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Master Thesis - Cand Merc. Finance & Investments

# The Optimal Time-Varying Factor Portfolio

A Mean-Variance Approach


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## Abstract

Timing the exposure to risk premia plays a huge role in active asset management as well as in academia in order to achieve the highest return for the lowest risk. This paper studies the optimal time-varying exposure to eight common equity factors from Jul. 1963 to Nov. 2022 via modern portfolio theory and volatility forecasting, while incorporating transaction cost.

The thesis finds that factor-timing can yield a higher Sharpe ratio than an equal-weight portfolio, as the means, variances and correlations of the factors vary over time. The effect is largest if the investor takes long-short factor bets besides a full investment in the market portfolio, which leads to a doubling of the Sharpe ratio after transaction cost, compared to the equal-weighted portfolio. For example, the investor benefits from investing in size during economic booms, quality and earnings during downturns, and can use value and investment as an inflation hedge. The culprit is that although factor-timing is beneficial theoretically, forecasting returns and the macroeconomy is extremely tricky, wherefore real-world implementation of factor-timing is hard and likely to result in excess risk. A valid implementation method is adjusting the asset allocation slightly to the underlying economy. The paper also finds that if the goal is to maximize the added alpha per unit of added idiosyncratic risk, the equal-weight portfolio substantially outperforms the factor-timing portfolio. Therefore, the benefit of factor-timing comes down to investor preferences. Estimating the volatility via GARCH-models improves the fit of the volatility and correlations, but still underperforms due to poorly fitted means. Hence, the investor can gain an edge by combining GARCH estimates with a model for mean returns.

The paper establishes that the success of factor-timing depends on the assumptions on transaction costs as well as forecasts of means and variances more so than the true link between factor returns and the economy. In theory, factor-timing does pay both over time and across business cycle stages, but the practical implementation is very tricky.

# Contents

# 1 Introduction

The topic of asset allocation is widely debated and of large interest among practitioners in the asset management industry as well as in finance academics. In essence, asset allocation revolves around how an investor should allocate his wealth across different asset classes (Lumholdt, 2018). The topic can be divided into strategic and tactical asset allocation, where the former evolves around the long-term allocation across asset classes and the latter focuses on short-term adjustments from the strategic weights to fit changes in the economy (Lumholdt, 2018). Common for all investments is that the investor seeks compensation for being exposed to certain persistent risks across assets (Linton, 2019). This thesis examines how to vary asset allocation weights to equity factors in the U.S. over time.

The basis for the factor research was laid by Fama & French (1992, 1993) developing a three-factor model, which hypothesized that the investor is compensated for size and value risk alongside the market risk of the underlying stocks. Later, Fama & French (2015) developed factors related to the investment- and earnings intensity of the firms, hypothesizing that firms with more robust earnings and more conservative investments yield higher returns.

Although these factors, on average, yield a positive return premium, their performance varies considerably over time. For example, Ilmanen et al. (2021) show that not only do the return and standard deviation of value and momentum vary over time, one can also time investments in the factors by betting on mean-reversion and using predictive regressions in linking the factors to the underlying economy. Similarly, Stanhope (2016) finds that the performance of the factors depends on the business cycle. Figure 1.1 illustrates that the daily value premium varies substantially. Hence, assuming that one can forecast the moves, a portfolio manager can, in theory, gain from time-varying factor weights compared to a static, strategic allocation strategy of, e.g., an equal-weighted portfolio.

The time-varying nature of the factors is of large interest to the portfolio manager given the transition from an economy of high growth to a high inflationary and relatively high interest rates environment. If one can forecast which factors do well in the new economic environment, it can enhance returns and alpha. This is the core aim of this thesis, namely, to examine if a portfolio manager can benefit from varying his weights in U.S. equity factors over time and if so, how he should do it. In doing so, the thesis takes a starting point in Markowitz (1952), while extending the framework with separate time-varying volatility estimates and transaction costs to make the results more realistic and applicable.

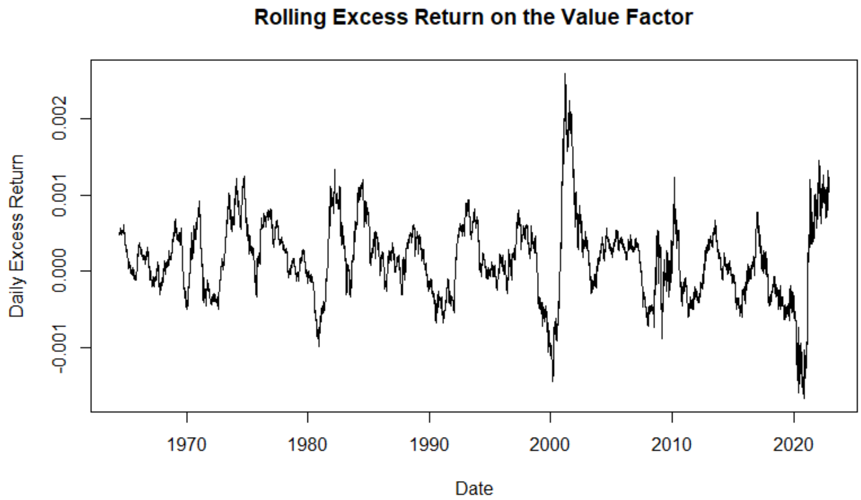


**Figure 1.1:** *Rolling daily excess return of the Fama & French (1993) value factor based on a 252-day window, from Jul. 1st, 1963, to Nov. 30th, 2022. Data: Daily per French (2023g). Illustration: The author*

## 1.1 Problem Statement

Based on the themes mentioned in the introduction, the thesis sets out to answer the following research question:

***"Can a factor-timing strategy increase the return/risk ratio compared to an equal-weight portfolio, and, if so, what is the optimal asset allocation strategy of factor investing in the U.S. stock market across time and stages of the business cycle?"***

To examine the overarching research question, the thesis develops the following sub-research questions:

RQ1: Is it possible for an investor to achieve superior performance, measured by risk-adjusted return, by timing factors compared to an equal-weight strategy?

RQ2: Given RQ1, is there a link between factor performance and the stages of the business cycle, and if so, what is the optimal allocation of U.S. equity factors across the business cycle?

RQ3: Given RQ1, how does the optimal asset allocation depend on transaction costs and can modelling of time-varying volatility improve performance of a factor-timed portfolio?

The basis for the research question relies on the empirically highly volatile nature of the factor premia seen in Figure 1.1. Similarly, Novy-Marx & Velikov (2016) show that the dynamic performance of factors changes substantially when incorporating transaction costs. Finally, as the mean-variance approach determines weights based on the variance-covariance matrix and means forecasting, the investor needs a model to forecast these. Rather than utilizing the standard approach of backwards-looking estimates, the thesis examines if GARCH models of volatility can improve the forecast and hence decision of how to allocate the factor weights (Linton, 2019).

### 1.2 Delimitations

The thesis delimits itself to examine factor-timing on equity factors in the U.S. stock market. Therefore, the thesis does not consider other assets or geographies, which means that the results are not generalizable outside the U.S. and equity. The thesis considers second-hand data on the factors, utilizing the factor returns of Fama & French (2015), Jegadeesh & Titman (1993), Frazzini & Pedersen (2014) and Asness et al. (2019). This means that the thesis does not consider the single stocks which make up the factors and, consequently, assumes that the investor can invest directly in the long-short portfolios. Further, the thesis limits itself to the data availability of returns. Appendix 8.3 shows that the data of Fama & French (2015), along with the COMPUSTAT data, start in Jul. 1963, whereas the AQR factors are only updated to and including Nov. 2022. Thus, the thesis limits itself to this period.

Although the study models the optimal time-varying portfolio, it does so by a series of static Markowitz (1952) portfolio allocations. Although a vast literature on dynamic portfolio allocation exists, it is not the aim of the thesis to develop, and use, said models. The reason is that doing so properly, while including transaction costs and time-varying volatility, requires a very large model, likely far from what is used in practice, and which is not achievable given the time constraints of the thesis. The appropriateness of the mean-variance model is further expanded in section 5.4. The goal of the thesis is not to develop bulls-eye estimates of the optimal factor-timing portfolio but to provide introductory insights that can uncover whether a factor-timing strategy is worth pursuing and, if so, where future research should focus.

### 1.3 Thesis Outline

The thesis is structured as follows. Chapter 2 accounts for the theoretical underpinnings of the thesis, here among modern portfolio theory, asset pricing and factor models. Further, the section explains the statistics and reasoning behind the GARCH models, both univariate and multivariate as well as outlines transaction cost theory.

Chapter 3 outlines the methodology. In particular, the chapter focuses on data collection and limitations therein, the technicalities in the portfolio- and factor construction, as well as the details behind the GARCH implementation. Another goal of the thesis is to establish the impact of methodological considerations on the results, which is especially relevant due to the concern for data mining in the asset pricing literature (Jensen et al., 2021). Therefore, the thesis employs both a rolling and expanding window for the mean-variance estimates, as well as various methods for accounting for economic cycles.

Chapter 4 constitutes the analysis. Section 4.1 presents the summary statistics and overall structure of the factor data, as well as examines the performance and correlation of the factors in different economic stages. Section 4.2.1 conducts predictive regressions of the return on economic variables by investigating whether the return of factors can be explained by the underlying economy. Further, section 4.2.2 performs an econometric assessment of the OLS method employed to suggest appropriate adjustments. Section 4.3 encompasses the main focal point of the research question, studying mean-variance optimization from various perspectives, both considering a naive full-period portfolio of factors, direct investment into the factor constituent portfolios, as well as factor-tilt portfolios. In the latter, the investor, in addition to a full investment in the market, takes time-varying factor bets. Section 4.4 includes transaction costs in the mean-variance estimation, while section 4.5 investigates how well the investor can capture said premia by investing in ETFs. Section 4.6 examines if a multivariate GARCH-model can improve the mean-variance estimates, and hence factor-timing. Chapter 5 discusses the results, linking them to previous research, while Chapter 6 concludes. The code for the analysis and results of the thesis can be found on the following GitHub link: `https://github.com/jskriverm/The-Optimal-Factor-Timing-Portfolio---Master-Thesis`, and references for the code is outlined in Appendix 8.12. The code is also attached as an additional document to this thesis. In the mathematical notation throughout the thesis, **bold** denotes matrices and vectors and an apostrophe, ('), denotes the transposed matrix or vector.

# 2 Theory and Literature Review

This section presents the theoretical foundation of the thesis.

## 2.1 Asset Allocation

Asset allocation describes how the investor should allocate his wealth across different assets and can be separated into three groups, strategic, tactical and dynamic asset allocation. Strategic asset allocation (SAA) establishes the optimal long-term allocation of the portfolio, based on a horizon of 10 years or longer and is often considered in a one-period model (Lumholdt, 2018). The classical portfolio of 60 % stocks and 40 % bonds is an example of an SAA-benchmark established with some bands that the weights can move within. SAA is, theoretically, based on forward-looking estimates of the return in the long horizon, which for equity originates from outlooks for the economy, dividend yield and price-to-earnings, but in reality, the estimation is often based on a long-term historical window assuming mean-reversion, as it is notoriously difficult to forecast returns properly (Lumholdt, 2018). Once the return in the coming period is determined, Markowitz (1952) is often used, despite its drawbacks, to determine the SAA-weights. This also means that the model, for optimal results, needs as many as 3,000 observations (Lumholdt, 2018).

Tactical asset allocation (TAA) is concerned with the short-term adjustment to changes in the economy and determines the deviations from the SAA-weights. The argument for TAA is that the economy goes through business cycles, where asset classes perform differently across the cycle stages (Lumholdt, 2018). During contractions, government bonds and high-quality credit outperform, while the investor should overweight high-yield bonds and stocks during recoveries and expansions (de Longis & Ellis, 2022). The weight adjustments for TAA are often estimated more loosely, for example, based on analyst estimates (Lumholdt, 2018).

Dynamic asset allocation (DAA) does not assume that the returns, variances and risk-free rates are constant over time, but determines the dynamic portfolio that maximizes the utility the investor derives from consumption. The simplest model is Merton (1971) considering a fixed time horizon, a constant risk-free rate, a concave utility function and no transaction costs (Munk, 2021). DAA can be extended to a hoard of realistic models including stochastic interest rates, stochastic returns and explicit consideration of transaction costs (Munk, 2017). Therefore, the dynamic models are more inclusive and realistic than the one-period SAA-models, but also require more complex modelling.

## 2.2 Markowitz and Modern Portfolio Theory

The modern portfolio theory takes its starting point in Markowitz (1952). Markowitz (1952) assumes that the investor wishes to optimize equation 2.1, where $\mathbf{\Sigma}$ denotes the variance-covariance matrix of returns for the assets considered, $\mathbf{i}$ is a vector of 1s, $\boldsymbol{\mu}$ is the vector of expected returns for the assets and $m$ is the return the investor wishes to have. Throughout, the thesis follows the notation of Linton (2019).

$$\min \mathbf{w}'\mathbf{\Sigma}\mathbf{w} \quad st.\mathbf{w}'\mathbf{i} = \mathbf{1} \wedge \boldsymbol{w}'\boldsymbol{\mu} = m \tag{2.1}$$

The theory shows that the investor should, depending on his risk aversion, invest along a mean-variance efficient frontier. Every time an asset is added, the frontier shifts to the northeast, as the worst that can happen is not investing at all in the new asset. If no risk-free asset exists, the investor optimizes equation 2.1, to which Appendix 8.1 shows equation 2.2 is the solution.

$$w(m) = \frac{\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}m - \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\mathbf{i}}{AC - B^2}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \frac{\boldsymbol{i}'\boldsymbol{\Sigma}^{-1}\boldsymbol{i} - \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{i}m}{AC - B^2}\boldsymbol{\Sigma}^{-1}\boldsymbol{i} \tag{2.2}$$

***Tangency Portfolio***

If a risk-free asset exists, the investor also derives a return from the risk-free asset and can invest in either that or the risky assets. Therefore, the investor optimizes equation 2.3:

$$\min_{\mathbf{w}} \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \text{ s.t. } r_f + \mathbf{w}(\boldsymbol{\mu} - r_f\mathbf{i}) = m \tag{2.3}$$

Appendix 8.1 shows that the optimal investment strategy is to invest according to equation 2.4:

$$\mathbf{w} = \frac{(\boldsymbol{\mu} - r_f)'\boldsymbol{\Sigma}^{-1}}{(\boldsymbol{\mu} - r_f)'\boldsymbol{\Sigma}^{-1}\mathbf{i}} \tag{2.4}$$

Thereafter, the investor chooses his investment in the tangency portfolio and the risk-free rate, respectively, based on his risk aversion.

***Minimum Variance Portfolio***

A final mean-variance portfolio is the minimum variance portfolio. Here, the investor does not care about the return but minimizes the variance subject to being fully invested, solving equation 2.5:

$$\min_{\mathbf{w}} \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \text{ subject to } \mathbf{w}'\mathbf{i} = 1 \tag{2.5}$$

Appendix 8.1 shows that the solution is equation 2.6, which solely depends on the variance-covariance matrix:

$$\frac{\mathbf{i}'\boldsymbol{\Sigma}^{-1}}{\mathbf{i}'\boldsymbol{\Sigma}^{-1}\mathbf{i}} = \mathbf{w} \tag{2.6}$$

***Critique of the Mean-Variance Approach***

While the mean-variance framework is intuitive and easily implementable, as it has a closed-form solution, it rests on unrealistic assumptions when employed dynamically (Munk, 2021). These are as follows:

1. The framework assumes that the investor gets utility from one point in time only, wherefore the assumption is that the expected return and variance are constant. In reality, the utility of the investors is dynamic and linked to consumption, wherefore the static assumption is a poor fit.

2. Even if the investor only considers the terminal wealth, he likely wishes to rebalance his portfolio over time, and, where transaction costs occur, these should be modelled explicitly.

3. The framework assumes that one can forecast the expected return and expected variance-covariance matrix with certainty. Although mathematics is clear on how to do the optimization, forecasting returns and their volatility is close to impossible.

## 2.3 The Efficient Market Hypothesis

The Markowitz (1952) portfolio assumes that markets are efficient per Fama (1970). The idea is that investors cannot earn superior returns in the market by having various degrees of information. In general, three forms of the efficient market hypothesis (EMH) exist.

*Weak form EMH:* The weak form EMH says that all information from previous prices is incorporated in current prices. Thus, the stock price should follow the random walk presented in equation 2.7:

$$P_t = P_{t-1} + \epsilon_t \tag{2.7}$$

This means that no autocorrelation exists in the return. Therefore, if returns can be described by an autoregressive (AR) model in equation 2.8, and $\phi$ is significant, the weak EMH is rejected (Linton, 2019):

$$r_t = \phi r_{t-1} + \epsilon_t \tag{2.8}$$

*Semi-strong EMH:* The semi-strong EMH says that all public information is included in stock prices. This means that the investor cannot earn superior returns based on public information such as accounting measures and that predictive regressions in the form of equation 2.9, where $\mathbf{x}$ is a vector of lagged variables, should have $\alpha_i = 0$ and non-significant betas (Linton, 2019).

$$r_{t+1} = \alpha_i + \boldsymbol{\beta_i}\mathbf{x}_t + \epsilon_{t+1} \tag{2.9}$$

As $\mathbf{x}$ may consist of macroeconomic variables, the semi-strong EMH also states that it is impossible to predict returns given the business cycle. The predictive regressions may be estimated via OLS, but an issue occurs if $x_t$ itself is a persistent stationary process of the form in equation 2.10:

$$x_{t+1} = \rho x_t + \eta_{t+1} \tag{2.10}$$

In the case that $\epsilon_{t+1}$ and $\eta_{t+1}$ are contemporaneously correlated, the regressors are not strictly endogenous, meaning the estimator is biased, which is called the Stambaugh bias (Linton, 2019).

*Strong EMH:* The strong EMH predicts that any information, be it publicly or privately, is already incorporated in the stock price. This form of the EMH is trickier to test as it involves testing whether private information can yield superior returns. Using this private information for trading is illegal as it constitutes insider trading.

## 2.4 Factor Models

### 2.4.1 Arbitrage Pricing Theory and Pervasive Factors

Ross (1976) introduced the Arbitrage Pricing Theory (APT), which stipulates that asset returns are a function of their common factors (Linton, 2019). This theory underlies the factor literature, and tells why certain risks should carry a return. The APT assumes that the factors follow a linear factor model. In the case of a risk-free asset, and with the $\alpha_f = 0$ assumption, the factors are traded portfolios, where the APT argues that the excess return of each asset can be written in equation 2.11, where $b_{ik}$ is the loading of asset $i$ on factor $k$.

$$r_i = \alpha_i + \sum_{k=1}^{K} b_{ik} f_k + \epsilon_i \tag{2.11}$$

This reduces the dimensionality in the portfolio problems, as all assets are allocated to a common factor, which ensures a more stable variance-covariance matrix, since $T >> N$ (Munk, 2021). A key assumption of the model is that all factors are pervasive, i.e., that the risk factor is important in the returns of many assets. If the factors are not pervasive, the price modelling will not work, and the variance-covariance matrix is 0 in the limit due to all risks being idiosyncratic (Linton, 2019).

### 2.4.2 Capital Asset Pricing Model

Based on the theory of EMH and the mean-variance efficient portfolio, Sharpe (1964) and Lintner (1965) define the Capital Asset Pricing Model (CAPM), which models the return of an asset as a function of the asset's risk exposure. Assuming that the market portfolio is the tangency portfolio in equilibrium, the return of all assets is a function of their covariance with the market portfolio, which is the key insight in the CAPM of equation 2.12:

$$E(r_i - r_f) = \frac{\mathrm{Cov}(r_i, r_m)}{\mathrm{Var}(r_m)} E(r_m - r_f) = \beta E(r_m - r_f) \tag{2.12}$$

### 2.4.3 Fama French Three Factor Model

Due to the CAPM not holding empirically, Fama & French (1993) develop the three-factor model, where the return is a function of a stock's market-, size- and value risk, which are defined as the security's loading to the size and value premia in section 3.3.1. The model assumes that size and value are pervasive factors such that the return can be expressed as equation 2.13:

$$E(r_i - r_f) = \alpha_i + \beta_{\mathrm{mkt}} E(r_m - r_f) + \beta_{\mathrm{i,size}} r_{\mathrm{size}} + \beta_{\mathrm{i,value}} r_{\mathrm{value}} \tag{2.13}$$

The reasoning behind the size and value premia is heavily debated. The original Fama & French (1992) paper, argues is that the value premium expresses the excess risk of the high book-to-market (B/M) portfolios. The researchers find that leverage is positively correlated with B/M. On the contrary, Lakonnishok et al. (1994) find that investors are extrapolating past earnings too far into the future, meaning that growth firms suffer from a too high valuation and that the value stocks are sold too cheaply. Kahneman & Tversky (1979) argue this is due to the human brain putting too much weight on recent developments rather than the historical average. Therefore, they argue that the premium is due to inefficiencies in the market rather than compensation for risk.

### 2.4.4 Momentum Factor

As an additional factor, Jegadeesh & Titman (1993) find that by buying the stocks rallying the most in the past 12 months, and selling past losers, the investor achieves a premium due to market inefficiency, arising from behavioral effects of underreaction to short-term information about the firm. Statistically, the investor benefits from autocorrelation in single stock returns as well as cross-autocorrelation between different stocks.

### 2.4.5 Fama French Five Factor Model

Novy-Marx (2013) expresses a critique that the Fama & French (1993) model does not capture the cross-section of returns when considering the difference in profitability among companies. Concretely, he argues that gross profitability provides an excellent hedge for value, as it involves buying productive assets and selling unproductive ones instead of buying cheap assets and selling expensive assets (Novy-Marx, 2013). Similarly, Titman et al. (2004) show that firms with high abnormal capital investments underperform firms with low abnormal capital investments due to overinvestment. As a result, Fama & French (2015) develop a five-factor model in equation 2.14, including RMW, the earnings factor, and CMA, capturing the overinvestment factor:

$$r_{it} - r_{ft} = \alpha_i + \beta_{i,mkt}(r_{mt} - r_{ft}) + \beta_{i,size}\mathrm{SMB}_t + \beta_{i,value}\mathrm{HML}_t + \beta_{i,earn}\mathrm{RMW}_t + \beta_{i,inv}\mathrm{CMA}_t + \epsilon_{it} \quad (2.14)$$

### 2.4.6 Additional Factors

In addition to the Fama & French (2015) factors, the finance literature has developed a hoard of factors to explain the anomalies in the cross-section of equity returns. This paper considers two of these, namely betting against beta (BAB) and the quality minus junk factor (quality). Asness et al. (2019) examine if investors pay more for quality firms than junk firms and define quality by expanding the Gordon growth formula to equation 2.15. The authors find that the quality return loads negatively on value and size, just as it fares well in market downturns.

$$\frac{P}{B} = \text{profitability} \cdot \text{payout} - \text{growth} \tag{2.15}$$

Frazzini & Pedersen (2014) creates the BAB factor as the linear relation between beta and return, the security market line, empirically is much flatter than the CAPM stipulates. The BAB factor is constructed as the return of low beta stocks less the return of high beta stocks, where both portfolios are levered to have the same beta by borrowing at the risk-free rate. The authors find that the BAB factor returns load negatively on the market return. Further, the factor is negatively related to the TED spread, hence, when the credit risk increases, the return of the BAB factor decreases. Section 3.3.1 outlines the construction of the factors.

## 2.5 GARCH Models

One major drawback of the mean-variance framework is the assumption that volatility is constant through time, just as the models often estimate volatility via backward estimation. Empirically, volatility is far from constant over time, and neither is it unrelated to stock returns. Schwert (2011) examines the volatility of U.S. stock returns from the late 1800s to 2009 and finds that, while the mean-monthly volatility level is stable at around 4%, volatility varies across time, spiking during recessions (Schwert, 2011). The author also finds that volatility is fat-tailed and presents in clusters. Hence, an optimal model for volatility is both time-varying and linked to the equity return in clusters with a fat-tail distribution. This is exactly the idea behind the General Autoregressive Conditional Heteroskedasticity (GARCH) model (Bollerslev, 1986). The model suggests that the volatility today is a function of the volatility yesterday and yesterday's squared return according to equation 2.16:

$$\sigma_t^2 = \omega + \sum_{k=1}^{p} \beta_k \sigma_{t-k}^2 + \sum_{j=1}^{q} \gamma u_{t-j}^2 \tag{2.16}$$

Often, the return is estimated via an autoregressive model, which indicates that an AR(2)-GARCH(1,1) model is described by equation 2.17 (Linton, 2019):

$$\text{AR}(2) - \text{GARCH}(1,1): \; y_t = c + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \epsilon_t \sigma_t, \; \sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \gamma u_{t-1}^2 \tag{2.17}$$

One problem with the standard GARCH is that it assumes symmetry of the news impact curve. The news impact curve illustrates how $\sigma_t^2$ is correlated with $y_{t-1}$, holding past levels of $\sigma_{t-1}^2$ constant (Linton, 2019). Empirically, there is evidence of a negative correlation between $y_t^2$ and $y_{t-j}$ and a positive correlation between $y_t^2$ and $y_{t+j}$, meaning that negative returns impact the volatility more than positive returns. As a result, the GJR-GARCH model takes an indicator variable for when past returns are negative following equation 2.18, which is also the model employed by the thesis (Glosten et al., 1993):

$$\sigma_t^2 = \omega + \beta_1 \sigma_{t-1}^2 + \gamma y_{t-1}^2 + \delta y_{t-1}^2 \mathbb{I}(y_{t-1} < 0) \tag{2.18}$$

***Multivariate GARCH***

In portfolio optimization, the interest is not only in the volatility of one asset over time. Rather, the investor is interested in the correlation and volatility of several assets. Therefore, the GARCH model used for portfolio selection must estimate the entire variance-covariance matrix. Consequently, the thesis considers a multivariate GARCH. Without estimating any restrictions on the matrix, the number of estimations is extremely large. Therefore, in practice, one needs to assume a specific form of multivariate GARCH (Linton, 2019).

One such model is the dynamic conditional correlation (DCC) model, which utilizes an explicit formula for the correlation between the assets. The DCC model by Sheppard & Engle (2001) assumes that the time-varying covariance matrix can be written following equation 2.19, where the standard deviation for each asset and point in time, $\sigma_{it}$, is estimated as a standard univariate GARCH. Each element of the matrix $R_t$ in equation 2.19 is then assumed to have the element following equation 2.20, where the correlation is calculated by standardizing the returns by their fitted value. The value of $q_{ij,t}$ for each asset at each time is estimated following the regression in equation 2.21 (Linton, 2019).

$$\Sigma_t = D_t R_t D_t, \quad D_t = \text{diag}(\sigma_t) \tag{2.19}$$

$$r_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}} \tag{2.20}$$

$$q_{ij,t} = c_{ij} + b_{ij} q_{ij,t-1} + a_{ij} \epsilon_{i,t-1} \epsilon_{j,t-1} \tag{2.21}$$

The issue with the DCC model is that it becomes computationally very intensive when the number of assets is large. Consequently, GARCH models emerged based on principal component analysis (PCA) being feasible for large covariance matrices (van der Weide, 2002). The Orthogonalized GARCH (O-GARCH) assumes that the asset returns can be transformed into a matrix of orthogonal vectors that can be seen as components driving the economy. The orthogonality, however, is restrictive, thus giving rise to the Generalized Orthogonal GARCH model (GO-GARCH), where the matrix linking the returns to the underlying economy can be any invertible matrix (van der Weide, 2002). The model assumes that an observed process $x_t$, in our case returns, can be described as a linear combination of components, $y_t$, which are then linked to the observed series via matrix $\mathbf{Z}$. $\mathbf{Z}$ is assumed constant over time and links the unobserved, uncorrelated, components with the observed variables. The GO-GARCH(1,1) model is the process of equation 2.22, where each component of the diagonal $\mathbf{H_t}$ matrix are themselves a GARCH(1,1) process as shown in equation 2.23:

$$x_t = \mathbf{Z}y_t, \qquad y_t \sim N(0, \mathbf{H_t}) \tag{2.22}$$

$$h_{i,t} = (1 + \alpha_i - \beta_i) + \alpha_i y_{i,t-1}^2 + \beta h_{i,t-1}, \quad i = 1, \ldots, m \tag{2.23}$$

Finally, this means that the conditional variance-covariance matrices that are used for the mean-variance optimization can be written as $\mathbf{Z H_t Z'}$ (van der Weide, 2002).

## 2.6 Transaction Costs

When trading assets, an investor must consider that the observed prices and theoretical returns cannot be achieved in practice. When trading, the investor incurs transaction costs which, grossly speaking, consist of the bid-offer-spread and strategic trading impact (Linton, 2019).

In reality, the prices are quoted by a dealer, who sets tradeable bid and ask prices different from the true theoretical price of the stock. The bid-ask spread arises as the market maker wishes to be compensated for providing liquidity, either as compensation to hold inventory or as a fee against adverse selection (Linton, 2019). Consequently, market participants can buy at slightly higher prices than the mid-price and sell at slightly lower prices. Novy-Marx & Velikov (2016) estimate the transaction costs. The authors assume a Roll (1984) model of bid-ask spreads, where the tradeable price, $P_t$, is the true price, $P_t^*$, plus half the bid-offer spread, $s$, in the trade direction, $Q_t$, as expressed by equation 2.24:

$$P_t = P_t^* + Q_t \frac{s}{2} \tag{2.24}$$

This means that the spread is a function of the autocovariance of the prices: $s = 2\sqrt{-Cov\left(X_t, X_{t-1}\right)}$. This, however, requires that the autocovariance of the first lag is negative, which is rarely the case empirically (Novy-Marx & Velikov, 2016). Instead, the authors configure the Roll (1984) model to include the market factor according to equation 2.25, and they estimate the spread via Bayesian statistics using a so-called Gibbs sampler.

$$\Delta P_t = c\Delta Q_t + \beta_m r_{mt} + \epsilon_t \tag{2.25}$$

The authors find that the transaction costs vary widely across the factors, being 5.66 bps and 5.45 bps for the size and value factors, respectively. Whereas size and value are turned over annually, the momentum factor is rebalanced monthly, which gives a time-average transaction cost of 48.4 bps per month.

Conversely, the majority of transaction costs stem from the strategic trading impact which measures how large the market impact of the investor's transaction is (Linton, 2019). A measure for such can be determined by the liquidity of the stock, for instance, Amihud (2002) liquidity measure, which considers the amount traded relative to the price movement in the stock.

# 3 Methodology and Data Collection

Chapter 3 presents the methodology of the thesis. Firstly, the research design is accounted for, and thereafter the data collection and -cleaning are outlined. The chapter ends by explaining factor, portfolio- and model constructions.

## 3.1 Research Design

The thesis takes a realist standpoint in its ontology, epistemology and axiology, as outlined by Saunders et al. (2012). The ontology is realist because the reality, in this case asset behavior and returns, is independent of thoughts and beliefs, which also means that the epistemology is realist as the observations, i.e., returns, provide credible data. As the thesis centers around different methods, it adopts a critical realist perspective, within which the methods influence reality. Throughout, the paper employs positive axiology, believing that the research is bias-free and solely basing its conclusions and studies on data rather than subjective beliefs. Thus, the analytical approach is highly data-driven, relying on statistics and large samples (Saunders et al., 2012). The research approach is deductive, collecting data to test hypotheses on a specific problem based on the generalist theories of portfolio allocation and volatility estimation.

## 3.2 Data Collection

The factor literature, generally, has two approaches to data collection. The first surrounds constructing the factors directly, so the dataset is panel data of individual securities and their financial attributes. This is the approach used by the original factor papers (Fama & French, 1993, 2015; Jegadeesh & Titman, 1993) as well as in studies examining the timing and allocation of factors (e.g. Ilmanen et al., 2021; Asness et al., 2013; Novy-Marx & Velikov, 2016). The methodology allows the researchers to estimate the factor premiums directly from the stocks, which, in the case of multiple factors, also sheds light on the optimal position in a certain stock at a given time. The approach, however, requires large data collection and -cleaning before one gets to the issue of timing the factors themselves. Further, there is a large risk of estimating a factor slightly differently than the established literature, which can have a large influence on the results.

Therefore, the other leg of factor-timing takes the factor premia as given based on the original factor studies. Such examples include Aghassi et al. (2022). Doing so does not allow for estimating the weights in individual securities, but it ensures alignment on the factor construction, as well as allows for faster examination of the factor-timing. As the focus of the thesis is not on the factor construction itself, but rather on the timing of said factors the thesis adopts the second approach. The discussion considers the impact of possible modifications to the factor construction framework.

### 3.2.1 Return Data and Transaction Costs

Table 8.3 in Appendix 8.3 presents the data sources utilized in the study along with the starting and end dates for the respective series. Ultimately, the common area of data availability decides the data period for which the analysis is performed. The main source of data is Kenneth French's database of the Fama & French (2015) daily returns, which includes return data on the size, value, earnings, investment and market factor from Jul. 1963 to and including Mar. 2023 at the time of writing (French, 2023f,h,d,c,m,a,j,l,k,g,i,b). All returns are in excess of the U.S. T-bill. Further, the study utilizes data from AQR on the BAB and quality factor (AQR, 2023a,d,b,c) whose availability is from Jul. 1$^{\text{st}}$, 1963, to November 30$^{\text{th}}$, 2022. This means that all mean-variance analyses are performed from Jul. 1$^{\text{st}}$, 1963, to November 30$^{\text{th}}$, 2022 using daily data.

For the study of transaction costs, the thesis utilizes the data of Novy-Marx & Velikov (2016), available from the website of Novy-Marx (2023) providing monthly estimates from Jul. 1963 to Dec. 2013. To not limit the transaction cost analysis to 2013, the thesis assumes that the estimates provided for Dec. 2013 stay constant till Nov. 2022. Section 3.3.4 outlines the full details of the transaction cost estimation.

### 3.2.2 Macroeconomic Data

To answer RQ2, the thesis needs data on the underlying economy. Welch & Goyal (2008) provide a comprehensive study on the factors that may predict the equity premium. The data is continuously updated on Goyal's website, with the current version holding data from the early 1900s to and including Dec. 2021, which is utilized for this thesis (Goyal, 2023). Further, the study follows the methodology of Ilmanen et al. (2021) and includes investor sentiment as constructed by Baker & Wurgler (2006). This is provided by Wurgler (2023) ranging from Jul. 1965 to Jun. 2022 (Wurgler, 2023). Concretely, the estimate is constructed as the first principal component of proxies to investor sentiment of previous studies, such as the NYSE share turnover, average of IPO first-day returns and the dividend premia (Baker & Wurgler, 2006). As BridgeWater (2012) hypothesizes that asset allocation is dependent on the inflationary environment, the thesis also includes the CPI-index from the Federal Reserve available monthly from Jan. 1947 to Jan. 2023 (FRED, 2023a).

To further answer RQ2, data on the business cycles is needed. Three methodologies are used in the literature, NBER-, OECD- and GDP classifications. The thesis uses all three classifications to establish which methodology has the highest predictive power on returns, and thus, which is the most appropriate for factor timing. The National Bureau of Economic Research (NBER) classifies recessions in the U.S., with the claim that a recession is *"a significant decline in economic activity that is spread across the economy and lasts more than a few months"* (NBER, 2023, para. 2). The exact process of the classification is not immediately clear, but the committee looks at the dept, diffusion and duration of the crisis to classify recession dates (NBER, 2023). From there, the thesis follows the method of Stanhope (2016) and classifies the 12 months before a recession as pre-recession, post-recession as the 12 months after a recession, and everything else as an expansion.

Alternatively, one may classify a recession as two consecutive periods of negative real GDP growth. Ilmanen et al. (2021) follow an adjusted approach, classifying the economic periods based on the direction of growth and the change in growth of the real U.S. GDP according to FRED (2023a). The authors only register a stage change when the growth rate and change in growth rates are above their 10-year rolling standard deviation, as indicated in Table 3.1, meaning that the economy stays longer in each stage of the business cycle.

| | Growth $> 0$ | Growth $< 0$ |
|---|---|---|
| Chg. Growth $> 0$ | Expansion | Recovery |
| Chg. Growth $< 0$ | Slowdown | Contraction |

**Table 3.1:** *Classification of business cycles by the GDP measure used by the thesis. Data as in FRED (2023d), methodology as in Ilmanen et al. (2021). Illustration: The author.*

Finally, the thesis considers the OECD composite indicators (CLIs). Concretely, the OECD indicator is constructed by collecting economic indicators hypothesized to either lead or lag GDP (OECD, 2023a). The indicator for the U.S. includes net new orders on durable goods, consumer confidence, weekly hours worked, manufacturing confidence, interest rate spread, NYSE share prices and work started for dwellings (OECD, 2022). The indicators are linked to the detrended GDP via the cross-covariance function, whereafter they are aggregated to create the OECD CLI. A score of 100 indicates that the economy is at its long-term growth trend. This also means that the cycles are equally spread around 100, leading, roughly, to an equal number of observations within each stage. The thesis follows the methodology of Stocks et al. (2022) and classifies the economic stages as per Table 3.2.

| | Level > 100 | Level < 100 |
|---|---|---|
| Chg. Level > 0 | Expansion | Recovery |
| Chg. Level < 0 | Slowdown | Downturn |

**Table 3.2:** *Classification of business cycles based on the OECD Composite Leading Indicators for the United States. Data as in OECD (2023a), method as in Stocks et al. (2022). Illustration: The author.*

As illustrated in Table 8.3 in Appendix 8.3, the economic analysis is capped by the sentiment data on the one end, being available from Jul. 1965, and the data of Goyal (2023) being available through Dec. 2021 only. Hence, the regression periods in the thesis are from Jul. 1965 to Dec. 2021, i.e., 2 years and 10 months shorter than the mean-variance optimization.

## 3.3 Applied Methodology

### 3.3.1 Factors

**3.3.1.1 Fama & French (2015) Five Factors** As the thesis centers around portfolio construction with factors, the construction of the factors themselves is of crucial importance. For the factors included in the five-factor model, the thesis follows the methodology of Fama & French (2015) constructing the factors as the average of the size-weighted factors. The factors are constructed using a double sort on the respective factor and size based on NYSE percentiles. The authors use independent sorts, which means that the number of stocks in each basket is not the same. Following the sorting in Table 3.3, the factor is constructed as in equation 3.1:

$$HML = Average\left(Small\ Value,\ Big\ Value\right) - Average\left(Small\ Growth,\ Big\ Growth\right) \tag{3.1}$$

| | Mkt. Cap > Median | Mkt. Cap < Median |
|---|---|---|
| B/M Percentile > 70 | Small Value | Big Value |
| B/M Percentile > 30 < 70 | Small Neutral | Big Neutral |
| B/M Percentile < 30 | Small Growth | Big Growth |

**Table 3.3:** *Bivariate, independent sorting methodology of the Fama & French (2015) portfolios. Method as in Fama & French (2015), data as in French (2023f). Illustration: The author.*

The return of each portfolio in Table 3.3 is calculated as the value-weighted return of the constituent stocks according to the market cap at the end of the previous year. Hence, the factor return is first a function of an independent sort across factor and size, then an aggregation of returns through value weights, and finally another average across sizes. Therefore, one may question whether the methodology is appropriate mathematically. The construction is concerning as an average of averages is never the population average, unless all sub-groups have the same number of observations, thus giving rise to Simpson's paradox. Simpson's paradox occurs where a trend may be present within groups, but when considering the whole dataset, the trend is not present, thus leading to conclude that a trend exists, where none, in fact, can be found (Sprenger & Weinberger, 2021). Nevertheless, as the thesis focuses on constructing the timing of said factors, and does not reconstruct the factors, this concern is ignored. The size factor itself is estimated as an average of all five factors of Fama & French (2015), hence utilizing an "aggregation and average" of the bivariate independent method. First, the size within each factor is constructed, such as equation 3.2 for value, whereafter the total size factor is constructed as an average of the individual "factor-size-factors" indicated by equation 3.3. Again there is concern for Simpson's paradox.

$$SMB_{Value} = \begin{array}{l} Average(Small\ Value,\ Small\ Neutral,\ Small\ Growth) - \\ Average(Big\ Value,\ Big\ Neutral,\ Big\ Growth) \end{array} \tag{3.2}$$

$$Size = \ Average\left(Size_{Value},\ Size_{Earnings},\ Size_{Investment}\right) \tag{3.3}$$

**3.3.1.2 Quality Factor** The quality factor follows Asness et al. (2019), where quality takes into account the profitability, growth and safety of the stocks. Appendix 8.2 presents the precise methodology and calculations. The authors use the methodology from Fama & French (1992) and construct variables in June of year $t$ based on the accounting measures reported in fiscal year $t-1$, hence using annual updating. The variables are each month converted into ranks on which a z-score is calculated following equation 3.4, where $x$ is the accounting measure of interest (e.g., ROE). The z-score of the entire profitability is then calculated as averages of the constituent z-scores, as illustrated in equation 3.5, whereafter the full quality score is again an average of averages of safety, growth and profitability, as illustrated by equation 3.6:

$$z\left(x\right) = \frac{\left(r - \mu_r\right)}{\sigma_r} \tag{3.4}$$

$$Profitability = Average\left(z_{GPOA},\ z_{ROE}, z_{ROA}, z_{CFOA}, z_{GMAR}, z_{ACC}\right) \tag{3.5}$$

$$Quality = \ Average\left(Profitability + Growth + Saftey\right) \tag{3.6}$$

The portfolio sorting of the quality factor follows Fama & French (1993, 2015) with the difference that Asness et al. (2019) use dependent sorts. Returns are value-weighted within each portfolio, and the quality measure is recalculated, and portfolios rebalanced, at the beginning of each month. The aforementioned methodology warrants discussion. Appendix 8.2 shows that almost all of the constituent variables of the quality factor are based on accounting data, which is updated annually following the Fama & French (1992) method. All else equal, this must mean that, when rebalancing the quality measure monthly, the only variables that change in 11/12 months must be the variables including market equity, i.e., ADJASSET, TLTA and WCTA as well as the Altman Z-score and Ohlson O-score. This means that 1) the number of stocks rebalanced each month must be relatively small and 2) the basis for rebalancing each month is tiny compared to the basis for doing so after the release of new annual data. Hence, in the interest of transaction costs and a cleaner method, one may suggest doing annual rebalancing instead. Again, due to the thesis focusing on established factors, it leaves such restructuring of the quality factor to future studies.

**3.3.1.3 BAB Factor** The construction of the BAB factor follows Frazzini & Pedersen (2014). Their data includes all stocks in CRSP, and beta is measured against the CRSP value-weighted market index. Betas are estimated for each stock following equation 3.7 before it is adjusted towards the cross-sectional mean to reduce the influence of outliers.

$$\beta_i^{ts} = \rho * \frac{\sigma_i}{\sigma_m} \tag{3.7}$$

The stocks are then sorted into either a "high beta" or "low beta" portfolio based on whether their beta is above or below the median, and the weights are scaled such that the lower (higher) the beta, the higher the weight in the low (high)-beta portfolio. This means that the return is calculated by 3.8:

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L}\left(r_{t+1}^L - r_f\right) - \frac{1}{\beta_t^H}\left(r_{t+1}^H - r_f\right) \tag{3.8}$$

Hence, the return is also here value-weighted. The value-weight is according to beta and not market value, as in Fama & French (2015), however, which means that the return is not excess return, as the portfolio is not dollar neutral. Therefore, to get the excess return, the thesis subtracts the risk-free rate from the BAB return.

**3.3.1.4 Momentum Factor** The momentum factor follows the Fama & French (2015) method of estimating the factor, which is slightly different from the original Jegadeesh & Titman (1993). Jegadeesh & Titman (1993) rank the portfolios by return in the lagged 2-12 months, sort the stocks in 10 deciles and construct the factor as the high return minus low return portfolio. French (2023i) uses the same approach as for the other factors using bivariate sorts on size and the 2-12 month lagged return with 30-40-30 breakpoints. The momentum return itself is calculated following equation 3.1, shifting "value" for "winners" and "growth" for "losers".

### 3.3.2 Regressions

The study employs regressions of the return on macroeconomic variables to answer RQ2 and examine the return drivers of the factors, as well as the linkage of the factor returns to the macroeconomic environments. The regressions take two forms, assessing both the predictive and contemporary nature of the indicators, following Ilmanen et al. (2021), presented by equations 3.9 and 3.10, respectively:

$$r_t = \alpha_t + \boldsymbol{\beta}_t' \mathbf{x}_{t-1} + \epsilon_t \tag{3.9}$$

$$r_t = \alpha_t + \boldsymbol{\beta}_t' \mathbf{x}_t + \epsilon_t \tag{3.10}$$

The contemporary regression is used for assessing RQ2, whether the current economic environment influences factor returns. The predictive equation also answers RQ1, whether the investor can predict returns. The study uses predictive regressions of both the factor returns themselves on the macroeconomic variables and the optimal mean-variance weights on said variables to assess the predictive power of the underlying economy from several viewpoints. Due to data availability, the regressions are based on monthly data from Jul. 1965 to Dec. 2021 and Aug. 1965 for the lagged regressions. The regression outcomes are also utilized in section 5.5 for discussing the optimal factor portfolio in the current economy.

### 3.3.3 Mean-Variance Optimization

The objective of all mean-variance models applied in the thesis, apart from the minimum variance portfolios, is to maximize the Sharpe ratio given full investment and the specified constraints for the problem considered. Hence, the general optimization problem to solve is specified in equation 3.11:

$$\max \left( \frac{\mathbf{w}'\boldsymbol{\mu}}{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}} \right) \text{ subject to } \mathbf{wi} = 1 \text{ and possibly other constraints} \tag{3.11}$$

The thesis takes a slight deviation from the common procedure of estimating the means and covariances. Normally, the means and covariances are based on full returns assuming a constant risk-free rate, as the Markowitz (1952) approach is static and assumed used for only one period (Linton, 2019). Here, the assumptions of constant returns and a constant risk-free rate are not upheld, and the variance is not invariant to drift, $R(W + c) \neq R(W) - c$. Therefore, estimating the covariances based on the full returns causes noise from the risk-free rate (Linton, 2019). Thus, the thesis estimates means and variances based on the excess return, such that $\boldsymbol{\mu}_{\text{estimated}_t} = \boldsymbol{\mu}_t - r_{f_t}$. This also means that the thesis finds the maximum Sharpe ratio on the excess returns, the maximum slope portfolio, to which equation 3.12 is the solution (Munk, 2021).

$$\pi(\bar{\boldsymbol{\mu}}) = \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}_{\boldsymbol{estimated}}}{\mathbf{i}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}_{\boldsymbol{estimated}}} \tag{3.12}$$

Substituting the expression for $\boldsymbol{\mu}_{\text{estimated}}$, exactly gives the formula for the tangency portfolio in equation 2.4, so maximizing the Sharpe ratio of the excess returns achieves exactly that. For this reason, the maximum slope portfolio of the excess returns is consistently referred to as the tangency portfolio throughout the thesis.

Throughout the analysis, the thesis compares different methods of doing the mean-variance optimization to establish which method provides the highest risk-adjusted return for the investor as well as which methods are practically implementable. In line with section 2.1 establishing that mean-variance optimization should be used over a very long horizon for SAA, the thesis first estimates the tangency portfolio over the full period, which is somehow cheating, as it assumes that the investor knows the returns in 2022 already in 1963. Therefore, the thesis also estimates a portfolio in the first 50 years and tests it out of sample in the remaining 9 years, equivalent to an investor standing in 2013 and using the last 50 years of data to determine the optimal weights going forward.

The thesis uses the same assumption of perfect foresight when estimating the tangency portfolio in each of the stages of the OECD business cycle, as the main purpose is to examine if there is a link between the stage of the business cycle and the optimal weights. However, as the business cycle data is released with one month lag, the thesis also considers a realistic implementation lagging the information about the economic stage one month. To further not assume prefect foresight of returns, the thesis examines the effect of TAA and perform small adjustments to an equal-weight portfolio depending on the business cycle.

To answer RQ1, the core of the thesis focuses on a time-varying factor portfolio established via the mean-variance framework. For each method, the thesis employs both an expanding and rolling estimation window with a minimum of 252 (one year at a daily interval) observations. The two methodologies are used as an expanding window allows for better estimation of the parameters when the parameters are constant (Bjerring et al., 2017). On the other hand, the estimates are not constant in practice, wherefore a rolling window provides the investor with more updated information, but with less certainty given fewer observations in the estimation. The moving mean-variance methodologies estimate the weights at the first trading day of the month, based on the backwards looking window being either expanding or rolling. All moving mean-variance portfolios hold the positions for one month, and then rebalance at the first trading day of the next month based on new estimates.

Contrary to earlier factor timing literature, the thesis focuses mainly on factor-tilt portfolios, where the investor, in addition to a 100% weight in the market, chooses positions in the factor portfolios. The reason is that viewing a portfolio of long-short factors becomes strange. For example, if the model suggests a weight of 50% in value and 50% in momentum, one can question if it means the investor should go long 50 cents in high-value stocks, short 50 cents worth of growth stocks, and the same with momentum. If that is the case, the total investment will be $2 * (50c - 50c) = 0$ USD. In this case, $x\%$ of the total investment will also always be 0. The investor can take infinite bets, his return is infinite, and all conventional risk-return measures are meaningless. In practice, one can argue that the portfolio of long-short factors is possible if considering positions in single stocks. As the same stocks are not included in all portfolios, the portfolio manager will trade a portfolio of the underlying stocks. Some of the factors increase the weight of a given stock, and some decrease it. As the thesis only considers factor portfolios, the intuition as to which trades shall be done is rather difficult to establish. Therefore, the paper focuses on the factor-tilt portfolios. Further, to avoid extreme weights, the absolute weights are capped at 100%, 200% and 500%, respectively.

As a further concrete implementable strategy, the thesis considers the investment into the underlying constituent portfolios of the factors, for example, both growth and value. As the data on the daily return on the constituent portfolios is not available for BAB and quality, the thesis only considers the Fama & French (2015) factors as well as momentum, wherefore it cannot be truly compared with the factor-market portfolio. The thesis compares the portfolios generated by the Fama & French (2015) bivariate 30-40-30 sorting with the univariate sorting classifying each "end" of a factor as the bottom and top decile of the univariate sorting. The goal is to study the impact of methodological considerations on results.

### 3.3.4 Transaction Costs

To account for the fact that the investor encounters transaction costs in the real world, the thesis includes transaction costs of constructing the factors, called "first-pass transaction costs", as well as rebalancing the portfolio of said factors called "second-pass transaction costs". The transaction costs are utilized on the factor-tilt portfolios as these are subject to the most elaborate analysis and constitute the most realistic investment approach.

The first pass transaction costs are estimated following Novy-Marx & Velikov (2016), as explained in section 2.6. As the data of Novy-Marx & Velikov (2016) ends in Dec. 2013, the thesis assumes that the transaction costs stay constant at the 2013 level until Nov. 2022. The assumption is not entirely correct, but as transaction costs decrease over time, it is deemed more appropriate than employing a time-series average (Novy-Marx & Velikov, 2016). It is beyond the scope of the thesis to estimate the transaction costs from the single stocks that comprise the factor returns. The thesis further makes assumptions regarding the transaction costs of the quality factor, as this is not part of Novy-Marx &

Velikov (2016). Khang et al. (forthcoming 2023) also use Novy-Marx & Velikov (2016) to estimate the transaction costs of the quality factor but do so more loosely without specifying the rationale and set the transaction costs of quality at a constant value of 10 bps. The thesis deems this measure inappropriate, as quality is rebalanced monthly, whereas value, with a cost of 5 bps, is rebalanced yearly. As Novy-Marx & Velikov (2016) demonstrate, frequent rebalancing heavily increases the transaction costs to at least the area of 50 bps per month. As quality is constructed as a combination of the measures used for value, profitability and investment, the thesis approximates the transaction costs for quality as the arithmetic average of the value, profitability and investment transaction costs, but where the costs are applied each month instead of each year, by back-filling the same cost 11 months before it is registered. Doing so, the average transaction cost for quality is 60.9 bps per month. On the other hand, as section 3.3.1 established that likely very few stocks are rebalanced each month in quality, it can be argued that the transaction costs are in reality lower. The thesis leaves such for further studies. The thesis uses daily data and assumes that the portfolio is rebalanced on the first day of the month, where it also incurs the full transaction cost.

The second-pass regressions quantify that a larger rebalancing is more expensive than a smaller one. This consideration is important as it may otherwise be beneficial to take massive factor bets with large swings each day to optimize return given the new, updated information set. The cost of rebalancing the portfolio is approximated assuming an ETF trades on each of the factors, and its transaction costs are estimated as the bid-offer spread, of this ETF. The average spread for the relevant ETFs, such as iShares Value Factor ETF, is around 20 bps (Yahoo Finance, 2023e). The 20 bps is scaled with the weight change. Equation 3.13 calculates the weight change (in percentage points), also considering the performance of the factor during the month-long holding period. This means that the equal-weighted portfolio is not free of transaction costs, as it must be scaled back to 1/N each month.

$$\Delta w_{1,2} = w_2 - (w_1 * (1 + r_1)) \tag{3.13}$$

Finally, the thesis includes a cost for shorting, again assuming ETFs trade on the factors. Li & Zhu (2022) estimate the lending fee associated with shorting ETFs to be 86 bps annually, meaning that for each month shorted, a lending fee of 7 bps is assumed paid for each 100% the factor is shorted. The fee is assumed paid at the beginning of the month for the coming month. In reality, the investor must construct the factors via short selling of the underlying stocks, to which the shorting costs can be much different from the 86 bps annually. This viewpoint is expanded in the discussion.

### 3.3.5 GO-GARCH Estimation

The thesis implements the GO-GARCH estimation of van der Weide (2002) and DCC-GARCH according to Sheppard & Engle (2001), as outlined in section 2.5. Both models use the GJR-GARCH estimation for the univariate volatilities. This means that unlike van der Weide (2002), the volatilities of the each component in equation 2.22 are estimated following 2.18 instead of equation 2.23.

The GARCH-models are all implemented using the "rugarch" and "rmgarch"-library in R based on the mathematical approach outlined in section 2.5 (Galanos, 2022b,a). The primary use of the model is fitting, and subsequently predicting, volatility, mean and correlations of the factor returns one period into the future. Therefore, unlike the other mean-variance optimizations, the thesis takes two approaches to forecasting the estimates. The first approach estimates the GARCH model on daily data with a backwards-looking window of 1256 trading days, meaning the first estimate is on Jul. 2nd, 1968. This ensures plenty of data for estimating the model, but it also requires that the model is refitted and, hence, that the portfolio is rebalanced daily, which is very expensive considering transaction costs. Consequently, the thesis considers an alternative GARCH model fitted on monthly data with a minimum estimate of 299 months. This makes the first prediction and, thus, mean-variance estimation, in July 1988. Consequently, the models are compared both on their full length and comparable length from Jul. 1st, 1988, to Dec. 1st, 2021. The monthly estimation saves transaction costs, but it has the drawback of being based on fewer observations. As the DCC-GARCH does not converge on daily data, only the GO-GARCH is utilized for the daily estimation. The model is estimated with a univariate AR(1) process for the filtration of each $h_{it}$, as opposed to the original paper (van der Weide, 2002), using constant filtration.

Throughout the analysis, the GARCH model is estimated based on gross factor returns, assuming the investor knows these and not the first-pass transaction costs. To account for transaction costs, the thesis, hence, just applies "second-pass" transaction costs for the GARCH model. The mean-variance optimization is done on the factor-tilt portfolios, estimating the weight at time $t$ on the GARCH forecast for time $t$. Further, to isolate the ability of GARCH to estimate the volatility and correlation, the performance of the GARCH model vs. the standard backwards-looking estimates is also applied on a minimum variance estimation following equation 2.6.

### 3.3.6 Active Manager Performance

A fundamental consideration of the overarching research question is how to measure performance and risk, hence, which risk-return measure the investor is interested in. Generally, there exist two branches of measures, one measuring the total return vs. the total risk and another measuring the performance against a return benchmark. The Sharpe ratio considers the excess return as a function of the excess volatility of an investment as seen in equation 3.14 (Sharpe, 1966). Hence, the measure is not a common candidate for factor investments, where the goal is performance against a benchmark, just as the measure, unrealistically, assumes normally distributed returns. The ratio, however, is good for evaluating the risk of single investments.

$$Sharpe\ Ratio_i = \frac{(\mu_i - r_f)}{\sigma_i} \tag{3.14}$$

As an alternative, the investor may consider the information ratio, which measures the excess alpha created relative to a benchmark over the excess risk relative to the benchmark, according to equation 3.15:

$$Information\ Ratio_i = \frac{\alpha_i}{\sigma_{\text{i,excess}}} \tag{3.15}$$

According to Grinold (1989), investment managers desire to have as high an information ratio as possible. Jacobs & Levy (1996) further establish that investors are "regret-averse", having their utility function negatively related to the excess risk relative to a position in the index, as they regret not taking the equal-weight index when things go haywire. In a sense, this is the same mechanism as ambiguity-averse investors not wishing to deviate too far from an optimal portfolio in the presence of unknown, stochastic returns, just as the ambiguity-aversion leads to less equity exposure than predicted by the mean-variance theory (Dimmock et al., 2016). With the presence of regret and ambiguity aversion, the investor measures the success of his investment by the information ratio.

As the information ratio is linked to a benchmark, and the Sharpe ratio to the full return and risk of an asset, the thesis assumes that strategies, where investors follow a benchmark, is evaluated relative to the information ratio, while investors without a benchmark prefer the Sharpe ratio.

## 3.4 Limitations

The main limitations pertain to the internal validity and generalizability of the results.

Regarding internal validity, a large premise for the thesis is that the methodology applied holds for the question at hand. Section 4.2.2 establishes, however, that the OLS used is not the most appropriate for the data structure. Further, the first-pass transaction costs not only rely on an external study, where the factors may be constructed slightly differently than the ones considered in this thesis, but the estimate is also crude for the quality factor and general transaction costs after 2013. This means that the results with transaction costs should be interpreted cautiously. The monthly GARCH models suffer from the same bias, as they are estimated on relatively little data due to data availability. This makes it difficult to extract the noise from missing data from real insights.

Similar limitations pertain to the use of the static mean-variance optimization for a dynamic problem. The assumption of constant returns and the investor only caring about utility at one final state is likely unrealistic, just as the monthly rebalancing of portfolios based on a rolling backwards-looking window of 252 days is not coherent with the long-term nature that the framework is originally developed for. On the other hand, as the thesis considers the development over time, the moving approach does provide insight and ensures that the mean-variance estimates are up to date.

Regarding the generalizability, the results are heavily dependent on the data period and scope, hence, cannot be generalized outside the U.S. equity later than Nov. 2022. Plenty of assumptions, as outlined in section in section 3.3.1, are implicitly made for each of the factor returns, for example, Fama & French (2015) uses independent sorts, while Asness et al. (2019) use dependent sorts, just as Asness et al. (2019) adjust their total assets in their accounting calculations as do Fama & French (2015) not. It is hypothesized that the influence is not small and that results would change with a different methodology for the factors. This is backed up by Asness et al. (2013), who change the Fama & French (2015) methodology slightly and estimate the HML Devil factor based on current instead of lagged market value, which materially alters the Fama & French (1993) results. The drawbacks of the methodology make it likely that the results are unique to this methodology and sample only.

# 4 Empirical Results

This Chapter outlines the results of the empirical analysis, analysing the research question and determining if factor-timing is worthwhile and if so, how it the timing should be implemented.

## 4.1 Summary Statistics

This section presents the performance of the factors. The section is based on daily returns. Further, as most of the thesis considers the factor-tilt portfolios, the section also presents the CAPM alphas and betas of the strategies. These are arguably, per section 3.3.6, the most relevant in measuring the active performance of the benchmark strategy.

4.1 presents the summary statistics for the factor returns. Excess returns and alphas are all different from 0 economically, and all but the size factor has excess returns statistically different from zero. This is the minimum requirement that a factor anomaly exists. Therefore, one may already question whether a size premium exists. However, as the size factor is well-established in the literature, the thesis continues assessing the factor. The largest anomaly is momentum with 2.8 bps per day, which is almost the same as the market. This is particularly impressive, as the market is a long-only portfolio, whereas the factors are long-short portfolios. At the other end of the spectrum lies size with a mean return of 0.7 bps per day, not statistically significant. The other factors have mean returns between 1.3 and 2.0 bps per day, but the standard deviation differs across the factors, with momentum having double the standard deviation of investment and quality. Therefore, there are early indications that some factors are riskier than others. The betas are all negative, meaning that the factors themselves have a negative exposure and provide a hedge against the market. Therefore, adding the factors to a positive market position, as done in section 4.3.2, seems worthwhile.

| | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB |
|---|---|---|---|---|---|---|---|---|
| Ex. Ret (%) | 0.027 | 0.007 | 0.015 | 0.014 | 0.014 | 0.028 | 0.018 | 0.020 |
| Std. Dev. (%) | 1.025 | 0.541 | 0.579 | 0.397 | 0.376 | 0.770 | 0.439 | 0.611 |
| Skewness | -0.531 | -0.708 | 0.301 | 0.346 | -0.343 | -1.311 | 0.244 | -0.527 |
| Kurtosis | 15.689 | 18.328 | 11.394 | 9.412 | 9.639 | 19.927 | 10.345 | 15.255 |
| Sharpe Ratio | 0.026 | 0.014 | 0.026 | 0.035 | 0.037 | 0.037 | 0.042 | 0.033 |
| CAPM Alpha (%) | 0.000 | 0.008 | 0.018 | 0.016 | 0.017 | 0.031 | 0.023 | 0.028 |
| CAPM Beta | 1.000 | -0.037 | -0.092 | -0.080 | -0.130 | -0.094 | -0.183 | -0.279 |
| T-stat | 3.220 | 1.694 | 3.238 | 4.247 | 4.496 | 4.513 | 5.093 | 4.006 |

**Table 4.1:** *Estimates of the moments of the daily factor excess returns considered in the data period spanning Jul. $1^{st}$, 1963, to Nov. $30^{th}$, 2022. Data: French (2023g,i) AQR (2023c,a). Illustration: The author.*

The third and fourth moments, skewness and kurtosis, show that the returns are a very far cry from normality. Momentum is the "riskiest" factor with the highest kurtosis and most negative skewness, indicating that the risk of a negative tail event is substantially higher for this factor than for, say, the closer-to-normally distributed earnings factor with a slight positive skewness. The skewness is also large and negative for size, market and BAB, indicating that the returns of these factors, in the tail, are more likely to be negative than positive. On the other hand, the skewness is positive for earnings, quality and value. It is too early to say, but this can indicate lower risk as well as these factors being attractive in periods of market turmoil. The fact that the skewness oscillates between positive and negative also indicates that there, at least theoretically, is a possibility to diversify the factors and mitigate downturns in one factor with upswings in another.

Figure 4.1 shows that the factors are good diversifiers for each other. For example, all the factors are negatively correlated with the market, as the negative beta also indicates. This is particularly the case for the investment, quality, and BAB factors. Another finding is that the returns of the investment-and value factor are positively correlated, i.e., value firms are also firms with low excess investments. On the other hand, momentum and value are negatively correlated speaking to the fact that combining value and momentum is a good strategy to enhance return while reducing risk. Quality is positively correlated with earnings, which means that high-earnings firms are quality firms and that both factors seem to be very good diversifiers of the market, i.e., work well in periods with low market returns. BAB is negatively correlated to the market, but positively correlated to earnings, investments, momentum, and quality, indicating that BAB may not be such a good diversifier when considering all other returns.

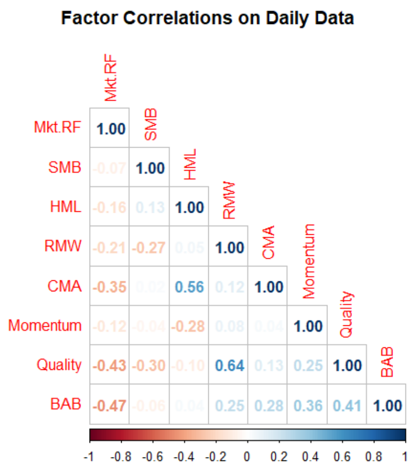


**Figure 4.1:** *Correlations of the factor excess returns estimated on the entire daily data sample period from Jul 1*$^{st}$*, 1963 to Nov. 30*$^{th}$*, 2022. Data: French (2023g,i) and AQR (2023a,c). Illustration: The author*

Figure 4.2 shows the correlation between size and value, value and momentum and quality and value over time using a 252-day rolling window. The figure illustrates that the correlations vary heavily, as was the case with the returns in Figure 1.1. Although value and momentum returns are overall negatively correlated, the correlation oscillates between +0.5 and -0.5. Hence, the correlation at a given point in time is likely to be either highly positive or highly negative, and if an investor based his weights on the long-term correlation of -0.28, he would be mispositioned most of the time. The other factor pairs exhibit similar movements, albeit not as strong. The correlation of size and value is relatively stable between 0 and 0.5, though going negative at certain periods. Compared with the quality-value correlation, the value-size correlation and value-quality correlations are vastly different at times around the 1970s as well as around the financial crisis and the Covid-19 crisis. This indicates that quality and size behave very differently to such crises, which is examined further in section 4.1.1. Figure 4.2 shows that, at least with perfect foresight, not only could it pay to diversify across factors, but a factor-timing strategy may also work and enhance alpha.

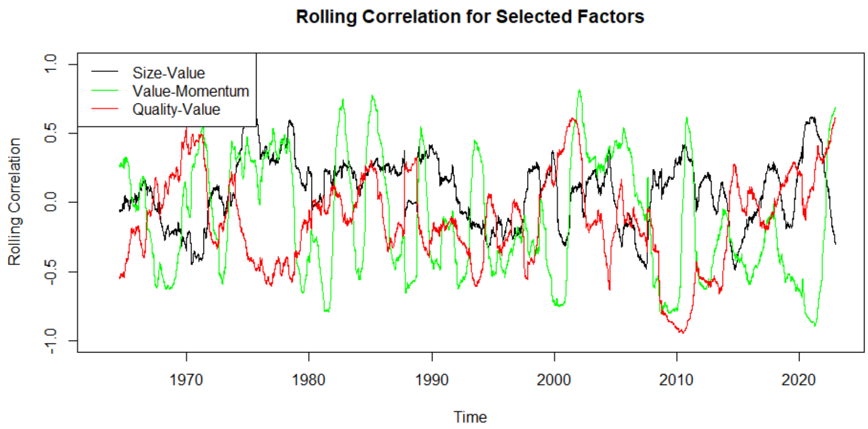


**Figure 4.2:** *Rolling 252-day correlation estimates beginning July 1*$^{st}$*, 1964, and ending November 30*$^{th}$*, 2022. Data: French (2023g,i) and AQR (2023a,c). Illustration: The author.*

### 4.1.1 Mean Returns of Factors across the Business Cycle Stages

Examining RQ1 and RQ2, the thesis wishes to establish if factors perform differently during different stages of the business cycles and whether the investor can benefit from adjusting his factor positions through the economic cycle. This section considers GDP, NBER and OECD classifications as specified in section 3.2.2.

Tables 4.2-4.4 consider the GDP-, NBER- and OECD measures, respectively. Surprisingly, the GDP measure shows that the market return during contractions is higher than during slowdowns. Reflecting on the construction of the indicator, the GDP measure changes, when the 10-year rolling standard deviation, on either growth or level, changes. This may indicate that recovery, as measured by GDP, contains an early part of a contraction and that the last part of contractions is an early part of a recovery, which creates the puzzling pattern due to spill-over effects. The lag of the GDP measure, hence, does not capture the true performance of the factors in the respective stages, especially not as there are indications that the largest movements happen at the beginning and end of a cycle. The suspicion is backed up by the NBER classification in Table 4.3, where the recession and pre-recession have equally low market returns, while the post-recession, i.e., recovery, is the one with the highest return. Similarly, the underperformance of the lagged OECD portfolio in section 4.3.1 tells that capturing the first month of an economic stage right is crucial.

Comparing Tables 4.2-4.4, further, shows that the number of periods in each stage of the cycle vastly differs across the classifications. The OECD classification in table 4.4 has roughly an equal number of periods in each stage, whereas the GDP measure has a very uneven distribution of periods with the far most being expansions and only a few observations falling in the recession bucket. This indicates that the OECD classification has higher statistical certainty than GDP, given that its classification is correct.

Turning towards the factors, the different methods also result in different conclusions, just as the factors themselves react very differently through the business cycle. Under the GDP measure, size does relatively well in all stages. Looking towards NBER and OECD, size is highly cyclical and does poorly in recessions/downturns and slowdowns/pre-recessions, whereas it thrives in recoveries and expansions.

Table 4.2, similarly, shows that the cyclicality of value is very different under the three classification schemes. While value performs well during all stages in the NBER classification, it does poorly during OECD downturns and best during OECD expansions. Hence, OECD indicates cyclicality, NBER the opposite and no clear link is evident from the GDP classification in Table 4.2. Therefore, the true cyclicality of value is difficult to assess, and it likely depends more on other factors than the pure business cycle. Section 4.2.1 studies this claim.

The earnings factor behaves rather differently. As classified by NBER and GDP, it has a positive return in all periods, though having lower returns during recoveries and expansions. The OECD classification expands the claim and shows that the earnings factor has a negative return in the booming recovery period. Intuitively, this makes sense. Firms with high earnings can be hypothesized to be more robust than firms with low earnings, wherefore the high-earnings firms must outperform their loss-making counterparts during economic hardships. However, in good economic times, all firms benefit, and having high earnings is not as large a necessity to woo investors.

Quality shows a similar anti-cyclicality across all three measures, having negative returns in the recovery phase and very high returns during slowdowns and recessions. In fact, the return during these periods is higher for quality than it is for earnings. These results mirror those of Asness et al. (2019), finding that the quality factor hedges against the market and is positively skewed in the distribution. The reason is, according to Asness et al. (2019), that quality benefits from a ”flight-to-quality”.

The investment factor, unlike the other factors, is a good all-round investment with no large, negative returns in any classifications. For the GDP measure, the investment factor looks anticyclical with the highest return during slowdowns and negative return during recoveries, which is also the case at NBER. On the other hand, the investment factor also does rather well in OECD expansions. It, thus, seems that investment depends on other factors than the economic cycle, as does value, which may explain the high correlation between value and investment as illustrated in Figure 4.1. Section 4.2.1

examines further the explanations for returns in value and investment.

Momentum has by far the largest absolute returns, yielding negative 4 bps per day during contractions in GDP and positive 3.3 and 3.4 bps during expansions and slowdowns. The same is the case for NBER, where it, during pre-recessions, returns a colossal 8.1 bps. The factor, however, does poorly in recoveries. This is contrary to Stocks et al. (2022), who find that momentum does well throughout all economic stages. Again, it is difficult to say something certain about the momentum cyclicality. Figure 4.1 suggests that momentum is relatively uncorrelated, and moves in different cycles, than the other factors. One may hypothesize that during slowdowns losers lose even more as the economy is entering poor economic times, and winners win. Therefore, momentum may be more related to the strength of the autocorrelation function more so than a general economic cyclicality. This is examined in section 4.3.4.

| EconStage | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB | Count |
|---|---|---|---|---|---|---|---|---|---|
| Contraction | 0.044 | 0.027 | -0.014 | 0.014 | 0.015 | -0.041 | 0.013 | 0.017 | 881 |
| Expansion | 0.029 | 0.004 | 0.010 | 0.011 | 0.010 | 0.034 | 0.019 | 0.018 | 11370 |
| Recovery | 0.131 | 0.049 | -0.009 | 0.010 | -0.001 | 0.009 | -0.028 | 0.025 | 574 |
| Slowdown | -0.020 | 0.005 | 0.062 | 0.031 | 0.038 | 0.033 | 0.029 | 0.028 | 2133 |

**Table 4.2:** *Daily excess returns in the percentage of the factors considered in different economic stages as classified by GDP following Ilmanen et al. (2021). Data: French (2023g,i) and AQR (2023a,c). Illustration: The author.*

| EconStage | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB | Count |
|---|---|---|---|---|---|---|---|---|---|
| Post-Rec. | 0.055 | 0.044 | 0.031 | 0.021 | 0.028 | -0.009 | -0.002 | 0.067 | 1772 |
| Pre-Rec. | -0.028 | -0.018 | 0.008 | 0.030 | 0.011 | 0.081 | 0.039 | -0.001 | 2013 |
| Pure-Exp. | 0.044 | 0.007 | 0.014 | 0.008 | 0.008 | 0.030 | 0.014 | 0.024 | 9391 |
| Recession | -0.027 | 0.001 | 0.017 | 0.017 | 0.036 | -0.005 | 0.037 | -0.025 | 1782 |

**Table 4.3:** *Daily excess returns in the percentage of the factors considered in different economic stages as classified by NBER following Stanhope (2016). Data: French (2023g,a) and AQR (2023a,c). Illustration: The author.*

| EconStage | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB | Count |
|---|---|---|---|---|---|---|---|---|---|
| Downturn | -0.016 | -0.016 | -0.009 | 0.017 | 0.012 | 0.025 | 0.036 | -0.034 | 3458 |
| Expansion | 0.052 | 0.024 | 0.031 | 0.015 | 0.021 | 0.021 | 0.008 | 0.057 | 4130 |
| Recovery | 0.082 | 0.039 | 0.018 | -0.005 | 0.009 | 0.007 | -0.015 | 0.042 | 2955 |
| Slowdown | 0.001 | -0.011 | 0.017 | 0.023 | 0.011 | 0.052 | 0.037 | 0.013 | 4415 |

**Table 4.4:** *Daily excess returns in the percentage of the factors considered in different economic stages as classified by OECD following Stocks et al. (2022).Data: French (2023g,i) and AQR (2023a,c). Illustration: The author.*

### 4.1.2 Correlation of Factors across the Business Cycle

In determining the optimal time-varying factor strategy, the correlations are just as important as the means. If correlations vary over time, time-varying weights are also a source of alpha. This section considers the correlation of the factor returns across the stages in the economic cycles via the three different classification methods, and Tables 8.2-8.10 in Appendix 8.4 present the correlations in recoveries, recessions and expansions for all measures, wherefore they should be seen in conjunction with this section.

Overall, the correlations are remarkably similar, comparing stages of the economy for all factors and methodologies. Few are more than 0.1 apart, and all are in the same direction across the three measures of the business cycle stage. Therefore, there is not much indication that the correlation changes with the methodology of the economic cycle, nor that it is very sensitive to which stage of the business cycle the economy is in. The results are surprising as Tables 4.2-4.4 suggest that returns vary within economic cycles and methodologies, just as Ilmanen et al. (2021) find that time-varying weights can add value, not due to time-varying means, but varying correlations. This study finds close to the polar opposite. One of the strongest differences in correlation across the business cycle stages pertains to the BAB factor. BAB is less strongly negatively (closer to 0) correlated with quality in contractions than expansions, whereas the earnings factor maintains the same positive correlation with BAB of around 0.23 in both expansions and contractions. This supports the story that BAB is cyclical, and quality is not, but that both factors become more similar in bad times, just as it highlights that despite the cyclicality being similar, earnings and quality are very different. BAB also, as the only factor, changes direction from being positively correlated with value during downturns to negatively correlated during recoveries when measured by OECD. Tables 8.9 and 8.10 also show that the correlation with momentum drops 0.15 from downturn to recovery. It is too early to speculate about the reasons, but it indicates that although the correlations are stable, there is something inherent in the BAB factor that makes its correlations more dependent on the stage of the cycle than the other factors.

***Sub-Conclusion on Summary Statistics and Cyclicality***

The analysis shows that both means and correlations change over time, wherefore a factor-timing strategy can, theoretically, add value. The excess returns vary much more between stages of the business cycle, than do the correlations. The factors perform differently in different business cycles, with BAB, the market and size being cyclical, earnings and quality anticyclical, while investment is a good all-round factor, whereas value and momentum does not have an obvious link to cyclicality, as they likely depend more on other factors.

## 4.2 Predictive Regressions

### 4.2.1 Regression Results

Section 4.1.1 established that varying factor weights to the stage of the business cycle is, theoretically, beneficial. To further analyze this claim, and thus answer RQ1, this section performs predictive regressions. Tables 8.11-8.13 in Appendix 8.5 present regressions of the factor returns on macroeconomic variables lagged one period, ensuring that the data is published and available for the investor before forecasting the return. Section 4.1.1 illustrated it is not trivial which classification is used, hence, Appendix 8.5 shows regressions for all three classifications.

Tables 8.11-8.13 illustrate that the regressions can only explain very little of the overall variance, with $R^2$ between 1% and 7%. Practically, this means that, although the regressions include many variables, the macroeconomy has little explanatory power. Consequently, Figure 8.21 in Appendix 8.11 presents that using rolling regressions to predict the return in the coming month results in very poor estimates of the actual returns. As Table 8.20 in Appendix 8.11 presents, the correlation between the predicted returns and actual returns for the factors is close to zero. Section 4.6, on the other hand, illustrates that using the regressions for predicting the future return in combination with GARCH-models for the volatility performs marginally better than utilizing a backwards-looking window. Therefore, the conclusion is that the macroeconomy can be used to predict returns, but to a limited extent. The analysis in Appendix 8.11 expands the view. The loadings must also be seen with caution, as there are likely many lurking variables that can influence the relationship uncovered.

Overall, the only variable consistently significant is investor sentiment. In all models, size is negatively related to lagged investor sentiment, while the returns of the earnings, quality and BAB factors are positively related to the sentiment index. This is in line with Ilmanen et al. (2021), finding that the defensive factor is positively related to the sentiment index, but the loadings of the sentiment index, somewhat contrasts the results of section 4.1.1, which indicates that size is cyclical, whereas quality and earnings are good in tough economic times. The thesis finds no apparent explanation for this, but the regression may simply be spurious given omitted variable bias, as the $R^2$ are very small.

Following Ilmanen et al. (2021), few other variables are significant. Comparing Tables 8.11-8.13 in Appendix 8.5, the differences in the classification methods are minor when looking at the macroeconomic variables, indicating no multicollinearity between the economic stages and the classifiers. The return of momentum loads significantly negatively on the default spread, positively on yield and inflation and negatively on the dividend/price ratio in all regressions, indicating that momentum does well in an inflationary environment with low credit risk, high rates and high dividend payouts. Such an environment is very close to the current U.S. economy as discussed in section 5.5. Also, value and investment load positively on inflation and yield, working as an inflation hedge. This also explains the lack of apparent cyclicality of value and investment. Rather than being dependent on the stage of the economy, they are influenced by the overall risk and rates market, which moves in different cycles than the economy itself. Finally, in all regressions, BAB loads positively on the term spread, but negatively on long-term yield, indicating that it benefits from a rising yield curve but requires that the long-term yield is low. These findings indicate that BAB does very well in ultra-low rates environments, which makes sense due to the embedded leverage of BAB.

The stages of the business cycle do not load consistently, in either direction or significance. Table 8.11 shows that in the OECD regression, the return of value is significantly cyclical, while the return of the quality factor loads significantly negatively on recovery indicating anti-cyclicality. Compared with the NBER classification in Table 8.12, value loads insignificantly negatively on expansion and positively on post-recession, indicating no cyclicality as such, whereas quality in the GDP-regression, in Table 8.13, loads positively on expansion and slowdown, revealing cyclical tendencies. Therefore, it is difficult to truly say something about the link of factor returns to the stages of the business cycle. The OECD and NBER regressions have the highest $R^2$ indicating they capture the variances the best, as was also the case in section 4.1.1 due to their larger number of observations. Therefore, these methods are utilized going forward.

Tables 8.14-8.15 in Appendix 8.5 consider how the return is linked to the current, instead of the lagged macroeconomic variables for NBER and OECD, respectively. The contemporary regressions examine RQ2, as they study the exact relationship between the returns and the current macroeconomy. Overall, the tables show the same results as their lagged counterparties, apart from the market variance being a significant determinant of the return. Here, the cyclical variables, size and BAB, have lower returns when the market volatility is higher, while investment and quality significantly benefit from high volatility, indicating that these variables can serve as a “safe haven” for the investor. Thus, the investor cannot use volatility to predict returns, but volatility is an important determinant for return. Therefore, the investor needs to forecast the volatility, which section 4.6 examines using GARCH estimation.

### 4.2.2 Assessment of OLS Assumptions

The thesis estimates the above regressions via OLS with standard errors adjusting for heteroskedasticity and autocorrelation, as is the common approach in the literature. This section studies the appropriateness of the method, as well as assess adjustments to OLS, which makes the results more applicable in the given setting. The section relies on Brooks (2008) and Linton (2019). All tests consider the contemporary OECD regression in table 8.15, but given that section 4.2.1 showed that the regression results are similar across methodologies, it is assumed that the insights hold for all regressions of returns on the macroeconomic variables illustrated in Tables 8.11-8.15.

For OLS to work, the estimator must be the Best Linear Unbiased Estimator (BLUE), which requires the following (Linton, 2019):

1. The errors satisfy $E\left(\epsilon_{t+1}|F_t\right) = 0$, where $F_t$ is the filtration process at time $t$. Hence, the conditional error must be 0.

2. The data-generating process must be linear.

3. No autocorrelation exists in the estimates.

Further, the conventional standard errors based on the t-distribution rely on the assumptions that:

4. The variance is finite, i.e., the data exhibits no heteroskedasticity.

5. The OLS-estimator is consistent. This is the case, asymptotically, when $E\left(\boldsymbol{\epsilon}'\boldsymbol{\epsilon}|X\right) = \sigma^2\mathbf{I}$, where $\mathbf{I}$ is the identity matrix and that $\lambda_{minimum}\left(\mathbf{X}'\mathbf{X}\right) \rightarrow \infty$, where $\lambda$ is the smallest eigenvalue of a symmetric matrix (Linton, 2019). A simpler version of this is to assume that the errors are normally distributed. However, this need not be the case as the Central Limit Theorem (CLT) can be evoked asymptotically.

1) is tricky to test given that the mean, by construction, is 0 when the regression includes an intercept, which it does here. One way to indirectly assess the issue is to test whether the residuals are on average 0 and linear. Addressing 2) and, by implication 1), the thesis plots the residuals vs. the fitted values and tests the significance of non-linearity via a Tukey test for additivity (Fox & Weisberg, 2019). The test adds a squared term of the coefficients to the regression and, subsequently, tests the probability that these squared terms are significantly different from 0. If they are, linearity is rejected. Table 4.5 shows the results of the Tukey test. The null hypothesis of linear returns is only rejected at the 5%-level of momentum and quality. For earnings, the hypothesis is rejected at the 10% level.

Figure 4.3 shows some degree of concavity in the relationship between the residuals and fitted values for momentum. Figure 8.1 in Appendix 8.6 shows that this arises from inflation and yields not being linear for momentum. Figure 4.3, further, brings the insight that the non-linearity is largely due to a few outliers. Therefore, one may argue that linearity can be improved by winsorizing the data to limit outliers. On the other hand, these are important returns for the investor, wherefore they are considered in the model. Figure 8.2 in Appendix 8.6 shows that value is much more an approximation to linearity because of the inflation and term spread being more linearly related to returns for this factor. As the linearity assumption is fulfilled for five out of eight factors, the thesis concludes that the assumption highly doubtful, questioning the core assumption of OLS.

| | Tukey T-stat | P-value |
|---|---|---|
| Size | 1.574 | 0.116 |
| Value | -1.421 | 0.155 |
| Earnings | 1.846 | 0.065 |
| Investment | -0.486 | 0.627 |
| Momentum | -4.920 | 0.000 |
| Quality | -3.023 | 0.002 |
| BAB | 0.288 | 0.773 |

**Table 4.5:** *Tukey-test of Fox & Weisberg (2019) for the contemporary OECD-regression of the factors considered. Data: Monthly from Jul. 1965 to Dec. 2021. Illustration: The author.*

| | Autocor. | P-value |
|---|---|---|
| Size | 0.052 | 0.092 |
| Value | 0.134 | 0.000 |
| Earnings | 0.131 | 0.000 |
| Investment | 0.093 | 0.004 |
| Momentum | 0.005 | 0.622 |
| Quality | 0.101 | 0.002 |
| BAB | 0.047 | 0.124 |

**Table 4.6:** *Durbin-Watson Test for autocorrelation in the residuals of the contemporary OECD-regression. Data: Monthly from Jul. 1965 to Dec. 2021. Illustration: The author.*

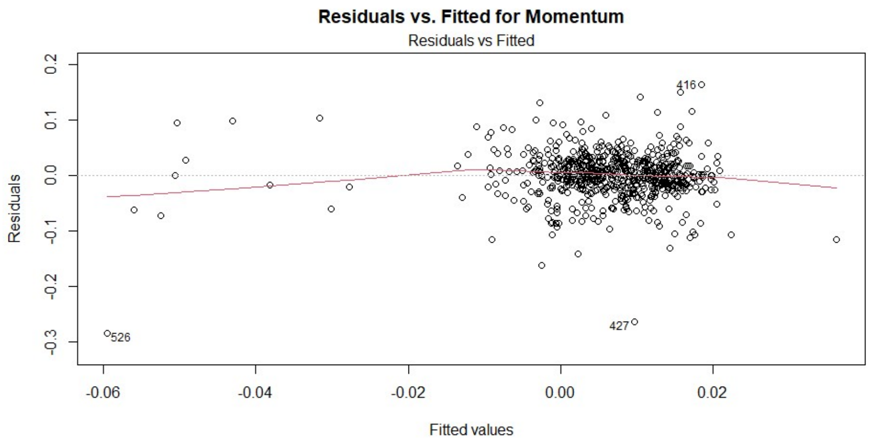


**Figure 4.3:** *Residuals vs. fitted values of the contemporary OECD-regression. Data: Monthly from Jul. 1965 to Dec. 2021. Illustration: The author.*

Testing 3) evolves around autocorrelation. If such exists, the variance grows over time, meaning $E(\boldsymbol{\epsilon}'\boldsymbol{\epsilon}|F) = \sigma\mathbf{I}$ is not fulfilled, whereby the estimator is not consistent (Linton, 2019). This is tested via the Durbin-Watson test (Brooks, 2008). Table 4.6 shows that the null hypothesis of no autocorrelation is rejected for the returns of the value, earnings, quality and investment factor at the 5% level. Only momentum cannot be rejected at conventional significance levels.

Figure 4.4 shows the errors vs. lagged errors for the fitted model on the value factor, where it is also clear that some degree of positive autocorrelation exists. Hence, $\mathbf{\Omega} = \sigma\mathbf{I}$ does not hold, i.e., the OLS estimator is biased in its estimates. An alternative would be Generalized Least Squares (GLS), which takes the assumption that $\mathbf{\Omega}$ can have covariance. However, due to its widespread use, the thesis focuses on OLS, but adjusts the standard errors for heteroskedasticity, assuming that $\mathbf{\Omega} = diag\,(\sigma_1, \ldots \sigma_n)$, in the Newey & West (1987) standard errors (Linton, 2019).

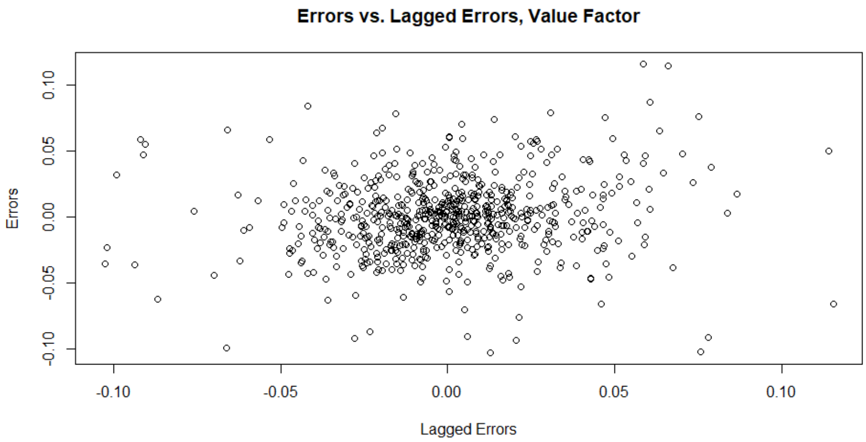


**Figure 4.4:** *Errors vs. lagged errors of the contemporary OECD-regression for the value factor. Data: Monthly from Jul. 1965 - Dec. 2021. Illustration: The author.*

Assumption 4) examines if there is finite variance in the errors. Table 4.7 shows that the null hypothesis of the Breusch-Pagan statistic, that the variance is finite, is rejected for all factors at the 1% level. Hence, the assumption of constant variance does not hold, wherefore the thesis adjusts the standard errors via Newey & West (1987) standard errors in all regressions. Table 4.7 also examines the fifth assumption of OLS. Unlike common perception, the errors need not be normal for OLS to hold if there is no autocorrelation. Table 4.7 rejects the null hypothesis of no autocorrelation in the residuals, which also means that normality is a very far cry from being the case as the QQ plots for value and momentum in figures 8.3-8.4 in Appendix 8.6 show. The return data is too heavy-tailed to be described by a normal distribution.

| | Breusch-Pagan Stat | P-value |
|---|---|---|
| Size | 59.269 | 0.000 |
| Value | 25.447 | 0.005 |
| Earnings | 52.690 | 0.000 |
| Investment | 64.254 | 0.000 |
| Momentum | 60.478 | 0.000 |
| Quality | 13.766 | 0.184 |
| BAB | 23.326 | 0.010 |

**Table 4.7:** *Breusch-Pagan test for residuals of the contemporary OECD-regressions for the factors considered. Data: Monthly from Jul. 1965 to Dec. 2021. Illustration: The author.*

| | Rho |
|---|---|
| Sentiment | 0.993 |
| DivPrice | 0.995 |
| E/P | 0.989 |
| Market Variance | 0.391 |
| Yield | 0.995 |
| Def. Spread | 0.964 |
| Inflation | 0.632 |
| Term Spread | 0.956 |

**Table 4.8:** *Rho coefficient of an AR(1) model for the independent variables of the contemporary OECD-regression. Data: Monthly from Jul. 1965 to Dec. 2021. Illustration: The author.*

A further assumption, when considering predictive regressions is that, although assuming the conditional moment restriction holds and $E\left(\epsilon_{t+1}|F_t\right) = 0$, the covariate process is uncorrelated, and none of the regressors are persistent. If they are, the Stambaugh bias occurs, which biases the estimators and calls for an adjustment of the standard errors (Linton, 2019). Table 4.8 shows that the regressors, apart from the market variance, are persistent. This is particularly the case for all fixed-income related variables as well as the dividend- and earnings variables. This indicates that even if the model assumes i.i.d. shocks, the Stambaugh bias exists, and the standard errors need to be adjusted.

| | VIF-Score |
|---|---|
| Sentiment | 1.369 |
| DivPrice | 3.990 |
| E/P | 2.991 |
| Market Variance | 1.233 |
| Yield | 3.093 |
| Def. Spread | 1.712 |
| Inflation | 1.507 |
| Term Spread | 1.359 |
| Slowdown | 1.839 |
| Expansion | 1.831 |
| Recovery | 1.654 |

**Table 4.9:** *VIF-score following Brooks (2008) for the variables in the contemporary OECD-regression. Data: Monthly from Jul. 1965 to Dec. 2021. Illustration: The author.*

Finally, to get robust results in regression, no multicollinearity must exist between the independent variables. Even though OLS works in the presence of multicollinearity, if the variables are not orthogonal, the inclusion of one variable, due to cross-covariance, influences the relation of the other variable, hence providing wrong inferences. Further, the presence of near-multicollinearity leads to high standard errors of the estimates, wherefore variables that may be significant on their own turn out not to be, when they are in a regression with other correlated regressors (Brooks, 2008). The VIF (Variance Inflation) test examines near-multicollinearity and is calculated as $VIF_i = \frac{1}{1-R_i^2}$ for variable i. A VIF of 1 indicates no correlation between predictor i and the remaining variables, and hence $\beta_i$ is not inflated. A VIF of 2 indicates that $\beta_i$ is inflated by 100% due to correlation with other factors in the model (PennState, 2018). Table 4.9 shows that all variables exhibit some degree of multicollinearity, but for the vast number of variables, the VIF-score is below even the most conservative thresholds for severe multicollinearity of 2.5. However, the yield, earnings to price (E/P) and dividend to price (D/P) are relatively highly correlated to the other variables. Further examination shows that these variables are highly correlated among themselves, with the E/P and D/P being correlated at 73%, yield and D/P at 72% and the yield and E/P at 63%. This intuitively makes sense, as when yields rise, the dividend that companies pay must rise as well to be attractive compared to fixed-income securities. Further, as earnings rise relative to price, because dividends are a function of earnings, dividends and earnings must be correlated as well. In summary, the presence of multicollinearity is not extremely severe in the sample, but the inference of the "dividend-like" variables must be interpreted with caution. For this reason, the regressions of the mean-variance weights on the macroeconomic variables in section 4.3.2 drops E/P and D/P to avoid multicollinearity.

Summing up, all assumptions of OLS seem violated. Linearity holds for five of the eight factor returns, but in general, OLS is a poor choice for the data at hand. This means that more robust statistical models such as GLS, are more applicable to the dataset. However, it is also clear that many of the drawbacks can be adjusted via Newey-West standard errors in combination with OLS. This makes the results of the thesis comparable to those of peers.

***Sub-Conclusion on Regressions***

The regressions shed light on the true link between the business cycles, economic variables, and returns. When controlling for the macroeconomic variables, the economic cycles support those of section 4.1.1 but it shows that the underlying macroeconomy, such as yield, inflation and default spread, explain more of the variance of returns than business cycles. In fact, the business cycles explain very little of the return. The analysis shows that value and investment are good inflation and yield-curve hedges, BAB wants ultra-low rates, while earnings and quality benefit from a weakening economy and size from a strong one. It is also clear, however, that the common approach of OLS is not the best for the data at hand, which means that the results must be interpreted with caution until more robust statistical methods have confirmed their validity.

### 4.3 Mean-Variance Optimization

The fact that the mean, variance and correlation of the factors vary heavily over time supports the idea that factor-timing could, theoretically, add alpha for the investor. Further, sections 4.1.1 and 4.2.1 show that some relation exists between macroeconomic variables and the factor returns, wherefore a time-varying factor portfolio linked to the economy may benefit the investor. The question pertains to how the weights should be scaled at each point in time, which is assessed through mean-variance analysis following equation 3.11. The thesis considers three variations of the approach. Firstly, a mean-variance portfolio of all factors, where the mean-variance framework is used over a long horizon as an SAA to see if it outperforms the equal-weight benchmark. To assess the time-varying weights, the thesis, based on the insights from section 4.1.1, slightly adjusts the weights based on the business cycle, thus utilizing a TAA approach. Secondly, the thesis examines a mean-variance portfolio of constant investment in the market and time-varying mean-variance optimal bets in the factors. As the factor bets are additional tilts of the overall portfolio, the shorter window of the mean-variance approach is warranted. Finally, the paper considers investments directly into the constituent portfolios of the factors also via a mean-variance approach.

Figure 4.5 shows the efficient frontier, illustrating that combining the factors in a portfolio improves the information set. Figure 4.5 also illustrates that any ideal portfolio holds a combination of the risk-free rate and the tangency portfolio depending on the risk-aversion of the investor. Another insight from the figure is that the factors lay in "clusters" of risk-return rewards. Quality, investment and earnings are relatively safe but with low excess returns, while BAB, size and value are riskier and, for the case of BAB, also provide higher returns.

The tangency portfolio in Table 4.10 is cheating, however, as it involves estimating the weight for 1963 based on returns in 2022. A more realistic alternative is an investor, who uses the first 50 years of data to optimize his mean-variance weights and then utilizes these weights from Jul 1$^{\text{st}}$, 2013 to Nov 30$^{\text{th}}$ 2022, the last 9 years out of sample. Table 4.10 presents the weights of the two tangency portfolios.

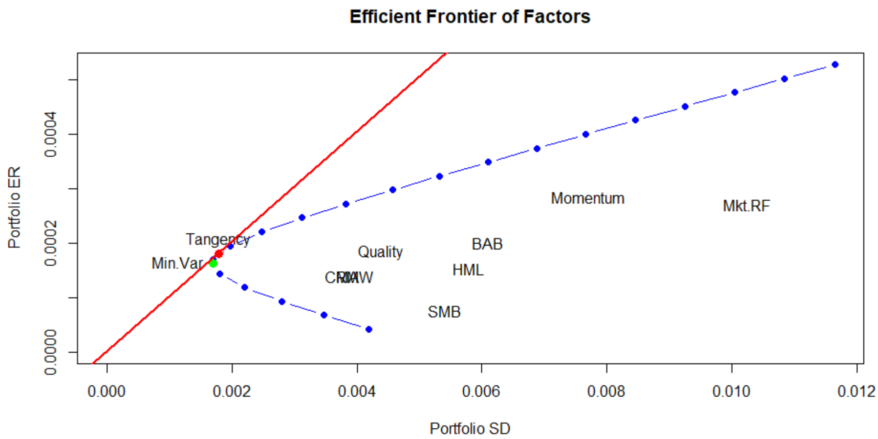


**Figure 4.5:** *Efficient frontier of the excess factor returns. Data: Daily from Jul 1st, 1963 to Nov. 30th, 2022 following French (2023g,i) and AQR (2023a,c). Illustration: The author, with code of Zivot (2019).*

| | Naive Tangency | In Sample Tangency |
|---|---|---|
| Market | 0.162 | 0.148 |
| Size | 0.133 | 0.132 |
| Value | 0.101 | 0.197 |
| Earnings | 0.019 | 0.006 |
| Investment | 0.148 | 0.076 |
| Momentum | 0.069 | 0.070 |
| Quality | 0.285 | 0.340 |
| BAB | 0.082 | 0.030 |

**Table 4.10:** *Mean-Variance optimal weights of the naive and in-sample tangency portfolio. Data: Daily, for naive portfolio, Jul. 1st, 1963-Nov. 30th, 2022, for in-sample ending Jun. 30th, 2013. Following French (2023g,i) and AQR (2023a,c). Illustration: The author.*

Table 4.10 shows that over a long horizon, the investor should not deviate too much from the equal-weight portfolio and hold a bit of everything. The quality factor warrants the largest exposure at 28.5% for the naïve portfolio and 34.0% in the first 50 years due to its good performance, in particular when the other factor strategies perform poorly, and therefore also has a negative correlation with the other factors per Figure 4.1. This is also the explanation for the earnings factor commanding a weight close to 0, as it is highly correlated, and has a lower return than quality. Table 4.10 reveals that the difference in using the full period, or the first 50 years, is not large with value having a tad higher weight if leaving out the last 9 years, investment a lower weight and quality a slightly higher weight. Had the estimation been done on 40 or 30 years, the weights would also have been similar. This supports section 2.1 that when the window is long enough, the factors revert to some mean, which is why the mean-variance approach is preferred over the long horizon.

Looking towards the performance of the strategies in Table 4.11, for the full, naïve period both the equal-weighted portfolio and tangency portfolio provide healthy returns of 73.8 bps and 73.9 bps per month, respectively, but the tangency portfolio does so with a lower standard deviation. Therefore, the tangency portfolio has a Sharpe ratio of 0.101, while the equal-weighted portfolio follows suit with a Sharpe ratio of 0.094. The tail risk, as measured by maximum drawdown, is very similar across the two portfolios at 10.694% and 10.290%. On the other hand, the kurtosis of the excess returns on the equal-weighted (tangency) portfolio is 18.67 (17.15), while the skewness is -0.94 (-0.56). Combining the statistics of the third and fourth moment with that of the second, the mean-variance optimal portfolio not only yields slightly higher returns but also lowers the average risk and tail risk of the investor. Considering the actively added performance to that of the market, the alpha is slightly lower for the tangency portfolio, but as the beta is higher, the idiosyncratic risk is lower, meaning that the information ratio is also higher for the naïve tangency portfolio. The beta is very close to 0 for either portfolio, meaning that the factor strategy is practically uncorrelated with the market. Therefore, by all conventional measures, a mean-variance approach with perfect foresight is more beneficial than equal weight. The euphory stops there, however. The out-of-sample tangency portfolio provides a slightly higher return than the equal-weight portfolio over the same out-of-sample period, but does so for a higher standard deviation, meaning that the Sharpe ratio is lower than the equal-weight portfolio. Similarly, the alpha of the out-of-sample tangency portfolio is higher than its equal-weight counterpart, but the idiosyncratic risk increases relatively more, leading to a lower information ratio, while the maximum drawdown simultaneously is higher. Therefore, once considering a realistic implementation of the mean-variance approach without perfect foresight, equal weight is better as a benchmark SAA. The story seems to be that, yes, in theory, one can optimize his long-term factor allocation via mean-variance analysis, but in practice, as returns, volatilities, and the state of the economy are hard to predict, the deviation from an equal-weight portfolio ends up underperforming.

| | Equal Full Period | Naive Tangency Full Period | OOS Equal | OOS Tangency |
|---|---|---|---|---|
| Mean Monthly Ret (%) | 0.738 | 0.739 | 0.379 | 0.397 |
| Excess Daily Ret (%) | 0.018 | 0.018 | 0.016 | 0.017 |
| Excess Std. Dev (%) | 0.192 | 0.178 | 0.282 | 0.324 |
| Sharpe Ratio | 0.094 | 0.101 | 0.056 | 0.051 |
| Drawdown (%) | -10.290 | -10.694 | -10.290 | -11.960 |
| CAPM Alpha (%) | 0.018 | 0.017 | 0.012 | 0.013 |
| CAPM Beta | 0.013 | 0.045 | 0.070 | 0.079 |
| Information Ratio | 0.099 | 0.128 | 0.061 | 0.055 |

**Table 4.11:** *Performance metrics of the naïve tangency portfolio and the equal-weight portfolio. Data: Daily Jul. 1$^{st}$, 1963-Nov. 30$^{th}$, 2022. Following French (2023g,i) and AQR (2023a,c). OOS = Out-of-sample. Illustration: The author.*

### 4.3.1 Optimal Tangency Portfolio in Economic Cycles

Another way to look at cyclicality is to determine the optimal tangency portfolio in the different stages during the economic cycle. As sections 4.1.1 and 4.2.1 determines that the OECD classification had the largest number of observations as well as performed the best in the predictive regressions, the section uses the OECD classification. Although it is again cheating, to compare the results with Table 4.11, the full period of returns in each economic stage is utilized.

Regarding the allocation of the business-cycle-stage-specific weights, the thesis takes three approaches. The first approach assumes that the investor knows which period he is currently in and adjusts his investments accordingly. This approach studies if there is a gain to adjusting the weights to the economic stage, and hence also if a link between the economic stage and the return exists. The second methodology takes a slightly more realistic route, as the OECD-indicator is released with one month lag, meaning that the indicator for March is released on April 5$^{\text{th}}$ (OECD, 2023b). Therefore, the realistic implementation is that the investor lags the indicator for one month. Here, the assumption still is, though, that the investor knows the optimal weights in each of the stages, only he is not required to know the stage he is currently in. As a final, and most realistic implementation, the thesis considers an investor that, based on either contemporary or lagged information about the stage of the business cycle, performs a slight tilt of the equal-weight portfolio in Table 4.10, i.e. adjusts via TAA. He does so, by recognizing which factors do well in each stage and adjusting the weight slightly, following Table 4.13. The Table is constructed such that the advantages of each factor are kept, while the adjustment is kept relatively minor, as is the rule in TAA. The TAA weights are subjective in the direction that benefits the individual factors, but lies within 0-25% allocation for all factors. This is equal to a practical equal-weight portfolio being allowed to deviate 100% from its original weights, but never goes short, which is fairly common in practice.

Table 4.12 both backs up the insights of section 4.1.1 as well as presents some differences to the results of the section. As indicated in section 4.1.1, the weight of quality is anticyclical, warranting a colossal weight of 204.6% during downturns, while the weight is 57.3% in slowdowns. In the recovery stage, the mean-variance approach suggests shorting quality by 2.2%, exactly as section 4.1.1 showed that the other factors do much better during recoveries than does quality during recoveries. Reconciling with section 4.1.1, quality is influenced much more by the business-stage-varying return than the correlation, which barely moved.

| | Expansion | Slowdown | Recovery | Downturn |
|---|---|---|---|---|
| Market | 0.265 | 0.095 | 0.221 | 0.136 |
| Size | 0.142 | 0.055 | 0.200 | -0.078 |
| Value | 0.143 | 0.173 | 0.087 | -0.141 |
| Earnings | -0.058 | -0.064 | 0.107 | -0.683 |
| Investment | 0.030 | 0.068 | 0.106 | 0.776 |
| Momentum | -0.065 | 0.158 | 0.044 | 0.371 |
| Quality | 0.151 | 0.573 | -0.022 | 2.046 |
| BAB | 0.391 | -0.060 | 0.257 | -1.427 |

**Table 4.12:** *Mean-variance weights in the OECD-business cycle stages. Data: Daily Jul. 1$^{st}$, 1963-Nov. 30$^{th}$, 2022. Following French (2023g,i) and AQR (2023a,c). Illustration: The author.*

| | Expansion | Slowdown | Recovery | Downturn |
|---|---|---|---|---|
| Market | 0.25 | 0.1 | 0.25 | 0.1 |
| Size | 0.15 | 0.05 | 0.20 | 0 |
| Value | 0.15 | 0.15 | 0.10 | 0.05 |
| Earnings | 0 | 0.10 | 0.05 | 0.10 |
| Investment | 0.10 | 0.10 | 0.10 | 0.25 |
| Momentum | 0 | 0.15 | 0.05 | 0.25 |
| Quality | 0.10 | 0.20 | 0 | 0.25 |
| BAB | 0.25 | 0.15 | 0.25 | 0 |

**Table 4.13:** *TAA-adjusted weights in the OECD business cycle stages. Data: Daily from Jul. 1$^{st}$, 1963-Nov. 30$^{th}$, 2022. Following French (2023g,i) and AQR (2023a,c). Illustration: The author.*

Surprisingly, the earnings factor should be sold short in all periods except recovery, where it provides higher returns than quality with a 0.6 correlation. The short-selling happens even though section 4.1.1 established that the earnings factor does best during downturns, where the mean-variance approach commands having a long weight of 204.6% in quality. Table 8.9 in Appendix 8.4 shows the two factors have a 0.639 correlation during downturns where earnings yield 1.7 bps per day, while quality, as per Table 4.4, yields 3.6 bps per day. Therefore, the investor can lower risk substantially, while earning an average return by going long quality and short earnings. The same is the reason for the low weight of earnings during slowdowns. Section 4.1.1 also showed that momentum had a high return in slowdowns and downturns, which, coupled with the fact that the correlation is only 0.28 with quality and 0.02 with investment, results in a large weight in these stages. This is the case as quality and investment command the highest weights in slowdowns and downturns, respectively.

As shown in section 4.1.1, the market, size and BAB returns are cyclical and require the highest weights during expansions and recoveries, although the market, being negatively correlated to all the other factors, warrants a stable positive weight throughout the economy. This, further, supports that a factor-tilt portfolio is a great way to add alpha to a full market investment. The optimal weight of BAB during downturns is rather extreme at -142.7%, which is due to the factor returning the lowest return of all factors considered at -3.4 bps per day. The fact that BAB is positively related to quality and momentum, which has large positive weights in downturns, further increases the short position.

Value and investment follow the results from section 4.1.1 and do not appear to be cyclical. Value commands a weight almost equal to 1/N in expansions and slowdowns only to being shorted with 1/N weight during downturns, especially due to its high correlation with investment. The investment factor comes out as a good overall investment, as section 4.1.1 also uncovered. Although its correlation with value is high, it provides diversification from the other factors and yields 1.3 bps per day during downturns, which warrants a high weight.

| | Contemp. OECD Adjusted | Lagged OECD Adjusted | Contemp. TAA | Lagged TAA |
|---|---|---|---|---|
| Mean Monthly Ret (%) | 1.571 | 1.404 | 1.017 | 0.967 |
| Excess Daily Ret (%) | 0.059 | 0.051 | 0.031 | 0.029 |
| Excess Std. Dev (%) | 0.562 | 0.555 | 0.247 | 0.261 |
| Sharpe Ratio | 0.104 | 0.091 | 0.127 | 0.111 |
| Drawdown (%) | -33.954 | -50.681 | -13.444 | -16.879 |
| CAPM Alpha (%) | 0.059 | 0.050 | 0.031 | 0.028 |
| CAPM Beta | -0.009 | 0.023 | 0.024 | 0.032 |
| Information Ratio | 0.103 | 0.094 | 0.138 | 0.124 |

**Table 4.14:** *Performance Metrics of the Business Cycle OECD-Portfolio. Contemp. OECD Adjusted = Contemporary OECD mean-variance portfolio, Lagged OECD Adjusted = OECD mean-variance portfolio with lagged OECD-stage. TAA = Adjustments per Table 4.13. Data: Daily from Jul. 1965 to Dec. 2021. Illustration: The author.*

Comparing Table 4.14 with Table 4.11, the investor can double the monthly return by employing business-cycle specific weights compared to an equal-weight portfolio, but the risk is also increased heavily. At the same time, utilizing the more realistic implementation, which lags the business cycle stage one-month, results in poorer performance than the fixed mean-variance weight portfolio in Table 4.11. The standard deviation of the contemporary portfolio increases from 17.9 bps to 56.2 bps, meaning that the Sharpe ratio is only a tad higher than the equal-weight portfolio. The risk is roughly the same for the lagged strategy, but here the return is lower, leading to a Sharpe ratio on par with that of the equal-weight portfolio. The tail risk is also much higher in the business-cycle portfolios, with the maximum drawdown being three times higher for the contemporary portfolio and five times higher for the lagged realistic portfolio. The business cycle portfolio adds more alpha compared to the equal-weight portfolio, but much more idiosyncratic risk arises as the business cycle portfolio is virtually uncorrelated with the market. Therefore, the information ratio in the contemporary portfolio is only a tad higher than the equal-weight portfolio in Table 4.11. Contrarily, the information ratio of the realistically lagged portfolio is lower than the equal-weight portfolio.

The reason for the large increase in risk is the rather extreme weights during downturns in quality, BAB and investment. Although they are optimized over the full period, these large bets mean that the return in a sub-period should go just a bit against the investor for the portfolio return to be disastrous. Such happened during the Mar. 2020 corona crisis. Given Mar. 2020 is classified as a downturn by OECD, extreme weights are employed. Initially, this goes extraordinarily well. On Mar. 9$^{th}$, 2020, the market drops 7.78% on corona-fear, momentum is up 3.22%, quality 1.75% and BAB rakes in a loss of 1.96%. From Mar. 9$^{th}$ through Mar. 18$^{th}$, the portfolio has returned a phenomenal 34.88%. From there, luck turns. On Mar. 19$^{th}$, the portfolio is down 5.48% following losses in quality and gains in BAB and, on Apr. 29$^{th}$, the portfolio is down 21.04% since its peak on Mar. 18$^{th}$. Therefore, the portfolio fitted to each business-cycle stage via mean-variance weights can yield much higher returns for the investor, but he must be prepared for large swings, and it can just as well be that the true returns go against the investor, in which case the losses are very large. The large difference between the contemporary and lagged portfolio also suggests that the largest gains are at the beginning of a stage, wherefore capturing these wrongly can be catastrophic for the investor. This is particularly likely in practice considering that the investor does not have perfect foresight of the economy, wherefore weights will not be perfectly suited for the current business cycle. In addition, the investor will not know the optimal weights based on returns 60 years in the future and will likely be estimated on a sort of expanding mean-variance optimizing window. This, very likely, makes the performance of the realistically implementable business cycle portfolio even worse.

Therefore, the small TAA portfolio adjustments, following Table 4.13, is an attractive alternative. The return is now 50% lower than it is under the mean-variance weights, but the standard deviation is also cut in half. Consequently, the Sharpe ratio is higher than for the equal-weight portfolio and the mean-variance optimized portfolios even after lagging the business cycle information one month. Additionally, although the alpha is cut in half, the idiosyncratic risk is reduced so greatly that the information ratio tops at 0.138, higher than any of the other portfolios. Further, the maximum drawdown is only a tad higher than the equal-weight portfolio and a third of the lagged mean-variance business-cycle optimized portfolio. Therefore, slightly adjusting the weights to the business cycle stage is highly beneficial, as the returns vary in stages of the business cycle, but as one can easily predict the cycle and return wrong, making large adjustments from the benchmark is too dangerous given stochastic returns.

### 4.3.2 Factor-Tilt Portfolios

A drawback with the portfolios in Tables 4.12 and 4.13 is the assumption that it is possible to invest directly in the factors. In practice, one must invest directly in the underlying stocks, and consider netting and aggregation effects, just as section 3.3 establishes it is not possible to logically take factor positions summing to 1. Consequently, a realistic implementation is an investor that is always invested 100% in the market portfolio and takes long-short factor bets as described in section 3.3. As benchmarks, the thesis considers a portfolio with a constant weight of 1 in the market and 1/7 in each of the other factors as well as a simple passive investment in the market.

Throughout, the thesis uses mean-variance optimization with a window of a minimum of 252-days to estimate the portfolios. The portfolios are estimated and rebalanced monthly, but uses daily data for the estimation. As described in section 3.3, the thesis uses both expanding and rolling methodologies, providing quick adjustments to current market events and stability, respectively. To avoid too extreme weights in especially the rolling methodology, the thesis employs weight caps following 3.3. The weight caps are utilized, as using the raw mean-variance estimates can lead to very extreme weights if either the mean returns are close to each other, or the correlation is close to zero in the estimates. Such large weights can cause extreme volatility in the portfolio, and mean that a small market move against the investor, can prove catastrophic. For this reason, one would either cap the mean-variance weights, as done in this study, or shrink the variance-covariance matrix to provide less extreme results. Ledoit & Wolf (2003) does the shrinkage via adjusting the variance-covariance matrix towards its diagonal matrix. Doing so avoids extreme weights, that would never be relevant in practice.

Table 4.15 shows the performance of the strategies with the abs(100%) to abs(500%) weight caps.

| | Equal | Roll. 1 | Exp. 1 | Exp. 2 | Roll. 2 | Exp. 5 | Roll. 5 | Market |
|---|---|---|---|---|---|---|---|---|
| Mon.Ret | 1.188 | 2.440 | 2.630 | 2.763 | 3.468 | 2.960 | 5.260 | 0.810 |
| Ex. Ret | 0.043 | 0.104 | 0.114 | 0.120 | 0.158 | 0.132 | 0.259 | 0.026 |
| SD. | 0.926 | 1.134 | 1.181 | 1.234 | 1.562 | 1.409 | 2.510 | 1.031 |
| Sharpe | 0.047 | 0.092 | 0.096 | 0.098 | 0.101 | 0.094 | 0.103 | 0.026 |
| DD. | -50.605 | -57.260 | -50.141 | -59.290 | -67.500 | -59.290 | -73.940 | -54.680 |
| Alpha | 0.020 | 0.094 | 0.099 | 0.112 | 0.146 | 0.125 | 0.251 | -0.000 |
| Beta | 0.872 | 0.384 | 0.541 | 0.298 | 0.422 | 0.252 | 0.327 | 1.000 |
| IR | 0.761 | 0.127 | 0.159 | 0.121 | 0.130 | 0.109 | 0.115 | 0.000 |

**Table 4.15:** *Performance of the factor-tilt portfolios with different weight caps. Data: Daily Jul. $1^{st}$, 1964-Nov. $30^{th}$, 2022. Following French (2023g,i) and AQR (2023a,c), model per section 3.3. Mon.Ret = Monthly full return in percent, Ex. Ret = Excess daily return in percent, SD = Standard deviation of Excess Returns in percent, DD = Maximum drawdown in percent, Alpha = Daily CAPM alpha in percent, Beta = CAPM beta IR = Information Ratio, Sharpe = Daily Sharpe ratio. 1-5 = weight caps of abs(100%) to abs(500%) respectively. Roll = Rolling Portfolio, Exp. = Expanding Portfolio Illustration: The author.*

Combining a long-market strategy with factor bets substantially increases the Sharpe ratio. Utilizing equal-weight bets in addition to a long market position increases the monthly return by 37 bps while lowering the standard deviation. Hence, the daily Sharpe ratio rises by over 60% compared to a passive market investment. Further, an equal-weighted factor portfolio does exceptionally well, capturing a daily alpha of 2.0 bps, while maintaining a beta of 0.872, meaning that the additional idiosyncratic risk taken is minor, which adds to a notable information ratio of 0.761. For the factor-tilt portfolios, the larger the bet is allowed to be, the lower the betas, which means that the idiosyncratic risk is positively related to the allowed bet size. This means, that although the alpha increases with the allowed bet, the information ratio decreases with the weight cap. Similarly, although the expanding portfolio provides less alpha than the rolling portfolio, its idiosyncratic risk is so low that it, for the low cap weights, returns a higher information ratio. As the investors following such a strategy have the market as a benchmark, they likely care about the information ratio as further expanded in section 5.1. Therefore, one may argue that equal weight is superior here.

On the other hand, if the investor cares about the return per unit of total risk or wishes to have a high return disregarding the risk, the factor-tilt portfolios provide a clear advantage. Capping the weights at 100%, the investor gets a 2.63% return monthly from the rolling portfolio, upping his daily Sharpe ratio substantially to 0.092 from 0.047. With it, however, also comes an increase of 20 bps in the daily standard deviation and a larger maximum drawdown. On the other hand, the kurtoses of all the rolling-factor-tilt portfolios are lower than the equal-weighted counterparts, e.g., 16.44 of the 200% portfolio, vs. 20.97 of the equal-weight portfolios, just as the factor-tilt portfolios are less negatively skewed. From this point of view, using factor-timing both limits the probability of observing extreme returns as well as increases the return. It is also worth mentioning that risk aversion does not play a role in the optimal allocation. These portfolios are the optimal allocation, and then the investor will, depending on the risk aversion, take positions in the factor portfolio and the risk-free rate. Only ambiguity aversion and regret aversion can change his wish of optimal weights.

As the weight cap increases, so does the standard deviation and maximum drawdown. With a cap of 200%, the mean monthly return is a mighty 3.468%, which comes at the price of a maximum drawdown of 67.5% and a standard deviation 70% higher than that of the equal-weight portfolio. With a 500% cap, the risk of the expanding portfolio almost stays the same, whereas the rolling portfolio becomes extremely risky. Albeit delivering a very healthy return of 5.26% per month, the standard deviation is 2.5 times that of the market, resulting in a Sharpe ratio only a tad higher than when the weights are capped at 100% and 200%. Also, in this data sample, the investor was "lucky" that his large bets paid off. It could just as well be that the bet went against the investor, to which the maximum drawdown of 73.94% is a testament.

Table 4.15 indicates that the rolling portfolio is much riskier than the expanding counterpart, yielding both higher returns and much higher risk. Figure 4.6 shows the development of the optimal time-varying weight value factor in the rolling and expanding factor-tilt portfolios, respectively. Both portfolios considered in figure 4.6, has a weight cap of abs (200%). The rolling portfolio takes much more extreme and volatile bets than the expanding portfolio. An example is that from Mar. to May 1980, the rolling portfolio adjusts its weight on the value factor from +200% to -149% and allocates to the earnings factor instead. The expanding portfolio also recognizes the poor performance of value but adjusts from its high of 200% later in October 1980 to reach a value weight of 185% in December. This means lower risk of getting a position completely wrong, but also that the investor misses out on opportunities in timing the factors, and can adjust his portfolio too late as further explored in section 4.4. One may also argue that the rolling methodology is more relevant for factor-timing, as it contains updated information, whereas the expanding window considers a long-term allocation strategy.

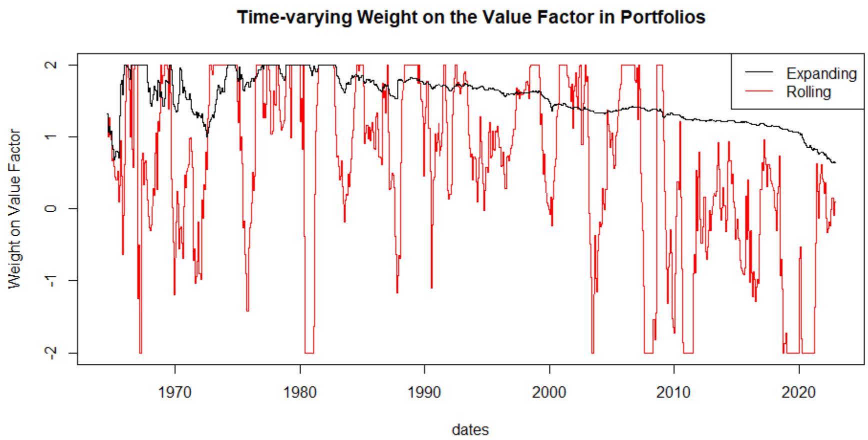


**Figure 4.6:** *Optimal mean-variance weight of the rolling and expanding factor-tilt portfolio with cap abs (200%). Data: daily from Jul 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022 following French (2023g,i) and AQR (2023a,c). Weights are updated monthly from Jul 1964 to Nov. 2022. Illustration: The author.*

Section 4.3.1 shows that due to the stochastic nature of returns, it is unwise to take massive factor positions, even though the positions are mean-variance optimal, as the backwards-looking mean-variance estimate is a poor predictor of future returns. Although the factor-tilt portfolios consider a static benchmark, and thus are different from the pure factor portfolios in section 4.3.1, it is of interest to the investor whether taking small bets is also superior in this context. Therefore, Table 4.16 considers the factor-tilt portfolio with a cap of only abs(30%), similar to a TAA deviation from a long-term equal-weight, as well as the subjective TAA weights of Table 4.13 adjusted to the business cycle, where the market weight is constantly 1. The weights of the factors are kept as in table 4.13.

Table 4.16 yields the interesting results. The excess return of the rolling mean-variance portfolio with cap abs (30%) is 1.9 bps higher per day than the equal-weight portfolio, while the standard deviation is 7 bps higher, yielding a higher Sharpe ratio than the equal-weight portfolio. The Sharpe ratio of the abs (30%) portfolio, however, is much worse than the rolling mean-variance portfolios with higher allowed bets. Similarly, although the alpha is twice that of the equal weight portfolio, the idiosyncratic risk is so high that the information ratio is some 0.5 lower than the equal-weight portfolio. The TAA portfolio, that adjusts its weights to the lagged business cycle, provides the opposite picture. The Sharpe ratio is a tad higher than the equal-weight portfolio, but lower than any of the mean-variance portfolios, primarily due to a low return. The information ratio of 0.781, however, is much higher than any of the mean-variance portfolios and even supersedes that of the equal-weight portfolio.

This tells the story that the mean-variance optimization and subjective TAA weights adjusted to the business cycle stage does two different jobs. The mean-variance approach improves the total return relative to the total risk, yielding very high returns for the risk-willing investor, who is also willing to trade heavily. The TAA portfolio far from delivers the same risk-return tradeoff but keeps the investor close to the benchmark. Under both circumstances, there is a gain from factor-timing. How the factor-timing should be carried out depends on the preferred risk measure, and hence ambiguity aversion, of the investor.

| | Lag. TAA Business Cycle Factor-tilt | Roll. Cap 0.3 |
|---|---|---|
| Mean Monthly Ret (%) | 1.308 | 1.569 |
| Excess Daily Ret (%) | 0.049 | 0.062 |
| Excess Std. Dev (%) | 0.934 | 0.994 |
| Sharpe Ratio | 0.053 | 0.062 |
| Drawdown (%) | -43.166 | -38.794 |
| CAPM Alpha (%) | 0.025 | 0.040 |
| CAPM Beta | 0.879 | 0.822 |
| Information Ratio | 0.781 | 0.275 |

**Table 4.16:** *Performance of the factor-tilt portfolios with TAA adjusted to the lagged business cycle stage and rolling mean-variance with abs(30%) cap. Data: daily Jul. $1^{st}$, 1964-Nov. $30^{th}$, 2022. Following French (2023g,i) and AQR (2023a,c), model per section 3.3. Illustration: The author.*

### 4.3.3 Regression of Factor-Tilt Weights on Macroeconomy

To examine the optimal time-varying factor bets more in detail, as well as how these are linked to the underlying economy, the thesis regresses the weights of the abs (200%) capped rolling and expanding portfolios on the lagged macroeconomic variables of Welch & Goyal (2008). If the macroeconomic variables can explain the weights, there is an argument for using the regressions for determining the optimal weights, as it indicates that the weights depend on the macroeconomy. Appendix 8.11 does the full analysis, and this section reports the main results. Tables 8.16-8.17 in Appendix 8.7 show the results for the rolling and expanding portfolios, respectively.

For the rolling portfolio, all weights load positively, and significantly at the 1% level, on their lagged return. The economic implications are also rather large, indicating that when the return on value has increased by 1%-point, the rolling portfolio should increase its weight by 5.6%-points, just as the weight for investment should go up 11.4%-points when its return has increased by 1%-point in the previous month. This indicates that the factors exhibit a momentum strategy themselves. Interestingly, the weights of the expanding portfolio far from load as strongly on lagged return, and BAB and quality do so insignificantly with quality indicating a contrarian strategy. This, again, is a testament to the fact that the expanding portfolio adjusts more slowly and, perhaps, also depends more on further lags as examined in section 4.3.4.

The weights of the expanding portfolio depend more on macroeconomic factors. For example, all factor returns now load significantly on yield with very large t-statistics and economic implications. All factors but BAB require higher weights when yields rise, which intuitively makes sense, as BAB relies on cheap funding opportunities to exploit the arbitrage via leverage, whereas the other strategies do not have an implied leverage. The explanations for the positive relation to yield for all other factors are examined in the discussion. The loading on yield is also significant and in the same direction for the rolling portfolio, but less significantly so.

Further, the default spread plays a larger role in the expanding than the rolling portfolio, showing that value, size and earnings benefit from a lower default spread, i.e., from less credit risk in society. Quality, on the other hand, benefits from more credit risk, further fueling the story of quality being a safe haven for investors. The same is the case for momentum and investment. Value and size command lower weights when the yield curve steepens, whereas the weight of quality should follow the first derivative of the yield curve, while inflation is only significantly positive (negative) for momentum (earnings). The $R^2$ also becomes much higher when considering the expanding portfolio. In general, Tables 8.16-8.17 show that the weights depend less on the concrete economic stage, and more so on the macroeconomic variables. Size is still cyclical as is BAB, but the loadings are not as significant as the ones for yield, whose loadings are much stronger than they were in the regressions of the return on the macroeconomic variables in table 8.11 in Appendix 8.5. The story, therefore, is that more so than returns, weights depend on the macroeconomy and in particular the current economy can be a reasonable determinant for the optimal weight tomorrow.

The analysis in Appendix 8.11 shows that an investment strategy based on predictive regressions of Table 8.17 can outperform a standard mean-variance approach. Assuming that the weights are estimated based on a rolling 84-month window, Table 8.19 shows that the portfolio trying to determine the expanding weights based on a rolling regression outperforms the mean-variance expanding portfolio both on Sharpe ratio (0.098 vs. 0.102) and information ratio (0.123 vs. 0.137) due to higher return and alpha. In fact, estimating the expanding weights with the rolling regression on the macroeconomy even outperforms the rolling mean-variance portfolio, which has an information ratio of 0.131.

Taken together with section 4.2.1, a story emerges that the optimal short-term weight increases with the previous performance of the factors, but in the long run understanding the rates market and the macroeconomic development is of more use. Further, linking the performance of the portfolio, in Appendix 8.11, based on predicted expanding weights in a rolling prediction, to the performance of the TAA-portfolio in section 4.3.1, signals that factor-timing makes sense if the weights are adjusted slowly to the underlying economy.

#### 4.3.4 Analysis of Autocorrelation in the Returns

Given the positive relationship between previous returns and weights, that section 4.3.2 illustrated, it must be that autocorrelation exists in the returns. If it does, it means that it is possible, to some extent, to forecast returns, wherefore a presence of autocorrelation is positive news and of large interest to the investor. A finding of autocorrelation can possibly also explain the behavioir of momentum over time. The investor is then interested in knowing which lags generate the autocorrelation, to determine which he should focus on, when seeking to predict returns. This section examines that question.

| | Test Stat | P-value |
|---:|---:|---:|
| Market | 99.365 | 0.499 |
| Size | 128.397 | 0.029 |
| Value | 138.801 | 0.006 |
| Earnings | 180.008 | 0.000 |
| Investment | 191.943 | 0.000 |
| Momentum | 191.340 | 0.000 |
| Quality | 153.347 | 0.000 |
| BAB | 132.725 | 0.016 |

**Table 4.17:** *Box-Pierce test for autocorrelation with 100 lags. The null-hypothesis is no autocorrelation. The p-value is based on White standard errors per Linton (2019). Data: Daily Jul. 1$^{st}$, 1963-Nov. 30$^{th}$, 2022. Following French (2023g,i) and AQR (2023a,c). Illustration: The author.*

Table 4.17 examines the claim of autocorrelation via the Box-Pierce test. One can claim that the Durbin-Watson test in Table 4.7 also tested for autocorrelation. However, the Box-Pierce test examines several lags of autocorrelation, whereas the Durbin-Watson test only considers one lag. Further, the Durbin-Watson test examines the autocorrelation in the residuals of a regression model, whereas the Box-Pierce test studies the autocorrelation of the series, in this case returns, itself. Therefore, the

Box-Pierce test says something about whether more returns than just the one yesterday can explain the return today, which is exactly what the investor is interested in if trying to predict returns. All Box-Pierce tests are done with White standard errors to correct for autocorrelation, following Linton (2019). Table 4.17 rejects the null hypothesis of no predictability at the 5% level for size and BAB and at the 1% level for all other factors. Only the market exhibits no autocorrelation. Figures 8.5-8.12 in Appendix 8.8 shed light on the origin of autocorrelation of the factors. Not surprisingly, most factors have a highly significant positive first lag, which exactly is what the momentum factor is built on. Also, the significant lags are, generally, positive, explaining the momentum behaviour of the weights. Figures 8.7, 8.11 and 8.12 show, however, that quality, BAB and size, exhibit a different autocorrelation structure, with BAB and size having many positive lags beyond lag 2, while quality shows several positive lags all through lag 100. The three factors also have more significant negative lags than their four counterparts. This can explain why the weight loading on the lagged return is insignificant on quality and BAB in the expanding regressions, as the relation to return is further lagged than simply the previous return. Coupled with the regressions, this makes the importance of previous returns lower for these strategies. Therefore, when seeking to predict returns, the investor must approach the problem differently depending on the factor in question.

Section 4.1.1 shows that momentum performed the best during slowdowns and expansions and hypothesizes that this is linked to autocorrelation being stronger in these stages. Table 4.18 presents the autocorrelation of the momentum factor across the four OECD stages. Although the momentum factor is a function of the autocorrelation and cross-covariance between the single stocks and the market, as presented by Lewellen (2002), the thesis does not have the single-stock data to calculate such. Therefore, as an approximation, the daily autocorrelation of the momentum factor itself along with the market is provided. If a strong positive autocorrelation exists there, it must also be a sign that positive autocorrelation exists among the single stocks. Table 4.18 shows that the autocorrelation is much lower and insignificant in recoveries than in the other three stages. In particular, during expansions and downturns, where momentum does well, the return of momentum itself shows strong autocorrelation. Figures 8.13-8.16 in Appendix 8.8 reveal that the reason, primarily, is the first lag, which is very significant in expansion and downturn, and barely is so for recovery. Table 4.19 shows similar results for the market return. As Lewellen (2002) establishes that underreaction leads to positive autocorrelation, this can indicate that in recoveries there is more optimism in the market. Hence, investors underreact less in recoveries, which harms momentum, as the return of momentum exactly feeds on underreaction. In downturns, skepticism means that the investors underreact severely driving the momentum premium.

| | Test Stat | P-value |
|---|---|---|
| Expansion | 146.689 | 0.002 |
| Slowdown | 165.189 | 0.000 |
| Downturn | 135.143 | 0.011 |
| Recovery | 108.900 | 0.255 |

**Table 4.18:** *Box-Pierce test for momentum with 100 lags in the OECD-cycle stages. The test is done with White standard errors. Data: Daily Jul. 1$^{st}$, 1963-Nov. 30$^{th}$, 2022, all observations in a given stage considered. Illustration: The author.*

| | Test Stat | P-value |
|---|---|---|
| Expansion | 106.59 | 0.31 |
| Slowdown | 117.39 | 0.11 |
| Downturn | 110.54 | 0.22 |
| Recovery | 88.82 | 0.78 |

**Table 4.19:** *Box-Pierce test for the market with 100 lags in the OECD-cycle stages. The test is done with White standard errors. Data: Daily Jul. 1$^{st}$, 1963-Nov. 30$^{th}$, 2022, all observations in a given stage considered. Illustration: The author*

### 4.3.5 Time-Varying Portfolios of Factor Constituent Portfolios

Another feasible strategy is to invest directly in the portfolios, which constitutes the factors. This enables the investor to have a total positive invested weight, which alleviates the problem that the total investment sums to zero. Further, it means that the investor can take same-direction bets in both ends of the long-short factor. This approach serves as an alternative to the factor-tilt portfolios and examines the robustness of timing factors via a mean-variance approach. To illustrate the importance of the choice of methods, the thesis considers the methodology used in Fama & French (2015), as well as univariate sorts of the bottom and top 10% equal to the sorting approach of Jegadeesh & Titman (1993). The two approaches are labelled "Individual Factors Portfolio" and "Univariate Factors Portfolio", respectively. Both strategies consider a portfolio of the factor constituents of the Fama & French (2015) factors with a cap of abs (200%) on each factor. In principle, one could argue that when the portfolio goes short on one end of a factor, say growth, it would correspond to going long on the other end, value. The portfolio is allowed to go short on both ends, however, as the two sides of a factor are not mutually exclusive. Value, for example, is the top 30% B/M stocks, and growth is the bottom B/M 30%, which means the investor could end up shorting if both "tails" of the distribution have become unattractive compared to the middle stocks.

Figure 4.7 illustrates the weights for the individual rolling factors portfolio, which are very volatile over time. It is not uncommon to see the rolling portfolio flip its bet from one extreme to the other, as is the example for the weight in the value factor from Oct. 1965 (-200%) to Feb. 1966 (+200%). Also, the figure shows that it is far from uncommon that the portfolio takes the same direction weights in each leg of the factor. The size of the value bet changes over time and, although the standard deviation of the weights for the expanding and rolling portfolios are very different, the bets move in the same direction. For example, during the 2010s, the rolling portfolio takes large negative value bets, and large growth bets, just as the expanding portfolio decreases its growth short and value long. Figure 4.8

shows that the direction of the bets at any given time is generally the same in the time-varying bets across methodology. Hence, the conclusion is that the same mechanisms are in play, the methodologies just show different ways of taking the bets.

Table 4.20 shows the performance of the individual portfolios. Since the weight in the market portfolio should not be a constant of 1, the most appropriate comparison, must be a strategy without a benchmark. Section 4.2.1 considers a such in the naive tangency portfolio, which had a Sharpe ratio of 0.101 daily for a mean monthly return of 73.9 bps. The univariate factor portfolios offer a similar Sharpe ratio just below that, while the individual portfolios offer a Sharpe ratio just above the naive tangency portfolio. The bases are much differnet, however, with the factor constituent portfolios yielding a higher return for a higher standard deviation and tail risk. Table 4.20 shows that the individual sorting, as used by Fama & French (2015), yields a higher Sharpe ratio than univariate sorts, as the risk of the individual sorting is much lower than the univariate counterpart. This is both the case when considering the standard deviation as well as the maximum drawdown which is a massive 90.71% for the univariate rolling portfolio.

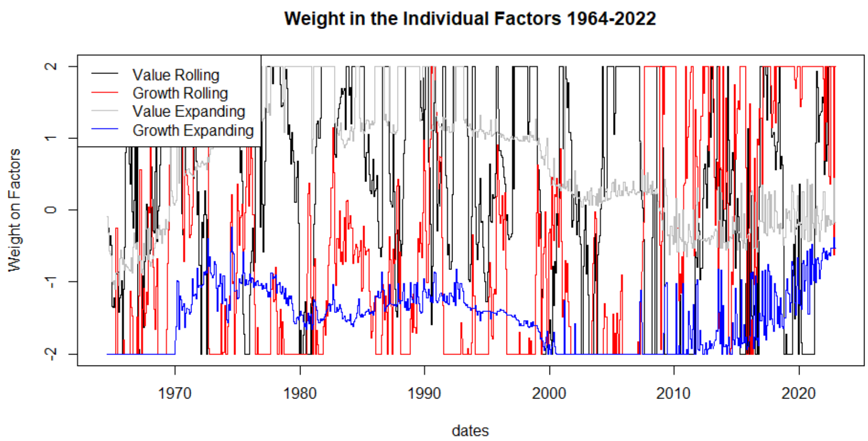


**Figure 4.7:** *Time-series development of the optimal mean-variance weights of the value constituents for the factor constituent portfolios following sorting of Fama & French (2015). Weights are monthly from Jul. 1964 to Nov. 2022. Data: Daily following French (2023g,a) and AQR (2023a,c). Illustration: The author.*

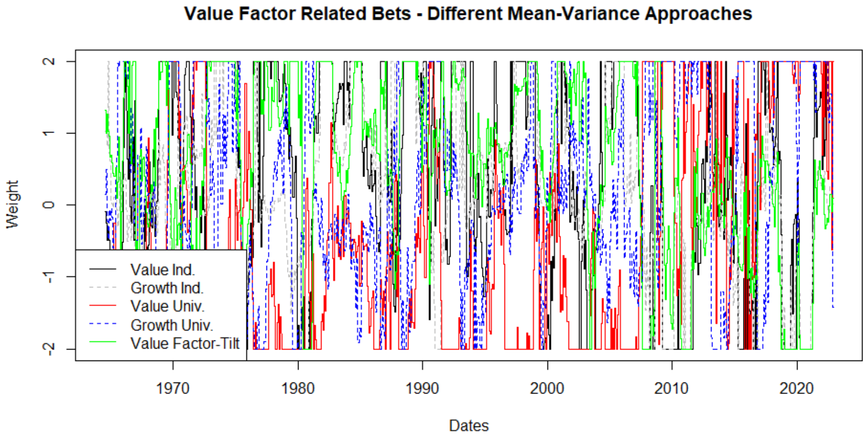


**Figure 4.8:** *Time-series development of the weights of mean-variance portfolios. "Ind." = individual sort, per Fama & French (2015). "Univ." = univariate sort per Jegadeesh & Titman (1993). "Factor Tilt" = Portfolios in 4.3.2. Monthly weights from Jul. 1964 to Nov. 2022. Data: Daily following French (2023g,i) and AQR (2023a,c). Illustration: The author.*

The Fama & French (2015) methodology yields both higher returns and lower standard deviation than the univariate approach, which could be the reason that Fama & French (2015) chose the methodology in the first place. When considering the expanding portfolio, the statistics for the univariate portfolio get better, but still not the standard of the individual portfolios. The reason, as Figure 4.8 shows, is that the weight bets of the univariate portfolios are more extreme than is the case for the individual sorting. Therefore, the thesis notes that if factor-timing is to be employed on a single portfolio basis, a capped weight in the Fama & French (2015) rolling methodology is preferred. Finally, comparing the equal-weighted portfolios estimated with both methodologies tells the same story as Table 4.15. If measured by the Sharpe ratio, the factor-timing contributes to a better risk-return tradeoff but, if the goal is to optimize the alpha per unit of risk, the best choice is by far an equal-weight portfolio due to lower idiosyncratic risk. Ultimately, the investor will likely choose between the strategies in Table 4.15 and the strategies in Table 4.20, as they are both actual implementable strategies. As the same factors are not considered in both strategies, they cannot be compared one to one, but measured by both Sharpe ratio and information ratio, the portfolio of individual factors is superior. Given that adding an asset can never decrease the Sharpe ratio, the individual factor strategy must be strictly preferred to the factor-tilt portfolio.

| | Ind. Roll. | Ind. Exp. | Univ. Roll. | Univ. Exp. | Eq. Univ. | Eq. Ind. |
|---|---|---|---|---|---|---|
| Mean Monthly Ret (%) | 3.596 | 3.338 | 3.345 | 3.274 | 1.124 | 1.226 |
| Excess Daily Ret (%) | 0.162 | 0.147 | 0.168 | 0.150 | 0.042 | 0.047 |
| Excess Std. Dev (%) | 1.471 | 1.260 | 2.320 | 1.648 | 1.120 | 1.080 |
| Sharpe Ratio | 0.110 | 0.117 | 0.072 | 0.091 | 0.038 | 0.043 |
| Drawdown (%) | -73.190 | -54.160 | -90.710 | -63.470 | -62.030 | -59.280 |
| CAPM Alpha (%) | 0.148 | 0.133 | 0.157 | 0.139 | 0.014 | 0.020 |
| CAPM Beta | 0.518 | 0.532 | 0.399 | 0.429 | 1.067 | 1.021 |
| Information Ratio | 0.158 | 0.188 | 0.082 | 0.115 | 0.739 | 0.716 |

**Table 4.20:** *Performance of the investments in the constituent factor portfolios. Ind = individual sorts per Fama & French (2015), Univ. = univariate sorts per Jegadeesh & Titman (1993). Data: Daily Jul. 1$^{st}$, 1964-Nov. 30$^{th}$, 2022. Following French (2023g,i) and AQR (2023a,c). Illustration: The author.*

***Sub-Conclusion on Mean-Variance Portfolios***

Section 4.3 shows that an investor can benefit from a time-varying portfolio of factors, as both excess returns and correlations vary over time. How the portfolio is implemented the best, however, depends on which risk-measure the investor is interested in, as well as requires proper due diligence in selecting the weights. Due to autocorrelation in the returns, the investor should increase his short-run weight in a factor when that factor has had high returns, although this effect is less pronounced for BAB and quality. Here, the investor should consider longer lags when doing his forecast. In the long run, he should base his weights on the macroeconomic links rather than short-term returns. Here, it pays to update the slowly adjusting expanding weights via a rolling methodology. All this speaks towards basing the long-term SAA-strategy on the underlying economy, and only adjusting this slowly and marginally, when the economy changes. The short-term TAA-tilts, however, can be done based on autocorrelation and the rolling weights. In general, a modestly adapting TAA-strategy from an equal-weight benchmark provides the best results if considering the information ratio, as the excess risk of the portfolio is kept at a minimum. The insight is that because returns are stochastic and close to impossible to predict, taking too large a risk, in the hope of capturing the future return, can lead to excessive risk and large losses. On the other hand, a time-varying mean-variance portfolio more than doubles the Sharpe ratio of factor-tilt portfolios and the factor constituent portfolios. Introducing transaction costs may prove to destroy the edge of the highly volatile mean-variance portfolios. Section 4.4 examines this question.

## 4.4 Factor-Timing with Transaction Costs

As outlined in section 3.3.4, this section considers the mean-variance approach including transaction costs. Building a full model of market impact is beyond the scope of this thesis, wherefore only the impact of price spreads and short-selling costs is considered. The impact of strategic trading is discussed qualitatively at the end of the section. The section considers factor-tilt portfolios.

The first level of transaction costs considers the cost to construct the factor portfolios following Novy-Marx & Velikov (2016). Table 4.21 shows the performance of the factors after accounting for the Novy-Marx & Velikov (2016) transaction costs. The transaction costs in 4.21 are presented as negative, to highlight them being a cost. Naturally, the mean return is lower, decreasing around 0.2 bps per day for the factors rebalanced annually and as much as 3 bps daily for the frequently balanced factors, namely momentum, quality and BAB, while alpha also decreases across the board. The excess return and alpha on momentum and quality even becomes negative, as their transaction costs exceed the 50-60 bps per month that the strategies generate on average. Earnings, value, and investment, which are only rebalanced annually, maintain much of their excess return after the transaction cost, making them more attractive than before. It is evident that because the transaction costs of the frequently rebalanced factors are so large, they become less attractive than their less frequently rebalanced counterparts. Therefore, including the transaction costs materially alters how the investor should invest.

| | Size | Value | Earnings | Investment | Momentum | Quality | BAB |
|---|---|---|---|---|---|---|---|
| TC Monthly (%) | -0.040 | -0.043 | -0.026 | -0.091 | -0.592 | -0.603 | -0.305 |
| Excess Ret wo TC (%) | 0.007 | 0.015 | 0.014 | 0.014 | 0.028 | 0.018 | 0.020 |
| Sharpe wo TC | 0.014 | 0.026 | 0.035 | 0.037 | 0.037 | 0.042 | 0.033 |
| Excess Ret w. TC (%) | 0.006 | 0.013 | 0.013 | 0.009 | 0.000 | -0.010 | 0.005 |
| Sharpe w TC | 0.010 | 0.023 | 0.032 | 0.025 | 0.000 | -0.023 | 0.009 |
| Alpha w TC | 0.007 | 0.016 | 0.015 | 0.013 | 0.003 | -0.005 | 0.013 |
| Beta w TC | -0.038 | -0.092 | -0.080 | -0.131 | -0.096 | -0.185 | -0.280 |
| Information Ratio w TC | 0.011 | 0.023 | 0.031 | 0.025 | 0.003 | -0.008 | 0.014 |

**Table 4.21:** *Performance metrics of factor returns after the first-pass transaction costs of Novy-Marx & Velikov (2016). The transaction costs follow section 3.3.4. Data: Daily Jul. 1$^{st}$, 1964-Nov. 30$^{th}$, 2022. Following French (2023g,i), AQR (2023a,c) and Novy-Marx & Velikov (2016). Illustration: The author.*

Table 4.22 backs the hunch and shows that the low-volatile, anti-cyclical earnings factor is now very attractive, commanding 74% of the capital allocation when considering the naïve tangency portfolio from section 4.3.1. This is due to the earnings factor being negatively correlated or uncorrelated with the other factors during a downturn, where it yields high returns. Quality, on the other hand, has fallen out of favor with a short position of 56.03% due to the large transaction cost. Comparing Table 4.10 with 4.22, it is surprising to see a relatively large positive weight on BAB, as it has very modest excess returns after considering transaction costs. The reason is that the correlation between BAB and earnings is only 0.25. Hence, BAB allows the investor to achieve diversification and obtain a return, when the return of his largest bet, earnings, is down.

| | Weight |
|---|---|
| Market | 0.160 |
| Size | 0.118 |
| Value | 0.021 |
| Earnings | 0.740 |
| Investment | 0.304 |
| Momentum | 0.044 |
| Quality | -0.560 |
| BAB | 0.174 |

**Table 4.22:** *Optimal weights of the naïve tangency portfolio after transaction costs per Novy-Marx & Velikov (2016). The transaction costs follow section 3.3.4. Data: Daily Jul. 1st, 1964-Nov. 30th, 2022. Following French (2023g,a), AQR (2023a,c) and Novy-Marx & Velikov (2016). Illustration: The author.*

Based on the new knowledge of transaction costs, the thesis estimates the optimal factor-tilt portfolio with abs (200%) weight. The new portfolio is calculated based on the returns including transaction cost in Table 4.21, as opposed to the analysis in section 4.3.2 focusing on gross returns. The assumption is that the investor knows both the return and the transaction costs of constructing the factor portfolios. When rebalancing the portfolio each month, the thesis considers a transaction cost of 20 bps per 100% the weight changes and a short fee following section 3.3.4.

Figure 4.9 shows the development of the mean-variance optimal weights in the value and momentum factors over time with and without transaction costs. The picture supports Table 4.21, as the bets vary in the same direction as the factor weights in section 4.3.2 without transaction costs, but the value bets are overall more positive compared to the clean returns, while the bets on momentum are constantly a bit below their level without transaction cost. Including transaction costs shifts the attractivity towards the "cheap-to-trade" factors although the market movements are the same as always.

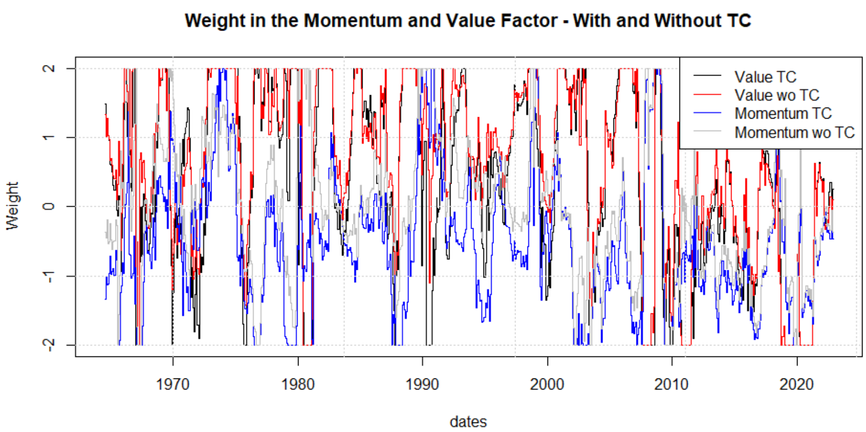


**Figure 4.9:** *Time-Series Weights of value and momentum in the factor-tilt portfolios with and without the first-pass transaction costs following Novy-Marx & Velikov (2016). Monthly weights from Jul. 1964 to Nov. 2022. Illustration: The author.*

Table 4.23 presents the performance of the mean-variance portfolios after considering both passes of transaction costs. Unsurprisingly, the return is around 50 bps lower monthly, than not considering the transaction costs, just as the daily standard deviation has increased by 7 bps for the rolling portfolio and a mighty 29 bps for the expanding portfolio given the extra constant risk of the transaction costs, which are not equal across factors. This means that the Sharpe ratio drops from 0.101 daily to 0.064 for the rolling portfolio, which makes the equally weighted portfolio relatively more attractive than without transaction cost. Further, the tail risk is now substantially higher for the expanding portfolio yielding a maximum drawdown of 80.67%, while the tail risk of the rolling portfolio is lower. On the other hand, the kurtoses of 14.91 and 12.86 are lower than their counterparts without transaction cost, which indicate less tail risk.

| | Rolling | Expanding | Equal-Weighted |
|---|---|---|---|
| Mean Monthly Ret (%) | 2.973 | 2.207 | 0.933 |
| Excess Daily Ret (%) | 0.104 | 0.089 | 0.031 |
| Excess Std. Dev (%) | 1.638 | 1.527 | 0.927 |
| Sharpe Ratio | 0.064 | 0.058 | 0.034 |
| Drawdown (%) | -58.253 | -80.666 | -53.262 |
| CAPM Alpha (%) | 0.088 | 0.070 | 0.008 |
| CAPM Beta | 0.592 | 0.708 | 0.871 |
| Information Ratio | 0.086 | 0.088 | 0.289 |

**Table 4.23:** *Performance of the factor-tilt portfolios with weight cap abs(200%) after the full transaction costs per section 3.3.4. Data: Daily Jul. $1^{st}$, 1964-Nov. $30^{th}$, 2022. Following French (2023g,i), AQR (2023a,c) and Novy-Marx & Velikov (2016). Illustration: The author.*

Although transaction costs hurt the performance of the expanding portfolio less than its rolling counterpart, as its weights are less volatile, the portfolio instead shows the drawback of a high maximum drawdown, as the slow adjustment does not adjust to changing market environments. In May 2007, the portfolio takes a large 181.42% bet in value and shorts quality by 177.36%, which has slowly been built up during the years from the poor after transaction costs performance of quality and good performance of value. This proves to be a horrendous decision, as quality rises and value drops, making the portfolio down 37% in Jan. 2008. The effect gets grimmer at the onset of the financial crisis, where quality benefits from a flight to quality and value suffer severe losses due to including many of the distressed banks. Hence, on March $6^{\text{th}}$, 2009, the portfolio is down 80% from its peak two years earlier. Along the ride, the short in quality is reduced and the bet on value ditto, but the adjustment is very slow, resulting in the sub-optimal portfolio. This shows a general tail risk with the expanding portfolio, which by chance happens to be higher in the transaction costs world. Comparing their performance from the beginning of the sample until May 2007, the Sharpe ratio of the expanding (rolling) portfolio is

0.0815 (0.0803), due to the lower risk of the expanding portfolio, indicating that with transaction costs a slower adjustment of the weights is attractive, as long as it does not harm the return substantially.

For the equal-weighted portfolio, the presence of transaction costs also hit hard. Even only considering the first level Novy-Marx & Velikov (2016) transaction costs, the return of the equal-weighted portfolio drops over 1 bps per day to 3.2 bps. The reason is that the factors now have much lower returns themselves. Hence, an equal-weighted strategy captures much less premium than when transaction costs are ignored. Introducing the second pass transaction costs reduces the daily return by an additional 0.09 bps. This means that the information ratio of the equal-weighted portfolio is reduced to 0.289. Still, this is higher than the information ratios of the timed portfolios, which is heavily reduced as the transaction costs slash alpha by 4 bps per day.

***Sub-Conclusion on Transaction Costs***

Transaction costs change the optimal factor-timing and TAA strategy. Factor-timing still adds value to the investor, and the direction of the positions in each factor is still the same over time. Stated differently, when the portfolio without transaction costs adds to the value factor, the one with transaction costs does as well, but the level of the factor weight is different. Factors that are rebalanced often are discouraged relative to annually rebalanced factors with lower costs, altering the portfolio in favour of earnings instead of quality. Further, the gain in the Sharpe ratio becomes smaller compared with the equal-weight portfolio, just as the equal-weight portfolio, even with transaction costs, outperforms on information ratio. The thesis relies on large assumptions for the analysis, but it supports that the investor must consider transaction costs and preferably do so in a model explicitly considering such.

## 4.5 Available Factor ETFs

As the transaction cost analysis assumes that ETFs trade for all factors, or at least that the cost of rebalancing the factors can be done at the same cost as trading the ETFs, it is relevant to examine the market for factor ETFs, as well as how well the ETFs captures the factors. Table 4.24 presents an overview of the longest-standing ETFs available to track the factors in question. The factor ETFs have only been around for a fraction of the time that the factors themselves have. Further, the factors are constructed of long-only positions, for instance, the value ETF invests in value companies and the momentum ETF in previous winners, just as no ETF exists for investment and earnings at all.

| Factor | ETF Name | Ticker | Inception Date |
|---|---|---|---|
| Value | iShares MSCI USA Value Factor ETF | VLUE | 18/04/2013 |
| Momentum | iShares MSCI USA Momentum Factor ETF | MTUM | 18/04/2013 |
| Size | iShares MSCI USA Size Factor ETF | SIZE | 18/04/2013 |
| Investment | NA | NA | NA |
| Earnings | NA | NA | NA |
| Quality | iShares MSCI USA Quality Factor ETF | QUAL | 18/07/2013 |
| BAB | iShares MSCI USA Min Vol Factor ETF | USMV | 20/10/2011 |
| Market | SPDR S&P 500 ETF Trust | SPY | 29/01/1993 |

**Table 4.24:** *Subset of the factor ETFs available, their ticket and the day they started trading. Data: Daily from inception to Nov 30$^{th}$, 2022 per Yahoo Finance (2023d,b,c,a,f,e). Illustration: The author.*

Table 4.25 shows the performance of the ETFs compared with the factors whose strategy they claim to duplicate, all time-series are compared for the same length, i.e., the ETF return from inception to Nov. 30$^{\text{th}}$, 2022. Intuitively, the market is captured very well in the ETF with a correlation of 97.7% between the ETF return and the return of the factor. For all other factors, the ETFs capture the factor returns extremely poorly. The correlation between the ETF return and the corresponding long-short factor return for size, value and momentum is between 25.6% and 17.4%, while for BAB, the ETF return is virtually uncorrelated with the factor it is meant to capture. The iShares Quality ETF is even negatively correlated to the AQR-quality factor indicating that the ETF captures something completely different from the factor it claims to follow. The mean returns and standard deviations are much higher for the ETFs than for the factors. When coupled with the fact that the alphas are more comparable and the betas are much higher and close to 1 for all ETFs, this is clear evidence that the ETFs employ a long-only strategy focused on the theme as opposed to a long-short strategy as the factors.

| | Size | Value | Momentum | Quality | BAB | Market |
|---|---|---|---|---|---|---|
| Mean Daily ETF Ret (%) | 0.047 | 0.042 | 0.055 | 0.047 | 0.048 | 0.035 |
| Mean Daily Factor Ret (%) | -0.002 | -0.003 | 0.009 | 0.027 | 0.035 | 0.036 |
| Daily Std. Dev ETF (%) | 1.137 | 1.214 | 1.242 | 1.130 | 0.897 | 1.193 |
| Daily Std. Dev Factor (%) | 0.639 | 0.863 | 1.065 | 0.600 | 0.696 | 1.186 |
| Correlation - Factor/ETF | 0.255 | 0.214 | 0.174 | -0.236 | 0.068 | 0.977 |
| CAPM Alpha ETF (%) | 0.001 | -0.007 | 0.005 | 0.002 | 0.009 | 0.000 |
| CAPM Alpha Factor (%) | -0.007 | -0.001 | 0.014 | 0.034 | 0.039 | 0.000 |
| CAPM Beta ETF | 0.907 | 0.965 | 0.994 | 0.960 | 0.725 | 0.983 |
| CAPM Beta Factor | 0.106 | -0.039 | -0.111 | -0.165 | -0.070 | 1.000 |

**Table 4.25:** *Performance of ETFs with the factors they claim to capture. Data is daily from inception to Nov. 30$^{th}$, 2022. See Table 4.24 for dates. Factor = the long-short factor, ETF = the ETF that claims to capture the factor. Illustration: The author.*

The insights from Tables 4.24 and 4.25 have two major implications for the investor. Firstly, the assumption in section 4.4, that the factors can be captured by ETFs, is very unrealistic. This means that the cost of rebalancing the factors also must be done on single stocks, following a similar procedure as the first pass costs by Novy-Marx & Velikov (2016), and likely is much higher than the 20-bps estimated. Section 5.3 expands on the point. Secondly, mean-variance optimal portfolios in section 4.3 are very different from the ones that an investor would choose when only focusing on ETFs. In a sense, the ETF strategy is similar to the investment in the individual and univariate portfolios considered in section 4.3.5, which showed a different strategy than the factor-tilt portfolios. Analyzing the optimal strategy using ETFs is beyond the scope of the thesis. Rather, it is noted that if the multifactor strategy is chosen using ETFs, the optimal portfolio is likely very different from the ones studied in section 4.3 and its sub-sections.

### 4.6 Predicting Returns and Volatility via GARCH

All the above analyses based their estimates of variances and means on historical volatility. However, volatility changes heavily over time, and historical volatility and return are by no means equal to the return and volatility of the next period (Schwert, 2011). Further, sections 4.3 and 4.4 have established that even though there is a gain to factor-timing theoretically, and a mean-variance approach works theoretically, the inputs must be estimated correctly and robustly for the gains to materialize. Therefore, this section estimates the variance-covariance matrices of the eight factors via GARCH models. The estimation of the model follows the outline in section 3.3.5, meaning that the first day of the daily estimation is Jul. 2$^{\text{nd}}$, 1968, and the first month of the monthly GARCH is Jul. 1$^{\text{st}}$, 1988.

The series on which GARCH is applied needs to be stationary. If they are not, finite unconditional variance does not exist in the model Linton (2019). Table 4.26 presents the Dickey-Fuller test for stationarity, where the null hypothesis is that the series exhibits a unit root, i.e., that the series is not stationary. The critical value of the distribution at the 1% level with the trend is -3.96 (Campbell et al., 1997). All test statistics are comfortably below that, so the null hypothesis is rejected, meaning that all time-series are stationary. Therefore, it is safe to continue with GARCH.

To assess the fit of the GO-GARCH(1,1), the thesis compares the absolute daily market excess returns with the volatilities estimated for the market excess return. Naturally, the fit is not one-to-one on the level, but it should be evident that when the absolute returns are extreme, the volatility is so as well. Figure 4.11 shows that the GO-GARCH(1,1) does a decent job. The volatility is a bit lower than the absolute returns and the spikes are not quite as dramatic as in reality, but the model captures the direction of the spikes and volatility clusters of the actual series very well. This is especially beneficial

| | Dickey-Fuller Statistic |
|---|---|
| Market | -25.020 |
| Size | -21.960 |
| Value | -22.935 |
| Earnings | -22.179 |
| Investment | -22.326 |
| Momentum | -23.729 |
| Quality | -21.951 |
| BAB | -22.456 |

**Table 4.26:** *Dickey-Fuller Tests for Stationarity. Data: Daily from Jul 1$^{st}$, 1963 to Nov 30$^{th}$, 2022. Illustration: The author.*

to the investor, as his backwards-looking model would never capture events such as Black Monday and the Financial Crisis, where the volatility spikes heavily. Figure 4.12 shows that the GARCH fitted AR(1) mean of returns is far from as volatile as the actual mean return. The directions are captured well, and spikes happen in both series at the same time, but the size of the spikes are a very far cry from reality, indicating that using the GARCH estimated means may lead to very faulty mean-variance solutions. Finally, Figure 4.10 presents the GO-GARCH(1,1) estimated correlation between the market excess return and the value factor compared to the rolling actual 200-day correlation on said variables. The picture is the same as for the means and variance. The overall trends are the same, but the actual time series exhibits even more volatility than the GARCH estimation. The conclusion is that the GO-GARCH(1,1) model estimates well the variables it was developed for, namely correlation and volatility, while the means need a different estimation method.

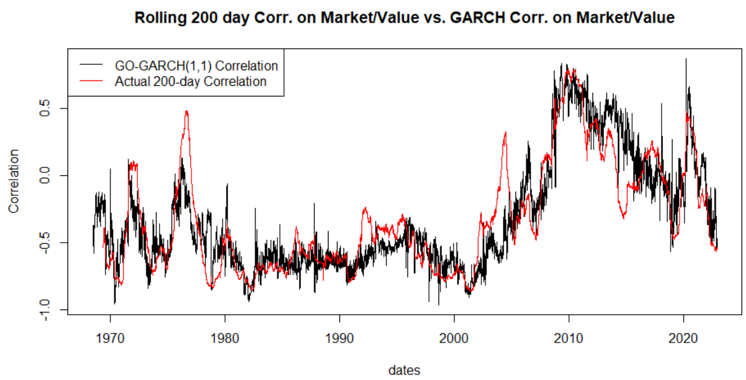


**Figure 4.10:** *Fit of the correlation between the market and value excess returns of the daily GO-GARCH(1,1) model. Data: Daily, estimates from Jul. 1$^{st}$, 1968 to Nov. 30$^{th}$, 2022. Model per sections 2.5 and 3.3.5 Illustration: The author.*

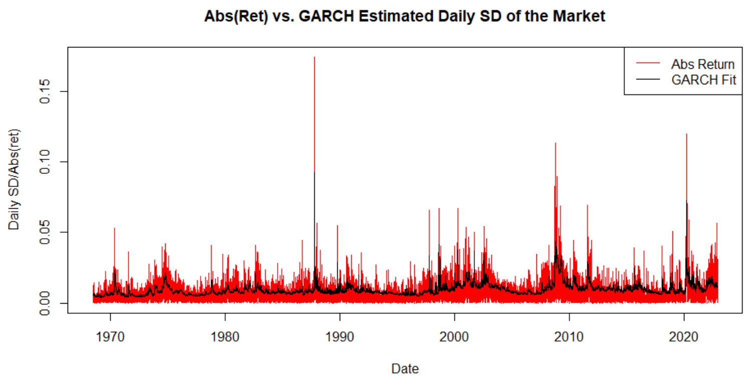


**Figure 4.11:** *Fit of the GO-GARCH(1,1) estimated daily volatility compared to the actual absolute daily returns. Data: Daily, estimates from Jul. 1$^{st}$, 1968 to Nov. 30$^{th}$, 2022. Model per sections 2.5 and 3.3.5 Illustration: The author.*

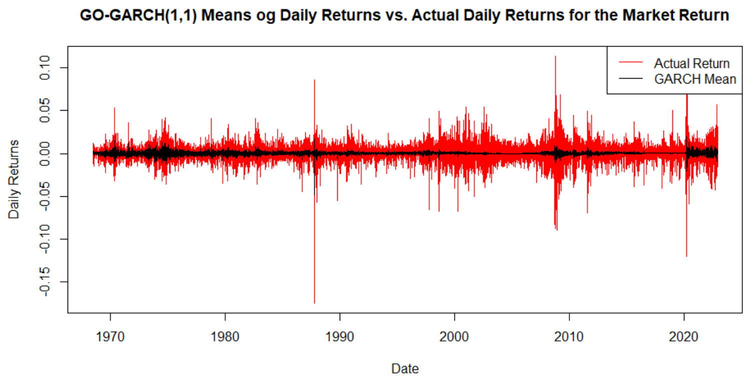


**Figure 4.12:** *Fit of the GO-GARCH(1,1) estimated daily means compared to the actual daily returns. Data: Daily, estimates from Jul. 1$^{st}$, 1968 to Nov. 30$^{th}$, 2022. Model per sections 2.5 and 3.3.5 Illustration: The author.*

To get a sense of the fit of the model, Table 4.27 presents the coefficients for the fitted GO-GARCH(1,1) on daily data. The beta, or ARCH parameter, is very large, indicating that the lagged squared return plays a large role in determining the volatility. The alpha estimator for the lagged volatilities, special for GARCH, is smaller, and so is the intercept, omega. Interestingly, the lagged volatility plays a larger role in the quality factor than for the other factors, which also is the only factor, apart from the market, having a rather large positive gamma, i.e., indicating a news curve where negative news influences the return more than does positive returns. This is interesting, as the hypothesized direction of the news curve is that the coefficient should be positive. For all other factors than quality and the market, positive news increase volatility more than negative news, just as momentum and BAB virtually have a symmetric news curve. One explanation can be that the factors are long-short portfolios. Hence, assuming a long-only position has a normal news curve, this is cancelled or even reversed by the short positions. The thesis does not investigate the phenomenon further, but it is relevant for the investor as it indicates negative tail risk shocks. On the other hand, subsequent estimations of the fit on the entire series with very small changes in the model assumptions fundamentally change the optimal fitting parameters, thus questioning how large a story can be told from the coefficients and their robustness.

| | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB |
|---|---|---|---|---|---|---|---|---|
| Omega | 0.004 | 0.003 | 0.007 | 0.003 | 0.003 | 0.004 | 0.016 | 0.010 |
| Alpha | 0.066 | 0.076 | 0.074 | 0.079 | 0.075 | 0.046 | 0.100 | 0.097 |
| Beta | 0.915 | 0.921 | 0.935 | 0.922 | 0.932 | 0.946 | 0.852 | 0.906 |
| Gamma | 0.032 | 0.002 | -0.030 | -0.006 | -0.019 | 0.009 | 0.063 | -0.021 |

**Table 4.27:** *GO-GARCH(1,1) coefficients for the daily GO-GARCH(1,1) model. Model fitted on daily data from Jul. $1^{st}$, 1963 to Nov $30^{th}$, 2022, entire period. Model per sections 2.5 and 3.3.5 Illustration: The author. Illustration: The author.*

Figures 8.17-8.19 in Appendix 8.9 show that the fit of the monthly estimation is worse than the daily counterpart. Figure 8.17 shows that the monthly volatilities are more extreme than the daily, which means that the spikes are rather difficult to estimate for the model. However, the DCC-GARCH performs well and captures the larger spikes better than the GO-GARCH(1,1). Appendix 8.9 also estimates an expanding GO-GARCH(1,1) estimate with a very poor fit. As the model gets more and more data, it estimates the spikes too late, as it adjusts slowly, and estimates too little volatility. Therefore, a rolling estimate is preferred. Figure 8.18 shows the fit to the means, which is very poor among all the monthly models. As a way to improve the model, the thesis considers both an AR(2) estimate of the mean as well as the predictive regressions, of section 4.2.1, based on a rolling window of 275 monthly observations, to fit the monthly GARCH estimates. These measures capture some of the volatility, but it does so at the wrong periods. Therefore, the fit is arguably even worse than the standard model means. Comparing the correlation fit of the models in Figure 8.19, the GO-GARCH(1,1) model is a very poor fit on monthly data compared to its daily counterpart. The peaks are estimated too early and too high and the expanding GO-GARCH(1,1) does an even more horrendous job. Only the DCC model has a reasonable fit, as was also the case for the monthly volatilities. Therefore, to utilize the GARCH model, a large data sample on daily data is needed. If the investor considers lower frequencies on monthly data, he needs to be cautious and at least consider the DCC model.

### 4.6.1 Performance of GARCH-estimated Factor-Tilt Portfolios

As the GARCH model only allows for forecasting one period ahead and not aggregating data, the investor must make a choice. He can either estimate weights based on monthly data, forecast one month, and use the weights for the coming month, or get a better estimate and use daily data, but then rebalance each day. Both approaches are presented, and the thesis assumes that the investor buys the factor-tilt portfolios throughout. The monthly estimation involves both the expanding and rolling GO-GARCH(1,1) as well as the DCC-GARCH.

Table 4.28 presents the performance of the models before transaction costs. In line with the better fit of the DCC model, the DCC performs better than the GO-GARCH(1,1) both when measured via Sharpe ratio and information ratio. The portfolio where the mean-estimates are based on the regressions in section 4.2.1 comes in a close second with only a slightly lower Sharpe ratio and information ratio than the DCC-model, primarily due to lower risk. This can indicate that despite the horrible fit, it is the best of the evils when predicting returns. The expanding model provides a better fit than the rolling GO-GARCH here, likely as the availability of more data enhances the model more than does the fact that it is based on stale observations. Further, the AR(2) model performs the worst with both the highest volatility and lowest return due to the mean being grossly misspecified, as was also uncovered in the previous section.

Looking at the daily estimates, the investor gets a phenomenal return and Sharpe ratio as compared to the monthly rebalancing considered through the thesis. The alpha and excess return is extremely large at 41 bps and 9.33% respectively, which is way above that of any of the mean-variance portfolios estimated via backwards-looking windows. The standard deviation is not much higher than for the monthly GO-GARCH(1,1) and lower than for the monthly DCC-GARCH, resulting in a daily Sharpe ratio of 0.264. It must be remembered that to achieve this phenomenal return, the portfolio must be rebalanced daily, inducing much higher transaction costs than the monthly portfolio. Whether this destroys its phenomenal performance is examined next.

| | Daily Rol. | Mon. Roll. GG | Mon. Roll. AR2 | Mon. Exp. GG | Mon. Roll. DCC | Reg. DCC |
|---|---|---|---|---|---|---|
| Mean Monthly Ret (%) | 9.328 | 0.108 | 0.092 | 0.119 | 0.128 | 0.119 |
| Excess Ret (%) | 0.421 | 0.106 | 0.097 | 0.119 | 0.128 | 0.118 |
| Excess Std. Dev (%) | 1.597 | 1.343 | 1.765 | 1.465 | 1.493 | 1.401 |
| Sharpe Ratio | 0.264 | 0.079 | 0.055 | 0.081 | 0.086 | 0.084 |
| Drawdown (%) | -54.450 | -71.172 | -66.462 | -61.927 | -67.313 | -57.037 |
| CAPM Alpha (%) | 0.410 | 0.088 | 0.077 | 0.102 | 0.107 | 0.097 |
| CAPM Beta | 0.419 | 0.519 | 0.568 | 0.487 | 0.580 | 0.551 |
| Information Ratio | 0.355 | 0.117 | 0.069 | 0.111 | 0.129 | 0.124 |

**Table 4.28:** *Performance metrics of GARCH-models. The daily GARCH-model forecasts daily from Jul. $1^{st}$, 1968, to Nov. $30^{th}$, 2022. Monthly models consider Jul, 1988, to Nov., 2022. Returns and data is daily. GG = GOGARCH(1,1), Reg. DCC = DCC volatility and means via predictive regressions in 4.2.1.* Illustration: The author.

### 4.6.2 GO-GARCH Portfolios with Transaction Costs

Table 4.29 shows that, after accounting for transaction costs, the phenomenal performance of the daily estimated GO-GARCH(1,1) is gone. The excess return becomes negative 1.14% per day, resulting in a Sharpe ratio of -0.541, and a loss of all the invested capital. Thus, the realistic daily GO-GARCH(1,1) mean-variance portfolio is of no interest to any sane investor.

Looking towards the monthly models, the loss from trading is much more modest, but the Sharpe ratio generally is worse than looking in the rear-view mirror. Although the DCC-model outperforms the GO-GARCH(1,1) on excess returns, the very poorly fitted expanding GO-GARCH(1,1) pairs the DCC-model on Sharpe ratio due to its lower volatility in the bets leading to lower volatility in the returns after transaction costs. Combining the DCC-model with the regression specified means improves the information ratio and Sharpe ratio to perform the best, again. However, none of the GARCH models outperforms the backwards-looking model on Sharpe ratio nor information ratio. Therefore, the GARCH models does not provide an edge to the investor over simple backwards estimation when using factor-timing and including transaction cost. One reason may be that the monthly GARCH models have much lower absolute means and standard deviations compared to the true series, biasing the weights for all factors. In other words, the weight change and the level of bets in the monthly GARCH are simply so low that the investor misses some of the opportunities presented. Combined with the fit of the means, the story is that the GARCH models can well capture the volatility and gain an edge over the backwards estimate, but not the means, wherefore the GARCH models may have their justice, just simply for estimating volatility and not means. This is examined in the minimum variance portfolios.

| | Daily GARCH | GG. Roll. | GG. Exp. | DCC GARCH | DCC Reg. | Backw. Mean-Var |
|---|---|---|---|---|---|---|
| Mean Monthly Ret (%) | -100.000 | 1.459 | 1.708 | 1.751 | 1.726 | 2.768 |
| Excess Daily Ret (%) | -1.140 | 0.067 | 0.081 | 0.083 | 0.080 | 0.135 |
| Excess Std. Dev (%) | 2.107 | 1.355 | 1.476 | 1.508 | 1.411 | 1.759 |
| Sharpe Ratio | -0.541 | 0.050 | 0.055 | 0.055 | 0.057 | 0.077 |
| Drawdown (%) | -100.000 | -73.855 | -65.280 | -70.410 | -64.130 | -69.378 |
| CAPM Alpha (%) | -1.148 | 0.049 | 0.064 | 0.063 | 0.060 | 0.120 |
| CAPM Beta | 0.434 | 0.513 | 0.481 | 0.575 | 0.545 | 0.414 |
| Information Ratio | -0.696 | 0.065 | 0.069 | 0.074 | 0.075 | 0.105 |

**Table 4.29:** *Performance metrics of GARCH-models along with the backwards-looking estimate, after TC. All models consider daily data from Jul. $1^{st}$, 1988, to Nov. $30^{th}$, 2022. Illustration: The author.*

### 4.6.3 Minimum Variance Portfolios

Section 4.6.2 showed that the GARCH models, no matter specification, could not properly forecast the mean of the returns, which limits the applicability in a mean-variance setting. To fully assess the root of the difficulty of the GARCH models, this sub-chapter considers the minimum-variance portfolios.

Table 4.30 shows the performance of the minimum variance portfolio estimated on monthly data, while Table 8.18 in Appendix 8.10 shows the results for the daily GO-GARCH (1,1) vs. a backwards-estimated model rebalanced each day. Table 8.18 clarifies that the GO-GARCH(1,1) yields a higher return for practically the same variance as the backwards-looking model. Including transaction costs, however, the daily GO-GARCH(1,1) yields a negative return due to the very large rebalancing of daily weights, while the backwards-looking minimum-variance portfolio "only" halves its before-transaction cost return. The explanation is that the GO-GARCH(1,1) model requires much larger daily rebalancing, on average between 3 and 5%-points daily, whereas the backwards-looking model only rebalances between 70 and 120 bps of the weights daily, wherefore the transaction costs naturally turn higher for the GO-GARCH model. Again, the story is that factor-timing pays, but the variations in the bets need to be small for the bets to pay off.

Table 4.30 cements that the GARCH models do well estimating the volatility before transaction costs. When estimated on monthly data, all GARCH models outperform the backwards-looking minimum-variance portfolio on Sharpe ratio and return. Because the portfolios are only rebalanced monthly, the transaction costs are smaller, which also means that the GARCH models outperform even after transaction costs.

| | Back. | GG-Roll. | GG-Exp. | DCC | Back. TC | Roll. TC | Exp. TC | DCC TC |
|---|---|---|---|---|---|---|---|---|
| Mon.Ret(%) | 0.987 | 1.468 | 1.365 | 1.419 | 0.946 | 1.406 | 1.302 | 1.366 |
| Ex.Ret (%) | 0.040 | 0.063 | 0.059 | 0.061 | 0.038 | 0.060 | 0.056 | 0.058 |
| SD (%) | 0.886 | 0.971 | 1.030 | 0.947 | 0.886 | 0.970 | 1.030 | 0.947 |
| Sharpe | 0.045 | 0.065 | 0.057 | 0.064 | 0.043 | 0.062 | 0.054 | 0.062 |
| DD (%) | -60.160 | -37.185 | -33.517 | -23.677 | -60.611 | -37.751 | -33.966 | -23.742 |
| Alpha (%) | 0.021 | 0.041 | 0.034 | 0.038 | 0.019 | 0.038 | 0.031 | 0.035 |
| Beta | 0.538 | 0.630 | 0.701 | 0.655 | 0.538 | 0.630 | 0.701 | 0.655 |
| IR | 0.077 | 0.163 | 0.148 | 0.188 | 0.070 | 0.151 | 0.136 | 0.176 |

**Table 4.30:** *Performance of Monthly Estimated Minimum Variance Portfolios. Data: Daily from Aug. 1988 to Nov. 2022. Back = Backwards Estimated means and variances with 252-day rolling window. GG = GO-GARCH(1,1), Exp. = Expanding Window, Roll. = Rolling Window. TC = Transaction Costs. Mon Ret. = Average monnthly return, Ex Ret. = Daily excess return, SD = Daily standard deviation of excess returns, Sharpe = Daily Sharpe ratio, DD = maximum Drawdown, Alpha = Daily CAPM alpha, Beta = Daily CAPM beta, IR = Daily Information Ratio. Illustration: The author.*

***Sub-Conclusion on GARCH-Models***

Section 4.6 answers RQ3 and establishes that a GARCH model cannot gain an edge in the Sharpe ratio compared to the backwards-looking estimate when considering standard mean-variance models. The thesis shows that a daily GO-GARCH(1,1) results in a very high Sharpe ratio of 0.264 daily, but that this is eaten up by the large transaction costs due to large daily rebalancing. Neither of the monthly GARCH-models outperforms the backwards-looking estimate after transaction costs, despite a reasonable fit of forecasted correlations and volatilities. The reason is that the GARCH-models forecast means horribly, even more so than a backwards-looking model, which means that any gain of factor-timing is destroyed by poor return forecasting. Therefore, a GARCH model is only useful if considering transaction costs and mean forecasting separately or used for minimum-variance portfolios.

# 5 Discussion

This chapter discusses the implications of the analysis for the investor as well as for further research.

## 5.1 Is It Possible to Gain Superior Returns by Factor-Timing?

RQ1 is concerned with whether an investor can find an asset allocation strategy to time factors and achieve a higher return-to-risk-reward than an equal-weighted portfolio can provide. Previous studies have agreed that it can be done, but doing so is very tricky.

Ilmanen et al. (2021) find that when a portfolio is built solely of factors, as done in section 4.3.1, factor-timing adds very little benefit. While the authors find that it is possible to beat the equal-weighted portfolio, by structuring weights on a combination of predictive regressions, z-scores on valuation spreads and momentum in the factor itself, the impact is very small. Using full-sample estimation the investor can increase his Sharpe ratio from 1.48 to 1.73. Considering an out-of-sample model, the Sharpe ratio increases a tiny 0.03 points to 1.51. The results echo those of Khang et al. (forthcoming 2023) examining momentum, value, and quality from 1963 to 2020. Examining 450 estimation methods, the authors conclude that it is very difficult to beat the 1/N equal-weighted portfolio producing returns in the 92nd percentile with relatively low risk.

The findings of the thesis are divided on the topic. Section 4.3.2 illustrates that time-varying factor-tilt portfolios based on a backwards-looking mean-variance window close to triples the Sharpe ratio compared to an equal-weight portfolio. Even after considering transaction costs, a rolling mean-variance weight doubles the Sharpe ratio compared to the equal-weight counterpart. Taking this approach, the investor has to be prepared for a heavily increased risk, however, as it is close to impossible to forecast future returns and get the right mean-variance weights. In the presence of investors being ambiguity- or regret-averse the additional risks are particularly relevant, because it has the potential to drive the investors towards the equal-weighted portfolio although the expectation of factor-timing is positive. Therefore, a TAA-approach that minorly adjusts the equal-weight portfolio to the business cycle or underlying economy is found to be worthwhile, as the risk is reduced considerably. These results second Asness (2017), who investigate value timing and find only a small benefit of a timed portfolio to that of an equal-weighted factor portfolio, as one ends up with larger bets on value than intended due to the stochastic returns being close to impossible to predict. Overall, a factor-timing strategy seems to add value, however.

On the other hand, the added alpha per unit of added idiosyncratic risk, given by the information ratio, is substantially lower for the mean-variance portfolios than the equal-weight portfolio across all methodologies. The TAA-approach outperforms the equal-weight portfolio both on Sharpe ratio and idiosyncratic risk, although the Sharpe ratio is nowhere near that of the mean-variance approaches. The discussion then is which risk measure the investor cares the most about. One argument is that, when considering the pure factor portfolios in section 4.3.1, the investor does not have the market as a benchmark, and cares about the total return relative to the total risk. When considering the factor-tilt portfolios, the investor has the market as a clear benchmark preferring the information ratio (Jacobs & Levy, 1996). Therefore, the discussion of whether factor-timing can yield superior gains comes down to a methodological discussion of what gains means to the investor. In the end, this depends on the structure of his individual utility function. Under any circumstances, it is clear that adding factors to a market portfolio increases return-to-risk, as the factor-premia do exist. This indicates that no matter the timing, the EMH is poorly fulfilled. Value, quality, earnings and investment rejects the semi-strong form, and momentum destroys the weak form.

## 5.2 Factor Performance and Link to the Macroeconomy

Having established that factor-timing works under certain conditions, RQ2 concerns how the factors and optimal mean-variance portfolio should be timed to the underlying macroeconomic variables and the stages during a business cycle. The literature is rather divided on the topic. Ilmanen et al. (2021) find no significant connection, while Baker & Wurgler (2006) and Aked (2020) find that the factor returns depend on the economy.

The results of the thesis are again dispersed. Comparing returns in different stages of the business cycle suggests that the link between stages in the cycle and returns depend on the economic indicator. The optimal mean-variance weights should be altered substantially in a downturn, focusing heavily on quality while short-selling the highly correlated earnings factor and the cyclical BAB factor. Utilizing the link, however, requires that the investor knows 1) the current business cycle and 2) the returns for all factors in all periods in that particular business cycle. "Only" requiring 2) and assuming the investor bases his investment on the business cycle of the past month, the performance is worse than that of the equal-weight portfolio due to increased risk, signalling that practical implementation of a business cycle stage varying portfolio is very hard. On the other hand, slightly adjusting the equal-weight portfolio as a mean of tactical asset allocation outperforms on both measures even after accounting for lag in the information about the business cycle stage. This approach, even, is the most realistic, as it does not require 1) nor 2), but only assumes that the investor knows the characteristics of each factor, which is very realistic assuming a professional portfolio manager. It seems there is a link between factor performance and the stage of the business cycle, but, following Aghassi et al. (2022), it is very hard to implement.

On the other hand, one may question how large the link to the business cycle is, when the effect is clearly the largest during downturns. Opposing Ilmanen et al. (2021), the thesis finds that the macroeconomic variables can to some extent predict returns better than the business cycle itself. For example, investment, value and momentum benefit from high yields and high inflation. One explanation may be Lettau & Wachter (2007) establishing that growth stocks are long duration, while value is short duration, meaning that the value factor is systematically short duration. By the same logic, high inflation, which tends to raise rates, should theoretically lead to a good relative performance of value. The link towards momentum is more subtle but can be explained through Hou et al. (2015), who find that the momentum factor can be dismantled into a combination of return on equity (ROE) and investment, and that momentum loads very positively and significantly on ROE while not so on investment. The only way to reconcile this is if discount rates, and hence yields, are high, such that the present values of the investments are discounted hard, leading to a high required ROE (Hou et al., 2015). In that sense, if the winners in momentum are more profitable, momentum should load positively on yield. BAB is negatively related to yield and inflation wanting ultra-low rates, which makes sense as the BAB factor has an inherent leverage in the factor, which becomes more expensive in higher yield environments. This also supports Frazzini & Pedersen (2014) that the BAB factor is negatively related to the TED-spread, as credit risk rises with poor economic times and makes borrowing more expensive.

Regressing the weights of the factor-tilt portfolio on the economic variables further supports that the underlying economy is much more important than the business cycle itself. When considering an expanding portfolio with a longer horizon, yield is significant for all variables, just as the default spread and inflation play a large role in determining the weights. The direction is largely the same as the regression of the returns. For the rolling portfolio, the results support Ilmanen et al. (2021) and suggest that the largest explanation for the weights in period $t$ is the return in $t-1$ due to the autocorrelation in the returns. Therefore, the optimal factor allocation seems to be a momentum strategy in itself, fully supporting Ilmanen et al. (2021). On the other hand, the $R^2$ are tiny and alphas often significant, indicating that the considered macroeconomic variables and stage of the business cycle cannot truly explain the return and weights.

In a sense, the results of the thesis mirror those of BridgeWater (2012), who finds that asset returns are a function of the growth and inflationary state the economy is in. For example, inflation-linked bonds fare well during high inflationary times, as does the value factor, while U.S. plain equities are good in growth times, as is size, and pure nominal bonds are good when inflation is falling, as is BAB. The general approach, thus, seems to be that the portfolio manager should adjust his factor-weights on two levels. On the long horizon, SAA-strategy, he should adjust the weights to the underlying economy, and adapt slowly as the economic landscape changes. On the short horizon, the tilt of his TAA-weights should adjust towards the factors that have performed the best in the previous period. All the while, the adjustments need to be minor to reduce risk.

## 5.3 Time-Varying Volatility and Transaction Cost

Having established that the investor can benefit from time-varying factor bets given the right framing of the portfolio, RQ3 estimates if the outperformance persists after transaction costs and if the investor can improve his information set with volatility forecasting.

The usage of GARCH models in the factor-timing literature is rather scarce. Examples include Siaw et al. (2017) examining a selection of mutual funds via a DCC-GARCH, and Alotaibi et al. (2022), who study a dataset of oil companies using the Copula GARCH. Both papers find that applying a GARCH model for mean-variance optimization yields better results than a backwards-looking methodology, but both papers also focus on the tail risk as measured by Worst Case Value at Risk and Value at Risk.

The daily GO-GARCH(1,1) is a good predictor for the daily variance-covariance matrix, less so for the mean. Applying the daily model leads to phenomenal results with a daily Sharpe ratio of 0.264, thrice that of the backwards-looking mean-variance portfolios rebalanced monthly, before transaction costs. Due to large daily rebalancing, all wealth is destructed when considering transaction costs, however. This problem is largely alleviated by the monthly GARCH-models, but these underperform due to misspecified means. On the other hand, the thesis supports the previous papers showing that a GARCH model can, potentially, do better than a backwards-looking mean-variance approach.

An issue also pertains to how the investment should be made in practice. The estimate of transaction costs, in a sense, mixes incomparable variables. The first-pass transaction costs per Novy-Marx & Velikov (2016) assumes that the investor trades all the underlying stocks, the second-pass transaction relies on the fact that the investor can trade these factors via an ETF. In practice, while ETFs exist, their ability to replicate the studied factor is horrid. Therefore, the true cost of rebalancing the portfolio, as well as the true shorting costs, are likely more expensive, especially for the more illiquid factors such as size and momentum. Firstly, the investor, in practice, must trade an underlying bucket of stocks. Hasbrouck (2009), on which Novy-Marx & Velikov (2016) is based, examines the effective transaction costs using the Gibbs sampler technique throughout the last century. While the median costs of around 10-15 bps are applied in this study, smaller stocks easily hit transaction costs above 1% (Hasbrouck, 2009). Hence, when an investor trades the size factor including small, illiquid stocks or the momentum factor going the same way as the rest of the market, the costs are likely much higher than 20 bps. In that sense, the rolling portfolios become even more expensive to trade, meaning that the equal-weighted alternative is more attractive. On the other hand, the underlying allocations may cancel each other out. Introducing transaction costs and time-varying volatility, therefore, complicates the picture and challenges if a factor-timing strategy pays off in reality, but also cements where a practitioner should have his focus.

## 5.4 Implications for Further Research

The limitations of the thesis lie in the methodology of constructing the factors as well as the implementation of a static mean-variance framework on what, essentially, is a dynamic problem. This problem becomes particularly large, as the mean-variance framework requires correct estimates of the volatility and means, which the thesis establishes is close to impossible to forecast. Therefore, future literature must take a fundamentally different starting point and consider dynamic models while redefining the factors to be comparable, i.e., utilizing the same breakpoints and sorts to ensure that the time-varying strategy is not methodologically dependent. Given that this paper only considers equity returns, it is also of interest to expand the analysis to other asset classes such as inflation-linked bonds, commodities, and straight-bonds (BridgeWater, 2012).

The dynamic model should consider methods to estimate the transaction costs more robustly. Collin-Dufresne et al. (2022) develop such a model based on stochastic returns and quadratic transaction costs, just as the authors consider risk aversion. In short, the model establishes that, in line with Merton (1971), it can be optimal to pick a position that hedges against the Markowitz (1952) portfolio, just as with transaction costs the optimal position only trades towards the Markowitz (1952) portfolio, depending on the correlation between the stock returns and shocks to expected returns. The insights are very similar to Gârleanu & Pedersen (2013), suggesting the investor should trade towards the "expected" Markowitz (1952) portfolio. The transaction costs can be estimated via a methodology such as Novy-Marx & Velikov (2016). However, the largest component of transaction costs comes in the strategic trading and price impact, as estimated through strategic trade models such as Kyle's lambda (Linton, 2019). To some extent, this is already implemented in Gârleanu & Pedersen (2013). The issue here is that it requires a market understanding of all stocks at all given times, which is a very large task.

Further, the thesis establishes that the standard methods used in the finance factor literature are of questionable use, as the true data does not fit the standard normal distribution nor the OLS assumptions. Therefore, examining the relations with more robust statistical methods such as GMM or even GLS is of much interest to future studies to validate the results of the thesis. Following this, the thesis only considers the maximum drawdown and standard deviation of the portfolios as a measure of risk, which assumes normality mathematically, a condition that is clearly not true. Therefore, it is of interest to the investors to test the performance using more robust risk measures such as Value at Risk and Expected Shortfall, which in some cases outperform the standard mean-variance approach (Lwin et al., 2017). A portfolio manager can usually sustain a rather high standard deviation in the short run, but the tail risk can bring the strategy and portfolio down, wherefore the measure is of interest.

Finally, the thesis shows that, while the GARCH model can help in problems only involving the variance-covariance matrix, it is defeated by the need for mean returns. Therefore, studies examining how to forecast mean returns, and combining these with GARCH estimates and transaction costs, are highly valuable.

## 5.5 The Current Optimal Factor Portfolio

The thesis set out to examine the optimal time-varying factor portfolio. Given that the factor portfolio depends on the underlying economy as well as previous returns, it is obvious to take a look at the current economic environment and establish which factors are currently desirable given the results of the thesis. As the data sample of the sentiment index ends in Jul. 2022, making a current prediction based on the regressions is beyond the scope of the thesis. Therefore, the discussion is qualitative. The most updated factor return data at the time of writing is Feb. 2023, wherefore the section considers an investor taking a position in Mar. 2023.

In the latter months of 2022 and the beginning of 2023, the rates market has seen the largest changes in many decades. The 3-month treasury bill rate soared from 5.6 bps annually on the first trading day in 2022 to 513.3 bps as of the market open, Apr. 17$^{\text{th}}$, 2023 (Investing.com, 2023a). The 20y yield increased from 205.8 bps to 392 bps in the same period (Investing.com, 2023b). This not only is an extreme increase but also creates a very peculiar situation, where the term premium is negative. This only happens rarely and has historically been a bullet-proof sign of recession. This is coupled with the OECD CLI indicators being below 100 and downwards trending, which it has been since Apr. 2022, indicating a downturn (OECD, 2023a). Simultaneously, inflation has soared from a YoY growth rate of 1.1% at the beginning of 2021 to 8.9% at its peak in July, 2022 only to slow down slightly at 4.9% in Feb., 2023. The default spread has widened from around 60 bps in mid-2021 to 110 bps in Feb. 2023, but the movement is, comparable to past widenings, small (FRED, 2023c,b). Section 4.3.2 also showed that the weights are positively related to the return of the factor in the past month. From mid-2021 to Jan. 2023 value, quality and earnings outperformed the other factors. In recent months, however, size and the market have begun to outperform (French, 2023f). This indicates a shift away from value and quality and towards size, which goes directly against the intuition of the underlying economy.

The current portfolio depends on whether the investor is utilizing the expanding or rolling methodology. On the one hand, the current state of the economy talks towards loading up on momentum, quality and earnings, which benefits from high yields, widening default spreads, high inflation and a low term-spread, exactly the economy we have now. Also, given the negative correlation between value and momentum, and the fact that value is positively related to high yields, indicates that the portfolio should be supplemented with value, just as investment was good in times with high inflation. The rolling portfolio, on the other hand, indicates the investor should load up on size. Therefore, the question is a tricky one to answer, which very well illustrates the challenge of timing factors and forecasting weights based on the economy. Therefore, the best strategy likely is to overweight quality, momentum, earnings and value, keep size and investment at its equal-weight level and underweight the other factors in a TAA-strategy. That way, the investor can benefit from adapting to the current economic stage, without being punished heavily, if he is wrong in his assumptions about the economy.

# 6 Conclusion

This thesis set out to establish if a factor-timing strategy in the U.S. equity factors could yield superior risk-adjusted performance to an equal-weighted portfolio, as well as determine the optimal allocation over time and stages in the economy.

The thesis used data on the excess return of eight factors: the market, value, size, investment, earnings, momentum, quality and BAB from 1963 to 2022. The thesis found the optimal factor-timing portfolio is more of a methodological question rather than a set-in-stone rule for how the weights should vary with time and the underlying economy.

The thesis uncovered that if considering a standard portfolio of long-short factors, factor timing only marginally improved the Sharpe ratio and information ratio, even with perfect foresight. In contrast, a rolling mean-variance approach in factor-tilt portfolios, having a constant weight of 100% in the market in addition to factor bets, could more than triple the Sharpe ratio before transaction costs. The factor-tilt portfolio, however, showed a much lower information ratio than the equal-weighted counterparts due to very high idiosyncratic risk. The culprit was to avoid too large and volatile factor weights, which increased risk and transaction costs. Nevertheless, a rolling methodology with weight caps performed better than the slowly adjusting expanding methodology. The conclusion is that factor-timing did pay if the investor was concerned with the total return relative to total risk, and was willing to endure large volatility and drawdown in the process. If the investor was focused on the risk-return relative to the market as a benchmark, factor-timing did not pay due to high idiosyncratic risk. The best investment strategy the investor could make was to invest in the constituent portfolios of the factors, i.e. both value and growth, which allowed for more investment opportunities, for example shorting both ends of a factor. The thesis, however, found that transaction cost drastically altered the optimal factor weights favoring the factors that were rebalanced rarely in their construction. Therefore, a robust estimation of transaction costs is warranted in future studies.

The results showed no clear link between factor returns and weights and the underlying stage of the business cycle. The OECD CLI Indicators provided the largest statistical link between the economy and returns. The only real benefit from varying the asset allocation to the business cycle was altering the weights towards quality and earnings in downturns. The thesis established that this arose due to the underlying macroeconomic variables explaining the returns and determining the optimal weights, rather than the stage of the business cycle itself capturing the variance of the factor weights and returns. The thesis found that the investor should focus on size in times of economic growth and on quality and earnings in economic hardships. Further, the value and investment factor could serve as inflation and rising yield hedges, while BAB should be utilized when long-term rates were very low. Momentum was good for an economy in slowdown or expansion, where the autocorrelation of returns was high. However, the link to the economy was only found to be valid in the long run. In the short run, the lagged return of the factors, due to autocorrelation, determined returns and weights. The thesis established that, theoretically, a link existed between the factor returns and the underlying economy but utilizing it in an investment strategy required perfect foresight of returns and business cycle stages, wherefore, the only valid option was to adjust the weights slowly and marginally to the underlying economy. Doing so, even with lagged information about the business cycle stage, outperformed the equal-weight benchmark. This can then be combined with short-term deviations in the TAA-strategy based on the factors that performed well in the previous period, utilizing the autocorrelation.

Throughout the thesis, it was clear that the true hurdle of factor-timing was to estimate the returns and variances correctly in the future. Using multivariate GARCH approaches provided a good fit for volatilities and correlations and could phenomenally outperform a backwards-looking estimation with a daily rebalancing schedule before transaction costs. However, the daily model resulted in a 100% loss for the investor due to the very frequent trading of large positions. When the models were estimated on monthly data, the performance was worse than a backwards-looking mean-variance approach due to very poor estimations of the mean returns. The thesis, therefore, determined that if the GARCH model should be used for anything else than minimum-variance portfolios, a more complex estimation of means coupled with an explicit model for transaction costs was needed.

Overall, factor-timing is, theoretically, beneficial and should be linked to the underlying macroeconomy, but implementing it requires forecasting returns, macroeconomic variables and robust transaction costs, which is a very large task awaiting future studies and close to impossible in practice.

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# 8 Appendix

## 8.1 Derivations

**Proof of equation 2.2 - Mean-Variance Optimal Portfolio**

We start with assuming the investor wishes top optimize equation 2.1:

This minimization problem can be solved via the Lagrange Multiplier, as there are here two constraints, we get that the equation has three unknowns, the weight, lambda and gamma:

$$\mathcal{L}\left(\boldsymbol{w},\lambda,\gamma\right)=\frac{1}{2}\mathbf{w'\Sigma w}+\gamma\left(m-\mathbf{w'}\boldsymbol{\mu}\right)+\lambda\left(1-\mathbf{w'i}\right) \tag{8.1}$$

We take the first derivative with respect to weight, wet it equal to 0 and isolate it for $w$.

$$\frac{\partial L}{\partial \mathbf{w}}=\mathbf{w'\Sigma}-\gamma\boldsymbol{\mu}-\lambda\mathbf{1} \tag{8.2}$$

$$\mathbf{w}=\gamma\boldsymbol{\mu}'\mathbf{\Sigma^{-1}}+\lambda\mathbf{i}'^{\mathbf{\Sigma^{-1}}} \tag{8.3}$$

8.3 is plugged into the constraints in 2.1 yielding 8.4 and 8.5

$$\left(\gamma\mu'\mathbf{\Sigma^{-1}}+\lambda\mathbf{i}'\mathbf{\Sigma^{-1}}\right)\mathbf{i}=1 \tag{8.4}$$

$$\left(\gamma\mu'\mathbf{\Sigma^{-1}}+\lambda\mathbf{i}'^{\mathbf{\Sigma^{-1}}}\right)\mu=m \tag{8.5}$$

We can then make life a bit easier, by naming the following constants

$$A=\mathbf{i}'\mathbf{\Sigma^{-1}i} \tag{8.6}$$

$$B=\boldsymbol{\mu}'\mathbf{\Sigma^{-1}i} \tag{8.7}$$

$$C=\boldsymbol{\mu}'\mathbf{\Sigma^{-1}}\mu \tag{8.8}$$

$$\Delta=AC-B^2 \tag{8.9}$$

Applying these, the 8.4 and 8.5 can be written as 8.10 and 8.11:

$$\gamma B+\lambda A=1 \tag{8.10}$$

$$\gamma C+\lambda B=m \tag{8.11}$$

Isolating for gamma in 8.11:

$$\gamma=\frac{m-\lambda B}{C} \tag{8.12}$$

Then we can take 8.12 and plug into 8.10 and isolating for lambda:

$$\frac{m - \lambda B}{C} B + \lambda A = 1 \tag{8.13}$$

$$\frac{Bm}{C} - \frac{\lambda B^2}{C} + \lambda A = 1 \tag{8.14}$$

$$\frac{Bm}{C} + \lambda \left( A - \frac{B^2}{C} \right) = 1 \tag{8.15}$$

$$1 - \frac{Bm}{C} = \lambda \left( A - \frac{B^2}{C} \right) \tag{8.16}$$

$$\lambda = \frac{1 - \frac{Bm}{C}}{\left( A - \frac{B^2}{C} \right)} \tag{8.17}$$

We multiply through with Constant C as defined in 8.8

$$\lambda = \frac{C - Bm}{AC - B^2} = \frac{C - Bm}{\Delta} \tag{8.18}$$

8.18 expresses lambda as a function of the known constants. Plugging 8.18 into 8.10 and isolating for gamma:

$$\gamma B + \left( \frac{C - Bm}{\Delta} \right) A = 1 \tag{8.19}$$

$$\gamma B = 1 - \frac{(CA - BAm)}{\Delta} \tag{8.20}$$

$$\gamma = \frac{1}{B} - \frac{\frac{CA}{\Delta}}{B} + \frac{Am}{\Delta} \tag{8.21}$$

$$\gamma = \frac{1}{B} - \frac{CA}{\Delta B} + \frac{Am}{\Delta} \tag{8.22}$$

$$\gamma = \frac{(Am - B)}{\Delta} \tag{8.23}$$

Plugging 8.23 back into 8.3, equation 2.2 is obtained:

**Q.E.D.**

**Proof of equation 2.3, the Tangency Portfolio**

Applying the Lagrange to equation 2.3 gives equation 8.24:

$$\mathcal{L}(\mathbf{w},\kappa) = \frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} + \left(m - r_f - \mathbf{w}'\left(\boldsymbol{\mu} - r_f\mathbf{i}\right)\right) \tag{8.24}$$

Taking the derivative w.r.t. the weight vector, setting it equal to 0 and isolating for **w**

$$\frac{\partial L(w,k)}{\partial \mathbf{w}} = \mathbf{w}'\boldsymbol{\Sigma} - \kappa\left(\boldsymbol{\mu} - r_f\mathbf{i}\right) \tag{8.25}$$

$$\mathbf{w} = \kappa\left(\boldsymbol{\mu} - r_f\mathbf{i}\right)\boldsymbol{\Sigma}^{\mathbf{-1}} \tag{8.26}$$

Plugging the expression for weight in 8.26 into the constraint in equation 2.3 and isolating for $\kappa$ yields 8.27

$$r_f + \kappa\left(\boldsymbol{\mu} - r_f\mathbf{i}\right)\boldsymbol{\Sigma}^{\mathbf{-1}}\left(\boldsymbol{\mu} - r_f\mathbf{i}\right) = m \tag{8.27}$$

$$\frac{m - r_f}{\left(\boldsymbol{\mu} - r_f\mathbf{i}\right)\boldsymbol{\Sigma}^{\mathbf{-1}}\left(\boldsymbol{\mu} - r_f\mathbf{i}\right)} = \kappa \tag{8.28}$$

This expression in 8.28 can then be plugged into 8.26 to yield 8.29

$$\mathbf{w} = \frac{m - r_f}{\left(\boldsymbol{\mu} - r_f\mathbf{i}\right)\boldsymbol{\Sigma}^{\mathbf{-1}}\left(\boldsymbol{\mu} - r_f\mathbf{i}\right)}\left(\boldsymbol{\mu} - r_f\mathbf{i}\right)\boldsymbol{\Sigma}^{\mathbf{-1}} \tag{8.29}$$

Now, we are almost there, but $m$ can also be expressed in terms of the other parameters. Inserting 8.29 into the constraint in equation 2.3 yields 8.30:

$$r_f + \frac{m - r_f}{(\boldsymbol{\mu} - r_f\mathbf{i})\boldsymbol{\Sigma}^{\mathbf{-1}}(\boldsymbol{\mu} - r_f\mathbf{i})}(\boldsymbol{\mu} - r_f\mathbf{i})\boldsymbol{\Sigma}^{\mathbf{-1}}(\boldsymbol{\mu} - r_f\mathbf{i}) = m \tag{8.30}$$

Now it is simply a matter of modelling around the equation to isolate for $m$

$$r_f + (m - r_f)\cdot\mathbf{1}\left(\boldsymbol{\mu} - r_f\mathbf{i}\right)\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\mu} - r_f\mathbf{i}\right) = m \tag{8.31}$$

Multiplying out in the parenthesis:

Subtracting m from both sides as well as adding $r_f$ and flipping sides

$$\begin{aligned} &r_f + m\left(\tfrac{1}{(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-r_f\mathbf{i})}\right)(\boldsymbol{\mu} - r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - r_f\mathbf{i}) - \\ &r_f\left(\tfrac{1}{(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-r_f\mathbf{i})}\right)(\boldsymbol{\mu} - r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - r_f\mathbf{i}) = m \end{aligned} \tag{8.32}$$

$$\begin{aligned} &r_f\left(\tfrac{1}{(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-R_f\mathbf{i})}\right)(\boldsymbol{\mu} - r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - r_f\mathbf{i}) - r_f = \\ &m\left(\tfrac{1}{(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-r_f\mathbf{i})}\right)(\boldsymbol{\mu} - r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - r_f\mathbf{i}) - m \end{aligned} \tag{8.33}$$

Factorizing $m$ out:

$$\begin{aligned} &r_f\left(\tfrac{1}{(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-r_f\mathbf{i})}\right)(\boldsymbol{\mu} - r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - r_f\mathbf{i}) - r_f = \\ &m\left(\left(\tfrac{1}{(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-r_f\mathbf{i})}\right)(\boldsymbol{\mu} - r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - r_f\mathbf{i}) - 1\right) \end{aligned} \tag{8.34}$$

And finally isolating for $m$.

$$m = \frac{r_f \left(\frac{1}{(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\mu-r_f\mathbf{i})}\right)(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-r_f\mathbf{i})-r_f}{\left(\left(\frac{1}{(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-r_f\mathbf{i})}\right)(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-r_f\mathbf{i})-1\right)} \tag{8.35}$$

The expression for $m$ in 8.35 can then be plugged into 8.29 yielding 8.36

$$\mathbf{w} = \frac{\frac{r_f\left(\frac{1}{(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-r_fi)}\right)(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-r_f\mathbf{i})-r_f}{\left(\left(\frac{1}{(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-r_fi)}\right)(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-r_f\mathbf{i})-1\right)}-r_f}{(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-r_fi)}(\boldsymbol{\mu}-r_f\mathbf{i})\boldsymbol{\Sigma}^{-1} \tag{8.36}$$

Reducing equation 8.36 results in the known tangency portfolio solution in equation 2.4 as also presented in 8.37

$$\mathbf{w} = \frac{1}{\mathbf{i}\Sigma^{-1}(\mu-f_f\mathbf{i})}\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-R_f\mathbf{i}) \tag{8.37}$$

**Q.E.D.**

**Proof of Equation 2.6**

The investor wish to solve equation 2.5. Applying the Lagrange technique yields equation 8.38

$$\min_{\mathbf{w}} \mathbf{w}\boldsymbol{\Sigma}\mathbf{w} \text{ st. } \mathbf{wi} = 1 \tag{8.38}$$

Applying the lagrange function:

$$\mathcal{L}(w,\lambda) = \mathbf{w}\boldsymbol{\Sigma}\mathbf{w} + \lambda(1-\mathbf{wi}) \tag{8.39}$$

Taking the first derivative with respect to weight and setting the expression equal to 0:

$$2\boldsymbol{\Sigma}\mathbf{w} - \lambda\mathbf{i} = 0 \tag{8.40}$$

$$\lambda\mathbf{i}2\boldsymbol{\Sigma}^{-1} = \mathbf{w} \tag{8.41}$$

Inserting 8.41 into the constraint in equation 2.5 yields 8.42, whereafter we isolate for $\lambda$ in 8.43

$$\lambda\mathbf{i}2\boldsymbol{\Sigma}^{-1}\mathbf{i} = 1 \tag{8.42}$$

$$\lambda = \frac{1}{\mathbf{i2\Sigma^{-1}i}} \tag{8.43}$$

The expression for $\lambda$ in 8.43 is insert into the expression for the weight in 8.41

$$\frac{1}{\mathbf{i2\Sigma^{-1}i}}\mathbf{i2\Sigma^{-1}} = \mathbf{w} \tag{8.44}$$

Reducing the expression yields equation 2.6, restated in 8.45 the solution for the minimum variance portfolio:

$$\frac{\mathbf{i\Sigma^{-1}}}{\mathbf{i\Sigma^{-1}i}} = \mathbf{w} \tag{8.45}$$

**Q.E.D.**

## 8.2 Technical Construction of the Quality Factor

This appendix shows the detailed calculation of the components of the quality factor. Hence, it outlines the relevant accounting variables, their updating and how each is constructed into the composite metrics used in quality.

### Profitability

Profitability is constructed as the combination of gross profits over assets (GPOA), return on equity (ROE), return on assets (ROA), cash flow over assets (CFOA), gross margin (GMAR) and earnings minus accruals (ACC). The authors use the methodology from Fama & French (1992) and construct variables in June of year t based on the accounting measures reported in fiscal year t-1. To compare the measures within profitability, the variables are each month converted into ranks on which a z-score is calculated. Hence, each month, let $\sigma_r$ and $\mu_r$ denote the cross-sectional mean at that month, respectively. Then, the z-score follows equation 8.46

$$z(x) = \frac{r - \mu_r}{\sigma_r} \tag{8.46}$$

Where $x$ is the accounting measure (e.g. ROE) of interest. The z-score of the entire profitability index is then used to calculate the profitability measure.

### Growth

The growth component is the change in the aforementioned variables such that the growth is calculated as the five-year growth concerning assets. I.e., the change in GPOA is 8.47:

$$\frac{(gp_t - r^{fat}t - 1) - (gpt - 5 - r^{fat}t - 6)}{att - 5} \tag{8.47}$$

Where $gp_t$ is the gross profit at time $t$, and $at_{t-1}$ is the total assets on the balance sheet at year $t-1$, i.e., the two Junes before the rebalancing date. The z-score is calculated following 8.46, based on the five-year cross-sectional ranking across the firms. Hence, the growth is calculated following 8.48

$$\text{Growth} = \text{Average}\left(z_{\Delta GPOA},\ z_{\Delta ROE},\ z_{\Delta ROA},\ z_{\Delta CFOA},\ z_{GMAR}\right) \tag{8.48}$$

### Saftey

Finally, the safety score is constructed as a composite of the z-scores on low beta (BAB), low leverage (LEV) low bankruptcy risk (O-Score and Z-score) and low ROE volatility (EVOL) as illustrated in equation 8.49:

$$\text{Safety} = \text{Average}\left(z_{BAB} + z_{LEV} + z_O + z_Z + z_{Evol}\right) \tag{8.49}$$

Betas are estimated as their negative beta calculated following the Frazzini & Pedersen (2014) methodology of estimating betas based not on the covariance with the market, but on the correlation and standard deviations of said variables as presented by equation 8.50:

$$\beta_i = \rho \cdot \frac{\sigma_i}{\sigma_m} \tag{8.50}$$

In the beta estimation in 8.50, standard deviations, $\sigma_m$ and $\sigma_i$, are one-year rolling estimates, while correlations, $\rho$, are five-year correlations of three-day overlapping returns to account for asynchronous trading (Frazzini & Pedersen, 2014).

The leverage variable, LEV, is calculated as the negative of total net debt to total assets, following equation 8.51, where t-1 indicates the current usage of COMPUSTAT data, i.e. in the previous June.

$$LEV_t = -\frac{Long\ Term\ Debt_{t-1} + Short\ Term\ Debt_{t-1} + Minority\ Interest_{t-1} + Pref.\ Stock_{t-1}}{Total\ Assets_{t-1}} \tag{8.51}$$

The Ohlson's O-score component of the safety score is calculated following equation (A61), where subscript "tc" denotes $t_current$ indicating that this should be updated each month. It is not entirely clear in the original paper, but given that the estimates are rebalanced monthly, and all other accounting variables are only updated yearly, it must be that the variables on which data arises each month are updated monthly. CPI is one such variable.

$$\begin{aligned} O = &- \left(-1.32 - 0.407 \cdot \log\left(\tfrac{ADJASSRET_t}{CPI_{tc}}\right) + 6.03 \cdot TLTA_{t-1}\ \text{-} \right. \\ &1.43 \cdot WCTA_t + 0.076 \cdot CLCA_{t-1} - 1.72 \cdot OENEG_{t-2} - 2.37 \cdot NITA_{t-1} \\ &\left. -1.83 \cdot FUTL_t + 0.285 \cdot INTWO_t - 0.521 \cdot CHIN_t \right) \end{aligned} \tag{8.52}$$

Each of the variables in equation 8.52 is defined following equation (8.53-8.59) all utilizing the same notation as in 8.52.

$$ADJASSET_{tc} = Total\ Assets_t + 0.1 \cdot (Market\ Equity_{tc} - Book\ Equity_t) \tag{8.53}$$

$$TLTA_{tc} = \frac{Book\ Value\ of\ Debt_t}{ADJASSET_{tc}} \tag{8.54}$$

$$WCTA_{tc} = \frac{Current\ Assets_t - Current\ Liabilities_t}{ADJASSET_{tc}} \tag{8.55}$$

$$CLCA_t = \frac{Current\ Liabilities_t}{Current\ Assets_t} \tag{8.56}$$

$$OENEG_t = \begin{cases} 1, & \text{if Total Liabilities}_t > \text{Total Assets}_t \\ 0, & \text{otherwise} \end{cases} \tag{8.57}$$

$$INTWO_t = \begin{cases} 1, & \text{if Net Income} t < 0 \text{ or Net Income} t - 1 < 0 \\ 0, & \text{otherwise} \end{cases} \tag{8.58}$$

$$CHIN_t = \frac{Net\ Income_t - Net\ Income_{t-1}}{|Net\ Income_t| + |Net\ Income_{t-1}|} \tag{8.59}$$

The Altman's Z-score component is calculated following equation 8.60:

$$Z = \frac{1.2 \cdot WC_t + 1.4 \cdot RE_t + 3.3 \cdot EBIT_t + 0.6 \cdot ME_{Tc} + Sales_t}{TA_t} \tag{8.60}$$

Where WC = Working Capital, RE = Retained Earnings, ME = Market Equity and TA = Total Assets Finally, the standard deviation of ROE, $E_{vol}$ is calculated as the standard deviation of quarterly data over 60 quarters. Hence, again, most of the estimation is based on annual accounting numbers.

## 8.3 Data Sources

| Data Variables | Source | Data Period |
|---|---|---|
| **Return Variables - Daily** | | |
| Return on FF5 Factors, Daily | French (2023g) | Jul 1st, 1963 to Mar. 31st 2023 |
| Momentum Return, Daily | French (2023i) | Jul 1st, 1963 to Mar. 31st 2023 |
| Quality Return, Daily | AQR (2023c) | Jul. 1st 1957 to Nov 30th, 2022 |
| BAB Return, Daily | AQR (2023a) | Jan. 12th, 1930 to Nov. 30th, 2023 |
| Risk Free Return, Daily | French (2023f) | Jul 1st 1963 to Mar. 31st, 2023 |
| Value Return Univ. | French (2023j) | Jul 1st 1927 to Mar. 31st, 2023 |
| Size Return Univ. | French (2023m) | Jul 1st 1927 to Mar. 31st, 2023 |
| Momentum Return Univ. | French (2023a) | Jul 1st 1927 to Mar. 31st, 2023 |
| Earnings Return Univ. | French (2023l) | Jul 1st 1963 to Mar. 31st, 2023 |
| Investment Return Univ. | French (2023k) | Jul 1st 1963 to Mar. 31st, 2023 |
| Value Return Ind. | French (2023b) | Nov 3rd 1926 to Mar. 31st, 2023 |
| Size Return Ind. | Calc per equations 3.2 & 3.3 | Jul 1st 1963 to Mar. 31st, 2023 |
| Earnings Return Ind. | French (2023e) | Jul 1st 1963 to Mar. 31st, 2023 |
| Investment Return Ind. | French (2023c) | Jul 1st 1963 to Mar. 31st, 2023 |
| Momentum Return Ind. | French (2023d) | Jul 1st 1963 to Mar. 31st, 2023 |
| **Return Variables - Monthly** | | |
| Return on FF5 Factors, Monthly | French (2023f) | Jan 1963 to Jan 2023 |
| Momentum Return, Monthly | French (2023h) | Jan 1927 to Jan 2023 |
| Quality Return, Monthly | AQR (2023d) | Jul. 1957 to Jan. 2023 |
| BAB Return, Monthly | AQR (2023b) | Dec. 1930 to Jan. 2023 |
| Risk Free Return, Monthly | French (2023f) | Jan 1963 to Jan 2023 |
| **Economic Variables** | | |
| Index Price | Goyal (2023) | Jan. 1871 to Dec. 2021 |
| Index Dividend Trailing 12m | Goyal (2023) | Jan. 1871 to Dec. 2021 |
| Dividend/Price | log(Div) − log(Index) | Jan. 1871 to Dec. 2021 |
| Dividend Yield | log(Div) − log(lag(Index)) | Feb. 1871 to Dec. 2021 |
| Index Earnings Trailing 12m | Goyal (2023) | Jan. 1871 to Dec. 2021 |
| Earnings/Price | log(Earn) − log(Index) | Jan. 1871 to Dec. 2021 |
| Index Variance | Goyal (2023) | Feb. 1885 to Dec. 2021 |
| Long Term Yield | Goyal (2023) | Jan. 1919 to Dec. 2021 |
| Return on AAA-Bonds | Goyal (2023) | Jan. 1919 to Dec. 2021 |
| Return on BAA-Bonds | Goyal (2023) | Jan. 1919 to Dec. 2021 |
| Default Spread | BAA Ret-AAA Ret | Jan. 1919 to Dec. 2021 |
| 3-month T-Bill Rate | Goyal (2023) | Jan. 1920 to Dec. 2021 |
| Term Spread | Long Term Yield-TBill | Jan. 1920 to Dec. 2021 |
| Inflation (CPI) | FRED (2023a) | Jan. 1947 to Feb. 2023 |
| Investor Sentiment | Wurgler (2023) | Jul. 1965 to Jun. 2022 |
| NBER Recessions | FRED (2023c) | Dec. 1854 to Feb. 2023 |
| OECD CLI | OECD (2023a) | Jan. 1955 to Feb. 2023 |
| Real U.S. GDP (Quarterly) | FRED (2023b) | Q1 1947 to Q3 2022 |
| **Transaction Costs** | | |

| | | |
|---|---|---|
| Factor Construction TC | Novy-Marx (2023) | Jan. 1963 to Dec. 2013 |
| **Factor ETF Prices (Compare With Table 20)** | | |
| Value ETF | Yahoo Finance (2023e) | Apr. 18th, 2013 to Mar. 30th, 2023 |
| Momentum ETF | Yahoo Finance (2023b) | Apr. 18th, 2013 to Mar. 30th, 2023 |
| Size ETF | Yahoo Finance (2023d) | Apr. 18th, 2013 to Mar. 30th, 2023 |
| Quality ETF | Yahoo Finance (2023c) | Jul. 18th, 2013 to Mar. 30th, 2023 |
| BAB ETF | Yahoo Finance (2023a) | Oct. 20th, 2011 to Mar. 30th, 2023 |
| Market ETF | Yahoo Finance (2023f) | Jan. 29th, 1993 to Mar. 30th, 2023 |

**Table 8.1:** *Data Sources used in the thesis. Univ. = univatiate sorts per Jegadeesh & Titman (1993) utilized in section 4.3.2. Ind. = individual sorts per Fama & French (2015)*

## 8.4 Correlation Across Economic Stages

This chapter presents the correlation matrices of daily returns in each of the business cycle stages across the three business cycle classifications used in the paper. The correlation tables are utilized in section 4.1.2

### GDP Classification

| | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB |
|---|---|---|---|---|---|---|---|---|
| Market | 1.000 | -0.078 | -0.151 | -0.207 | -0.351 | -0.129 | -0.418 | -0.451 |
| Size | -0.078 | 1.000 | 0.106 | -0.274 | 0.014 | -0.034 | -0.300 | -0.050 |
| Value | -0.151 | 0.106 | 1.000 | 0.045 | 0.553 | -0.251 | -0.085 | 0.079 |
| Earnings | -0.207 | -0.274 | 0.045 | 1.000 | 0.128 | 0.105 | 0.656 | 0.260 |
| Investment | -0.351 | 0.014 | 0.553 | 0.128 | 1.000 | 0.062 | 0.145 | 0.292 |
| Momentum | -0.129 | -0.034 | -0.251 | 0.105 | 0.062 | 1.000 | 0.257 | 0.340 |
| Quality | -0.418 | -0.300 | -0.085 | 0.656 | 0.145 | 0.257 | 1.000 | 0.390 |
| BAB | -0.451 | -0.050 | 0.079 | 0.260 | 0.292 | 0.340 | 0.390 | 1.000 |

**Table 8.2:** *Correlations of the factors considered during an expansion as classified by the GDP-measure per Ilmanen et al. (2021) and section 3.2.2. Data: Daily for all trading days that occurred in an expansion from Jul. 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022 per French (2023g,i); AQR (2023a,c) Illustration: The author.*

| | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB |
|---|---|---|---|---|---|---|---|---|
| Market | 1.000 | -0.093 | -0.183 | -0.209 | -0.377 | -0.080 | -0.422 | -0.513 |
| Size | -0.093 | 1.000 | 0.239 | -0.164 | 0.085 | -0.187 | -0.223 | -0.056 |
| Value | -0.183 | 0.239 | 1.000 | 0.187 | 0.621 | -0.404 | -0.092 | -0.049 |
| Earnings | -0.209 | -0.164 | 0.187 | 1.000 | 0.220 | -0.058 | 0.609 | 0.238 |
| Investment | -0.377 | 0.085 | 0.621 | 0.220 | 1.000 | -0.068 | 0.119 | 0.245 |
| Momentum | -0.080 | -0.187 | -0.404 | -0.058 | -0.068 | 1.000 | 0.161 | 0.385 |
| Quality | -0.422 | -0.223 | -0.092 | 0.609 | 0.119 | 0.161 | 1.000 | 0.430 |
| BAB | -0.513 | -0.056 | -0.049 | 0.238 | 0.245 | 0.385 | 0.430 | 1.000 |

**Table 8.3:** *Correlations of the factors considered during an contraction as classified by the GDP-measure per Ilmanen et al. (2021) and section 3.2.2. Data: Daily for all trading days that occurred in an expansion from Jul. 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022 per French (2023g,i); AQR (2023a,c) Illustration: The author.*

| | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB |
|---|---|---|---|---|---|---|---|---|
| Market | 1.000 | 0.028 | -0.052 | -0.166 | -0.344 | -0.161 | -0.442 | -0.531 |
| Size | 0.028 | 1.000 | 0.284 | -0.240 | 0.076 | -0.174 | -0.326 | -0.272 |
| Value | -0.052 | 0.284 | 1.000 | -0.039 | 0.536 | -0.452 | -0.222 | -0.323 |
| Earnings | -0.166 | -0.240 | -0.039 | 1.000 | 0.068 | 0.070 | 0.605 | 0.139 |
| Investment | -0.344 | 0.076 | 0.536 | 0.068 | 1.000 | -0.016 | 0.068 | 0.150 |
| Momentum | -0.161 | -0.174 | -0.452 | 0.070 | -0.016 | 1.000 | 0.159 | 0.569 |
| Quality | -0.442 | -0.326 | -0.222 | 0.605 | 0.068 | 0.159 | 1.000 | 0.411 |
| BAB | -0.531 | -0.272 | -0.323 | 0.139 | 0.150 | 0.569 | 0.411 | 1.000 |

**Table 8.4:** *Correlations of the factors considered during a recovery as classified by the GDP-measure per Ilmanen et al. (2021) and section 3.2.2. Data: Daily for all trading days that occurred in a recovery from Jul. 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022 per French (2023g,i); AQR (2023a,c) Illustration: The author.*

### NBER Classifications

| | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB |
|---|---|---|---|---|---|---|---|---|
| Market | 1.000 | -0.074 | -0.164 | -0.221 | -0.351 | -0.128 | -0.432 | -0.438 |
| Size | -0.074 | 1.000 | 0.100 | -0.268 | 0.011 | -0.041 | -0.299 | -0.044 |
| Value | -0.164 | 0.100 | 1.000 | 0.064 | 0.568 | -0.233 | -0.081 | 0.100 |
| Earnings | -0.221 | -0.268 | 0.064 | 1.000 | 0.149 | 0.127 | 0.653 | 0.269 |
| Investment | -0.351 | 0.011 | 0.568 | 0.149 | 1.000 | 0.073 | 0.157 | 0.296 |
| Momentum | -0.128 | -0.041 | -0.233 | 0.127 | 0.073 | 1.000 | 0.292 | 0.347 |
| Quality | -0.432 | -0.299 | -0.081 | 0.653 | 0.157 | 0.292 | 1.000 | 0.409 |
| BAB | -0.438 | -0.044 | 0.100 | 0.269 | 0.296 | 0.347 | 0.409 | 1.000 |

**Table 8.5:** *Correlations of the factors during a pure-expansion as classified by the NBER-measure per Stanhope (2016) and section 3.2.2. Data: Daily for all trading days that occurred in a pure-expansion from Jul. 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022 per French (2023g,i); AQR (2023a,c). Illustration: The author.*

| | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB |
|---|---|---|---|---|---|---|---|---|
| Market | 1.000 | -0.170 | -0.244 | -0.156 | -0.371 | -0.077 | -0.390 | -0.527 |
| Size | -0.170 | 1.000 | 0.232 | -0.215 | 0.130 | -0.149 | -0.226 | -0.052 |
| Value | -0.244 | 0.232 | 1.000 | 0.134 | 0.587 | -0.390 | -0.021 | -0.028 |
| Earnings | -0.156 | -0.215 | 0.134 | 1.000 | 0.115 | -0.029 | 0.604 | 0.209 |
| Investment | -0.371 | 0.130 | 0.587 | 0.115 | 1.000 | -0.062 | 0.080 | 0.234 |
| Momentum | -0.077 | -0.149 | -0.390 | -0.029 | -0.062 | 1.000 | 0.109 | 0.403 |
| Quality | -0.390 | -0.226 | -0.021 | 0.604 | 0.080 | 0.109 | 1.000 | 0.362 |
| BAB | -0.527 | -0.052 | -0.028 | 0.209 | 0.234 | 0.403 | 0.362 | 1.000 |

**Table 8.6:** *Correlations of the factors during a recession as classified by the NBER-measure per Stanhope (2016) and section 3.2.2. Data: Daily for all trading days that occurred in a recession from Jul. 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022 per French (2023g,i); AQR (2023a,c). Illustration: The author.*

| | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB |
|---|---|---|---|---|---|---|---|---|
| Market | 1.000 | 0.023 | -0.066 | -0.236 | -0.377 | -0.152 | -0.465 | -0.563 |
| Size | 0.023 | 1.000 | 0.127 | -0.352 | 0.008 | -0.001 | -0.332 | -0.180 |
| Value | -0.066 | 0.127 | 1.000 | -0.024 | 0.484 | -0.390 | -0.222 | -0.143 |
| Earnings | -0.236 | -0.352 | -0.024 | 1.000 | 0.099 | -0.004 | 0.655 | 0.259 |
| Investment | -0.377 | 0.008 | 0.484 | 0.099 | 1.000 | -0.005 | 0.120 | 0.257 |
| Momentum | -0.152 | -0.001 | -0.390 | -0.004 | -0.005 | 1.000 | 0.222 | 0.378 |
| Quality | -0.465 | -0.332 | -0.222 | 0.655 | 0.120 | 0.222 | 1.000 | 0.449 |
| BAB | -0.563 | -0.180 | -0.143 | 0.259 | 0.257 | 0.378 | 0.449 | 1.000 |

**Table 8.7:** *Correlations of the factors during a post-recession as classified by the NBER-measure per Stanhope (2016) and section 3.2.2. Data: Daily for all trading days that occurred in a post-recession from Jul. 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022 per French (2023g,i); AQR (2023a,c). Illustration: The author.*

### OECD Classification

| | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB |
|---|---|---|---|---|---|---|---|---|
| Market | 1.000 | 0.000 | -0.117 | -0.229 | -0.374 | -0.132 | -0.446 | -0.446 |
| Size | 0.000 | 1.000 | 0.106 | -0.293 | -0.014 | -0.045 | -0.339 | -0.036 |
| Value | -0.117 | 0.106 | 1.000 | 0.020 | 0.522 | -0.322 | -0.120 | 0.052 |
| Earnings | -0.229 | -0.293 | 0.020 | 1.000 | 0.150 | 0.107 | 0.668 | 0.231 |
| Investment | -0.374 | -0.014 | 0.522 | 0.150 | 1.000 | 0.044 | 0.169 | 0.272 |
| Momentum | -0.132 | -0.045 | -0.322 | 0.107 | 0.044 | 1.000 | 0.265 | 0.347 |
| Quality | -0.446 | -0.339 | -0.120 | 0.668 | 0.169 | 0.265 | 1.000 | 0.378 |
| BAB | -0.446 | -0.036 | 0.052 | 0.231 | 0.272 | 0.347 | 0.378 | 1.000 |

**Table 8.8:** *Correlations of the factors considered during an expansion as classified by the OECD-measure per Stocks et al. (2022). Data: Daily for all trading days that occurred in an expansion from Jul. 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022 per French (2023g,i); AQR (2023a,c). Illustration: The author.*

| | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB |
|---|---|---|---|---|---|---|---|---|
| Market | 1.000 | -0.054 | -0.239 | -0.201 | -0.368 | -0.123 | -0.416 | -0.556 |
| Size | -0.054 | 1.000 | 0.107 | -0.281 | 0.032 | 0.016 | -0.279 | -0.066 |
| Value | -0.239 | 0.107 | 1.000 | 0.073 | 0.607 | -0.228 | -0.100 | 0.109 |
| Earnings | -0.201 | -0.281 | 0.073 | 1.000 | 0.126 | 0.067 | 0.639 | 0.306 |
| Investment | -0.368 | 0.032 | 0.607 | 0.126 | 1.000 | 0.034 | 0.132 | 0.306 |
| Momentum | -0.123 | 0.016 | -0.228 | 0.067 | 0.034 | 1.000 | 0.253 | 0.293 |
| Quality | -0.416 | -0.279 | -0.100 | 0.639 | 0.132 | 0.253 | 1.000 | 0.444 |
| BAB | -0.556 | -0.066 | 0.109 | 0.306 | 0.306 | 0.293 | 0.444 | 1.000 |

**Table 8.9:** *Correlations of the factors considered during a downturn as classified by the OECD-measure per Stocks et al. (2022). Data: Daily for all trading days that occurred in a downturn from Jul. 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022 per French (2023g,i); AQR (2023a,c). Illustration: The author.*

| | Market | Size | Value | Earnings | Investment | Momentum | Quality | BAB |
|---|---|---|---|---|---|---|---|---|
| Market | 1.000 | -0.147 | -0.162 | -0.172 | -0.344 | -0.111 | -0.431 | -0.496 |
| Size | -0.147 | 1.000 | 0.209 | -0.225 | 0.036 | -0.137 | -0.239 | -0.080 |
| Value | -0.162 | 0.209 | 1.000 | 0.044 | 0.559 | -0.323 | -0.128 | -0.087 |
| Earnings | -0.172 | -0.225 | 0.044 | 1.000 | 0.120 | 0.082 | 0.589 | 0.218 |
| Investment | -0.344 | 0.036 | 0.559 | 0.120 | 1.000 | 0.013 | 0.130 | 0.271 |
| Momentum | -0.111 | -0.137 | -0.323 | 0.082 | 0.013 | 1.000 | 0.261 | 0.449 |
| Quality | -0.431 | -0.239 | -0.128 | 0.589 | 0.130 | 0.261 | 1.000 | 0.447 |
| BAB | -0.496 | -0.080 | -0.087 | 0.218 | 0.271 | 0.449 | 0.447 | 1.000 |

**Table 8.10:** *Correlations of the factors considered during a recovery as classified by the OECD-measure per Stocks et al. (2022). Data: Daily for all trading days that occurred in a recoveryfrom Jul. 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022 per French (2023g,i); AQR (2023a,c). Illustration: The author.*

## 8.5 Macroeconomic Regressions

| | *OECD Lagged Regression* | | | | | | |
|---|---|---|---|---|---|---|---|
| | Value | Size | Investment | Earnings | Momentum | Quality | BAB |
| Sentiment | 0.002 | −0.003** | 0.001 | 0.004*** | −0.001 | 0.004*** | 0.005*** |
| | (0.002) | (0.002) | (0.001) | (0.001) | (0.002) | (0.001) | (0.002) |
| D/P | −0.002 | 0.002 | −0.006 | −0.002 | −0.022** | −0.007 | −0.001 |
| | (0.008) | (0.007) | (0.005) | (0.007) | (0.010) | (0.005) | (0.009) |
| E/P | −0.006 | −0.001 | −0.002 | 0.001 | 0.018* | 0.004 | 0.005 |
| | (0.005) | (0.004) | (0.003) | (0.003) | (0.011) | (0.005) | (0.005) |
| Var | −0.112 | 0.057 | 0.160 | 0.120 | −0.378 | 0.184 | −0.457 |
| | (0.218) | (0.271) | (0.148) | (0.232) | (0.460) | (0.374) | (0.370) |
| Yield | 0.153 | −0.022 | 0.099* | −0.017 | 0.218** | 0.039 | −0.170 |
| | (0.106) | (0.082) | (0.058) | (0.083) | (0.100) | (0.057) | (0.116) |
| Def.Spread | −0.514 | 0.421 | −0.066 | 0.020 | −0.993** | −0.100 | 0.574 |
| | (0.363) | (0.340) | (0.249) | (0.223) | (0.495) | (0.270) | (0.400) |
| Inflation | 0.868** | −0.376 | 0.687** | 0.324 | 0.095 | 0.369 | −0.047 |
| | (0.369) | (0.399) | (0.287) | (0.337) | (0.536) | (0.335) | (0.431) |
| TermSpread | −0.033 | −0.003 | −0.043 | 0.130* | 0.125 | 0.080 | 0.326*** |
| | (0.104) | (0.085) | (0.066) | (0.075) | (0.136) | (0.071) | (0.106) |
| Expansion | 0.007* | 0.005 | 0.001 | 0.001 | −0.003 | −0.004 | 0.016*** |
| | (0.003) | (0.003) | (0.002) | (0.003) | (0.005) | (0.003) | (0.004) |
| Slowdown | 0.004 | 0.001 | −0.001 | −0.001 | 0.003 | −0.001 | 0.006* |
| | (0.003) | (0.004) | (0.002) | (0.002) | (0.005) | (0.003) | (0.004) |
| Recovery | 0.010** | 0.006 | 0.003 | −0.004* | −0.0002 | −0.007** | 0.013*** |
| | (0.004) | (0.004) | (0.003) | (0.002) | (0.005) | (0.003) | (0.004) |
| Constant | −0.034 | 0.004 | −0.033 | −0.001 | −0.027 | −0.012 | 0.002 |
| | (0.036) | (0.028) | (0.022) | (0.031) | (0.035) | (0.019) | (0.044) |
| $R^2$ | 0.0276 | 0.0129 | 0.0163 | 0.0226 | 0.0394 | 0.0482 | 0.0713 |

*Note:* $^{*}$p<0.1; $^{**}$p<0.05; $^{***}$p<0.01

**Table 8.11:** *Regression of the monthly return on lagged economic variables, OECD-classification following Stocks et al. (2022). Data: Monthly from Jul. 1965 to Dec. 2021*

| | *NBER Lagged Regression* | | | | | | |
|---|---|---|---|---|---|---|---|
| | Value | Size | Investment | Earnings | Momentum | Quality | BAB |
| Sentiment | 0.002 | −0.003** | 0.001 | 0.004*** | −0.002 | 0.004*** | 0.005*** |
| | (0.002) | (0.002) | (0.001) | (0.001) | (0.002) | (0.001) | (0.002) |
| D/P | −0.004 | 0.001 | −0.007 | −0.003 | −0.022** | −0.008 | −0.003 |
| | (0.009) | (0.007) | (0.005) | (0.007) | (0.011) | (0.005) | (0.010) |
| E/P | −0.005 | 0.0001 | −0.001 | 0.004 | 0.021* | 0.006 | 0.005 |
| | (0.006) | (0.004) | (0.003) | (0.003) | (0.011) | (0.005) | (0.005) |
| Var | −0.245 | −0.066 | 0.093 | 0.118 | −0.407 | 0.215 | −0.569 |
| | (0.257) | (0.283) | (0.140) | (0.229) | (0.447) | (0.375) | (0.385) |
| Yield | 0.163 | −0.015 | 0.095* | −0.021 | 0.202* | 0.037 | −0.150 |
| | (0.110) | (0.082) | (0.057) | (0.083) | (0.108) | (0.056) | (0.125) |
| Def.Spread | −0.483 | 0.441 | −0.146 | −0.347 | −1.422*** | −0.511* | 0.834** |
| | (0.389) | (0.364) | (0.266) | (0.236) | (0.538) | (0.280) | (0.411) |
| Inflation | 0.817** | −0.411 | 0.593** | 0.161 | −0.274 | 0.214 | 0.049 |
| | (0.392) | (0.396) | (0.273) | (0.348) | (0.541) | (0.357) | (0.405) |
| TermSpread | −0.119 | −0.079 | −0.084 | 0.200** | 0.318** | 0.180** | 0.233* |
| | (0.115) | (0.113) | (0.073) | (0.083) | (0.149) | (0.082) | (0.137) |
| Pure-Expansion | −0.002 | −0.002 | −0.004 | −0.006* | −0.007 | −0.008** | 0.008 |
| | (0.004) | (0.005) | (0.004) | (0.003) | (0.006) | (0.003) | (0.006) |
| Pre-Recession | −0.008 | −0.008 | −0.006 | −0.001 | 0.009 | −0.001 | 0.0002 |
| | (0.006) | (0.006) | (0.005) | (0.004) | (0.008) | (0.003) | (0.006) |
| Post-Recession | 0.006 | 0.003 | 0.002 | −0.001 | −0.002 | −0.006 | 0.015** |
| | (0.005) | (0.006) | (0.004) | (0.004) | (0.008) | (0.004) | (0.007) |
| Constant | −0.031 | 0.006 | −0.026 | 0.008 | −0.011 | −0.004 | −0.006 |
| | (0.039) | (0.029) | (0.022) | (0.031) | (0.039) | (0.019) | (0.048) |
| $R^2$ | 0.0255 | 0.0118 | 0.0237 | 0.026 | 0.0478 | 0.0459 | 0.054 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

**Table 8.12:** *Regression of the monthly return on lagged economic variables, NBER-classification following Stanhope (2016). Data: Monthly from Jul. 1965 to Dec. 2021*

| | *GDP Lagged Regression* | | | | | | |
|---|---|---|---|---|---|---|---|
| | Value | Size | Investment | Earnings | Momentum | Quality | BAB |
| Sentiment | 0.001 | −0.003** | 0.001 | 0.004*** | −0.0003 | 0.004*** | 0.004** |
| | (0.002) | (0.001) | (0.001) | (0.001) | (0.002) | (0.001) | (0.002) |
| D/P | −0.001 | 0.001 | −0.005 | −0.002 | −0.023** | −0.007 | −0.005 |
| | (0.009) | (0.007) | (0.005) | (0.007) | (0.010) | (0.005) | (0.010) |
| E/P | −0.008 | 0.0001 | −0.003 | 0.002 | 0.019* | 0.004 | 0.006 |
| | (0.005) | (0.004) | (0.003) | (0.003) | (0.011) | (0.005) | (0.005) |
| Var | −0.272 | −0.043 | 0.090 | 0.137 | −0.309 | 0.280 | −0.743* |
| | (0.256) | (0.283) | (0.129) | (0.236) | (0.465) | (0.395) | (0.443) |
| Yield | 0.151 | −0.020 | 0.091 | −0.015 | 0.226** | 0.041 | −0.146 |
| | (0.106) | (0.081) | (0.057) | (0.085) | (0.104) | (0.057) | (0.122) |
| Def.Spread | −0.162 | 0.299 | 0.070 | −0.050 | −1.188** | −0.121 | 0.357 |
| | (0.375) | (0.363) | (0.273) | (0.244) | (0.490) | (0.298) | (0.487) |
| Inflation | 0.789** | −0.487 | 0.647** | 0.355 | 0.099 | 0.464 | −0.233 |
| | (0.363) | (0.415) | (0.283) | (0.332) | (0.524) | (0.337) | (0.430) |
| TermSpread | −0.004 | 0.004 | −0.032 | 0.159** | 0.108 | 0.088 | 0.398*** |
| | (0.106) | (0.087) | (0.064) | (0.069) | (0.139) | (0.070) | (0.117) |
| Expansion | 0.006 | −0.004 | −0.0003 | −0.002 | −0.001 | 0.001 | −0.010 |
| | (0.007) | (0.006) | (0.006) | (0.004) | (0.007) | (0.004) | (0.006) |
| Slowdown | 0.013* | −0.006 | 0.004 | 0.002 | −0.008 | 0.003 | −0.008 |
| | (0.007) | (0.006) | (0.006) | (0.005) | (0.008) | (0.005) | (0.008) |
| Recovery | 0.003 | 0.008 | −0.001 | −0.005 | 0.004 | −0.007 | −0.003 |
| | (0.006) | (0.008) | (0.005) | (0.005) | (0.010) | (0.006) | (0.008) |
| Constant | −0.040 | 0.011 | −0.033 | −0.002 | −0.023 | −0.016 | 0.010 |
| | (0.036) | (0.029) | (0.022) | (0.031) | (0.037) | (0.019) | (0.046) |
| $R^2$ | 0.0235 | 0.0113 | 0.018 | 0.0222 | 0.0399 | 0.0409 | 0.0455 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

**Table 8.13:** Regression of the monthly return on lagged economic variables, GDP-classification following Ilmanen et al. (2021). Data: Monthly from Jul. 1965 to Dec. 2021

| | *Current NBER Regression* | | | | | | |
|---|---|---|---|---|---|---|---|
| | Value | Size | Investment | Earnings | Momentum | Quality | BAB |
| Sentiment | 0.003* | −0.004** | 0.003** | 0.004*** | −0.001 | 0.005*** | 0.006*** |
| | (0.002) | (0.001) | (0.001) | (0.001) | (0.002) | (0.001) | (0.002) |
| D/P | −0.001 | −0.001 | −0.003 | −0.0001 | −0.016 | −0.001 | 0.003 |
| | (0.009) | (0.006) | (0.005) | (0.006) | (0.010) | (0.005) | (0.009) |
| E/P | −0.001 | −0.004 | 0.003 | 0.005* | 0.021* | 0.009** | 0.004 |
| | (0.006) | (0.004) | (0.003) | (0.003) | (0.012) | (0.004) | (0.005) |
| Var | −0.429 | −1.522*** | 0.401*** | 0.261 | 0.366 | 1.150*** | −1.763*** |
| | (0.565) | (0.208) | (0.120) | (0.203) | (0.734) | (0.288) | (0.205) |
| Yield | 0.090 | 0.020 | 0.036 | −0.032 | 0.142 | −0.014 | −0.234** |
| | (0.102) | (0.075) | (0.055) | (0.078) | (0.109) | (0.057) | (0.113) |
| Def.Spread | −0.577 | 0.649* | −0.373 | −0.294 | −1.593** | −0.697** | 0.845** |
| | (0.403) | (0.332) | (0.260) | (0.242) | (0.674) | (0.275) | (0.376) |
| Inflation | 0.927** | −0.515 | 0.519* | −0.528 | −0.015 | −0.304 | 0.059 |
| | (0.402) | (0.382) | (0.306) | (0.363) | (0.804) | (0.390) | (0.482) |
| TermSpread | 0.035 | 0.020 | −0.001 | 0.142* | 0.168 | 0.135* | 0.361*** |
| | (0.106) | (0.116) | (0.068) | (0.080) | (0.140) | (0.078) | (0.122) |
| Pure-Expansion | −0.003 | −0.004 | −0.006 | −0.003 | −0.004 | −0.004 | 0.008 |
| | (0.005) | (0.005) | (0.004) | (0.004) | (0.006) | (0.003) | (0.006) |
| Pre-Recession | −0.007 | −0.006 | −0.007 | 0.004 | 0.006 | 0.003 | 0.006 |
| | (0.006) | (0.005) | (0.005) | (0.004) | (0.007) | (0.003) | (0.006) |
| Rost-Recession | 0.004 | 0.001 | 0.002 | 0.001 | −0.001 | −0.004 | 0.014* |
| | (0.006) | (0.006) | (0.004) | (0.004) | (0.009) | (0.004) | (0.007) |
| Constant | −0.040 | 0.011 | −0.033 | −0.002 | −0.023 | −0.016 | 0.010 |
| | (0.036) | (0.029) | (0.022) | (0.031) | (0.037) | (0.019) | (0.046) |
| $R^2$ | 0.0301 | 0.0594 | 0.0309 | 0.0294 | 0.0363 | 0.1004 | 0.1225 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

**Table 8.14:** *Regression of the monthly return on current economic variables, NBER-classification following Stanhope (2016). Data: Monthly from Jul. 1965 to Dec. 2021*

| | *OECD Current Regression* | | | | | | |
|---|---|---|---|---|---|---|---|
| | Value | Size | Investment | Earnings | Momentum | Quality | BAB |
| Sentiment | 0.003* | −0.003** | 0.003* | 0.004*** | −0.001 | 0.005*** | 0.006*** |
| | (0.002) | (0.001) | (0.001) | (0.001) | (0.002) | (0.001) | (0.002) |
| D/P | 0.002 | 0.0004 | −0.001 | 0.001 | −0.016 | −0.001 | 0.005 |
| | (0.008) | (0.006) | (0.005) | (0.006) | (0.010) | (0.005) | (0.008) |
| E/P | −0.003 | −0.005 | 0.001 | 0.002 | 0.019* | 0.007* | 0.003 |
| | (0.005) | (0.004) | (0.003) | (0.003) | (0.011) | (0.004) | (0.005) |
| Var | −0.288 | −1.377*** | 0.484*** | 0.224 | 0.365 | 1.082*** | −1.667*** |
| | (0.561) | (0.179) | (0.136) | (0.203) | (0.748) | (0.282) | (0.199) |
| Yield | 0.074 | 0.018 | 0.032 | −0.027 | 0.153 | −0.012 | −0.252** |
| | (0.098) | (0.075) | (0.055) | (0.079) | (0.101) | (0.056) | (0.108) |
| Def.Spread | −0.473 | 0.696** | −0.150 | 0.030 | −1.307* | −0.361 | 0.720** |
| | (0.394) | (0.302) | (0.237) | (0.222) | (0.676) | (0.261) | (0.366) |
| Inflation | 1.014*** | −0.393 | 0.639** | −0.383 | 0.203 | −0.200 | 0.055 |
| | (0.373) | (0.402) | (0.302) | (0.325) | (0.787) | (0.355) | (0.455) |
| TermSpread | 0.069 | 0.047 | 0.016 | 0.045 | 0.042 | 0.035 | 0.367*** |
| | (0.100) | (0.082) | (0.066) | (0.078) | (0.125) | (0.073) | (0.100) |
| Expansion | 0.009*** | 0.005* | 0.004 | −0.00004 | −0.002 | −0.003 | 0.016*** |
| | (0.003) | (0.003) | (0.003) | (0.003) | (0.005) | (0.003) | (0.004) |
| Slowdown | 0.004 | 0.0001 | 0.0002 | 0.001 | 0.003 | 0.001 | 0.008** |
| | (0.003) | (0.004) | (0.002) | (0.002) | (0.005) | (0.003) | (0.004) |
| Recovery | 0.008** | 0.006* | 0.002 | −0.004 | −0.0005 | −0.006** | 0.012*** |
| | (0.004) | (0.004) | (0.003) | (0.002) | (0.005) | (0.003) | (0.004) |
| Constant | −0.040 | 0.011 | −0.033 | −0.002 | −0.023 | −0.016 | 0.010 |
| | (0.036) | (0.029) | (0.022) | (0.031) | (0.037) | (0.019) | (0.046) |
| $R^2$ | 0.0363 | 0.0638 | 0.0226 | 0.0242 | 0.0337 | 0.1067 | 0.1405 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

**Table 8.15:** *Regression of the monthly return on current economic variables, OECD-classification following Stocks et al. (2022). Data: Monthly from Jul. 1965 to Dec. 2021*

## 8.6 Tests of OLS Assumptions

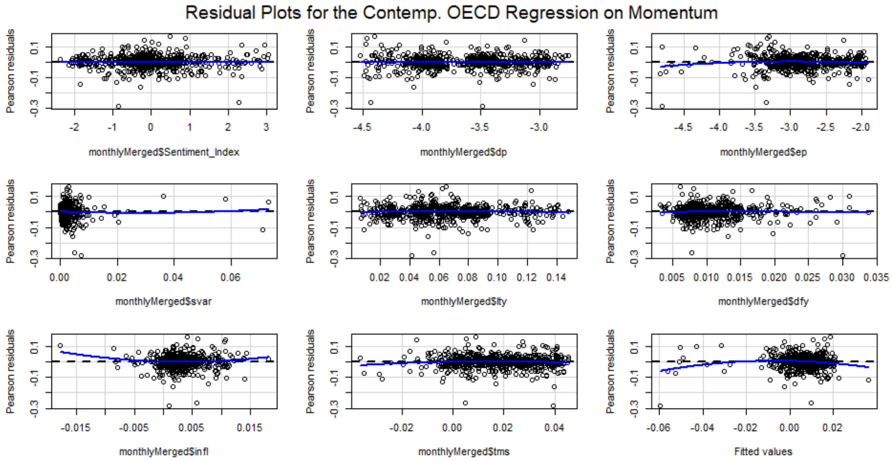


**Figure 8.1:** *Residuals plot for the contemporary OECD regression on momentum considered in table 8.15. The x-axis are the values of the variables, the y-axes the Pearson residuals. From the upper left going to lower right: Sentiment Index, Dividend/Price Ratio of the S&P500 index, Earnings/Price Ratio of the S&P500 index, Squared returns of the S&P500, long-term yield, Default Spread, inflation, Term spread and fitted values vs. residuals. For sources see 8.3. Illustration: The author.*

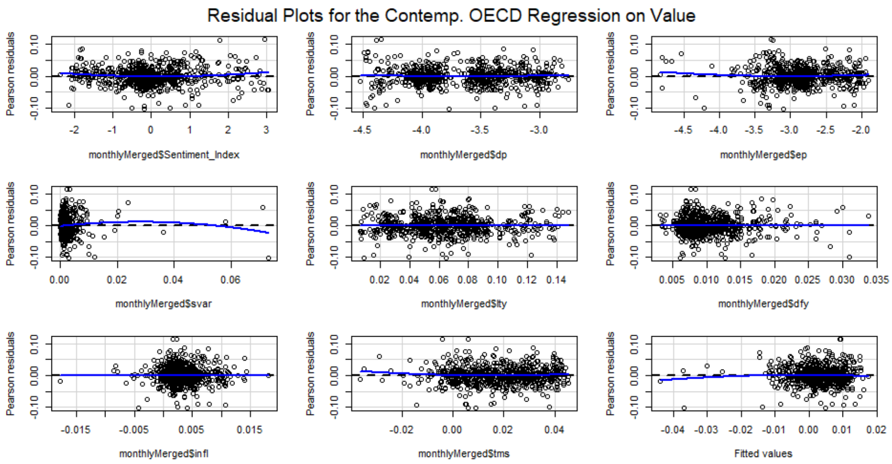


**Figure 8.2:** *Residuals plot for the contemporary OECD regression on value considered in table 8.15. The x-axis are the values of the respective variables, the y-axes the Pearson residuals of the model. The variables presented are as follows from upper left going to lower right: Sentiment Index, Dividend/Price Ratio of the S&P500 index, Earnings/Price Ratio of the S&P500 index, Squared returns of the S&P500, long-term yield, Default Spread, inflation, Term spread and fitted values vs. residuals. For sources see 8.3. Illustration: The author.*

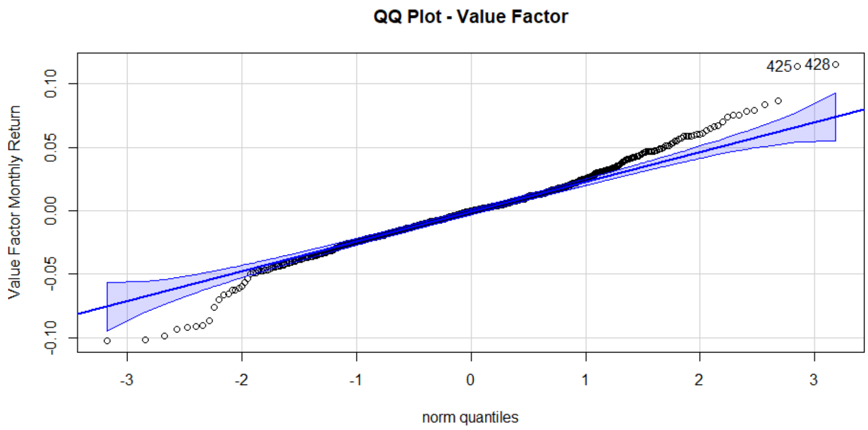


**Figure 8.3:** *Quantile-Quantile (QQ) plot for the residuals of the regression of the value return on the current macroeconomic variables, per 8.15. Data is monthly from Jul. 1965 to Dec. 2021. Illustration: The author.*

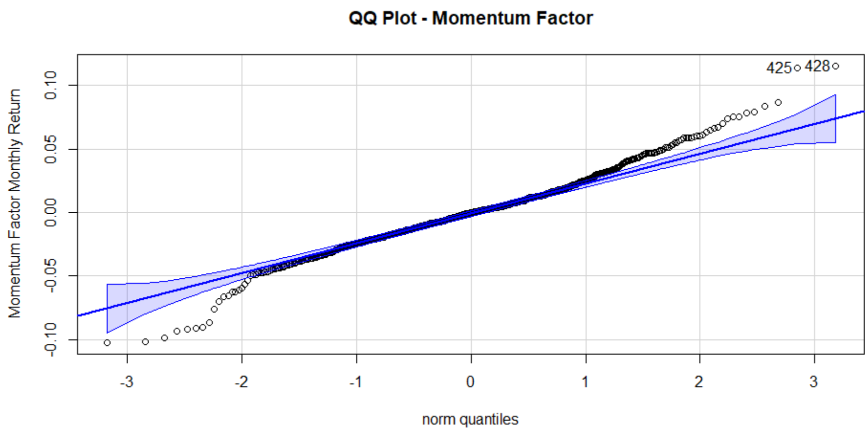


**Figure 8.4:** *Quantile-Quantile (QQ) plot for the residuals of the regression of the momentum return on the current macroeconomic variables, per 8.15. Data is monthly from Jul. 1965 to Dec. 2021. Illustration: The author.*

## 8.7 Regression of Factor-Tilt Weights

| | *Rolling Weights on Lagged Macroeconomic Variables* | | | | | | |
|---|---|---|---|---|---|---|---|
| | Value | Size | Investment | Earnings | Momentum | Quality | BAB |
| Sentiment | −0.060 | −0.212*** | −0.101 | 0.027 | 0.034 | 0.036 | 0.027 |
| | (0.077) | (0.082) | (0.084) | (0.089) | (0.066) | (0.076) | (0.103) |
| Var | 11.479 | 5.883 | 13.614 | 7.704 | 22.315 | 22.462** | −45.133** |
| | (21.693) | (8.974) | (16.582) | (13.071) | (16.401) | (9.570) | (17.814) |
| Yield | 23.717*** | 15.680*** | 6.863 | 5.930* | 10.797*** | 10.364*** | −13.852*** |
| | (3.571) | (2.568) | (4.174) | (3.172) | (3.168) | (3.373) | (3.757) |
| Def.Spread | −62.043** | −46.590*** | 0.070 | 23.934 | −21.378 | −6.609 | −2.196 |
| | (30.945) | (15.060) | (20.984) | (20.314) | (24.718) | (18.123) | (30.506) |
| Term.Spread | −8.066 | 5.382 | 7.520 | 16.373*** | −14.038*** | −7.547 | 34.048*** |
| | (6.417) | (5.014) | (6.852) | (5.875) | (5.092) | (6.234) | (7.767) |
| Inflation | −46.767** | −15.689 | 15.543 | 0.351 | −10.393 | −12.007 | 5.184 |
| | (23.708) | (18.505) | (24.796) | (24.346) | (19.413) | (19.805) | (24.037) |
| Expansion | 0.321 | 0.289* | −0.264 | 0.036 | −0.635*** | −0.404* | 0.644*** |
| | (0.213) | (0.152) | (0.244) | (0.234) | (0.193) | (0.209) | (0.233) |
| Slowdown | 0.106 | 0.045 | −0.388 | 0.242 | −0.582*** | −0.323* | 0.369 |
| | (0.217) | (0.177) | (0.237) | (0.188) | (0.191) | (0.167) | (0.231) |
| Recovery | −0.279 | 0.241 | −0.091 | 0.332 | −0.617*** | −0.088 | 0.032 |
| | (0.218) | (0.148) | (0.201) | (0.208) | (0.180) | (0.150) | (0.257) |
| Market | −0.747 | 0.011 | 2.633** | 0.567 | −0.348 | 0.630 | −0.268 |
| | (1.242) | (0.769) | (1.061) | (1.026) | (0.804) | (0.850) | (1.078) |
| Ret. Lag | 5.746*** | 6.405*** | 11.354*** | 11.245*** | 4.077*** | 6.610*** | 4.756** |
| | (2.001) | (1.336) | (2.275) | (2.257) | (0.895) | (2.204) | (1.985) |
| Constant | −0.043 | −0.116 | 0.059 | −0.716** | 0.228 | 0.948*** | 0.421 |
| | (0.334) | (0.237) | (0.306) | (0.288) | (0.251) | (0.248) | (0.359) |
| Adjusted $R^2$ | 0.2948 | 0.2577 | 0.0765 | 0.152 | 0.2556 | 0.1502 | 0.3377 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

**Table 8.16:** *Regression of the rolling factor-tilt weights, cap abs (200%) on lagged macroeconomic variables, OECD per Stocks et al. (2022). Data: monthly from Aug. 1965 to Dec. 2021. Illustration: The author.*

| | *Expanding Weights on Lagged Macroeconomic Variables* | | | | | | |
|---|---|---|---|---|---|---|---|
| | Value | Size | Investment | Earnings | Momentum | Quality | BAB |
| Sentiment | −0.082*** | 0.022 | −0.081*** | −0.067*** | −0.024 | −0.011 | 0.176*** |
| | (0.018) | (0.038) | (0.025) | (0.021) | (0.015) | (0.009) | (0.033) |
| Var | 2.353 | −3.361 | 1.003 | 4.937** | 2.655* | −0.008 | −1.045 |
| | (1.527) | (2.581) | (2.866) | (2.116) | (1.441) | (0.337) | (2.305) |
| Yield | 9.731*** | 5.967*** | 4.367*** | 14.379*** | 0.903*** | 0.387* | −6.682*** |
| | (0.485) | (0.599) | (0.922) | (0.879) | (0.316) | (0.223) | (0.551) |
| Def.Spread | −15.649*** | −5.608 | 6.512 | −33.085*** | 7.048*** | 1.009 | −6.193 |
| | (3.507) | (5.251) | (4.508) | (5.883) | (2.396) | (1.228) | (4.934) |
| Term.Spread | −2.324*** | −7.512*** | 1.134 | 0.286 | −1.249* | 0.579* | −0.624 |
| | (0.855) | (1.705) | (1.363) | (2.086) | (0.698) | (0.351) | (1.778) |
| Inflation | −5.549 | −1.075 | 12.321* | −9.781* | 13.553*** | −1.134 | −3.409 |
| | (3.416) | (6.441) | (6.639) | (5.685) | (3.114) | (2.245) | (7.409) |
| Expansion | −0.059* | 0.034 | −0.093* | −0.105* | −0.026 | −0.008 | 0.081 |
| | (0.033) | (0.068) | (0.049) | (0.063) | (0.025) | (0.011) | (0.054) |
| Slowdown | −0.071** | −0.023 | −0.113** | −0.118** | −0.037* | −0.006 | 0.070 |
| | (0.031) | (0.060) | (0.045) | (0.054) | (0.023) | (0.012) | (0.051) |
| Recovery | −0.003 | 0.074 | −0.021 | 0.060 | −0.018 | −0.009 | −0.010 |
| | (0.032) | (0.063) | (0.046) | (0.044) | (0.028) | (0.007) | (0.050) |
| Market | 0.009 | −0.405 | 0.478** | 0.748*** | −0.110 | −0.032 | 0.206 |
| | (0.156) | (0.265) | (0.223) | (0.218) | (0.121) | (0.052) | (0.202) |
| Ret. Lag | 0.448* | 1.373*** | 1.069* | 0.961* | 0.279** | 0.139 | −0.232 |
| | (0.236) | (0.518) | (0.569) | (0.561) | (0.109) | (0.168) | (0.310) |
| Constant | 1.188*** | 0.988*** | 0.287*** | 0.062 | 0.362*** | 1.950*** | 0.482*** |
| | (0.059) | (0.102) | (0.070) | (0.065) | (0.039) | (0.023) | (0.111) |
| Adjusted $R^2$ | 0.7555 | 0.3468 | 0.3842 | 0.6324 | 0.2914 | 0.0607 | 0.5179 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

**Table 8.17:** *Regression of the expanding factor-tilt weights, cap abs (200%) on lagged economic variables, OECD per Stocks et al. (2022). Data: monthly from Aug. 1965 to Dec. 2021. Illustration: The author.*

## 8.8 Autocorrelation of Daily Factor

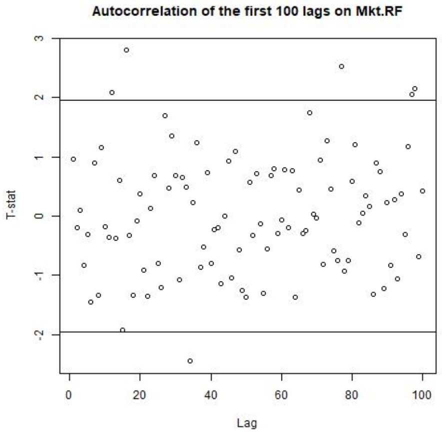


**Figure 8.5:** *Significance of autocorrelations on market excess return. White standard errors. Data: Daily from Jul 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022. Illustration: The author.*

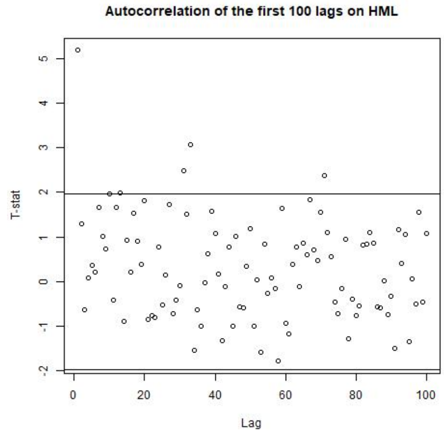


**Figure 8.6:** *Significance of autocorrelations from value excess return. White standard errors. Data: Daily from Jul 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022. Illustration: The author.*

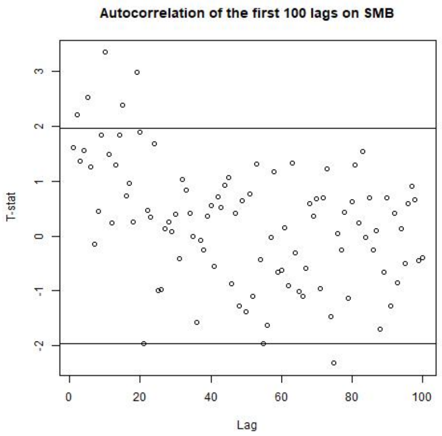


**Figure 8.7:** *Significance of autocorrelations on size excess return. White standard errors. Data: Daily from Jul 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022. Illustration: The author.*

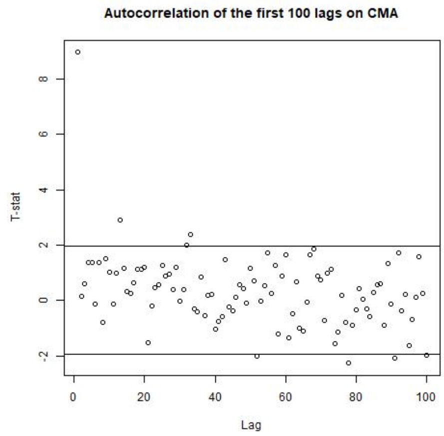


**Figure 8.8:** *Significance of autocorrelations on investment excess return. White standard errors. Data: Daily from Jul 1$^{st}$, 1963 to Nov. 30$^{th}$, 2022. Illustration: The author.*

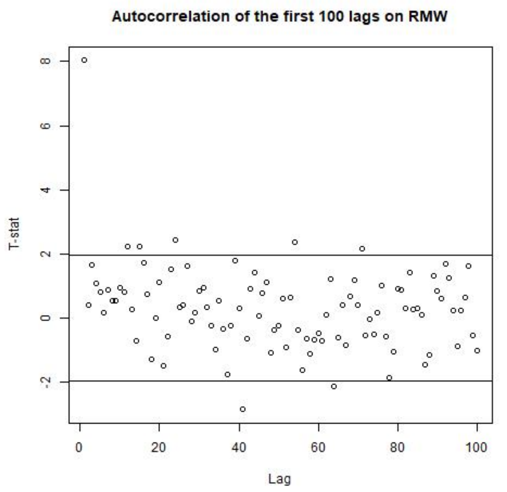


**Figure 8.9:** *Significance of autocorrelations from lag 1-100 on the excess returns of the earnings factor return. White standard errors. Data: Daily from Jul 1st, 1963 to Nov. 30th, 2022. Illustration: The author.*

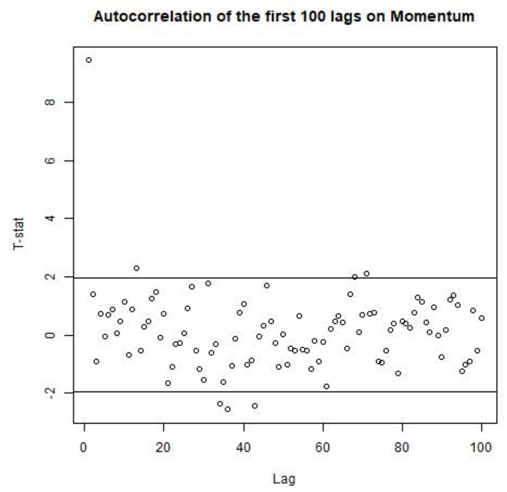


**Figure 8.10:** *Significance of the autocorrelations from lag 1-100 on the excess returns of the momentum factor return. White standard errors. Data: Daily from Jul 1st, 1963 to Nov. 30th, 2022. Illustration: The author.*

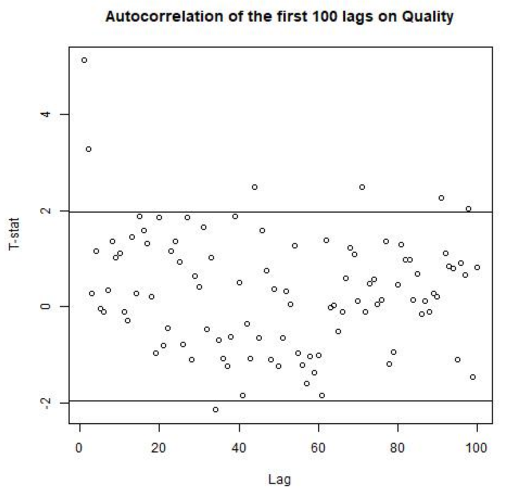


**Figure 8.11:** *Significance of the autocorrelations from lag 1-100 on the excess returns of the quality factor return. White standard errors. Data: Daily from Jul 1st, 1963 to Nov. 30th, 2022. Illustration: The author.*

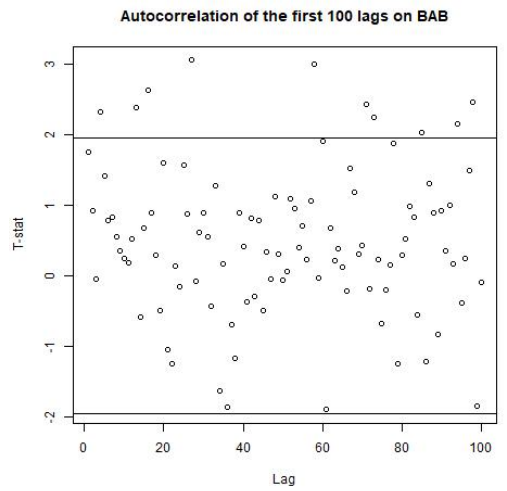


**Figure 8.12:** *Signifiance of the autocorrelations from lag 1-100 on the excess returns of the BAB factor return. White standard errors. Data: Daily from Jul 1st, 1963 to Nov. 30th, 2022. Illustration: The author.*

Autocorrelation of momentum across the OECD business cycle stages

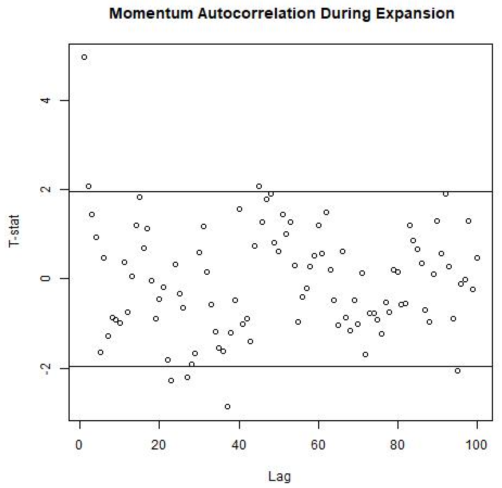


**Figure 8.13:** *Significance of the autocorrelations from lag 1-100 on the excess returns of momentum in an OECD-expansion. White standard errors. Data: Daily in expansions from Jul 1st, 1963 to Nov. 30th, 2022. Illustration: The author.*

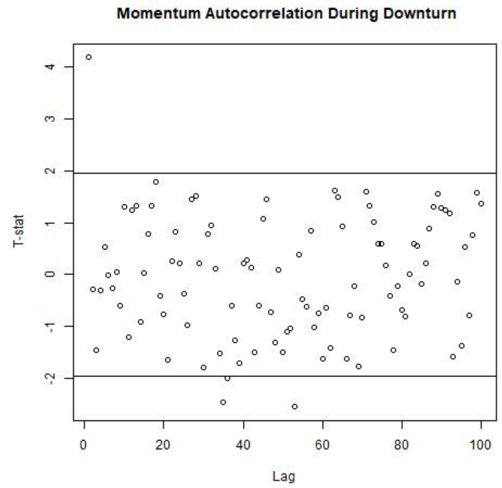


**Figure 8.14:** *Significance of the autocorrelations from lag 1-100 on the excess returns of momentum in an OECD-downturn. White standard errors. Data: Daily in downturn from Jul 1st, 1963 to Nov. 30th, 2022. Illustration: The author.*

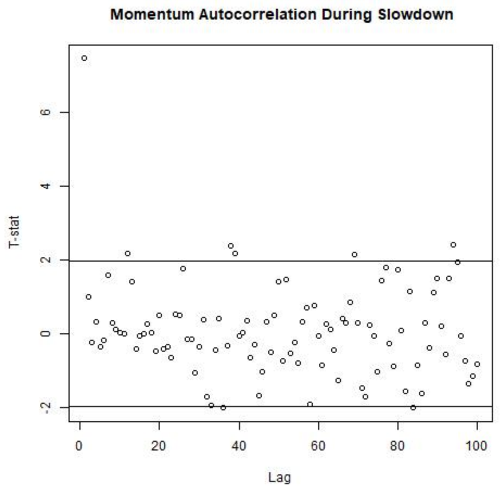


**Figure 8.15:** *Significance of the autocorrelations from lag 1-100 on the excess returns of momentum in an OECD-slowdown. White standard errors. Data: Daily in slowdown from Jul 1st, 1963 to Nov. 30th, 2022. Illustration: The author.*

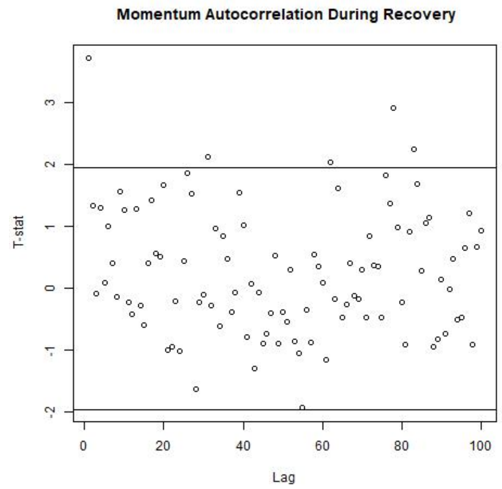


**Figure 8.16:** *Signficance of the autocorrelations from lag 1-100 on the excess returns of momentum in an OECD-recovery. White standard errors. Data: Daily in recovery from Jul 1st, 1963 to Nov. 30th, 2022. Illustration: The author.*

## 8.9 Monthly GARCH Estimation

This section presents the fit of the monthly GARCH-estimation

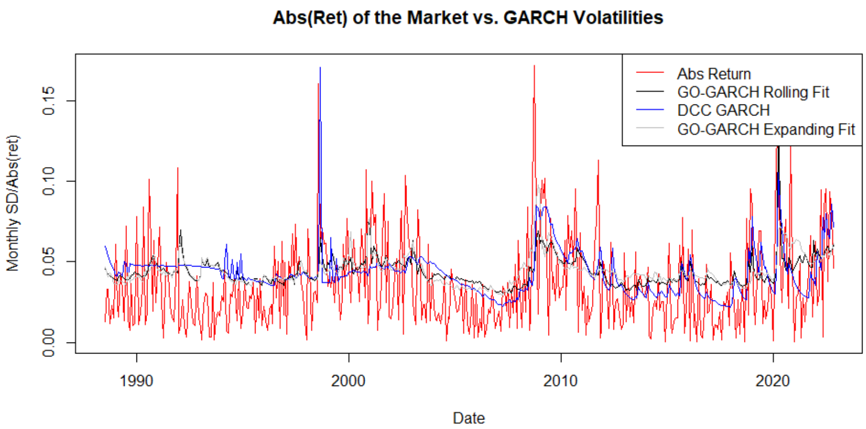


**Figure 8.17:** *Fit of the monthly GARCH volatilities to the absolute monthly return, which is used as a proxy for volatility. The models consider monthly data from Jul. 1988 to Nov. 2022. Illustration: The author.*

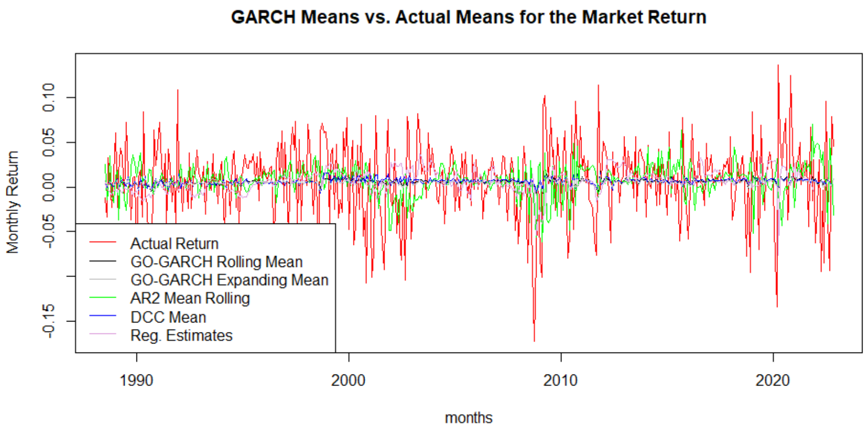


**Figure 8.18:** *Fit of the monthly GARCH means for the market return to the actual monthly return. The models consider monthly data from Jul. 1988 to Nov. 2022. Illustration: The author.*

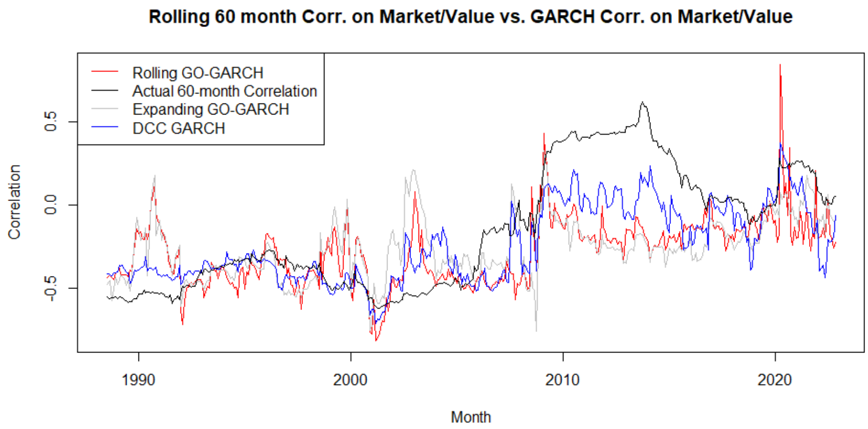


**Figure 8.19:** *Fit of the monthly correlation of the GARCH-models vs. the actual 60-month correlation. The models consider monthly data from Jul. 1988 to Nov. 2022. Illustration: The author.*

## 8.10 Minimum Variance Portfolios

This section presents the performance of the daily estimated minimum variance portfolios based on the GARCH-models in section 4.6.3

| | Back. Est. | Daily GG | Back. w. TC | GG w. TC |
|---|---|---|---|---|
| Mean Monthly Ret (%) | 1.179 | 1.478 | 0.808 | -0.346 |
| Excess Daily Ret (%) | 0.041 | 0.055 | 0.024 | -0.031 |
| Excess Std. Dev (%) | 0.751 | 0.742 | 0.753 | 0.758 |
| Sharpe Ratio | 0.055 | 0.075 | 0.032 | -0.041 |
| Drawdown (%) | -61.400 | -48.015 | -65.525 | -92.302 |
| CAPM Alpha (%) | 0.029 | 0.044 | 0.011 | -0.043 |
| CAPM Beta | 0.469 | 0.440 | 0.468 | 0.439 |
| Information Ratio | 0.114 | 0.158 | 0.045 | -0.145 |

**Table 8.18:** *Performance of Daily Estimated Minimum Variance Portfolios. Data: Daily from Aug. 1988 to Nov. 2022. Illustration: The author.*

## 8.11 Analysis of Predicting Weights

### Predicting Weights

Sections 4.2.1 and 4.3.2 shows that the returns of the factor portfolios have some link to the underlying economy, and that when regressing the mean-variance optimal weights on the macroeconomy, rather high $R^2$ result. This appendix uses the predictive regressions for exactly that, predicting the weights in the next period based on the macroeconomic variables in this period.

The regressions assume that the investor needs at least 84 months of data before he can start to make the predictions to have enough statistical power. From there, the investor can either try to predict the rolling weights or expanding weights of the abs (200%) factor-tilt. The section also considers two different methods of regression, namely that the window for regression is rolling 84 months or expanding starting with 84 months expanding one month per month. The performance is considered from Aug. 1972 to Dec. 2021. The performance, as compared to the mean-variance portfolios, tells something about how tightly knit the macroeconomy is to the factors.

Table 8.19 shows the performance of the portfolios with the predicted weights as well as their mean-variance counterparties as a benchmark. Not surprisingly, the portfolio weights based on expanding mean-variance estimation, which had the highest $R^2$, performs much better than does the rolling weights. The best performance is captured by the rolling regression for expanding weights, i.e. taking updated information to refit the regression, but doing so for the weights that are the most dependent on the macroeconomy. Doing so yields a Sharpe ratio on par with the mean variance rolling methodology and actually a tad higher than the mean-variance expanding weights. The information ratio, further, is higher on the rolling regression for expanding weights than any of the other portfolios due to lower idiosyncratic risk than its counterparts. The mean return is higher and standard deviation roughly the same as the mean-variance expanding portfolio.

| | Exp, Exp Reg. | Exp, Roll Reg | Roll, Roll Reg | Roll, Exp Reg | MV Roll | MV Exp |
|---|---|---|---|---|---|---|
| Mon. Ret (%) | 3.085 | 3.010 | 1.623 | 2.636 | 3.576 | 2.832 |
| Ex Ret (%) | 0.139 | 0.132 | 0.093 | 0.118 | 0.163 | 0.124 |
| SD (%) | 1.482 | 1.298 | 2.450 | 1.499 | 1.599 | 1.263 |
| Sharpe | 0.093 | 0.102 | 0.038 | 0.079 | 0.102 | 0.098 |
| DD (%) | -62.793 | -56.934 | -98.753 | -82.141 | -67.500 | -59.286 |
| Alpha (%) | 0.125 | 0.121 | 0.090 | 0.103 | 0.151 | 0.115 |
| Beta | 0.446 | 0.389 | 0.117 | 0.493 | 0.413 | 0.302 |
| IR | 0.124 | 0.137 | 0.039 | 0.106 | 0.131 | 0.123 |

**Table 8.19:** *Performance of Portfolios based on Predicted Weights based on tables 8.17 and 8.16. Data is daily from Aug. 1972 to Dec. 2021. The portfolio names are to be read as follows: Exp = Expanding, Roll = Rolling. The first word is the methodology of constructing the mean-variance weights, which are sought explained in the regression, the second is the methodology used in the predictive regression of weights on the macroeconomy. MV = Standard Mean-variance backwards looking approach as utilized in the majority of section 4.3. Ex Ret = Daily excess return, Mon Ret = Mean monthly full return, SD = Standard deviation of excess returns, DD = Maximum drawdown, Alpha = CAPM alpha, Beta = CAPM beta, IR = Daily information ratio. Illustration: The author.*

Looking at the performance of the predicted portfolio of rolling weights, this is much, much worse. The average Sharpe ratio is 0.038, which is close to a third of its mean-variance counterpart, and the information ratio is equally small at 0.039. The mean return is small compared to the counterparties, and the risk is extreme with a standard deviation of 2.45% per day and an extreme maximum drawdown of 98.753%. The large risk is due to very large weight predictions, in absolute terms. For example, during April, 2020, the regression suggests going long 1,108% in the investment factor, long 987% in the momentum factor and short sell the value factor by 600%. Such results in daily losses as large as 73% during the month. The result is also expected due to the rolling regressions only capturing a fraction of the total variance compared to that captured by the expanding regressions.

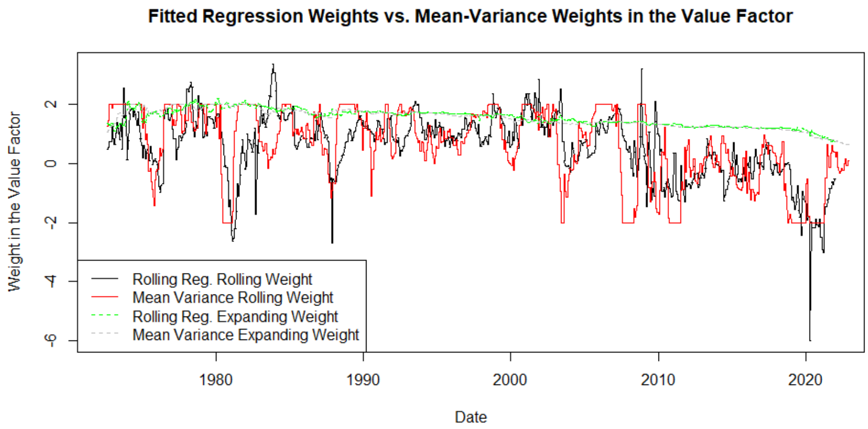


**Figure 8.20:** *Weights of the value factor for the rolling and expanding mean-variance approach in section 4.3.2 and the fit of the predictive regressions to that weight with rolling and expanding windows with a minimum of 84 months in the estimate. Data: Monthly weights from Aug. 1972 to Dec. 2021. Illustration: The author.*

Figure 8.20 shed lights on the reason of the poor performance of the rolling portfolio. The fit is horrible, primarily due to the regression taking, at times, very large bets where the mean-variance approach is capped. However, it seems to adjust more slowly than is the case for the mean-variance approach, which is positive considering transaction costs and practical implementation. The expanding weight, on the other hand, moves very little, wherefore it is easy to explain its variability via the changing macroeconomic variables, which is seen in the better fit of said regression compared to the rolling.

The conclusion, therefore, seems to be that the predictive regressions can be used with reasonable accuracy for determining the expanding portfolio weights and doing so with an updating, rolling regression window can even outperform the rolling mean-variance approach. This is testament to the weights depending differently on the economy over time. However, figure 8.20 also shows the story that the investor prefers slowly adjusting measures, even if it means missing some return. Therefore, the optimal investment strategy for a portfolio manager is to adjust to the underlying macroeconomy but do so slowly adjusting weights rather than quick adjustments to the changing markets. Hence, a risk-adjusted strategy is a good idea.

**Predicting Returns**

Another approach to which the predictive regressions can be used is to predict not the weights, but the returns of the factors. Here, the regressions in section 4.2.1 are used. The section focusses on the OECD-prediction in table 8.11. The regression is fitted with a rolling window based on either 84 months worth of observations to match the regressions for the weights, or 275 months worth of observations to match the estimation period for the GARCH-models. Table 8.11 shows that the $R^2$ are very low, hence suggesting that the fit must be horrible. Figure 8.21 backs the suspicion, but also shows, if compared with figure 8.18, that the fit is as least as good as the fits of the means obtained from an AR(2) model and the GARCH-estimated means. Naturally, the prediction with 275 obs is more slowly adjusting, which leads to less radical return estimates, and also means that the large misspecifications seen in the prediction based on 84 months, is not present. On the other hand, it does not capture the peaks of the series as the short prediction does somehow. Table 8.20 also shows that the correlations between the actual returns and the two prediction methods is not widely different, some is better under one measure and others under the other. However, the correlations are very poor, with most of the returns practically being uncorrelated with their forecasts from the regression models. This is further support that it is meaningless to use the macroeconomic variables to say anything about factor returns

| | Prediction w. 84 obs | Prediction w. 275 obs |
|---|---|---|
| Market | -0.012 | 0.021 |
| Size | -0.008 | 0.034 |
| Value | 0.108 | 0.149 |
| Investment | -0.010 | -0.044 |
| Earnings | 0.044 | 0.088 |
| Momentum | 0.011 | -0.031 |
| Quality | 0.095 | 0.095 |
| BAB | 0.194 | 0.187 |

**Table 8.20:** *Correlation of the predicted returns based on a rolling version of table 8.11. Data is daily from Aug. 1972 to Dec. 2021 for the 84-obs window and daily from Jul. 1988 to Dec. 2021 for the 275 obs-window. Illustration: The author.*

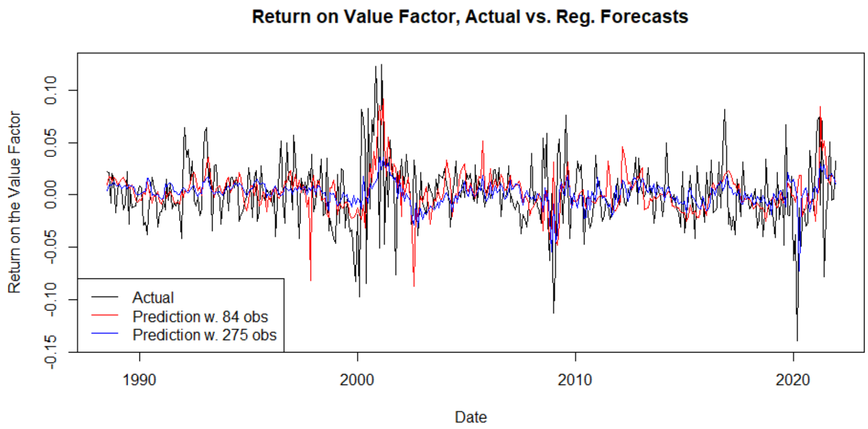


**Figure 8.21:** *The predicted value return based on the regressions in 8.11 with 84 obs rolling and 275 obs rolling respectively. Monthly data from Aug. 1988 to Dec. 2021. Illustration: The author.*

As this estimate is no better than the mean-estimates used in the GARCH-section of the paper, one can try to estimate the optimal mean-variance weights by coupling the regression forecasts with the monthly GARCH-estimates. As section 4.6.1 found that the DCC-GARCH performed best on the monthly data, this is utilized. Tables 4.28 and 4.29 in section 4.6.1 show that the methodology performs marginally worse (better) than the DCC estimated mean-model before (after) transaction costs, when the regression estimates are based on 275 observations. Table 8.21 shows the performance of the 84 obs regression vs. the 275 obs, and also presents that the gain from using a regression to predict factor returns is poor, due to the very poor $R^2$ of the models

| | 84 Obs wo TC | 275 Obs wo TC | 84 Obs w TC | 275 Obs w TC |
|---|---|---|---|---|
| Mean Monthly Ret (%) | 2.376 | 2.535 | 1.304 | 1.726 |
| Excess Daily Ret (%) | 0.114 | 0.118 | 0.064 | 0.080 |
| Excess Std. Dev (%) | 1.586 | 1.401 | 1.601 | 1.411 |
| Sharpe Ratio | 0.072 | 0.084 | 0.040 | 0.057 |
| Drawdown (%) | -50.670 | -57.040 | -55.980 | -64.130 |
| CAPM Alpha (%) | 0.086 | 0.097 | 0.037 | 0.060 |
| CAPM Beta | 0.715 | 0.551 | 0.708 | 0.545 |
| Information Ratio | 0.110 | 0.124 | 0.046 | 0.075 |

**Table 8.21:** Performance of Regression Estimated Portfolios With and Without Transaction Costs, Data: Daily from Aug. 1972 to Dec. 2021 for the 84-obs window and daily from Jul. 1988 to Dec. 2021 for the 275 obs-window. Illustration: The author.

Overall, the regression based on 275 observations, as presented in section 4.6, performs the best both before and after transaction costs with a higher return for a lower standard deviation, just as the information ratio is higher. The 84 obs prediction is hit much harder by the transaction costs than its counterpart, making the difference between them even larger after considering the transaction costs. Reflecting back on figure 8.21, the explanation is that the short regression overestimates the weights too often and has a much larger volatility in the weight adjustments than does the 275 obs regression, which leads to higher transaction costs and a higher risk to get the prediction very wrong. The performance of the 275 obs regression, in fact, is better than any of the other methodologies after considering transaction costs, which suggests that if one is basing a mean-variance approach on monthly data, this may be the best of the worst ways to do it. The story again is, however, that the investor needs an elaborate model for means and transaction costs as well as is best served by slowly adjusting asset allocation weights.

## 8.12 R-Code

The analysis of this thesis has been performed in R-Studio. The R-code and developed functions can be found in GitHub on the following link, which also contains any needed data in a .csv format:
`https://github.com/jskriverm/The-Optimal-Factor-Timing-Portfolio---Master-Thesis`

Part of the code resides on functions developed by other authors as well as professors at Copenhagen Business School. A list of references are found in the below.

1. Mean-Variance plots of the efficient frontier in figure 4.5: Zivot (2019)

2. Construction of the monthly variance-covariance matrices and mean vectors, backwards looking 252 days at the beginning of each month. These are later used for constructing mean-variance weights. (Shenanigans, 2019).

3. Constructing test of autocorrelations with White standard errors in section 4.3.4. Varneskov (2022) as provided in the Master's elective course "Financial Econometrics" at Copenhagen Business School, Fall, 2022.

4. Implementation of Newey & West (1987) standard errors in R. Hanck et al. (2023).