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QALYs, DALYs, and HALYs: A unifying framework for the evaluation of population health[☆]

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ABSTRACT

We provide a unifying framework for the evaluation of population health. We formalize several axioms for social preferences over distributions of health. We show that a specific combination of those axioms characterizes a large class of *population health evaluation functions* combining concerns for quality of life, quantity of life and health shortfalls. We refer to the class as (unweighted) aggregations of *health-adjusted life years* (HALYs). Two focal (and somewhat polar) members of this family are the (unweighted) aggregations of *quality-adjusted life years* (QALYs), and of *disability-adjusted life years* (DALYs). We also provide new characterization results for these focal members that enable us to scrutinize their normative foundations and shed new light on their similarities and differences.

1. Introduction

The ability to assess the effect on a population of specific health interventions is crucial for decisions concerning resource prioritization and financing in the health care sector. Social scientists, health services researchers, and operations researchers alike have long been concerned with developing appealing quantitative measures to evaluate the health of a population.¹ Mortality indicators (such as life expectancy) were typically used first. Although they are still of great importance nowadays, there has been a growing consensus to combine them with morbidity indicators. By now, it is widely accepted that the benefit a patient derives from a particular health care intervention can be defined according to two natural dimensions: quality of life and quantity of life. Pliskin et al. (1980) developed the so-called *quality-adjusted life years* (in short, QALYs), which offer a straightforward procedure to combine the two natural dimensions. It is arguably the most widely accepted methodology in the economic evaluation of health care nowadays, and a reference standard in cost-effectiveness analysis (e.g., Gold et al., 1996). Nevertheless, alternative health outcome measures are also popular. A special emphasis goes to the so-called *disability-adjusted life years* (in short, DALYs), primarily a measure of disease burden, which arose in the early 1990s (e.g., World Bank, 1993) as a result of an effort to quantify the global burden of

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¹ Early instances are Fanshel and Bush (1970) and Torrance (1976). For comprehensive surveys, the reader is referred to Dolan (2000), Gold et al. (2002) or Murray et al. (2002), Zweifel et al. (2009), among others.

premature death, disease, and injury and to make recommendations that would improve health, particularly in developing nations. DALYs have been extensively studied ever since (e.g., Murray, 1994; Murray and Acharya, 1997; Anand and Hanson, 1998). As with QALYs, they have also been systematically used in applied work (e.g., Murray et al., 2012, 2015; Kyu et al., 2018) and remain extremely popular for a wide variety of cases as of today (e.g., Briggs and Vassall, 2021; Gianino et al., 2021; Chapman et al., 2022; Xiong et al., 2022). QALYs and DALYs have usually been contrasted (e.g., Sassi, 2006; Martinez et al., 2019; Feng et al., 2020). Nevertheless, both measures can actually be seen as (admittedly, polar) instances of *Health Adjusted Life Years* (HALYs), an umbrella term for a family of measures endorsing concerns for health attainments as well as health shortfalls (e.g., Gold et al., 2002).²

We provide in this paper axiomatic foundations for HALY-based measures. Axiomatic foundations of population health measures are crucial to guide policy maker's choices among them. Nevertheless, the literature has not paid sufficient attention to them. To wit, although they have been established for basic families of QALY-based measures of population health, this has not been the case for measures involving a health shortfall. In this paper, we aim to fill that gap upon introducing a framework broad enough to allow for the analysis of measures based on both approaches. In doing so, we are also able to investigate the normative principles underlying models in which the health of a population is either measured in terms of (health) attainments, shortfalls or both.

In our model, we assume that society has preferences over distributions of (average) health states, lifetime spans and reference lifetimes in a population, and we determine specific combinations of axioms that characterize different measures for the evaluation of population health, dubbed *population health evaluation functions* (in short, PHEFs). More precisely, we assume that the distribution of health in a population is defined by a collection of triplets, each indicating the status that an individual of the population achieves in the health dimension (quality of life), the time dimension (quantity of life), as well as the (individual) reference lifetime. The framework that we set up thus allows us to approach the problem from both a health asset view and a health gap view. As a result, we are able to axiomatize population health evaluation functions concerned with the loss of life and/or the accumulation of disability or ill health, as well as those concerned with the health gains.

Our approach builds upon the framework introduced in Hougaard et al. (2013). Therein, a number of population health evaluation functions, such as the (time linear) QALY and HYE (acronym for *Healthy Years Equivalent*) population health evaluation functions, concerned with the accumulation of health, are characterized axiomatically. Our generalization of that framework allows us to characterize not only those population health evaluation functions but also others taking a 'health gap' approach, including the (time linear) DALY population health evaluation function. Furthermore, by characterizing the two types of population health evaluation functions side by side, we are able to highlight the similarities and differences between the two approaches. The similarities and differences between the QALY and DALY measures (with a special emphasis on whether there is an impact from using the latter) have previously been discussed (e.g., Sassi, 2006; Airolidi and Morton, 2009; Morton, 2010). Our results add new insights to this discussion. We present general classes of population health evaluation functions that are able to encompass both approaches into one. In particular, we present a one-parameter family of population health evaluation functions that compromise (via linear combination) between the time linear QALY and time linear DALY. We shall refer to this as the HALY family (of population health evaluation functions).

Our first result is precisely a characterization of this family by the combination of a pack of basic structural axioms (dubbed "COMMON") with two additional axioms. One (*lifetime invariance at common health*) states that the policy maker should be indifferent between adding (the same amount of) lifetime to one individual or another, provided both experience the same health status (although probably different lifetimes) and the gap between lifetime and reference lifetime is kept fixed. Another (*reference age invariance at common health*) states that the evaluation of a population health distribution in which the reference lifetime of an individual is increased by a certain amount does not depend on whether this is done for one individual or another, as long as the two individuals have the same health state.

We also show that adding just one axiom (out of a pair of dual independence axioms) to those listed above allows us to characterize the focal elements within the above-mentioned family that correspond to the time linear QALY and time linear DALY population health evaluation functions. This shows that both (polar) measures actually satisfy a large set of common axioms (those characterizing the HALY family).

Finally, we provide additional results characterizing more general families of population health evaluation functions encompassing the HALY-based measures.

We should stress that our axiomatic characterizations help disentangling the properties on which certain population health evaluation functions rest. This also allows us to contrast their descriptive and normative support. For instance, DALY-type measures are frequently being used in the economic evaluation of health care programs. This implies descriptive support for the axioms on which these measures rely (as stated by our results). But it does not discard that a policy maker might find some of these axioms normatively inadequate. Thus, it becomes a pressing issue to understand the principles over which standard measures in the economic evaluation of health care programs rely.

Our paper obviously lies within the literature on health economics dealing with the axiomatic foundations of health measures. As such, it is connected to the sizable literature on decision theory making use of multiattribute utility functions, pioneered by Debreu (1960), Fishburn (1965), Raiffa (1968) and Keeney (1974), among others. We should, nevertheless, stress that we do not presuppose in our analysis the existence of an individual utility function to evaluate health.³ Our paper also connects to the population ethics

² The notion of acceptable health as a reference point in health priority setting is, for instance, explored by Wouters et al. (2015, 2017).

³ This is in line with Hougaard et al. (2013) and Moreno-Ternero and Østerdal (2017). There exist earlier contributions within the health economics literature providing axiomatic foundations for QALY-based measures of population health, but they presume the existence of individual health-related utility (QALY) measures (e.g., Bleichrodt, 1995, 1997) or individual preference relations over quality and quantity of life (e.g., Østerdal, 2005).

literature which is a mostly axiomatic literature concerned with population evaluation functions of generic utilities and reference levels (e.g., Blackorby et al., 1995, 1997, 2005).

A novelty of our formal analysis is to augment the concept of population health distributions to include (individual) reference lifetimes, beyond the standard dimensions of quality of life and quantity of life.⁴ This opens the door to explore possible connections between evaluations on attainment and shortfall health. The discussion on attainment and shortfall inequality (e.g., Sen, 1992) has been particularly lively within health economics (e.g., Erreygers, 2009; Lambert and Zheng, 2011). In measuring inequality of a bounded variable such as health status, one can focus on attainments or shortfalls. Both look at the same situation, but from a different point of view. Thus, they can move in opposite directions. Nevertheless, the health economics literature has focussed on the requirement that both are measured *consistently*, which leads towards strong consequences. In the context of our paper, this would translate, for instance, into *consistent* evaluations of health care programs when assessed via the QALYs they generate and the DALYs they generate.

The rest of the paper proceeds as follows. In Section 2, we introduce the framework, some basic instances of population health evaluation functions, as well as a list of seven ‘COMMON’ axioms that will be used for all the results in the paper. In Section 3, we provide characterizations for the three focal classes of population health evaluation functions mentioned above. Section 4 provides additional characterization results for more general families. Section 5 concludes. All proofs have been deferred to an Appendix.

2. The preliminaries

Imagine a policy maker who has to compare distributions of health for a population of fixed size $n \geq 3$. Let us identify the population (society) with the set $N = \{1, \dots, n\}$. The health of each individual in the population is described by a triplet $h_i = (a_i, t_i, r_i)$, where $a_i \in A$ is a health state, $t_i \in T = [0, +\infty)$ is the lifetime, and $r_i \in T$, such that $r_i \geq t_i$, is a reference lifetime. We can think of r_i as the aspirational number of life years for individual i , i.e., a target or an expectation under the best possible conditions.⁵

The set of possible health states, A , is defined generally enough to encompass all possible health states for everybody in the population. We emphasize that A could be any set without a particular mathematical structure. As such, the domain could for instance be interpreted as encompassing time trajectories rather than fixed levels of health (provided the length of that trajectory is determined by t_i).⁶ Even though we do not impose a specific mathematical structure on the set A , we assume that it contains a specific element, a_* , referred to as *full health*, which is interpreted as the absence of any disease and disability whatsoever. As such, a_* refers to a universal concept (thus considered the same for all individuals).⁷

A population health distribution (or, simply, a health profile) $h = [h_1, \dots, h_n] = [(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)]$ specifies the health triplet of each individual in society.⁸ Denote the set of all possible health profiles by H . The policy maker’s preferences (or social preferences) over health profiles are expressed by a preference relation $\succeq$, to be read as “at least as preferred as”. As usual, $>$ denotes strict preference and $\sim$ denotes indifference. Assume the relation $\succeq$ is a weak order, i.e., it is complete (for each health profiles h, h' , either $h \succeq h'$, or $h' \succeq h$, or both) and transitive (if $h \succeq h'$ and $h' \succeq h''$ then $h \succeq h''$).

We stress that the two main interpretations of the inputs in our model are the following. On the one hand, the triplet (a_i, t_i, r_i) could be identifying an individual having a chronic (or “average”) health state a_i throughout a lifetime of length t_i , with a reference lifetime r_i . In this sense, the model can be used to measure the overall amount of health in the population, as well as to measure its total burden of disease. On the other hand, we may also think of the scenario in which a policy maker launches an intervention and this intervention results (as of today) in a health state a_i , for a (remaining) period of time t_i , with a (remaining) reference lifetime r_i , for each individual i in the target group. Different interventions can then be compared on the basis of their resulting distributions.

2.1. Population health evaluation functions

A *population health evaluation function* (PHEF) is a real-valued function $P : H \rightarrow \mathbb{R}$. We say that P represents $\succeq$ if it holds that, for each pair $h, h' \in H$, $P(h) \geq P(h')$ if and only if $h \succeq h'$. Note that if P represents $\succeq$ then any strictly increasing transformation of P would also do so.

The following population health evaluation function, which we call (*aggregated*) *time-linear QALY*, evaluates population health distributions by means of the unweighted aggregation of individual QALYs in society, or, in other words, by the weighted (through health levels) aggregate time span the distribution yields. Formally,

$$P^q[h_1, \dots, h_n] = P^q[(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] = \sum_{i=1}^n q(a_i)t_i, \quad (1)$$

where $q : A \rightarrow [0, 1]$ is a function satisfying $0 \leq q(a_i) \leq q(a_*) = 1$, for each $a_i \in A$.

⁴ As such, our move is reminiscent of the so-called *baseline* rationing (e.g., Hougaard et al., 2012), which enriches the standard claim problems (e.g., O’Neill, 1982) to account for additional references in the allocation process. Similarly, Ju et al. (2021) recently augmented the standard model for the allocation of greenhouse gas emissions to account for historical emissions, which can also play the role of references in the allocation process.

⁵ The assumption $r_i \geq t_i$ is in line with a truncation approach. The CDC, for instance, truncates life expectancy at the reference age 75 (e.g., Steenland and Armstrong, 2006). Although not essential for our analysis, it is convenient to interpret some of the axioms in our model.

⁶ Such an interpretation would involve that some of the axioms we present next would be less straightforward to understand. That is why we stick to the basic interpretation considered above.

⁷ Note that a_* can be interpreted as the reference health state to which all individuals aspire. That is why in our model we only consider references with respect to lifetimes.

⁸ For ease of exposition, we establish the notational convention that $h_S \equiv (h_i)_{i \in S}$, for each $S \subset N$.

The (*aggregated*) *time-linear DALY* population health evaluation function evaluates population health distributions by means of the unweighted aggregation of individual gaps from reference lifetime and QALYs in society. Formally,

$$P^d[h_1, \dots, h_n] = P^d[(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] = \sum_{i=1}^n (q(a_i)t_i - r_i), \quad (2)$$

where $q : A \rightarrow [0, 1]$ is a function satisfying $0 \leq q(a_i) \leq q(a_*) = 1$, for each $a_i \in A$.

Equivalently, (2) can be expressed as follows.

$$P^d[h_1, \dots, h_n] = P^d[(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] = - \sum_{i=1}^n (q(a_*) - q(a_i))t_i - \sum_{i=1}^n (r_i - t_i),$$

where $q : A \rightarrow [0, 1]$ is a function satisfying $0 \leq q(a_i) \leq q(a_*) = 1$, for each $a_i \in A$. That is, the (*aggregated*) *time-linear DALY* population health evaluation function evaluates population health distributions by means of the reverse unweighted aggregation of individual quality losses (with respect to full health) and lifetime gaps (with respect to reference lifetime). That is, the lower individual quality losses and reference-lifetimes gaps, the higher the value achieved by the (*aggregated*) *time-linear DALY* population health evaluation function.

The previous two population health evaluation functions are instances of the class introduced next. This class encompasses various interpretations of the importance of reference lifetime and health shortfalls, while capturing both quality and quantity of time. That is why we refer to the class as Health Adjusted Life Years (HALY) population health evaluation functions.⁹ Formally,

$$P^h[h_1, \dots, h_n] = P^h[(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] = \sum_{i=1}^n (q(a_i)t_i - \alpha r_i), \quad (3)$$

where $q : A \rightarrow [0, 1]$ is a function satisfying $0 \leq q(a_i) \leq q(a_*) = 1$ for each $a_i \in A$.

The parameter α reflects the relative importance of reference lifetimes with respect to (quality-adjusted) actual lifetimes in the population health evaluation function. At the risk of stressing the obvious, note that when $\alpha = 0$ at (3) we obtain the QALY population health evaluation function (in which reference lifetimes play no role whatsoever), whereas when $\alpha = 1$ at (3) we obtain the DALY population health evaluation function.

2.2. COMMON axioms

We now present a set of seven (common) axioms that can be considered as basic axioms for social preferences in the current context.¹⁰ They are all satisfied by the population health evaluation functions introduced above.

The first one, *anonymity*, states that a permutation of individuals should not matter for the evaluation of the population health. Formally, let Π^N denote the class of bijections from N into itself. Then,

ANON: $h \sim h_\pi$ for each $h \in H$, and each $\pi \in \Pi^N$.

The second axiom, *separability*, says that if the distribution of health in a population changes only for a subgroup of individuals, the relative evaluation of the two corresponding distributions should only depend on that subgroup.¹¹ Formally,

SEP: $[h_S, h_{N \setminus S}] \succeq [h'_S, h'_{N \setminus S}] \Leftrightarrow [h_S, h'_{N \setminus S}] \succeq [h'_S, h'_{N \setminus S}]$, for each $S \subseteq N$, and each pair $h, h' \in H$.

Continuity indicates that, for fixed distributions of health states, small changes in lifetimes and/or references should not lead to large changes in the evaluation of the population health distribution. Formally,

CONT: Let $h, h' \in H$, and $h^{(k)}$ be a sequence in H such that, for each $i \in N$, $h_i^{(k)} = (a_i, t_i^{(k)}, r_i^{(k)}) \rightarrow (a_i, t_i, r_i) = h_i$. If $h^{(k)} \succeq h'$ for each k then $h \succeq h'$, and if $h' \succeq h^{(k)}$ for each k then $h' \succeq h$.

The next pair of axioms refers to the focal state of full health (a_*).

Full health superiority says that replacing an individual's health status by full health cannot hurt the evaluation of the population health. Formally,

FHS: $[(a_*, t_i, r_i), h_{N \setminus \{i\}}] \succeq h$, for each $h = [h_1, \dots, h_n] \in H$ and each $i \in N$.

Lifetime monotonicity at full health says that if an individual is at full health, then a higher lifetime for that individual (keeping the reference lifetime fixed) is strictly better. Formally,

LMFH: Let $h \in H$ and $i \in N$ such that $t_i > t'_i$. Then $[(a_*, t_i, r_i), h_{N \setminus \{i\}}] > [(a_*, t'_i, r_i), h_{N \setminus \{i\}}]$.

The last two basic axioms we consider deal with the other focal situation of zero lifetime.

⁹ As mentioned above, this has already been used as an umbrella term for QALY and DALY-like measures of population health (e.g., Gold et al., 2002).

¹⁰ They all extend to this general setting the axioms in Hougaard et al. (2013).

¹¹ This axiom dismisses interaction effects. It nevertheless underlies the use of incremental analysis in cost-effectiveness analysis, which implies that individuals for whom two treatments yield the same health should not influence the relative evaluation of these treatments (e.g., Gold et al., 1996; Turpcu et al., 2012). And it also has a long tradition of use in models of welfare economics (e.g., Moulin, 1988).

Positive lifetime desirability states that moving an individual from zero lifetime to positive lifetime (for a given health state and reference) is a societal improvement. Thus, it excludes health states that are considered worse than death. Formally,

PLD: $h \succsim [h_{N \setminus \{i\}}, (a_i, 0, r_i)]$, for each $h = [h_1, \dots, h_n] \in H$ and $i \in N$.

The *social zero condition* says that the health state of an individual with zero lifetime is irrelevant for the evaluation of the health distribution. Miyamoto et al. (1998) introduced this axiom for individuals. We extend it to social decisions in our setting, with the caveat that references remain fixed. Formally,

ZERO: For each $h \in H$ and each $i \in N$ such that $t_i = 0$, and each $a'_i \in A$, $h \sim [h_{N \setminus \{i\}}, (a'_i, 0, r_i)]$.

In what follows, we refer to the set of axioms introduced above as the *common structural axioms* (in short, **COMMON**).

3. Axiomatic foundations for HALYs, QALYs and DALYs

Our first result says that the class of HALY population health evaluation functions (P^h) introduced above is characterized by the combination of the common structural axioms described above plus the following two specific axioms. First, *lifetime invariance at common health*, which says that the policy maker should be indifferent between adding (the same amount of) lifetime to one individual or another, provided both experience the same health status (although probably different lifetimes) and the gap between lifetime and reference lifetime is kept fixed. Second, *reference age invariance at common health*, which says that the evaluation of a population health distribution in which the reference lifetime of an individual is increased by a certain amount does not depend on whether this is done for one individual or another, as long as the two individuals have the same health state. Formally,

LICH: For each $h \in H$, each $c > 0$, and each pair $i, j \in N$, such that $a_i = a_j = a$,

$$[(a, t_i + c, r_i + c), (a, t_j, r_j), h_{N \setminus \{i, j\}}] \sim [(a, t_i, r_i), (a, t_j + c, r_j + c), h_{N \setminus \{i, j\}}].$$

RICH: For each $h \in H$, each $c > 0$, and each pair $i, j \in N$, with $a_i = a_j = a$,

$$[(a, t_i, r_i + c), (a, t_j, r_j), h_{N \setminus \{i, j\}}] \sim [(a, t_i, r_i), (a, t_j, r_j + c), h_{N \setminus \{i, j\}}].$$

Theorem 1. *The following statements are equivalent:*

1. $\succsim$ is represented by a HALY population health evaluation function.
2. $\succsim$ satisfies **COMMON**, **LICH**, and **RICH**.

One might argue that the class of HALY population health evaluation functions characterized above is too large and it would make sense to consider only its members arising from convex combinations of its most focal members (the DALY and QALY population health evaluation functions). This is equivalent to restricting the parameter α at (3) to the range $[0, 1]$. It turns out that such a sub-class can be characterized adding two natural axioms to those listed at Theorem 1. The first axiom states that if an individual enjoys full health with a certain lifetime t equal to its reference lifetime, then increasing t cannot hurt the population evaluation. The second axiom states that decreasing the lifetime gap for an individual enjoying full health (by means of her reference) cannot hurt the population evaluation.

More precisely, *joint monotonicity at full health* says that if an individual enjoys full health with a certain lifetime equal to its reference lifetime, then increasing them equally cannot hurt the population evaluation. *Gap monotonicity at full health* says that decreasing the reference while keeping the lifetime fixed, for an individual enjoying full health (by means of her reference) cannot hurt the population evaluation. Formally,

JMPH: For each $h \in H$, each $c > 0$, and each $i \in N$, such that $a_i = a_*$, and $t_i = r_i$,

$$[(a_*, t_i + c, r_i + c), h_{N \setminus \{i\}}] \succsim [(a_*, t_i, r_i), h_{N \setminus \{i\}}].$$

GMPH: For each $h \in H$, each $c > 0$, and each $i \in N$, such that $a_i = a_*$,

$$[(a_*, t_i, r_i), h_{N \setminus \{i\}}] \succsim [(a_*, t_i, r_i + c), h_{N \setminus \{i\}}].$$

Corollary 1. *The following statements are equivalent:*

1. $\succsim$ is represented by a HALY population health evaluation function with $\alpha \in [0, 1]$.
2. $\succsim$ satisfies **COMMON**, **LICH**, **RICH**, **JMPH**, and **GMPH**.

We now consider another specific axiom. *Independence of reference lifetime at full health* says that when an individual lives in full health, the reference lifetime (and therefore the gap between lifetime and reference lifetime) is irrelevant for the evaluation of population health. Formally,

IRFH: For each $h \in H$, each $i \in N$, and each $r'_i \geq t_i$,

$$[(a_*, t_i, r_i), h_{N \setminus \{i\}}] \sim [(a_*, t_i, r'_i), h_{N \setminus \{i\}}].$$

As the next result states, if we add *independence of reference lifetime at full health* to the axioms at [Theorem 1](#), we characterize the QALY population health evaluation function (P^q).¹²

Theorem 2. *The following statements are equivalent:*

1. $\succsim$ is represented by an (aggregated) time-linear QALY population health evaluation function.
2. $\succsim$ satisfies COMMON, LICH, RICH, and IRFH.

Finally, *gap invariance at full health* says that when an individual enjoys full health, only the gap between lifetime and reference lifetime matters, not lifetime per se. Formally,

GIFH: For each $h \in H$ and each $i \in N$,

$$[(a_*, 0, r_i), h_{N \setminus \{i\}}] \sim [(a_*, t_i, r_i + t_i), h_{N \setminus \{i\}}].$$

If we now add *gap invariance at full health*, instead of *independence of reference lifetime at full health*, to the axioms at [Theorem 1](#), we characterize the DALY population health evaluation function (P^d). In other words, our next result says that P^d is characterized by the combination of the common structural axioms, *lifetime invariance at common health*, *reference age invariance at common health* and *gap invariance at full health*.

Theorem 3. *The following statements are equivalent:*

1. $\succsim$ is represented by an (aggregated) time-linear DALY population health evaluation function.
2. $\succsim$ satisfies COMMON, LICH, RICH, and GIFH.

A simple inspection of the statements of the previous two theorems, allows us to infer that the characterizations of two allegedly polar procedures for the evaluation of population health, such as (time-linear) QALYs and DALYs, actually share many axioms: the so-called COMMON axioms (anonymity, separability, continuity, full health superiority, lifetime monotonicity at full health, positive lifetime desirability, and the social zero condition), as well as *lifetime invariance at common health* and *reference age invariance at common health*. They only differ in two: the time-linear QALY requires *independence of reference lifetime at full health*, whereas the time-linear DALY requires *gap invariance at full health*. These two axioms are rather different. As a matter of fact, and as the next result shows, they are incompatible when simply adding a third axiom (within the COMMON pack): *lifetime monotonicity at full health*.

Corollary 2. *The axioms LMFH, IRFH, and GIFH are incompatible.*

4. Going beyond time linearity

In this section, we obtain axiomatic foundations for more general classes of population health evaluation functions that include HALYs. They all rely on the so-called *healthy years equivalent*, formally introduced next.

Let $f : A \times T^2 \rightarrow \mathbb{R}_+$ be a continuous function with respect to its second and third variable, such that

- $0 \leq f(a_i, t_i, r_i) \leq t_i$, for each $(a_i, t_i, r_i) \in A \times T^2$,
- $h \sim [(a_*, f(a_i, t_i, r_i), r_i)_{i \in N}]$, for each $h = [h_1, \dots, h_n] = [(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] \in H$.

Following [Mehrez and Gafni \(1989\)](#), we refer to f as the *Healthy Years Equivalent* (HYE) function.

One might find it plausible to generalize the class of HALY population health evaluation functions to account for (separable) functional forms relying on the gap between *healthy years equivalent* (instead of QALYs) and reference lifetimes. It turns out that such a family could be characterized by just adding one of the axioms introduced in the previous section (*gap invariance at full health*) to the list of COMMON structural axioms.

Let $P^l : H \rightarrow \mathbb{R}$ be such that, for each $h = [h_1, \dots, h_n] = [(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] \in H$,

$$P^l[h_1, \dots, h_n] = P^l[(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] = \sum_{i=1}^n g(f(a_i, t_i, r_i) - r_i), \quad (4)$$

where $g : \mathbb{R}_- \rightarrow \mathbb{R}$ is a strictly increasing continuous function, and f is the HYE function. We shall refer to P^l as *generalized HYE loss population health evaluation function*.

Theorem 4. *The following statements are equivalent:*

1. $\succsim$ is represented by a generalized HYE loss population health evaluation function.
2. $\succsim$ satisfies COMMON and GIFH.

¹² This result can be seen as an extension of Theorem 2 in [Hougaard et al. \(2013\)](#) to our setting.

If instead of *gap invariance at full health*, the only axiom we add to the list of COMMON structural axioms is *independence of reference lifetime at full health*, then we characterize a class of (separable) population health evaluation functions conveying a transformation of *healthy years equivalent*. Formally, let $P^f : H \rightarrow \mathbb{R}$ be such that, for each $h = [h_1, \dots, h_n] = [(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] \in H$,

$$P^f[h_1, \dots, h_n] = P^f[(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] = \sum_{i=1}^n g(f(a_i, t_i, r_i)), \quad (5)$$

where $g : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a strictly increasing continuous function, and f is the HYE function. We shall refer to P^f as *generalized HYE* population health evaluation function.

Theorem 5. *The following statements are equivalent:*

1. $\succsim$ is represented by a generalized HYE population health evaluation function.
2. $\succsim$ satisfies COMMON and IRFH.

Finally, we provide a characterization for the most general (separable) functional form, relying on the *healthy years equivalent* and reference lifetimes.¹³ Formally, let $P^g : H \rightarrow \mathbb{R}$ be such that, for each $h = [h_1, \dots, h_n] = [(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] \in H$,

$$P^g[h_1, \dots, h_n] = P^g[(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] = \sum_{i=1}^n g(f(a_i, t_i, r_i), r_i), \quad (6)$$

where $g : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ is a continuous function (strictly increasing in its first variable), and f is the HYE function. We shall refer to P^g as a *generalized HALY* population health evaluation function.

Theorem 6. *The following statements are equivalent:*

1. $\succsim$ is represented by a generalized HALY population health evaluation function.
2. $\succsim$ satisfies COMMON.

5. Conclusion

We have presented in this paper an axiomatic approach to the evaluation of population health, when individuals' statuses are described in the health dimension (quality of life), the time dimension (quantity of life), as well as the (individual) reference lifetime. We have characterized two focal representations of social preferences over population health distributions; namely, the time-linear QALY and DALY representations. Those representations, which are widely used in applied work for the economic evaluation of health care programs, were initially considered as polars. Some of their differences and similarities were addressed in the literature (e.g., Sassi, 2006; Airoldi and Morton, 2009; Morton, 2010). We have seen in this paper that the two representations actually share a solid common ground, being both focal elements of a class of population health evaluation functions (dubbed HALY), which we have also characterized. Typically, QALYs and DALYs will not provide *consistent* evaluations of health care programs. But if they do, this will also be the case with all members of the HALY family. Note also that if the reference lifetime (r_i) is fixed, then both DALYs and QALYs (as well as all members of the HALY family) are informationally equivalent. This crucially relies on the time linearity all these representations share.

We have also provided further insights, obtaining additional characterizations of more general HALY-related population health evaluation functions. We note that all population health evaluation functions we characterize impose unweighted aggregation across individuals, an aspect usually criticized in the health economics literature by its lack of concern for distributive justice (e.g., Wagstaff, 1991). That is why more general population health evaluation functions, such as the ones mentioned at the end of the previous section, may be of interest to capture such a concern. A natural course of action is the so-called Bergsonian approach, which can be traced back to Bergson (e.g., Burk, 1936).¹⁴ In a health economics context, power functions of QALYs were introduced, at an individual level, by Pliskin et al. (1980). The concept was also studied by Wagstaff (1991), Williams (1997), Østerdal (2005) and Hougaard et al. (2013), among others.¹⁵ Nevertheless, power functions of DALYs (or other population health evaluation functions involving reference lifetimes, such as those captured by the HALY family) have not been proposed yet in the literature.¹⁶ There exist different plausible alternatives to do so (e.g., taking powers of DALYs, HALYs, or simply lifetimes, or shortfalls between references and lifetimes). The axiomatic characterization for those resulting functions requires considerable work. That is why we leave it for further research.

Appendix. Proofs of the results

Proof of Theorem 6. We start with this proof, as it will be instrumental for the remaining ones.

¹³ The result can be seen as an extension of Theorem 1 in Hougaard et al. (2013) to our setting.

¹⁴ See also Moulin (1988), Chapter 2) for further details.

¹⁵ The so-called rank-dependent QALY model (Bleichrodt et al., 2004, 2005) is another approach to incorporate equity concerns in population health evaluation functions. Likewise, there exist recent developments in distributional cost-effectiveness analysis with the same goal (Cookson et al., 2020).

¹⁶ Note that all these power functions would keep the additively separable structure, thus obeying in particular the axiom of separability, as well as the remaining COMMON axioms we consider in this paper.

Suppose first that $\succsim$ is represented by a PHEF satisfying (6). We start by noticing that, from inspection of (6), it follows immediately that ANON and SEP hold. CONT holds because f and g are continuous functions themselves. As $0 \leq f(a_i, t_i, r_i) \leq t_i$, it follows that $f(a_i, 0, r_i) = 0$, implying ZERO. Thus, as $f(a_i, 0, r_i) = 0 \leq f(a_i, t_i, r_i)$, this in turn implies PLD. Furthermore, as $h \sim [(a_*, f(a_i, t_i, r_i), r_i)_{i \in N}]$, it follows that if $h_i = [(a_*, t_i, r_i)]$, $h'_i = [(a_*, t'_i, r_i)]$, and $t_i > t'_i$, then $f(a_*, t_i, r_i) = t_i > t'_i = f(a_*, t'_i, r_i)$, which implies that $[h_i, h_{N \setminus \{i\}}] > [h'_i, h_{N \setminus \{i\}}]$, so LMFH holds. Finally, as $h \sim [(a_*, f(a_i, t_i, r_i), r_i)_{i \in N}]$, and $f(a_i, t_i, r_i) \leq t_i$, it follows from LMFH that $[(a_*, t_i, r_i)_{i \in N}] \succeq [(a_*, f(a_i, t_i, r_i), r_i)_{i \in N}] \sim h$. Thus, FHS holds.

Conversely, suppose $\succsim$ satisfies COMMON. We start by showing that there exists a function $f : A \times T^2 \rightarrow \mathbb{R}$ such that f is continuous with respect to its second and third variable and such that for each $h = [h_1, \dots, h_n] = [(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i, r_i), r_i)_{i \in N}],$$

where $0 \leq f(a_i, t_i, r_i) \leq t_i$ for each $(a_i, t_i, r_i) \in A \times T^2$. Note that this part of the proof follows along similar lines as the proofs of existence of HYE in Østerdal (2005) and Hougaard et al. (2013) for individual or social health profiles, respectively, without reference lifetime.

First, we prove that for each $h \in H$ and each $i \in N$, there exists $t_i^* \in T$, such that

$$h \sim [(a_*, t_i^*, r_i), h_{N \setminus \{i\}}].$$

If $t_i = 0$, then it follows from ZERO that $t_i^* = t_i = 0$. Therefore, let $t_i > 0$. By contradiction, assume that t_i^* does not exist. Then, $T = A \cup B$, where

$$A = \{s \in T \mid h > [(a_*, s, r_i), h_{N \setminus \{i\}}]\},$$

$$B = \{s \in T \mid [(a_*, s, r_i), h_{N \setminus \{i\}}] > h\}.$$

We show first that both A and B are non-empty sets. By FHS, $[(a_*, t_i, r_i), h_{N \setminus \{i\}}] \succeq h$, implying that either $t_i^* = t_i$ (a contradiction), or $t_i \in B$. By PLD and ZERO, it follows that either $t_i^* = 0$ (a contradiction), or $0 \in A$.

Now, by CONT, A and B are open sets. Thus, as $A \cap B = \emptyset$, it follows that T is not a connected set, which is a contradiction. Therefore, t_i^* exists, and due to LMFH, it is uniquely determined. By SEP, we can determine each t_i^* separately. Therefore, let $f_i : A \times T^2 \rightarrow \mathbb{R}$ be such that $f_i(a_i, t_i, r_i) = t_i^*$ for each $i \in N$. By ANON, $f_i() = f_j() = f()$ for each pair $i, j \in N$. By CONT, f is continuous with respect to its second and third variables. And, as $t_i \in B$ and $0 \in A$, $0 \leq f(a_i, t_i, r_i) \leq t_i$, so the range of f is a connected subset of $\mathbb{R}$. Furthermore,

$$h \sim [(a_*, f(a_i, t_i, r_i), r_i)_{i \in N}].$$

Thus, using the notation $h^* = [(a_*, f(a_i, t_i, r_i), r_i)_{i \in N}]$, there exists an induced social preference relation $\succsim^*$ such that $h \succeq h'$ if and only if $h^* \succeq^* h'^*$. By CONT and SEP, it follows that $\succsim^*$ is continuous on its domain and satisfies separability across individuals. It then follows by application of Theorem 3 in Debreu (1960) that there exists a continuous function $g : \mathbb{R}_+^2 \rightarrow \mathbb{R}$, such that

$$h^* \succeq h'^* \Leftrightarrow \sum_{i=1}^n g(f(a_i, t_i, r_i), r_i) \geq \sum_{i=1}^n g(f(a'_i, t'_i, r'_i), r'_i).$$

Moreover, by LMFH, g is strictly increasing in its first variable.

Proof of Theorem 1. We focus on the non-trivial implication. Suppose $\succsim$ satisfies COMMON, LICH, and RICH. Then, by Theorem 6, $\succsim$ is represented by a generalized HALY PHEF. That is,

$$P[h_1, \dots, h_n] = \sum_{i=1}^n g(f(a_i, t_i, r_i), r_i),$$

where $g : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ is a continuous function, strictly increasing in its first variable, and $f : A \times T^2 \rightarrow \mathbb{R}_+$ is the HYE function.

Let $\varphi : A \times T^2 \rightarrow \mathbb{R}$ be such that $\varphi(a_i, t_i, r_i) = g(f(a_i, t_i, r_i), r_i)$, for each $(a_i, t_i, r_i) \in A \times T^2$. Assume, without loss of generality, that $\varphi(\bar{a}, 0, 0) = 0$ for some $\bar{a} \in A$. Let $a \in A$. By iterated application of LICH and RICH and the transitivity of $\succsim$, as well as ZERO,

$$\begin{aligned} \sum_{i=1}^n \varphi(a, t_i, r_i) &\stackrel{[\text{LICH}]}{=} \varphi(a, \sum_{i=1}^n t_i, r_i + \sum_{j \neq i} t_j) + (n-1)\varphi(a, 0, r_j - t_j) \\ &\stackrel{[\text{RICH}]}{=} \varphi(a, \sum_{i=1}^n t_i, \sum_{i=1}^n r_i) + (n-1)\varphi(a, 0, 0) \\ &\stackrel{[\text{ZERO}]}{=} \varphi(a, \sum_{i=1}^n t_i, \sum_{i=1}^n r_i) + (n-1)\varphi(\bar{a}, 0, 0) \\ &= \varphi(a, \sum_{i=1}^n t_i, \sum_{i=1}^n r_i). \end{aligned} \tag{7}$$

Let $i, j \in N$. It then follows from the above, as well as LICH, RICH and ZERO, that

$$\varphi(a, t_i, r_i) + \varphi(a, t_j, r_j) \stackrel{[\text{LICH}]}{=} \varphi(a, t_i + t_j, r_i + r_j) + \varphi(a, 0, r_j - t_j)$$

$$\begin{aligned}
& \stackrel{[\text{RICH}]}{=} \varphi(a, t_i + t_j, t_i + t_j) + \varphi(a, 0, r_i - t_i + r_j - t_j) \\
& \stackrel{[(7)]}{=} \varphi(a, t_i, t_i) + \varphi(a, t_j, t_j) + \varphi(a, 0, r_i - t_i) + \varphi(a, 0, r_j - t_j) \\
& \stackrel{[\text{ZERO}]}{=} \varphi(a, t_i, t_i) + \varphi(a_*, 0, r_i - t_i) + \varphi(a, t_j, t_j) + \varphi(a_*, 0, r_j - t_j).
\end{aligned}$$

Therefore, $\varphi(a, t_i, r_i)$ can be decomposed as follows:

$$\varphi(a, t_i, r_i) = \varphi(a, t_i, t_i) + \varphi(a_*, 0, r_i - t_i). \quad (8)$$

Next, define the function $\phi : A \times T^2 \rightarrow \mathbb{R}$ such that $\phi(a_i, t_i) = \varphi(a, t_i, t_i)$, for each $(a_i, t_i) \in A \times T$. Let $a \in A$. Then, $\sum_{i=1}^n \phi(a, t_i) = \phi(a, \sum_{i=1}^n t_i)$, for each $t_i \in T$. In particular, $\phi(a, t_1 + t_2) = \phi(a, t_1) + \phi(a, t_2)$ for each pair $t_1, t_2 \in T$, which is precisely one of Cauchy's canonical functional equations. As $\phi(a, \cdot)$ is a continuous function, it follows that the unique solutions to such an equation are the linear functions (e.g., Aczel, 2006; page 34). More precisely, there exists a function $\hat{q} : A \rightarrow \mathbb{R}$ such that

$$\varphi(a, t, t) = \phi(a, t) = \hat{q}(a)t,$$

for each $a \in A$, and each $t \in T$. It follows from FHS that $\hat{q}(a_*) \geq \hat{q}(a)$.

Let $h_i = (a_*, 0, r_i) \in A \times T^2$. From (7), it follows that

$$\sum_{i=1}^n \varphi(a_*, 0, r_i) = \varphi(a_*, 0, \sum_{i=1}^n r_i).$$

Next, define the function $\psi : T \rightarrow \mathbb{R}$ such that $\psi(r_i) = \varphi(a_*, 0, r_i)$, for each $r_i \in T$. Then, $\psi(r_1) + \psi(r_2) = \psi(r_1 + r_2)$, for each pair $r_1, r_2 \in T$. As before, as ψ is a continuous function, it follows that the unique solutions to such an equation are the linear functions. Therefore, there exists $\beta \in \mathbb{R}$ such that

$$(a_*, 0, r_i) = \psi(r_i) = \beta r_i,$$

for each $r_i \in T$.

Thus, by (8),

$$\varphi(a, t_i, r_i) = \hat{q}(a)t_i + \beta(r_i - t_i) = (\hat{q}(a) - \beta)t_i + \beta r_i,$$

for each $a \in A$ and each $(t_i, r_i) \in T^2$ such that $r_i \geq t_i$, where $\hat{q} : A \rightarrow \mathbb{R}$ is a function satisfying $\hat{q}(a) \leq \hat{q}(a_*)$, for each $a \in A$. To conclude, let $\alpha = -\beta$ and $q : A \rightarrow \mathbb{R}$ be such that $q(a) = \frac{\hat{q}(a) - \beta}{\hat{q}(a_*) - \beta}$, for each $a \in A$. By PLD and LMFH, it follows that $1 = q(a_*) \geq q(a) \geq 0$, for each $a \in A$. Then, we may write:

$$\varphi(a, t_i, r_i) = q(a)t_i - \alpha r_i,$$

where $\alpha \in \mathbb{R}$, and $0 \leq q(a) \leq q(a_*) = 1$, for each $a \in A$, as desired. ■

Proof of Theorem 2. We focus on the non-trivial implication. Suppose $\succsim$ satisfies COMMON, LICH, RICH, and IRFH. Then, by Theorem 1, $\succsim$ is represented by a HALY PHEF, i.e.,

$$P[h_1, \dots, h_n] = P[(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] = \sum_{i=1}^n (q(a_i)t_i - \alpha r_i).$$

By IRFH,

$$[(a_*, t_i, r_i), h_{N \setminus \{i\}}] \sim [(a_*, t_i, r'_i), h_{N \setminus \{i\}}],$$

for each $i \in N$, and each pair $r'_i \neq r_i$ such that $r_i, r'_i \geq t_i$. Equivalently,

$$\begin{aligned}
P[(a_*, t_i, r_i), h_{N \setminus \{i\}}] &= q(a_*)t_i - \alpha r_i + \sum_{j \neq i} (q(a_j)t_j - \alpha r_j) \\
&= q(a_*)t_i - \alpha r'_i + \sum_{j \neq i} (q(a_j)t_j - \alpha r_j) = P[(a_*, t_i, r'_i), h_{N \setminus \{i\}}],
\end{aligned}$$

from where it follows that $\alpha = 0$. ■

Proof of Theorem 3. We focus on the non-trivial implication. Suppose $\succsim$ satisfies COMMON, LICH, RICH, and GIFH. Then, by Theorem 1, $\succsim$ is represented by a HALY PHEF, i.e.,

$$P[h_1, \dots, h_n] = P[(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] = \sum_{i=1}^n (q(a_i)t_i - \alpha r_i).$$

By GIFH,

$$[(a_*, 0, r_i), h_{N \setminus \{i\}}] \sim [(a_*, t_i, r_i + t_i), h_{N \setminus \{i\}}],$$

for each $i \in N$. Equivalently,

$$\begin{aligned} P[(a_*, 0, r_i), h_{N \setminus \{i\}}] &= -\alpha r_i + \sum_{j \neq i} (q(a_j) t_j - \alpha r_j) \\ &= t_i - \alpha(r_i + t_i) + \sum_{j \neq i} (q(a_j) t_j - \alpha r_j) = P[(a_*, t_i, r_i + t_i), h_{N \setminus \{i\}}], \end{aligned}$$

from where it follows that $\alpha = 1$. ■

Proof of Corollary 1. We focus on the non-trivial implication. Suppose $\succsim$ satisfies COMMON, LICH, RICH, JMPH, and GMPH. Then, by Theorem 1, $\succsim$ is represented by a HALY PHEF, i.e.,

$$P[h_1, \dots, h_n] = P[(a_1, t_1, r_1), \dots, (a_n, t_n, r_n)] = \sum_{i=1}^n (q(a_i) t_i - \alpha r_i).$$

By GMPH, for each $h \in H$, each $i \in N$, and each $c > 0$,

$$[(a_*, t_i, r_i), h_{N \setminus \{i\}}] \succsim [(a_*, t_i, r_i + c), h_{N \setminus \{i\}}].$$

Equivalently,

$$\begin{aligned} P[(a_*, t_i, r_i), h_{N \setminus \{i\}}] &= t_i - \alpha r_i + \sum_{j \neq i} (q(a_j) t_j - \alpha r_j) \\ &\geq t_i - \alpha(r_i + c) + \sum_{j \neq i} (q(a_j) t_j - \alpha r_j) = P[(a_*, t_i, r_i + c), h_{N \setminus \{i\}}], \end{aligned}$$

from where it follows that $\alpha \geq 0$. By JMPH, for each $h \in H$, each $c > 0$, and each $i \in N$, such that $a_i = a_*$, and $t_i = r_i$,

$$[(a_*, t_i + c, r_i + c), h_{N \setminus \{i\}}] \succsim [(a_*, t_i, r_i), h_{N \setminus \{i\}}].$$

Equivalently,

$$\begin{aligned} P[(a_*, t_i + c, r_i + c), h_{N \setminus \{i\}}] &= t_i + c - \alpha(r_i + c) + \sum_{j \neq i} (q(a_j) t_j - \alpha r_j) \\ &\geq t_i - \alpha r_i + \sum_{j \neq i} (q(a_j) t_j - \alpha r_j) = P[(a_*, t_i, r_i), h_{N \setminus \{i\}}], \end{aligned}$$

from where it follows that $\alpha \leq 1$. ■

Proof of Corollary 2. Assume, by contradiction, that $\succsim$ satisfies LMFH, IRFH, and GIFH. Let $h = [h_1, \dots, h_n] = [(a_*, t_1, r_1), \dots, (a_*, t_n, r_n)] \in H$. Assume that $t_j > 0$ for, at least an individual $j \in N$. Let $\hat{r} = \max\{r_1, \dots, r_n\}$. By repeated use of IRFH and transitivity,

$$h = [(a_*, t_1, r_1), \dots, (a_*, t_n, r_n)] \sim [(a_*, t_1, \hat{r}), \dots, (a_*, t_n, \hat{r})].$$

By repeated use of GIFH and transitivity,

$$[(a_*, t_1, \hat{r}), \dots, (a_*, t_n, \hat{r})] \sim [(a_*, 0, \hat{r} - t_1), \dots, (a_*, 0, \hat{r} - t_n)].$$

By repeated use of IRFH and transitivity,

$$[(a_*, 0, \hat{r} - t_1), \dots, (a_*, 0, \hat{r} - t_n)] \sim [(a_*, 0, r_1), \dots, (a_*, 0, r_n)].$$

Thus, by transitivity,

$$h = [(a_*, t_1, r_1), \dots, (a_*, t_n, r_n)] \sim [(a_*, 0, r_1), \dots, (a_*, 0, r_n)],$$

which contradicts LMFH. ■

Proof of Theorem 4. We focus on the non-trivial implication. Suppose $\succsim$ satisfies COMMON and GIFH. Then, by Theorem 6, $\succsim$ is represented by a generalized HALY PHEF. In particular, for each $h \in H$, $h \sim h^* \equiv [(a_*, f(a_i, t_i, r_i), r_i)_{i \in N}]$. Thus, there exists an induced social preference relation $\succsim^*$ such that $h \succsim h'$ if and only if $h^* \succsim^* h'^*$. By CONT and SEP, it follows that $\succsim^*$ is continuous on its domain and satisfies separability across individuals. It then follows by application of Theorem 3 in Debreu (1960) that there exists a continuous function $\hat{g} : \mathbb{R}_+^2 \rightarrow \mathbb{R}$, such that

$$h^* \succsim h'^* \Leftrightarrow \sum_{i=1}^n \hat{g}(f(a_i, t_i, r_i), r_i) \geq \sum_{i=1}^n \hat{g}(f(a'_i, t'_i, r'_i), r'_i).$$

By GIFH, $[(a_*, f(a_i, t_i, r_i), r_i)_{i \in N}] \sim [(a_*, 0, r_i - f(a_i, t_i, r_i))_{i \in N}]$. Thus, $h \sim [(a_*, 0, r_i - f(a_i, t_i, r_i))_{i \in N}]$, for each $h \in H$. Let $g : \mathbb{R}_- \rightarrow \mathbb{R}$ be such that $g(x) = \hat{g}(0, -x)$, for each $x \in \mathbb{R}_-$. By construction and LMFH (part of COMMON), g is continuous and strictly increasing. Altogether,

$$h \succsim h' \Leftrightarrow \sum_{i=1}^n g(f(a_i, t_i, r_i) - r_i) \geq \sum_{i=1}^n g(f(a'_i, t'_i, r'_i) - r'_i),$$

as desired. ■

Proof of Theorem 5. We focus on the non-trivial implication. Suppose $\succsim$ satisfies COMMON and IRFH. Then, as in the previous proof, there exists a continuous function $\hat{g} : \mathbb{R}_+^2 \rightarrow \mathbb{R}$, such that

$$h^* \succsim h'^* \Leftrightarrow \sum_{i=1}^n \hat{g}(f(a_i, t_i, r_i), r_i) \geq \sum_{i=1}^n \hat{g}(f(a'_i, t'_i, r'_i), r'_i).$$

By IRFH, $\hat{g}(f(a_i, t_i, r_i), r_i) = \hat{g}(f(a_i, t_i, r_i), r'_i)$ for all $r'_i \neq r_i$. Let $g : \mathbb{R}_+ \rightarrow \mathbb{R}$ be the corresponding univariate function. Then, g is continuous and, by LMFH, strictly increasing. Furthermore,

$$h \succsim h' \Leftrightarrow \sum_{i=1}^n g(f(a_i, t_i, r_i)) \geq \sum_{i=1}^n g(f(a'_i, t'_i, r'_i)),$$

as desired. ■

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