Game On
Social Networks and Markets

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Game on: Social networks and markets☆

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A B S T R A C T

I present closed-form solutions for prices, portfolios, and beliefs in a model where four types of investors trade assets over time: naive investors who learn via a social network, "fanatics" possibly spreading fake news, and rational short- and long-term investors. I show that fanatic and rational views dominate over time, and their relative importance depends on their following by influencers. Securities markets exhibit social network spillovers, large effects of influencers and thought leaders, bubbles, bursts of high volume, price momentum, fundamental momentum, and reversal. The model sheds new light on the GameStop event, historical bubbles, and asset markets more generally.

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Communication in social networks has influenced investors since equity trading started in trading clubs connected to Amsterdam Stock Exchange. In fact, London Stock Exchange started in coffee houses in the 17th century, and the social dynamics in these coffee houses gave rise to such events as the South See Bubble.[1] More recently, social media have made social networks larger and more observable to researchers, but how do social networks affect asset markets?

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[1] De la Vega (1688) and Petram (2014) describe how Amsterdam Stock Exchange started in 1602 with social elements. Standage (2006) ch. 8 describes how London's stock trading started in what the author calls the "coffeehouse internet," pointing out that "the drama of the South Sea Bubble, a fraudulent investment scheme that collapsed in September 1720, ruining thousands of investors, was played out in coffeehouses."

This paper presents a model of how investment ideas can propagate through a social network and affect market behavior and prices. When investors learn through their social network, they can disagree for extensive time periods, and their disagreement is time-varying and predictable based on the social network. This disagreement can generate a trading frenzy with a spike in turnover, high volatility, price momentum as a bubble builds, and a value effect as the bubble bursts. Rational investors exploit network dynamics and may initially ride the bubble, but eventually bet on reversal to fundamentals. As an illustration of these insights of the model, the paper presents the dramatic events related to the GameStop stock in early 2021 as well as broader connections to historical bubbles and market dynamics.

To study social network effects in the simplest possible way, I introduce rational agents and financial markets into an otherwise standard DeGroot (1974) model.[2] In the DeGroot model, people update their beliefs by listening to other people in their social network. While this method of updating may initially be rational, the continued updating over multiple rounds of communication does not take into account that the same information may echo back many times (DeMarzo et al., 2003). While standard network models assume that everyone behaves in this naive way, I introduce rational learners into the model to capture the effects of sophisticated professional investors (or arbitrageurs).

Characterizing rational behavior in a network can be highly complex,[3] but I show that the updating of rational agents becomes very tractable when they can use the rational strategy of listening to everyone. The tractability afforded by my setting allows me to solve all agents' beliefs at all times in closed form, and, further, allows me to derive closed-form solutions for portfolios and market prices, as well as their limits as time increases.

The model has several important implications for how social networks affect opinions and asset prices. First, the model shows how rational agents can have a large effect on prices, both via their own trades and via their effect on the general opinion. Rational agents are initially extremely flexible, listening to all available information, and paying no special attention to their own initial view. However, once a rational agent has processed all the available information, the agent becomes completely "stubborn" in the sense that seeing a different view presented again and again does not sway the rational agent if the different view does not contain any new information. Therefore, rational

agents quickly become stubborn. Hence, they keep repeating their rational view, and this anchored opinion has an increasing influence on naive investors over time. This effect, which I denote the "stubbornness of truth," makes the market rational in the long term when the only stubborn agents are the rational ones.

However, "fanatic" agents who are stubborn about their own personal view, however irrational, can also have a large influence over time if others are willing to listen. In the long run, all agents' views converge to a combination of the various stubborn views, but investors differ in their reliance on rational or fanatic views. I show that the aggregate importance (or "thought leadership") of each stubborn view is the sum-product of the attention it gets from its followers and the "influencer values" of their followers. Influencer values can be easily computed, and I show how thought leadership and influencer values affect asset prices.[4]

The resulting differences of opinions lead to trading activity, but the trading activity dies down over time as views stabilize (since trading arises from view changes). Nevertheless, investors' portfolios differ, even in the long run, in contrast to the prediction of the standard capital asset pricing model (CAPM). These network effects can further lead to high prices (bubbles), low prices (anti-bubbles or deep value), and large and prolonged price swings.

The model can therefore help explain pervasive market effects such as price momentum and reversal effects (see Asness et al. 2013 and references therein), large trading volume with poor performance of the retail investors who trade the most (Odean, 1999), the relation between volume and momentum (Lee and Swaminathan, 2000), and excess volatility (Shiller, 1981) driven by chat in social media (Antweiler and Frank, 2004). Also, while rational investors and efficient prices react almost immediately to earnings announcements and other news, investors learning via a social network react only gradually, so the model can also help explain post-earnings announcement drift (Ball and Brown, 1968) and other kinds of fundamental momentum and announcement effects.

While there already exist theories for several of these phenomena,[5] the key distinguishing feature of my theory is that it predicts that trading behavior spreads via a social network. This specific prediction of the theory is confirmed by Bailey et al. (2018) who use Facebook data to show

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[2] In doing so, I contribute to the network literature by developing a tractable dynamic model of financial markets with network effects, showing the separate equilibrium effects of influencers, thought leaders, naive agents, and rational forward-looking agents who exploit predictability arising from network dynamics, including momentum and reversal effects. Surveys on network economics include Jackson (2010) and Golub and Sadler (2016), and networks and trading are studied in a rational expectations equilibrium by Colla and Mele (2010), Ozsoylev and Walden (2011), and Walden (2019) and in an over-the-counter setting by Babus and Kondor (2018).

[3] E.g., DeMarzo et al. (2003) state that "We should emphasize that the calculations that agents must perform even in this simple case where the network is common knowledge can be very complicated" (p. 927).

[4] Thought leadership is a generalization of the concept "social influence" used in the literature following DeGroot (1974) while influencer values are new to the literature, to the best of my knowledge. Stubborn agents have also been studied by Ghaderi and Srikant (2014) and others; I contribute by showing that rational agents also become "stubborn," by showing how influencers affect stubborn agents' thought leadership, and by developing implications for financial markets.

[5] Differences of opinions can generate bubbles (see Harrison and Kreps, 1978 for a seminal model) and turnover (Harris and Raviv, 1993). Theories of momentum and reversal based on psychological biases include Barberis et al. (1998), Daniel et al. (1998), and Hong and Stein (1999). Models of information percolation via random search (rather than a network) include Duffie et al. (2009), Andrei and Cujean (2017), Burnside et al. (2016) who apply such a framework to housing booms and busts, and Hirshleifer (2020) who assumes that each random interaction is associated with a "social transmission bias." Han et al. (2022) show how social transmission bias can spread active strategies.

that people with friends who experienced recent house price gains increase their housing market expectations and "buy larger houses and pay more for a given house." Further, investors' local social network can also help explain their local bias in their equity investments, and the resulting social network effects have an impact on firm values (Kuchler et al., 2020).

To test the model's predictions on thought leadership, suppose that fanatics cheering for a stock are more likely to reside in the same county as the firm's headquarter (e.g., the executives of the firm). Then these fanatics can elevate prices more if they have greater thought leadership, that is, are more connected to investors, and Kuchler et al. (2020) create a measure of Facebook friends between the firm's county and institutional investors' counties, denoted the firm's "social proximity to capital." They find that firms with stronger social proximity to capital indeed have higher institutional ownership and higher valuations. Also, Cookson et al. (2020) provide evidence of echo chamber effects in investor beliefs.

Social networks have also been shown empirically to affect equity market participation of retail investors (Hong et al., 2004; Brown et al., 2008; Kaustia and Knüpfer, 2012), affect the portfolios of money managers (Hong et al., 2005; Cohen et al., 2008) and retail investors (Bhamra et al., 2021), and potentially serve as a useful source of information (Chen et al., 2014). Moreover, social media facilitate pump-and-dump schemes in cryptocurrencies (Li et al., 2020) and professional traders' discussions on social media have been central to litigation of financial market misconduct (see, e.g., Financial Times, 11/12/14, "Traders' forex chatroom banter exposed").

In the GameStop case, investors on social media signaled their stubborn commitment to buying and holding the stock via the meme "diamond hands." They spurred each other via Reddit, Twitter, YouTube, and other social media, and signaled an extreme view of the potential valuation via the "rocket" meme. The price increased when Elon Musk, an influencer on social media (and much more), tweeted a link to the Reddit site hosting the most fanatic GameStop traders, WallStreetBets. The stock price increased by orders of magnitude at enormous trading volume and volatility, coinciding with an increase in social media attention.

Eaton et al. (2021) exploit platform outages for the broker used by many GameStop traders, Robinhood, to identify the causal effects of retail traders on financial markets. The paper finds that exogenous negative shocks to Robinhood participation leads to lower return volatility among stocks favored by Robinhood investors. Barber et al. (2022) also study trades by customers of Robinhood, reporting that "intense buying by Robinhood users forecast negative returns."

GameStop can be viewed as the latest example in a long history of bubbles, and my model sheds new light on the "anatomy" of bubbles described by Kindleberger (2000) and Shleifer (2000), ch. 6.2. As illustrated in Table 1, the anatomy starts with an "initial displacement," which is captured in my model by fanatic agents who observe positive news and decide to focus all their attention on this positive aspect of the asset.

Next, the price increases further due to "speculation for price increases" by sophisticated short-term traders. Short-traders are also modelled by De Long et al. (1990), but in my model their speculation is based on an understanding of predictability due to the network communication. In the spirit of my model, Soros (2003) reports riding bubbles based on the communicated sentiment among investors, e.g.: "if an idea was appealing enough to attract me on first hearing, it was likely to have the same effect on others. If, on further investigation, I found it to be flawed I could always turn around and liquidate my position with a profit provided I was not the last one to hear it" (p. 36).[6]

The bubble is then sustained as a "larger and larger group of people seeks to become rich without a real understanding of the processes involved."[7] This "mania" is captured in my model via the social network that enables the idea to spread across people, detailing this "emulation" step by step.

The bubble is further expanded by an "authoritative blessing," captured in my model via the effect of influencers. When an influencer listens to fanatics, the influencer spreads the idea to a broader group of people, further increasing the price (as Elon Musk did in the GameStop case).

At the bubble's late stage, rational "insiders sell out," just as rational investors do in my model.[8] The final stage is a "crash," which can start from a "revelation of a swindle" or a "revulsion."[9] I capture the former by modeling the possibility that the true asset value is revealed, which leads to an abrupt crash (or "panic"). I capture the latter via fanatics who are optimistic due to the positive displacement, but not entirely stubborn. In this case, their initial optimism creates a bubble, but, as they learn the rational truth over time, their optimism turns to revulsion, and the bubble peters out. As seen in Table 1, these model elements also describe the GameStop case and help explain several key asset pricing anomalies (as discussed earlier).

The related literature also includes models of "rational bubbles," in which bubbles may exist simply because they can (in some of multiple equilibria). In contrast, I model the economics underlying the anatomy of bubbles as people learn via their network. My framework is also fundamentally different from standard models of "private information" where people learn from prices. Said simply, my framework captures the situation where your cab driver tells you about the opportunity in bitcoin that he learned about from his friends, not from the price. People in my model seek to learn the facts directly via communication and their trades inform the price – not the other way

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[6] Likewise, De la Vega (1688) states that "means are not lacking to recognize what political or business opinions are held by persons of influence. He who makes it his business to watch these things conscientiously, without blind passion and irritating stubbornness, will hit upon the right thing innumerable times, though not always" (p. 41).

[7] Kindleberger (2000) p.16, who traces the ideas of "overtrading" to Adam Smith and John Stuart Mill, also cites Minsky for the element of expansion of credit, from which I abstract.

[8] Brunnermeier and Nagel (2004) provide evidence that hedge funds initially rode the internet bubble and eventually reduced or reversed their positions, just like the short-term investors in my model.

[9] Kindleberger (2000) p.17.

Table 1

Model implications. The first column contains the key elements of my model, showing how an investment idea starts in the model (A), initially affects prices (B), evolves over time (C,D), and how rationality ultimately sets in (E,F). The second column shows how these model elements capture the stylized evolution of historical bubbles, that is, the bubble anatomy of Kindleberger (2000) and Shleifer (2000). The third column shows how each element played a role in the GameStop case. Finally, the last column shows how these model elements help explain certain general asset-pricing effects in financial markets.

Model elements	Bubble anatomy	GameStop 2021	Asset pricing
A. Investors receive news, fanatics focus on one element	Initial displacement	Retail investors focus on plan to pivot online	Announcement effects, e.g., post-earnings drift
B. Short-term investors bet on network spillovers	Speculation	Some institutional investors are long	Momentum and fundamental momentum
C. Opinions spread through the network	Mania and emulation	More and more people hear about GameStop	Local bias and network spillover effects
D. Influencers follow a fanatic	Authoritative blessing	Elon Musk tweets a link to WallStreetBets	Excess volatility
E. Rational traders bet on reversal	Insiders sell out	Institutional investors sell	Value investing
F. Fundamentals are revealed or fanatics gradually learn	Crash: revelation or revulsion	Drops in January, March earnings announcement, etc.	Long-run reversal and value effect

around. My model thus captures situations in which information is "out there" to be found by rational investors, but naive investors nevertheless learn gradually and imperfectly via their network.[10]

Consistent with my premise, Shiller and Pound (1989) report survey evidence that "interpersonal communications are very important in investor decisions" and Shiller (2001) argues that "word-of-mouth communications, either positive or negative, are an essential part of the propagation of speculative bubbles." Shiller (2001) also considers a standard epidemic model to capture this phenomenon, discussing how people can "infect" each other with an investment idea. Hirshleifer (2020) considers a time-varying infection rate or "buzz" and, more broadly, calls for a greater focus on social interaction in finance. My model contributes to this approach by deriving equilibrium prices when different rational and fanatic ideas attract attention from investors via their communication in a social network in which some people are more influential than others (rather than a single idea spreading via random matching among identical myopic investors).

Lastly, my paper is related to the emerging literature on expectation formation (see, Bordalo et al., 2019 and references therein). My model shows how inter-personal communication can spread the intra-personal judgment biases documented in this literature.

In summary, this paper contributes to the literature by developing a simple model of rational and naive investors who interact via a social network in order to trade assets.[11]

The model yields closed-form beliefs, prices, and portfolios, and provides a new mechanism of a number of financial market properties such as social-network spillover effects, the effects of influencers and thought leaders, momentum, reversal, high trading volume, the anatomy of bubbles in general, and the GameStop case in particular.

1. Model

The economy has N investors who communicate with each other and trade an asset in discrete time indexed by $t = 0, 1, 2, \ldots$, as described in this section.

Asset and signals. The asset has a supply of shares given by s. Its fundamental value is given by $v + u(t) \in \mathbb{R}$, where $u(t)$ is a publicly observed random walk and v is an unobserved random variable that investors try to learn about. The random walk has innovations with constant variance given by $\sigma_u^2 = \text{Var}(u(t) - u(t-1))$. We can think of $u(t)$ as the value of assets in place (e.g., GameStop's retail stores) and v as the value of a new uncertain investment opportunity (e.g., GameStop's opportunity to sell games online).

At time 0, each person i starts with a signal about the value v given by the random variable $x_i(0) = v_i$. This signal gives each agent a useful, but incomplete, piece of information about v. Collectively, all agents have full information about v, which is modeled via the relation $v = \sum_{i=1}^{N} \kappa_i v_i$, where the known weights sum to one, $\sum_{i=1}^{N} \kappa_i = 1$. Each weight, κ_i, is a measure of the importance of agent i's signal; for example, if all agents receive signals of the same importance, then the weights are equal, $\kappa_i = \frac{1}{N}$. In any event, agents have an incentive to communicate with others to learn about v.[12]

The unobserved value, v, is revealed each time period with probability π and remains unknown with probability

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[10] Tirole (1982) shows that bubbles cannot exist in many situations in which all agents are fully rational, while Allen et al. (1993) show that rational bubbles can exist when people have private information, short-sale constraints, and trade is not common knowledge.

[11] Relative to the seminal paper by DeMarzo et al. (2003), my contribution is to consider rational and stubborn agents, to define and analyze influencers and thought leaders, and to combine the network model with an economic equilibrium in an asset market. As cited earlier in the introduction, the paper is also related to behavioral finance (see survey by Hirshleifer, 2015). One challenge to behavioral finance is that investor mistakes would not matter in aggregate if they were uncorrelated, but a standard response is that people make correlated mistakes because they have common biases. My response is simply: people talk. Mackay (1850) starts his classic treatment of the "madness of crowds" by stating that "Popular delusions began so early, spread so widely, and have

lasted so long, that instead of two or three volumes, fifty would scarcely suffice to detail their history." See Kuchler et al. (2020) for a survey on the empirical finance literature on social networks.

[12] The information structure is slightly simpler than the standard information-theoretic framework (e.g., Hellwig, 1980) in which the signals equal the true v plus random noise. The standard framework yields the same solution, except that one must adjust the definitions of x_r and σ_u^2, as shown in Appendix A.3.

$1 - \pi$. Said differently, v is revealed at the random time τ, with a geometric distribution. When v is revealed, the price equals its total value, $v + u(\tau)$. For example, the firm could be acquired or liquidated for $v + u(\tau)$, or the firm could continue with investors knowing the fundamental value $v + u(t)$ when $t \geq \tau$. The objective of the model is to understand how beliefs, trading, and prices evolve before the value is revealed.[13]

Naive and rational learning in a social network. People communicate with each other as follows. At each time t, everyone states their current views, collected in the vector $x(t) = (x_1(t), \ldots, x_N(t))'$, to everyone who listens.[14] The economy consists of people with two methods of paying attention, "rational learners" and boundedly-rational ones denoted as "naive" for brevity. Each naive learner i selectively follows a subset of people that he views as most informative or most entertaining. Further, he uses the same method for updating each round. Specifically, he uses the row vector $A_i \in \mathbb{R}_{\geq 0}^{1 \times N}$ to make the update, such that his view in the next time period becomes

$$x_i(t + 1) = A_i x(t) \qquad (1)$$

where the weights add up to one, $\sum_j A_{ij} = 1$. The network is therefore characterized by A, which is called the *adjacency matrix* (or weight matrix) in the DeGroot (1974) model. The i'th row, A_i, can have many zeros, representing all the people that i does not "follow." The non-zero elements represent the list of people that i "follows" and the amount of attention paid to each of them. Similarly, the i'th column contains the list of i's "followers." DeMarzo et al. (2003) show that the first-round updating can be seen as rational Bayesian updating given that an agent only listens to the people with non-zero weights. Hence, the naivety of this investor comes from his selective listening and from the fact that the agent keeps using the same method of updating, which can be justified based on imperfect recall (Molavi et al., 2018). In particular, the naive agent does not take into account that the same information may be received many times, which DeMarzo et al. (2003) denote as "persuasion bias." Naive agents who only listen to themselves $A_{jj} = 1$ are denoted as fanatics, and they play a special role in my analysis.

In the literature that follows the standard DeGroot model, everyone is naive, but I also consider rational learners to capture the effects of sophisticated investors. A rational learner i listens to everyone in the first round of communication (as this is the rational way to listen with unbounded attention).[15] Based on hearing everyone's views, the rational learner updates her view to $x_i(1) = x_r$,

where the rational view x_r is given by:

$$x_r = E(v | x_1(0), \ldots, x_N(0)) = (\kappa_1, \ldots, \kappa_N)x(0) = v \qquad (2)$$

Naturally, the rational view uses the information contained in all signals, and here the weighted average of all signals in fact reveals v for simplicity. Further, the rational person never changes her view about v after the first round since no new information arrives, $x_i(t) = x_i = x_r$ for all $t \geq 1$. In other words, a rational agent i can also be seen as updating her views using the adjacency matrix as in (1), but she initially uses the row κ' and thereafter she uses $A_i = e_i$, where $e_i = (0, \ldots, 0, 1, 0, \ldots, 0)$ is the i'th unit vector.

Portfolios and prices. Trading starts at time 1, after the first round of communication. At any time t after the value of v has been revealed (i.e., $t \geq \tau$), the price naturally equals the commonly known total value, $v + u(t)$. Before revelation, the endogenous price of the asset at time t is denoted by $p(t)$. So the price can be written as

$$price(t) = p(t)1_{(t < \tau)} + [v + u(t)]1_{(t \geq \tau)} \qquad (3)$$

All agents take this price as given and the equilibrium market price is determined such that the total demand equals the supply of shares s, that is,

$$s = \sum_{i=1}^{N} d_i(t) \qquad (4)$$

where $d_i(t)$ is the demand of agent i at time t, which depends on the price as described next.

The agents behave as either long-term investors or short-term traders, where long-term investors think in terms of the long-term asset value at revelation, whereas short-term investors think in terms of the one-period expected gains each time period. Specifically, naive, fanatic, and some of the rational learners behave as long-term investors, while the remaining rational learners behave as short-term investors — so the economy has these four types of agents (naive, fanatic, rational long-term, rational short-term).

Any naive, fanatic, or rational long-term investor i chooses his investment demand to maximize his mean-variance utility at the revelation time, τ. To compute this utility, note that the investor earns a profit per share of $v + u(\tau) - p(t)$ from buying at time t for $p(t)$ and selling at time τ when the asset is worth $v + u(\tau)$. Hence, the utility of buying d_i shares is

$$\max_{d_i} d_i E_t[x_i(t) + u(\tau) - p(t)]$$

$$- \frac{1}{2w_i}\text{Var}_t[d_i(x_i(t) + u(\tau) - p(t))] \qquad (5)$$

where v is replaced by $x_i(t)$ because rational investors have learned the true value of v at time 1, and naive and fanatic investors behave as if their current belief is also the true value. The utility also depends on w_i, which is investor i's absolute risk tolerance, that is, the ratio of the agent's wealth and relative risk aversion, $w_i = wealth_i / \gamma_i$. To ease

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[13] The model is intended to be as simple as possible, but note that the model could easily be extended in several ways. For example, while the model is focused on a single asset, the analysis is straightforward to extend to the case of any number of assets as shown in Appendix A.4 (by letting $x_i(t)$ be a row vector with investor i's views about the different assets, implying that $x(t)$ becomes a matrix of all investors' views about all assets).

[14] If agent i prefers to stay silent, then this can be captured by having an adjacency matrix A in which column i has zeros outside the diagonal, but I assume that rational agents still have a way of getting this information. The conclusion discusses pump-and-dump and other forms of strategic communication.

[15] If rational agents could only listen to a subset of agents, they could still learn all signals in finite time under certain conditions, after which time the model would be the same.

the discussion, I refer to w_i simply as wealth, so risk aversion can be taken to be equal across agents, say $\gamma_i = 1$, or simply think of the word "wealth" as shorthand for risk-bearing capacity.[16]

Maximizing the utility (5) with a risk given by $\text{Var}_t(u_\tau) = \sigma_u^2/\pi$, the investor's optimal portfolio is seen to be[17]

$$d_i(t) = \frac{\pi w_i}{\sigma_u^2}(x_i(t) + u(t) - p(t)) \qquad (6)$$

Naturally, the asset demand increases in the perceived gap between the value and the price, and the price sensitivity is larger if the agent is wealthier (larger w_i), the value is realized sooner (larger π), and if the risk (σ_u) is smaller.

Turning to rational short-term traders, any such agent i maximizes her one-period mean-variance utility, which depends on the difference between the current price and the price in the next time period. The expected price in the next time period is the average of the price without and with revelation of the fundamental value, weighted by their respective probabilities, $1 - \pi$ and π:

$$E_t(price(t+1)) = (1-\pi)E_t(p(t+1)) + \pi(x_r + u(t)) \qquad (7)$$

Given this expected price and the one-period risk of $\sigma_u^2 = \text{Var}_t(u(t+1) - u(t))$, the demand of any short-term investor is[18]

$$d_i(t) = \frac{w_i}{\sigma_u^2}[(1-\pi)E_t(p(t+1)) + \pi(x_r + u(t)) - p(t)] \qquad (8)$$

What is special about the short-term investors is that they seek to exploit predictable price changes due to the network spillover effects in the equilibrium price.

Notation. In summary, the economy has four types of agents who communicate to learn about an asset that they are trading: Naive investors, fanatics, rational long-term investors, and rational short-term traders. Agents are ordered such that the first N_n agents are naive, the next N_f are fanatics, the next N_l are long-term investors, and the last N_s are short-term investors, where $N = N_n + N_f + N_l + N_s$.

The latter three types are denoted as "hardheaded" (h) since these investors keep constant views after time 1. The column vector of all views can be decomposed as $x(t) = (x_n(t); x_h)$, where $x_n(t) = (x_1(t); \ldots; x_{N_n}(t))$ contains the naive views and $x_h = (x_f; x_l; x_s)$ contains the hardheaded views. Note that x_h is not indexed by t since these hardheaded views are constant after time 1, but I use the notation $x_h(0)$ for the initial views of these investors.

I use a similar notation for the vector of all agents' wealth, $w = (w_1, \ldots, w_N)'$ and its naive and hardheaded components, $w = (w_n; w_h)$, where $w_h = (w_f; w_l; w_s)$.

All notation is summarized in Table A.1 in the appendix.

2. Belief formation in a social network

To get some intuition for how beliefs are formed over time in a social network, I first consider the special cases in which all agents are naive (Section 2.1), or all but one agent are naive (Sections 2.2–2.3). Then I show how beliefs are formed in the main model (Section 2.4) when naive agents are influenced by many different fanatic and rational agents.

2.1. Social influence when everyone is naive and connected

Suppose first that everyone is naive, which is the classic DeGroot model. In this case, agents' views after t rounds of communication is

$$x(t) = A^t x(0) \qquad (9)$$

If all agents are "strongly connected" (everyone indirectly influences everyone) and listen to themselves, then DeGroot (1974) and DeMarzo et al. (2003) show that a unique $z \in \mathbb{R}^N$ exists such that $z'A = z'$ and, as $t \to \infty$,[19]

$$x(t) = A^t x(0) \to 1_N z' x(0) \qquad (10)$$

In other words, everyone ends up with the consensus view $z'x(0)$. The consensus view weights the different agents' signals by their "social influence" z_i, whereas the rational solution would weight all agents' signals by the precision κ_i. In particular, person i's social influence, z_i, is the weighted-average of the social influence of everyone that listens to i, that is, $z_i = \sum_j A_{ij} z_j$. So a person becomes influential by having the ear of influential people. This eigenvector property is also the idea behind Google's PageRank.

2.2. A fanatic in an echo chamber: stubborn fake news

Consider next the case in which one agent is completely stubborn and all other agents are naive as above. The stubborn agent can be interpreted as a fanatic or someone who deliberately tries to shut down all alternative views. In particular, suppose that agent N is stubborn, meaning that N'th row of A is the N'th unit vector, $A_N = e_N'$. Mathematically, this corresponds to a Markov chain with an absorbing state, and opinions evolve as follows

$$x(t) = A^t x(0) \to 1_N e_N' x(0) = 1_N x_N(0) \qquad (11)$$

meaning that everyone ends up having the same view as the stubborn person (a special case of Proposition 1 below). So, in this case, the stubborn agent becomes the

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[16] Differences in real-world investors' risk tolerance is mostly driven by wealth since relative risk aversion varies only by one order of magnitude (γ_i is usually between 1 and 10) while invested wealth varies by many orders of magnitude (between \$1000 and billions of dollars).

[17] The risk is $\text{Var}_t(u_\tau) = \text{Var}(\Delta u_\tau + \ldots + \Delta u_{t+1}) = E(\Delta u_\tau + \ldots + \Delta u_{t+1})^2 = \sigma_u^2 E(\tau - t) = \sigma_u^2/\pi$, based on the optional stopping theorem and using $E(\tau - t) = \sum_{k=1}^{\infty} k(1-\pi)^{k-1}\pi = 1/\pi$.

[18] When computing the risk of price changes, short-term investors focus on $\text{Var}_t(u(t+1) - u(t))$ and are risk-neutral with respect to uncertainty about the revelation time, for simplicity. Note that the model retains its tractability for any linear demand.

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[19] Strongly connected means that A is irreducible, that is, any agent j can influence any other agent i (specifically, agents $k_1, \ldots, k_z$ exist such that $A_{ik_1} > 0, A_{k_1 k_2} > 0, \ldots, A_{k_z j} > 0$). Further, A is aperiodic since people listen to themselves, $A_{jj} > 0$, meaning that infinite cycles cannot arise. Hence, A corresponds to an irreducible aperiodic positive matrix, which ensures convergence of views via the Perron-Frobenius Theorem, and z can also be seen as the stationary distribution of the corresponding Markov chain.

only person with social influence as he listens to no one but himself. What happens is that, each time period, the other agents move a little toward the stubborn agent (and toward each other) and, since the stubborn agent never moves, all other agents end up moving closer and closer to the stubborn view. Said differently, the naive agents keep hearing the stubborn view, and other views influenced by the stubborn view, so this situation can be seen as an "echo chamber" in which these agents are cut off from rational opinions and fail to realize that the stubborn opinion echoes through the system many times.

This phenomenon does not arise with Google's PageRank, because a website does not get credit for linking to itself, so, perhaps as a result, social influence via stubbornness is not the standard case in economics. However, stubbornness is important in this model because people can be stubborn and, in fact, rationality is a form of stubbornness, as discussed next.

2.3. Rationality in an echo chamber: the stubbornness of truth

Suppose alternatively that one agent is fully rational, while the rest are naive and connected. Recall that, after one round of communication, the rational agent already knows the "truth," that is, the best possible estimate of the value, v, given by x_r. Hence, from time 1 and onwards, the rational agent does not further update her views. Said differently, the rational behaves as if she is completely stubborn. Therefore, by the same logic as above, views converge as follows

$$x(t) \rightarrow 1_N x_r \tag{12}$$

meaning that everyone ends up having the same rational view.

Note that fanaticism and rationality look similar to a casual observer since such agents behave similarly in all rounds of communication except the first one. Indeed, a rational person is completely flexible in the first round of communication, seeking out all sources of information, having no special attachment to her own information, and logically aggregates all of this information. However, once all the information is aggregated, the rational person has no interest in hearing the same views again, does not budge to hearing a particular view repeated many times, and simply sticks to the same opinion forever (or at least until truly new information arrives).

Golub and Jackson (2010) describe another way to ensure rationality, even when all agents are naive. They show "that all opinions in a large society converge to the truth if and only if the influence of the most influential agent vanishes as the society grows." While a mathematically beautiful benchmark, the condition that the most influential agent has a vanishingly small influence seems clearly violated in the modern world of social media where, for example, Kim Kardashian has more than 300 million followers on Instagram as of this writing. Hence, for rationality to prevail, the mechanism developed here based on the presence of rational agents and the stubbornness of truth may have a better chance of success.

2.4. Fanaticism vs. rationality: main model of a social network

In the most general and realistic case, the economy has some of all the types of agents: naive, fanatic, and rational. This framework is the focus of the rest of the paper. After time 1, all rational agents are completely stubborn since they know the truth, and fanatics are stubborn for other reasons. So a central ingredient to the model is understanding the dynamics of an economy in which some agents are hardheaded ("h") and others are naive.

Recall that agents are ordered such that the first agents N_n are naive and the rest are hardheaded — specifically, the next N_f are fanatics, the last $N_l + N_s$ ones are rational. In this case, the adjacency matrix has the form:

$$A = \begin{pmatrix} A_{nn} & A_{nh} \\ 0 & I \end{pmatrix} \tag{13}$$

where $A_{nn} \in \mathbb{R}^{N_n \times N_n}$ is a matrix that determines how the naive agents listen to each other, and $A_{nh} \in \mathbb{R}^{N_n \times (N_f + N_l + N_s)}$ is a matrix that determines how they listen to the hardheaded agents. The lower rows of A consist of a matrix of zeros and the identity matrix since each hardheaded agent only listens to himself. The top rows of A must satisfy the following natural assumption, which is imposed from now on.

Assumption 1. Any agent i is hardheaded (rational or fanatic) or influenced by a hardheaded agent j, either directly (i.e., $A_{ij} > 0$) or indirectly (i.e., there exist agents $k_1, \ldots, k_z$ such that $A_{ik_1} > 0, A_{k_1 k_2} > 0, \ldots, A_{k_z j} > 0$).

This assumption says that any naive agent either listens to one of the hardheaded agents or listens to someone who listens to someone who does. Whereas the literature is focused on "strongly connected" agents (Section 2.1), my focus on "hardheaded-connected" agents is dictated by the existence of rational agents and possibly also fanatic agents.

Example 1. To illustrate how the model works, consider the following numerical example:

$$\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_f \\ x_r \end{pmatrix} = \begin{pmatrix} 70\% & 0 & 0 & 20\% & 10\% \\ 40\% & 40\% & 0 & 10\% & 10\% \\ 40\% & 0 & 40\% & 10\% & 10\% \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(t-1) \\ x_2(t-1) \\ x_3(t-1) \\ x_f \\ x_r \end{pmatrix} \tag{14}$$

As seen in the first row, agent 1 gives 70% weight to his own previous opinion, no weight to each of the other two naive agents, 20% to the fanatic agent, and 10% to the rational. Agents 2 and 3 also give 40% weight to their own respective views, 40% weight to agent 1, none to each other, and 10% to the fanatic and rational agents. The zeros in this example illustrate in a simplified way that, in the real world, each row has millions of entries – one for each person – but usually all but a few hundred entries are zero as most people only follow a limited number of others. Nevertheless, some columns can have millions of non-zero entries as some people have many followers. For example, agent 1 in this example is an "influencer" in the sense that

other agents pay strong attention to his views. Finally, the last two rows in (14) show that the views of the fanatic and a rational learner (agents 4 and 5) do not change after time 1.

The first step in solving the model is to determine the vector of naive views, $x_n(t)$, at each point in time. Naturally, the vector of naive views depends on itself, on the vector of beliefs by fanatics and rational agents after the rational agents have learned the truth, $x_h = (x_f; x_l; x_s)$, and their views before rational agents have learned anything, $x_h(0) = (x_f; x_l(0); x_s(0))$. The following proposition shows that the hardheaded views dominate over time as these opinions continue to "echo" through the population. All proofs are in the appendix.

Proposition 1 (Network belief spillover and convergence). *The views of naive agents at time 1 is $x_n(1) = A_{nn}x_n(0) + A_{nh}x_h(0)$ and, for $t = 2, 3, \ldots$, their views are*

$$x_n(t) = A_{nn}^{t-1}x_n(1) + \sum_{k=0}^{t-2} A_{nn}^k A_{nh}x_h \qquad (15)$$

In the limit as $t \to \infty$, each naive agent's view is a convex combination of the views of fanatics and rational agents

$$x_n(t) \to (I - A_{nn})^{-1}A_{nh}x_h \qquad (16)$$

The first part of the proposition shows how the naive agents are initially influenced by each other and by rational and fanatic views (Eq. (15)). The second part of the proposition shows how the naive investors end up with views that are a mixture of the fanatic and rational views, x_h. The relative weights given to the views of different fanatic and rational people have a natural interpretation as discussed next.

2.5. Influencers versus thought leaders

Recall that the standard strongly-connected DeGroot model of Section 2.1 has the property that everyone reaches a consensus in the long run, which implies a simple measure of social influence, namely the weight of agent *i*'s opinion in the consensus. Social influence works differently, however, in my setting of Section 2.4 with hardheaded-connected agents. In this case, people never reach a consensus, but the focus on the consensus can be replaced by a focus on the *average opinion*, defined as

$$\bar{x}(t) = \sum_{i=1}^N \frac{w_i}{w.} x_i(t) = \frac{1}{w.} w'x(t) \qquad (17)$$

The opinions are weighted by each agent's wealth w_i, using the notation $w = (w_1, \ldots w_N)'$ for the vector of all agents' wealth and $w. = \sum_{i=1}^N w_i$ for the aggregate wealth. I focus on wealth-weighted opinions since this is what matters for prices (as shown in the next section), but the results in this section obviously hold for any weighting vector *w*, including equal-weighted.

Proposition 1 shows that each agent's opinion converges over time, so the average opinion also has a well-defined limit. This limit, the "long-run average opinion," is important to the equilibrium. The long-run average opinion is determined by agents' *thought leadership*, as defined next.

Definition 1 (Thought leadership). The vector of all agents' thought leaderships is

$$\theta' = \frac{1}{w.} w' \lim_{t \to \infty} A^t \qquad (18)$$

which adds to one, $\sum_{j=1}^N \theta_j = 1$, and provides agents' weights in the long-run overall view:

$$\bar{x}(t) \to \theta' x(1) = \sum_{j=1}^N \theta_j x_j(1) \qquad (19)$$

Proposition 1 shows that naive agents' views disappear in the long term (except in how they initially affect rational views) so their thought leadership is zero. Nevertheless, the communication of naive agents still matters, because this communication determines the thought leadership of the various fanatic and rational views. To capture this idea, I introduce the following definition of the *influencer value* of any naive agent.

Definition 2 (Influencer value). The vector μ of naive agents' influencer values is

$$\mu' = \mu'A_{nn} + \frac{w_n'}{w.} = \frac{w_n'}{w.}(I - A_{nn})^{-1} \qquad (20)$$

The first equation shows that each influencer value is the sum of how much the agent influences others (computed as the sum-product of how much people listen and their influencer values, $\mu'A_{nn}$) and himself (weighted by his wealth as a fraction of total wealth). The second equation shows how influencer values can be computed easily. Importantly, influencer values determine hardheaded agents' thought leadership as follows.

Proposition 2 (Influencers and thought leaders). **A.** *The thought leadership of any naive agent is zero, $\theta_i = 0$, and the vector of thought leaderships of hardheaded agents, θ_h, depends on how much attention they get from naive agents, A_{nh}, and their influencer values, μ:*

$$\theta_h' = \mu'A_{nh} + \frac{w_h'}{w.} \qquad (21)$$

B. *If naive agent i increases his following A_{ij} of hardheaded agent j by ε at the expense of a lower following of other hardheaded agents, then*

$$\frac{\partial \theta_j}{\partial \varepsilon} = \mu_i \qquad (22)$$

The intuition behind this result is that hardheaded agents become thought leaders due to their commitment to their views. Influencers, on the other hand, are not committed to any particular view, and their fluid opinions end up without thought leadership. Nevertheless, naive agents affect which hardheaded agents have most thought leadership. Specifically, (21) shows that the thought leadership of any hardheaded agent *j* is:

$$\theta_j = \sum_{i=1}^{N_n} \mu_i A_{ij} + \frac{w_j}{w.} \qquad (23)$$

The last term, $\frac{w_j}{w.}$, simply means that the hardheaded agent influences his own opinion, which counts towards the overall opinion based on his own share of aggregate wealth. More interesting, the first term says that thought leadership comes from the sum-product of the following of naive agents, A_{ij}, and the followers' influencer values, μ_i. Hence, a thought leader can increase his impact by acquiring the following of an "influencer," that is, an agent with a large influencer value. In fact, the influencer value is precisely the impact on the thought leadership of a greater following, as seen in (22).

Interestingly, my framework implies separate roles for influencers and thought leaders, while these are intertwined in the standard strongly-connected DeGroot model of Section 2.1. In the strongly-connected case, an agent essentially becomes a thought leader by being an influencer, that is, by having many influential followers. What is different here, is that the fluid opinion of an influencer changes over time and ends up being dominated by the views of thought leaders. Hence, over time, an influencer's impact comes from influencing the relative popularity of the various thought-leading ideas.[20]

To understand this at a deeper level, note that the influencer values are wealth-weighted sums of the columns in the "echo matrix," $(I - A_{nn})^{-1}$ as seen in (20). The echo matrix can be written as

$$(I - A_{nn})^{-1} = \sum_{k=0}^{\infty} A_{nn}^k \tag{24}$$

so agent i's influencer value can be seen as an infinite sum. In other words, if i follows a hardheaded agent, then this changes the overall opinion via a series of terms that capture how the idea spreads, step by step. The first term is the effect on agent i's own view, which is simply the agent's wealth multiplied by 1 (coming from the i'th column in $A_{nn}^0 = I$). The next term captures the agent's direct following among other naive investors (i.e., the i'th column of A_{nn}). Then comes his indirect following (i.e., the i'th column of A_{nn}^2), his next-order indirect following (i.e., the i'th column of A_{nn}^3), and so on. In other words, the echo matrix provides the overall effect (or echo) of a hardheaded agent influencing a naive agent, and, through him, other naive agents.

An implication of Proposition 2 is that the economy is more rational if there are more rational people to begin with, if fanatics have views closer to the rational view, and if the naive people listen more to the rational ones, especially if the influential naive people do so. So education makes the economy more rational if it teaches naive people to be rational or to listen to those who are, and education of influencers is particularly effective.

As another example, religious beliefs may become thought leading partly by committing to a fixed text such as the bible. Simultaneously, science becomes thought leading by collecting as much data as possible and committing to a scientific understanding of the laws of nature. Continuously preaching these relatively fixed principles creates thought leadership.

Example 1, **continued.** Revisiting the numerical example in Eq. (14), we can calculate the influencer values of agents 1, 2, and 3 using Eq. (20) assuming, for example, that all agents have equal wealth. In this case, their influencer values are 1.56, 0.33, and 0.33, respectively. Naturally, agent 1 has the largest influencer value since other people pay most attention to him. The thought leadership of the fanatic is 57.8% based on (21), and the thought leadership of the rational agent is 42.2%. The fanatic has a larger thought leadership because of her larger following by the influencer.

It is interesting to consider what happens if agent 1 increases his following of the fanatic by 1 percentage point (i.e., from 20% to 21%) at the expense of a lower following of the rational agent (from 10% to 9%). This increases the thought leadership of the fanatic to 59.3%. The increase in the fanatic agent's thought leadership of 59.3%-57.8%=1.56% equals agent 1's influencer value 1.56 times the 1% view change, which helps explain the meaning of influencer value. If instead agent 2 increases his following of the fanatic by 1 percentage point (from 10% to 11%) at the expense of a lower following of the rational, then the fanatic's thought leadership only increases by 0.33%, again matching the 0.33 influencer value of agent 2 times the 1% view change.

The next section shows that influencer values and thought leadership are also useful in quantifying the social network effects on prices.

3. Market behavior with a social network

3.1. Prices with a social network

When the fundamental asset value is revealed, the price equals the fundamental, $v + u_t$, and, before that time, the equilibrium price is determined by equalizing the supply s and the total demand. The equilibrium condition (4) can be written as:

$$p(t) = (1 - c) \left[\frac{w_{n.}}{w.} \bar{x}_n(t) + \frac{w_{f.}}{w.} \bar{x}_f + \frac{w_{l.} + w_{s.}}{w.} x_r + u(t) - \frac{s\sigma_u^2}{\pi w.} \right] + c E_t(p(t+1)) \tag{25}$$

which depends on the wealth-weighted average view among naive investors, $\bar{x}_n(t)$, the weighted average view among fanatics, $\bar{x}_f$, and the aggregate wealth of naive agents ($w_{n.}$), fanatics ($w_{f.}$), long-term rationals ($w_{l.}$), and short-term rationals ($w_{s.}$), relative to that of all agents ($w.$). Further, the constant c is defined as

$$c = \frac{1}{1 + \frac{w.}{w_{s.}} \frac{\pi}{1-\pi}} \in (0, 1) \tag{26}$$

Iterating this equation forward to infinity yields the equilibrium price.

Proposition 3 (Rational price). *If all wealth is in the hands of rational long-term or short-term investors (i.e., $w_i = 0$ for all*

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[20] Mathematically, in the strongly-connected DeGroot model, social influence is determined via $z' = z'A$. In my framework, it also holds that $\theta' = \theta'A$, but this relation does not pin down θ because of the $N - N_n$ dimensions of this eigenspace. Instead, thought leadership is determined via similar relations, (20)–(21).

naive and fanatic agents), then the equilibrium price is

$$p_r(t) = x_r + u(t) - \frac{s\sigma_u^2}{\pi w.} \qquad (27)$$

The rational price, $p_r(t)$, is the expected fundamental value, $x_r + u(t)$, less a risk premium that depends on the supply of shares (s), the fundamental risk (σ_u^2), the speed of revelation(π), and the aggregate wealth (or risk-bearing capacity) of investors ($w.$). This rational price is a useful benchmark when considering the equilibrium price in the presence of naive investors and fanatics. As seen in the next proposition, the general price also depends on the average view among naive investors, $\bar{x}_n(t)$, and among fanatics, $\bar{x}_f$, weighted by their wealth.[21]

Proposition 4 (Network effects on price). *The equilibrium price before the fundamental value is revealed is the sum,*

$$p(t) = p_r(t) + p_n(t), \qquad (28)$$

of the rational price, $p_r(t)$, and the following social network price component:

$$p_n(t) = \frac{w_n.(1-c)}{w.} \sum_{k=0}^{\infty} c^k (\bar{x}_n(t+k) - x_r) + \frac{w_f.}{w.} (\bar{x}_f - x_r) \qquad (29)$$

where c and p_r are given in (26),(27). This equilibrium price is unique under the transversality condition that $p_t - u_t$ is bounded in t state by state. As $t \to \infty$, the network part of the price converges to

$$p_n^\infty = \sum_{j=N_n+1}^{N_n+N_f} \theta_j(x_j - x_r) \qquad (30)$$

which is the fanatics' mispricings, $x_j - x_r$, weighted by their thought leadership, θ_j.

The market price when investors learn through a social network can deviate significantly from the rational price. In particular, Eq. (29) shows how the price depends on all investors' views, which are determined via their social network interactions. These network effects are seen from the price dependence on the naive view $\bar{x}_n(t)$, which can be traced back to the investors' original views and their spillover through the network as seen from Proposition 1. The price also depends on expected future network effects, since the rational short-term investors anticipate these future network effects and adjust their asset demand accordingly, as seen from the terms depending on $\bar{x}_n(t+k)$ with $k > 0$ in Eq. (29).

Over the longer term, naive investors' views are completely dominated by the fanatic and rational views, and therefore the long-term prices depend on these hard-headed views and their thought leaderships, as seen in (30). These social dynamics can generate prices way above the fundamentals (bubbles) and way below fundamentals (anti-bubbles, or deep value). Naturally, the bubble

gets larger if fanatics have more extreme valuations. Interestingly, this effect is larger if the fanatics have greater thought leadership.

Proposition 5 (Fanatic effect on price). *The price increases in the valuation, x_j, of fanatic agent j, and this price sensitivity is larger if the agent has greater thought leadership θ_j:*

$$\frac{\partial p_n^\infty}{\partial x_j} = \theta_j \qquad (31)$$

Another aspect of the social network dynamic is the effect of influencers.

Proposition 6 (Influencer effect on price). *If naive agent i with influencer value μ_i increases his following A_{ij} of fanatic agent j by ε at the expense of a lower following of a rational agent, then the effect on price is:*

$$\frac{\partial p_n^\infty}{\partial \varepsilon} = \mu_i (x_j - x_r) \qquad (32)$$

This proposition shows that a naive agent can affect the price by elevating the thought leadership of a fanatic. This effect is naturally larger if the naive agent has a greater influencer value. For example, when Elon Musk tweeted a link to WallStreetBets with its fanatically high valuation of GameStop, the price of GameStop increased.

In summary, Propositions 4–6 show that the price deviates more from the rational price when (i) a larger fraction of wealth is held by the fanatic or naive; (ii) fanatic views are more extreme, (iii) fanatics have higher thought leadership because more naive people listen to them, people devote more attention to fanatics, wealthier people listen, or influencers listen, thus affecting other naive investors indirectly; or (iv) investors are uncertain about a larger fraction of the stock value (i.e., v is a larger part of $v + u_t$).

Example 1, **continued.** Suppose that the rational expected value of v is $x_r = 400$ and the supply is $s = 0$, such that the rational price is $p_r(t) = 400 + u(t)$ regardless of σ_u^2 and π as seen in (27). If the fanatic assigns a value to the asset of $x_f = x_r + 100 = 500$, then the long-term network price can be calculated from (30) to be $p_n^\infty = 57.8$, corresponding to a long-term total price before revelation of $457.8 + u(t)$, which is 57.8% toward the valuation of the fanatic relative to that of the rational because of the fanatic's 57.8% thought leadership (as calculated in the end of Section 2.5).

If the fanatic increases his valuation to 600, then the long-term price increases to $515.6 + u(t)$, a price increase of 57.8. This price increase is a 100 times the fanatic's thought leadership of 57.8%, consistent with (31). If instead the rational view changes by 100, this leads to a price change of 42.2, which is a smaller price move due to the lower thought leadership of the rational agent.

To illustrate the importance of influencers, suppose that agent 1 increases his following of the fanatic by 10 percentage points. Then the long-term network price increases from $p_n^\infty = 57.8$ to 73.3. As seen from (32), the price increase of 15.6 can be computed directly as $\Delta p_n^\infty = (\Delta A_{1f})\mu_1(x_f - x_r) = 10\% \times \mu_1 \times 100 = 10\mu_1 = 15.6$, where Δ indicates change and agent 1's influencer value, $\mu_1 = 1.56$, is calculated in Section 2.5. If

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[21] The weighted average opinion of naive agents is computed as $\bar{x}_n(t) = \frac{1}{w_n.} w_n' x_n(t)$ and similarly for $\bar{x}_f$.

instead agent 2 increases his following of the fanatic by 10 percentage points, then the price increases only by 3.3, again 10 times the influencer value, $\Delta p_n^\infty = 10\mu_2$. The price is clearly more sensitive to the actions of agent 1 because of his larger influencer value.

3.2. Value, momentum, trading volume, and volatility

The model has interesting predictions for asset returns, defined as $r(t+1) = \Delta price(t+1)$, where Δ means the change over time and the price is given in (3). The expected return is $E_t(r(t+1)) = 0$ after the revelation of v since the price becomes a random walk after that time. Before revelation, $t < \tau$, the expected return is:

$$
\begin{aligned}
E_t(r(t+1)) &= (1-\pi)E_t(\Delta p(t+1)) \\
&\quad + \pi(x_r + u(t) - p(t)) \\
&= (1-\pi)\underbrace{\Delta p_n(t+1)}_{momentum} + \pi\underbrace{b(t)}_{value}
\end{aligned}
\tag{33}
$$

because $\Delta p(t+1) = \Delta p_n(t+1) + \Delta u(t+1)$ and $\Delta u(t+1)$ is unpredictable. We see that returns are predicted by value and momentum factors. To understand this, note that the second term is the "valuation metric," defined as the difference between the fundamental value and the price, $b(t) := x_r + u(t) - p(t)$, similar to book-to-price and other metrics used by value investors. This value metric is multiplied by the probability of revelation, that is, the mean reversion toward the fundamental value.

The first term is the future momentum of the network effect, Δp_n, which can be predicted by rational investors. This network momentum is multiplied by the probability that the network dynamics continue, $1-\pi$, that is, no revelation (or the bubble does not burst). Interestingly, this network momentum can be proxied by standard price momentum. Indeed, given that the network effect is a smooth function of time, the recent network price momentum may proxy for its future momentum, $\Delta p_n(t+1) \cong \Delta p_n(t)$. Further, the recent price momentum is a noisy, but unbiased, estimate of this network momentum, $r(t) = \Delta p(t) = \Delta p_n(t) + \Delta u(t)$. Hence, the model can help explain why value and momentum predict returns.

This finding, which relies on an approximation, can be made precise in specific settings. For example, consider the case in which naive agents have equal wealth, they listen to each other in a symmetric way (their adjacency matrix can be written as $A_{nn} = a_0 1_{N_n} 1'_{N_n} + a_1 I$ for scalers $a_0, a_1 \in \mathbb{R}$), and there exist non-zero numbers of naive and fanatic agents ($N_n > 0$ and $N_f > 0$). The model has a particularly simple solution in this "symmetric naive case" as shown in appendix, leading to the following result.

Proposition 7 (Value and momentum effects). *In the symmetric naive case, there exists a number $a > 0$ such that the value-momentum strategy, $b(t) + a\Delta p(t)$, has a positive expected return, $E[(b(t) + a\Delta p(t))r_{t+1}] > 0$, for all t.*

It is also interesting to consider how short-term investors trade in light of the network dynamics driving the price. The demand (8) of short-term investors before reve-

lation can be rewritten as

$$
d_i(t) = \frac{(1-\pi)w_i}{\sigma_u^2}\Delta p_n(t+1) + \frac{\pi w_i}{\sigma_u^2}b(t)
\tag{34}
$$

which is proportional to the expected return in (33). The first term shows that a short-term investor has an incentive to "ride" a bubble in the sense that the short-term investor looks at the expected network price change if the fundamental is not revealed. This first term captures the "momentum of the bubble," a more sophisticated form of momentum trading than looking simply at past prices changes, $\Delta p(t)$, but I refer to this term also as a form of "momentum trading."

The second term in (34) shows that a short-term investor also worries about the magnitude of the bubble, realizing that a revelation of the fundamental will burst the bubble, leading to an expected price move equal to the difference between the price and the fundamental, $b(t) = x_r + u(t) - p(t)$. At any point in time, short-term traders act as either momentum traders, value investors, or both (i.e., $d_i(t)$ must have the same sign as the first term in (34), the second term, or both) and the next proposition shows how their portfolios evolve in the simple case of symmetry.

Proposition 8 (Value and momentum trading). *Consider an economy in the symmetric naive case as defined above. In case of a positive bubble, $p_n^\infty > 0$, short-term investors initially buy a rising undervalued asset (value and momentum investing), continue to hold when the asset becomes over-valued (momentum), and finally shorts when the over-valuation is large enough (value on the short side). Short-term investors only go through the latter one or two phases (momentum buying and value shorting) depending on the initial price $p(1)$ (i.e., depending on the initial signals).*

Conversely, in case of an anti-bubble, $p_n^\infty < 0$, the short-term investor initially shorts a falling over-valued asset (momentum and value shorting), continues to short as the asset becomes cheap (momentum), and eventually buys when the asset is cheap enough (value), or only goes through the latter one or two phases depending on $p(1)$.

Short-term traders face a trade off between riding a bubble and betting on its burst. Proposition 8 shows that this trade off leads them to initially ride the bubble when the network component of the price drifts quickly upward due to strong network-spillover effects and, at the same time, the cost of a potential bursting bubble is limited due to its moderate initial size. Eventually, however, naive investors' views converge as the echo resides, implying that further price moves become small, and, moreover, the size of the bubble grows large, so the short-term traders ultimately trade against the bubble. Further, social network effects increase the trading volume and create another source of price variation, as shown next.

Proposition 9 (Spike in volume and excess volatility). *With naive and fanatic agents, the trading volume is greater but dies down over time (until the fundamental is revealed when final trading may happen) and the valuation metric $b(t)$ varies more, relative to when all agents are rational.*
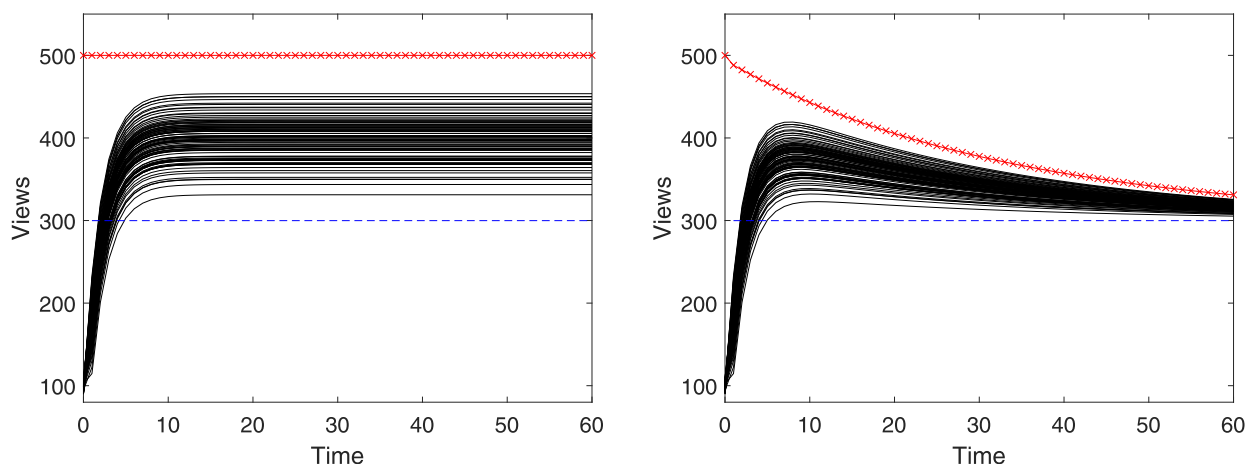
Fig. 1. Investor beliefs over time. The left panel shows how investors' beliefs about the fundamental asset value change over time in Example 2. The line on top (identified with cross-marks) shows the fanatic view, the dotted line shows the rational view, and the solid curves show the naive investors' views. Naive investors converge to different limits because of their differing social ties to fanatic or rational agents. The right panel shows the same, except that "fanatics" are not completely fanatic, they put 3% weight on the rational view.

The model has the property that, initially when investors receive signals that lead to disagreement, trading volume is high (Proposition 9) and the momentum effect is strong (Proposition 7), creating a bubble that later reverses. So momentum is strong when the trading volume is high, and this interaction between volume and momentum is consistent with the empirical findings of Lee and Swaminathan (2000). Indeed, these authors find that momentum is strongest among high-volume stocks (their Table II) and that this momentum return partially reverses after 5 years (Table VI).

The model also has implications for when bubbles burst and trading volume dies down. The price can stay a bubble as long as fanatics remain unmoved by rational opinion and fundamentals remain unrevealed. Indeed, agents disagree until revelation in contrast to standard DeGroot models in which all agents converge to the same opinion exponentially fast (see Section 2.1). Such long-term disagreement leads to a long-term bubble in the price and long-term differences in portfolios across agents. But, interestingly, even as the bubble stays high, the trading volume declines fast.[22] This decline in trading volume happens as the level of bubble steadies and opinions settle over time because trading arises from new – not existing – differences of opinions. This prediction appears consistent with the GameStop case as discussed in Section 4. Portfolios are discussed further in the context of many assets in Appendix A.4.

Example 2. The economy has $N = 100$ investors with equal wealth $w_i = .2$, 96 of which are naive, 2 are fanatics, 1 is a long-term investor, and 1 is a short-term investor. The supply of shares is normalized to $s = 1$ and the asset's funda-

mental value is $v + u(t)$, where $u(t) = \Delta u(1) + \ldots + \Delta u(t)$ with $\Delta u(t) \sim N(0, 2^2)$. Investors' initial signals are $x_i(0) = \bar{v} + \varepsilon_i$, where $\bar{v} \sim N(100, 0^2)$ and $\varepsilon_i \sim N(0, 5^2)$, except that the fanatics draw a signal of $x_f = 500$. The unobserved value is $v = \sum_{i=1}^{100} \kappa_i x_i(0)$, where each fanatic j have weight $\kappa_j = 25\%$ while other agents share the remaining weight, 50%/98 each. Revelation happens with probability $\pi = 4\%$. After receiving these initial signals, the agents communicate in a social network. Any naive agent i puts a weight of $A_{ii} = 50\%$ on his own previous view, splits 10% across naive agents, and, once and for all, randomly allocates the remaining 40% across fanatic and rational agents. Rational agents initially listen to everyone, learn the truth $x_r = v \cong 300$, and keep this updated view from time 1 and onwards. I consider two versions of fanatic investors: In the "fully fanatic" case, they only listen to themselves, $A_{jj} = 1$; in the "nearly fanatic" case, they put 3% weight on the rational view (i.e., $A_{jj} = 97\%$ and $A_{j,100} = 3\%$), so strictly speaking they are very self-reliant optimists rather than true fanatics.

Fig. 1 shows how investors' beliefs evolve over time. The left panel shows the views when fanatics are fully fanatic, and the right panel shows the nearly fanatic case. The "initial displacement" (using Kindleberger's terminology as discussed in the introduction) is that fanatics receive positive news. Rational investors learn about this displacement at time 1, but their rational update is smaller than that of fanatics, who overweight their own signals. Naive investors learn about this positive news only gradually as they communicate via their social network. Hence, naive views start too low, grow as they learn about the initial displacement, and eventually end up between the rational and fanatic view. Some naive investors develop views close to the fanatic view while others become close to rational, depending on who they listen to the most, but all naive investors are too optimistic in the end – a "mania." In the left panel with full fanaticism, beliefs converge to different limits (long-term polarized opinions), while all

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[22] Beliefs evolve based on A^t, which has an eigenvalue of 1 and other eigenvalues of smaller magnitudes. Denoting the magnitude of the second largest eigenvalue by $\phi \in (0, 1)$, we see that belief dynamics and other endogenous quantities converge to their limit at the same speed as ϕ^t converges to zero.
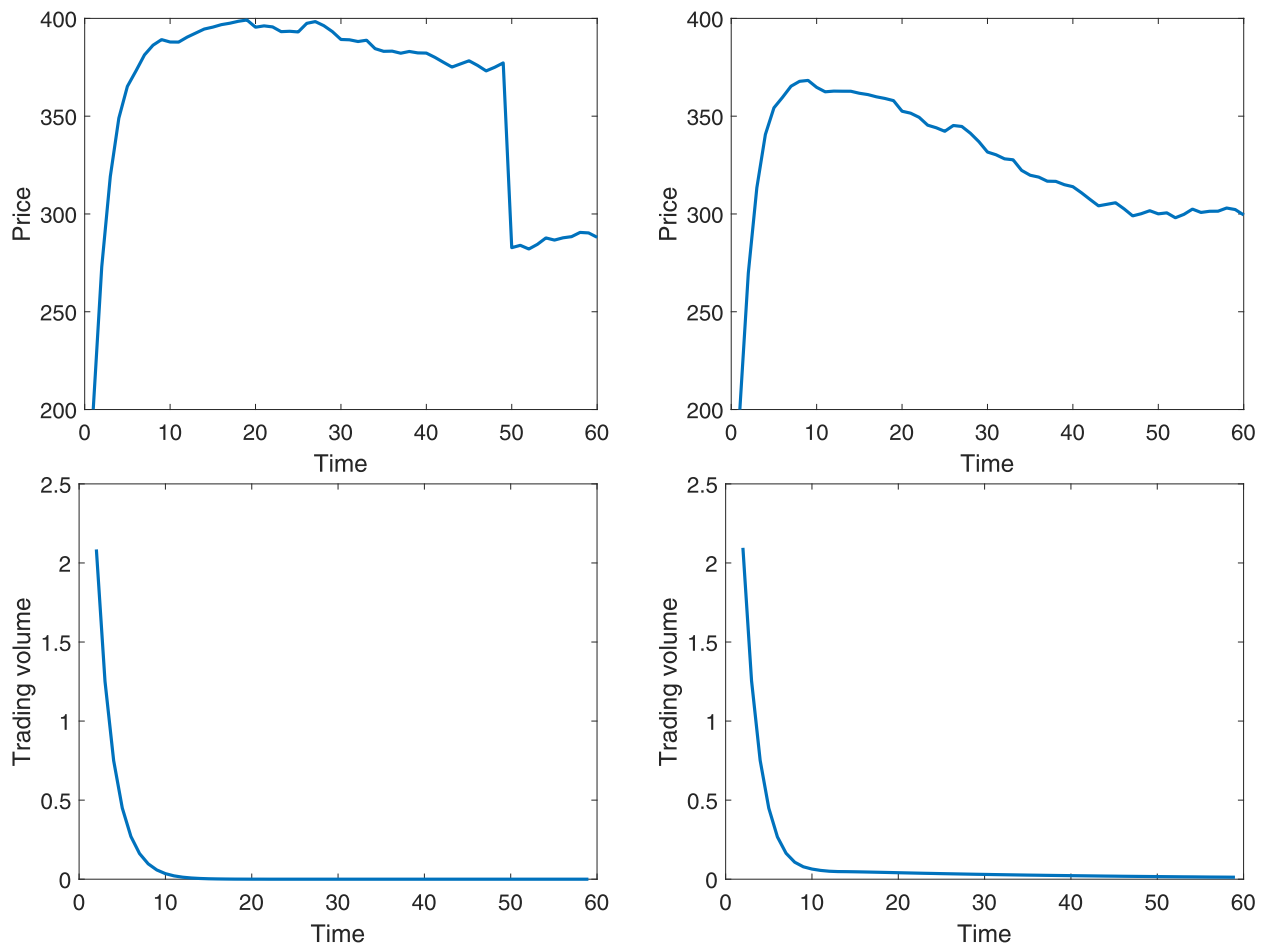
Fig. 2. Price bubble and volume. The top left panel shows the evolution of the asset price over time in Example 2 when fanatics are fully fanatic and the bubble bursts at time 50 when the fundamental value is revealed. The top right figure shows the price when fanatics put a small weight on the rational view, so the bubble peters out even though the fundamental value is not revealed over the shown time period. The bottom panes show the trading volume with fully fanatic agents (left) and fanatics who put a small weight on the rational view (right).

views eventually converge to the rational view in the right panel when fanatics eventually learn the truth as a revulsion against their extreme views sets in.

The top panels of Fig. 2 show the resulting equilibrium price. As before, the left panel illustrates the case when fanatics are fully fanatic, and the right panel shows the nearly fanatic case. Further, in the left panel, the bubble bursts at time 50 because the fundamental value is revealed at this time. In the right panel, the fundamental value is never revealed over this time range, but the bubble nevertheless peters out as fanatics' optimism turns to revulsion. The figures illustrate how the price starts too low as the initial news is incorporated into prices only gradually, the price grows into a bubble as naive investors are swayed by fanatics, and ultimately the bubble bursts (left panel) or peters out (right panel).

The bottom panels of Fig. 2 show the total trading volume. The figure illustrates how trading volume is initially very high, but the trading volume dies down even before the bubble bursts.

Finally, Fig. 3 shows the asset position of rational short-term traders. These traders initially buy the asset aggressively for two reasons: (a) the price starts below the rational value since naive investors have not yet understood the full importance of the initial displacement; and (b) the price is trending up as naive investors are expected to grow increasingly optimistic. As the asset becomes expensive, the short-term investors continue their momentum trading with a reduced position size. Eventually, as the bubble grows too large, short-term investors start shorting the asset.

4. A case study of GameStop

The case of GameStop offers a window into the effects of social networks on asset markets due to its well-publicized and extreme events. While the introduction cites scientific evidence in support of the model, GameStop offers an interesting illustration that may spur ideas for future research, hopefully ideas that also apply in less ex-
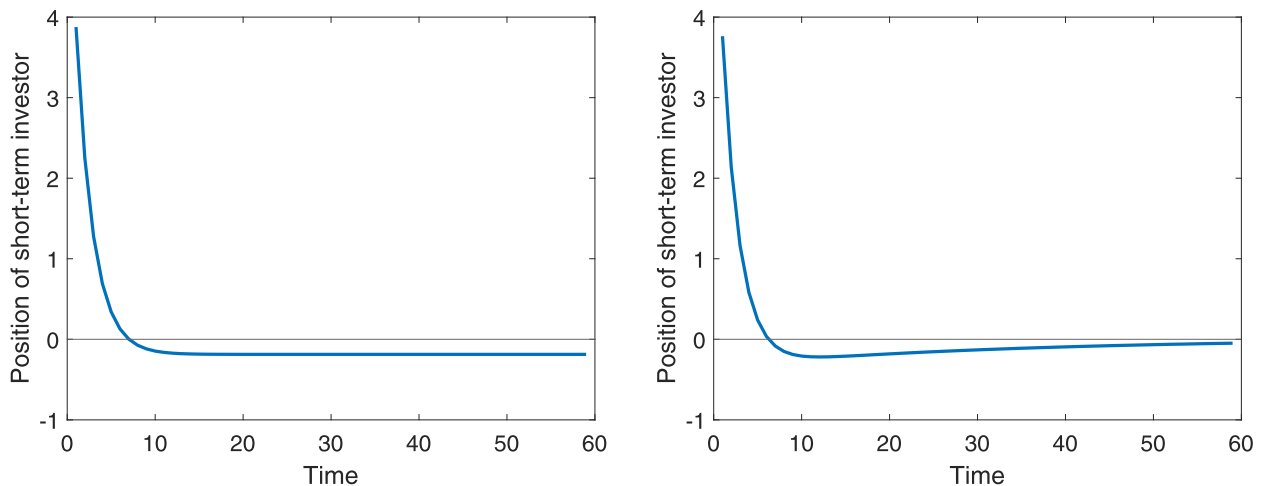
Fig. 3. Asset position of short-term investors. This figure shows the asset position of short-term investors over time in Example 2 assuming no revelation. Fanatics are fully fanatic in the left panel and nearly fanatic in the right panel. Short-term investors are initially long the asset, acting as momentum traders who ride the bubble. Later in time, short-term investors act as value investors who bet of a price reversion toward fundamentals when the bubble bursts.

treme cases such that they can help explain the general principles of social market behavior.

Gamified trading of GameStop. In early 2021, the world's largest video game retailer, GameStop, was struggling as games were increasingly sold online and the retail industry was hit by the Covid-19 crisis. Nevertheless, a group of investors caught an almost fanatic liking to the stock, a trend that was reinforced through social networks. Keith Gill (better known under various social media aliases) became the most followed of these traders and he expressed an unwavering belief in the stock at all observed price levels — so he can be represented in the model as one of the fanatics. Gill and others believed that GameStop could pivot online and capture a share of the large digital software distribution market while simultaneously benefiting from their physical locations with the help of recent block investor and board member, Ryan Cohen, and others (the initial displacement in Kindleberger's terminology).

The most well-publicised group of retail investors interested in GameStop communicated through the platform Reddit, in the community (subreddit) called WallStreet-Bets. Some investors believed like Gill that GameStop was undervalued, while others stated an intention to buy the share because of its large short interest of just over 100% of shares outstanding at the beginning of the year (or around 140% of all floating shares). Some GameStop investors resented shortsellers or expressed a view that pushing up the stock price could lead to a short squeeze, but by March 2021 the focus on shortsellers dissipated. The extreme view of the stock price was reinforced with rocket memes and by labelling GameStop a YOLO trade. Similarly in the model, the fanatics have a larger effect on the price if they express a more extreme valuation, $\bar{x}_f$, as seen in Proposition 4 and this effect is larger if the fanatics have more thought leadership (Proposition 5).

As another sign of the importance of social media, the interest in GameStop was spurred by a tweet by Elon Musk on January 26, with the single word "Gamestonk!!" along with a link to WallStreetBets. In the model, Elon Musk can be represented as an influencer who chooses to follow the fanatic. That is, the agent i representing Musk puts a positive weight, $A_{if} > 0$, on the fanatic's view, and Musk is an influencer in the sense that many other investors j follow Elon Musk, $A_{ji} > 0$. In the model, when an influencer follows a fanatic with a very bullish view, this increases the stock price (Proposition 6), as it did in the case of GameStop. In the terminology discussed in the introduction, Musk's tweet can be seen as an "authoritative blessing."

This bullish sentiment on GameStop communicated via social networks was translated into actual trading activity. Trading by inexperienced traders was facilitated by a competitive online brokerage industry offering zero-commission trading, led by the new broker, Robinhood, which uses a business model based on payment for order flow from market makers such as Citadel. Robinhood and other brokers sought to make trading more broadly available to a wide range of investors in a fun way, which lead to the accusation that trading became "gamified."

Price, volume, volatility, and social media interest. As seen in Fig. 4.A, GameStop had been trading at less than $20 per share through 2020, but increased dramatically in the beginning of 2021. The stock started the year at $19 per share and hit an intra-day high of $483 on January 28, a 25 fold increase with little news. The price dropped to $40 in February, but it increased significantly again in late February, and stayed above $100 for the rest of 2021.

The extraordinary volatility visible in Fig. 4.A is shown more explicitly in Fig. 4.B, which plots the 20-day close-to-close realized volatility, annualized by multiplying by $\sqrt{250}$. Realized volatility peaked at over 700%, an exceptionally large price volatility. The high volatility coincided with an enormous trading volume as GameStop temporarily became one of the most traded stocks in the world despite the modest size of the company. Fig. 4.C shows the daily turnover computed as the daily trading volume in
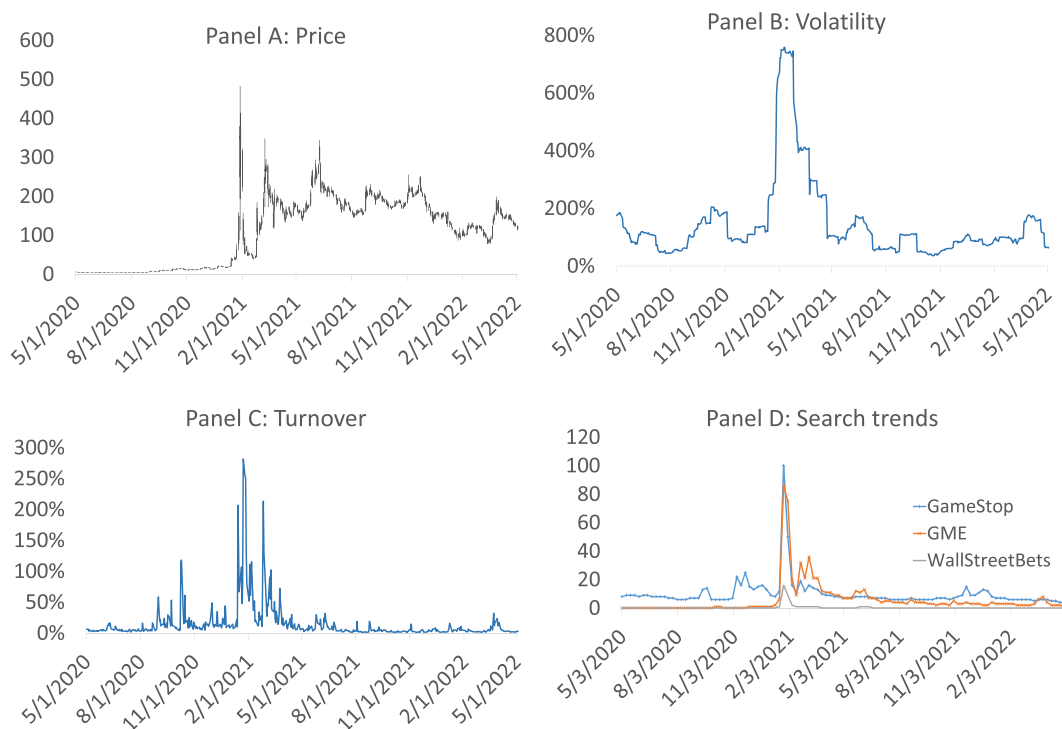
Fig. 4. GameStop price, volatility, turnover, and interest, 5/1/2020-5/3/2022. Panel A shows the GameStop stock price over time (open-high-low-close), Panel B shows the realized volatility (20-day annualized volatility), Panel C shows the daily turnover, and Panel D shows the social media interest as proxied by Google search trends for GameStop, GME, and WallStreetBets.

shares divided by the number of shares outstanding. The daily trading volume peaked at over 200% on January 22, meaning that all the company's shares were traded more than twice each day. The spikes in trading volume and volatility also coincided with an increase in social media interest in GameStop as proxied by Google searches for GameStop, its ticker GME, and WallStreetBets as seen in Fig. 4.D. Interestingly, while prices remained elevated relative to 2020 through the end of the sample period, trading volume and volatility came down to more normal levels similar to those in 2020, consistent with the model as explained in the end of Section 3.2.

Other effects. The real world is almost always more complex than any stylized theoretical model, and the case of GameStop is no exception. The trading frenzy almost surely started in social media similarly to the model, but ultimately several effects played a role in the meteoric rise in the price. First of all, retail investors bet on GameStop both by buying the stock and by buying call options. Call options have embedded leverage, allowing investors to multiply their gains or losses many times for the same dollar investment (Frazzini and Pedersen, 2022). When end-investors buy call options, they are sold by market makers who hedge their risk by buying the stock. The hedge ratio (called the "delta") increases when the stock price increases (and this change in the delta is called the "gamma"). Therefore, an increasing stock price leads to buying from option market makers, leading to further stock price increases, and so on (a "gamma squeeze"). In other words, buying call options is similar to a pre-

programmed trading strategy, where the end-investor buys more and more shares if the price rises. In the model, this strategy corresponds to letting the demand-sensitivity of naive investors depend on the stock price (or their trading gains), an interesting extension of the model.

As the stock price of GameStop rose in late January, some shortsellers were simultaneously forced to close their positions, buying shares that they earlier borrowed. This reduction in short positions further pushed the price upward (a "short squeeze"). This short squeeze likely played a role in the price spike in January, but it was not a major driver of the subsequent price rise that started in late February 2021, suggesting an importance of social network effects.

Further, some newspapers reported possible buying by institutional investors, similar in spirit to the short-term investors in the model. As the short-term investors in the model, these investors may have chosen to ride the bubble, thus contributing to the increase in price. Other sophisticated investors were focused on the long-term value, shorting the stock or simply selling their positions.

In summary, GameStop started the year 2021 with a market capitalization of $1.2B and reached a high on Jan. 28 of $34B. While certainly an economically meaningful rise in value, GameStop's peak market capitalization was only 0.07% of US equities, still a small corner of the overall equity market, even when including the other meme stocks that also rose in value at the time. GameStop raised "$1,672.8 million in gross equity capital through an at-the-

market offering"[23] in 2021, more than their market value at the beginning of the year. An at-the-market offering means that GameStop sold shares directly into the market, perhaps targeting naive investors rather than selling to institutional investors via a conventional bookbuilding. One of the other meme stocks, AMC, also raised about $300m in an at-the-market offering in January 2021, illustrating how price displacements can have real effects as shares sales affect the survival and operation of these businesses.

Link to the model. The GameStop saga had many of the elements included in the model: an investment thesis spreads via a social network (Proposition 1), fanatic views gain prominence over time (Proposition 2), the contagious investment idea leads to network effects on prices (Propositions 4), which starts a bubble (Propositions 5), prices rise further as influencers link to the fanatics (Proposition 6), sophisticated momentum investors ride the bubble while value investors bet against it (Propositions 7,8), and high trading volume and volatility ensue (Propositions 9), but die down faster than the price bubble. Further, this episode illustrates both the general anatomy of bubbles and a magnified version of more common asset pricing effects (as seen in Table 1), one that allows us to more easily observe how social networks affect markets.

5. Conclusions and further directions

I present a simple model of a financial market in which some investors are rational and others learn through a social network. The model can help explain a number of observed phenomena such as social network effects in portfolio holdings, excess volatility, momentum and reversal effects, meme trading, the effects of repeated news, and spillover of expectations and transaction prices across people with social links. I study the events of GameStop in 2021 in light of the model, showing how extreme price moves were related to extreme trading volume and social media attention. Hence, belief formation via social networks may both affect the normal day-to-day fluctuations in asset prices and the extreme events connected to bubbles and crashes.

Social networks have been prevalent throughout history, but modern social media are changing the nature of these networks and making them more observable, which opens up new research possibilities to test the model predictions using data on networks and market behavior. If social network effects are a force behind pervasive dynamics throughout global equity, bond, currency, commodity, cryptocurrency, and housing markets such as value, momentum, and excess volatility, then the GameStop affair is only the tip of the iceberg, a very extreme tip that is currently seen more clearly than the part hidden under the surface. I end by discussing further implications and future directions.

Diamond hands. People who want to affect the opinion of others try to get attention, but may also enforce a fanatic stubbornness. In social media related

to trading, such a stubborn willingness to keep buying and holding a stock is called "diamond hands" (often written with diamond and hand emojis). In the case of GameStop, Keith Gill – a trader who made an impact on social media – ended his testimony in the hearing of the U.S. House Committee on Financial Services by saying: "In short, I like the stock," signaling his continued commitment to the position. The model shows that a stubborn view can come to have a significant effect on naive investors (Proposition 1), leading to bubbles (Propositions 5).

Rocket and YOLO trades. A stubborn view has a larger effect if it is more extreme (Propositions 4). Social-media investors sometimes signal such an extreme view of the potential rise in price via a rocket emoji and, the traders of GameStop on WallStreetBets had a special category for YOLO (you only live once) trades.

Gamification of trading. Some investors are trading with broker apps with game-like features while chatting on social media with their social connections. Hence, another interpretation of the model is that it may capture such a gamification of trading and future research may shed more light on the effects of gamification.

Meme trading frenzy. A "meme" is an idea that becomes a fad and spreads by means of imitation in a social network, in the spirit of the model considered in this paper. The meme can lead to differences of opinions across variations of the meme (i.e., across different fanatics in the model) and relative to rational investors, leading to a trading frenzy (Proposition 9). This can happen even long after any news has arrived or a meme is originated, since the meme can take a long time to gather momentum in the social network. See Shiller (2017) for other economic effects of viral "narratives" and Hirshleifer (2020) for the related idea of "social transmission bias."

Echo chambers. When people communicate in a closed system insulated from rebuttal, their opinions are amplified via confirmation bias, referred to as an "echo chamber." This situation can be captured in the model by letting a subset of the naive agents listen only to each other and one of the fanatics. In this case, these naive agents end up having the same opinion as this particular fanatic. The other agents in the economy have a mixture of the rational view and all the other views, but they can also be influenced by the echo chamber if they listen to members of the chamber. Hence, such an echo chamber can amplify the size of a bubble, especially if the echo chamber includes – or influences – many people, wealthy people, or risk-tolerant ones.

Size of wallet vs. number of followers. In traditional models of finance, people influence prices based on the size of their wealth. Interestingly, when investors learn through their social network, even a non-investor (or a penniless investor) can wield a large influence on prices if the person is a thought leader or influencer, especially if the followers are

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[23] Annual Report on Form 10-K for the fiscal year ending January 29, 2022.

wealthy or consists of sufficiently many small investors. So an "important agent" has a completely different meaning in traditional models and in this model of a social network!

Influencer values. Influencers are being used in the marketing of financial securities as well other products. For example, Kim Kardashian has promoted the cryptocurrency Ethereum Max, the American football player Tom Brady has endorsed the cryptocurrency platform FTX, and other fulltime "finfluencers" are paid to promote financial products.[24] My model can be used to compute the value of paying an influencer for such promotion, which can be modeled as creating new links in the adjacency matrix, A.

Excess volatility. Since the evolution of the network component of the price is largely divorced from fundamentals, asset prices vary for reasons unrelated to news about their fundamentals (Proposition 9), consistent with Shiller (1981), (1984). Volatility would be even larger in an extension of the model in which naive investors trade at random times, such that the price at any time would depend on the mix of investors trading at that moment.

Value and momentum. Social network effects can lead to price momentum and subsequent reversal toward fundamentals (also called a value effect) that are seen across many asset classes and global markets (Asness et al., 2013). Indeed, the build-up of naive investor demand can lead to momentum effects, and the eventual price reversal as the fundamental value is revealed leads to a value premium, and it would be interesting to empirically link these patterns more directly to social networks.

Repeat news. In the model, repeating old news can move prices, especially if the repeat news is displayed prominently to many people, as naive investors keep reacting to the same information, consistent with the evidence of Huberman and Regev (2001) and Tetlock (2011).

Investor communication and fake news. In the model, fanatics may be spreading fake news, but they are assumed to do so because they truly hold this view. More broadly, all agents' report their true views, consistent with the assumption that they are price-takers — and consistent with the idea that most investors simply have an honest talk with their friends in order to figure things out. Nevertheless, an interesting extension would consider agents' incentive to communicate strategically.

Pump and dump. An opportunistic investor could try to profit by first buying an asset, and then pretend to have a fanatic bullish view on the asset in order to create a wave of buying by his followers, leading to a price increase. If the opportunistic investor then sells despite talking up the asset, he is engaging in "pump and dump," an illegal form of market manipulation. This behavior can be captured in the

model as follows. An investor splits herself into two parts, say i and j: The fanatic part i "pumps" up the price via a strong attention-grabbing view ($A_{ki} > 0$ for many k), but does not trade (zero wealth to this part, $w_i = 0$); The rational part is a short-term investor who trades ($w_j > 0$), but does not speak ($A_{kj} = 0$ for all $k \neq j$). The rational side initially rides the pump and eventually "dumps"—so viewed as a team, these agents pump and dump.

Short squeeze. A bubble driven by social media effects can be greatly exacerbated if short-sellers are forced to close their positions due to share recalls or risk controls. This paper abstracts from this effect here for simplicity—see Brunnermeier and Pedersen (2005) for a model of short squeezes and other forms of predatory trading, Duffie et al. (2002) for a model of securities lending, and Gârleanu et al. (2021) for a model of how fears among short sellers can become self-fulfilling. Future research may explore how short-sale frictions interact with network effects.

Social network effects and local bias. Social network effects mean that people are affected by those they are connected to, and, indirectly by the connections of their connections, and so on. People often interact with others in their local area and work place, which can be captured in the structure of the adjacency matrix A. So if people hear about local stocks via their social network, this mechanism can contribute to investments having home bias (French and Poterba, 1991), local bias (Coval and Moskowitz, 1999), and own-firm bias. Kuchler et al. (2020) provide evidence that social network effects contribute to local bias and affect firm values, and this effect could be studied further in an extension of the model with many stocks (see Appendix A.4).

Who buys early or late into a bubble? In the model, the earliest big buyers are the most extreme fanatics (due to their bullish views) and the short-term investors (due to their realization that a bubble is forming) while naive investors initially hold small positions. Fanatics continue to be large owners until the crash, but they gradually reduce their positions as prices rise — so they sell into the crash, but still experience large losses. Naive investors who are influenced by fanatics continue to buy the asset until the crash as these investors grow increasingly optimistic. The last buyers are naive investors influenced by fanatics, especially those who learn the fanatic view only slowly, e.g., via indirect connections to the fanatic due to a peripheral position in the social network.

Attention-grabbing profits. Fanatic views may spread more quickly when their proponents are seen profiting from their views. Hence, an interesting extension of the model would let the adjacency matrix A at each time depend on the past profits of the different agents. Given that fanatics tend to profit early on as their views start affecting the price, this extension could generate larger bubbles as the fanatic

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[24] Bloomberg 9/17/21, "Wall Street Influencers Are Making $500,000, Topping Even Bankers."

profits would grab attention, the attention would increase profits, and so on.

Firm communication. Firms use their communication and advertising to try to boost their stock prices (Lou, 2014). Such behavior could be studied in an extension of the model in which firms participate in the social network or affect the revelation probability π. For example, a firm has a much stronger incentive to reveal its value or help propagate the rational view when its stock price is undervalued vs. overvalued. Hence, a negative bubble may have a higher revelation probability π and a larger thought leadership of rationals aided by the firm, thus leading to shorter and smaller price distortions. In contrast, positive bubbles may be larger, more prevalent, and initiated by the firm itself, especially when the firm needs to raise capital or insiders need to sell out.

Central bank communication. Central banks use their communication to ensure the transmission of their monetary policy. They may also use communication strategies to lean against bubbles and improve market efficiency — in the language of the model, they try to increase the thought leadership of the rational view. Indeed, many central banks have social media accounts and they are very deliberate with their communication. Central bank communication via a social network is thus another interesting avenue of future research.

Many funds. While standard asset pricing theory predicts that investors just need a few investment funds (e.g., two-fund separation), there exist more mutual funds and ETFs than stocks in the real world. This finding is consistent with my model if investors follow a larger number of fanatics than there are stocks (see Appendix A.4).

Appendix A

A1. Overview of notation

Table A.1
Overview of notation.

Symbol	Meaning
v	Part of the fundamental value that agents try to learn about
$u(t)$	Part of the fundamental value that agents observe
$\sigma_u^2 =$ Var$(u(t) - u(t-1))$	Variance of the iid increments of the random walk $u(t)$
$x_i(0) = v_i$	Agent i's initial signal about the value v
κ_i	Importance of i's signal since $v = \sum_{i=1}^N \kappa_i v_i$ and $\sum_{i=1}^N \kappa_i = 1$
$x(t) = (x_i(t))_{i=1,\ldots,N}$	Vector of agents' views about the value v at time t
τ	The random time when v is revealed to all agents

(*continued on next page*)

Table A.1 (*continued*)

Symbol	Meaning
π	Probability that v is revealed at t given no revelation yet
$A = (A_{i,j})_{i,j=1,\ldots,N}$	Adjacency (or weight) matrix (how people update their views)
θ_i	Agent i's thought leadership
μ_i	Agent i's influencer value
$p(t)$	Price before revelation
$p(t) = p_r(t) + p_n(t)$	Price split into its rational and network components
s	Supply of shares
$d_i(t)$	Asset demand of agent i
N	Total number of agents
$N = N_n + N_f + N_l + N_s$	Naive, fanatic, long-term rational, and short-term rational
$x(t) = (x_n(t); x_h)$	Views split into those of naive and hardheaded agents
$x_h = (x_f; x_l; x_s)$	Hardheaded agents split by fanatic, long-term, short-term,
x_r	The rational view, $x_r = E(v\|x_1(0), \ldots, x_N(0) = v$
$\bar{x}(t)$	Average view of all agents at time t
$\bar{x}_n(t)$	Average naive view at time t
$\bar{x}_f$	Average fanatic view at any time
w_i	Wealth of agent i
$w = (w_n; w_h)$	Wealth vector split into naive and hardheaded agents
$w = (w_n; w_f; w_l; w_s)$	Wealth vector split into naive, fanatic, long-term, short-term
$w.$	Total wealth of all agents
$w. = $ $w_{n.} + w_{f.} + w_{l.} + w_{s.}$	Total wealth of naive, fanatic, long-term, short-term
$b(t) = x_r + u(t) - p(t)$	Valuation metric similar to book-to-price
$1_N, e_i, I$	Vector of ones, i'th standard unit vector, identity matrix
Δ	Change over time, e.g., $\Delta p(t+1) = p(t+1) - p(t)$

A2. Proofs

Proof of Proposition 1. Beliefs evolve as follows for $t > 1$

$$x(t) = A^{t-1}x(1) = \begin{pmatrix} A_{nn}^{t-1} & \sum_{k=0}^{t-2} A_{nn}^k A_{nh} \\ 0 & I \end{pmatrix} x(1)$$

$$\to \begin{pmatrix} 0 & (I - A_{nn})^{-1} A_{nh} \\ 0 & I \end{pmatrix} x(1) \tag{A.1}$$

Here, the second equality can be seen via induction. The convergence result follows from summing the geometric series in the upper-right block matrix, and the upper-left block matrix converges to zero, $A_{nn}^{t-1} \to 0$, because all of its eigenvalues are strictly less than one.

To see this property of the eigenvalues, let $M = A_{nn}$, which is a non-negative matrix with $\sum_j M_{i,j} \leq 1$ for all i, with strict inequality for at least one i since at least one naive agent listens to a hardheaded one. Suppose first that M is an irreducible matrix. Then the Perron-Frobenius Theorem shows that M has a strictly positive left eigenvector ξ corresponding to the largest eigenvalue λ, that is, $\xi' M = \lambda \xi'$ with $\xi_i > 0$ for all i. Therefore, $\lambda \sum_i \xi_i = \sum_i (\xi' M)_i = \sum_i \sum_j \xi_j M_{j,i} = \sum_j \xi_j \sum_i M_{j,i} < \sum_j \xi_j$, implying that $\lambda < 1$.

If instead M is reducible, then it can be written on the normal form

$$KMK^{-1} = \begin{pmatrix} M_1 & * & \dots & * \\ 0 & M_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & * \\ 0 & \dots & 0 & M_h \end{pmatrix} \tag{A.2}$$

where K is a permutation matrix, each M_k is a square matrix that is either irreducible or zero, and the spectrum of M is the union of the spectra of the M_i. Further, each M_i is a matrix of non-negative numbers with rows that sum to at most 1, and at least one row that sums to strictly less than 1 (since otherwise no one in this group of agents is influenced by a hardheaded agent, in violation of assumption 1). Therefore, the argument above shows that all eigenvalues are strictly less than 1.

Finally, the naive views are convex combinations of the hardheaded views in the limit for two reasons. First, the naive views are clearly non-negative linear combinations of hardheaded views as the weights are limits of non-negative weights. Second, the weights sum to unity because $\lim A^t 1_N = 1_N$, implying that $(I - A_{nn})^{-1} A_{nh} 1_{N-N_n} = 1_{N_n}$ using (A.1). □

Proof of Proposition 2. A. As $t \to \infty$, the average opinion, $\bar{x}(t)$, converges to

$$\bar{x}(t) \to \frac{1}{w.} w_n'(I - A_{nn})^{-1} A_{nh} x_h + \frac{1}{w.} w_h' x_h$$

$$= \left(\mu' A_{nh} + \frac{1}{w.} w_h' \right) x_h \tag{A.3}$$

using (A.1), which shows that θ_h' is given by the last parenthesis.

Note that the sum of the thought leaderships is 1 (as stated in Definition 1) since each row in A^t sums to 1, a property that is preserved in the limit as t goes to infinity, and so $\theta' 1 = \frac{1}{w.} w' \lim_{t \to \infty} A^t 1 = \frac{1}{w.} w' 1 = \frac{w.}{w.} = 1$.

B. If a naive agent i increases his following of j at the expense of other hardheaded agents, then A_{nn} remains unchanged and, therefore, influencers values μ remain unchanged. Hence, the result follows from differentiating (23) using $\partial A_{ij} / \partial \varepsilon = 1$ and $\partial A_{kj} / \partial \varepsilon = 0$ for $k \neq i$. □

Proof of Propositions 3 and 4. The equilibrium price is determined by equalized the supply s and the total demand:

$$s = \sum_{i \text{ naive}} d_i(t) + \sum_{i \text{ fanatic}} d_i(t) + \sum_{i \text{ long-term rational}} d_i(t)$$

$$+ \sum_{i \text{ short-term rational}} d_i(t) \tag{A.4}$$

To derive a convenient expression for this equilibrium condition, recall the notation that $w_n = (w_i)_{i=1,\dots,N_n}$ is the column vector of wealth of the naive agents, and, similarly, w_f, w_l, w_s, and w are the vectors of wealth of, respectively, fanatic, long-term, short-term, and all agents. Further, $w_n. = 1'w_n$ is the total wealth of all naive agents, and similarly for the other groups. With this notation, the equilibrium condition can be written as

$$s\sigma_u^2 / \pi = (w_n' x_n(t) + w_n. u(t) - w_n. p(t))$$

$$+ (w_f' x_f + w_f. u(t) - w_f. p(t))$$
$$+ w_l. (x_r + u(t) - p(t))$$
$$+ w_s. \left[\frac{(1-\pi)}{\pi} E_t(p(t+1) - p(t)) + (x_r + u(t) - p(t)) \right] \tag{A.5}$$

Isolating $p(t)$ on the left-hand side gives

$$p(t) = \frac{w_n.}{w. + \frac{1-\pi}{\pi} w_s.} \bar{x}_n(t)$$

$$+ \frac{w_f.}{w. + \frac{1-\pi}{\pi} w_s.} \bar{x}_f + \frac{w_l. + w_s.}{w. + \frac{1-\pi}{\pi} w_s.} x_r \tag{A.6}$$

$$+ \frac{\frac{1-\pi}{\pi} w_s.}{w. + \frac{1-\pi}{\pi} w_s.} E_t(p(t+1)) + \frac{w.}{w. + \frac{1-\pi}{\pi} w_s.} u(t)$$

$$- \frac{w.}{w. + \frac{1-\pi}{\pi} w_s.} \frac{s\sigma_u^2}{\pi w.}$$

$$= (1-c) \left[\frac{w_n.}{w.} \bar{x}_n(t) + \frac{w_f.}{w.} \bar{x}_f + \frac{w_l. + w_s.}{w.} x_r + u(t) - \frac{s\sigma_u^2}{\pi w.} \right]$$

$$+ cE_t(p(t+1))$$

$$= (1-c) \left[\bar{x}(t) + u(t) - \frac{s\sigma_u^2}{\pi w.} \right] + cE_t(p(t+1))$$

where the constant c is defined as $c = \frac{\frac{1-\pi}{\pi} w_s.}{w. + \frac{1-\pi}{\pi} w_s.}$, thus proving (25).

Iterating this equation forward yields the equilibrium price using standard difference equation methods and the sum of a geometric series. In particular, one can eliminate the future price by discounting the future versions of the rest of the right-hand side by c. Note the discounted future price converges to zero because of the premise that (the network part of) the expected price is bounded and because this bounded value is discounted by $c \in (0, 1)$, thus yielding a unique solution (i.e., using the standard transversality condition). To see this, note that (25) has the following structure, where I collect several terms under the umbrella called $b_t = (1-c)[\bar{x}(t) + u(t) - s\sigma_u^2/(\pi w.)]$:

$$p_t = b_t + cE_t(p_{t+1}) = b_t + cE_t(b_{t+1} + cp_{t+2})$$

$$= b_t + E_t(cb_{t+1} + c^2[b_{t+2} + p_{t+3}])$$

$$= b_t + E_t(cb_{t+1} + c^2 b_{t+2} + c^3 b_{t+3} + \dots)$$

Using that $1 + c + c^2 + \dots = 1/(1-c)$, the equilibrium price is calculated as:

$$p(t) = \frac{w_n. (1-c)}{w.} \sum_{k=0}^{\infty} c^k \bar{x}_n(t+k) + \frac{w_f.}{w.} \bar{x}_f$$

$$+ \frac{w_l. + w_s.}{w.} x_r + u(t) - \frac{s\sigma_u^2}{\pi w.} \tag{A.7}$$

When $w_n. = w_f. = 0$, the above expression yields the equation for the rational price (since, in this case, $w. = w_l. + w_s.$). To get the network part of the price, one subtracts the rational price from (A.7), using that $\frac{w_n. (1-c)}{w.} \sum_{k=0}^{\infty} c^k + \frac{w_f.}{w.} + \frac{w_l. + w_s.}{w.} = 1$.

The convergence result as $t \to \infty$ now follows from (19) using the notation $\bar{x}^\infty$ for the limit of each variable:

$$p(t) - \left(u(t) - \frac{s\sigma_u^2}{\pi w.} \right)$$

$$\rightarrow \frac{w_{n\cdot}}{w_\cdot}\bar{x}_n^\infty + \frac{w_{f\cdot}}{w_\cdot}\bar{x}_f + \frac{w_{l\cdot}+w_{s\cdot}}{w_\cdot}x_r$$

$$= \bar{x}^\infty = \sum_{j=N_n+1}^{N}\theta_j x_j \tag{A.8}$$

□

Proof of Proposition 5. The price sensitivity to each fanatic value is seen by differentiating (30). □

Proof of Proposition 6. If naive agent i with influencer value μ_i increases his following A_{ij} of fanatic agent j by ε at the expense of a lower following of a rational agent, then the effect on price is:

$$\frac{\partial p_n^\infty}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\left(\sum_{j=N_n+1}^{N_n+N_f}\theta_j(x_j-x_r)\right) = \mu_i(x_j-x_r)$$

where the last equality uses that the fanatic agent's thought leadership increases as follows:

$$\frac{\partial \theta_j}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\mu_i(A_{ij}+\varepsilon) = \mu_i$$

based on (21), the thought leadership of other fanatics is unchanged, and the thought leadership of rational agents is decreased accordingly. □

Solving the symmetric naive case (used in Propositions 7,8). In this case, it holds that $1'_{N_n}A_{nn} = 1'_{N_n}(a_0 1_{N_n}1'_{N_n}+a_1 I) = \lambda 1'_{N_n}$, where $\lambda = a_0 N_n + a_1$. Further, $\lambda \in (0,1)$ since symmetry implies that $A_{nn}1_{N_n} = \lambda 1_{N_n}$ and the row sums of A_{nn} must be less than one since agents are hardheaded connected. Hence, the average view of naive investors is:

$$\bar{x}_n(t) = \frac{1}{N_n}1'_{N_n}x_n(t) = \frac{1}{N_n}1'_{N_n}A_{nn}^{t-1}x_n(1)$$

$$+\frac{1}{N_n}1'_{N_n}\sum_{k=0}^{t-2}A_{nn}^k A_{nh}x_h = \lambda^{t-1}\bar{x}_n(1)$$

$$+\frac{1}{N_n}\sum_{k=0}^{t-2}\lambda^k 1'_{N_n}A_{nh}x_h = \lambda^{t-1}\bar{x}_n(1)+\left(1-\lambda^{t-1}\right)\bar{x}_n^\infty \tag{A.9}$$

where $\bar{x}_n^\infty = \frac{1}{N_n}1'_{N_n}A_{nh}x_h/(1-\lambda)$. Therefore, the first term in the price Eq. (A.7) can be rewritten as follows, leaving out the constant term $\frac{w_{n\cdot}(1-c)}{w_\cdot}$,

$$\sum_{k=0}^\infty c^k\bar{x}_n(t+k) = \sum_{k=0}^\infty c^k\left(\lambda^{t+k-1}\bar{x}_n(1)+(1-\lambda^{t+k-1})\bar{x}_n^\infty\right)$$

$$= \frac{\lambda^{t-1}}{1-c\lambda}(\bar{x}_n(1)-\bar{x}_n^\infty)+\frac{\bar{x}_n^\infty}{1-c}$$

so the price is

$$p(t) = \frac{\lambda^{t-1}(1-c)}{1-c\lambda}\frac{w_{n\cdot}}{w_\cdot}(\bar{x}_n(1)-\bar{x}_n^\infty)+\frac{w_{n\cdot}}{w_\cdot}\bar{x}_n^\infty+\frac{w_{f\cdot}}{w_\cdot}\bar{x}_f$$

$$+\frac{w_{l\cdot}+w_{s\cdot}}{w_\cdot}x_r+u(t)-\frac{s\sigma_u^2}{\pi w_\cdot} \tag{A.10}$$

Therefore, the network component of the price is

$$p_n(t) = \frac{\lambda^{t-1}(1-c)}{1-c\lambda}\frac{w_{n\cdot}}{w_\cdot}(\bar{x}_n(1)-\bar{x}_n^\infty)$$

$$+\frac{w_{n\cdot}}{w_\cdot}(\bar{x}_n^\infty-x_r)+\frac{w_{f\cdot}}{w_\cdot}(\bar{x}_f-x_r) \tag{A.11}$$

Proof of Proposition 7. The momentum in the current and future network price are linked as, $\Delta p_n(t+1) = \lambda\Delta p_n(t)$, as seen from (A.11). Hence, expected returns (33) can be written as follows in the symmetric case

$$E_t(r_{t+1}) = 1_{(t<\tau)}[(1-\pi)\lambda\Delta p_n(t)+\pi b(t)] \tag{A.12}$$

Further, note that the past return, $r(t) = \Delta p(t) = \Delta p_n(t)+u(t)$ is a noisy, but unbiased, proxy for $\Delta p_n(t)$, and that the noise $u(t)$ is independent of the future shock to returns, $r_{t+1}-E_t(r_{t+1})$, and of the current valuation ratio, $b(t) = x_r+u(t)-p(t) = \frac{s\sigma_u^2}{\pi w_\cdot}-p_n(t)$. So this is a standard errors-in-variables problem, where a noisy regressor will bias downward its regression coefficient (and also affect the other regression coefficient, potentially increasing it). However, here the variables are not stationary in time so, to rely on standard statistics, we either need to repeat the model many times or consider a cross-section of many stocks. Rather than considering such extensions, we simply compute the expected profit of the value-momentum strategy, $d(t) = a\Delta p(t)+b(t)$, with a weight on momentum given by $a = \lambda(1-\pi)/\pi \in (0,1)$ using (A.12):

$$E[d(t)r_{t+1}] = E[(a\Delta p(t)+b(t))E_t(r_{t+1})]$$

$$= E[(a(\Delta p_n(t)+u(t))+b(t))1_{(t<\tau)}$$

$$\times[(1-\pi)\lambda\Delta p_n(t)+\pi b(t)]]$$

$$= \pi E\left[1_{(t<\tau)}(a\Delta p_n(t)+b(t))^2\right]$$

$$+\pi E\left[u(t)1_{(t<\tau)}(a\Delta p_n(t)+b(t))\right]$$

$$= \pi E\left[1_{(t<\tau)}(a\Delta p_n(t)+b(t))^2\right] > 0 \tag{A.13}$$

using that $u(t)$ is independent of τ, $p_n(t)$, and $b(t)$. □

Proof of Proposition 8. The general statement before the proposition that short-term investors always end up being value investors follows from the fact that the price absent revelation converges to a limit as seen in Proposition 4. This convergence means that momentum profits, captured in the first part of Eq. (34), converge to zero over time while profits from trading on reversal, captured in the second part of Eq. (34), are bounded away from zero.

In the naive symmetric case, the price and its network component are given by (A.10),(A.11) as shown above. Hence, if $(\bar{x}_n(1)-\bar{x}_n^\infty) < 0$, then, before revelation, the network price is rising monotonically in time t and network price changes $p_n(t+1)-p_n(t)$ are monotonically falling over time, and vice versa if $(\bar{x}_n(1)-\bar{x}_n^\infty) > 0$. Therefore, Eq. (34) shows that the position of short-term investors is falling in the first case and rising in the latter case. □

Proof of Proposition 9. Regarding trading volume, when all agents are rational, then all agents establish their positions at time 1 and keep these positions throughout, so volume is minimal. With naive and fanatic agents, trading volume is generally positive as naive agents continue to update their views, but, as $t \rightarrow \infty$, their views converge so that view changes approach zero, leading to a diminishing volume, except at the revelation time.

Regarding price variability, with only rational agents, the price is $p(t) = x_r + u(t) - \frac{s\gamma\sigma_u^2}{N\pi}$ so the valuation metric $b(t)$ is constant over time until the revelation. With naive and fanatic agents, the valuation metric varies over time due to changes in the network part of the price. This network effect creates another source of price variation in addition to the price variation arising from changes in u and the realization of τ. □

A3. Alternative information structure

Suppose that the signals are written as a sum of a common random variable y and independent noise terms ε_i:

$$x_i(0) = y + \varepsilon_i \tag{A.14}$$

This specification is consistent with my model if we let the true value be

$$v = y + \sum_{i=1}^{N} \kappa_i \varepsilon_i. \tag{A.15}$$

since, in this case, we have

$$\sum_{i=1}^{N} \kappa_i x_i(0) = y + \sum_{i=1}^{N} \kappa_i \varepsilon_i = v. \tag{A.16}$$

If instead signals continue to be given by (A.14), but the common component is the true value, $v = y$, then we have the standard information-theoretic framework (e.g., Hellwig 1980). So we see that the difference is that, the average "noise" coming from the ε_i's is part of the true value in my model, but not in the standard framework.

However, we can reinterpret my model as being consistent with the standard information framework simply by changing the definition of two parameters (x_r and σ_u^2) while leaving everything else in the paper the same!

To analyze the standard framework, let the true value be v and the signals be given by (A.14) with $v = y$ such that $v \sim N(\bar{v}, \sigma_v^2)$ and $\varepsilon_i \sim N(0, \sigma_{\varepsilon_i}^2)$ are independent. Then the rational view x_r is:

$$x_r = E(v|x_1(0), \dots, x_N(0)) = \bar{v} + \Sigma_{vx}\Sigma_{xx}^{-1}(x(0) - \bar{v}1_N)$$
$$= \kappa_0\bar{v} + \kappa'x(0) \tag{A.17}$$

where $\kappa_0 = 1 - \Sigma_{vx}\Sigma_{xx}^{-1}1_N$ and $\kappa' = (\kappa_1, \dots, \kappa_N) = \Sigma_{vx}\Sigma_{xx}^{-1}$, and I use the standard notation that $\Sigma_{vx} = (\text{Cov}(v, x_1(0)), \dots, \text{Cov}(v, x_N(0)))$ and Σ_{xx} is the variance-covariance matrix of $x(0)$. We see that the rational view is an average of the prior $\bar{v}$ and the signals, where more precise signals (with less noise) receive more weight. When all signals are equally precise, $\sigma_{\varepsilon_i}^2 = \sigma_\varepsilon^2$ for all i, the rational view has weights

$$\kappa_0 = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + N\sigma_v^2} \text{ and } \kappa_i = \frac{\sigma_v^2}{\sigma_\varepsilon^2 + N\sigma_v^2} \text{ for } i = 1, \dots, N \tag{A.18}$$

This rational update is very similar to my model, except for two properties, both of which are easy to handle: (i) First, $\kappa_0 + \sum_{i=1}^{N} \kappa_i = 1$ so the rational weights do not add up to one when summed across the agents, $\sum_{i=1}^{N} \kappa_i < 1$; (ii) Second, agents cannot fully learn the true value, $x_r \neq v$.

Regarding (i), note that, after time 1, rational agents again use weights that add to one since any rational agent puts 100% weight on her own previous opinion. Hence, the weighting matrix continues to have the standard form in all time periods $t > 1$, which is all that matters for the solution of the belief dynamics in my model. The rational update at time 1 can be anything, including of the form (A.17), without affecting anything in the model except the definition of x_r. While in the model considered in the body of the paper, it holds that $x_r = v$, I use the notation x_r rather than v to ensure the generality of the framework. Hence, we can instead define x_r by (A.17) or whatever comes out of your favorite information framework.

Regarding (ii) that agents cannot learn the true value, $x_r \neq v$, this property increases the variance when solving each agent's portfolio problem. For example, the risk penalty in (5) becomes

$$\frac{1}{2w_i}\text{Var}_t[d_i(v + u(\tau) - p(t))]$$
$$= \frac{d_i^2}{2w_i}[\text{Var}(v|x(0)) + \text{Var}_t(u_\tau)]$$
$$= \frac{d_i^2}{2w_i}\left[\sigma_v^2 - \Sigma_{vx}\Sigma_{xx}^{-1}\Sigma_{xv} + \frac{\sigma_u^2}{\pi}\right]$$
$$=: \frac{d_i^2\tilde{\sigma}_u^2}{2w_i\pi} \tag{A.19}$$

where $\tilde{\sigma}_u^2$ is defined by the last equation. So we get back to the same solution by simply replacing σ_u^2 by $\tilde{\sigma}_u^2$ to reflect to additional risk coming from the unpredictable part of v. (Or, alternatively, one can change the definition of the wealth, w_i.) For example, when signals are equally precise ($\sigma_{\varepsilon_i}^2 = \sigma_\varepsilon^2$), then $\text{Var}(v|x(0)) = \kappa_0\sigma_v^2$ such that

$$\tilde{\sigma}_u^2 = \sigma_u^2 + \kappa_0\sigma_v^2\pi \tag{A.20}$$

In summary, the model can be solved in exactly the same way using the standard information-theoretic framework — the only change is the definitions of x_r and σ_u^2.

I end this section by noting that one might in fact prefer that the rational agents use weights that add to one even in the first time period, as in my model, for economic rather than mathematical reasons. Indeed, this is a property that underlies all updating in the literature that follows DeGroot (1974). More importantly, it might be slightly more compelling that the boundedly rational agents behave "almost" like the rational ones. In other words, when the rational agents in my model use weights that sum to one, then it might seem more plausible that the boundedly rational agents do the same. Said differently, if a rational agent puts weight on the prior, then should not the boundedly rational agents be allowed to put weight on the prior too (along with putting weight on the subset of other agents that they listen to)? My model avoids this issue, but it can be addressed here too.

Of course, the simplest solution is to say that the boundedly rational agents just ignore the prior. This behavior is, in fact, almost like the rational behavior when the number of agents, N, is large. Indeed, as $N \to \infty$, the optimal weight on the prior vanishes, $\kappa_0 \to 0$, and the total weight across agents adds to one, $\sum_{i=1}^{N} \kappa_i \to 1$, as seen

in (A.18). Further, the residual risk also vanishes such that $\tilde{\sigma}_u^2 \to \sigma_u^2$ as seen in (A.20). Intuitively, with many agents, then agents actually almost do learn the true v. So, when N is large (the realistic case), there is essentially no difference between my framework and the standard information framework.

A more sophisticated answer is that we can have a uninformative prior about v such that $\bar{v}$ disappears and we get closer to my model again. Alternatively, one can have an (additional) agent who always has a belief equal to the prior, $\bar{v}$. This agent can have a wealth of zero so that this agent is just a device used in belief formation, but not relevant for trading. Hence, when we add up the weights including this agent, then the rational weights again add up to 1, and naive agents are allowed to put some weight on the prior as well. Again, this is a special case of the model considered in the body of the paper.

A4. Many assets: many-fund separation

The model can be extended to an economy with many assets in a straightforward way. This extension provides insights on investors' portfolios that cannot be meaningfully discussed with just one asset.

To extend the model in this way, suppose that the economy now has $L > 1$ assets. The time-t fundamental value of all assets is given by $v + u(t)$ as before, but now $v, u(t) \in \mathbb{R}^{1 \times L}$ are row vectors. Any agent i starts with a time-0 view about all assets given by $x_i(0) = v_i \in \mathbb{R}^{1 \times L}$, rational agents learn v at time 1, and otherwise agents' views evolve as $x(t) = Ax(t-1)$ until revelation, where $x(t) \in \mathbb{R}^{N \times L}$ is now a matrix of all agents' views about all assets.

The simplest case is that the shocks $u(t)$ are independent and identically distributed across assets, each with a variance of $\sigma_u^2 \in \mathbb{R}$ as before. In this case, the demand equations are unchanged, and, hence, so are the equilibrium prices. In particular, the portfolio of any naive agent i is

$$d_i(t) = \frac{\pi w_i}{\sigma_u^2}(x_i(t) + u(t) - p(t)) \qquad (A.21)$$

where $d_i(t), p(t) \in \mathbb{R}^{1 \times L}$ are now a vectors. Further, using Proposition 1, the view of naive agent i converges $x_i(t) \to b_i x_h$ as $t \to \infty$, where $x_h \in \mathbb{R}^{N_h \times L}$ is matrix of hardheaded views, $b_i = e_i'(I - A_{nn})^{-1}A_{nh} \in \mathbb{R}^{1 \times N_h}$ contains the weights placed on the various hardheaded views, N_h is the number of hardheaded agents, and $e_i \in \mathbb{R}^{1 \times N_n}$ is the i'th unit vector. Hence, the portfolio of naive agent i becomes closer and closer to

$$\frac{\pi w_i}{\sigma_u^2}(b_i x_h + u(t) - p(t))$$

$$= \frac{\pi w_i}{\sigma_u^2}\sum_{j=1}^{N_h} b_{i,j} \underbrace{(x_{N_n+j} + u(t) - p(t))}_{\text{portfolio of } j\text{'th hardheaded agent}} \qquad (A.22)$$

that is, the difference between $d_i(t)$ and (A.22) converges to zero. Here, the equality uses that $\sum_j b_{i,j} = 1$ because the naive agent's view is a convex combination of hardheaded view as per Proposition 1. So we see that, over time, the portfolio of naive agent i becomes a combination of the portfolios of the N_h hardheaded agents (scaled by wealth and risk).

In other words, in the long term, all agents' portfolios satisfy $N_h + 1$ fund separation, that is, all agents hold a combination of the risk-free asset and the various portfolios of the hardheaded agents. If there are more fanatics than stocks, there could be more relevant funds than stocks, just like there exist more mutual funds and ETFs than stocks.

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