative return over the period, with less volatility. The total return of the portfolio over the period is substantially lower than the return of the portfolio with cryptocurrencies or 33%.

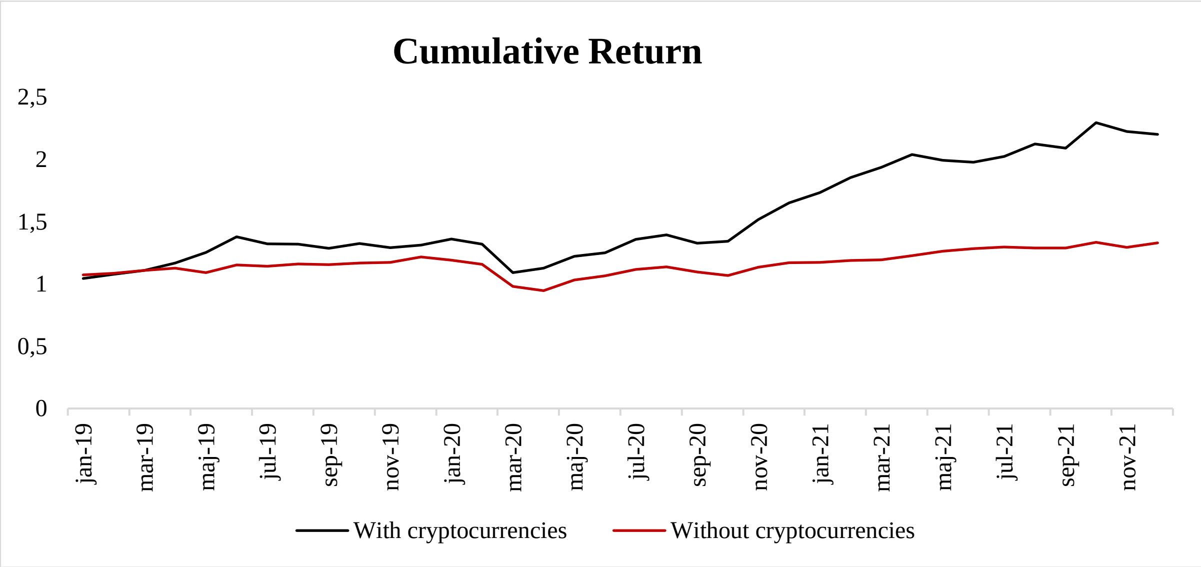

Figure 19: Cumulative Return of the Equally Weighted portfolio

A new HWM is reached for the portfolio with an investment in cryptocurrencies in June 2019, followed by a 13-month period of drawdowns. The portfolio experiences the largest loss after March 2020, as well as the maximum drawdown after the same month of 20.86%, attributable to the market uncertainty amid the COVID-19 outbreak. In the latter part of the sample period, less significant drawdowns occur, and the portfolio yields the maximum return in November 2020.

The portfolio without an investment in cryptocurrencies reaches a new HWM after December 2019, followed by a 15-month period of drawdowns. The month with the largest loss is March 2020 of -15.41%. The maximum drawdown occurs after April 2020, of 22.29%. In the later part of the sample period, the portfolio experiences less fluctuations in return, as well as shorter periods of drawdowns. A graph showing the high-water mark and drawdowns of each portfolio can be seen in Appendix 5. Table 17 shows statistics on weights allocated to the securities in the portfolios over the period.

Portfolio Allocations – Equally Weighted Portfolio

With CC						
EW	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%
Std. Dev	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Minimum	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%
Maximum	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%
Without CC						
EW	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	20.00%	20.00%	20.00%	20.00%	20.00%	0.00%
Std. Dev	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Minimum	20.00%	20.00%	20.00%	20.00%	20.00%	0.00%
Maximum	20.00%	20.00%	20.00%	20.00%	20.00%	0.00%

Table 17: Equally Weighted Portfolio allocations

The table shows the average allocation to the assets included in the analysis. As the portfolio is equally weighed (1/N), the same weight is allocated to each asset and the standard deviation of the weights is equal to 0.00%.

6.6 Empirical Results – Analysis 6

This section reports the empirical results of Analysis 6, which examines the effect of enabling an investment in an index of cryptocurrencies in a Risk Parity portfolio in an out-of-sample setting. First, return statistics and performance measures are summarized and discussed, as well as graphical illustrations of portfolio performance over time. Statistics of portfolio weights during the holding period are then considered to illustrate the allocation to each portfolio constituents, with an emphasis towards the allocation to cryptocurrencies.

Risk Parity Portfolio		
	With CC	**Without CC**
Average monthly excess return	0.64%	0.48%
Annualized excess return	7.64%	5.79%
St. Dev of Monthly Excess Returns	1.89%	1.81%
Annualized St. Dev of Excess Returns	6.56%	6.26%
Maximum Return	3.93%	3.48%
Minimum Return	-6.18%	-5.87%
Monthly Sharpe Ratio	0.34	0.27
Annualized Sharpe Ratio	1.16	0.93
Monthly Sortino Ratio	0.57	0.44
Annualized Sortino Ratio	1.97	1.54
Maximum Drawdown	6.24%	5.87%
Number of Losing Months	11	12

Table 18: Return statistics of the Risk Parity portfolio

Both portfolios have a positive average excess return, with relatively low standard deviation compared to previous analyses. The low standard deviations are expected, as the Risk Parity portfolio construction naturally directs higher allocations to less risky securities and lower allocations to more risky securities. The portfolio including cryptocurrencies has a higher average excess return, and marginally higher standard deviation. The average return of the portfolio without cryptocurrencies is however considerably lower considering how little difference there is between the two portfolios standard deviations.

The larger decline in average excess return of the portfolio without cryptocurrencies compared to the smaller decline in standard deviation results in a lower Sharpe Ratio, an annualized Sharpe Ratio of 0.93 compared to 1.16 in the portfolio with cryptocurrencies. By the same token, the portfolio without an investment in cryptocurrencies has an annualized Sortino Ratio of 1.54, compared to the annualized Sortino Ratio of 1.97 for the portfolio with cryptocurrency investment. The performance metrics are all on an acceptable level, but a better risk-return trade-off has been achieved for the same or higher level of excess returns in previous analyses, such as Analysis 4. Figure 20 shows the cumulative return of the Risk Parity portfolio without cryptocurrencies, with January 2019 set as 1.

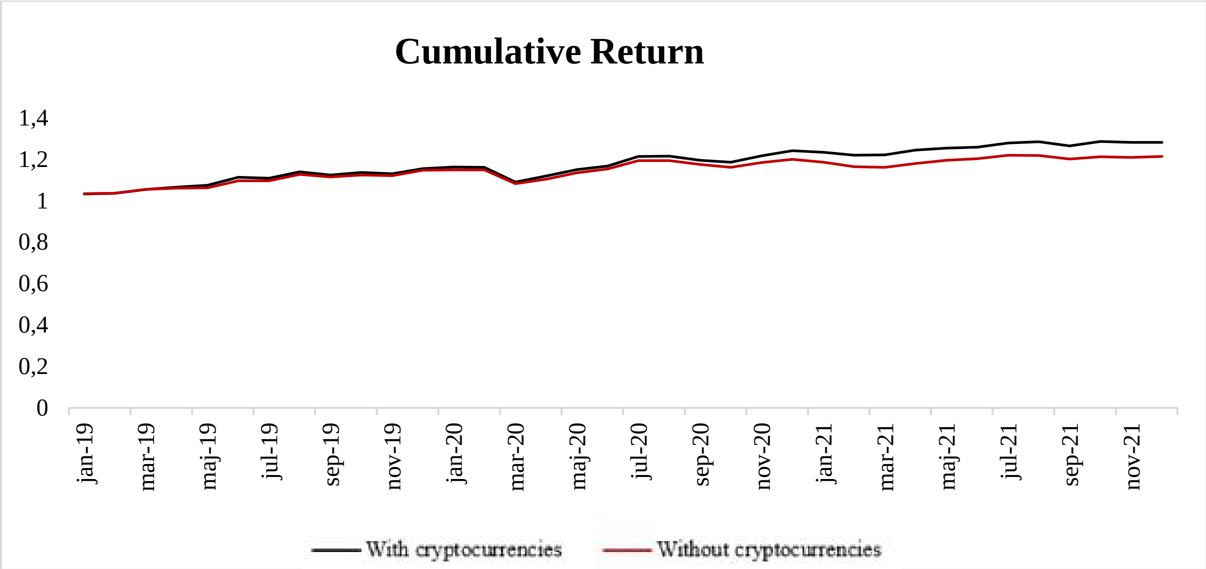

Figure 20: Cumulative Return of the Risk Parity portfolio

As expected, when looking at the portfolios´ descriptive return statistics, the portfolios have a stable return with low volatility. Until the market turbulence period of early 2020, almost no drawdowns occur in either portfolio. The portfolio with cryptocurrencies recovers swiftly after the maximum drawdown after March 2020 and does not have long sustained period of drawdowns after that period. The total return of the portfolio over the whole period was 28.13%

After the drawdown of March 2020, the portfolio without cryptocurrencies also recovers quickly, however, in contrast with the Risk Parity portfolio with cryptocurrency investment, this portfolio does have longer periods of small drawdowns, such as during 2021, when the portfolio endures small drawdowns for most of the year, as the return is not high enough to recover from a loss of more than -1% in January and February. Over the whole period, the total return of the portfolio is 21.31%. Table 19 shows the weights allocated to each security in the portfolios.

Portfolio Allocations – Risk parity portfolio

With CC						
RPP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	11.35%	42.25%	4.27%	28.18%	12.34%	1.60%
Std. Dev	0.60%	1.47%	0.56%	1.07%	0.80%	0.21%
Minimum	10.58%	38.82%	3.69%	27.11%	10.84%	1.23%
Maximum	12.44%	43.86%	5.17%	30.68%	13.75%	1.82%
Without CC						
RPP	**GSPC**	**GOVT**	**USO**	**LQD**	**GLD**	**CRIX**
Mean allocation	11.54%	42.94%	4.34%	28.63%	12.54%	0.00%
Std. Dev	0.59%	1.56%	0.56%	1.03%	0.83%	0.00%
Minimum	10.77%	39.40%	3.75%	27.60%	10.97%	0.00%
Maximum	12.62%	44.65%	5.24%	31.15%	14.00%	0.00%

Table 19: Risk Parity portfolio allocations

As the CRIX index has a very low allocation in the portfolio with cryptocurrencies, expectedly, the weights are very similar between the portfolios. The least risky securities are allocated the highest percentage of capital, GOVT and LQD in both portfolios. As the riskiest one of the assets, the CRIX index is allocated a very small portion of the portfolio.

Another thing to note is that the weights are very stable, as seen in the low standard deviations of allocations and the small spread between the minima and maxima allocated to the securities, especially compared to previous analyses. This stems from the usage of expanding estimation window to compute volatility, as over time, the volatility stabilizes and consequently, the weights stabilize with time. What is interesting is the considerable difference in the performance metrics of the two portfolios, the Sharpe and Sortino Ratios are 24.73% and 27.92%, respectively, higher in the Risk Parity portfolio with cryptocurrencies compared to the portfolio without them. This is especially interesting considering the low allocation to cryptocurrencies in the portfolio with cryptocurrencies and the overall similarities in the weights allocated to the securities in the two portfolios.

6.7 Empirical Results – Summary Table

Summary of performance metrics

	Sharpe Ratio		Sortino Ratio		Average excess return	
	With CC	**Without CC**	**With CC**	**Without CC**	**With CC**	**Without CC**
Global Min. Var. Portfolio	1.29	1.28	3.35	3.27	4.23%	4.16%
Tangency Portfolio	0.81	0.78	2.31	1.50	20.27%	11.73%
Optimal Portfolio γ = 5	1.31	0.94	3.21	1.95	53.20%	34.02%
Optimal Portfolio γ = 10	1.43	1.06	3.71	2.31	28.70%	19.09%
Optimal Portfolio γ = 15	1.54	1.18	4.18	2.67	20.52%	14.11%
BS Optimal Portfolio γ= 5	1.34	1.02	3.24	2.04	30.41%	14.99%
BS Optimal Portfolio γ = 10	1.56	1.27	4.02	2.71	17.68%	9.57%
BS Optimal Portfolio γ = 15	1.70	1.45	4.52	3.24	13.16%	7.84%
Equally Weighted Portfolio	1.42	0.68	2.57	1.01	27.58%	9.69%
Risk Parity Portfolio	1.16	0.93	1.97	1.54	7.64%	5.79%

Table 20: Summary of performance metrics, annualized

Table 20 illustrates a summary of the performance metrics relevant to the hypotheses in the analysis for all the portfolios constructed, both with and without cryptocurrencies. All metrics are annualized. Section 7, Discussion, provides a detailed evaluation of each hypothesis and the performance metrics of the portfolios constructed.

7 Discussion

This section further examines the Empirical Results of the analysis, presented in section 6. The Discussion section is structured as follows: First, a general discussion of the analysis´ research question and contribution to the academic literature is provided, after which the results are evaluated with regards to the hypotheses laid forward in section 4 and compared to existing literature on the topic, as well as discussing whether the results find evidence for or against the hypotheses formulated. Finally, the limitations of the analyses are noted.

Since cryptocurrencies entered the investment scene, they have attracted substantial interest from investors, both retail and professional, governments and corporations. The foundation of this interest is twofold, first, because cryptocurrencies differ from almost all traditional investment assets in the way that they are in most cases independent of a company, a government or an individual, in other words, there is no issuer. One could then think that cryptocurrencies could possibly resemble a commodity. However, a traditional commodity has an obvious value or usage. This is not the case with cryptocurrencies, as their value has still not been fully defined and they are rarely used in everyday transactions such as paying for goods and services. The consequence of cryptocurrencies not being tied to a government are that they are independent of monetary policy, as an example, some cryptocurrencies are scarce, meaning it is not possible to print more of them.

The second foundation of interest in cryptocurrencies is that they have since their surfacing had a monumentally high price appreciation and returns compared to other financial asset classes. However, the very high returns have been combined with extreme volatility. Based on these characteristics, it is evident that more risk averse investors would prefer to not invest a large share of their portfolios in such an asset.

However, these characteristics built the foundation of the research performed in this analysis. There exists a large theoretical foundation of portfolio theory, mostly built on Markowitz (1952), which relies on two parameters: expected returns and variances. Constructing portfolios based on the rigorous foundations and including cryptocurrencies as an investable asset to see if it could enhance the portfolio performance in the context of risk adjusted returns, it was deemed necessary to perform the analysis across portfolio construction methods as well as risk aversion, to make the results as robust and applicable as possible, however within the limitations of the analysis.

These foundations lead to the research question of this analysis:

Does an investment in cryptocurrencies enhance the performance of optimized portfolios?

Because of the relatively limited time period since cryptocurrencies surfaced as an investment asset compared to other asset classes, the literature on the subject is not a very vast one, albeit growing fast. A majority of the literature is centers around the effects of adding Bitcoin to an optimized portfolio, as well as its capabilities to act as a safe haven asset or a hedge during periods of financial markets turbulence. As the cryptocurrency with the most coverage, the highest market capitalization and the longest price history of the major cryptocurrencies, it is understandable that most of the literature up until this point has had Bitcoin as its focal point.

This research takes a different approach compared to only including one cryptocurrency in the portfolios. A portfolio including indices that represent a broad selection of securities within the given asset class and naturally includes a degree of diversification through limitation of unsystematic risk and only one asset of another asset class, cryptocurrencies, does not fully represent the advantages or disadvantages of investing in that given asset class. Rather, this analysis combines a well-diversified portfolio of indices representing major asset classes with a similar instrument for cryptocurrencies, the CRIX index. Though the index is not directly tradeable, the index constituents and their weights are easily accessible and reproduceable. Using an index as a proxy for cryptocurrencies makes the instruments used in the analysis more comparable and can further capture the effect of including cryptocurrencies in a well-diversified portfolio.

20 portfolios are constructed using six different portfolio construction techniques, 10 portfolios without an investment in cryptocurrencies and 10 portfolios with an investment in cryptocurrencies. The techniques used in this analysis are the Global Minimum Variance portfolio, the Tangency portfolio, the Optimal portfolio for three levels of risk aversion, the Optimal portfolio using a Bayes-Stein approach to shrink the estimators with three levels of risk aversion, the Equally Weighted portfolio and finally the Risk Parity portfolio. This broad portfolio construction is an attempt to make the results robust across techniques and investors' preferences and risk appetite. All portfolios are constructed out-of-sample, to make the

portfolio construction more realistic than in an in-sample setting and expanding estimation windows are used. The drawbacks of the portfolio construction and general discussion about the criticism of Markowitz´s framework has been discussed in detail in section 5.6.

7.1 Evaluation of Hypothesis 1

The first hypothesis laid forward in this analysis centers around a basic performance metric, returns.

H1: An investment in cryptocurrencies increases returns of optimized portfolios.

Here, plain excess returns are the only metric that is looked at, as it builds the foundations of further analysis of performance metrics. If returns do not prove to be higher in portfolios including cryptocurrencies, their high volatility are very likely to result in lower performance metrics when looking at more sophisticated metrics.

The results of all analyses are unanimous with regards to H1, that an investment in cryptocurrencies increases returns of optimized portfolios, as all portfolios including cryptocurrencies have a higher annualized excess return when comparing it to the corresponding portfolio without cryptocurrencies. On average, the annualized excess returns of the portfolios including cryptocurrencies are 69.87% higher than the annualized excess returns of the corresponding portfolio without cryptocurrencies. This is a large difference and does support H1 in a strong way.

The smallest difference in annualized excess returns between two portfolios is expectedly in the Global Minimum Variance portfolio. The Global Minimum Variance portfolio with an investment in cryptocurrencies has annualized excess returns of 4.23% compared to an annualized excess return of 4.16% in the portfolio without cryptocurrencies, a difference of 0.07 percentage points or an increase of 1.68%. The Global Minimum Variance portfolio´s construction naturally makes this an expected result, as it allocates very small weights to the riskiest assets. The average allocation to cryptocurrencies is 0.05%, so not much difference is illustrated in the results of the portfolio.

On the other hand, the largest difference between the annualized excess returns of portfolios is found in the Equally Weighted portfolio, where the annualized average excess return is 27.58% for the portfolio including an investment in cryptocurrencies compared to an annualized average excess return of 9.69% for the portfolio without an investment in cryptocurrencies, a 184.62% increase in return when including cryptocurrencies in the portfolio, a very large difference as the return is almost three times higher including cryptocurrencies. Another instance of over 100% increase in return when including cryptocurrencies in a portfolio is in the Bayes-Stein Optimal portfolio with a risk aversion coefficient of $\gamma = 5$, where the portfolio with had an annualized average excess return of 30.41% compared to 14.99% in the portfolio not including cryptocurrencies, a 102.87% increase. Cryptocurrencies have a fair share of weight allocated to them on average in both portfolios, with 16.67% weight allocation in all months in the Equally Weighted portfolio and an average allocation of 12.66% in the Bayes-Stein Optimal portfolio with $\gamma = 5$.

The increase in returns when including cryptocurrencies in an optimized portfolio is in line with previous research on the topic, as Platanikis & Urquhart (2020) also conclude that an investment in Bitcoin did increase returns of all portfolios for all levels of risk aversion. They constructed portfolios based on eight approaches and three levels of risk aversion. Kajtazi & Moro (2019) use a mean-CVaR optimization technique for four portfolios with different constraints and show that including Bitcoin in such portfolios increases the return of every portfolio. Similarly, Wu & Pandey (2014) show that the inclusion of Bitcoin in portfolios increases the return of every portfolio.

The literature reviewed is unanimous with regards to cryptocurrencies' ability to increase the returns of portfolios. If a security has high average returns compared to other securities in a portfolio, it is natural to expect that a positive weight allocated to that given security leads to an increase on average in the portfolio´s return. As all portfolios constructed in this analysis allocate a positive weight to cryptocurrencies on average, the portfolios are successful in reaping the benefits of including cryptocurrencies in a well-diversified portfolio. However, even though cryptocurrencies do have a very high average return and an obvious ability to increase the return of portfolios, there is another side to the high average returns, which is their very high volatility. The portfolio return with regards to volatility is the focal point of Hypothesis 2, as returns must be viewed in a wider scope than just plain returns to evaluate performance.

7.2 Evaluation of Hypothesis 2

The second hypothesis laid forward in this analysis takes a deeper look at the returns of the portfolios by looking at the standard deviation of returns as well as the Sharpe Ratio of the portfolios, as performance should be assessed on a risk-adjusted basis, not only on plain returns. This leads to the second hypothesis:

H2: An investment in cryptocurrencies increases the volatility of optimized portfolios, but the increase in average return offsets the increase in volatility, resulting in a higher Sharpe Ratio.

Hypothesis 2 is two-fold. First, the standard deviations of the portfolios are analysed and compared, after which the Sharpe Ratios are analysed and compared. It has now been supported that an investment in cryptocurrencies in optimized portfolios does increase the returns of the portfolios, but the returns are only one component of investment performance. If standard deviations increase to the point that the return per unit of risk is less, then an inclusion of cryptocurrencies in a portfolio is not necessarily an attractive option for investors. This is measured by the Sharpe Ratio of the portfolios.

First, looking at the annualized standard deviations of the average excess returns of the portfolios, the results are almost unanimous that the standard deviation of excess returns increases with an investment in cryptocurrencies. The standard deviation does not decline with an investment in cryptocurrencies on any occasion, but the Global Minimum Variance portfolio does not show a difference in the standard deviation of excess return when investing in cryptocurrencies, as it allocates little capital to them, as noted in previous sections.

On average, the increase in annualized standard deviations of excess returns after including cryptocurrencies in the corresponding portfolios is 31.79%. This supports the first part of H2, that the standard deviation increases with the inclusion of cryptocurrencies. Looking past the Global Minimum Variance portfolio where there is no difference, the smallest difference in the standard deviation is found in the Optimal Portfolio with a risk aversion coefficient of $\gamma = 15$. The annualized standard deviation of excess returns is 11.98% for the portfolio without an investment in cryptocurrencies compared to 13.36% with cryptocurrencies, an increase of 1.38 percentage points or 11.52%. The portfolio allocates 7.5% weight on average to cryptocurrencies.

On the other hand, the highest increase in standard deviation when including cryptocurrencies is found in the Tangency portfolio, where the annualized standard deviation of excess returns increases from 15.01% without an investment in cryptocurrencies to 25.09% with cryptocurrencies, a 67.16% increase in standard deviation. The Tangency portfolio allocates 8.87% of capital to cryptocurrencies on average. The increased standard deviation in the Tangency portfolio also stems from the very unstable and aggressive weights in the portfolio, not only the inclusion of cryptocurrencies. This can be seen in the weight allocated to cryptocurrencies on average which is not much higher than in the Optimal Portfolio with γ = 15, which has the least increase in standard deviation when including cryptocurrencies in the portfolio.

The increased volatility when adding cryptocurrencies to the portfolios is however outweighed by the increase in average excess returns when looking at Sharpe Ratios. For all portfolios, the Sharpe Ratio increases when adding cryptocurrencies to the portfolios. On average, the increase in the Sharpe Ratio is 31.44% across all portfolios. The results thus unanimously point to a better risk-return trade-off in the portfolios with cryptocurrencies. This supports H2 in a strong way. The portfolio with the highest Sharpe Ratio in the analysis is the Bayes-Stein Optimal portfolio with a risk aversion coefficient of γ = 15, which has an annualized Sharpe Ratio of 1.7. On the other hand, the portfolio with the lowest Sharpe Ratio is the Equally Weighted portfolio without cryptocurrencies, 0.68 annualized. Of the 20 portfolios constructed, only five have an annualized Sharpe Ratio below 1, which is generally considered a threshold for good performance. Of the five, only one included cryptocurrencies.

The smallest difference in the Sharpe Ratio is again in the Global Minimum Variance portfolio, of 1.29 with cryptocurrencies compared to 1.28 without it. As already mentioned, these results are expected as it allocates small weights to cryptocurrencies. The largest increase in the Sharpe Ratio between corresponding portfolios is found in the Equally Weighted portfolio, where the Sharpe Ratio increases by 108.82% when including cryptocurrencies, from 0.68 to 1.42, a considerable increase. Other portfolios with substantial Sharpe Ratio increases are the three Optimal Portfolios for all risk aversion coefficients as well as the Bayes-Stein Optimal portfolio with γ = 5.

7.3 Evaluation of Hypothesis 3

The third hypothesis proposed in this analysis takes a deeper look at the risk adjusted returns. H3 looks at the standard deviation of the downside return, more specifically the Sortino Ratio. With the introduction of the Sharpe Ratio an enormous contribution was made to modern investment theory as investors seek to find the portfolios promising the greatest expected return for a given level of risk. The Sharpe Ratio is one of the most commonly used ratios to calculate the risk adjusted return, however professionals argue that the standard deviation, where both the upside and downside deviations are considered, is not necessarily the most relevant measure of risk for every investment situation (Sortino and Meer, 1991).

Sortino and Meer (1991) advocated the use for downside risk in investment decisions. Therefore, this analysis aims to answer the third hypothesis:

H3: An investment in cryptocurrencies reduces the downside standard deviation of a portfolio, resulting in a higher Sortino Ratio.

Hypothesis 3 is two-fold. That is, the discussion will concentrate on if an investment in cryptocurrencies increases the Sortino Ratio and if that increase is driven by the reduction in the standard deviation of downside excess returns.

In this analysis, the Sortino Ratio is computed to give a better view of the risk adjusted performance of the 20 portfolios constructed. Due to cryptocurrencies' low or negative correlation to other financial assets during their relatively short history (Aslandis, Bariviera & Martinez, 2019), it can be expected that adding cryptocurrencies to the portfolios in this analysis lowers the downside variation of the portfolios and increases the Sortino Ratio.

In this analysis, the CRIX Index has relatively low correlations with all the other asset classes. The lowest correlation between CRIX and the other assets is with GOVT of -0.06 and the highest correlation is with S&P500 of 0.22. This is in line with the studies of Briére, Oosterlinck, and Szafarz (2015) and Guesmi et al. (2019), which conclude that Bitcoin has a low correlation with other financial assets during the periods studied.

The results in this analysis show that an investment in cryptocurrencies increases the Sortino Ratio in all cases. The portfolio with the highest Sortino Ratio is the Bayes-Stein Optimal Portfolio with γ = 15, a high Sortino Ratio of 4.52. The Sortino Ratio of the corresponding portfolio without an investment in cryptocurrencies is 3.24, an increase of 39.5% when adding cryptocurrencies to the portfolio. Nevertheless, this is not the portfolio with the largest increase in the Sortino Ratio. The Equally Weighted portfolio including cryptocurrencies has a Sortino Ratio of 2.57, but the same portfolio without cryptocurrencies has a ratio of 1.01, an increase of 154.5% when including cryptocurrencies which is the largest increase in the Sortino Ratio in the analysis.

In addition to proposing that an investment in cryptocurrencies increases the Sortino Ratio, the first part of Hypothesis 3 proposes that an investment in cryptocurrencies reduces the downside standard deviation of a portfolio. Although the Sortino Ratio increases for all 10 portfolios when including an investment in cryptocurrencies, the standard deviation of the downside excess return increases in six cases out of 10. More specifically, the standard deviation of the downside increases for the Tangency portfolio, The Bayes-Stein Optimal portfolio for all coefficients of risk aversion (γ = 5,10,15), the Equally Weighted portfolio, and the Risk Parity portfolio when including an investment in cryptocurrencies, indicating a larger variation in downside returns. Therefore, there is not a clear support for the first part of Hypothesis 3. However, as the Sortino Ratio increases in all cases, it is evident that the increase is driven by the growth in returns, which clearly outweighs the increase in the standard deviation of the downside when it occurs.

The average increase in the Sortino Ratio across all portfolios when investing in cryptocurrencies is 56.73%. The results therefore indicate that adding cryptocurrencies to the optimized portfolios improves the downside risk adjusted excess returns in all cases. This is in line with the results of the analyses by Wu and Pandey (2014), Briere, Oosterlinck and Szafarz (2015), Eisl, Gasser and Weinmayer (2015), Gangwal (2017), Agarwal et al. (2018), Kajtazi and Moro (2019), Platanikis and Urquhart (2020), and Bakry et al. (2021) which are unanimous regarding the ability of cryptocurrencies to improve downside risk adjusted excess returns, all concluding that adding Bitcoin to a portfolio increases the Sortino Ratio.

7.4 General Discussion Points

The results of all analyses are unanimous regarding the three proposed hypotheses. The authors find evidence that supports each hypothesis. Regarding H1, the results show that an investment in cryptocurrencies increases the returns of the optimized portfolios as all portfolios including cryptocurrencies have higher annualized excess returns compared to corresponding portfolios without cryptocurrencies. Additionally, the Sharpe Ratio of each portfolio increases with an investment in cryptocurrencies, supporting H2 in a strong way. Even though strong support for the first part of H3 was not found, as the downside variation increases in six cases out of 10 when adding an investment in cryptocurrencies, the results show strong evidence that the Sortino Ratio increases with an investment in cryptocurrencies. Therefore, all performance metrics analysed increase with an investment in cryptocurrencies.

All portfolios allocate a positive weight to the CRIX Index on average. It is interesting to look at the weight allocated to cryptocurrencies across the different portfolios. On average across the portfolios constructed, a weight of 9.89% is allocated to cryptocurrencies. Even though that is a fairly small weight on average, it has a substantial effect on the portfolios, as all performance metrics increase for every portfolio. It can also be noted that the average weight to cryptocurrencies is lower than one could expect, due to the asset's high returns. This further shows that even though the results from this analysis indicate that investing in cryptocurrencies enhances the performance of the portfolios, an investor should not invest all her money in cryptocurrencies, as the weights range from 0.05%-22.41%. This is in line with the studies by Wu and Pandey (2014), Damianov and Elsayed (2020), and Bakry et al. (2021).

8 Conclusion

Cryptocurrencies are still a young phenomenon, and it is likely that they will continue to develop in the coming years as the world takes faster and bigger steps towards technological progression, but how they will develop is widely debated. From an investment point of view, an investment in more developed cryptocurrencies has in recent years been in most cases very profitable, which from a fiat currency perspective is unusual, as investors would not expect anything near the realized cryptocurrency returns when investing in a fiat currency, and neither the extreme volatility. These characteristics, extreme returns and very high volatility lay an interesting foundation for this analysis; would it be beneficial to include these assets in optimized portfolios of traditional asset classes? Traditional financial instruments are tied to an underlying company or government which can be fundamentally analysed to attempt to value the security, as well as being subject to strict regulation. None of this applies to cryptocurrencies, which is another interesting characteristic of cryptocurrencies in the context of an investment asset.

Even though some literature exists on adding Bitcoin to optimized portfolios, the literature is young and dependent on a short period of data. As the literature on including a bundle of cryptocurrencies to optimal portfolios is scarce, the authors seek to answer the research question and add to the literature by including a wider range of developed cryptocurrencies than just Bitcoin in the analysis.

To answer the research question, this analysis constructs and simulates 20 portfolios based on five optimization techniques, the Global Minimum Variance portfolio, the Tangency portfolio, the Optimal portfolio with three different risk aversion coefficients, the Bayes-Stein Optimal portfolio with three different risk aversion coefficients and the Risk Parity portfolio, as well as the Equally Weighted portfolio. For each portfolio construction method, a portfolio is simulated with and without an investment in cryptocurrencies. Performance of the portfolios is measured out-of-sample with monthly rebalancing, resulting in 36 rebalances for each portfolio.

The results of the analysis show evidence of the benefits of including a bundle of cryptocurrencies in an optimized portfolio. The simple average excess return increases in all portfolios when including cryptocurrencies, on average increasing by 69.87%. However, returns are not the only metric to look at when evaluating portfolio performance, as risk needs to be considered. When investigating Sharpe Ratios, the results show evidence for the benefit of including cryptocurrencies in optimized portfolios, as on average the Sharpe Ratio increases by 31.79% when including cryptocurrencies. The evidence is even clearer when analysing the Sortino Ratios of the portfolios, as on average, the Sortino Ratio increases by 56.73% when including cryptocurrencies in the portfolios. The research question of this analysis is:

Does an investment in cryptocurrencies enhance the performance of optimal portfolios?

To be able to answer the research question, three hypotheses are developed and empirically tested. The results show evidence that supports all three hypotheses. Based on the evidence and the hypotheses, an investment in cryptocurrencies enhances the performance of optimized portfolios, as all common performance metrics increase with the inclusion of cryptocurrencies in the portfolios. An interesting component of the results is that to achieve better portfolio performance, the weights allocated to cryptocurrencies are not extreme. The smallest average weight allocated to cryptocurrencies is 0.05% in the Global Minimum Variance portfolio whereas the largest average weight allocation to cryptocurrencies is 22.41% in the Optimal portfolio with $\gamma = 5$.

It should be noted and kept in mind that the results of this analysis are subject to some limitations as outlined in section 5.10. First, the results are dependent on the assets and time period studied in this analysis. The portfolio optimization procedure is sensitive to both the time period and the assets included in the analysis, as the results are likely to be considerably different with another composition or of assets and another time period. Further, the estimation period is relatively short at the beginning of the sample, as time series data for the CRIX index is limited because of its young age, but it grows as time passes with the expanding estimation window. It is also assumed in the analysis that investors can observe prices and contemporaneously both compute the next optimal weight allocations and trade on these prices. In this analysis, transactions costs are also neglected.

Possibilities for further research on the topic are vast. First, with the passage of time, more data will be available for researchers to include in their analysis. The data available for cryptocurrencies is still scarce, compared to the decades of data available for other traditional asset classes, so with more data available, the results can be tested in a more robust way with respect to time. Inclusion of more assets and asset classes can also make the results more robust.

As stated before, these results are dependent on the specific assets chosen in this analysis. The portfolio theory and portfolio optimization literature are vast. Further research could be conducted using more optimization techniques, for example using the mean-CVaR approach, the Black-Litterman optimization method, and many others. Finally, more constraints can be added and removed from the analysis. A popular constraint is to restrict short-selling or to enforce minimum positions in all assets, as well as a wide range of other possible constraints. This is however left for further research.

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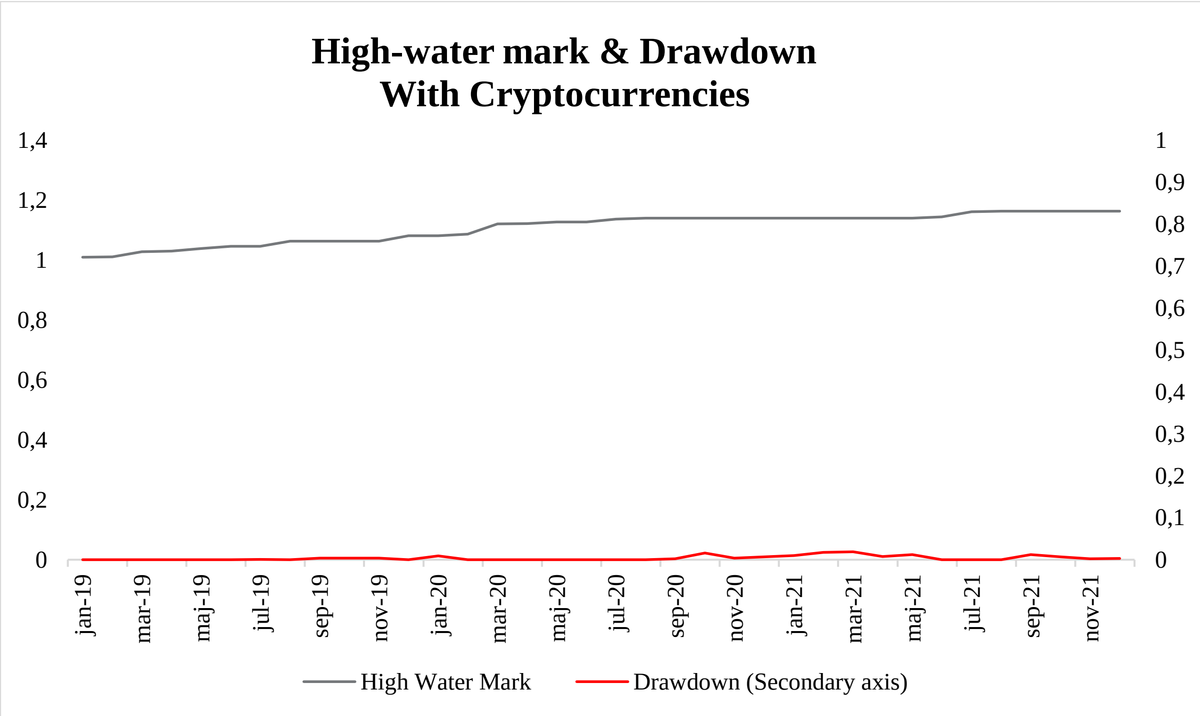
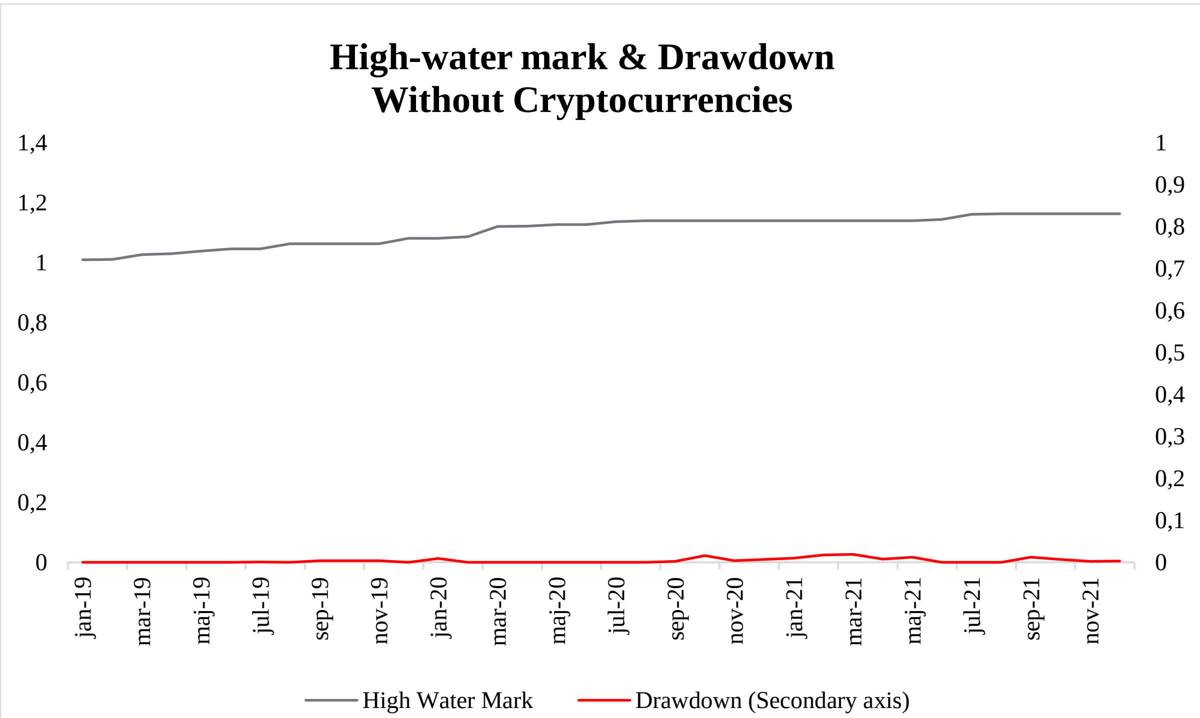
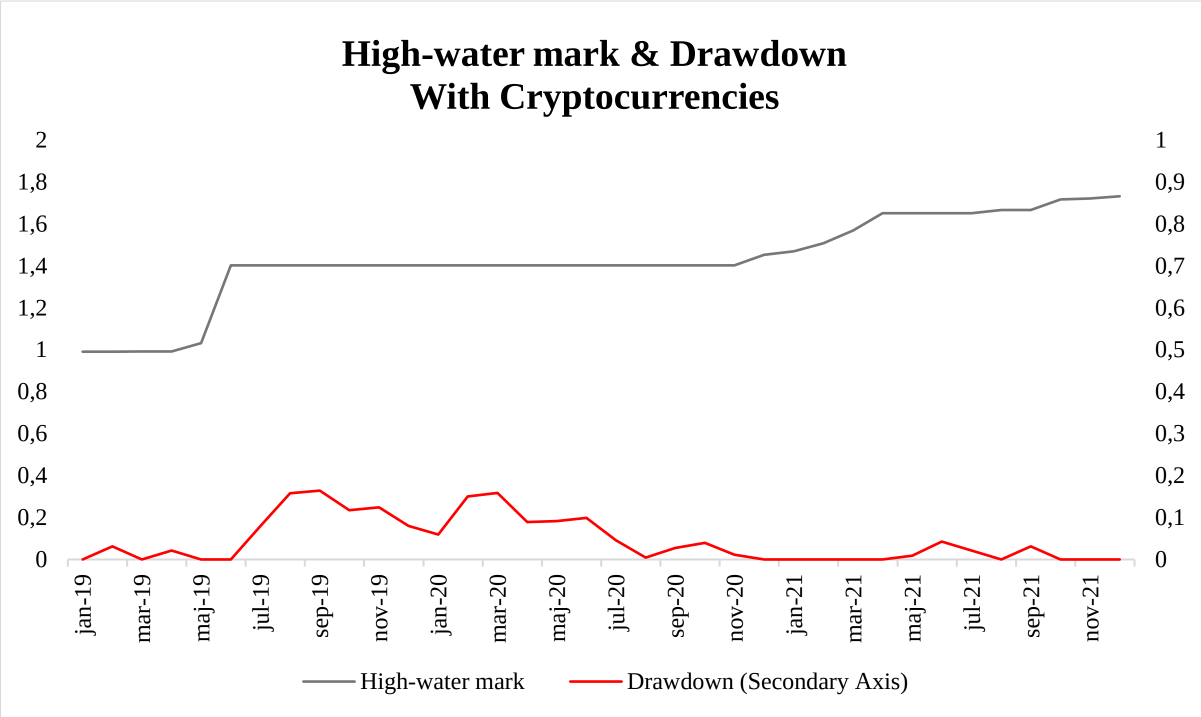
Appendix

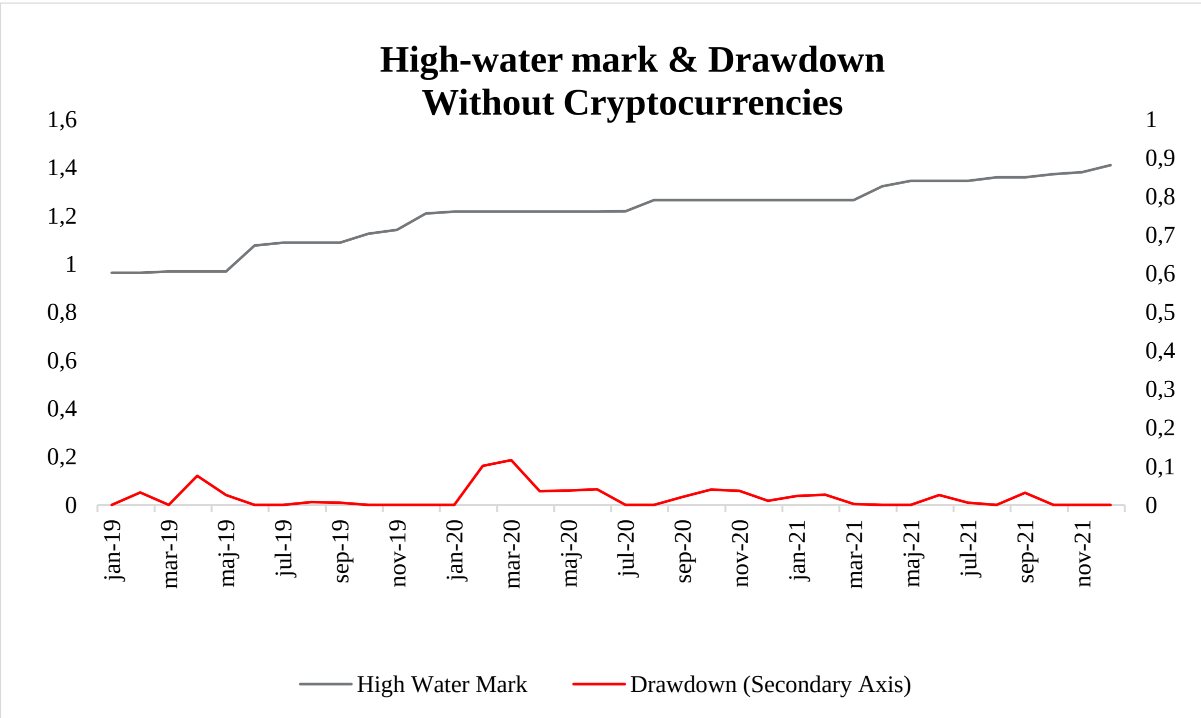
Appendix 1 – HWM and DD Global Minimum Variance portfolio



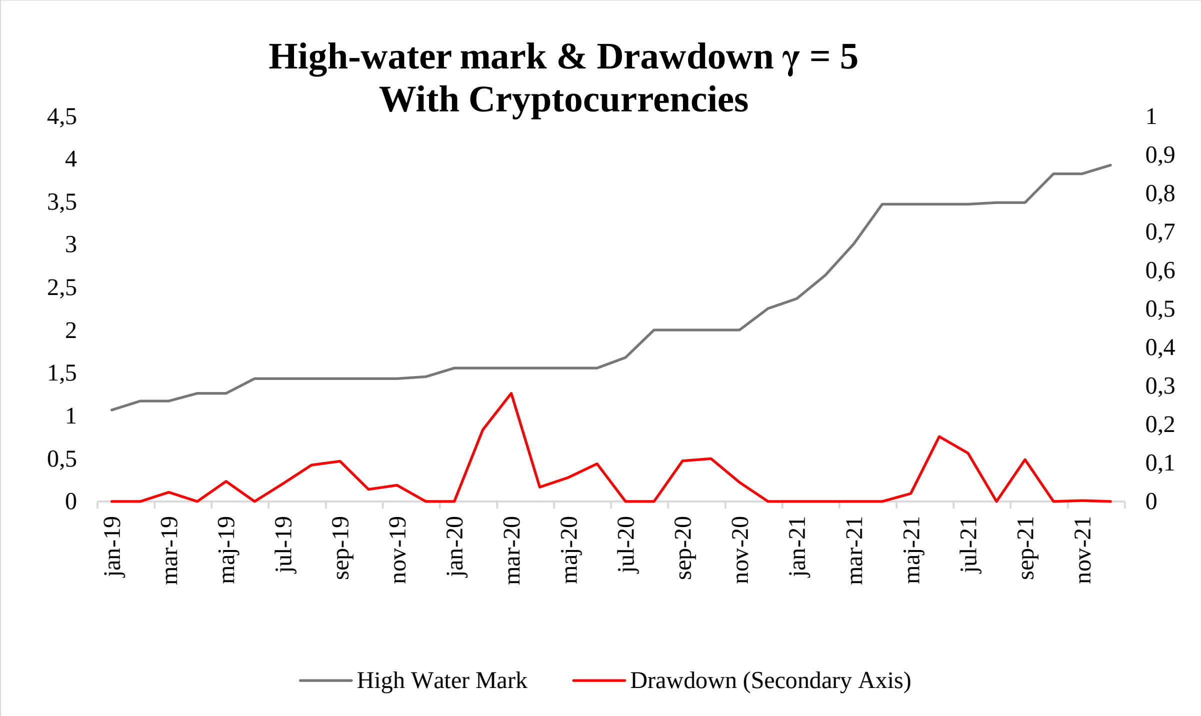


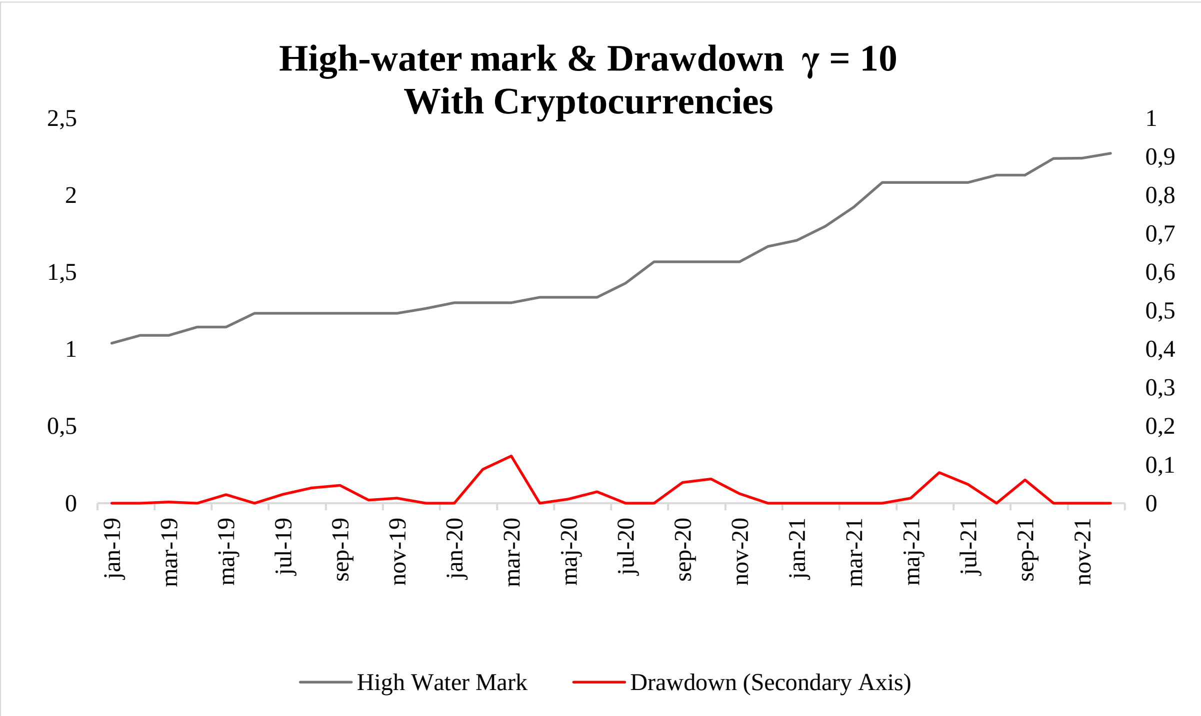
Appendix 2 – HWM and DD Tangency portfolio

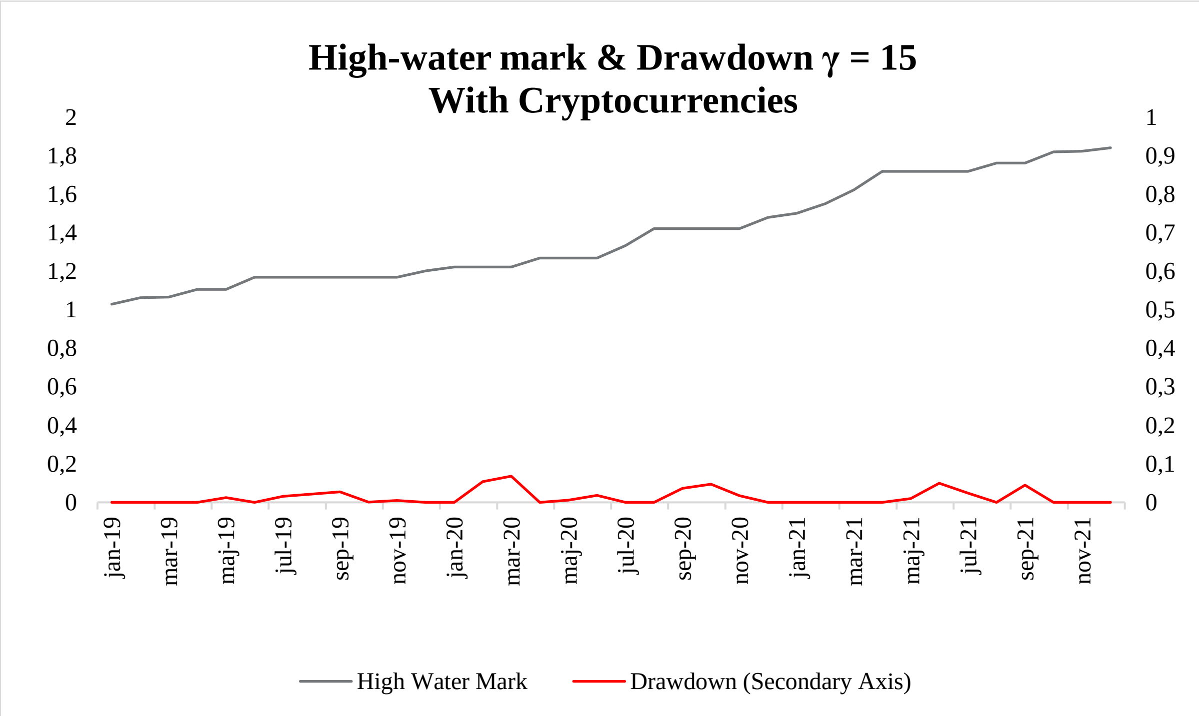




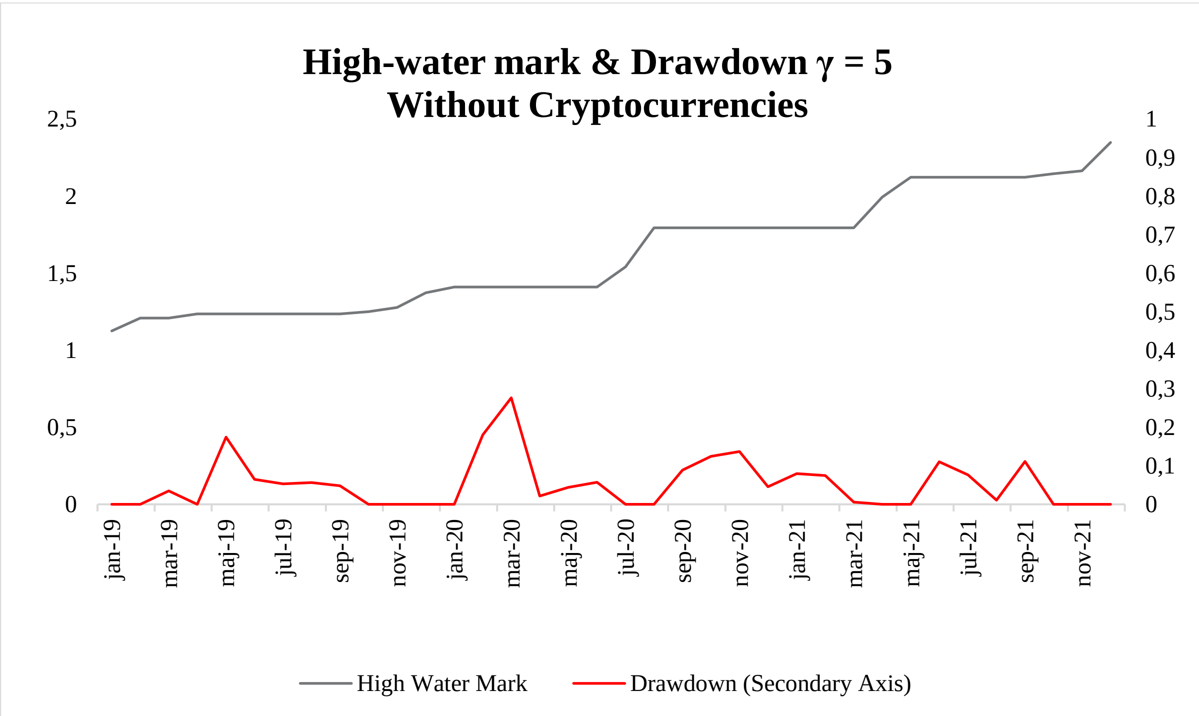
Appendix 3 – HWM and DD Optimal portfolio



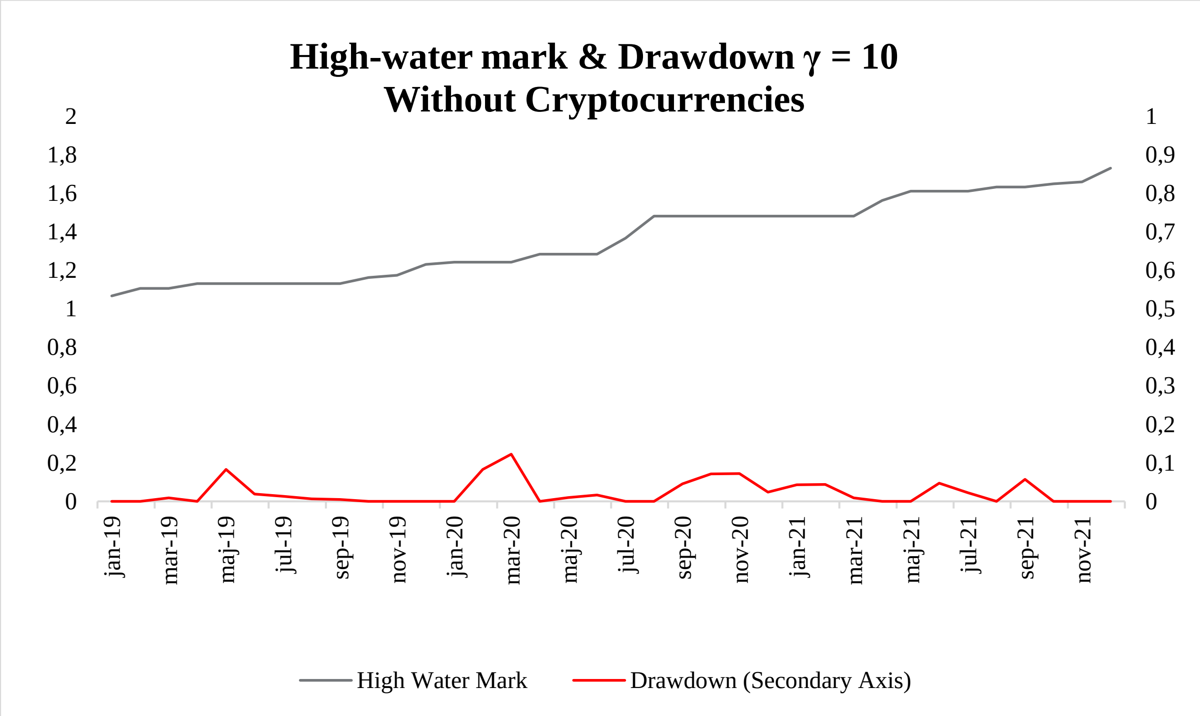




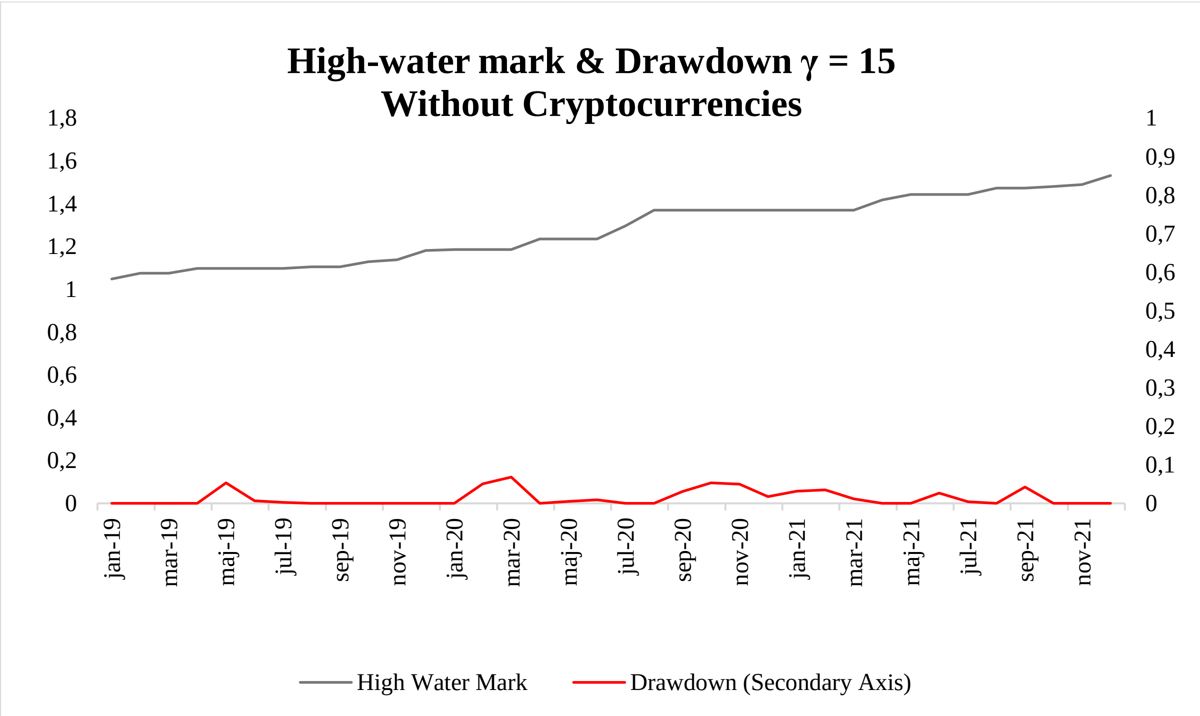
High-water mark & Drawdown γ = 15
With Cryptocurrencies
2
1,8
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
jan-19
mar-19
maj-19
jul-19
sep-19
nov-19
jan-20
mar-20
maj-20
jul-20
sep-20
nov-20
jan-21
mar-21
maj-21
jul-21
sep-21
nov-21
High Water Mark
Drawdown (Secondary Axis)



High-water mark & Drawdown γ = 5
Without Cryptocurrencies
2,5
2
1,5
1
0,5
0
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
jan-19
mar-19
maj-19
jul-19
sep-19
nov-19
jan-20
mar-20
maj-20
jul-20
sep-20
nov-20
jan-21
mar-21
maj-21
jul-21
sep-21
nov-21
High Water Mark
Drawdown (Secondary Axis)

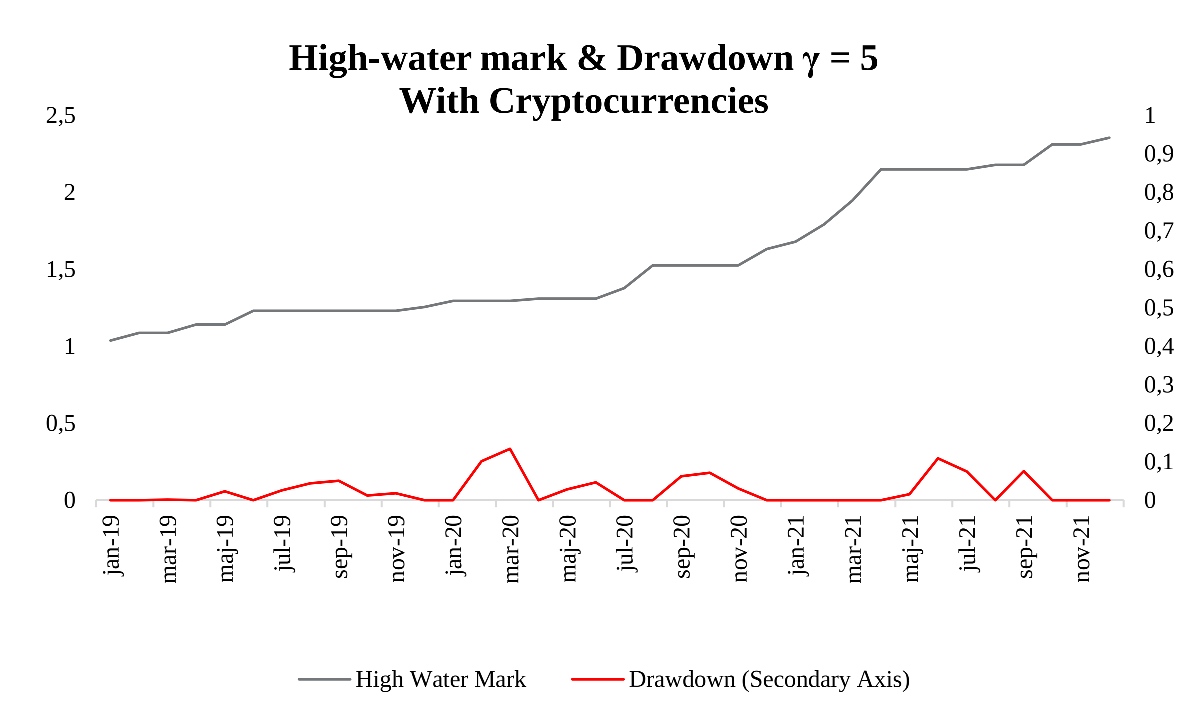


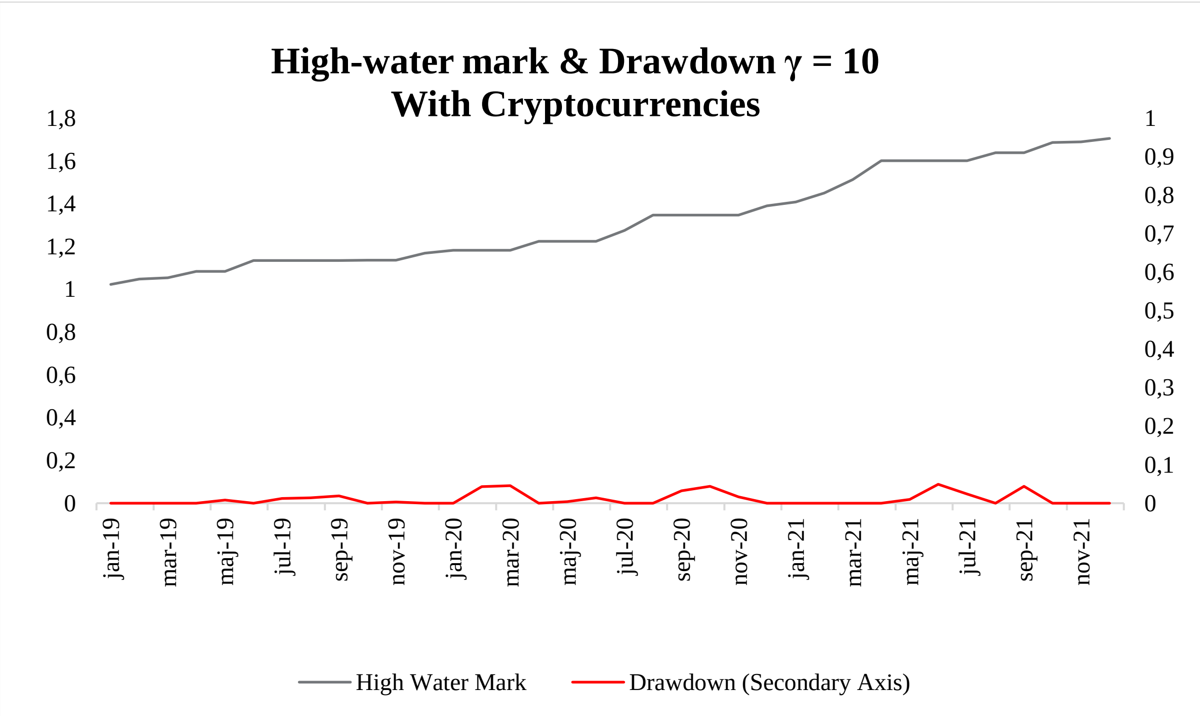
High-water mark & Drawdown γ = 10
Without Cryptocurrencies
2
1,8
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
jan-19
mar-19
maj-19
jul-19
sep-19
nov-19
jan-20
mar-20
maj-20
jul-20
sep-20
nov-20
jan-21
mar-21
maj-21
jul-21
sep-21
nov-21
High Water Mark
Drawdown (Secondary Axis)

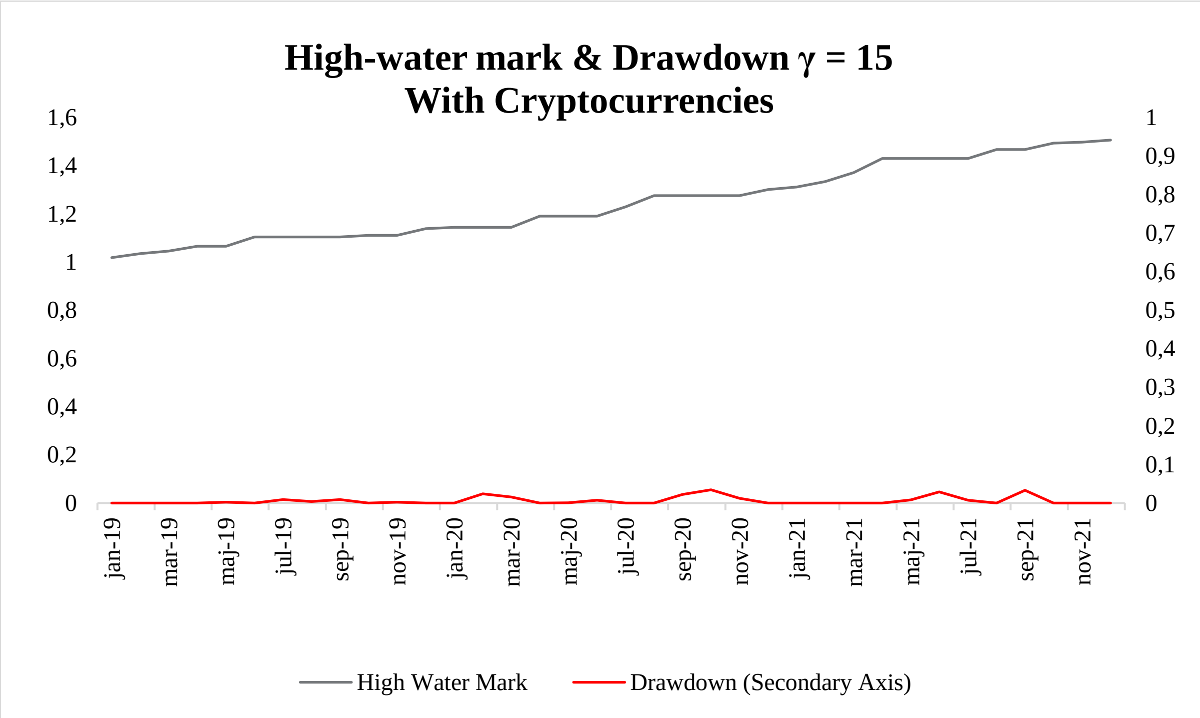


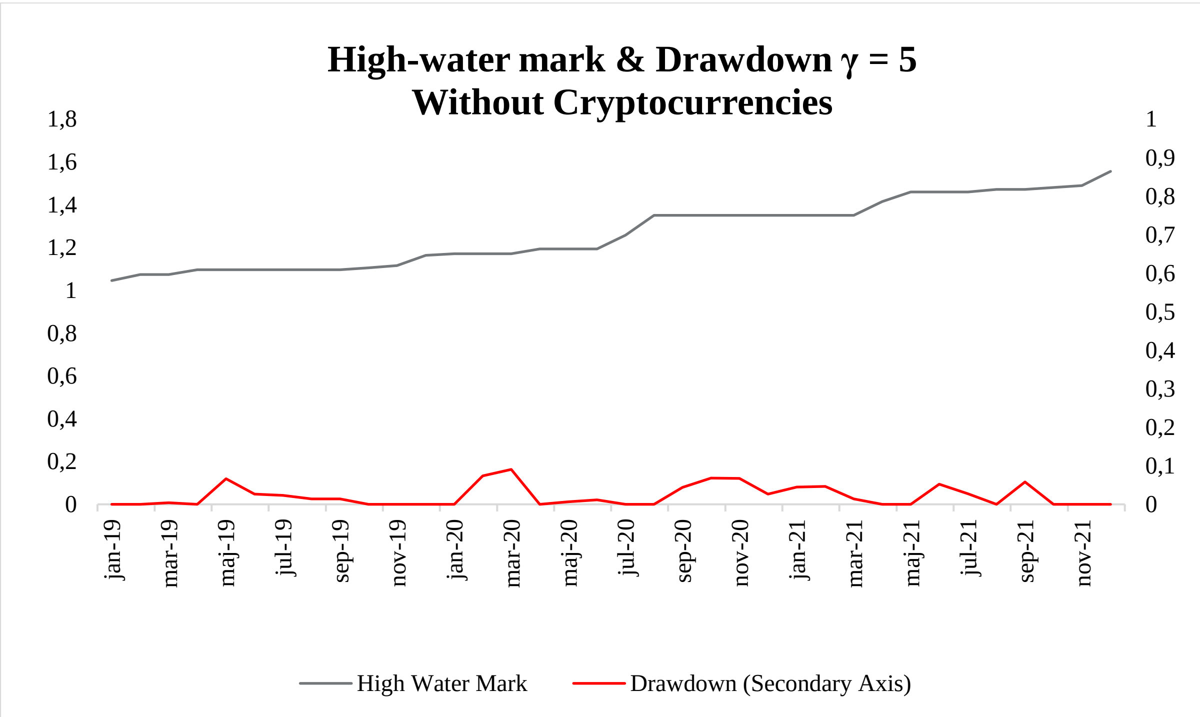
High-water mark & Drawdown γ = 15
Without Cryptocurrencies
1,8
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
jan-19
mar-19
maj-19
jul-19
sep-19
nov-19
jan-20
mar-20
maj-20
jul-20
sep-20
nov-20
jan-21
mar-21
maj-21
jul-21
sep-21
nov-21
High Water Mark
Drawdown (Secondary Axis)

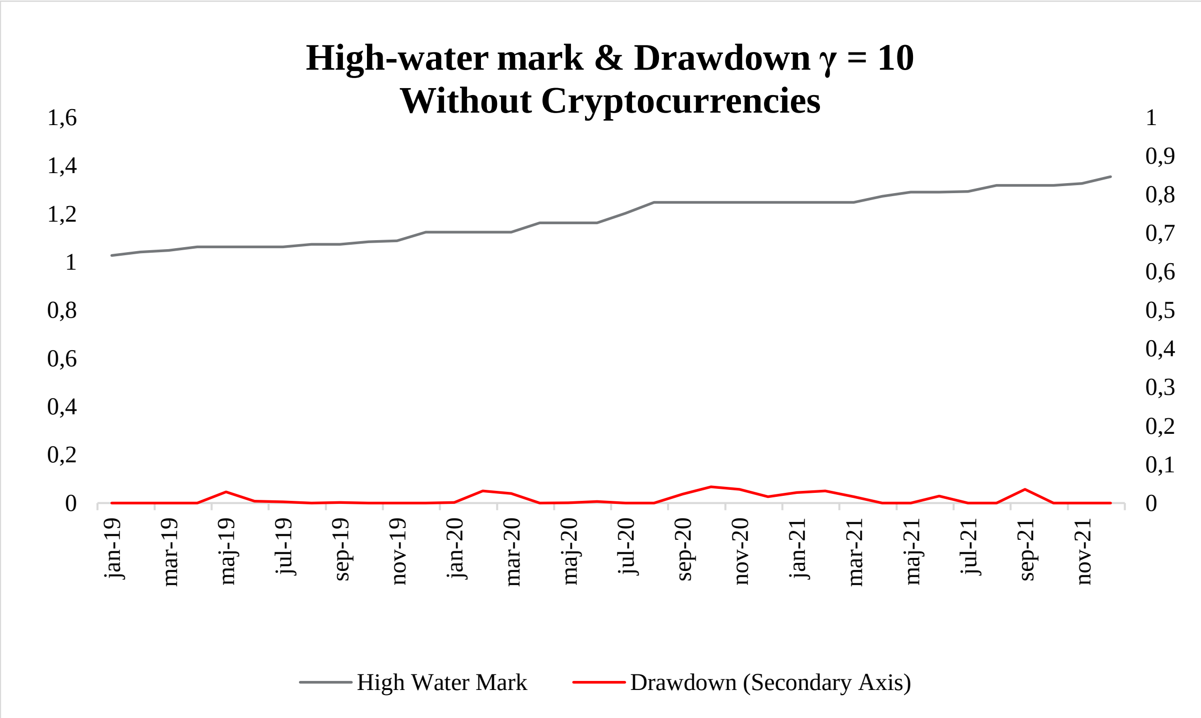
Appendix 4 – HWM and DD Bayes-Stein Optimal portfolio

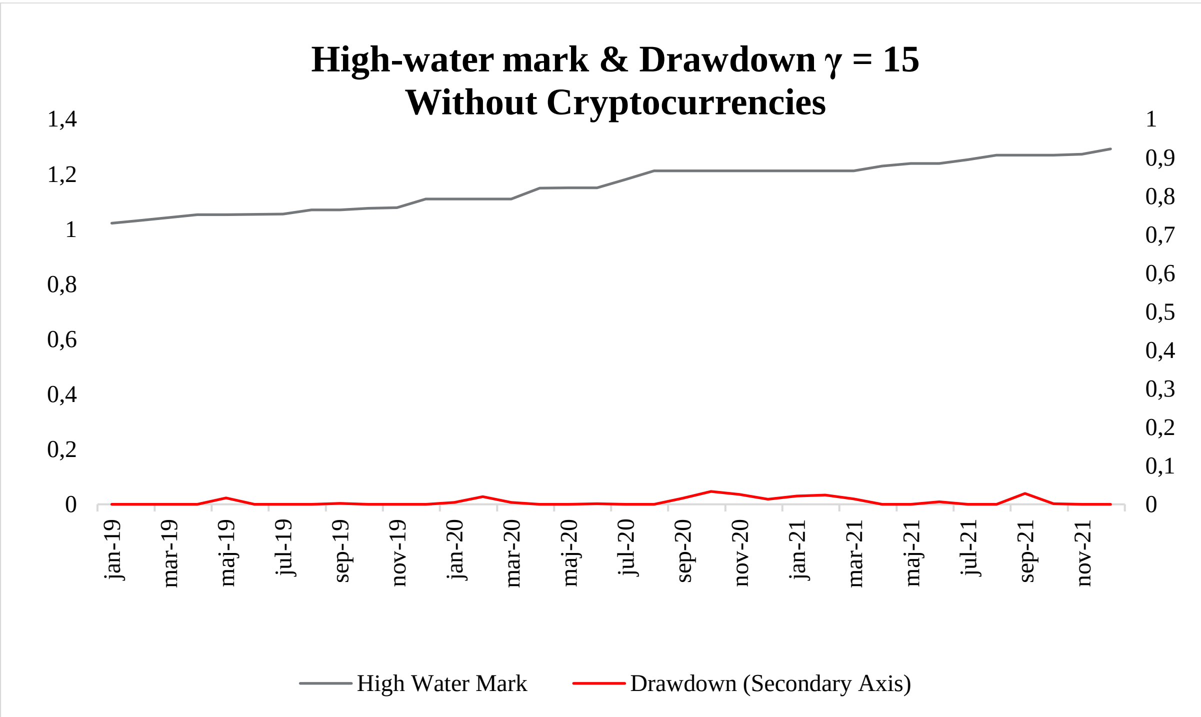




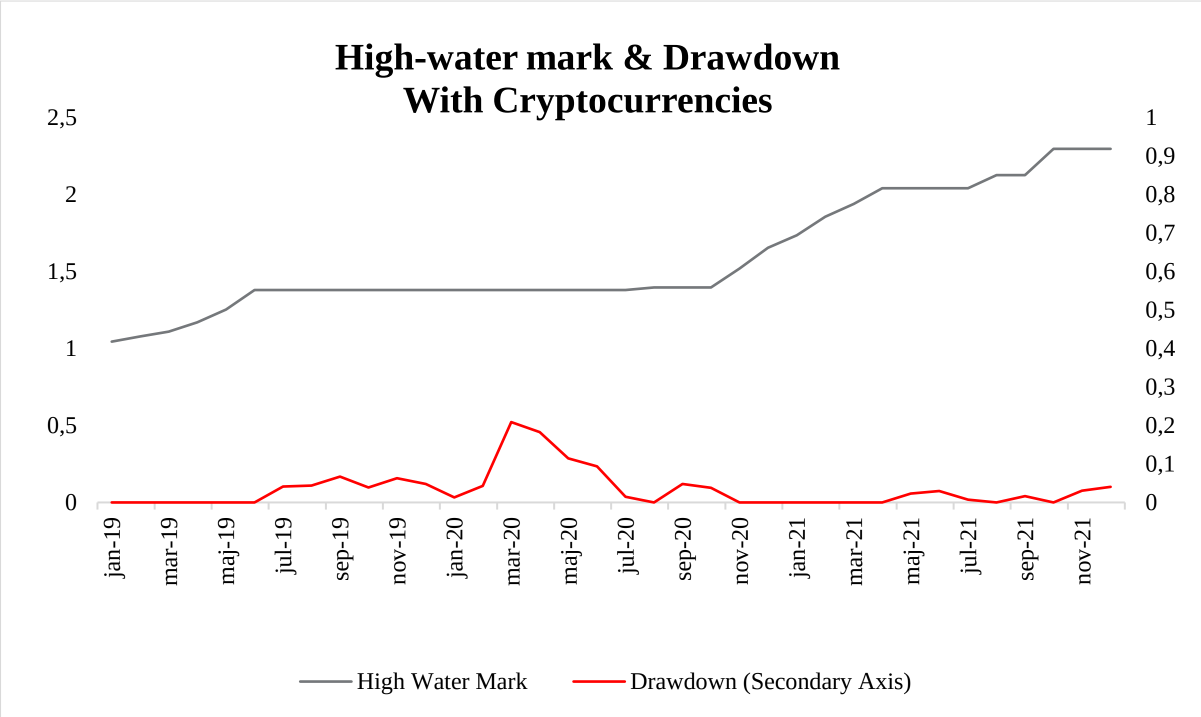

High-water mark & Drawdown γ = 15
With Cryptocurrencies
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
jan-19
mar-19
maj-19
jul-19
sep-19
nov-19
jan-20
mar-20
maj-20
jul-20
sep-20
nov-20
jan-21
mar-21
maj-21
jul-21
sep-21
nov-21
High Water Mark
Drawdown (Secondary Axis)

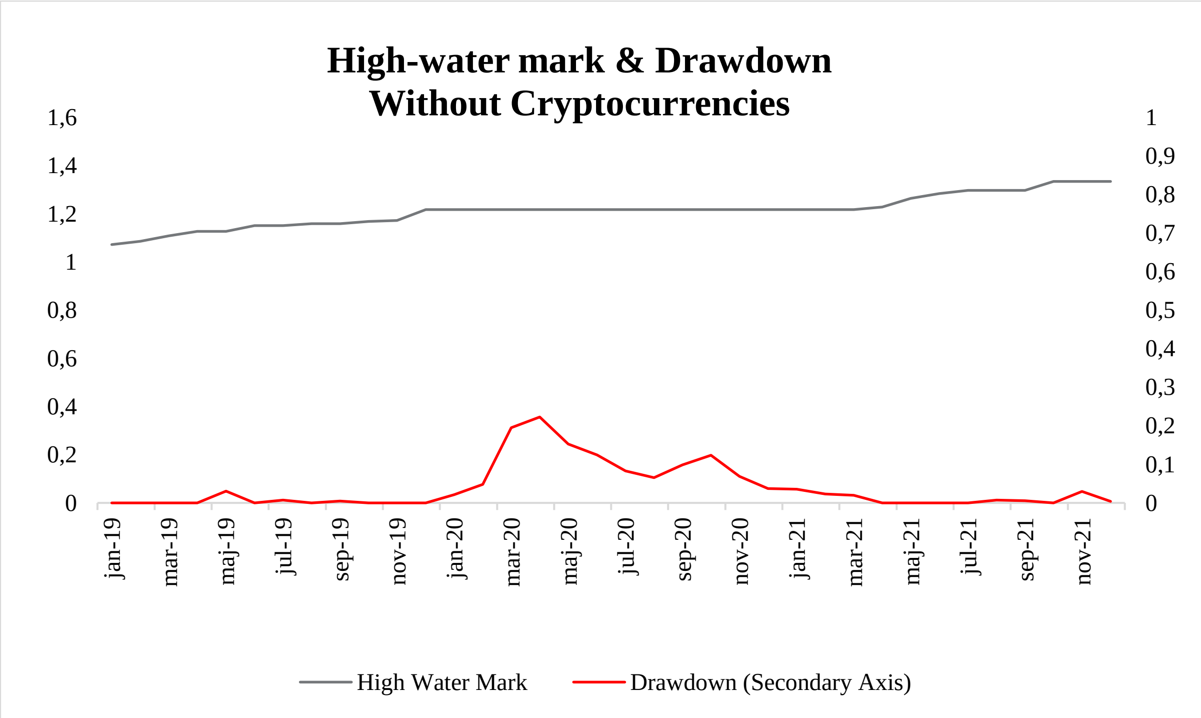

High-water mark & Drawdown γ = 5
Without Cryptocurrencies
1,8
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
jan-19
mar-19
maj-19
jul-19
sep-19
nov-19
jan-20
mar-20
maj-20
jul-20
sep-20
nov-20
jan-21
mar-21
maj-21
jul-21
sep-21
nov-21
High Water Mark
Drawdown (Secondary Axis)


High-water mark & Drawdown γ = 10
Without Cryptocurrencies
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
jan-19
mar-19
maj-19
jul-19
sep-19
nov-19
jan-20
mar-20
maj-20
jul-20
sep-20
nov-20
jan-21
mar-21
maj-21
jul-21
sep-21
nov-21
High Water Mark
Drawdown (Secondary Axis)

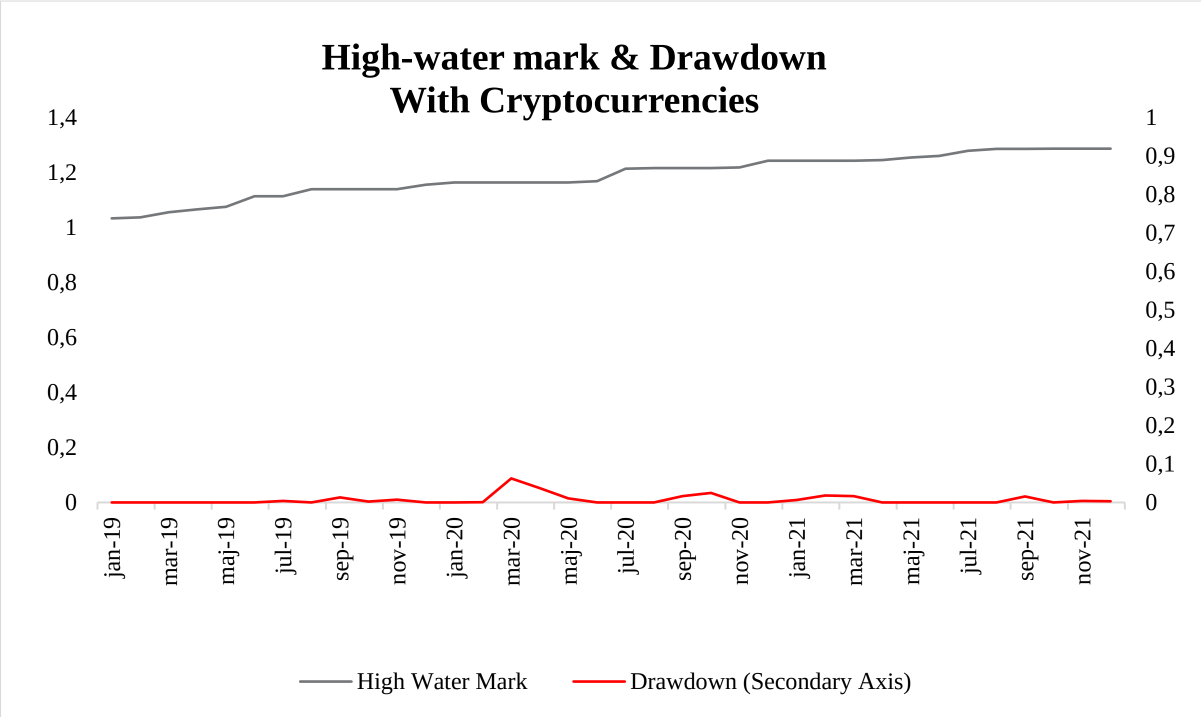

High-water mark & Drawdown γ = 15
Without Cryptocurrencies
1,4
1,2
1
0,8
0,6
0,4
0,2
0
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
jan-19
mar-19
maj-19
jul-19
sep-19
nov-19
jan-20
mar-20
maj-20
jul-20
sep-20
nov-20
jan-21
mar-21
maj-21
jul-21
sep-21
nov-21
High Water Mark
Drawdown (Secondary Axis)

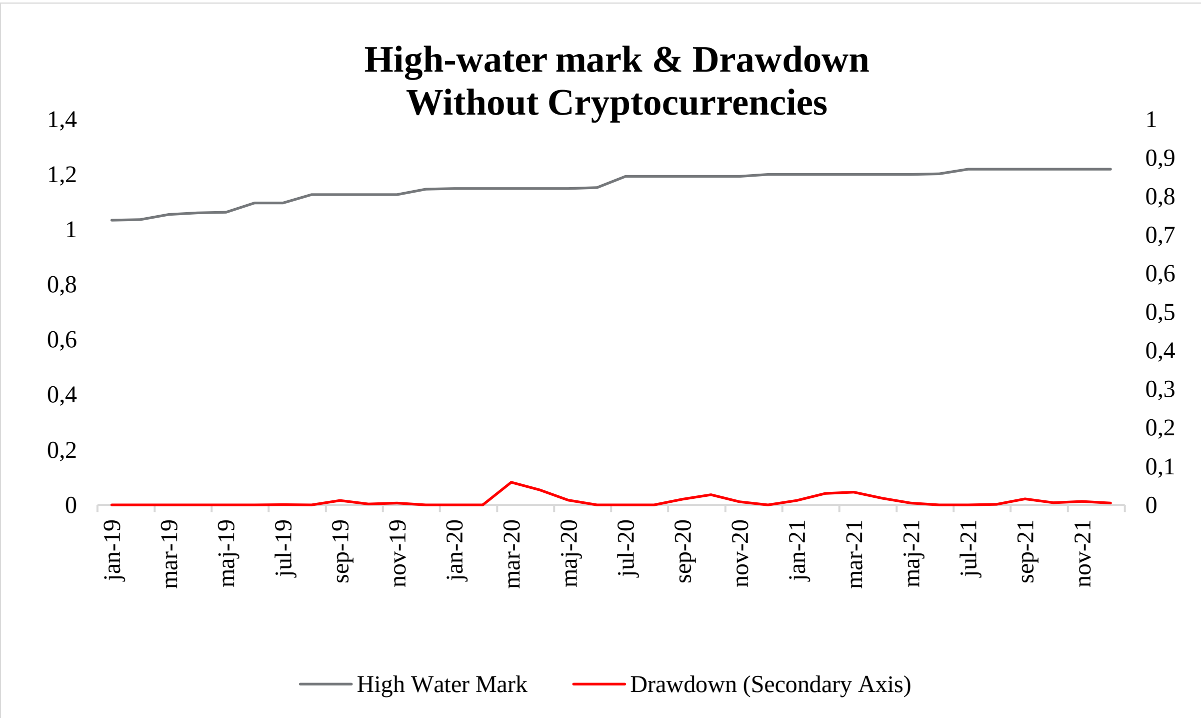
Appendix 5 – HWM and DD Equally Weighted portfolio





Appendix 6 – HWM and DD Risk Parity portfolio





Appendix 7 - Dataset

Date	GSPC	GOVT	USO	LQD	GLD	CRIX
Mar 01, 2016	1,937.09	25.79	71.84	115.11	118.74	36.91
Apr 01, 2016	2,056.62	25.76	75.28	118.65	116.08	37.45
May 01, 2016	2,067.17	25.69	89.92	119.95	123.78	39.21
Jun 01, 2016	2,093.94	25.71	92.96	119.28	115.97	47.89
Jul 01, 2016	2,099.34	26.30	92.16	122.99	127.66	58.65
Aug 01, 2016	2,173.15	26.19	76.96	123.32	128.57	54.50
Sep 01, 2016	2,171.33	26.02	81.52	123.20	124.67	50.50
Oct 01, 2016	2,164.33	26.07	88.32	122.92	125.32	54.48
Nov 01, 2016	2,128.68	25.67	85.20	120.43	122.80	59.95
Dec 01, 2016	2,200.17	24.90	90.56	115.98	111.11	62.56
Jan 01, 2017	2,251.57	24.88	95.84	116.95	109.62	80.10
Feb 01, 2017	2,285.59	24.92	91.36	116.78	114.66	81.56
Mar 01, 2017	2,380.13	24.91	92.08	117.61	117.98	100.83
Apr 01, 2017	2,362.34	25.03	84.96	117.64	118.69	111.76
May 01, 2017	2,388.50	25.16	81.52	118.57	120.21	154.41
Jun 01, 2017	2,415.65	25.27	80.08	119.69	120.17	335.80
Jul 01, 2017	2,431.39	25.23	76.40	120.34	116.54	375.64
Aug 01, 2017	2,477.10	25.21	81.36	120.63	120.44	356.25
Sep 01, 2017	2,474.42	25.46	76.80	121.19	126.01	605.66
Oct 01, 2017	2,521.20	25.25	81.36	120.96	121.17	525.98
Nov 01, 2017	2,583.21	25.12	88.24	121.05	120.98	688.62
Dec 01, 2017	2,645.10	25.09	93.20	120.81	120.94	1,068.24
Jan 01, 2018	2,683.73	25.08	96.56	121.32	124.66	1,717.95
Feb 01, 2018	2,816.45	24.73	104.64	119.91	127.18	1,440.15
Mar 01, 2018	2,715.22	24.50	98.08	116.56	124.15	1,306.46
Apr 01, 2018	2,633.45	24.64	103.84	116.90	126.65	743.50
May 01, 2018	2,642.96	24.41	109.52	114.83	123.90	1,193.32
Jun 01, 2018	2,718.70	24.50	107.28	114.87	122.58	939.49
Jul 01, 2018	2,704.95	24.61	120.00	114.41	118.18	672.27
Aug 01, 2018	2,821.17	24.36	112.72	114.90	115.63	809.61
Sep 01, 2018	2,896.96	24.52	118.80	114.86	112.75	667.54
Oct 01, 2018	2,926.29	24.27	124.00	114.50	112.40	633.14
Nov 01, 2018	2,717.58	24.13	110.88	111.71	116.33	580.22
Dec 01, 2018	2,790.50	24.28	90.16	111.31	116.60	358.06
Jan 01, 2019	2,476.96	24.82	75.36	112.82	121.35	343.56
Feb 01, 2019	2,702.32	24.85	91.20	116.12	124.89	309.95
Mar 01, 2019	2,798.22	24.70	95.60	115.74	123.48	351.93
Apr 01, 2019	2,848.63	25.12	100.96	118.45	122.40	375.00
May 01, 2019	2,952.33	25.07	106.24	119.00	121.03	464.78
Jun 01, 2019	2,751.53	25.65	90.40	120.75	124.09	736.11
Jul 01, 2019	2,971.41	25.76	99.52	124.21	131.56	983.62

Date	GSPC	GOVT	USO	LQD	GLD	CRIX
Aug 01, 2019	2,980.32	25.77	94.96	124.31	132.42	773.39
Sep 01, 2019	2,909.01	26.58	88.72	128.37	144.96	706.60
Oct 01, 2019	2,983.69	26.20	91.12	126.56	138.07	621.79
Nov 01, 2019	3,050.72	26.22	91.60	127.38	142.21	692.94
Dec 01, 2019	3,143.85	26.02	94.24	127.07	137.32	578.83
Jan 01, 2020	3,244.67	26.42	102.40	128.34	143.86	523.93
Feb 01, 2020	3,235.66	26.50	86.00	130.58	148.66	697.33
Mar 01, 2020	2,974.28	27.24	77.36	131.87	150.00	664.23
Apr 01, 2020	2,498.08	28.16	33.92	122.55	148.20	485.37
May 01, 2020	2,869.09	28.08	19.12	128.70	158.00	667.17
Jun 01, 2020	3,038.78	27.91	25.72	131.40	162.92	698.35
Jul 01, 2020	3,105.92	27.91	28.30	134.15	167.05	677.69
Aug 01, 2020	3,288.26	28.19	29.15	137.95	185.05	873.53
Sep 01, 2020	3,507.44	27.87	30.64	135.25	186.99	928.99
Oct 01, 2020	3,385.87	27.89	27.76	134.26	178.71	827.03
Nov 01, 2020	3,296.20	27.66	25.16	133.96	177.45	993.86
Dec 01, 2020	3,645.87	27.41	30.92	138.05	169.76	1,462.50
Jan 01, 2021	3,764.61	27.33	33.00	137.89	181.97	1,993.76
Feb 01, 2021	3,731.17	27.09	35.77	135.37	175.02	2,564.23
Mar 01, 2021	3,842.51	26.45	41.30	131.64	163.03	3,473.24
Apr 01, 2021	3,992.78	26.19	41.47	130.46	161.56	4,346.49
May 01, 2021	4,191.98	26.34	43.56	131.13	167.61	5,064.14
Jun 01, 2021	4,216.52	26.34	46.58	131.44	178.76	3,637.79
Jul 01, 2021	4,300.73	26.54	51.21	134.05	166.65	3,207.44
Aug 01, 2021	4,406.86	26.90	50.35	135.92	169.30	3,477.27
Sep 01, 2021	4,528.80	26.83	47.36	135.33	169.94	4,645.16
Oct 01, 2021	4,317.16	26.54	52.62	133.23	164.27	4,158.44
Nov 01, 2021	4,610.62	26.37	57.91	132.61	167.38	5,880.45
Dec 01, 2021	4,602.82	26.55	49.09	132.77	166.84	5,688.17
Jan 01, 2022	4,778.14	26.33	53.97	131.78	168.86	4,554.93