Copenhagen Business School

Master's Thesis

Exploring the Diversification Benefits of Frontier Markets in Global Portfolios

---

Authors:

Carl Philip Fiane (150311)

MSc Economics and Business Administration – Accounting, Strategy and Control

Thomas Alexander Juell (129020)

MSc Economics and Business Administration – Accounting, Strategy and Control

Supervisor:

Lars Sønnich Pørksen

15 of May 2023

Number of Pages: 90

Characters with Spaces: 159 382

Abstract

Frontier markets represent a fast-growing segment of international equity markets. Attractive fundamentals, diversification benefits and undervaluation characterize the segment. Despite these opportunities, investors generally avoid frontier markets due to their perceived riskiness. This thesis aims to research frontier equity markets and their applicability in an international portfolio for an American investor. More specifically, the thesis intents to investigate the risk-reducing and performance-enhancing properties of frontier markets.

The authors construct nine optimal portfolios subject to different allocation restrictions to evaluate the individual portfolios' performance. The portfolios are constructed using Markowitz's mean-variance, Estrada's mean-semivariance, and a Black-Litterman-inspired method. Portfolios are then optimized with respect to Sharpe- and Sortino-ratio and evaluated on the basis of return, risk and performance metrics. Two different sample periods are introduced to test the portfolios for consistency. The portfolios were evaluated and compared to each other as well as a benchmark portfolio.

The study finds that frontier equity markets can serve as a viable diversification tool for American investors aiming for higher risk-adjusted returns. The portfolios consistently yield similar outcomes, and frontier market-inclusive portfolios achieves the highest performance metrics. However, the findings suggest that the risk-reducing abilities of frontier markets are inconsistent. Compared to the benchmark portfolio, the performance is, however, varied. The authors conclude that frontier markets possess performance-enhancing properties in an international portfolio.

Table of Contents

1. Introduction

1.1. Background

Investors today find themselves in a constantly evolving economic environment where increased globalization is at the forefront of development. Advancing technologies, globalization of national economies, liberalization of national financial and capital markets, and increased competition among providers of intermediary services have been the main driving forces behind the recent increased interconnectedness and globalization of finance (Häusler, 2002). As such, international investors constantly search for new markets to provide return opportunities and diversification benefits. Emerging markets have for many years been considered the best option for international investors, as the markets have promising growth opportunities and are less connected to other global markets. However, emerging markets are becoming more and more interconnected with other global equity markets as a result of the rapid globalization (FTSE, 2014).

Frontier markets have emerged as a new investment opportunity for international investors. These markets possess many of the same qualities as emerging markets yet remain less integrated in the global equity market. Frontier markets have attractive fundamentals, lower volatility than generally perceived and a historically high growth in GDP. Furthermore, rapid urbanization and a large young population and future labor force will facilitate economic and financial growth for several years to come (FTSE Russel, 2014). In addition, frontier markets have historically traded at a discount and consistently delivered good performance in both risk- and dollar-adjusted returns (Sivabalan, 2018), making them attractive to foreign investors.

Frontier markets are also recognized as a risk diversifier. The markets exhibit low correlation to developed and emerging markets, thus presenting the potential for risk diversification (Loefstrand, 2022). The increasing globalization of emerging markets has left a gap in the market for diversifiable markets, which frontier markets have the potential to fill due to the markets remaining local (FTSE, 2014). In contrast to frontier markets, emerging markets are strongly influenced by international investors, whereas frontier markets remain segregated and less affected by global events (Boulter & Stein, 2022).

Despite the promising fundamentals in frontier markets, studies in the area remain limited (Uludag & Ezzat, 2016). As such, frontier markets constitute an exciting field of exploration due to the continued globalization of financial markets and the existing research gap. Therefore, this thesis will investigate the relationship between developed and frontier markets to determine the degree of integration between the markets. The authors will subsequently analyze the effects of including frontier markets in an international portfolio to determine frontier markets' potential performance and diversification benefits.

1.2. Research Question

This thesis aims to investigate the potential diversification benefits of frontier equity markets (hereby "frontier markets" or "FM") from the perspective of an American investor. More specifically, the impact of frontier equity markets on portfolio variance and expected return will be examined. As such, the relationship between FM and other equity markets will be researched, after which three optimal portfolios will be constructed using Markowitz's (1952) mean-variance approach, Estrada's (2008) heuristic mean-semivariance approach, and a final approach inspired by Black-Litterman (1992). The research contributes to academia and the international investment community by providing valuable insights into the dynamics of frontier markets and its applicability in portfolio composition, an area that has received less attention in recent research. Thus, throughout the remainder of this thesis the authors will attempt to answer following research question:

"*To what extent are frontier equity markets applicable as a diversification tool to reduce volatility or improve risk-adjusted returns associated with international portfolios for an American investor?"*

In order to narrow down the field of research and facilitate a structure approached to the research question, the following sub-questions are formulated:

1. *Are frontier equity markets weakly correlated with developed markets?*
2. *Do frontier markets reduce portfolio volatility when combined with developed markets in an international portfolio?*
3. *Does the inclusion of frontier markets in an international portfolio improve the portfolio's risk-adjusted returns?*

1.3. Research Philosophy and Design

As business researchers it is important to be aware of the philosophical commitments undertaken through the choice of research strategy, as this ultimately shapes our behavior and understanding of what is being investigated (Johnson & Clark, 2006; Saunders et al., 2020). As stated in the research question, this thesis aims to examine the potential diversification benefits of including frontier equity markets in an international portfolio. Hence, a central part in answering this question will be to investigate the relationships and correlation coefficients between developed equity markets and markets that are less developed, i.e., frontier markets. Further, the relationship between frontier markets and portfolio risk and performance will be scrutinized. The research takes its point of departure from Markowitz's (1952) modern portfolio theory, which allows for the formulation of generalized presumptions concerning the chosen field of research that can be investigated through thorough quantitative research.

Given the nature and scope of this paper, the research is conducted through the lens of Popper's *critical rationalism*, a branch within scientific knowledge that arose in response to logical positivism (Koch, 2013). According to Popper, scientific knowledge distinguishes itself by falsifiability rather than empirical verifiability, meaning a truly scientific theory is formulated in such a way that it can be refuted on the basis of experience. Hence, a central element in critical rationalism is the practice of formulating "basic sentences", or hypotheses, that can serve as a premise in the empirical testing of a theory (Leezenberg & De Vries, 2018).

Additionally, this thesis makes use of the *deductive research approach* commonly associated with the methods of critical rationalism and quantitative research. According to Saunders et al. (2020), quantitative research examines relationships between variables, measured through numerical and statistical analysis and graphical techniques. As such, this paper is concerned with testing the theories of modern financial and portfolio theory as opposed to developing a new theory, which is present in an inductive approach. Further, deductive reasoning occurs when the conclusion is derived logically from a set of theory-derived premises, in which the conclusion becomes true when all the premises are true (Ketokivi & Mantere, 2010; Saunders et al., 2020). Furthermore, the research purpose is defined as *exploratory* as the research question concerns a topic that is relatively limited in studies. The authors aim to contribute to the field of research by deepening the understanding of frontier equity markets and their mechanisms.

1.4. Thesis Structure

The rest of the thesis is organized as follows. Section 2 introduces the theoretical concepts and frameworks that form the basis of the analysis, whereas Section 3 presents a literature review on topics relevant to the field of investigation. Section 4 and 5 describe the data as well as the methods applied for the analysis. Section 6 addresses the empirical findings of the analysis. Section 7 presents a discussion of the obtained results, a critical reflection of the applied methods and recommendations for future research. Lastly, Section 8 explicitly answers the research question and concludes the thesis.

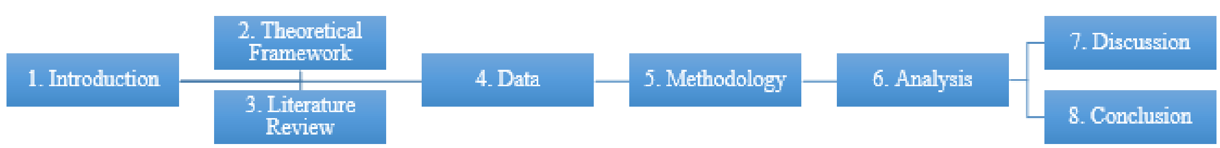

Figure 1: Thesis Structure (Source: Own contribution)

2. Theoretical Framework

The following section presents the relevant theories on which the analysis is built and provides a fundamental understanding of the general concepts of investment theory and modern finance. Hence, this section aids in establishing formal expectations and presumptions about what the analysis might find. Additionally, the section introduces basic return and risk properties, performance metrics and other measures relevant to the methodology and analysis of the thesis.

2.1. Modern Portfolio Theory

Modern portfolio theory (MPT), also referred to as mean-variance analysis, was first introduced by Harry Markowitz (1952) in his paper on portfolio selection. He argues that selecting a portfolio may be divided into two stages. The first stage starts with observation and experience and ends with beliefs about the future performance of available securities, whereas the second stage starts with relevant beliefs about future performance and ends with the choice of portfolio. The fundamental concept of MPT is the tradeoff between risk and return, also referred to by Markowitz as the "expected returns – variance of returns" rule. Assuming investors are rational and risk-averse, they will choose the portfolio that maximizes the expected return at a given level of systematic risk. However, this portfolio is not necessarily the one with the minimum variance, and investors can alternatively choose to minimize portfolio risk at a given expected return. In other words, investing is a compromise between risk and expected return. Markowitz provides a conceptual framework for asset allocation of a portfolio subject to the investor's risk preferences, in which statistical properties such as return variance and standard deviation define the financial risks of an isolated security or portfolio (Markowitz, 1952).

A vital component of the theory is diversification, i.e., to disperse investments across relatively many securities and industries to reduce idiosyncratic risk. However, Markowitz (1952) notes that diversification cannot eliminate all variance, but by measuring the covariance between assets, the investor can adjust portfolio weights accordingly to reduce the portfolio's overall risk while maintaining a desired level of expected returns. At each level of risk, the portfolio that provides the optimal expected return is efficient. Together, the set of mean-variance efficient portfolios forms the efficient frontier in the shape of a hyperbola. The shape of the curve is determined by the correlation between the securities, where a higher positive value gives a rounder shape. Rational risk-averse investors will thus only invest in portfolios that fall on the efficient frontier and maximize utility, as a higher expected return cannot be obtained without incurring increased risk.

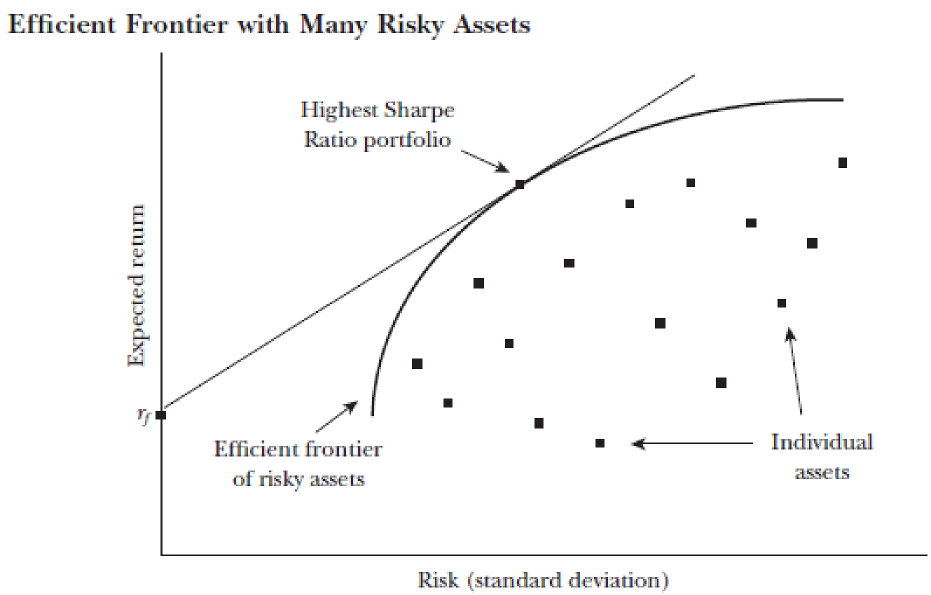

Figure 2: Efficient Frontier with Many Risky Assets (Source: Perold, 2004)

When the risk-free asset is introduced, the capital allocation line (CAL) appears, creating a new efficient frontier for the investor. The tangency portfolio has the highest Sharpe ratio and is positioned at the point where CAL is tangent to the efficient frontier (Munk, 2021).

Markowitz's theory is subject to several assumptions about the investor, the fundamental assumption being that investors are rational and risk-averse individuals who subscribe to the expected returns – variance of returns rule. Thus, their investment decision is based only on the expected return and variance of the investment over the time horizon in question. To satisfy this assumption returns on risky assets must be normally distributed. Furthermore, modern portfolio theory relies on the efficient market hypothesis (see 2.2. Efficient Market Hypothesis) and assumes a frictionless market where investors share identical views and expectations about future returns and can access the same information. The final assumption is that investors do not influence the supply and demand of the risky asset, i.e., they are price-takers (Munk, 2021).

Despite its theoretical importance and extensive adoption among academics and practitioners, Markowitz's framework has been criticized for stating unrealistic assumptions about financial markets and the real world (Vaclavik & Jablonsky, 2012). In particular, the model's use of the variance as a measure of risk implies that investors are indifferent between abnormally high and abnormally low returns. However, investors are more concerned with downside risk and view this as non-desirable. This has led to the emergence of post-modern portfolio theory (PMPT), which concerns the semi-variance of expected asset returns. Moreover, the assumption of normally distributed returns can be challenged, as the lognormal probability distribution better approximates financial instruments' returns (Vaclavik & Jablonsky, 2012). However, Munk (2021) notes that if gross returns of individual securities are lognormally distributed, the gross return of any portfolio would not be lognormally distributed since the sum of lognormal variables is not lognormal. Munk further emphasizes the estimation issues of mean-variance optimization and how its output is quite sensitive to the magnitude of its input.

2.2. Efficient Market Hypothesis

The Efficient Market Hypothesis (EMH) states that security prices instantaneously adjust to all newly available information, implying that no investor can gain abnormal returns through technical or fundamental analysis; only by accepting above-average risk can investors earn above-average returns. As such, markets follow a "random walk" pattern in which subsequent price changes represent random departures from previous prices since new information is, by definition, unpredictable (Malkiel, 2003).

In general, market efficiency concerns the extent to which security prices reflect available information. An "efficient" market is characterized by prices that always fully reflect available information under frictionless conditions. A frictionless market entails all market participants agreeing on the implications of current information, information is freely available to all participants, and there are no transaction costs. Such a market is not descriptive of markets met in practice; however, Fama (1970) emphasizes that these conditions are sufficient for market efficiency rather than necessary. Although these conditions are not necessarily sources of market efficiency, they are potential sources. Given that they all exist to some extent in real-world markets, the concept of different degrees of efficiency was introduced in the literature (Fama, 1970).

Weak form market efficiency suggests that current stock prices reflect all historical returns data, implying that technical analysis cannot be used to predict future stock price movements to gain abnormal returns. Semi-strong efficiency states that prices reflect past prices and efficiently adjust to all publicly available information, such as annual earnings announcements or stock splits. Those who support this theory maintain that trading tactics relying on fundamental analysis of information accessible to the public will not yield returns greater than the overall market. Lastly, strong form efficiency states that all public and privately held information is reflected in the stock price and that not even inside investors can outperform the market (Fama, 1970).

Malkiel (2003) mentions three schools of thought that challenge EMH and the belief that stock prices are, to some extent, predictable, the first being momentum investing. In a truly efficient market, short-term serial correlations among stock prices should be zero, yet several studies have empirically contradicted this notion. Therefore, momentum investors believe that certain price patterns persist over time and can be discovered through a combination of technical and fundamental analysis. Malkiel (2003) argues that these findings may not be economically significant despite their statistical significance and further stresses that momentum strategies do not perform well in all markets. The second argument against EMH emerges from behavioral finance and states that investors overreact to some events and underreact to others. The author rebuts the argument by citing research indicating that overreaction is as typical as underreaction and that subsequent continuation of excess returns is as common as subsequent reversals, i.e., what appears to be a trend may as well be a random occurrence. The last school of thought challenging EMH is fundamental analysis. Supporters of this argument claim that certain valuation ratios such as initial dividend yield and P/E multiples can predict future stock results. However, these measures do not consistently predict stock performance in all periods, meaning the efficient market hypothesis is not violated. Also, occasional anomalies do not violate EMH, yet they lose predictive power when discovered and do not hold in the long run (Malkiel, 2003).

As mentioned in Section 2, some potential risks associated with investing in frontier markets include restrictions imposed on foreign investors, small and illiquid stock markets, less functioning or missing financial institutions, and often significant transaction costs and asymmetrical information. Hence, these markets fall short in terms of efficiency, and many are only efficient in the weak form (see Section 3.4.1). Furthermore, EMH implies that the market cannot be outperformed since current prices reflect all available information and that only the "market" portfolio is efficient. Frontier markets represent only a tiny portion of the total world market cap and world GDP, so it could be argued that FM should represent at least a similar allocation in a diversified portfolio (FTSE Russell, 2014).

2.3. Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) was developed in the early 1960s by Treynor (1962), Sharpe (1964), Lintner (1965a, b) and Mossin (1966). Building on the theory of Markowitz (1952), the model was the first coherent framework to explicitly express the positive relationship between the risk of an investment and its expected return. Also referred to as the single-index model, the CAPM is a fundamental contribution to the understanding of the determinants of asset prices and has, since its introduction, become a cornerstone in modern financial theory. A vital premise of the model is that investors should only be compensated for the systematic risk of holding the asset and not for otherwise diversifiable risk (Perold, 2004). The expected return of an asset is given by

$$E(r_i) = r_f + \beta_i[E(r_m) - r_f] \tag{1}$$

Where:

r_f is the risk-free rate

β_i is the beta of asset i

$E(R_m)$ is the expected return on the market portfolio

Equation (1) states that the expected return of an asset equals the risk-free rate and the product of the beta of the asset and the market (equity) risk premium, the latter being the excess return required by an investor on a diversified portfolio (Damodaran, 2022). The risk-free rate expresses the expected return on the risk-free asset, and CAPM assumes that investors can borrow or lend capital at this rate with complete certainty. Furthermore, the market portfolio is the portfolio that comprises all available shares of each risky asset, where the value of the market portfolio equals the total value of all risky assets. Asset prices and thus expected returns are set in equilibrium, i.e., when all investors decide to hold the exact supply of assets. Under the assumption that investors share the same investing universe and agree on the risk-free rate and the efficient frontier of risky assets, investors agree on the tangency portfolio's composition. Thus, in equilibrium, the portfolio of risky assets with the highest Sharpe ratio must be the market portfolio, i.e., the tangency portfolio (Perold, 2004; Munk, 2021).

Beta is an integral part of the CAPM as it measures the asset's volatility of returns relative to the market portfolio. Hence, beta is the only asset-specific determinant of the asset's risk premium, which according to CAPM, makes it the correct risk measure for each asset (Munk, 2021). The asset beta is defined as:

$$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)} \tag{2}$$

Where:

$Cov(r_i, r_m)$ is the covariance between asset i and the market portfolio m

$Var(r_m)$ is the variance of the market portfolio

By definition, the market portfolio has a beta of 1 and the risk-free rate has a beta of 0. As such, any asset with a beta value greater than 1 has more volatile returns than the market portfolio, and the investor would accordingly require higher expected returns. If the beta value of a risky asset is between 0 and 1, adding a marginal share of the asset to a portfolio will increase the Sharpe ratio of the portfolio if the asset's alpha is positive, i.e., if its risk premium satisfies

$$E(r_i) - r_f > \beta\left[E(r_m) - r_f\right] \tag{3}$$

The portfolio has the highest possible Sharpe ratio if $E(r_i) - r_f = \beta[E(r_m) - r_f]$ for every asset, in other words, if the risk premium for each asset is equal to the product of beta and the risk premium for the portfolio as a whole (Perold, 2004).

The CAPM rests on four main assumptions, the first being that capital markets are perfect and efficient. The following assumption states that investors have mean-variance preferences and the same investment horizon. Further, all investors have access to the same investment opportunities, and lastly, investors have homogenous expectations about asset returns, the standard deviation of returns and the correlation among asset returns (Perold, 2004). Despite its widespread application, the CAPM has been criticized for its fundamentally unrealistic assumptions that fail to describe the world particularly well. Several empirical tests of the model have sought to determine CAPM's predictive power by examining past stock returns and the volatility of returns. Findings include that beta as a measure of risk appears to be related to past returns; however, empirically, it is difficult to distinguish between the effects of total and systematic risk.

Furthermore, the empirical Security Market Line (SML) appears less steeply sloped than its theoretical counterpart, implying that low-beta assets provide slightly higher returns than CAPM would predict. Nevertheless, the data suggest a positive linear relationship between past returns and beta, which conforms to the model's predictions (Mullins, 1982). Critics have also challenged the proposition that there is only one relevant type of risk: systematic risk. Hence, risks such as liquidity, political, and credit risks are not considered. It is also worth noting that the assumption of a truly risk-free asset is unrealistic as there will always be some degree of default risk present.

2.4. Black-Litterman Model

The Black-Litterman asset allocation model was created by Fischer Black and Robert Litterman. The model was built to construct portfolios that overcome the problem of unintuitive, highly-concentrated portfolios. The Black-Litterman model, therefore, enables investors to combine their unique views regarding the performance of various assets with the market equilibrium in a manner that results in intuitive, diversified portfolios (Idzorek, 2007). Contrary to modern portfolio theory, the Black-Litterman model uses observed market data and investors' projections about future expected returns. The Black-Litterman model can therefore be considered an extension of modern portfolio theory that accounts for expectations about future performance.

The Black-Litterman Model recognizes that the true values of excess returns and covariances are generally unknown. However, the variances and covariances do not typically vary much over time and can be estimated quite precisely from the return time series. Therefore, it is possible to assume that the actual variance and covariance are known (Munk, 2021). Excess returns are, however, much more challenging to estimate and are more likely to change over time due to market-wide risk premiums and market sensitivity. The theory has some prior point estimates for the excess return, yet these remain only estimates (Munk, 2021).

Due to the difficulties of estimating precise and reliable estimates for expected returns from past performance, the Black-Litterman model suggests using CAPM-implied estimates (Munk, 2021).

The Black-Litterman approach model allows for combining the prior expected excess returns with a set of objective views on the return. The view predicts the excess return on one risky asset or a linear combination of excess returns on the risky asset (Munk, 2021). The Black-Litterman model combines the market equilibrium expected returns with investor views to generate a new vector of returns (Idzorek, 2007).

The vector of expected returns is the most important input in the mean-variance optimization model (Idzorek, 2007). A slight increase in the expected return on one of the portfolio's assets can force half of the assets from the portfolio (Best & Grauer, 1991; Idzorek, 2007). The Black-Litterman model uses "equilibrium" return as a natural starting point. The equilibrium returns are derived using a reverse optimization method in which the vector of implied excess equilibrium returns is extracted from available information (Idzorek, 2007). The equilibrium returns can therefore be derived based on a standard market excess return.

2.5. Return and Risk Properties

The risk-return tradeoff is an integral concept in portfolio theory and modern financial theory otherwise. The following section presents relevant measures and properties that will be used to evaluate the performance characteristics of the constructed portfolios. Reviewing portfolios across the same set of metrics allows for the assessment of the portfolios' relative performance viewed against a benchmark portfolio, thus contributing to the analysis and the assessment of its results.

2.5.1. Return

Historical asset returns may prove helpful in determining future returns. Time series of past returns are frequently used by analysts who try to estimate the type or shape of the distribution of future returns (Munk, 2021). Therefore, this thesis utilizes historical returns of the relevant indices as a reference point for expected returns, but it is also a necessary step in calculating the corresponding risk measures. As such, historical return is calculated on a monthly basis using past monthly index returns throughout the dataset and is given by $R_t = \frac{P_t}{P_{t-1}} - 1$

(4)

Where:

P_t and P_{t-1} is the price at period t and $t-1$

Equation 4 represents the return of a security at a given period in time. Given several observations, the arithmetic mean of historical returns can be calculated and annualized to provide easier interpretation and a basis for comparison. The extracted dataset is formatted on a monthly basis, which is why the compound growth rate is raised to the power of 12:

$$\bar{R} = \frac{1}{n}\left(\sum_{t=1}^{n} R_t\right), \quad n = 1, \dots, 81 \tag{5}$$

$$\bar{R}_A = (1 + R)^{12} - 1 \tag{6}$$

Where:

$n = 81$ is the number of observations in the dataset

However, under the assumption of mean-variance preferences, the investor only considers the expectation and the variance of returns on the portfolio over a given time horizon when choosing among different portfolios. The expected portfolio return is found by the proportional weight of each asset in the investment multiplied by the respective asset return (Munk, 2021). The expected return of a portfolio with assets is calculated as follows:

$$E[R_P] = \sum_{i=1}^{n} w_i E[R_i] \tag{7}$$

Where:

w_i is the weight of assets i and R_i is the return of asset i

The expected return of a single security is traditionally derived through CAPM, but the thesis utilizes an alternative approach for this application. Thus, the expected return is calculated as the sum of a country-specific equity risk premium and the risk-free rate. The decision to deviate from CAPM is discussed further in Section 4.

2.5.2. Volatility

The riskiness of an investment is determined by its volatility of returns, i.e., to what extent actual returns deviate from expected mean returns. Higher volatility is often associated with riskier investments. Variance and standard deviation of returns are common measures for quantifying risk and are frequently derived from the historical mean return over the analyzed period (Munk, 2021). The standard deviation is given as the square root of the variance of returns:

$$\sigma_R = \sqrt{\sum_{t=1}^{T} (R_t - \bar{R})^2} \tag{8}$$

Where:

R_t is the return at time t

$\bar{R}$ is the historical mean return

As with historical return, the standard deviation is annualized for easier comparison across markets and portfolios:

$$\sigma_A = \sigma\sqrt{12} \tag{9}$$

In a portfolio context, the standard deviation is measured using the weight and variance of each asset in combination with the covariance of all assets. The standard deviation for the portfolio is thus calculated as follows:

$$\sigma_p = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j \neq i}^{n} w_i w_j Cov(R_i, R_j)} \tag{10}$$

Where:

σ_i^2 is the variance of asset i

$Cov(R_i, R_j)$ is the covariance between asset i and j

2.5.3. Skewness and Kurtosis

Skewness measures the lack of symmetry of a distribution and is therefore relevant for the risk assessment of historical returns (Munk, 2021). The skewness is representative of the distribution relative to the normal distribution, which has a skewness of 0. A positively skewed return distribution implies a mean return greater than the median, whereas negative skewness implies a mean return less than the median. The skewness of a distribution is given by

$$Skew[R] = \frac{E[(R - \bar{R})^3]}{\sigma_R^3} \tag{11}$$

Another quantity that describes the shape of the return distribution is its kurtosis, which describes the probability of getting extreme returns or outliers. Kurtosis evaluates the thickness of the distribution’s tails and is therefore a measure of how much of the variance of returns arises from extreme values (Stock & Watson, 2020). A normal distribution has a kurtosis value of 0, whereas a positive kurtosis entails fatter tails implying a higher probability of large positive and negative return realizations. An investment with a negative kurtosis is associated with less risk than an investment with a positive one, holding all things equal. Kurtosis is calculated as follows:

$$Kurt[R] = \frac{E[(R - \bar{R})^4]}{\sigma_R^4} - 3 \tag{12}$$

2.5.4. Value at Risk

Value at risk (VaR) is the most important of the tail risk measures as it measures the potential loss on an asset or portfolio over a defined period for a given confidence interval (Munk, 2021). VaR can be quantified either in monetary units or in the percentage of the portfolio's initial value, and confidence levels are often chosen at 95% or 99%. Regarding historical returns and standard deviation of returns, the VaR represents the maximum negative return in percentage for one month.

For the application of this thesis, the 95% value at risk will be utilized. Value at risk is calculated as:

$$VaR_p = q_{95}$$

(13)

The value at risk is proportional to the standard deviation of the normal distribution and is therefore not of great informational value. However, as an additional risk measure, the VaR is especially relevant for distributions with fatter left tails, as these distributions will have the most significant potential loss (Munk, 2021). Furthermore, as a measure of risk exposure, commercial and investment banks often use VaR to capture the potential loss on portfolios from adverse market movements for a specific period (Damodaran, n.d.).

2.5.5. Downside Risk

Downside risk, or lower partial standard deviation, is the negative risk an investor incurs when investing. The downside risk is the standard deviation below the average return. It is calculated with the semi-variance, which is argued to be a better measure of risk, given that regular standard deviation includes upside risk (Estrada, 2008). The semi-variance of returns must be found and squared to compute downside risk. The semi-variance of an asset can be expressed as (Estrada, 2008):

$$\sigma_i^{LP} = \left(\frac{1}{n}\right) * \sum_{n=1}^{n} [Min(R_i - \bar{R}, 0)]^2$$

(14)

Where:

n is the total number of observations below the mean

R_i is the observed return where returns < average return ($\bar{R}$)

The value is considered only if R_i is less than $\bar{R}$, in which case the difference between the two values is used as input. However, if the daily return is higher than the mean return, the value becomes zero. Lower partial standard deviation is annualized using the same formula as standard deviation and is multiplied by the square root of the number of periods.

2.5.6. Maximum Drawdown

Maximum drawdown (MDD) represents the maximum observed loss from a peak to a trough during the analyzed period and is used to evaluate the downside risk of the constructed portfolios (Sortino & van der Meer, 1991). A low MDD value implies lower risk as return fluctuations are smaller and thus more stable. In contrast, a high MDD value is associated with more risk and significant return fluctuations. Risk-averse investors would therefore favor a portfolio with frequent small movements as opposed to a portfolio with fewer, albeit greater, movements in returns. Maximum drawdown is derived through the following formula:

$$MDD_P = \frac{TV_P - HWM_P}{HWM_P} \tag{15}$$

Where:

TV_P is the through value of the portfolio

HWM_P is the high-water mark, or peak value, of the portfolio

2.6. Performance Measures

The constructed portfolios will be evaluated based on their risk-adjusted returns. More specifically, the Sharpe ratio and Sortino ratio are used to analyze the respective performance of each portfolio.

2.6.1. Sharpe Ratio

The Sharpe ratio is a commonly used measure that quantifies the risk-return tradeoff of an investment. As such, it measures the expected excess return per unit of risk. The ratio is closely related to the concepts of MPT and CAPM, which state that investors should be compensated for enduring more risk. The Sharpe ratio is defined as the ratio between the risk premium and the standard deviation (Munk, 2021) and is given by:

$$Sharpe\ Ratio = \frac{E[R_P] - r_f}{\sigma_P} \tag{16}$$

Where:

$E[R_P]$ is the expected portfolio return

r_f is the risk-free rate

σ_P is the portfolio standard deviation

The Sharpe ratio can be used to locate the tangency portfolio on the efficient frontier, which is the portfolio that yields the optimal ratio and thereby the highest risk-adjusted return. However, the optimal portfolio is not necessarily the portfolio with the highest expected return but the highest risk-adjusted return. Further, the Sharpe ratio uses the standard deviation of returns to quantify risk, yet it could be argued that it might not be the best measure for risk. Standard deviation includes both positive and negative deviations from the expected value, meaning the result accounts for upside and downside risk. At the same time, most investors would only be concerned with the possibility of loss. It can therefore be argued that only the negative dispersion or the downside risk should be included in a risk measure, which has led to the emergence of the Sortino ratio (Munk, 2021).

2.6.2. Sortino Ratio

The Sortino ratio modifies the Sharpe ratio and represents the expected return per unit of lower partial standard deviation. By construction, the ratio excludes upside risk as this is generally not perceived as unfavorable to investors. The downside risk is computed by only utilizing the realization below the expected value or the risk-free return over the same period (Munk, 2021). The Sortino ratio is given as follows:

$$Sortino\ ratio = \frac{E[R_P] - r_f}{\sigma_P^{LP}} \tag{17}$$

Where:

DR_p is the standard deviation of the downside returns

σ_P^{LP} is the lower partial standard deviation of the portfolio

The Sortino ratio is used to evaluate the performance of portfolios by only considering the downside risk or negative returns. A conservative or risk-averse investor would prefer a portfolio with a higher Sortino ratio.

2.7. Correlation and Covariance

The first part of the analysis will calculate the correlation coefficient for global equity markets, providing insight into the relationship and co-movements between the respective individual markets. Correlation is defined as the covariance between products X and Y divided by their standard deviation (Stock & Watson, 2020) and is given by:

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(x) * Var(Y)}} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} \tag{18}$$

Correlation as a measure is unit free given that values or units in the numerator are equal to values or units in the denominator. Hence, the variables (X, Y) are uncorrelated if the correlation is 0. Further, the correlation will take on a value of $-1 < \rho_{X,Y} < 1$ (Stock & Watson, 2020). In correlation analysis, the quantities are considered symmetrical (Lindley, 1990). For the application of this thesis, frontier equity markets are categorized as the dependent variable, responding to the changes in the other explanatory variables (Yan & Su, 2009). Correlation coefficients are judged based on Taylor (1990), who states that there are systems that roughly categorize the coefficient based on value. Values less than 0,35 generally represent a low or weak correlation between the subjects, values between 0,36 and 0,67 represent a moderate or modest correlation, and values above 0,67 represent a high or strong correlation, whereas values above 0,90 exhibit a very high or strong correlation.

The covariance between assets must be derived before calculating the constructed portfolios' standard deviation. The covariance determines the co-movement between two assets, i.e., the degree of the linear relation between two random variables (Munk, 2021). A positive covariance indicates that the variables move in the same direction, whereas a negative covariance indicates opposing movement. The covariance between the two assets i and j is given as:

$$Cov(R_i, R_j) = E(R_i R_j) - E(R_i) * E(R_j) \tag{19}$$

Where:

R_i and R_j represent the return of assets i and j

$E(R_i)$ and $E(R_j)$ are the historical mean return of assert i and j

The covariance is annualized in order to evaluate and compare the portfolios between sample periods:

$$Annual\ Cov(R_i, R_j) = Cov(R_i, R_j) * 12 \tag{20}$$

3. Literature Review

The following section is dedicated to a comprehensive review of existing literature on topics relevant to the chosen field of research. It includes literature on equity market classification, developed and frontier markets, portfolio optimization and international diversification. The identified literature forms the foundation of the paper and will aid in recognizing potential research gaps within the field.

Articles were identified and selected on the basis of various criteria to reduce potential selection or publication bias. When searching for literature, the authors relied heavily on credible search tools and databases such as Copenhagen Business School's Libsearch function and Business Source Ultimate to ensure access to a broad range of academic journals. Relevant keywords like *frontier/emerging markets*, *portfolio*, *optimization/optimizing*, *diversification*, *market efficiency/EMH* etc. were utilized to locate potentially relevant literature. Furthermore, the authors prioritized peer-reviewed articles ahead of non-peer-reviewed articles.

3.1. Equity Market Classification

Leading providers of market indices such as Morgan Stanley Capital International (MSCI) and FTSE Russell evaluate and classify equity markets into market groups based on different criteria. MSCI has developed a framework that aims to reflect the views and practices of the international investment community, which is used to classify equity markets into "Developed", "Emerging", "Frontier" and "Standalone" markets (MSCI, 2022). The classification is based on three main criteria, namely 1) a country's economic development, 2) size and liquidity requirements, and 3) market accessibility criteria. Economic development entails the sustainability of economic development and is only used to separate developing markets. Size and liquidity determine the securities that meet the minimum requirements according to MSCI. Lastly, the market accessibility criterion aims to reflect international investors' experience concerning investing in a given market. It includes five criteria: openness to foreign ownership, ease of capital flows, efficiency of operational framework, availability of investment instruments, and stability of institutional framework (MSCI, 2022).

Developed markets are characterized by high income, openness to foreign ownership, ease of capital flows and efficient and established institutions. Examples of developed markets are Denmark, Japan and the United States. In contrast, emerging markets have a suitable degree of openness to foreign ownership, reasonable ease in capital flows, and fair stability of market institutions. Examples of emerging markets are Brazil, Greece and China. Lastly, frontier markets are characterized by low levels of liquidity and smaller market capitalizations, as well as being restrictive for foreign investors. Examples of frontier markets are Serbia, Morocco and Vietnam (MSCI, 2022).

Graham et al. (2013) define emerging markets as markets with low to middle-income per capita and frontier markets as a subset of emerging markets. From an investing point of view, the two markets possess distinct differences in potential benefits and drawbacks. According to Graham et al. (2013), frontier economies are at a stage of development similar to where emerging markets found themselves 10 to 15 years ago. Frontier markets will likely follow the same economic trajectory as emerging markets, making them desirable investment prospects for the international investment community. Furthermore, frontier markets exhibit a low correlation with other international equity markets. Although individual frontier markets are volatile in and of themselves, their addition to a more extensive portfolio should reduce portfolio volatility (Graham et al., 2013).

3.2. Benefits of International Diversification

Levy and Sarnat (1970) investigate the potential gains from international diversification and provide a method for empirically determining optimal international portfolio composition. Using common stock data from 28 countries from 1951-1967, the authors test for international diversification benefits and find that including developing countries in the opportunity set materially improves the risk-return position of an American investor. Given the low correlation between markets, low-yielding foreign investments in emerging and frontier market countries may significantly reduce the overall portfolio variance. Additionally, the ex-post results of the study suggest that restrictions on international trade significantly affect the pattern of security returns and permit inefficient markets to persist (Levy & Sarnat, 1970).

Driessen and Laeven (2007) explore how the benefits of international portfolio diversification differ across countries using monthly stock market index returns from 52 countries from 1985-2002. The authors find substantial regional and global diversification benefits for domestic investors in developed and emerging economies; however, the benefits of international portfolio diversification are more significant for developing countries than for developed countries. Additionally, the study finds country risk to be a good determinant of diversification benefits, i.e., countries with higher country risk have greater potential for diversification benefits. Lastly, diversification benefits have decreased throughout the sample period, which can be explained by improvements in country risk over time (Driessen & Laeven, 2007). Similarly, Chiou (2008) examines the benefits of international diversification for domestic investors in various countries and finds that investors in less developed countries benefit more from global and regional diversification. The effect is more remarkable for East Asia and Latin America, and the author also reports that global diversification benefits decrease as financial markets become more integrated (Chiou, 2008).

Gupta, Jithendranathan and Sukumaran (2011) estimate the potential benefits of frontier market diversification from the viewpoint of an Australian investor. Using an ADCC GARCH model to analyze changes in correlations between Australia and ten frontier markets between 1997-2011, the authors construct optimal portfolios and find an increase in mean returns from 3.98% to 12.43% when frontier markets are included in the portfolio. When restricted to a minimum of 50% investment in Australia, including frontier markets still provides significant increases in mean returns (Gupta et al., 2011). Sukumaran, Gupta and Jithendranathan (2015) examine the potential benefits of diversification into frontier markets for an Australian investor compared to a U.S. investor. The authors estimate time-varying correlations of returns using a conditional GARCH model and construct optimal portfolios subject to several restrictions to test for diversification benefits. Frontier markets demonstrate higher standard deviations in the sample period, implying that the markets are more volatile than developed markets. Furthermore, Australia and the U.S. correlate weakly to frontier markets. The low degree of correlation can be explained by the low integration of frontier markets into global markets, which are thus less affected by global economic trends in comparison to emerging and developed markets. However, the study found that the market correlation fluctuates over time and can be influenced by external factors such as global economic conditions and investor sentiment (Sukumaran et al., 2015).

Spiru and Qin (2016) study the potential for diversification in frontier markets by integrating 15 Central and Eastern European (CEE) and Middle Eastern and North African (MENA) frontier markets. The study utilizes data from 2005-2015 and employs correlation and cointegration techniques to determine short- and long-run correlations. The authors find a greater degree of integration at a regional level; however, at a global level, the selected CEE frontier markets were more correlated with their developed counterparts than markets in the MENA region. This implies that MENA markets may have more potential for diversification benefits.

3.3. Portfolio Selection

Portfolio selection is the theory of maximizing returns for a given risk profile (Markowitz, 1959). Financial markets are becoming more complex and challenging to navigate, which is why it is increasingly important for investors to consider different factors and aspects of markets to increase their returns. Investors attach wealth to stocks in exchange markets, and most prefer a combination of different stocks since a single stock carries inherent risk. Therefore, portfolio selection is an essential topic of investigation (Li et al., 2017; Wu et al., 2019). In general, portfolio selection aims to achieve specific long-term targets by allocating wealth to a set of assets (Li et al., 2015).

Odier and Solnik (1990) examine the issue of international asset allocation and how to allocate investments across different countries while minimizing risk and maximizing returns. The traditional approach to international asset allocation involves investing in a mix of different countries' stock and bond markets. Odier and Solnik (1990) argue that this strategy is inadequate due to highly correlated markets. The study suggests another strategy which includes investing in currencies and commodities that are less correlated with countries' stocks and bonds (Odier & Solnik, 1990).

Frontier markets offer a unique aspect for portfolio selection as their distinctive characteristics compared to other equity markets provide diversification potential for investors. Berger, Pukthuanthong and Yang (2011) find that frontier markets are weakly correlated with other global markets. Adding frontier markets to an international portfolio can thereby improve diversification and enhance returns (Berger et al., 2011).

3.4. Frontier Markets

Uludag and Ezzat (2016) define frontier markets as being at the furthest edge of the acceptable investment horizon, beyond which markets are no longer suitable for investment. As such, frontier markets are exposed to more risk and barriers, which may discourage and inhibit foreign investments. On the other hand, they offer significant profits and growth potential, thus attracting foreign investors (Uludag & Ezzat, 2016). Berger, Pukthuanthong and Yang (2011) define frontier markets as smaller, less accessible, yet still investible countries in the developing world. Frontier markets have received increased attention in recent years and are widely recognized as an exciting investment asset. Despite recent awareness, more research needs to be conducted to investigate the dynamics of frontier markets.

Frontier markets are characterized by illiquidity and significant transaction costs. Marshall et al. (2013) identify transaction costs up to three times as large in frontier markets compared to the US. when using high-frequency tick data from 19 markets. The authors conclude that the diversification benefits investors receive from the investment do not compensate for the transaction costs when portfolios are rebalanced at intervals of less than three months. Speidell (2018), however, emphasizes the improved conditions of financial markets in frontier countries, such as the presence of global audit firms that apply IFRS standards, resulting in greater transparency and liquidity in frontier markets. Nguyen et al. (2021) report on the impacts of financial reporting quality and corporate governance mechanisms in Vietnam and provide evidence that firms with high-quality financial reports and high state ownership tend to have lower idiosyncratic risk.

3.4.1. Market Efficiency

Amoah (2020) provides a meta-analysis of existing research reports involving the efficient market hypothesis (EMH) theory in emerging and frontier markets to determine how these equity markets conform to EMH. Additionally, the author seeks to identify determinants of efficiency in emerging and frontier markets. Of developing economies, the paper investigated studies of both market capitalization and EMH of stock exchanges in India, South Africa, Kenya, Ghana and Nigeria. Amoah (2020) concludes that emerging and frontier markets empirically lack the characteristics of efficient markets. Furthermore, the paper emphasizes that efficient market growth in these economies is prohibited by the lack of stock market awareness, hardly any financial education, and cultural influences. Market size is identified as a crucial factor in making markets efficient. However, the author notes that random walk characteristics were present in smaller-sized markets such as Argentina, Turkey and Indonesia. Other identified determinants are openness to foreign ownership, ease of capital inflow/outflow, competitive landscape, efficient operational framework, and stability of institutional framework, similar to the MSCI market classification methodology. Lastly, Amoah (2020) recommends that frontier markets need abrupt changes in institutional and operational framework in order to become more efficient and sustain stable financial markets.

Another study by de Villiers et al. (2020) examines the weak-form market efficiency hypothesis for 8 African frontier markets between 2001 and 2017. The authors perform root testing procedures that are robust to nonlinearities and smooth structural breaks. The research was conducted to determine whether stock market participants can use historical data on share returns to predict future streams of returns as a means of gaining abnormal profits. Using data of closing prices of stock returns for the respective stock exchanges, the authors find that most African frontier markets are not market efficient, in the weak-form sense, regardless of whether daily or weekly series are employed. However, exceptions were identified for the stock exchanges in Kenya and Botswana, which are weak-form efficient. Thus, the research supports the notion that market participants can devise strategies using different data frequencies to "beat the market" and obtain abnormal investment returns.

3.4.2. Investing in Frontier Markets

Identifying relevant value drivers provides insight into the markets' exposure and factors affecting the equity market. De Groot et al. (2012) analyze the cross-section of individual stock returns in 24 frontier markets over a 12-year period. The authors find that frontier equity markets are mainly driven by market liquidity, firm size, book-to-market ratio, momentum and country-specific factors (de Groot et al., 2012). Another study by Zaremba and Maydybura (2019) explores stock pricing in frontier equity markets and tries to determine whether the pricing is integrated or segmented. The researchers find that stocks are more segmented than integrated and that the cross-sectional relationship between stock returns and market characteristics is differentiated between markets (Zaremba & Maydyburda, 2019).

Recently, interest in frontier markets in the investment community has increased despite being a segment characterized by inaccessibility and uncertainty. Berger, Pukthuanthong and Yang (2011) researched the benefits and risks of investing in frontier markets. Potential benefits include diversification for international investors, growth potential due to frontier markets' potential for high economic growth rates, and untapped potential, as many frontier markets are undervalued compared to their emerging and developed counterparts. Among risk factors associated with investments in frontier markets, the authors highlight stock market volatility, low levels of liquidity making it challenging to buy and sell assets efficiently, and political and economic instability (Berger et al., 2011).

Rahman, Shien and Sadique (2013) investigate the relationship between investor sentiment and stock returns in the frontier market of Bangladesh. The analysis is based on a time-varying sentiment index and monthly data from 2001 to 2012. It provides empirical results of a significant positive relationship between shifts in investor sentiment and excess market returns. Additionally, the data suggest an asymmetric effect of the extent of bullish or bearish changes in sentiment on conditional volatility of returns. The overall findings of the paper suggest that shifts in investor sentiment in a frontier equity market represent a systematic risk factor in which the equilibrium is priced (Rahman et al., 2013).

An article written by Blanco (2013) investigates the world economic growth rates and discovers large economic growths within the frontier market categorization. Blanco (2013) finds that sizeable economic growth rates are no longer exclusive to the BRICS countries and that smaller, lesser-known economies such as Nigeria and Vietnam are critical players in global economic growth. He further uncovers that growth rates are expected to continue for decades. However, the article states that economic growth is still highly dependent on low labor costs and abundant natural resources. To mitigate problems with commodity dependence and ensure continued economic growth, many of these economies have created sovereign wealth funds and tech hubs (Blanco, 2013), illustrating frontier markets' willingness and ability to develop their economies and move away from dependent industries.

In a research article from 2014, FTSE Russell makes a case for frontier markets as a promising investment opportunity. The report argues that FM have historically higher GDP growth rates than their developed and emerging counterparts. Further, the markets have favorable demographics with relatively larger and younger populations. Frontier markets are also in an immediate urbanization process combined with rapid technological advances. These fundamentals, combined with low labor costs, present an attractive investment destination for large manufacturing companies, resulting in increased economic growth (FTSE Russel, 2014). The report further argues that frontier markets possess lower volatility than generally perceived. Frontier markets have historically been less volatile than developed and emerging markets. This could be due to the markets' relatively high degree of independence from other global markets, a trait that is difficult to find in an increasingly globalized world. Lastly, frontier markets still trade at a discount compared to their developed and emerging counterparts (FTSE Russel, 2014).

3.4.3. Correlation with Other Markets

Speidell and Krohne (2007) examine the relationship between frontier and developed equity markets and locate a low correlation between the two markets. Marshall et al. (2013) build upon this with their study and report a low correlation between frontier stock markets and developed and emerging market returns. Girad and Sinha (2008) find that frontier markets provide greater return potential and further diversification benefits compared to larger emerging markets. Another study by Samarakoon (2011) investigates the contagion effect of the US financial crisis on emerging and frontier equity markets. The study examines 22 emerging and frontier markets by applying various econometric techniques and finds strong interdependencies between the respective markets. The author notes that the effect is stronger during the crisis than pre-crisis and is more substantial for emerging equity markets than frontier markets (Samarakoon, 2011).

Baumöhl and Lyócsa (2014) explore the volatility and dynamic conditional correlation of emerging and frontier markets worldwide. The study utilizes daily stock market returns of 25 emerging and 22 frontier markets from January 2005 to May 2014. The study finds that both emerging and frontier markets experienced increased volatility during the financial crisis of 2008-2009. Emerging markets were shown to be more volatile than frontier markets. The study further suggests that the dynamic conditional correlation between emerging and frontier markets was generally low, indicating that the two market groups were not highly correlated. Lastly, the study discovered a significant increase in conditional correlation in periods of crisis for neighboring markets. The authors suggest that diversification across regions and geographical locations can help mitigate some risks associated with investments in frontier markets (Baumöhl & Lyócsa, 2014).

3.5. Investment Strategy

There are many different widely accepted investment strategies within the investment community. Lekovic (2018) discusses the utilization of investment diversification as an investment strategy to reduce risk. The article emphasizes the importance of investing across a broad range of assets and securities to help reduce the risk of loss. Lekovic (2018) continues explaining how different asset classes possess unique risk and return characteristics and that diversifying between them can create balance in a given portfolio. The article further discusses the concept of correlation and how correlation is essential to investment diversification. Assets with low correlation can help reduce the portfolio's overall risk, as they are less likely to move in the same direction simultaneously (Lekovic, 2018). Therefore, building a broad and well-diversified portfolio can be essential to minimize the risks associated with investing in a given market. As found previously, investments in frontier markets are inherently more volatile and subject to significant fluctuations. The volatility in frontier markets, combined with stronger illiquidity and higher transaction cost, makes it even more important for investors to have a well-diversified portfolio in case of large variations in equity prices for the short term.

4. Data

The following section provides an overview of the data utilized in the analysis, including the process of sourcing, cleaning and preparing the data, and a definition of the sample periods. A detailed explanation of the data is presented to secure the quality and accuracy of the data, which is crucial to the success of any analysis. Data cleaning and preparation involves filtering, structuring and transforming the raw data into a format suitable for analysis. Finally, details on the sample periods used in the analysis are described, including the rationale for selecting the specific time frames and any considerations revolving around this.

4.1. Data Collection and Data Quality

In order to build the analysis and attempt to meet the objectives of the thesis, the authors are dependent on acquiring financial data that are both relevant and accurate. Secondary data are considered the most appropriate datatype given the quantitative nature of the research. Further, secondary data are preferred to primary data as the resources spent acquiring the data are far smaller. However, it remains a disadvantage of using secondary data in that the data may be collected for a purpose that does not match the needs and objectives of this thesis. This is immediately not of concern as the majority of data used is in a raw numerical format, which preserves the objectivity of the data. Additionally, the authors have no real control over data quality, which stresses the importance of sourcing from widely recognized and generally accepted data providers (Saunders et al., 2020).

The dataset is retrieved from financial databases like Refinitiv Eikon (2023) and MSCI and consists of country indices provided by MSCI. Refinitiv Eikon offers the world's most comprehensive historical database for financial data across all major asset classes dating back to the 1950s (Refinitiv, n.d.). Provided by the London Stock Exchange Group, the data are considered to be of high accuracy and credibility. Further, MSCI is deemed a highly reliable and accurate provider of financial market data, with more than 50 years of expertise in research, data and technology within equity markets and portfolio management (MSCI, n.d.[a]). With that being said, a case could be made for using FTSE or S&P as providers of frontier market data, as these actors also have similar equity market classifications to that of MSCI – of the 18 frontier countries included in this paper, 16 are classified by all three actors as frontier equity markets.

The dataset is supplemented by data from numerous sources. Equity risk premiums are collected from Aswath Damodaran, who annually estimates equity risk premiums per country. The risk-free rate is also gathered from Investing.com, a top global financial website that provides financial data on stocks, commodities, bonds and more. Statistical databases such as the World Bank are also utilized when extracting relevant figures. The authors have exercised caution when sourcing secondary data from various sources to ensure the validity of the thesis' findings. The thesis makes exclusive use of globally recognized institutions to extract relevant data. Overall, the dataset and its reliability and validity are considered satisfactory for the research.

4.2. Index Data

This paper uses index data to analyze the diversification benefits of frontier markets. In doing so, the authors assume that investing in an index serves as a proxy for investing in a country's respective equity market. The analysis covers developed and frontier equity markets in all world markets and is based on the MSCI Country Classification. Developed markets are represented through the geographical segments Americas, Europe & Middle East, and Pacific, whereas frontier markets are represented through Europe, Africa, Middle East, and Asia (MSCI, n.d.[b]). In total, this study analyzes 23 developed markets and 18 frontier markets. Table 1 displays the markets included in the study in Developed and Frontier markets.

	Developed Markets			Frontier Markets		
Americas	*Europe & Middle East*	*Pacific*	*Europe*	*Africa*	*Middle East*	*Asia*
Canada	Austria	Australia	Croatia	Kenya	Bahrain	Bangladesh
United States	Belgium	Hong Kong	Estonia	Mauritius	Jordan	Pakistan
	Denmark	Japan	Lithuania	Morocco	Oman	Vietnam
	Finland	New Zealand	Kazakhstan	Nigeria		
	France	Singapore	Romania	Tunisia		
	Germany		Serbia			
	Ireland		Slovenia			
	Israel					
	Italy					
	Netherlands					
	Norway					
	Portugal					
	Spain					
	Sweden					
	Switzerland					
	United Kingdom					

Table 1: Countries Included in the Study (Source: Own contribution)

Values are given at a monthly frequency and denominated as gross US dollars, meaning dividend reinvestment and price movements are considered. Denoting values in USD provides consistency between the calculated expected returns and various risk rates used throughout the paper, as well as accounting for the issue of currency hedging by assuming that returns are converted into USD at the appropriate period’s spot foreign exchange rate (Stevenson, 2001).

4.2.1. Benchmark Index

The constructed portfolios will be compared to each other as well as a benchmark portfolio. A benchmark serves as an objective basis for comparison and allows the authors to observe the effects of introducing frontier markets on return and risk metrics. Given that the extracted dataset is denominated in US dollars, the S&P 500 index is an appropriate benchmark. S&P 500 includes 500 leading companies and covers approximately 80% of available market capitalization. The index is widely regarded as the best single gauge of large-cap US equities (Dow Jones, 2023) and is, therefore, a good benchmark for evaluating and comparing investments. S&P 500 is exclusively reserved for American companies, where Information Technology, Health Care and Financials constitute the largest sector weights in the index. Data from S&P 500 has been gathered from Refinitv Eikon (2023). Total returns at a monthly frequency are utilized within the same timeframe as the MSCI indices.

4.3. Data Preparation

The dataset is categorized by country and date to facilitate the research and analysis. Monthly observations are preferred over daily observations as daily observations may be prone to duplicate values. This is especially relevant for frontier markets, which suffer from illiquidity. Monthly observations mitigate the risk of encountering duplicate values and may provide a better and more realistic view of market movements. Hence, monthly data are more accurate for calculating correlation, covariance and standard deviation of the markets. Furthermore, specific markets included in MSCI's frontier market categorization have been excluded from the analysis due to a lack of data or other extraordinary conditions distorting the analysis. Sri Lanka has been ruled out of the dataset due to the economic collapse in 2022. Iceland and WAEMU[1] have also been removed due to a lack of available data from Refinitiv Eikon (2023).

Furthermore, the raw index data is logarithmically transformed to derive log returns. Log returns are preferred to simple returns for several reasons: I) logarithmic returns can be interpreted as continuously compounded returns, II) continuously compounded logarithmic returns are time additive, which is preferable when working with multi-period returns, III) logarithmic returns prevent security prices from taking on negative values, and IV) logarithmic returns are approximately equal to simple returns. Despite several good arguments in favor of using log returns to assess investment returns over longer periods of time, there has been debate over which method is the most appropriate to assess returns (Gregoriou, 2015). For the application of this thesis, however, log returns are utilized when optimizing portfolios.

[1] The West African Economic and Monetary Union (WAEMU) consists of the following countries: Benin, Burkina Faso, Ivory Coast, Guinea-Bissau, Mali, Niger, Senegal and Togo (MSCI, n.d.[b]).

4.4. Normality

Modern portfolio theory assumes that returns are normally distributed, an assumption that only holds if each individual stock's return is normally distributed. Under general conditions, the central limit theorem states that $\bar{Y}$ is sufficiently approximated by a normal distribution when the number of observations is large. The number of observations for $\bar{Y}$ to be approximately normally distributed depends on the underlying distribution of Y_i that constitutes the average (Stock & Watson, 2020). Considering the scope of this paper, a visualization of each market's normal distribution is not feasible. Therefore, the return distribution of the entire population is presented in Figure 3. The authors conclude that the population's return distribution is normal, but arguably slightly skewed to the right. Thus, from this point on, the assumption of normally distributed returns is deemed satisfied.

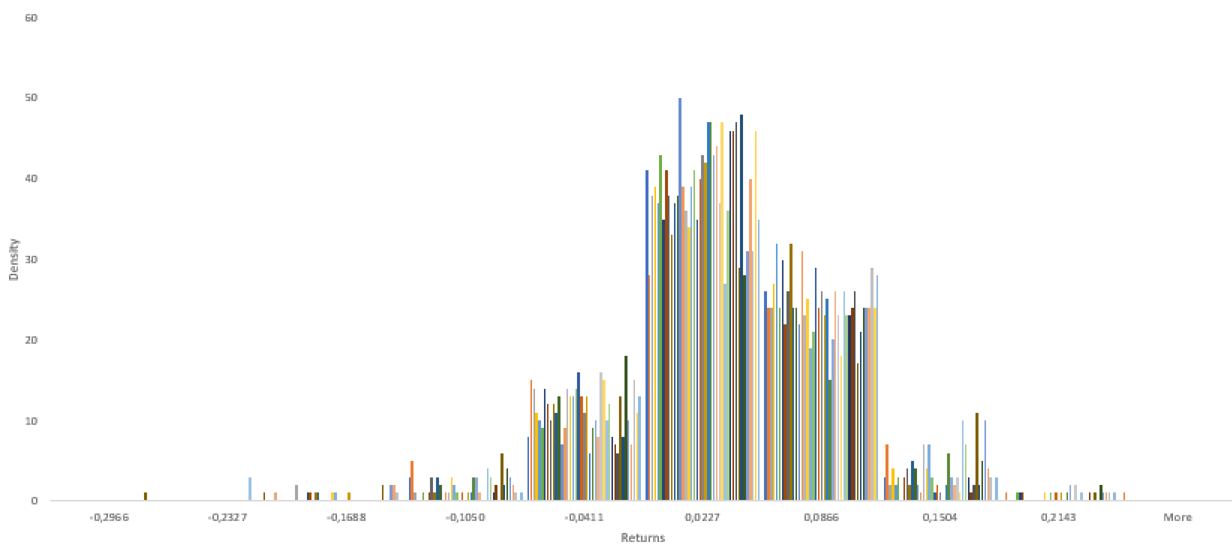

Figure 3: Return Distribution (Source: Own Contribution)

4.5. Risk-Free Rate

The risk-free rate is utilized to calculate expected returns and performance measurements such as each constructed portfolio's Sharpe- and Sortino ratio. Hence, the risk-free rate constitutes an integral element in this research paper, and proper consideration should be taken when determining its value. For an asset to be considered truly free of risk, i.e., for the expected return to always equal the actual return with certainty, two conditions must be satisfied. First, there can be no risk of default, and second, there can be no reinvestment risk (Damodaran, 2008). Long-term government bonds are often used as a proxy for the risk-free rate as governments control the printing of money and are most likely to fulfill their obligations, meaning the risk of default for governments is minuscule. However, this is not the case for every country – the risk of a government defaulting is inherently greater in a frontier market than in a well-established economy like the US.

Furthermore, the risk-free rate must be consistent with how cash flows are defined, mainly because the risk-free rate is not a suitable vehicle for conveying concerns about country-specific risk (Damodaran, 2008). Given that this thesis utilizes data denominated in US dollars and that expected returns are calculated in this currency (see 4.6. Expected Returns), the faithful proxy for the risk-free rate is the US treasury bond. Thus, for the application of this thesis, the average monthly yield on the 1-year US treasury bond between 30.04.2014 and 31.12.2020 is used as a proxy for the risk-free rate. It is noted that the period corresponds to the defined sample period (see 4.7. Sample Periods). Historical rates are retrieved from Investing.com:

$$r_f^{US} = 1.0240\% \tag{21}$$

4.6. Expected Returns

For the sake of optimizing portfolios under Markowitz's mean-variance framework, it is necessary to calculate each asset's expected returns, in this case for each country index. Efficient market theory and the random walk hypothesis suggest that subsequent changes in asset prices are random and unpredictable, meaning historical returns are not indicative of future returns on assets. As such, the historical performance of the gathered index data is not applicable for portfolio optimization. A conventional and widely accepted method for computing expected returns is CAPM, a single-index model consisting of the risk-free asset, the market risk premium and the security beta. However, as mentioned in Section 2.5.1., this paper defines expected returns as the sum of the country-specific risk premium and the risk free-asset:

$$E(R_i) = MRP_C + r_f$$

(22)

Where:

MRP is the market risk premium for country C

r_f is the risk-free rate

Thus, the expected return on any investment equals the sum of the risk-free rate and a market risk premium that compensates for the risk (Damodaran, 2022a). Risk premiums for countries included in this paper are extracted from Damodaran (2020), who estimates the long-term country equity risk premium by adding a country risk premium to a mature market premium; in his case, the implied equity risk premium of the S&P 500. The country premium reflects the additional risk in a specific market, and a standard proxy for this premium is the default spread on sovereign bonds or the CDS market. The dataset did not include premiums for Singapore, Bangladesh, Jordan or Mauritius. The difference in premiums between 2023 and 2020 values was first calculated for each country to derive the premiums as of 2020, after which an average was generated for each market segment. The average was then subtracted from the respective country's equity premium as of 2023, extracted from Damodaran (2023).

Next, the risk-free rate was added to arrive at the expected returns, which subsequently was adjusted for the average currency movement between the US dollar and each respective currency. For example, the Euro depreciated by 2,85% on average when measured against USD between 2014 and 2020, resulting in an adjusted expected return for EU countries of:

$$E\left(R_i^{Adj}\right) = E(R_i) \times (1 - 0{,}0285)$$

(23)

Given that these countries share the same currency, the expected return of each EU country is adjusted by the same factor. It is also noted that some countries peg their currencies to the USD, including Bahrain, Hong Kong, Jordan and Oman, meaning no real currency movement was reported in the relevant period. The United States is a major trading partner for oil nations in the Middle East. Currency pegs help stabilize exchange rates, thus promoting trade, foreign investments and growth. Expected returns per country are listed in Table 2. For complete calculation, see Appendix 2.

Developed Countries		Frontier Countries	
Risk-free rate	*1,02%*	*Risk-free rate*	*1,02%*
Australia	6,01%	Bahrain	8,65%
Austria	6,08%	Bangladesh	9,59%
Belgium	6,18%	Croatia	7,23%
Canada	6,19%	Estonia	6,67%
Denmark	6,04%	Jordan	11,29%
Finland	6,08%	Kazakhstan	6,06%
France	6,22%	Kenya	10,57%
Germany	6,05%	Lithuania	6,84%
Hong Kong	6,66%	Mauritius	6,99%
Ireland	6,23%	Morocco	7,33%
Israel	6,93%	Nigeria	9,20%
Italy	7,77%	Oman	9,46%
Japan	6,45%	Pakistan	10,61%
Netherlands	6,09%	Romania	7,11%
New Zealand	6,12%	Serbia	7,29%
Norway	5,81%	Slovenia	7,14%
Portugal	6,62%	Tunisia	9,83%
Singapore	6,15%	Vietnam	7,76%
Spain	6,68%		
Sweden	5,90%		
Switzerland	6,20%		
United Kingdom	6,13%		
United States	6,22%		

Table 2: Expected Returns per Country. Source: Own contribution.

The issue with using CAPM to estimate expected returns for frontier markets, and why another method is utilized in this thesis, relates to whether the traditional CAPM can be adopted in a global market. Some theorists and practitioners believe that all assets, regardless of where they are traded, should face the same global equity risk premium and that differences in risk are represented by differences in beta values (Damodaran, 2022b). The argument implies that if Nigerian stocks are riskier than US stocks, they should have higher betas and expected returns. However, when betas are estimated against global equity indices such as MSCI, equities in smaller, less developed and riskier countries consistently report lower betas than developed market equities. As such, using these betas with a global equity risk premium results in lower costs of equity for emerging and frontier markets, which distorts the notion that these markets inherently are riskier (Damodaran, 2022b).

4.7. Sample Periods

The sample period is defined as the time interval between two successive sampling points (Rao et al., 2016). It establishes the time frame for which data used in the analysis have been collected. Thus, any data outside the sample period is not considered. For the application of this thesis, an overall sample period from 30.04.2014 to 30.12.2022 has been chosen. April 2014 is the earliest point in time at which data were available for all country indices included in this study. Therefore, the first observation in 2014 is considered sufficient and equally representable for all markets.

Furthermore, the sample period is divided into two sub-periods: an in-sample period and an out-of-sample period. This aims to evaluate whether the testing result differs significantly between periods. The in-sample period begins on 30.04.2014 and ends on 31.12.2020, constituting roughly 6.5 years of monthly observations, which is deemed sufficient. By including 2020 in the in-sample period, the period incorporates the impacts of the Covid-19 pandemic, which was still in full effect by the end of 2020. This allows for investigating how the different equity markets responded to fiscal shocks. However, the authors are wary that the period is not representative of normal equity market movements and that this may distort the results.

Additionally, the out-of-sample period lasts from 31.12.2020 to 30.12.2022. The period will test the constructed portfolios and evaluate their application in global equity markets outside the in-sample period. The portfolios will be evaluated on the same performance and risk metrics as the in-sample portfolios to uncover similarities and differences in the performance. The in- and out-of-sample performance will be evaluated relative to the benchmark portfolio S&P 500. The optimal portfolios are formed based on rolling monthly rebalancing of portfolios. As such, portfolios are rebalanced each month to the monthly data retaining the optimized portfolio weights.

5. Methodology

The following chapter outlines the methodology used by the authors throughout the thesis to clarify how the collected data has been processed and used to construct optimal portfolios consisting of developed and frontier markets. As such, this section constitutes the foundation of the subsequent empirical analysis and interpretation of its results. The section elaborates on the portfolio construction and the methods utilized. The section then describes how the constructed portfolios are optimized for different performance metrics.

5.1. Construction Methods

This chapter outlines the methods used for constructing the portfolios in the study. The methods are chosen based on their reliability and relevance to the data under examination. The primary objective of portfolio construction is to construct and optimize portfolios to examine the diversification effects of frontier markets. To this end, nine portfolios will be constructed, categorized into three optimization methods, with three portfolios per method. Each optimization method will impose placement constraints regarding the markets available for inclusion. The initial portfolio constructed for each optimization method will exclusively include developed markets. The second portfolio will combine developed and frontier markets, with 80% of the allocation dedicated to developed markets. The third and final portfolio will extend the second portfolio, but with a 50% restriction on developed markets. This approach will enable the authors to gradually assess the portfolios' performance by increasing the frontier market allocation proportion. This methodology aims to improve comprehension of the impacts of frontier markets on international portfolios.

5.1.1. Mean-Variance Portfolio

The first set of portfolios are constructed using Markowitz's (1952) "portfolio selection". The analysis relies on Markowitz's theory to construct different portfolios for the developed and frontier markets. This method provides a mechanism for the selection of portfolios of securities in a manner that trades off the expected returns and risk of potential portfolios (Cornuejols & Tütücü, 2007). Expected return can be expressed as method can be expressed as:

$$E[R_P] = x_1 * E(R_1) + \cdots + x_n * E(R_n) = E(R)^n x \tag{24}$$

And standard deviation as:

$$\sigma_p = \sqrt{\sum_{i\,j} \rho_{ij}\sigma_i\sigma_j x_i x_j = x^T \Sigma x} \tag{25}$$

Where:

$E(R)$ is the expected asset return

σ the standard deviation

ρ_{ij} represent the correlation coefficient of the returns between asset S_i and S_j

A portfolio is considered efficient if it has the maximal expected return among all portfolios with the same variance or the minimum variance among all portfolios with at least a specific expected return. The collection of efficient portfolios creates the efficient frontier of the portfolio universe (Cornuejols & Tütücü, 2007).

5.1.2. Mean-Semivariance Portfolio

The mean-semivariance portfolio optimization method represents an extension of Markowitz's mean-variance approach. Although semi-variance is widely regarded as a more reasonable measure of risk than variance, portfolios are typically optimized using the traditional approach. According to Estrada (2008), Markowitz initially favored semi-variance as a risk measure.

The mean-semivariance method is similar to Markowitz's construction method but is based on the semi-variance of returns, which measures the downside risk of investments. By utilizing this method, the authors can optimize portfolios for the maximum Sortino ratio, yielding valuable insights into the behavior of frontier markets and their effects on international portfolios. Expected portfolio return can be expressed as:

$$E[R_P] = x_1 * E(R_1) + \cdots + x_n * E(R_n) = E(R)^n x \tag{26}$$

and

$$\sigma_p^{LP} = \sqrt{\sum_{i\,j} \rho_{ij} \sigma_{ij}^{LP} x_i x_j = x^T \sum x} \tag{27}$$

where

$$\sigma_{ij0}^{LP} = \left(\frac{1}{n}\right) * \sum_{n=1}^{n} \left[Min(R_{in} - \bar{R}, 0) * Min\left(R_{jn} - \bar{R}, 0\right)\right] \tag{28}$$

where

R is the asset return and E(R) is the expected asset return

ρ_{ij} represent the correlation coefficient of the returns between asset S_i and S_j

The construction of the formula is designed such that when a return exceeds the mean, the formula returns a value of zero. In contrast, when a return falls below the mean, the difference between the return and the mean is significant. Different from the mean-variance approach of Markowitz, the mean-semivariance method calculates the expected return using the same technique but measures downside risk using a covariance matrix derived from assets' lower partial standard deviation.

5.1.3. Maximum Black-Litterman Light

The third construction method takes inspiration from the Black-Litterman model and will be referred to as the Black-Litterman Light Portfolio Construction Approach. The approach utilizes different expected returns based on the Black-Litterman assumption that all world markets are in equilibrium. Black and Litterman (1992), He and Litterman (1999) and Litterman (2003) explore several alternative return forecasts, including historical returns, equal "mean" returns for all assets, and risk-adjusted equal mean returns (Idzorek, 2007). Hence, the expected return in this optimization method will be calculated based on the historical standard deviation for each market, a universal Sharpe ratio, and the risk-free rate. The universal Sharpe ratio is derived from the Sharpe ratio of the MSCI World Index, which incorporates 23 developed markets globally. The Sharpe ratio for the MSCI World Index based on data from 30.04.2014 to 31.12.2020 is 0,5791. The individual country's expected return is thereby calculated as follows:

$$E[R_i] = (SH_W * \sigma_i) + R_f \tag{29}$$

Where:

SH_W is the Sharpe ratio for the MSCI World Index

σ_i is the standard deviation of market i

R_f is the risk-free rate

Based on these derived expected returns, the portfolios are further optimized for the maximum Sharpe ratio. The calculations are identical to those of the first Sharpe ratio optimization method. This optimization method provides information regarding the individual markets' diversification properties.

5.1.4. Portfolio Construction Limitations

The mean-variance and semi-variance portfolio construction methods are linear, meaning the construction of portfolios is subject to certain limitations. First, the portfolio construction does not consider any short-sell options. This is important to consider as the results from the analysis might not represent the optimal allocation with the exclusion of short-selling options, and a portfolio constructed with short-selling options might generate different results. Second, the portfolios constructed are not leveraged, and the entire portfolio must always equal 100%. This assumes that investors invest all available capital and cannot borrow or find more capital. Therefore, the portfolio can be at most 100% of placement.

5.2. Portfolio Optimization

The portfolios are optimized to maximize or minimize specific performance or risk metrics, which allows for evaluating the portfolio’s value-creating ability and assessing frontier markets’ effect on portfolio performance. The optimization methods are chosen based on the degree of information they individually provide and how the results complement each other. Three portfolios are constructed for each optimization method to evaluate the gradual effect of frontier market inclusion.

5.2.1. Maximum Risk-Adjusted Return

The first optimization method utilized in the analysis is maximized risk-adjusted return. The Sharpe ratio measures the risk-adjusted return and provides insight into the portfolio's risk-return tradeoff. The Sharpe ratio explicitly measures the ratio between the risk premium and the standard deviation (Munk, 2021). The portfolio is constructed using the following formula:

$$Max(Sharpe\ ratio) = w^T \mu - w^T \sum w \tag{30}$$

Where:

w is the vector of portfolio weights

μ expresses the expected return of the assets in the portfolio

Σ is the covariance matrix

The formula is also subject to:

$$\sum_{i}^{k} w_i = 1 \ and \ w_i \geq 0$$

(31)

Equation (31) ensures that the sum of investments in the portfolio is 100% and that no short sales are allowed in the portfolio (Clarke et al., 2011). The optimized Sharpe ratio portfolios will provide valuable information regarding the effect of frontier markets on international portfolios. The individual portfolios expected return, standard deviation and other risk metrics will be considered to evaluate the performance thoroughly.

5.2.2. Maximum Sortino Ratio

A portfolio built to maximize the Sortino ratio is used to evaluate the portfolio's performance while only looking at the negative returns or the lower partial standard deviation. In contrast to the mean-variance portfolio and Sharpe ratio, the mean semi-variance portfolio only focuses on the portfolio's downside risk. The Sortino ratio is maximized using the following formula:

$$Max(Sortino\ ratio) = w^T\mu - w^T \sum w$$

(32)

Where:

w is the vector of portfolio weights

μ expresses the expected return of the assets in the portfolio

Σ is the covariance matrix for the negative returns

The formula is also subject to:

$$\sum_{i}^{k} w_i = 1 \ and \ w_i \geq 0$$

(33)

Condition (33) ensures that the sum of investments in the portfolio is 100% and that no short-selling is included (Clarke et al., 2011). The Sortino optimized portfolios can provide additional information to the results from the optimized Sharpe ratio portfolios. Since the portfolios are optimized based on negative downside risk, the results will provide different results compared to the Sharpe ratio and help the authors further understand the effects.

6. Analysis

The study is divided into two primary analyses to uncover the potential diversification benefits of frontier markets. The first section entails a correlation analysis of broader global equity indices and the individual countries included in the study. This initial analysis seeks to elucidate the relationship between developed and frontier equity markets to establish some preliminary assumptions about the potential diversification benefits of frontier markets.

The second analysis employs the previously mentioned construction and optimization methods to construct nine optimal portfolios. The results of this analysis will provide a deeper understanding of the effect of frontier markets on international portfolios. The model's consistency and predictive abilities are evaluated by testing the constructed portfolios in an in-sample and out-of-sample period. Furthermore, the results of the constructed portfolios from the sample periods will be compared to each other and a benchmark portfolio.

6.1. Correlation Analysis

In this part of the thesis, market correlations will be investigated. The analysis will provide insight into how global markets are impacted by each other and which markets are more independent regarding international movements. The individual equity market correlation breakdown aims to provide insight into the specific market correlations and give some preliminary information regarding portfolio optimization.

6.1.1. Global Equity Markets Correlation

Analyzing the correlation between the global MSCI indices is relevant for constructing a risk-mitigating portfolio between frontier equity markets and other global equity markets. The global markets most interesting in this analysis are the overarching categorized markets. These markets include emerging, frontier, and developed markets, as well as the benchmark portfolio S&P 500. Frontier markets' relationship with these market categorizations is essential as most investors investing in frontier markets will already be exposed to many of these markets. A high correlation between the markets will constitute a low diversification benefit and higher exposure to the same risk factors. A lower correlation coefficient will, on the other hand, indicate potential diversification benefits.

The analysis is based on the MSCI Frontier Markets Index. The MSCI Frontier Markets Index captures large and mid-cap representation across 28 Frontier Market countries (MSCI, 2023a). The index includes 96 constituents and covers 85% of each country's free float-adjusted market capitalization. The index is weighted by sector, with Financial (38,01 %) and Communication Services (11,5 %) being the biggest. Vietnam (26,6 %), Morocco (9,92 %), and Kazakhstan (8,92 %) have the most significant weight for individual countries (MSCI, 2023a). The MSCI Frontier Markets Index attempts to represent the equity development in frontier markets across countries and industries, providing a complete outlook of frontier markets.

The MSCI Frontier Market Index is compared to the MSCI Emerging Markets Index, which captures large and mid-cap representation across 24 emerging market countries (see Appendix 4). The index has 1 373 constituents and covers approximately 85% of each country's free float-adjusted market capitalization (MSCI, 2023b). The MSCI Emerging Markets Index has Financials (21,65 %), Information Technology (20,23 %), and Consumer Discretionary (13,61 %) as the most heavily weighted sectors. China (32,11 %), Taiwan (15,26 %), and India (13,23 %) are the most weighted countries in the index (MSCI, 2023b).

The MSCI World Index represents the developed equity market. The index captures large and mid-cap representations of 23 developed market countries (See Appendix 4). The index has 1 509 constituents and covers approximately 85% of free float-adjusted market capitalization in each country (MSCI, 2023c). The MSCI World Index is primarily weighted by sectors such as Information Technology (21,25 %), Financials (14,61 %), and Health Care (13,31 %). The United States weighs 67,67 %, while Japan and UK constitute the second and third most significant contributors at weights of 6,13 % and 4,43 %, respectively (MSCI, 2023c).

The S&P 500 index includes 500 leading companies and covers approximately 80% of available market capitalization. The index is widely regarded as the best single gauge of large-cap US equities (Dow Jones, 2023) and is, therefore, a good benchmark for evaluating and comparing investments. The index is exclusively reserved for American companies. Significant sector weights include Information Technology, Health Care, and Financials.

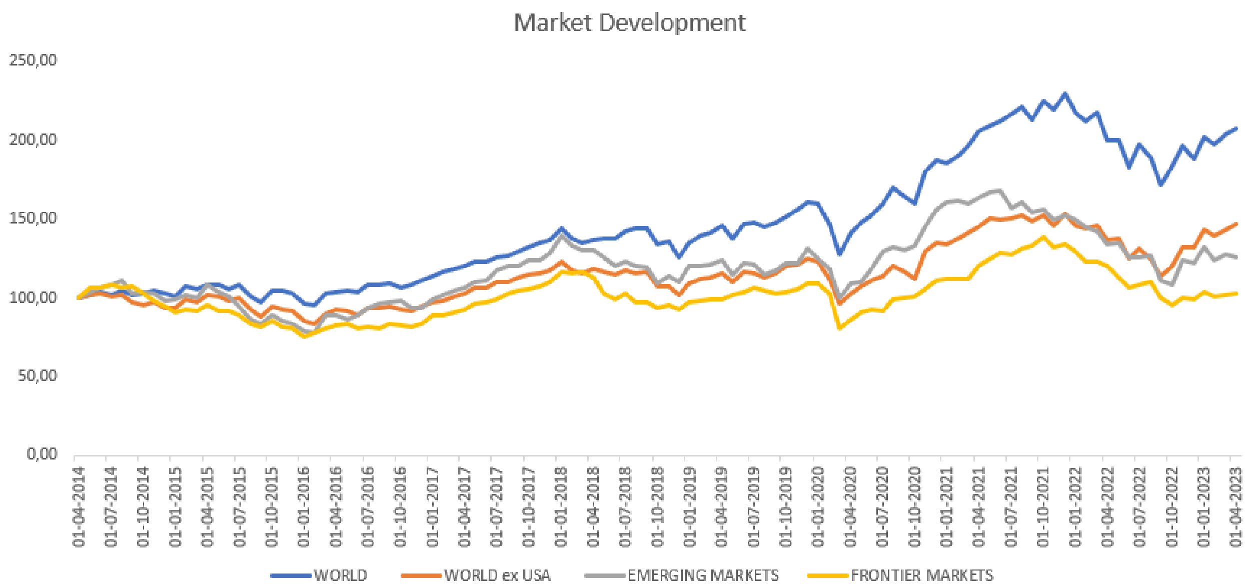

Figure 4: Global Markets Development (Source: Own Contribution)

Figure 4 shows the development of the MSCI Frontier Markets Index, MSCI Emerging Markets Index, and MSCI World Index, as well as the benchmark portfolio since 2014. The benchmark portfolio has exhibited the most significant growth, while MSCI Frontier Markets have grown significantly less.

While frontier markets have yet to have exceptional development as developed markets, it is interesting to look at the development compared to each other. Frontier markets seem to be far less affected by the movements of developed markets. This observation supports previous statements regarding the low correlation between frontier markets and other global markets. Frontier markets and emerging markets do, however, have a stronger relationship, which should be investigated further.

In order to perform a correlation analysis of the relevant market indices, data is collected and compared for a given time period. The in-sample period, 30.04.2014 to 31.12.2020, is used for the correlation analysis.

	S&P 500	Froniter Markets	Emerging Markets	World	World ex. USA
S&P 500	1				
Froniter Markets	0,1843	1			
Emerging Markets	0,2205	0,5025	1		
World	0,9909	0,2973	0,2936	1	
World ex. USA	0,7100	0,7353	0,5548	0,7981	1

Table 3: Global Equity Market Correlation (Source: Own Contribution)

The correlation analysis confirms many of the observations from the graph of equity market development for the three market indices. The correlation coefficient between the MSCI Frontier Markets Index and the benchmark portfolio is 0,1843. Taylor (1990) explains how the correlation coefficient can be interpreted. He indicates that correlation coefficients are difficult to interpret but that there are some labeling systems to categorize the values roughly. A value below 0,35 (in absolute value) generally represents a low or weak correlation between variables (Taylor, 1990).

The MSCI Frontier Markets Index further exhibits a low correlation with the MSCI World Index at 0,2973. However, the MSCI Frontier Markets and MSCI Emerging Markets indices are seemingly more related. The correlation analysis indicates a correlation coefficient of 0,5025 between the two indices. According to Taylor (1990), the coefficient represents a modest or moderate correlation. Taylor (1990) defines coefficients between 0,36 and 0,67 as modest or moderate, and the result from the analysis fits well within the categorization.

The analysis of frontier markets' relationship with emerging and developed markets shows that frontier markets are somewhat isolated from other global markets. The low correlation with the MSCI World Index is fascinating as the low correlation indicates a possible risk-mitigating strategy for a portfolio by combining the two indices. On the other hand, emerging markets are moderately correlated and will have a smaller diversification benefit. Furthermore, the emerging markets index has a higher correlation with global markets (0,2936), indicating that the index will move more similarly to the World Index simultaneously.

Interestingly, the frontier markets correlation with the MSCI World ex. US index is significantly higher. Frontier markets may therefore correlate more with other developed markets than the US. Emerging markets have a lower correlation with the MSCI World ex. US Index, indicating that emerging markets could be a better diversification tool for investors outside the US.

6.1.2. Individual Market Correlation

The second analysis builds upon the findings in the market classification index correlation. The first analysis found that frontier markets have a low correlation with global developed markets. An extensive correlation analysis of all equity markets included in the study is conducted to explore these findings further. The analysis aims to provide insight into how the different markets impact each other and which markets the portfolios could include.

The first analysis, as stated earlier, investigates the correlation between the developed markets included in the study. Figure 5 highlights correlation coefficients between the markets, with the highest correlation in red and the lowest or negative correlations in blue. The strength of color represents how far away from the average the correlation is. The analysis shows some predictable as well as some surprising results. France and Germany have the strongest correlation with other developed markets, with an average of 0,79. It can therefore be assumed that these markets offer little diversification benefit between themselves, and an optimal portfolio would not have large stakes in more than one of the markets.

On the other hand, Israel, New Zealand, and Hong Kong have low correlations with the rest of the developed markets. The markets are geographically separated from other developed markets, which could explain the lower-than-average correlation. On the other hand, Denmark also possesses a low correlation coefficient while being highly intertwined with European trade connections. Denmark's low correlation with the other developed markets could therefore present a good diversification benefit for European investors.

Additionally, certain relationships are worth mentioning. France and Germany have the highest market correlation, with a 0,93 correlation. These markets can therefore be said to move, more or less, identical to each other. Israel and Portugal have the lowest correlation of any market, with a correlation of 0,37.

	Australia	Austria	Belgium	Canada	Denmark	Finland	France	Germany	Hong Kong	Ireland	Israel	Italy	Japan	Netherlands	New Zealand	Norway	Portugal	Singapore	Spain	Sweden	Switzerland	United Kingdom
Austria	0,79																					
Belgium	0,69	0,75																				
Canada	0,83	0,80	0,69																			
Denmark	0,54	0,58	0,64	0,51																		
Finland	0,72	0,72	0,79	0,65	0,66																	
France	0,74	0,88	0,83	0,77	0,67	0,84																
Germany	0,75	0,85	0,80	0,76	0,70	0,84	0,93															
Hong Kong	0,71	0,63	0,58	0,61	0,46	0,57	0,64	0,63														
Ireland	0,64	0,74	0,64	0,68	0,61	0,66	0,79	0,80	0,44													
Israel	0,61	0,54	0,55	0,59	0,53	0,57	0,56	0,62	0,43	0,54												
Italy	0,68	0,84	0,75	0,69	0,64	0,75	0,91	0,86	0,56	0,70	0,49											
Japan	0,68	0,74	0,70	0,67	0,63	0,71	0,77	0,81	0,64	0,60	0,61	0,70										
Netherlands	0,71	0,83	0,86	0,74	0,73	0,79	0,91	0,91	0,62	0,79	0,61	0,83	0,76									
New Zealand	0,72	0,56	0,57	0,67	0,49	0,54	0,58	0,61	0,63	0,53	0,56	0,48	0,53	0,60								
Norway	0,74	0,79	0,70	0,81	0,54	0,72	0,80	0,74	0,58	0,62	0,44	0,72	0,71	0,72	0,54							
Portugal	0,58	0,71	0,61	0,58	0,65	0,66	0,75	0,72	0,44	0,58	0,37	0,71	0,60	0,74	0,49	0,67						
Singapore	0,76	0,75	0,67	0,75	0,46	0,66	0,76	0,73	0,82	0,52	0,49	0,69	0,70	0,68	0,64	0,71	0,51					
Spain	0,71	0,87	0,76	0,74	0,60	0,74	0,92	0,86	0,59	0,68	0,53	0,92	0,72	0,85	0,51	0,73	0,70	0,71				
Sweden	0,73	0,82	0,76	0,76	0,71	0,81	0,90	0,91	0,64	0,81	0,58	0,82	0,76	0,86	0,61	0,78	0,67	0,75	0,81			
Switzerland	0,64	0,70	0,75	0,65	0,77	0,75	0,81	0,83	0,59	0,66	0,59	0,74	0,71	0,84	0,60	0,67	0,71	0,62	0,73	0,80		
United Kingdom	0,79	0,85	0,82	0,83	0,60	0,80	0,91	0,86	0,68	0,79	0,57	0,82	0,77	0,85	0,64	0,85	0,69	0,79	0,84	0,85	0,80	
United States	0,82	0,78	0,79	0,84	0,61	0,78	0,81	0,84	0,63	0,74	0,71	0,69	0,78	0,83	0,68	0,73	0,56	0,73	0,69	0,82	0,76	0,83

Figure 5: DM Correlation (Source: Own contribution)

The second correlation analysis examines the correlation between developed markets and frontier markets. The markets are significantly less correlated with each other than the initial correlation between the world indices suggested. The average correlation for developed markets is 0,70, while the average between developed and frontier markets is 0,41. This further validates the claim that frontier markets possess a lower correlation with developed markets and can be used to achieve diversification benefits.

The highest correlation between frontier and developed markets is Slovenia, Lithuania, Croatia, and Romania. These possess relatively high correlations with developed markets. The findings suggest that geographical location is a vital determinant regarding markets' correlation and that markets in close proximity to each other move in similar patterns. Based on the findings, it can be assumed that these markets would not provide significant diversification benefits and that the optimized model will not include these markets. On the other hand, Jordan, Bangladesh, and Tunisia have the lowest correlation with developed markets. Other markets may impact the markets less due to geographical location and fewer trade interactions. The findings suggest that Jordan, Bangladesh, and Tunisia will be essential assets regarding market diversification.

	Bahrain	*Bangladesh*	*Croatia*	*Estonia*	*Jordan*	*Kazakhstan*	*Kenya*	*Lithuania*	*Mauritius*	*Morocco*	*Nigeria*	*Oman*	*Pakistan*	*Romania*	*Serbia*	*Slovenia*	*Tunisia*
Bangladesh	0,19																
Croatia	0,44	0,16															
Estonia	0,35	0,08	0,53														
Jordan	0,05	0,13	-0,04	-0,08													
Kazakhstan	0,37	0,28	0,49	0,38	0,08												
Kenya	0,32	0,19	0,29	0,41	0,03	0,29											
Lithuania	0,48	0,13	0,58	0,65	0,03	0,48	0,52										
Mauritius	0,42	0,25	0,56	0,64	-0,09	0,40	0,29	0,40									
Morocco	0,47	0,30	0,63	0,63	0,02	0,45	0,33	0,62	0,68								
Nigeria	0,26	0,24	0,33	0,32	0,09	0,42	0,38	0,36	0,35	0,37							
Oman	0,39	0,25	0,32	0,33	-0,16	0,25	0,20	0,29	0,48	0,46	0,19						
Pakistan	0,46	0,19	0,47	0,44	0,11	0,28	0,38	0,58	0,46	0,57	0,28	0,40					
Romania	0,49	0,20	0,64	0,51	-0,09	0,48	0,37	0,56	0,46	0,44	0,40	0,44	0,45				
Serbia	0,43	0,14	0,54	0,43	0,06	0,44	0,20	0,45	0,48	0,57	0,43	0,36	0,32	0,56			
Slovenia	0,44	0,12	0,72	0,64	-0,16	0,47	0,41	0,68	0,58	0,61	0,45	0,39	0,49	0,65	0,61		
Tunisia	0,26	0,10	0,33	0,24	-0,06	0,20	0,29	0,37	0,23	0,35	0,12	0,03	0,10	0,32	0,31	0,39	
Vietnam	0,43	0,28	0,42	0,50	0,06	0,37	0,37	0,48	0,45	0,53	0,35	0,44	0,53	0,53	0,37	0,46	0,26

Figure 6: FM Correlation (Source: Own contribution)

	Australia	Austria	Belgium	Canada	Denmark	Finland	France	Germany	Hong Kong	Ireland	Israel	Italy	Japan	Netherlands	New Zealand	Norway	Portugal	Singapore	Spain	Sweden	Switzerland	United Kingdom	United States
Bahrain	0,47	0,49	0,38	0,52	0,31	0,36	0,45	0,36	0,41	0,41	0,24	0,43	0,34	0,42	0,29	0,52	0,43	0,45	0,44	0,43	0,42	0,48	0,46
Bangladesh	0,19	0,11	0,13	0,20	0,12	0,17	0,14	0,12	0,06	0,11	0,17	0,17	0,11	0,12	0,02	0,20	-0,02	0,08	0,13	0,09	0,11	0,16	0,24
Croatia	0,62	0,64	0,53	0,61	0,45	0,57	0,62	0,62	0,50	0,49	0,40	0,56	0,59	0,61	0,59	0,62	0,57	0,64	0,60	0,63	0,59	0,63	0,58
Estonia	0,65	0,67	0,57	0,61	0,50	0,48	0,58	0,58	0,39	0,46	0,44	0,58	0,47	0,58	0,51	0,55	0,53	0,53	0,61	0,55	0,47	0,55	0,54
Jordan	-0,08	-0,17	-0,27	-0,15	-0,14	-0,13	-0,23	-0,16	-0,09	-0,04	-0,11	-0,16	-0,16	-0,18	-0,16	-0,19	-0,09	-0,15	-0,25	-0,19	-0,12	-0,14	-0,13
Kazakhstan	0,54	0,48	0,34	0,52	0,31	0,37	0,41	0,46	0,36	0,35	0,36	0,39	0,42	0,41	0,38	0,46	0,30	0,50	0,47	0,38	0,35	0,42	0,49
Kenya	0,39	0,49	0,46	0,45	0,36	0,42	0,43	0,43	0,40	0,34	0,18	0,44	0,38	0,43	0,25	0,37	0,39	0,46	0,42	0,40	0,34	0,44	0,43
Lithuania	0,64	0,69	0,62	0,67	0,60	0,62	0,63	0,64	0,48	0,57	0,44	0,63	0,48	0,66	0,52	0,60	0,59	0,61	0,60	0,64	0,55	0,58	0,64
Mauritius	0,59	0,60	0,41	0,61	0,25	0,45	0,58	0,50	0,37	0,46	0,35	0,56	0,38	0,44	0,37	0,53	0,38	0,51	0,59	0,47	0,35	0,58	0,44
Morocco	0,63	0,66	0,46	0,60	0,39	0,48	0,58	0,54	0,39	0,44	0,38	0,63	0,44	0,50	0,43	0,55	0,42	0,55	0,63	0,55	0,38	0,51	0,49
Nigeria	0,35	0,42	0,27	0,36	0,34	0,32	0,38	0,33	0,19	0,39	0,23	0,44	0,31	0,36	0,17	0,42	0,31	0,34	0,45	0,34	0,28	0,38	0,30
Oman	0,42	0,38	0,25	0,42	0,29	0,38	0,42	0,38	0,31	0,27	0,28	0,42	0,42	0,31	0,25	0,49	0,28	0,37	0,38	0,38	0,35	0,39	0,40
Pakistan	0,59	0,58	0,51	0,61	0,31	0,54	0,54	0,56	0,43	0,53	0,54	0,52	0,49	0,50	0,46	0,54	0,36	0,56	0,51	0,55	0,45	0,62	0,59
Romania	0,57	0,63	0,59	0,54	0,48	0,55	0,65	0,61	0,45	0,47	0,37	0,60	0,55	0,64	0,50	0,63	0,57	0,58	0,61	0,57	0,64	0,63	0,61
Serbia	0,48	0,48	0,22	0,40	0,30	0,32	0,41	0,37	0,24	0,35	0,11	0,46	0,28	0,32	0,31	0,42	0,45	0,39	0,48	0,34	0,34	0,41	0,31
Slovenia	0,69	0,70	0,62	0,65	0,64	0,69	0,72	0,69	0,54	0,54	0,47	0,70	0,57	0,68	0,62	0,70	0,69	0,65	0,74	0,66	0,66	0,70	0,59
Tunisia	0,26	0,22	0,17	0,27	0,19	0,23	0,23	0,22	0,13	0,21	0,05	0,25	0,08	0,24	0,16	0,24	0,31	0,14	0,24	0,22	0,19	0,18	0,16
Vietnam	0,62	0,56	0,48	0,52	0,36	0,49	0,51	0,49	0,46	0,44	0,41	0,47	0,57	0,52	0,37	0,52	0,44	0,53	0,47	0,46	0,44	0,52	0,57

Figure 7: DM and FM Correlation (Source: Own contribution)

There are some other specific relationships worth mentioning. Spain and Romania have the highest correlation with 0,74. The relationship could be because both countries rely highly on European trade, as Spain and Romania export 71,51% and 80,40% of all goods to Europe, respectively (OEC, 2021). It can therefore be assumed that the economies are subject to the same changes in the market and will be affected similarly to changes in the economic landscape in Europe. Jordan and Belgium, on the other hand, possess the lowest correlation coefficient. The countries possess quite different trade structures, which may be the reason for the low correlation. Only 7,36% of Jordan's export is to Europe, and the majority of trade is with Asia. This removes Jordan from much of the economic outcomes European and Western countries experience during economic downfalls.

The initial correlation analysis provides insight into the global equity markets' relationships and how they are correlated. The analysis confirms the author's initial sub-question regarding frontier markets' correlation with developed markets, as the global and individual market correlation analyses provide the same results. The analysis further enables the authors to make certain assumptions regarding the next section of the analysis. According to previous research, countries with lower correlation coefficients should be more attractive in a risk-reducing portfolio. The correlations of the markets suggest that markets such as Israel, New Zealand, and Hong Kong will have a significant stake in the developed markets portfolios, as the markets have significantly lower correlations with the rest of the developed markets and, therefore, should possess risk-reducing properties.

Regarding the frontier markets and their correlation coefficient with developed markets, Jordan, Bangladesh, and Tunisia, which all possess negative correlation coefficients, should, based on previous literature and assumptions, have more significant stakes in the frontier market, including portfolios. It is, however, essential to remember that certain countries have low correlation coefficients with one another despite having a generally high coefficient on average. Furthermore, the return/risk profile is a vital aspect of portfolio construction, and the inclusion of markets is highly dependent on this. It will, however, be interesting to investigate the portfolio compositions in the next section to investigate whether or not the initial assumptions are upheld.

6.2. Portfolio Analysis

In the following section, portfolios are constructed and compared for the two market combinations. The analysis will investigate the strengths and weaknesses of the selected markets and how diversification between the markets can mitigate some of the risks associated with undiversified investing.

6.2.1. Descriptive Statistics

The summary statistics in Table 4 illustrate the historical risk and return properties of developed and frontier markets between 2014 and 2020. Developed markets present a good investment opportunity as several markets exhibit high returns per unit of risk. The best performers in the time period are the United States, Denmark, and the Netherlands, with a Return/Risk profile of 0,92, 0,77, and 0,66, respectively. The worst performers include Spain and Portugal, who experienced negative log returns during the period. Norway and the United Kingdom both experienced returns close to zero. Austria recorded the most significant volatility of all developed markets, with a standard deviation of 28,73%. However, developed markets' overall return per unit of risk is positive and indicates steady market increases from 2014 to 2020.

Country	Mean return	Standard deviation	Return/Risk	Skewness	Kurtosis
Australia	4,04%	20,53%	0,1967	-1,4497	7,0236
Austria	1,36%	28,73%	0,0474	-0,7550	5,5792
Belgium	1,03%	20,03%	0,0513	-0,3043	2,7333
Canada	3,75%	18,35%	0,2043	-0,9939	4,9535
Denmark	11,39%	14,82%	0,7685	-0,1884	-0,5674
Finland	7,56%	16,53%	0,4575	-0,0689	1,1702
France	5,50%	18,04%	0,3050	-0,0941	3,7805
Germany	3,62%	19,20%	0,1886	-0,2920	1,0750
Hong Kong	7,39%	18,14%	0,4076	-0,5144	0,3438
Ireland	4,95%	17,89%	0,2770	-0,5257	0,9286
Israel	1,11%	20,20%	0,0547	-0,5233	0,7784
Italy	0,72%	22,79%	0,0314	-0,2785	3,6156
Japan	8,39%	13,47%	0,6225	-0,2388	1,0028
Netherlands	10,45%	15,80%	0,6613	-0,2012	0,1168
Norway	9,95%	18,89%	0,5264	0,2053	0,0222
New Zealand	0,06%	21,81%	0,0027	-0,2365	1,5396
Portugal	-0,22%	19,97%	-0,0109	-0,1479	0,3878
Singapore	1,92%	19,32%	0,0995	-0,5282	3,1142
Spain	-1,71%	21,75%	-0,0787	0,0401	4,9826
Sweden	5,59%	16,92%	0,3303	-0,1586	0,9944
Switzerland	7,08%	12,50%	0,5667	-0,4376	-0,1411
UK	0,09%	16,35%	0,0057	-0,4805	2,5205
US	13,36%	14,49%	0,9216	-0,5505	2,0292
Average	4,67%	18,55%	0,2886	-0,3792	2,0862

Table 4: Summary Statistics of Monthly Log-Returns for DM (Source: Own contribution)

The unequivocal majority of developed countries exhibited negatively skewed log returns between 2014 and 2020, with the exception of Spain and New Zealand. Negative skewness indicates a higher susceptibility to negative shocks in the market, whereas positive skewness indicates a higher susceptibility to positive shocks. The most extreme values are seen in Australia and Canada, with a skewness of -1,45 and -0,99, respectively.

Table 5 illustrates the descriptive statistics for frontier markets. An interesting observation is that the return per unit of risk is negative for the frontier markets. Frontier markets have -0,04 return per unit of risk. The ratio indicates that frontier markets have not been an attractive investment opportunity in the in-sample period. Jordan (-0,5910) and Pakistan (-0,3741) have large negative ratios, impacting the overall market average. There are, however, markets with positive returns per unit of risk. Romania (0,3252) and Vietnam (0,3186) present higher positive return/risk ratios than the developed markets' average ratio.

Country	Mean return	Standard deviation	Return/Risk	Skewness	Kurtosis
Bahrain	-1,17%	21,57%	-0,0544	-0,4105	3,9748
Bangladesh	2,22%	19,41%	0,1142	0,4180	2,4623
Croatia	5,51%	15,04%	0,3660	-0,3168	1,3083
Estonia	2,09%	22,65%	0,0922	-0,8072	5,8270
Jordan	-10,87%	22,38%	-0,4857	-4,1464	29,3834
Kazakhstan	6,33%	29,12%	0,2175	-1,0452	2,2638
Kenya	7,58%	20,36%	0,3722	-0,2693	0,5150
Lithuania	2,96%	17,44%	0,1699	-0,5488	4,0932
Mauritius	-5,08%	24,33%	-0,2088	-3,6619	24,5354
Morocco	4,17%	16,98%	0,2458	-2,6358	15,7403
Nigeria	-8,44%	30,93%	-0,2729	-0,8205	1,4328
Oman	-0,47%	15,38%	-0,0309	-0,4819	1,5512
Pakistan	-7,20%	28,16%	-0,2557	-1,6202	8,0419
Romania	12,83%	24,03%	0,5337	-0,5415	0,8446
Serbia	-0,78%	22,67%	-0,0342	-1,0313	3,0401
Slovenia	7,35%	21,08%	0,3489	-0,3432	2,1794
Tunisia	4,07%	15,57%	0,2616	0,3681	0,9882
Vietnam	5,22%	22,06%	0,2367	-0,7990	3,4086
Average	1,46%	21,62%	0,0898	-1,0385	6,1995

Table 5: Summary Statistics of Monthly Log-Returns for FM (Source: Own contribution)

Table 5 further presents a similar skewness for frontier markets to developed markets. Most markets exhibit negative skew and are more susceptible to negative market shocks. There are specific markets, such as Jordan (-2,7579), Mauritius (-2,7563), and Morocco (-1,7563), contributing to most of the negative skewness. However, the overall market skewness can be considered moderate at a value of -0,56.

6.2.1.1. *Discussion of Findings*

The preliminary analysis (descriptive statistics) presents some interesting findings. The statistics show that developed markets, for the selected period, present a better investment opportunity for the market as a whole. Most individual developed markets have positive returns per unit of risk, and the overall market has a return per unit of risk of 0,1985. Frontier markets, on the other hand, present a less attractive investment opportunity as the overall market have a return per unit of risk of -0,4. There are, however, specific markets within the frontier markets that present good investment opportunities.

6.2.2. Optimization Results

This section presents the findings of the constructed portfolios, which are based on the asset allocation optimization methods used for developed and frontier markets. The assets are allocated through optimization based on portfolio construction methods for the in-sample period. The markets included in each portfolio, and their performance are presented in this section.

The first set of portfolios, constructed through the mean-variance approach and optimized for the Sharpe ratio, will hereby be referred to as P1. The second set of portfolios, constructed according to Estrada's mean-semivariance method and optimized for Sortino, will be called P2. The last set of portfolios, constructed using different expected returns and optimized for the Sharpe ratio, will be referred to as P3. The portfolio sets are further optimized according to certain allocation restrictions. The first portfolio in each portfolio set is optimized with a restriction of solely utilizing developed markets. These portfolios will be referred to as Portfolio A. The second portfolio, optimized with an 80% developed market restriction, will be called Portfolio B. The last portfolio, optimized with a 50% developed market restriction, will be referred to as Portfolio C. Therefore, the first portfolio in the first set, constructed using Markowitz, optimized for Sharpe, and only using developed markets, will be referred to as P1-A.

6.2.2.1. MVP Sharpe Optimized

Portfolio P1-A, consisting solely of developed markets, have the most significant allocations in Switzerland (51,82%), Japan (39,55%), and Hong Kong (5,54%). The Swiss market has a standard deviation of 12,42%, the lowest among developed markets. Japan has a standard deviation of

13,39%, while Hong Kong has a standard deviation of 18,03%. The portfolio also includes minor placements in Denmark and Israel.

Portfolio P1-B is similarly allocated across developed markets. Switzerland and Japan are the most significant developed market allocations, whereas allocations to frontier markets are spread between Jordan (16,69%) and Tunisia (3,31%). Jordan has above average standard deviation, while Tunisia has one of the lowest standard deviations for frontier markets. However, both markets have above-average return/risk ratios, with Tunisia having the largest of all markets.

Portfolio P1-C is optimized with a 50% frontier market restriction, resulting in a portfolio with similar traits to the previous portfolios constructed. Switzerland, Japan, Jordan, and Tunisia are all still prevalent in the portfolio. More interestingly, Oman (8,19%) and Bangladesh (4,58%) have significant allocations, which could be explained by their extensive Return/Risk profiles combined with relatively low covariance with other markets.

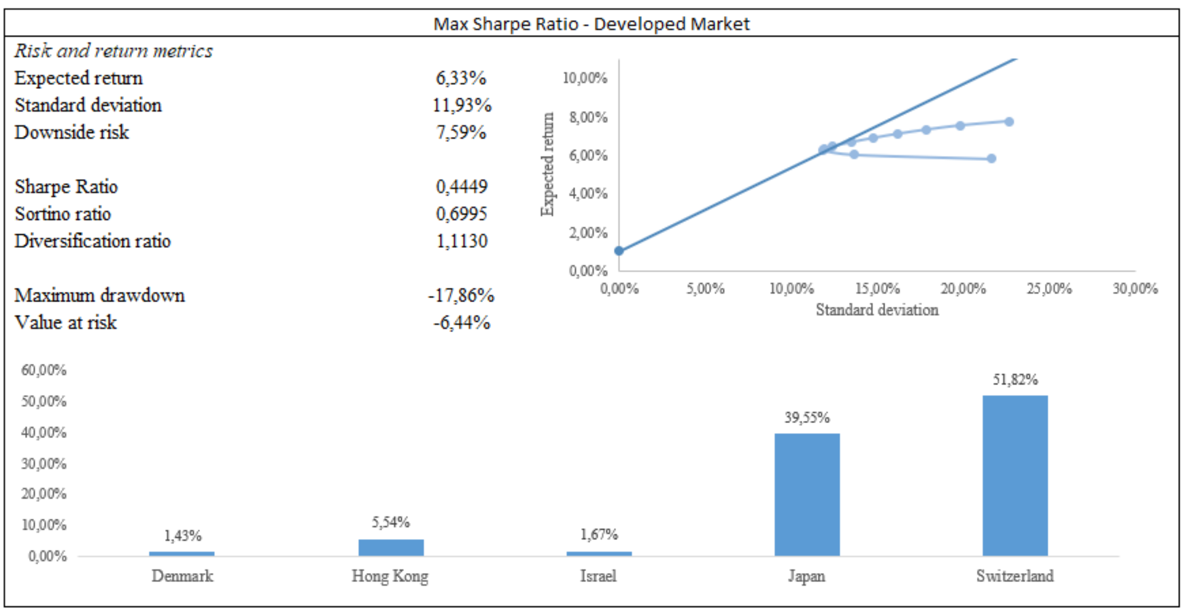

Figure 8: Optimized Portfolio P1-A (Source: Own contribution)

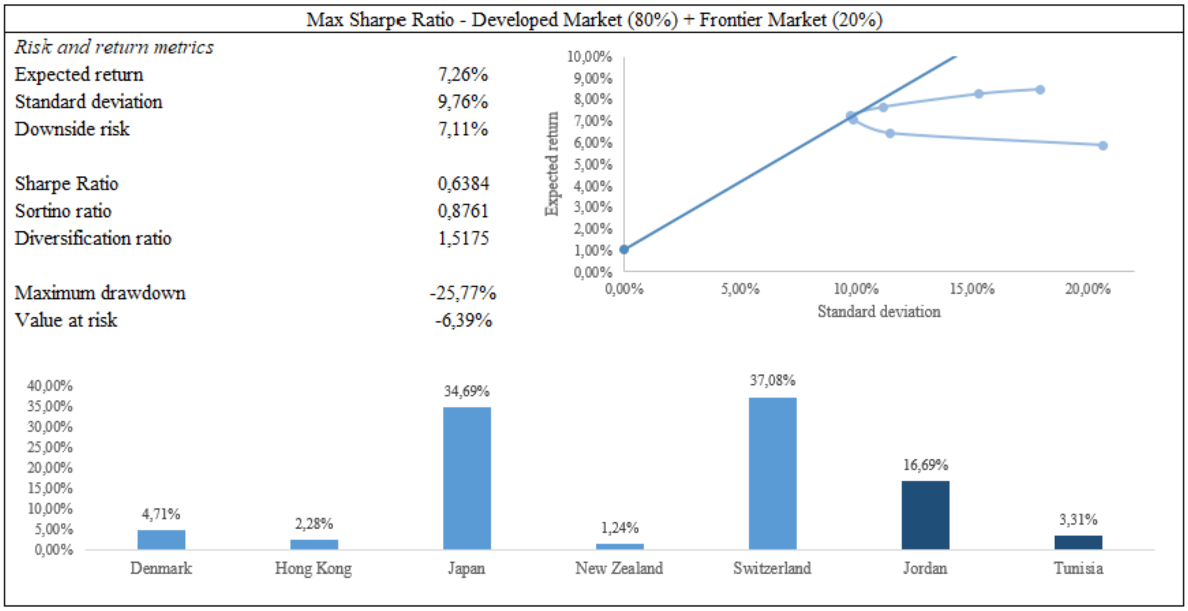

Figure 9: Optimized Portfolio P1-B (Source: Own contribution)

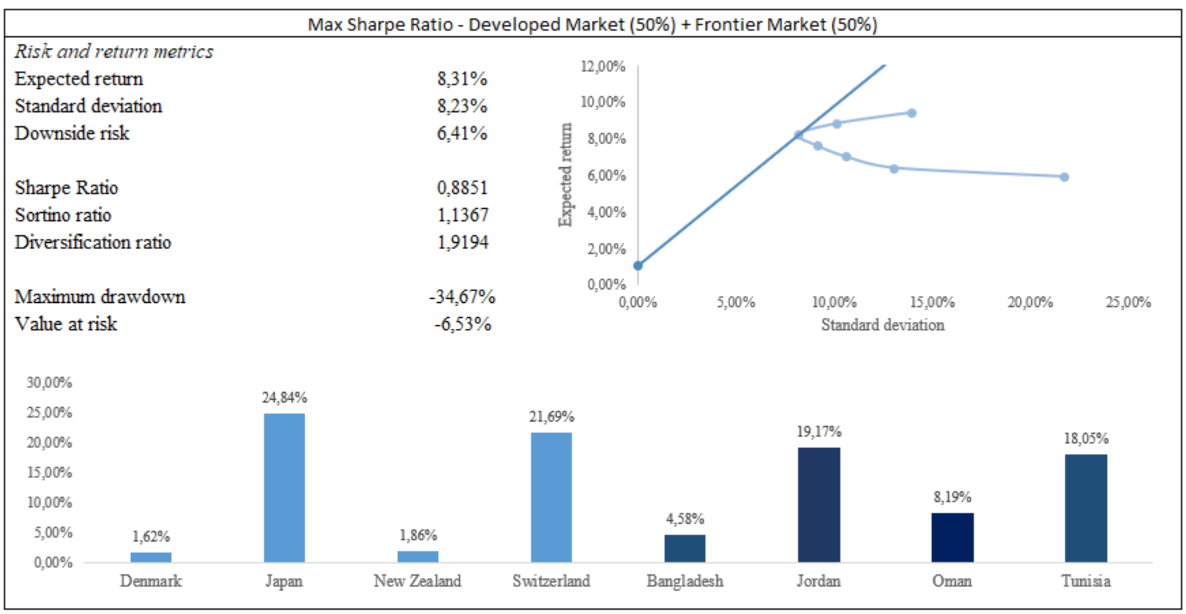

Figure 10: Optimized Portfolio P1-C (Source: Own contribution)

To fully comprehend the portfolios' composition, examining the covariance of the markets is essential. Switzerland has the lowest covariance of all developed markets, which could explain the large allocation in this market. Japan and Denmark also have the second and third lowest covariance among developed markets. Jordan possesses the lowest covariance of all markets and is the only market with an overall negative covariance. Tunisia and Bangladesh have the second and third lowest of all markets included in the study. Oman and New Zealand also have comparatively low covariances. These markets' Return/Risk profile also seems to be a determining factor for market inclusion. Switzerland has the highest Return/Risk profile of all developed markets, with Japan and Denmark having above-average return/risk profiles. Tunisia has the highest return/risk profile of all markets, and Jordan and Bangladesh also have high profiles. The findings of the first set of portfolios indicate that portfolio selection is not exclusively based on expected return and standard deviation but also the covariance coefficient of individual markets. The descriptive statistics show that the three large allocations have above-average Return/Risk profiles. However, despite having a significantly lower Return/Risk profile than Tunisia, Jordan has a large allocation, suggesting that covariance is also a significant determinant.

The Portfolio 1 results indicate that the Return/Risk ratio, combined with the individual countries' covariance values, determines the optimal portfolio. The higher expected return due to larger equity risk premiums in frontier markets and relatively similar volatilities between the two segments make frontier markets attractive investment opportunities. Furthermore, the frontier markets possess lower covariance with other markets, which reduces the overall portfolio risk and creates a higher portfolio performance. However, further investigating the other optimization methods to explore frontier markets and their impact on international portfolios is important.

It can also be observed that the efficient frontier shifts upwards and to the left as a larger proportion of frontier markets are introduced in the portfolio. The expected return increases, and the standard deviation decreases, resulting in a gradually higher Sharpe ratio. These findings indicate that frontier markets are positively correlated with the Sharpe ratio and that increases in the proportion of frontier markets result in higher Sharpe ratios. Interestingly, the maximum drawdown, representing the largest peak-to-trough drop throughout the period, increases with frontier markets. The increase in drawdown is significant and can potentially be explained by frontier markets being more volatile, exposing investors to larger possible losses. Lastly, the value-at-risk also increases with the introduction of frontier markets, albeit on a smaller scale than the maximum drawdown.

The initial portfolio set indicates that including frontier markets in an international portfolio increases expected returns while reducing portfolio volatility. Therefore, including frontier markets is related to an increase in the Sharpe ratio and, thus, higher portfolio performance. However, the risk metrics suggest that frontier markets also expose investors to larger potential losses as maximum drawdown and value-at-risk increase with the incorporation of frontier markets.

6.2.2.2. MSVP Sortino Optimized

The second portfolio optimization method optimizes based on the downside risk associated with equity markets. The portfolios are optimized to maximize the Sortino ratio, which measures the downside risk-adjusted return. Portfolio P2-A possesses some similarities to the previously optimized portfolios. Nevertheless, there are some distinct differences. Like the first set of portfolios, Switzerland and Japan are prevalent in the portfolio composition. Japan has the most significant stake in the portfolio, indicating that Japan has a lower downside risk than Switzerland, which had the largest allocation in the previous set of portfolios. Denmark also has a significantly higher allocated stake than the previous portfolio set. New Zealand and Portugal also have significant allocations in the portfolio. This is interesting as New Zealand had minor allocations, and Portugal was not included in the Sharpe ratio optimization portfolios. Portugal has a below-average return/risk rate, which again indicates that Portugal has a lower downside risk-adjusted return compared to other developed markets.

Portfolio P2-B shows similarities to Portfolio P1-B. The frontier markets included in the portfolio are Tunisia, Jordan, and Bangladesh. The large allocation in the Tunisian market could be due to the market having a significantly higher return/risk profile than other markets. The findings further suggest that the Tunisian market has low downside risk.

The allocation of Portfolio P2-C indicates that both the Tunisian and Bangladeshi equity markets have low downside risk compared to their peers. The Bangladeshi market has a significantly larger allocation in the Sortino portfolio than the Sharpe ratio portfolio. The findings suggest that the downside risk-adjusted return and the covariance are significant determinants in the portfolio's construction.

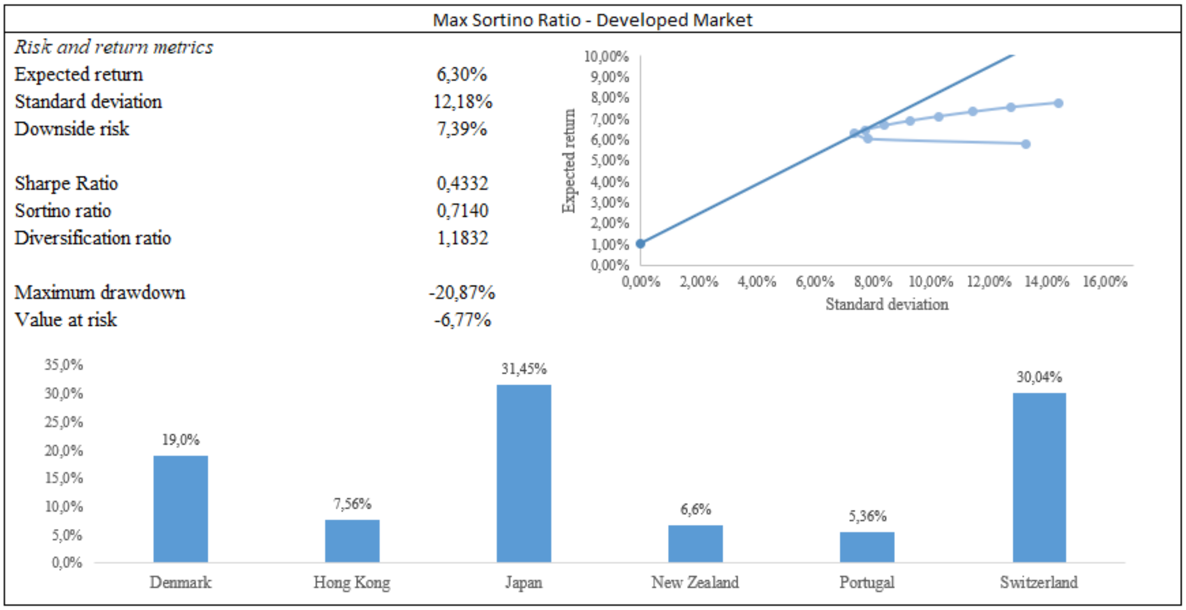

Figure 11: Optimized Portfolio P2-A (Source: Own contribution)

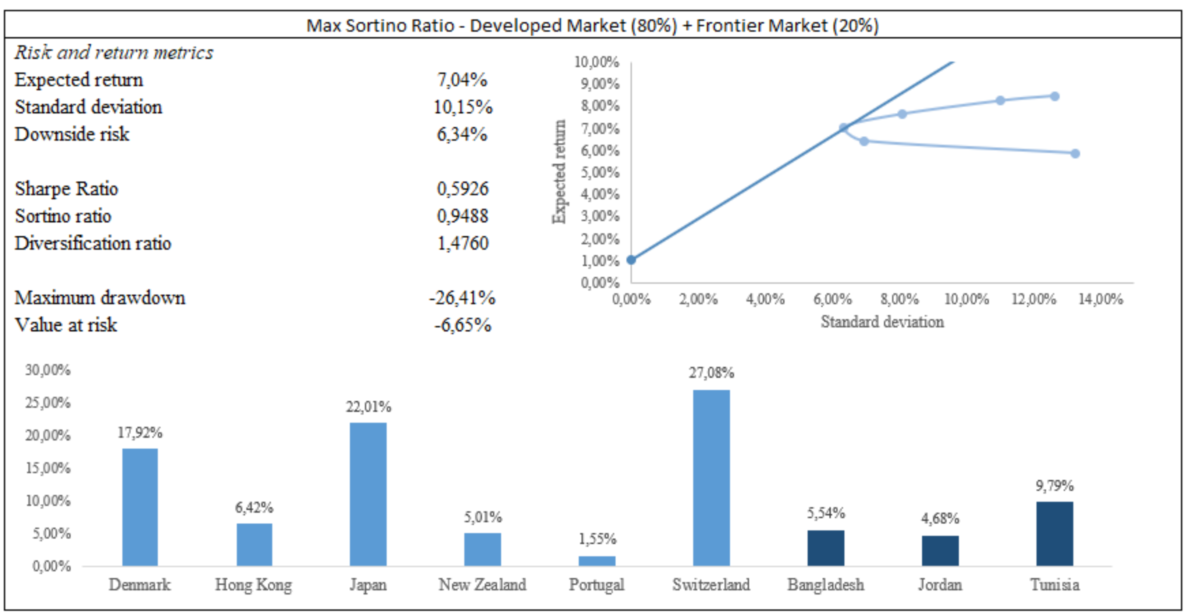

Figure 12: Optimized Portfolio P2-B (Source: Own contribution)

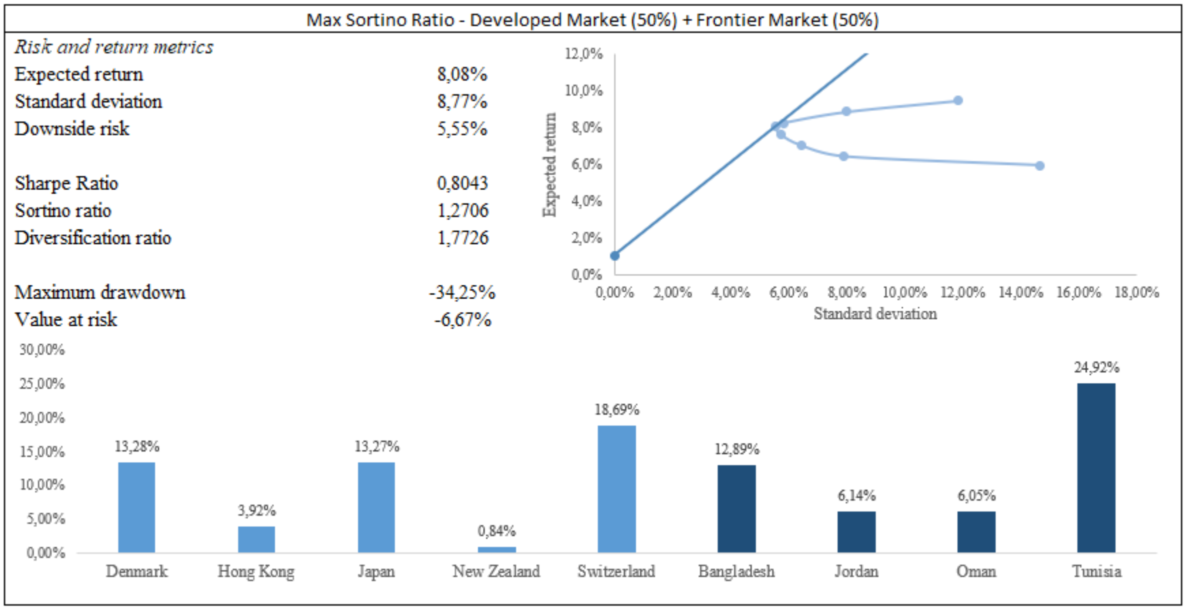

Figure 13: Optimized Portfolio P2-C (Source: Own contribution)

The portfolio set furthermore has a similar efficient frontier development. The tangency portfolio can be observed shifting upward to the left with the introduction of frontier markets. The portfolios achieve higher expected returns and lower downside risk as frontier market allocation increases. The results, similar to the last portfolio, indicate that frontier markets can improve the performance of international portfolios. The portfolio set exhibits increasing maximum drawdown with frontier markets, indicating that investors with assets in frontier markets are exposed to larger fluctuations. The value at risk is fairly unchanged in the three portfolios, with smaller increases and decreases between portfolios.

The Sortino ratio optimized portfolios expand on the findings from those optimized for maximum Sharpe ratio. The allocation of equity markets shows that the standard deviation for individual markets is not necessarily the same as the lower partial standard deviation. This is evidenced as the Sortino optimized portfolios favor Denmark to a much larger degree as opposed to the Sharpe ratio. The portfolio set further exhibits similar diversification traits as the portfolios achieve a higher diversification benefit with frontier markets.

6.2.2.3. *Black-Litterman Sharpe Optimized*

The last optimization method is optimized with inspiration from the Black-Litterman model. The portfolios will provide insight into the individual market's ability to reduce the overall risk through diversification. This is interesting to investigate, as the markets producing the highest Sharpe and Sortino Ratios in the previous optimization methods might differ due to the Return/risk profiles. A higher degree of diversification ability in selected markets could also allow investors to accomplish higher returns with the same level of risk.

Portfolio P3-A, optimized using solely developed markets, shows some interesting findings. The portfolio is noticeably different from the Portfolio P1-A and P2-A. The portfolio's most significant allocations are Hong Kong (22,64%), Israel (22,47%), and Portugal (19,04%). Denmark, Ireland, New Zealand, and Norway also have significant allocations. The large difference between the portfolio and the previously optimized portfolios can, as mentioned, most likely be attributed to the optimization model being based on a different set of expected returns and the optimization being more focused on the diversification ability of the individual markets.

Therefore, the equity markets chosen are likely based on the equity markets' return/risk profile and the covariance value. Higher volatility and low covariance values will allow for more diversifiable risk. Israel, Hong Kong, and Portugal share relatively high standard deviations and low covariance values. Ireland and New Zealand are furthermore markets with low covariance. The markets do, however, have average or below-average standard deviations. Lastly, Norway has a quite high covariance value and high standard deviation. The high standard deviation might explain the market's inclusion in the portfolio.

Portfolio P3-B, with a 20% frontier market restriction, shows a broad diversification across developed markets. The equity markets chosen are broadly spread both geographically and industrially. Jordan is the only frontier market included in the portfolio. Jordan has a relatively average standard deviation for frontier markets but has the significantly lowest covariance value of all countries. Jordan, as previously stated, is the only country with an overall negative covariance. It is interesting to see the Belgian market having a significant stake in the portfolio, as it was not included in the developed market portfolio. This could be due to the Belgian market having one of the lowest market covariances with Jordan.

Like the previous portfolios, Portfolio P3-C shows broad diversification across several markets. Jordan has the significantly highest allocation, while Bangladesh, Nigeria, Oman, and Tunisia are introduced to the portfolio. Bangladesh, Oman, and Tunisia have similar to Jordan's low covariances, and their inclusion is therefore highly dependent on this. On the other hand, Nigeria does not have a noticeable low covariance and a covariance slightly below average. Interestingly, Nigeria has the highest standard deviation of all markets studied. This could contribute to explaining the market's inclusion in the portfolio, as a higher standard deviation would create a larger diversifiable risk.

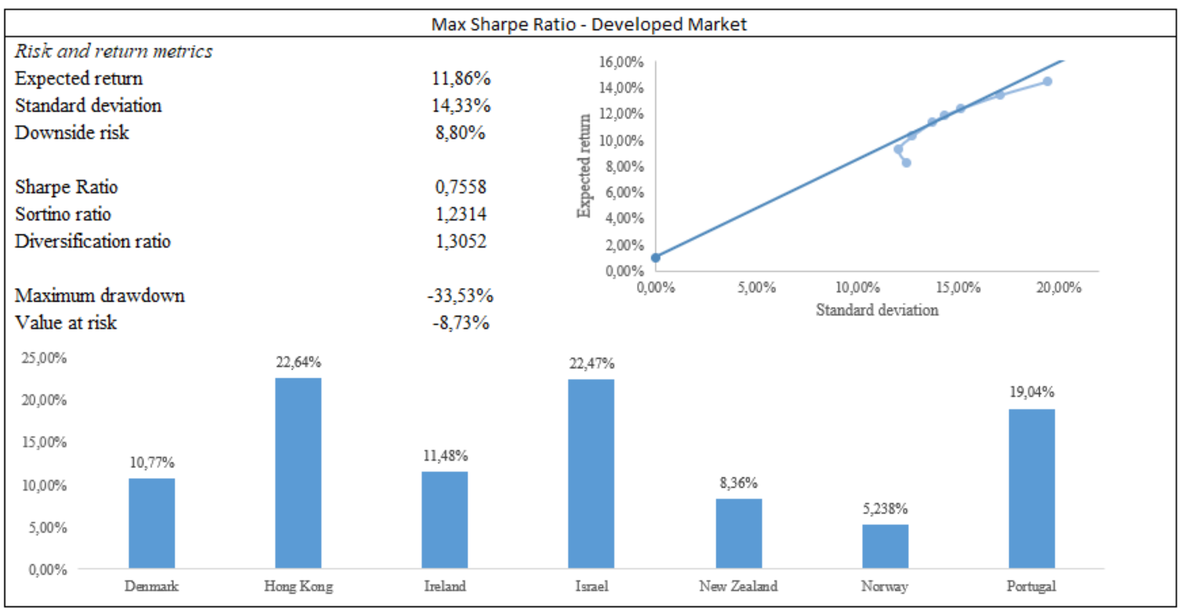

Figure 14: Optimized Portfolio P3-A (Source: Own contribution)

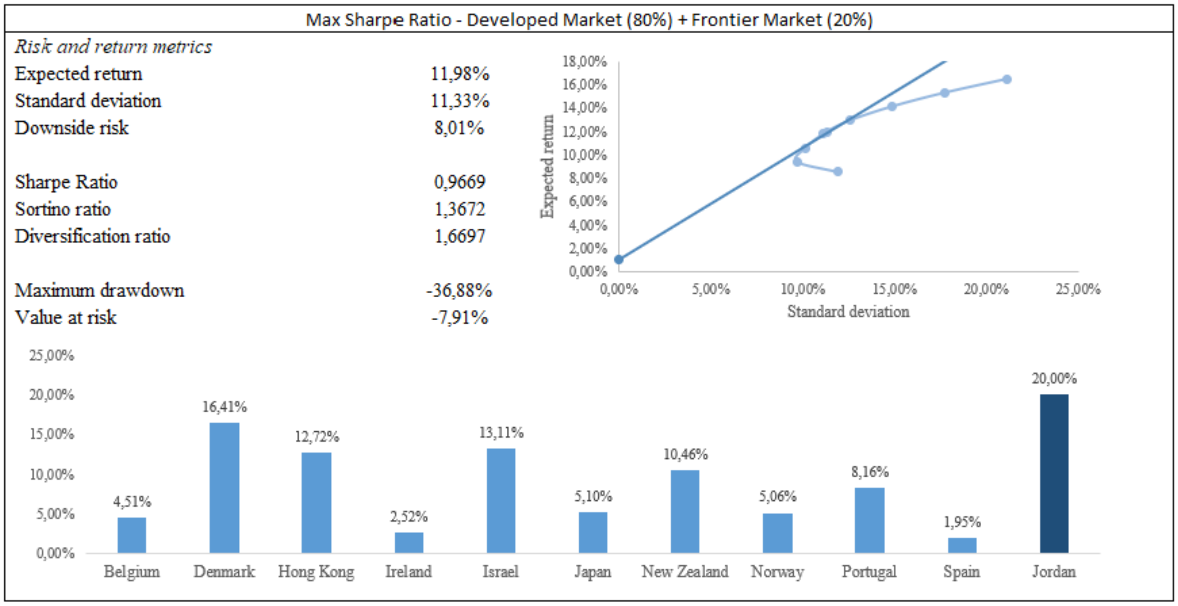

Figure 15: Optimized Portfolio P3-B (Source: Own contribution)

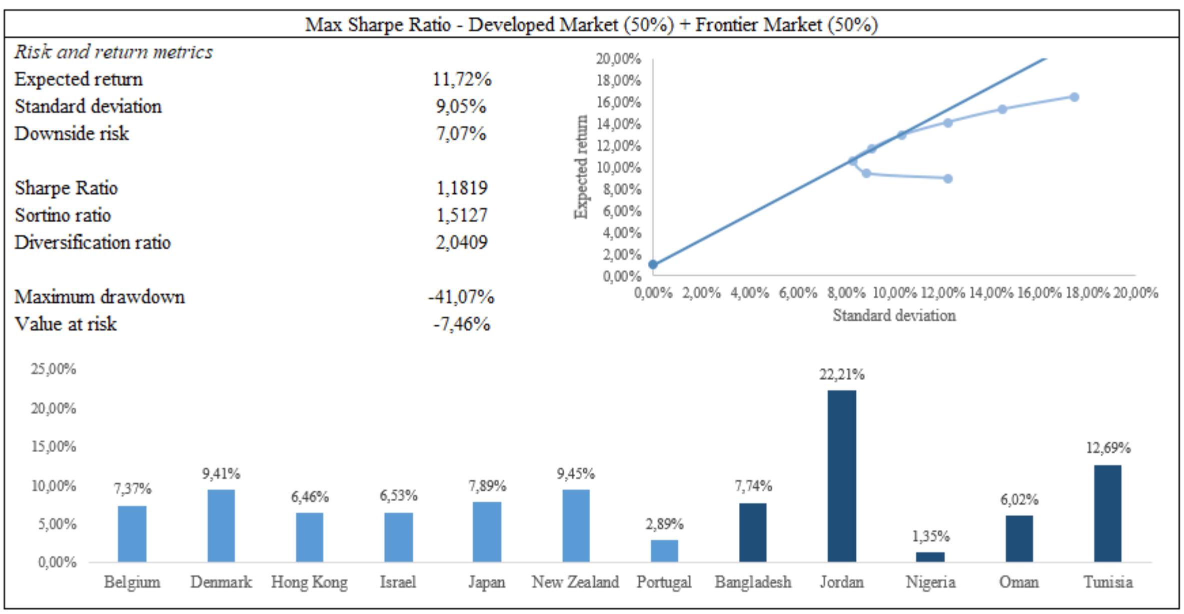

Figure 16: Optimized Portfolio P3-C (Source: Own contribution)

Due to the optimization method and the expected returns being calculated based on a market equilibrium assumption, the change in portfolio expected return is minor. The expected return deviates slightly between portfolios but not significantly or in a particular trend. The portfolios' standard deviation and downside risk can, however, be seen to be significantly reduced as frontier markets are introduced. This can be seen through the portfolios' increasing Sharpe and Sortino trend. The maximum drawdown has a similar pattern as the previous portfolio sets, as the maximum drawdown increases with frontier markets. However, the value at risk is slightly decreased as frontier markets are introduced in the portfolio.

This section has presented the results of the portfolio construction based on the asset allocation optimization methods for developed and frontier markets. Nine portfolios were constructed and optimized using Markowitz's mean-variance optimization, Estrada's mean semi-variance optimization method, and a Black-Litterman-inspired approach. The first portfolios were comprised solely of developed markets, while the second and third portfolios include frontier markets with different placement restrictions. The findings suggest that portfolio selection is based not only on expected return and standard deviation but also on the covariance coefficient of individual markets. The results further indicate that the return/risk ratio, the downside risk-adjusted return, and the covariance of individual markets are determinants of the optimal portfolio. The composition of markets shows that attractive investments are identified based on higher expected return, lower risk or downside risk, and low covariance with other markets. The findings further suggest that including frontier markets increases the portfolio's expected return and reduces the standard deviation and downside risk, resulting in higher Sharpe and Sortino ratios.

6.2.3. In-Sample Performance

The in-sample period is the time period on which the model is trained and bases its predictions on. The model is tested on historical data to create optimal portfolios based on specific criteria. In this section of the thesis, the findings from the in-sample period are presented and evaluated. The in-sample period examined spans from 30.04.2014 to 31.12.2020. Based on the markets' movements and expected return, the model builds the optimal portfolio according to the specific restrictions.

The constructed portfolios are then evaluated based on actual performance during the period and compared to each other and a benchmark portfolio.

The first set of portfolios, optimized for maximum Sharpe ratio, show some interesting results. Portfolio P1-B achieves the highest Sharpe ratio of all the constructed portfolios; portfolio P1-C achieves the second highest Sharpe ratio, whereas portfolio P1-A achieves the lowest ratio. These results are, to some extent, congruent with the findings from the previous section, as frontier markets seem to have a positive effect on portfolio performance. The result can be explained by the portfolios achieving different returns in the sample period while increased diversification results in comparatively similar standard deviation.

The Sharpe ratios were lower than the benchmark portfolio, i.e., the S&P 500. The initial sub-question posed by the authors concerned whether including frontier markets in international portfolios improve portfolio performance. The findings appear inconsistent with this notion, as including frontier markets improves the portfolio's Sharpe ratio yet fails to outperform the benchmark portfolio. However, the US market and S&P 500 outperformed most markets during the period. Therefore, the benchmark portfolio may not be the most appropriate case for comparison. Nevertheless, comparing the portfolios to each other provides valuable insights.

Portfolio P1				
	S&P 500	DM	DM (80%) + FM (20%)	DM (50%) + FM (50%)
Performance metrics				
Return	99,37%	61,31%	66,53%	62,42%
Annual Return	10,76%	7,34%	7,85%	7,45%
Standard Deviation	14,34%	12,34%	12,04%	11,46%
Downside Risk	11,11%	8,54%	8,23%	8,14%
Sharpe Ratio	0,6791	0,5119	0,5666	0,5607
Sortino Ratio	0,8766	0,7398	0,8296	0,7898
Risk metrics				
Maximum Drawdown	-20,00%	-17,55%	-15,65%	-15,31%
Skewness	-0,3603	-0,2764	-0,2774	-0,3148
Kurtosis	1,6939	0,1643	0,0422	0,1783
Value at Risk	-6,60%	-6,40%	-5,63%	-5,36%

Table 6: In-sample Portfolio 1 Results (Source: Own contribution)

Further examining the portfolio's performance and risk metrics can provide valuable insights. The standard deviation is consistent with the sub-question stating that including frontier markets would reduce portfolio volatility – as more and more markets are incorporated into the portfolio; the standard deviation gradually reduces. Compared to the S&P 500, which experienced a volatility of 14,36%, Portfolio P1-C has a standard deviation of 11,46%. This illustrates the concepts of modern portfolio theory, which suggest that unsystematic risk is diversifiable. Furthermore, downside risk follows a downward trend as frontier markets are gradually introduced in the portfolio, which contradicts the notion that frontier markets are more prone to downside risk.

The benchmark portfolio has the highest maximum drawdown of all portfolios. The drawdown reduces with diversification and further when frontier markets are introduced. Investors can utilize the measure to evaluate the riskiness of a portfolio and the potential downside of investments. The results indicate that including frontier markets reduces the maximum drawdown and downside risk. These results are contradictory to the findings from the model, as the model found that the introduction of frontier markets was associated with significant increases in maximum drawdown. The skewness of the constructed portfolios is similar to the benchmark and moderately negative. The skewness indicates that the portfolios can experience frequent smaller positive returns and a few large losses. This is congruent with the sample-period, as the period contains positive periods as well as large market crashes such as the Covid-19 pandemic.

The kurtosis of the constructed portfolios is significantly lower than the benchmark portfolio. A high kurtosis indicates that the benchmark portfolio might experience more extreme price fluctuations compared to the constructed portfolios. This conforms with the standard deviation as well as the maximum drawdown. The value at risk metric shows that the value at risk with a 95% probability follows similar movements as the portfolios' standard deviation and maximum drawdown. Value at risk is reduced with diversification and is significantly reduced in the constructed portfolios. This indicates that reduced value at risk is correlated with diversification and the inclusion of frontier markets.

The second set of portfolios, optimized to maximize the Sortino ratio, presents results that align with and contradict the previous findings. Including frontier markets in the portfolio has the same effect on the portfolio as previously. Returns are, however, mainly unchanged through the treeconstructed portfolios, and the Sharpe and Sortino ratio is increased due to decreases in standard deviations and downside risk. Portfolio P2-C archives the highest Sharpe and Sortino ratio of the constructed portfolio. Interestingly, all the constructed portfolios achieve higher Sortino ratios than the benchmark portfolio. These results further confirm the findings from the model and the previous set of portfolios.

Portfolio P2				
	S&P 500	DM	DM (80%) + FM (20%)	DM (50%) + FM (50%)
Performance metrics				
Return	99,37%	76,94%	77,05%	76,27%
Annual Return	10,76%	8,82%	8,83%	8,76%
Standard Deviation	14,34%	12,69%	12,61%	12,31%
Downside Risk	11,11%	8,27%	8,12%	7,93%
Sharpe Ratio	0,6791	0,6145	0,6194	0,6284
Sortino Ratio	0,8766	0,9432	0,9614	0,9758
Risk metrics				
Maximum Drawdown	-20,00%	-14,42%	-14,34%	-14,08%
Skewness	-0,3603	-0,2295	-0,2188	-0,2077
Kurtosis	1,6939	-0,0948	-0,1480	-0,1747
Value at Risk	-6,60%	-5,88%	-5,81%	-5,75%

Table 7: In-sample Portfolio 2 Results (Source: Own contribution)

The risk metrics for the Sortino ratio optimized portfolios are similar to those of the Sharpe ratio optimized portfolios. The maximum drawdown decreases with diversification and further with frontier markets. All portfolios have negative skewness values, which are moderate, within the normal range and similar to the previous portfolio set. The kurtosis of the constructed portfolios is significantly lower than that of the benchmark portfolio. This suggests that the constructed portfolios have a lower probability of experiencing large positive or negative returns, consistent with the findings regarding reduced standard deviation. Lastly, value at risk, similar to previous findings, decreases with diversification.

The third set, of portfolios is optimized for maximum Sharpe ratio using expected returns derived from the assumption that all markets have the same Sharpe ratio, yield distinct results from the previous two sets. This could be attributed to the equity markets being optimized using Sharpe equal expected returns, and allocations are chosen based on diversification ability rather than return/risk profile. Nevertheless, the findings demonstrate some similarities with the previous portfolios. Portfolio P3-C achieves the highest Sharpe ratio of all constructed portfolios, while Portfolio P3-B achieves the highest Sortino ratio. The portfolio returns are fairly similar, while the standard deviation and downside risk are reduced with diversification. Interestingly, the standard deviation increases from the benchmark portfolio to Portfolio P3-A. The standard deviation is, however, subsequently decreased with further diversification and the introduction of frontier markets.

Portfolio P3				
	S&P 500	DM	DM (80%) + FM (20%)	DM (50%) + FM (50%)
Performance metrics				
Return	99,37%	64,90%	69,51%	66,81%
Annual Return	10,76%	7,69%	8,13%	7,87%
Standard Deviation	14,34%	15,78%	14,26%	13,64%
Downside Risk	11,11%	10,62%	9,29%	9,03%
Sharpe Ratio	0,6791	0,4875	0,5703	0,5772
Sortino Ratio	0,8766	0,6275	0,7651	0,7584
Risk metrics				
Maximum Drawdown	-20,00%	-18,89%	-17,29%	-17,57%
Skewness	-0,3603	-0,3111	-0,2515	-0,2601
Kurtosis	1,6939	0,2179	0,2552	0,4076
Value at Risk	-6,60%	-6,22%	-5,52%	-5,34%

Table 8: In-sample Portfolio 3 Results (Source: Own contribution)

The maximum drawdown development is similar to the previous portfolios and is reduced with diversification. Interestingly, the results indicate that the drawdown is lowered by introducing frontier markets and then increased in Portfolio P3-C. The maximum drawdown results are inconstant, and it is difficult to draw a clear conclusion regarding the portfolios. The skewness of the portfolios is similar to the previous ones and close to that of the benchmark market.

The kurtosis of the portfolios is also comparable to earlier findings, with the constructed portfolios having significantly lower kurtosis than the benchmark portfolio. Lastly, the value at risk metric is similarly reduced as with the previous portfolios.

The in-sample results are highly congruent with the model and indicate that introducing frontier markets increases an international portfolio's performance. The increase in performance is mainly due to the portfolios’ ability to reduce standard deviation rather than higher returns. The risk metrics further validate the findings, as frontier markets seem correlated with reductions in maximum drawdown and value at risk.

6.2.4. Out-of-Sample Performance

The out-of-sample test is a crucial part of the analysis as it allows the authors to test the validity and reliability of the model and in-sample findings. The out-of-sample period is outside the period for which the model has been trained and will therefore test the model's ability to make accurate predictions in real-world scenarios. In this section of the thesis the findings from the out-of-sample period are presented. The study aims to evaluate the individual portfolios' performance in the out-of-sample period and, specifically, to what degree the findings from the in-sample period are represented in the out-of-sample period. The examined period spans from 31.12.2020 to 30.12.2022, during which the portfolio value is measured by the portfolio weights derived from the in-sample period.

The findings obtained during the out-of-sample period are both congruent and conflicting to those obtained during the in-sample period. For the portfolios optimized on the Sharpe ratio, the benchmark portfolio demonstrates the highest returns throughout the sampling period and is the only portfolio with positive returns. Portfolio P1-A is the worst performer with a return of negative 4,05%, whereas portfolios P1-B and P1-C experience a return of negative 1,59% and 2,71%, respectively. Portfolio P1-A is also the portfolio with the lowest risk-adjusted return among the constructed portfolios. The highest Sharpe ratio is achieved by Portfolio P1-B, it is however still negative. It is also noted that Portfolio P1-C achieves a higher Sharpe ratio than Portfolio P1-A, which indicates that frontier markets, up to a degree, improve portfolio performance.

Furthermore, the volatility of the portfolios mirrors the results obtained during the in-sample period, implying a negative relationship between diversification and standard deviation. Portfolio P1-B achieves the highest standard deviation of all constructed portfolios, which also could explain its returns. Overall, the constructed portfolios' volatility is lower than the S&P 500. Portfolio P1-A does, however, achieve a lower standard deviation than Portfolio P1-B in period, which contradicts previous findings.

Portfolio P1	S&P 500	DM	DM (80%) + FM (20%)	DM (50%) + FM (50%)
Performance Metrics				
Return	2,22%	-4,05%	-1,59%	-2,71%
Standard Deviation	18,77%	15,10%	15,95%	15,19%
Downside Risk	11,29%	9,08%	9,43%	9,00%
Sharpe Ratio	0,0638	-0,3358	-0,1638	-0,2455
Sortino Ratio	0,1060	-0,5587	-0,2770	-0,4141
Risk Metrics				
Maximum Drawdown	-24,77%	-24,53%	-25,22%	-24,59%
Skewness	-0,287	0,247	-0,057	-0,224
Kurtosis	-0,944	0,521	-0,284	-0,752
Value-at-Risk	-8,74%	-5,95%	-7,02%	-7,18%

Table 9: Out-of-sample Portfolio 1 Results (Source: Own contribution)

The maximum drawdown exhibits similar values across all portfolios, with Portfolio P1-A achieving the lowest value. These findings contradict the results from the in-sample period but are more congruent with the initial results from the model. Furthermore, the benchmark portfolio exhibits a negative skew, which indicates that the investment is susceptible to negative shocks in the market, or so-called "black swan events." The skewness in the Portfolio P1-A indicates less exposure to such events and would be attractive to risk-averse investors.

The benchmark portfolio has a large negative kurtosis. This indicates that the benchmark portfolio has a "thinner tail" and a lower probability of extreme return fluctuations. The constructed portfolios have smaller both positive and negative kurtosis. The kurtosis can, however, be considered within the scope of normal distribution. Portfolio P1-A further achieves the lowest value-at-risk. Including the frontier market increases the value-at-risk, with Portfolio P1-C achieving the highest value-at-risk of the constructed portfolios.

The out-of-sample results of the Sortino ratio optimized portfolio set are more congruent with the previous results. Portfolio P2-C achieves the highest return. Interestingly, as opposed to the findings in the in-sample period, diversification is correlated with higher returns. At the same time, the standard deviation and downside risk are more or less similar in all constructed portfolios. Therefore, the increase in performance is due to higher returns with similar volatility.

Portfolio P2	S&P	DM	DM (80%) + FM (20%)	DM (50%) + FM (50%)
Performance Metrics				
Return	2,22%	2,51%	3,07%	3,59%
Standard Deviation	18,77%	17,67%	17,81%	17,68%
Downside Risk	11,29%	10,02%	10,05%	10,72%
Sharpe Ratio	0,0638	0,0842	0,1150	0,1449
Sortino Ratio	0,1060	0,1486	0,2037	0,2391
Risk Metrics				
Maximum Drawdown	-24,77%	-25,89%	-25,81%	-25,39%
Skewness	-0,29	0,06	0,06	-0,062
Kurtosis	-0,94	0,39	0,21	-0,044
Value-at-Risk	-8,74%	-7,29%	-7,45%	-7,55%

Table 10: Out-of-sample Portfolio 2 Results (Source: Own contribution)

The maximum drawdown exhibits inconsistent results as the drawdown is more or less consistent throughout the diversification. The skewness of the portfolios is positive for the optimized Sortino ratio. The findings conform to the optimization method as downside risk is reduced and, therefore, also the portfolio´s susceptibility to negative shocks in the market. The kurtosis further shows that the constructed portfolios achieve positive values, which contradicts the standard deviation results. The kurtosis can however be consider small and needs to be considered in combination with standard deviation and that all normal is within the acceptable scope. The value at risk is decreased by diversification but slightly increased by frontier markets.

The last set of optimized portfolios exhibits much of the same results as the in-sample portfolios. The portfolio's return experiences an initial drop from the benchmark portfolio to Portfolio P3-A. The return does, however, increase in the frontier market portfolios, with Portfolio P3-B experiencing the highest returns of all portfolios. Portfolio P3-B subsequently achieves the highest Sharpe and Sortino ratio.

Compared to the benchmark, the standard deviation increases in the Portfolio P3-A but is reduced by introducing frontier markets. Portfolio P3-B exhibits the same standard deviation as the benchmark portfolio with a higher return, indicating that frontier markets can create higher returns for the same risk profiles. However, the downside risk profile is slightly higher in Portfolio P3-B, indicating that the downside risk of frontier markets might be higher than the benchmark portfolio and that risk-averse investors should carefully consider investments in frontier markets.

	Portfolio P3			
	S&P	DM	DM (80%) + FM (20%)	DM (50%) + FM (50%)
Performance Metrics				
Return	2,22%	-1,85%	2,55%	1,25%
Standard Deviation	18,77%	19,87%	18,77%	17,98%
Downside Risk	11,29%	11,03%	11,73%	10,95%
Sharpe Ratio	0,0638	-0,1446	0,0812	0,0127
Sortino Ratio	0,1060	-0,2606	0,1300	0,0208
Risk Metrics				
Maximum Drawdown	-24,77%	-31,11%	-26,90%	-26,79%
Skewness	-0,287	1,419	0,630	0,582
Kurtosis	-0,944	5,280	2,663	2,237
Value-at-Risk	-8,74%	-5,62%	-5,77%	-5,92%

Table 11: Out-of-sample Portfolio 3 Results (Source: Own Contribution)

The maximum drawdown portrays some interesting results as the drawdown significantly increases from the benchmark portfolio to Portfolio P3-A. The drawdown is, however, reduced with the introduction of frontier markets but never becomes lower than the benchmark portfolio. This conforms with the findings regarding downside risk, as the potential negative consequences of frontier market investment can be higher than the benchmark portfolio. The results contradict the in-sample period as the constructed portfolios have significantly higher skewness values than the in-sample results. This is highly skewed and indicates that the portfolios experience several smaller losses and few large positive returns. The kurtosis of the portfolios is further significantly higher than the other portfolios. This means the portfolios have a higher probability of experiencing large positive and negative returns. This result is substantiated by the maximum drawdown, which is higher than the benchmark and the previous portfolios. Lastly, the value-at-risk results are similar to the last portfolio set as the value-at-risk decreases with diversification but increases with frontier markets' introduction.

The findings from the out-of-sample period present some interesting findings. The portfolios exhibit congruence and contradiction with the model and the in-sample results.

6.3. Analysis Summary

The results of the analysis provide valuable insights into the performance of portfolios that include frontier markets. The results from the in-sample and out-of-sample periods exhibit both congruence and conflict with the sub-questions of the thesis.

The Sharpe ratio optimized portfolios exhibited a correlation between the introduction of frontier markets and increased portfolio performance. The increase in performance is quantified through increases in the Sharpe ratio. Higher returns with a similar standard deviation across the constructed portfolios could explain the increase. The portfolios also exhibited decreases in maximum drawdown and value-at-risk, indicating the frontier markets risk reducing abilities. The out-of-sample results saw similar results. The portfolio returns were correlated with frontier markets, and a higher proportion of frontier markets in the portfolio resulted in less negative returns. The standard deviation experienced smaller deviations across portfolios but in no apparent trend or direction. The same results could be seen for the maximum drawdown, while the value at risk was reduced with diversification but subsequently increased with frontier markets.

The Sortino ratio-based portfolio optimization method presented similar results as the Sharpe ratio portfolio sets. The in-sample findings observed showed that the constructed portfolios perform very similarly. However, a slight performance increase can be observed as frontier markets are introduced. This is true for both Sharpe and Sortino. The portfolios achieve higher Sortino ratios than the benchmark portfolio, indicating that diversification and frontier markets possess downside risk-reducing abilities. This is further emphasized as the maximum drawdown is significantly reduced with diversification, and the value at risk is also reduced. The out-of-sample findings provide similar results to the in-sample period.

Interestingly, the introduction of frontier markets is correlated with an increased return, while standard deviation and downside risk are more or less stagnant across the portfolio. The Sharpe and Sortino ratio increase with higher returns and similar volatility. The maximum drawdown is similar across the constructed portfolios, with all constructed portfolios achieving a higher maximum drawdown than the benchmark portfolio. The value at risk is reduced with diversification but with a light increase with the introduction of frontier markets.

The last set of optimized portfolios was optimized according to the Black-Litterman assumption of market equilibrium, and the expected returns used were derived from an equal Sharpe ratio. The in-sample portfolios exhibited many of the same results as the previous portfolio sets. Introducing frontier markets in the portfolio is positively correlated with higher Sharpe ratios. Portfolio P3-C achieved the highest Sharpe ratio of the constructed portfolios. In contrast with the previous portfolio sets, there is no clear trend regarding how frontier markets increase portfolio performance. The performance can best be described as being achieved through the optimal combination of returns and volatility. The maximum drawdown is reduced with frontier markets, but the drawdown is slightly larger in Portfolio P3-C compared to the Portfolio P3-B. The value-at-risk follows a more apparent reduction with diversification and the introduction of frontier markets. The out-of-sample findings further validate the previous findings. The portfolio performance is increased with frontier markets, and the Portfolio P3-B achieves the highest Sharpe ratio of all portfolios.

Interestingly, the maximum drawdown is significantly higher than the other portfolios constructed. The maximum drawdown can, however, be observed to be reducing as frontier markets are introduced. On the other hand, the value at risk is increased from Portfolio P3-A to Portfolio P3-C.

7. Discussion

This thesis set out to investigate frontier equity markets and their potential to provide diversification benefits in an international portfolio. As such, the authors posed three research sub-questions to facilitate the analysis and assist in answering the main research question. The following section critically reflects on the findings of the conducted study in light of relevant theory and the reviewed literature. Potential discrepancies between the results and literature will then be discussed to explain differences in results. Next, a discussion of the methods applied is presented. The section concludes with recommendations for further research on frontier equity markets and their role in the global financial market.

7.1. Discussion of Results

The first sub-question, "Are frontier equity markets weakly correlated with developed markets?", relates to the fact that frontier markets, being less globalized, would exhibit a weaker correlation with other global markets. Graham et al. (2013), Berger et al. (2011), Speidell and Krohne (2007) and Marshall et al. (2013) found that frontier markets exhibited low correlation with other international equity markets. Sukumaran et al. (2015) found that frontier markets correlated weakly with Australia and USA. Spiru and Qin (2016) further found that Central and Eastern European frontier markets had a higher correlation with developed markets than frontier markets in the MENA region.

The thesis correlation analysis found that frontier markets were weakly correlated with the benchmark market of the S&P 500. Emerging markets also exhibited a low correlation with the benchmark market, but, as expected, frontier markets had the lowest correlation. The market correlation was further tested with other global equity markets, illustrating that frontier markets also exhibited a low correlation with the MSCI World Index. The findings were similar to those of Graham et al. (2013), Berger et al. (2011), Speidell and Krohne (2007) and Marshall et al. (2013). Surprisingly, frontier markets had a moderately high correlation with the MSCI World ex. USA index, which could indicate that the US market is responsible for much of the low correlation. The finding was congruent with that of Sukumaran et al. (2015). Lastly, countries in the MENA region were found to have generally low correlations with developed markets, as Jordan, Tunisia and Oman had some of the lowest average correlation coefficients, similar to the previous findings of Spiru and Qin (2016). As such, the findings of the correlation analysis were highly conformant with previous literature.

The high conformity between the thesis findings and previous literature could potentially be explained by frontier markets' distinct characteristics and properties. Frontier market economies are often dependent on different industries compared to developed markets and are therefore affected differently by changing industry-specific landscapes. It can be seen that geographical location is an essential determinant for the correlation, as European and more closely located frontier markets possess a higher correlation with developed markets. This can also be due to trade relationships and economic dependence. (oil)

The market-specific correlation analysis found that frontier markets were less correlated with other developed markets as well as each other. Based on the findings from both the global equity markets analysis and the individual country equity market analysis, it can be concluded that frontier equity markets have a weaker correlation to other equity markets and have a weaker correlation compared to developed markets.

The second sub-question concerns whether “frontier markets reduce portfolio volatility when combined with developed markets in an international portfolio?”. Modern portfolio theory proposes that diversification, in general, reduces portfolio risk. Given the low correlation between FM and DM, cf. Section 6.1, introducing frontier countries to a global portfolio should provide risk-reducing abilities. Levy & Sarnat (1970) suggest low-yielding foreign investments in EM and FM may significantly reduce portfolio variance. FTSE Russell (2014) reports that FM are historically less volatile than developed and emerging markets, whereas Sukumaran et al. (2015) find higher standard deviations for frontier markets. Odier & Solnik (1990) claim that diversification through investing in international stocks and bonds is inadequate and suggest that currencies and commodities are more suitable diversification tools. Hence, extant literature is divided on whether FM reduces portfolio variance as these markets are generally perceived as risky investments.

Findings from the analysis suggest that, in most cases, including frontier markets reduces standard deviation and lower partial standard deviation, supporting the posed sub-question. Portfolios P1 and P2 in the in-sample period exhibit risk-reducing abilities when frontier markets are gradually introduced to the portfolios. This applies to both standard deviation and downside risk. Portfolio P3 shows slightly different results but with similar tendencies. The DM portfolio exhibits greater standard deviation and downside risk than the benchmark. However, both risk measures decrease with the gradual increase of FM in the portfolio.

On the other hand, the out-of-sample period displays less consistent risk measures. Portfolio P3 was the only portfolio that mirrored the results from the in-sample period. However, there is no apparent relationship between diversification and (downside) risk, and the results are inconclusive. Further examination of maximum drawdown and value-at-risk in both sample periods indicates similar patterns as observed with the other risk measures, and no valuable conclusion can be drawn from the results. The inconclusiveness of the out-of-sample results could be explained by how portfolios are optimized, i.e., with respect to risk-adjusted returns and not volatility alone.

Based on findings from the analysis, both in and out-of-sample, it can be concluded that frontier markets possess some risk-reducing properties when combined with developed markets in an international portfolio. The results are, to some degree, inconsistent, which could potentially devalue the findings. However, the reviewed literature also shows some inconsistency in results regarding whether frontier markets actually reduce portfolio risk, yet there is little doubt of their potential. As such, the overarching findings of this thesis suggest that the inclusion of FM in an international portfolio can reduce the portfolio's overall volatility, thus conforming to extant literature.

However, it is important to consider the restrictions of the portfolio constructions, which are limited to 20% and 50% frontier market involvement in the overall portfolio. The findings are, therefore, only representable for portfolios with these weightings. Frontier markets, on average, exhibit higher individual volatility, and investors should be aware of this. The portfolios can achieve lower volatility by combining markets with low covariance, thus constructing a diversified portfolio.

The third sub-question was, "Does the inclusion of frontier markets in an international portfolio improve the portfolio's risk-adjusted return?". The intuition behind the question is that the potential for economic growth rates, low labor costs and isolated equity markets are attractive characteristics and that frontier markets inhabit great potential for improving international portfolio performance. Segregation from other global equity markets in a rapidly globalizing world was further assumed to be beneficial in regard to diversification.

Levy and Sarnat (1970) investigated the potential benefits of international diversification. The authors found that including developing countries in the portfolio opportunity set improves the risk-return position for an American investor. Sukumaran et al. (2011) studied the potential diversification benefits of frontier markets for an Australian investor. The study found that the inclusion of frontier markets significantly increases the mean return of the optimal portfolio. Girad and Sinha (2008) also found that frontier markets could provide greater return and diversification benefits.

The analysis suggests a positive relationship between the inclusion of frontier markets and portfolio performance as measured by risk-adjusted returns, i.e., Sharpe and Sortino ratios. Frontier markets also possess downside risk-reducing abilities, as reflected in lower values of maximum drawdown and value-at-risk. The results provide valuable insights into the performance of portfolios that include frontier markets and can guide investors in their portfolio diversification decisions. The findings thus share similarities with previous research conducted by Levy and Sarnat (1970), Sukumaran et al. (2011) and Girad and Sinha (2008).

The in-sample period illustrates how different optimization methods yield different results. Portfolio B achieves the highest Sharpe and Sortino ratio in the Sharpe-optimized method. The Sortino optimization sees Portfolio C achieving the highest Sharpe and Sortino ratio. In the last optimization method, Portfolio C achieves the highest Sharpe ratio while Portfolio B achieves the highest Sortino ratio. However, the portfolios do not exceed the ratios of the benchmark portfolio. The common factor for the constructed portfolios is, nevertheless, that a frontier market inclusive portfolio achieves the highest performance ratio.

The out-of-sample period further validates the in-sample results as the portfolios inclusive frontier markets achieved higher risk-adjusted returns than the DM portfolios. Portfolio B achieves the highest Sharpe and Sortino ratio among the Sharpe-optimized portfolios. The Sortino-optimization has Portfolio C with the highest in both performance metrics. In the last optimization method, Portfolio B achieves the highest Sharpe ratio while Portfolio C achieves the highest Sortino ratio.

Based on the in-sample and out-of-sample findings, it can be concluded that frontier markets improve an international portfolio's performance. However, the specific proportion of frontier markets that should be allocated in an international portfolio is debatable. The 80% and 50% restricted portfolios achieve the highest Sharpe and Sortino rates using different optimization methods, making it difficult to conclude the optimal frontier market allocation. Furthermore, the results were highly similar to those of Spiru and Qin (2016). The FM included in the constructed portfolios were mainly from the MENA region. The countries' low covariances with developed markets and high return/risk profiles made them attractive assets for international diversification.

It should be noted that the sample periods chosen for the analysis were affected by extraordinary economic developments due to events such as the Covid-19 pandemic in the in-sample period and the invasion of Ukraine in the out-of-sample period. These events may have caused the equity markets to move uncharacteristically and distorted some of the analysis findings. However, these are examples of local and global events that affect the global equity markets and should never be removed from an analysis as such events happen occasionally.

7.2. Discussion of Methods

The findings of the analysis are a result of the culmination of choices made by the authors. As such, it is crucial to consider and reflect on how certain choices may have affected the results and their potential implications on the thesis.

The decision to use MSCI indices as proxies for global equity markets rests on the data's reliability, consistency, availability and overall quality. However, whether or not MSCI indices are efficient proxies for equity markets can be discussed. The MSCI Indices are tracking indices that attempt to track the specific equity market as best as possible. Other methods of acquiring data could have been more appropriate and led to different and potentially more robust results. The most obvious choice is to source data directly from stock exchanges in countries included in the study. Another option would be to use a country's GDP growth (%) as a proxy for equity markets. However, an article by MSCI investigated the link between GDP growth and equity returns and found large deviations between the two, concluding that GDP growth was not a viable replication of equity market returns. Based on this and the reliability of MSCI, the MSCI indices were chosen in the study.

All indices used in the analysis are denominated in American dollars (USD). By utilizing USD indices, the return values of the indices deviate from their nominal value due to currency effects. As such, the indices provide different results compared to results that would have been obtained from using local currency. The US currency has, in recent years, outperformed other currencies. The utilized indices are, therefore, subject to projecting returns greater than the actual returns of the respective equity market. Using local currency indices could have led to different results in the analysis. However, the analysis is angled from the perspective of an American investor, and the method used is therefore considered the most representative.

The choice of sourcing data in a monthly format is also relevant to the results obtained. Unlike monthly observations, weekly and daily observations were inconsistent and contained large amounts of duplicates. The high level of illiquidity in some of the frontier markets caused many duplicates in the data that ultimately had significant effects on the historical returns and standard deviations of the markets. This created an unprecise and non-representative outlook of the frontier markets. Monthly data were therefore chosen as the data mitigated many of the problems associated with the other data and still provided sufficient data for an in-depth analysis.

Lastly, the analysis uses Markowitz's mean-variance approach and other extended versions such as Estrada and Black-Litterman. However, there are other methods that arguably are more accurate at modeling optimal portfolios, such as factor models. Factor models attempt to explain the risk premium on any individual asset by the asset's exposure to a few common priced factors (Munk, 2021). The Fama-French factor models, for instance, concern stock returns and attempt to uncover the determining factors of stock prices. Using the Fama-French framework could provide exciting findings regarding the determinants of cross-sectional variations in frontier equity markets, which could then be compared to developed markets. However, due to the objective and aim of this thesis, Markowitz's mean-variance approach was deemed the most appropriate.

7.3. Recommendations for Future Research

The sample periods in the analysis are distinguished by high degrees of market uncertainties. Research of frontier markets in steadier periods might provide different findings than those obtained, as the results from the chosen sample periods may suffer from being uncharacteristic. Therefore, future research should examine how frontier markets perform in international portfolios under "normal conditions". Given that "frontier markets" as a viable investment option is a relatively recent phenomenon, financial data is limited, and empirical studies are scarce. Further quantitative research would benefit the international investment community, policymakers and financial institutions as it would assist in deciphering the dynamics of underdeveloped equity markets.

Using other construction- and optimization methods could further generate intriguing results. Portfolio construction using the abovementioned factor models could elaborate on the findings and substantiate the understanding of frontier markets. The thesis optimized portfolios using Sharpe and Sortino ratios subject to specific allocation restrictions. These allocations were, to a degree, arbitrarily appointed and only represent a small portion of the opportunity set. There are, however, many other possible optimization methods that could provide other interesting findings. Optimizing in regard to a certain return given a set risk profile could provide more detailed findings regarding the markets' risk-reducing properties.

Lastly, it could be interesting to investigate diversification across different asset classes and market categorizations. The thesis investigated the diversification benefits of including frontier markets in an international portfolio but was limited to stocks and equity markets. However, there are potential diversification effects of combining other asset classes and frontier markets, as proposed by Odier & Solnik (1990). This could provide further information regarding the diversification properties of frontier markets.

8. Conclusion

This thesis investigates the diversification benefits of frontier markets in a global portfolio from the perspective of an American investor. As such, three questions were sought answered. First, the correlation between frontier equity markets and developed equity markets was analyzed. The second question concerned frontier markets’ risk-reducing abilities in a globally diversified portfolio context. The final question examined whether there exists a positive relationship between the inclusion of frontier markets and the risk-adjusted return of an international portfolio. The research objectives are studied through various construction and optimization methods resulting in nine optimal portfolios. Subsequently, the portfolios are evaluated on return, risk and performance metrics and compared to a benchmark portfolio.

The authors find that including frontier markets in a global portfolio is positively correlated with increased performance and that frontier equity markets are a viable diversification tool for an American investor seeking higher risk-adjusted returns. Portfolios are tested in different sample periods showing similar results. For all constructed portfolio sets, the frontier market-inclusive portfolios achieve the highest performance metric. However, when compared to the benchmark, the results vary. Furthermore, the authors fail to identify a consistent relationship between risk-adjusted performance and the proportion of FM-inclusion. Therefore, the optimal frontier market allocation is uncertain. Portfolio performance compared to the benchmark portfolio is inconsistent and inconclusive.

The correlation analysis aligned with the findings of previous literature suggesting that frontier markets possess a low correlation with other international equity markets. The low correlation between FM and DM can potentially be explained by the majority of frontier markets having distinct characteristics and properties. These markets are often financially, industrially and geographically separated from developed markets, as opposed to European frontier markets, which exhibit a high correlation with developed markets. Additionally, the results of the analysis suggest that, in some cases, the inclusion of frontier markets reduces standard deviation and lower partial standard deviation, providing support for the posed sub-question. Results were, however, inconsistent, and the authors cannot confidently provide a clear conclusion regarding the risk-reducing properties of FM. Furthermore, the analysis found that frontier markets positively impact portfolio performance measured by the Sharpe and Sortino ratio. The constructed portfolios exhibited higher performance with FM compared to the DM-restricted portfolio. However, the optimal proportion of frontier market-inclusion is uncertain as the portfolio's performance differed between optimization methods.

The findings of this study have practical implications for investors looking to diversify their portfolios internationally. Frontier markets offer an opportunity for investors to increase the performance of their portfolios by adding an asset class that has a low correlation with developed markets. Therefore, investors should consider including frontier markets in their investment portfolios as a diversification tool due to their performance-enhancing potential. However, it is essential to emphasize that investing in frontier markets is not without risks. These markets are often characterized by high political and economic instability, low liquidity and limited information transparency. Therefore, investors must conduct thorough due diligence and risk assessment before investing.

Overall, this study contributes to the existing literature on frontier markets and portfolio diversification. The results of the study provide insights into the role of frontier markets in increasing portfolio performance, which can be useful for investors, fund managers and policymakers. The study findings can also be a valuable resource for future research on frontier markets and portfolio diversification.

9. References

Data

Damodaran, A. (2020). *Risk Premiums for Other Markets*. https://pages.stern.nyu.edu/~adamodar/pc/archives/ctryprem19.xls

Damodaran, A. (2023). *Risk Premiums for Other Markets*. https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ctryprem.html

Investing.com (2023). *United States 1-Year Bond Yield Historical Data*. Retrieved April 20, 2023, from https://www.investing.com/rates-bonds/u.s.-1-year-bond-yield-historical-data

MSCI (2023). *End of day data Country*. Retrieved April 25, 2023, from https://www.msci.com/end-of-day-data-country

Refinitiv Eikon (2023). *MSCI Country Indices (USD)*. Retrieved April 25, 2023, from Refinitiv

Refinitiv Eikon (2023). *S&P 500 Total Return Index (USD)*. Retrieved April 25, 2023, from Refinitiv

Literature

Amoah, R. (2020). Efficient Market Hypothesis. Are Emerging and Frontier Markets Sufficient to be Efficient? Meta- Analysis. *Journal of Emerging Issues in Economics, Finance & Banking, 9*(1), 3080–3093.

Ang, A., Liu, J., & Schwarz, K. (2020). Using Stocks or Portfolios in Tests of Factor Models. *Journal of Financial and Quantitative Analysis, 55*(3), 709–750. https://doi.org/10.1017/S0022109019000255

Baumoehl, E., & Lyocsa, S. (2014). Volatility and dynamic conditional correlations of worldwide emerging and frontier markets. *Economic Modelling, 38,* 175–183. https://doi.org/10.1016/j.econmod.2013.12.022

Berger, D., Pukthuanthong, K., & Jimmy Yang, J. (2011). International diversification with frontier markets. *Journal of Financial Economics, 101*(1), 227–242. https://doi.org/10.1016/j.jfineco.2011.02.009

Berk, J., & DeMarzo, P. (2021). *Corporate Finance, Enhanced, Global Edition* (5th ed.). Pearson International Content. https://bookshelf.vitalsource.com/books/9781292366562

Blanco, T. G. (2013). Frontier markets: a world of opportunities. *ESADEgeo Position Paper*, (30). https://dx.doi.org/10.2139/ssrn.2245075

Boulter, E., Stein, D. (2022, December 2). The opportunity and risk of Frontier Markets. *Money for the rest of Us*. https://moneyfortherestofus.com/frontier-markets/#frontier-markets-vs-emerging-markets

Chiou, W.-J. P. (2008). Who benefits more from international diversification? *Journal of International Financial Markets, Institutions & Money, 18*(5), 466–482. https://doi.org/10.1016/j.intfin.2007.07.002

Choueifaty, Y., & Coignard, Y. (2008). Toward Maximum Diversification. *Journal of Portfolio Management, 35*(1), 40–51. https://doi.org/10.3905/JPM.2008.35.1.40

Chunhachinda, P., de Boyrie, M. E., & Pavlova, I. (2019). Measuring the hedging effectiveness of commodities. *Finance Research Letters, 30*, 201–207. https://doi.org/10.1016/j.frl.2018.09.012

Clarke, R., de Silva, H., & Thorley, S. (2011). Minimum-Variance Portfolio Composition. *Journal of Portfolio Management*, *37*(2), 31–45. https://doi.org/10.3905/jpm.2011.37.2.031

Cornuejols, G., & Tütüncü, R. (2007). *Optimization methods in finance*. Cambridge University Press.

Damodaran, A. (2008). *What is the riskfree rate? A Search for the Basic Building Block*. https://www.stern.nyu.edu/~adamodar/pdfiles/papers/riskfreerate.pdf

Damodaran, A. (2022a). *Equity Risk Premiums (ERP): Determinants, Estimation and Implications - A Post-Crisis Update*. https://dx.doi.org/10.2139/ssrn.4066060

Damodaran, A. (2022b). *Damodaran, Aswath, Country Risk: Determinants, Measures and Implications - The 2022 Edition*. https://dx.doi.org/10.2139/ssrn.4161010

Damodaran, A. (n.d.). *Value at Risk (VAR)*. https://pages.stern.nyu.edu/~adamodar/pdfiles/papers/VAR.pdf

de Boyrie, M. E., & Pavlova, I. (2018). Equities and Commodities Comovements: Evidence from Emerging Markets. *Global Economy Journal*, *18*(3). https://doi.org/10.1515/gej-2017-0075 de Groot, W., Pang, J., & Swinkels, L. (2012). The cross-section of stock returns in frontier emerging markets. *Journal of Empirical Finance, 19*(5), 796–818. https://doi.org/10.1016/j.jempfin.2012.08.007

de Villiers, D., Apopo, N., Phiri, A., & McMillan, D. (2020). Unobserved structural shifts and asymmetries in the random walk model for stock returns in African frontier markets. *Cogent Economics & Finance*, *8*(1), 1–20. https://doi.org/10.1080/23322039.2020.1769348

Driessen, J., & Laeven, L. (2007). International portfolio diversification benefits: Cross-country evidence from a local perspective. *Journal of Banking & Finance*, *31*(6), 1693-1712. https://doi.org/10.1016/j.jbankfin.2006.11.006

Estrada, J. (2008). Mean-Semivariance Optimization: A Heuristic Approach. *Journal of Applied Finance : JAF, 18*(1), 57–.

Fama, E. F. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work. *The Journal of Finance*, *25*(2), 383–417. https://doi.org/10.2307/2325486

Fisher, J. D., & D'Alessandro, J. (2021). Portfolio Upside and Downside Risk—Both Matter. *Journal of Portfolio Management*, *47*(10), 158–171. https://doi.org/10.3905/jpm.2021.1.263

FTSE Russell (2014). *Frontier Markets – Accessing the next frontier*. https://www.ftserussell.com/research/frontier-markets-accessing-next-frontier

Girard, E., & Sinha, A. (2008). *Risk and return in the next frontier*. *Journal of Emerging Market Finance*, *7*(1), 43-80. https://doi.org/10.1177/097265270700700103

Graham, G., Emid, A., Milner, K., Maclean, A., Vokey, P., & Lopez, M. (2013). *Investing in frontier markets : opportunity, risk and role in an investment portfolio* (1st edition). John Wiley & Sons Canada, Ltd.

Gyamfi, E. N., Kyei, K. A., & Gill, R. (2017). Market Efficiency of African Stock Markets: A Meta-Analysis. *Journal of Developing Areas, 51*(4), 69–80. https://doi.org/10.1353/jda.2017.0088

Hausler, G. (2002). The globalization of finance. *Finance & Development*, *39*(1), 10–12. https://www.imf.org/external/pubs/ft/fandd/2002/03/hausler.htm

Hudson, R. S., & Gregoriou, A. (2015). Calculating and comparing security returns is harder than you think: A comparison between logarithmic and simple returns. *International Review of Financial Analysis*, *38*, 151–162. https://doi.org/10.1016/j.irfa.2014.10.008

Idzorek, T. (2007). 2 - A step-by-step guide to the Black-Litterman model: Incorporating user-specified confidence levels. In *Forecasting Expected Returns in the Financial Markets* (pp. 17–38). Elsevier Ltd. https://doi.org/10.1016/B978-075068321-0.50003-0

Jorion, P. (1985). International Portfolio Diversification with Estimation Risk. *The Journal of Business (Chicago, Ill.), 58*(3), 259–278. https://doi.org/10.1086/296296

Kang, S. H., McIver, R., & Yoon, S.-M. (2016). Modeling Time-Varying Correlations in Volatility Between BRICS and Commodity Markets. *Emerging Markets Finance & Trade, 52*(7), 1698–1723. https://doi.org/10.1080/1540496X.2016.1143248

Koch, C. A. (2013). Kritisk Rationalisme. I Fuglsang, L., Bitsch Olsen, P., & Rasborg, K. (red.), *Videnskabsteori i samfundsvidenskaberne : på tværs af fagkulturer og paradigmer* (3. udgave, s. 79-109). Samfundslitteratur.

Leezenberg, M., & De Vries, G. (2018). Logical Empiricism and Critical Rationalism. In *History and Philosophy of the Humanities: An Introduction* (pp. 71-100). Amsterdam University Press. doi:10.1017/9789048539338.004

Levy, H., & Sarnat, M. (1970). International Diversification of Investment Portfolios. *The American Economic Review, 60*(4), 668–675.

Li, B., Hoi, S. C. H., Sahoo, D., & Liu, Z.-Y. (2015). Moving average reversion strategy for on-line portfolio selection. *Artificial Intelligence, 222*, 104–123. https://doi.org/10.1016/j.artint.2015.01.006

Loefstrand, J. (2022). Are frontier markets still the great untapped opportunity*? T. Rowe Price.* https://www.troweprice.com/financial-intermediary/pt/en/thinking/articles/2022/q3/are-frontier-markets-still-the-great-untapped-opportunity.html

Malkiel, B. G. (2003). The Efficient Market Hypothesis and Its Critics. *The Journal of Economic Perspectives*, *17*(1), 59–82. https://doi.org/10.1257/089533003321164958

Markowitz, H. (1952). PORTFOLIO SELECTION. *The Journal of Finance (New York), 7*(1), 77–91. https://doi.org/10.1111/j.1540-6261.1952.tb01525.x

Markowitz, H. (1991). FOUNDATIONS OF PORTFOLIO THEORY. *The Journal of Finance (New York), 46*(2), 469–477. https://doi.org/10.2307/2328831

Markowitz, H. M. (1991). *Portfolio selection : efficient diversification of investments*. (2. ed.). Blackwell.

Marshall, B. R., Nguyen, N. H., & Visaltanachoti, N. (2015). Frontier market transaction costs and diversification. *Journal of Financial Markets (Amsterdam, Netherlands)*, *24*, 1–24. https://doi.org/10.1016/j.finmar.2015.04.002

MSCI Barra Research. (2010). *Is There a Link Between GDP Growth and Equity Returns*? https://www.msci.com/documents/10199/a134c5d5-dca0-420d-875d-06adb948f578

MSCI. (2023a). *MSCI Frontier Markets Index (USD) Factsheet*. https://www.msci.com/documents/10199/f9354b32-04ac-4c7e-b76e-460848afe026

MSCI. (2023b). *MSCI Emerging Markets Index (USD) Factsheet*. https://www.msci.com/documents/10199/c0db0a48-01f2-4ba9-ad01-226fd5678111

MSCI. (2023c). *MSCI World Index (USD) Factsheet*. https://www.msci.com/www/fact-sheet/msci-world-index/05830501

MSCI. (n.d.[a]). *About Us*. https://www.msci.com/who-we-are/about-us

MSCI. (n.d.[b]). *Market classification*. https://www.msci.com/our-solutions/indexes/market-classification

Mullins, D. W. (1982). Does the capital asset pricing model work? *Harvard Business Review*, *60*(1), 105–.

Munk, C. (2021). Financial markets and investments. *Copenhagen, Denmark: Lecture notes*.

Nguyen, L. T. M., Le, D., Tran, T., & Dang, T. (2021). Financial Reporting Quality, Corporate Governance, and Idiosyncratic Risk: Evidence from a Frontier Market. *Australasian Accounting, Business & Finance Journal*, *15*(4), 28–46. https://doi.org/10.14453/aabfj.v15i4.3

OEC. (2021). *Romania Country Profile*. https://oec.world/en/profile/country/rou

OEC. (2021). *Spain Country Profile*. https://oec.world/en/profile/country/esp

Pätäri, E., Ahmed, S., John, E., & Karell, V. (2019). The changing role of emerging and frontier markets in global portfolio diversification. *Cogent Economics & Finance*, *7*(1), 1701910–. https://doi.org/10.1080/23322039.2019.1701910

Perold, A. F. (2004). The Capital Asset Pricing Model. *The Journal of Economic Perspectives*, *18*(3), 3–24. https://doi.org/10.1257/0895330042162340

Pla-Santamaria, D., & Bravo, M. (2013). Portfolio optimization based on downside risk: a mean-semivariance efficient frontier from Dow Jones blue chips. *Annals of Operations Research*, *205*(1), 189–201. https://doi.org/10.1007/s10479-012-1243-x

Plenborg, T., & Kinserdal, F. (2021). *Financial statement analysis : valuation - credit analysis - performance evaluation* (2. ed.). Fagbokforlaget.

Rahman, M. A., Shien, L. K., & Sadique, M. S. (2013). Swings in sentiment and stock returns: Evidence from a frontier market. *International Journal of Trade, Economics and Finance, 4*(6), 347.

Rao, C., Govindaraju, V., Gudivada, V. N., & Raghavan, V. V. (2016). *Cognitive computing: Theory and applications* (Vol. 35). Elsevier.

Refinitiv. (n.d.). *Refinitiv Eikon*. https://www.refinitiv.com/en/products/eikon-trading-software#specs-and-download

S&P Dow Jones Indices. (2023). *S&P 500*. https://www.spglobal.com/spdji/en/indices/equity/sp-500/#overview

Samarakoon, L. P. (2011). Stock market interdependence, contagion, and the U.S. financial crisis: The case of emerging and frontier markets. *Journal of International Financial Markets, Institutions & Money, 21*(5), 724–742. https://doi.org/10.1016/j.intfin.2011.05.001

Saunders, M.N. K., Thornhill, A., & Lewis, P. (2020). *Research Methods for Business Students* (8th ed.). Pearson International Content. https://bookshelf.vitalsource.com/books/9781292208800

Sivabalan, S. (2018). *If you thought frontier investing was high risk, here's comfort*. Bloomberg.com. https://www.bloomberg.com/news/articles/2018-02-22/if-you-thought-frontier-investing-was-high-risk-here-s-comfort#xj4y7vzkg

Sortino, F. & van der Meer, R. (1991). DOWNSIDE RISK: Capturing what's at stake in investment situations. *Journal of Portfolio Management, 17*(4), 27–31. https://doi.org/10.3905/jpm.1991.409343

Speidell, L. (2011). Frontier Markets Equity Investing: Finding the Winners of the Future. *CFA Institute Research Foundation M2011-2*. https://ssrn.com/abstract=2616254

Speidell, L. S. (2018). Practical Applications of Looking Both Ways: A Quarter Century in Emerging and Frontier Markets. *Practical Applications of Institutional Investor Journals, 6*(1), 1–4. https://doi.org/10.3905/pa.6.1.277

Speidell, L., & Krohne, A. (2007). The Case for Frontier Equity Markets. *Journal of Investing, 16*(3), 12–22. https://doi.org/10.3905/joi.2007.694758

Spiru, A., & Qin, Z. (2016). Investing on the Edge: Exploring the opportunities for diversification in frontier markets. In *Handbook of Frontier Markets* (pp. 123-162). Academic Press.

Stevenson, S. (2001). Emerging markets, downside risk and the asset allocation decision. *Emerging Markets Review, 2*(1), 50–66. https://doi.org/10.1016/S1566-0141(00)00019-4

Sukumaran, A., Gupta, R., & Jithendranathan, T. (2015). Looking at new markets for international diversification: frontier markets. *International Journal of Managerial Finance, 11*(1), 97–116. https://doi.org/10.1108/IJMF-05-2013-0057

Taylor, R. (1990). Interpretation of the Correlation Coefficient: A Basic Review. *Journal of Diagnostic Medical Sonography*, *6*(1), 35–39. https://doi.org/10.1177/875647939000600106

Uludag, B. K., & Ezzat, H. (2016). Chapter 5 - Are Frontier Markets Worth the Risk? *In Handbook of Frontier Markets* (pp. 67–80). Elsevier Inc. https://doi.org/10.1016/B978-0-12-809200-2.00005-1

Vaclavik, M., & Jablonsky, J. (2012). Revisions of modern portfolio theory optimization model. *Central European Journal of Operations Research, 20*(3), 473–483. https://doi.org/10.1007/s10100-011-0227-2

Wu, W., Chen, J., Xu, L., He, Q., & Tindall, M. L. (2019). A statistical learning approach for stock selection in the Chinese stock market. *Financial Innovation (Heidelberg), 5*(1), 1–18. https://doi.org/10.1186/s40854-019-0137-1

Zaremba, A., & Maydybura, A. (2019). The cross-section of returns in frontier equity markets: Integrated or segmented pricing? *Emerging Markets Review, 38*, 219–238. https://doi.org/10.1016/j.ememar.2019.02.003

10. List of Equations, Figures and Tables

List of Equations

List of Figures

List of Tables

11. Appendices

Normality Check

Australia

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	3
-0,0411	8
0,0227	41
0,0866	26
0,1504	3
0,2143	0
More	0

Austria

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	5
-0,0411	15
0,0227	28
0,0866	24
0,1504	7
0,2143	1
More	1

Belgium

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	2
-0,1050	1
-0,0411	14
0,0227	38
0,0866	24
0,1504	2
0,2143	0
More	0

Cananda

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	0
-0,0411	11
0,0227	39
0,0866	27
0,1504	4
0,2143	0
More	0

Denmark

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	0
-0,0411	10
0,0227	37
0,0866	32
0,1504	2
0,2143	0
More	0

Finland

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	1
-0,0411	9
0,0227	43
0,0866	24
0,1504	3
0,2143	1
More	0

France

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	1
-0,1050	0
-0,0411	14
0,0227	35
0,0866	30
0,1504	0
0,2143	1
More	0

Germany

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	1
-0,1050	1
-0,0411	12
0,0227	41
0,0866	22
0,1504	3
0,2143	1
More	0

Hong Kong

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	3
-0,0411	10
0,0227	38
0,0866	26
0,1504	4
0,2143	0
More	0

Ireland

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	1
-0,1050	1
-0,0411	12
0,0227	33
0,0866	32
0,1504	2
0,2143	0
More	0

Israel

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	1
-0,1050	3
-0,0411	11
0,0227	37
0,0866	24
0,1504	5
0,2143	0
More	0

Italy

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	2
-0,0411	13
0,0227	38
0,0866	24
0,1504	4
0,2143	0
More	0

Japan

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	0
-0,0411	7
0,0227	50
0,0866	22
0,1504	2
0,2143	0
More	0

Netherlands

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	1
-0,0411	9
0,0227	39
0,0866	31
0,1504	1
0,2143	0
More	0

New Zealand

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	1
-0,0411	14
0,0227	36
0,0866	23
0,1504	7
0,2143	0
More	0

Norway

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	1
-0,1050	3
-0,0411	13
0,0227	34
0,0866	25
0,1504	4
0,2143	1
More	0

Portugal

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	1
-0,1050	2
-0,0411	13
0,0227	39
0,0866	19
0,1504	7
0,2143	0
More	0

Singapore

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	1
-0,0411	14
0,0227	41
0,0866	21
0,1504	3
0,2143	1
More	0

Spain

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	0
-0,0411	16
0,0227	35
0,0866	29
0,1504	1
0,2143	0
More	0

Sweden

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	1
-0,0411	13
0,0227	40
0,0866	24
0,1504	2
0,2143	1
More	0

Switzerland

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	0
-0,0411	11
0,0227	43
0,0866	26
0,1504	1
0,2143	0
More	0

United kingdom

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	1
-0,1050	1
-0,0411	13
0,0227	42
0,0866	23
0,1504	0
0,2143	1
More	0

United States

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	1
-0,0411	6
0,0227	47
0,0866	25
0,1504	2
0,2143	0
More	0

Bahrain

Bin	Frequency
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	3
-0,0411	9
0,0227	47
0,0866	15
0,1504	6
0,2143	1
More	0

Bangladesh	
Bin	*Frequency*
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	3
-0,0411	10
0,0227	43
0,0866	20
0,1504	3
0,2143	2
More	0

Croatia	
Bin	*Frequency*
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	1
-0,0411	8
0,0227	44
0,0866	26
0,1504	2
0,2143	0
More	0

Estonia	
Bin	*Frequency*
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	0
-0,0411	16
0,0227	37
0,0866	23
0,1504	3
0,2143	2
More	0

Jordan	
Bin	*Frequency*
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	0
-0,0411	15
0,0227	47
0,0866	18
0,1504	1
0,2143	0
More	0

Kazakhstan	
Bin	*Frequency*
-0,2966	0
-0,2327	3
-0,1688	0
-0,1050	4
-0,0411	10
0,0227	27
0,0866	26
0,1504	10
0,2143	1
More	0

Kenya	
Bin	*Frequency*
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	3
-0,0411	12
0,0227	36
0,0866	23
0,1504	7
0,2143	0
More	0

Lithuania	
Bin	*Frequency*
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	1
-0,0411	8
0,0227	46
0,0866	23
0,1504	3
0,2143	0
More	0

Mauritius	
Bin	*Frequency*
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	2
-0,0411	7
0,0227	46
0,0866	24
0,1504	1
0,2143	1
More	0

Morocco	
Bin	*Frequency*
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	0
-0,0411	6
0,0227	47
0,0866	26
0,1504	2
0,2143	0
More	0

Nigeria	
Bin	*Frequency*
-0,2966	1
-0,2327	1
-0,1688	2
-0,1050	6
-0,0411	13
0,0227	29
0,0866	17
0,1504	11
0,2143	1
More	0

Oman	
Bin	*Frequency*
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	2
-0,0411	8
0,0227	48
0,0866	21
0,1504	2
0,2143	0
More	0

Pakistan	
Bin	*Frequency*
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	4
-0,0411	18
0,0227	28
0,0866	24
0,1504	5
0,2143	2
More	0

Romania	
Bin	*Frequency*
-0,30	0
-0,23	0
-0,17	2
-0,10	3
-0,04	10
0,02	31
0,09	24
0,15	10
0,21	1
More	0

Serbia	
Bin	*Frequency*
-0,2966	0
-0,2327	1
-0,1688	2
-0,1050	2
-0,0411	7
0,0227	40
0,0866	24
0,1504	4
0,2143	1
More	0

Slovenia	
Bin	*Frequency*
-0,2966	0
-0,2327	0
-0,1688	1
-0,1050	1
-0,0411	15
0,0227	31
0,0866	29
0,1504	3
0,2143	1
More	0

Tunisia	
Bin	*Frequency*
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	0
-0,0411	11
0,0227	46
0,0866	24
0,1504	0
0,2143	0
More	0

Vietnam	
Bin	*Frequency*
-0,2966	0
-0,2327	0
-0,1688	0
-0,1050	1
-0,0411	13
0,0227	35
0,0866	28
0,1504	3
0,2143	1
More	0

Expected Return Calculation

Country	Equity Risk Premium (CDS)	Country Risk Premium	Risk-free Rate	Expected Return	Currency Adjustment	Currency Adjusted Expected Return
Australia	5,34%	0,14%	1,02%	6,37%	94,41%	6,01%
Austria	5,24%	0,04%	1,02%	6,26%	97,15%	6,08%
Belgium	5,34%	0,14%	1,02%	6,37%	97,15%	6,18%
Canada	5,39%	0,19%	1,02%	6,41%	96,56%	6,19%
Denmark	5,20%	0,00%	1,02%	6,22%	97,11%	6,04%
Finland	5,24%	0,04%	1,02%	6,26%	97,15%	6,08%
France	5,38%	0,18%	1,02%	6,40%	97,15%	6,22%
Germany	5,20%	0,00%	1,02%	6,22%	97,15%	6,05%
Hong Kong	5,64%	0,44%	1,02%	6,66%	99,99%	6,66%
Ireland	5,39%	0,19%	1,02%	6,41%	97,15%	6,23%
Israel	5,87%	0,67%	1,02%	6,90%	100,54%	6,93%
Italy	6,97%	1,77%	1,02%	7,99%	97,15%	7,77%
Japan	5,45%	0,25%	1,02%	6,47%	99,61%	6,45%
Netherlands	5,25%	0,05%	1,02%	6,27%	97,15%	6,09%
New Zealand	5,38%	0,18%	1,02%	6,40%	95,60%	6,12%
Norway	5,25%	0,05%	1,02%	6,27%	92,65%	5,81%
Portugal	5,79%	0,59%	1,02%	6,81%	97,15%	6,62%
Singapore	5,22%	0,02%	1,02%	6,25%	98,52%	6,15%
Spain	5,85%	0,65%	1,02%	6,87%	97,15%	6,68%
Sweden	5,21%	0,01%	1,02%	6,24%	94,64%	5,90%
Switzerland	5,20%	0,00%	1,02%	6,22%	99,54%	6,20%
United Kingdom	5,39%	0,19%	1,02%	6,41%	95,61%	6,13%
United States	5,20%	0,00%	1,02%	6,22%	100,00%	6,22%
Bahrain	7,63%	2,43%	1,02%	8,65%	100,00%	8,65%
Bangladesh	8,71%	3,51%	1,02%	9,73%	98,53%	9,59%
Croatia	6,40%	1,20%	1,02%	7,43%	97,33%	7,23%
Estonia	5,84%	0,64%	1,02%	6,86%	97,15%	6,67%
Jordan	10,27%	5,07%	1,02%	11,29%	100,00%	11,29%
Kazakhstan	6,21%	1,01%	1,02%	7,24%	83,67%	6,06%
Kenya	9,91%	4,71%	1,02%	10,93%	96,68%	10,57%
Lithuania	6,01%	0,81%	1,02%	7,04%	97,17%	6,84%
Mauritius	6,29%	1,09%	1,02%	7,31%	95,54%	6,99%
Morocco	6,47%	1,27%	1,02%	7,50%	97,75%	7,33%
Nigeria	9,82%	4,62%	1,02%	10,85%	84,83%	9,20%
Oman	8,43%	3,23%	1,02%	9,46%	100,00%	9,46%
Pakistan	10,48%	5,28%	1,02%	11,51%	92,21%	10,61%
Romania	6,40%	1,20%	1,02%	7,43%	95,72%	7,11%
Serbia	6,50%	1,30%	1,02%	7,52%	96,96%	7,29%
Slovenia	6,32%	1,12%	1,02%	7,34%	97,15%	7,14%
Tunisia	9,78%	4,58%	1,02%	10,80%	91,04%	9,83%
Vietnam	6,86%	1,66%	1,02%	7,89%	98,44%	7,76%
Average	6,43%	1,23%	1,02%	7,46%	96,50%	7,18%

Currency Adjustment Rate

Country Name	2014	2015	2016	2017	2018	2019	2020	Average Yearly Change (%)	Adjustment Rate
Australia	1,09	1,20	1,37	1,33	1,29	1,40	1,49	5,59%	94,41%
Austria	0,75	0,90	0,90	0,89	0,85	0,89	0,88	2,85%	97,15%
Bahrain	0,38	0,38	0,38	0,38	0,38	0,38	0,38	0,00%	100,00%
Bangladesh	77,72	77,70	78,26	79,12	82,12	84,03	84,79	1,47%	98,53%
Belgium	0,75	0,90	0,90	0,89	0,85	0,89	0,88	2,85%	97,15%
Canada	1,10	1,28	1,33	1,30	1,30	1,33	1,34	3,44%	96,56%
Croatia	5,75	6,86	6,81	6,62	6,28	6,62	6,61	2,67%	97,33%
Denmark	5,61	6,73	6,73	6,60	6,31	6,67	6,54	2,89%	97,11%
Estonia	0,75	0,90	0,90	0,89	0,85	0,89	0,88	2,85%	97,15%
Finland	0,75	0,90	0,90	0,89	0,85	0,89	0,88	2,85%	97,15%
France	0,75	0,90	0,90	0,89	0,85	0,89	0,88	2,85%	97,15%
Germany	0,75	0,90	0,90	0,89	0,85	0,89	0,88	2,85%	97,15%
Hong Kong	7,75	7,75	7,76	7,79	7,84	7,84	7,76	0,01%	99,99%
Ireland	0,75	0,90	0,90	0,89	0,85	0,89	0,88	2,85%	97,15%
Israel	3,58	3,89	3,84	3,60	3,59	3,56	3,44	-0,54%	100,54%
Italy	0,75	0,90	0,90	0,89	0,85	0,89	0,88	2,85%	97,15%
Japan	105,94	121,04	108,79	112,17	110,42	109,01	106,77	0,39%	99,61%
Jordan	0,71	0,71	0,71	0,71	0,71	0,71	0,71	0,00%	100,00%
Kazakhstan	179,19	221,73	342,16	326,00	344,71	382,75	412,95	16,33%	83,67%
Kenya	87,92	98,18	101,50	103,41	101,30	101,99	106,45	3,32%	96,68%
Lithuania	0,75	0,90	0,90	0,89	0,85	0,89	0,88	2,83%	97,17%
Mauritius	30,62	35,06	35,54	34,48	33,93	35,47	39,35	4,46%	95,54%
Morocco	8,41	9,76	9,81	9,69	9,39	9,62	9,50	2,25%	97,75%
Netherlands	0,75	0,90	0,90	0,89	0,85	0,89	0,88	2,85%	97,15%
New Zealand	1,21	1,43	1,44	1,41	1,45	1,52	1,54	4,40%	95,60%
Nigeria	156,98	193,05	253,49	305,79	306,08	325,00	356,90	15,17%	84,83%
Norway	6,30	8,06	8,40	8,27	8,13	8,80	9,42	7,35%	92,65%
Oman	0,38	0,38	0,38	0,38	0,38	0,38	0,38	0,00%	100,00%
Pakistan	103,00	101,43	104,34	104,81	110,04	136,48	158,24	7,79%	92,21%
Portugal	0,75	0,90	0,90	0,89	0,85	0,89	0,88	2,85%	97,15%
Romania	3,35	4,01	4,06	4,05	3,94	4,24	4,24	4,28%	95,72%
Serbia	88,41	108,81	111,28	107,76	100,18	105,25	103,16	3,04%	96,96%
Singapore	1,27	1,37	1,38	1,38	1,35	1,36	1,38	1,48%	98,52%
Slovenia	0,75	0,90	0,90	0,89	0,85	0,89	0,88	2,85%	97,15%
Spain	0,75	0,90	0,90	0,89	0,85	0,89	0,88	2,85%	97,15%
Sweden	6,86	8,43	8,56	8,55	8,69	9,46	9,21	5,36%	94,64%
Switzerland	0,92	0,96	0,99	0,98	0,98	0,99	0,94	0,46%	99,54%
Tunisia	1,70	1,96	2,15	2,42	2,65	2,93	2,81	8,96%	91,04%
United Kingdom	0,61	0,65	0,74	0,78	0,75	0,78	0,78	4,39%	95,61%
United States	1,00	1,00	1,00	1,00	1,00	1,00	1,00	0,00%	100,00%
Vietnam	21.148,00	21.697,57	21.935,00	22.370,09	22.602,05	23.050,24	23.208,37	1,56%	98,44%

MSCI Index Contents

MSCI Frontier Markets Index	MSCI World INDEX	MSCI Emerging Markets Index
Bahrain	Australia	Brazil
Bangladesh	Austria	Chile
Brukina Faso	Belgium	China
Benin	Canada	Colombia
Croatia	Denmark	Czech Republic
Estonia	Finland	Egytpt
Guniea-Bissau	France	Greece
Iceland	Germany	Hungary
Ivory Coast	Hong Kong	India
Jordan	Ireland	Indonesia
Kenya	Israel	Korea
Kazakhstan	Italy	Kuwait
Mauritius	Japan	Malaysia
Mali	Netherlands	Mexico
Mauritius	New Zealand	Peru
Niger	Norway	Phillipines
Nigeria	Portugal	Poland
Oman	Singapore	Qatar
Pakistan	Spain	Saudi Arabia
Romania	Sweden	South Africa
Serbia	Switzerland	Taiwan
Senegal	United Kingdom	Thailand
Slovenia	United States	Turkey
Sri Lanka		United Arab Emirates
Togo		
Tunisia		
Vietnam		

Covariance-Variance Matrix

	Australia	Austria	Belgium	Canada	Denmark	Finland	France	Germany	Hong Kong	Ireland	Israel	Italy	Japan	Netherlands
Australia	0,0416	0,0459	0,0280	0,0311	0,0162	0,0240	0,0272	0,0294	0,0261	0,0231	0,0249	0,0314	0,0186	0,0226
Austria	0,0459	0,0815	0,0424	0,0416	0,0245	0,0337	0,0451	0,0461	0,0325	0,0376	0,0308	0,0543	0,0282	0,0370
Belgium	0,0280	0,0424	0,0396	0,0250	0,0186	0,0260	0,0295	0,0304	0,0207	0,0227	0,0221	0,0337	0,0187	0,0270
Canada	0,0311	0,0416	0,0250	0,0333	0,0136	0,0195	0,0253	0,0265	0,0201	0,0220	0,0216	0,0285	0,0163	0,0212
Denmark	0,0162	0,0245	0,0186	0,0136	0,0217	0,0159	0,0176	0,0196	0,0123	0,0160	0,0157	0,0212	0,0124	0,0169
Finland	0,0240	0,0337	0,0260	0,0195	0,0159	0,0270	0,0246	0,0263	0,0170	0,0193	0,0189	0,0279	0,0155	0,0204
France	0,0272	0,0451	0,0295	0,0253	0,0176	0,0246	0,0322	0,0319	0,0206	0,0252	0,0200	0,0371	0,0184	0,0257
Germany	0,0294	0,0461	0,0304	0,0265	0,0196	0,0263	0,0319	0,0364	0,0218	0,0272	0,0237	0,0372	0,0206	0,0273
Hong Kong	0,0261	0,0325	0,0207	0,0201	0,0123	0,0170	0,0206	0,0218	0,0325	0,0142	0,0157	0,0230	0,0154	0,0176
Ireland	0,0231	0,0376	0,0227	0,0220	0,0160	0,0193	0,0252	0,0272	0,0142	0,0316	0,0191	0,0282	0,0143	0,0221
Israel	0,0249	0,0308	0,0221	0,0216	0,0157	0,0189	0,0200	0,0237	0,0157	0,0191	0,0403	0,0224	0,0164	0,0191
Italy	0,0314	0,0543	0,0337	0,0285	0,0212	0,0279	0,0371	0,0372	0,0230	0,0282	0,0224	0,0513	0,0212	0,0296
Japan	0,0186	0,0282	0,0187	0,0163	0,0124	0,0155	0,0184	0,0206	0,0154	0,0143	0,0164	0,0212	0,0179	0,0160
Netherlands	0,0226	0,0370	0,0270	0,0212	0,0169	0,0204	0,0257	0,0273	0,0176	0,0221	0,0191	0,0296	0,0160	0,0247
New Zealand	0,0277	0,0298	0,0214	0,0231	0,0136	0,0166	0,0197	0,0219	0,0212	0,0177	0,0212	0,0202	0,0134	0,0177
Norway	0,0327	0,0488	0,0304	0,0321	0,0173	0,0257	0,0312	0,0307	0,0225	0,0238	0,0191	0,0355	0,0205	0,0245
Portugal	0,0236	0,0400	0,0241	0,0208	0,0191	0,0217	0,0267	0,0271	0,0158	0,0206	0,0146	0,0321	0,0159	0,0229
Singapore	0,0299	0,0411	0,0255	0,0264	0,0131	0,0207	0,0261	0,0267	0,0284	0,0179	0,0189	0,0301	0,0179	0,0206
Spain	0,0312	0,0539	0,0325	0,0291	0,0191	0,0262	0,0356	0,0355	0,0230	0,0261	0,0229	0,0451	0,0209	0,0289
Sweden	0,0252	0,0393	0,0255	0,0232	0,0177	0,0224	0,0270	0,0293	0,0194	0,0241	0,0196	0,0314	0,0170	0,0227
Switzerland	0,0163	0,0250	0,0186	0,0147	0,0141	0,0153	0,0181	0,0197	0,0131	0,0146	0,0147	0,0207	0,0119	0,0163
United Kingdom	0,0263	0,0395	0,0266	0,0247	0,0144	0,0212	0,0267	0,0268	0,0201	0,0227	0,0187	0,0303	0,0168	0,0217
United States	0,0240	0,0321	0,0227	0,0221	0,0130	0,0184	0,0208	0,0232	0,0164	0,0190	0,0206	0,0224	0,0150	0,0189
Bahrain	0,0206	0,0302	0,0161	0,0203	0,0097	0,0125	0,0174	0,0145	0,0157	0,0156	0,0105	0,0209	0,0098	0,0142
Bangladesh	0,0075	0,0061	0,0051	0,0072	0,0034	0,0053	0,0049	0,0046	0,0020	0,0038	0,0066	0,0076	0,0029	0,0035
Croatia	0,0188	0,0273	0,0159	0,0166	0,0099	0,0140	0,0167	0,0176	0,0134	0,0130	0,0120	0,0188	0,0118	0,0144
Estonia	0,0298	0,0431	0,0257	0,0249	0,0166	0,0178	0,0236	0,0249	0,0159	0,0183	0,0200	0,0298	0,0142	0,0207
Jordan	-0,0035	-0,0110	-0,0119	-0,0062	-0,0045	-0,0049	-0,0091	-0,0070	-0,0038	-0,0014	-0,0049	-0,0079	-0,0046	-0,0064
Kazakhstan	0,0318	0,0396	0,0193	0,0275	0,0132	0,0178	0,0211	0,0254	0,0186	0,0178	0,0210	0,0256	0,0164	0,0186
Kenya	0,0163	0,0283	0,0185	0,0164	0,0109	0,0141	0,0157	0,0166	0,0147	0,0123	0,0074	0,0200	0,0103	0,0137
Lithuania	0,0227	0,0341	0,0215	0,0210	0,0152	0,0176	0,0195	0,0211	0,0151	0,0176	0,0152	0,0246	0,0112	0,0179
Mauritius	0,0293	0,0415	0,0196	0,0268	0,0089	0,0177	0,0251	0,0229	0,0163	0,0196	0,0171	0,0305	0,0122	0,0166
Morocco	0,0217	0,0317	0,0153	0,0184	0,0097	0,0132	0,0175	0,0173	0,0117	0,0131	0,0130	0,0239	0,0099	0,0134
Nigeria	0,0222	0,0367	0,0164	0,0203	0,0154	0,0161	0,0209	0,0191	0,0105	0,0211	0,0141	0,0307	0,0129	0,0173
Oman	0,0129	0,0164	0,0078	0,0117	0,0064	0,0095	0,0115	0,0110	0,0085	0,0074	0,0086	0,0145	0,0086	0,0074
Pakistan	0,0334	0,0464	0,0286	0,0311	0,0129	0,0250	0,0273	0,0297	0,0217	0,0262	0,0303	0,0329	0,0183	0,0218
Romania	0,0278	0,0431	0,0282	0,0233	0,0169	0,0217	0,0280	0,0279	0,0195	0,0200	0,0178	0,0323	0,0176	0,0241
Serbia	0,0221	0,0308	0,0100	0,0164	0,0099	0,0117	0,0168	0,0159	0,0097	0,0140	0,0049	0,0234	0,0085	0,0114
Slovenia	0,0294	0,0421	0,0260	0,0248	0,0198	0,0237	0,0272	0,0277	0,0203	0,0201	0,0197	0,0333	0,0160	0,0225
Tunisia	0,0084	0,0097	0,0052	0,0076	0,0043	0,0058	0,0064	0,0064	0,0037	0,0057	0,0017	0,0087	0,0017	0,0058
Vietnam	0,0276	0,0351	0,0209	0,0209	0,0118	0,0177	0,0199	0,0207	0,0181	0,0173	0,0180	0,0232	0,0166	0,0179

	New Zealand	Norway	Portugal	Singapore	Spain	Sweden	Switzerland	United Kingdom	United States	Bahrain	Bangladesh	Croatia	Estonia
Australia	0,0277	0,0327	0,0236	0,0299	0,0312	0,0252	0,0163	0,0263	0,0240	0,0206	0,0075	0,0188	0,0298
Austria	0,0298	0,0488	0,0400	0,0411	0,0539	0,0393	0,0250	0,0395	0,0321	0,0302	0,0061	0,0273	0,0431
Belgium	0,0214	0,0304	0,0241	0,0255	0,0325	0,0255	0,0186	0,0266	0,0227	0,0161	0,0051	0,0159	0,0257
Canada	0,0231	0,0321	0,0208	0,0264	0,0291	0,0232	0,0147	0,0247	0,0221	0,0203	0,0072	0,0166	0,0249
Denmark	0,0136	0,0173	0,0191	0,0131	0,0191	0,0177	0,0141	0,0144	0,0130	0,0097	0,0034	0,0099	0,0166
Finland	0,0166	0,0257	0,0217	0,0207	0,0262	0,0224	0,0153	0,0212	0,0184	0,0125	0,0053	0,0140	0,0178
France	0,0197	0,0312	0,0267	0,0261	0,0356	0,0270	0,0181	0,0267	0,0208	0,0174	0,0049	0,0167	0,0236
Germany	0,0219	0,0307	0,0271	0,0267	0,0355	0,0293	0,0197	0,0268	0,0232	0,0145	0,0046	0,0176	0,0249
Hong Kong	0,0212	0,0225	0,0158	0,0284	0,0230	0,0194	0,0131	0,0201	0,0164	0,0157	0,0020	0,0134	0,0159
Ireland	0,0177	0,0238	0,0206	0,0179	0,0261	0,0241	0,0146	0,0227	0,0190	0,0156	0,0038	0,0130	0,0183
Israel	0,0212	0,0191	0,0146	0,0189	0,0229	0,0196	0,0147	0,0187	0,0206	0,0105	0,0066	0,0120	0,0200
Italy	0,0202	0,0355	0,0321	0,0301	0,0451	0,0314	0,0207	0,0303	0,0224	0,0209	0,0076	0,0188	0,0298
Japan	0,0134	0,0205	0,0159	0,0179	0,0209	0,0170	0,0119	0,0168	0,0150	0,0098	0,0029	0,0118	0,0142
Netherlands	0,0177	0,0245	0,0229	0,0206	0,0289	0,0227	0,0163	0,0217	0,0189	0,0142	0,0035	0,0144	0,0207
New Zealand	0,0353	0,0219	0,0181	0,0230	0,0206	0,0194	0,0140	0,0194	0,0183	0,0117	0,0007	0,0167	0,0217
Norway	0,0219	0,0470	0,0289	0,0297	0,0342	0,0285	0,0180	0,0299	0,0227	0,0240	0,0085	0,0202	0,0268
Portugal	0,0181	0,0289	0,0394	0,0195	0,0299	0,0222	0,0175	0,0222	0,0159	0,0182	-0,0009	0,0168	0,0238
Singapore	0,0230	0,0297	0,0195	0,0369	0,0296	0,0241	0,0148	0,0246	0,0202	0,0186	0,0028	0,0182	0,0231
Spain	0,0206	0,0342	0,0299	0,0296	0,0467	0,0294	0,0196	0,0294	0,0215	0,0202	0,0053	0,0194	0,0298
Sweden	0,0194	0,0285	0,0222	0,0241	0,0294	0,0283	0,0168	0,0233	0,0200	0,0155	0,0028	0,0159	0,0207
Switzerland	0,0140	0,0180	0,0175	0,0148	0,0196	0,0168	0,0154	0,0162	0,0137	0,0112	0,0027	0,0110	0,0131
United Kingdom	0,0194	0,0299	0,0222	0,0246	0,0294	0,0233	0,0162	0,0264	0,0193	0,0166	0,0049	0,0152	0,0202
United States	0,0183	0,0227	0,0159	0,0202	0,0215	0,0200	0,0137	0,0193	0,0207	0,0142	0,0065	0,0125	0,0175
Bahrain	0,0117	0,0240	0,0182	0,0186	0,0202	0,0155	0,0112	0,0166	0,0142	0,0460	0,0079	0,0140	0,0167
Bangladesh	0,0007	0,0085	-0,0009	0,0028	0,0053	0,0028	0,0027	0,0049	0,0065	0,0079	0,0372	0,0046	0,0035
Croatia	0,0167	0,0202	0,0168	0,0182	0,0194	0,0159	0,0110	0,0152	0,0125	0,0140	0,0046	0,0223	0,0179
Estonia	0,0217	0,0268	0,0238	0,0231	0,0298	0,0207	0,0131	0,0202	0,0175	0,0167	0,0035	0,0179	0,0507
Jordan	-0,0067	-0,0091	-0,0038	-0,0066	-0,0122	-0,0071	-0,0033	-0,0050	-0,0041	0,0026	0,0054	-0,0012	-0,0041
Kazakhstan	0,0207	0,0291	0,0170	0,0277	0,0296	0,0183	0,0126	0,0197	0,0206	0,0228	0,0159	0,0213	0,0246
Kenya	0,0096	0,0163	0,0155	0,0180	0,0184	0,0138	0,0086	0,0144	0,0124	0,0139	0,0074	0,0086	0,0189
Lithuania	0,0169	0,0224	0,0201	0,0202	0,0224	0,0188	0,0119	0,0164	0,0159	0,0179	0,0043	0,0150	0,0253
Mauritius	0,0167	0,0280	0,0181	0,0238	0,0310	0,0190	0,0104	0,0227	0,0155	0,0219	0,0117	0,0203	0,0346
Morocco	0,0137	0,0201	0,0142	0,0178	0,0230	0,0155	0,0079	0,0139	0,0120	0,0169	0,0099	0,0158	0,0240
Nigeria	0,0096	0,0282	0,0189	0,0202	0,0300	0,0177	0,0106	0,0189	0,0131	0,0169	0,0143	0,0153	0,0222
Oman	0,0071	0,0162	0,0084	0,0108	0,0124	0,0097	0,0067	0,0098	0,0089	0,0128	0,0073	0,0074	0,0114
Pakistan	0,0240	0,0325	0,0202	0,0301	0,0308	0,0259	0,0156	0,0282	0,0236	0,0278	0,0104	0,0197	0,0274
Romania	0,0223	0,0327	0,0268	0,0264	0,0315	0,0227	0,0190	0,0244	0,0210	0,0253	0,0092	0,0229	0,0275
Serbia	0,0131	0,0206	0,0202	0,0167	0,0233	0,0129	0,0095	0,0149	0,0100	0,0207	0,0060	0,0183	0,0216
Slovenia	0,0243	0,0316	0,0288	0,0261	0,0335	0,0233	0,0171	0,0239	0,0178	0,0196	0,0050	0,0225	0,0304
Tunisia	0,0047	0,0081	0,0097	0,0041	0,0081	0,0056	0,0036	0,0044	0,0035	0,0087	0,0031	0,0077	0,0084
Vietnam	0,0154	0,0245	0,0191	0,0223	0,0223	0,0171	0,0121	0,0187	0,0181	0,0200	0,0120	0,0138	0,0248

	Jordan	Kazakhstan	Kenya	Lithuania	Mauritius	Morocco	Nigeria	Oman	Pakistan	Romania	Serbia	Slovenia	Tunisia	Vietnam
Australia	-0,0035	0,0318	0,0163	0,0227	0,0293	0,0217	0,0222	0,0129	0,0334	0,0278	0,0221	0,0294	0,0084	0,0276
Austria	-0,0110	0,0396	0,0283	0,0341	0,0415	0,0317	0,0367	0,0164	0,0464	0,0431	0,0308	0,0421	0,0097	0,0351
Belgium	-0,0119	0,0193	0,0185	0,0215	0,0196	0,0153	0,0164	0,0078	0,0286	0,0282	0,0100	0,0260	0,0052	0,0209
Canada	-0,0062	0,0275	0,0164	0,0210	0,0268	0,0184	0,0203	0,0117	0,0311	0,0233	0,0164	0,0248	0,0076	0,0209
Denmark	-0,0045	0,0132	0,0109	0,0152	0,0089	0,0097	0,0154	0,0064	0,0129	0,0169	0,0099	0,0198	0,0043	0,0118
Finland	-0,0049	0,0178	0,0141	0,0176	0,0177	0,0132	0,0161	0,0095	0,0250	0,0217	0,0117	0,0237	0,0058	0,0177
France	-0,0091	0,0211	0,0157	0,0195	0,0251	0,0175	0,0209	0,0115	0,0273	0,0280	0,0168	0,0272	0,0064	0,0199
Germany	-0,0070	0,0254	0,0166	0,0211	0,0229	0,0173	0,0191	0,0110	0,0297	0,0279	0,0159	0,0277	0,0064	0,0207
Hong Kong	-0,0038	0,0186	0,0147	0,0151	0,0163	0,0117	0,0105	0,0085	0,0217	0,0195	0,0097	0,0203	0,0037	0,0181
Ireland	-0,0014	0,0178	0,0123	0,0176	0,0196	0,0131	0,0211	0,0074	0,0262	0,0200	0,0140	0,0201	0,0057	0,0173
Israel	-0,0049	0,0210	0,0074	0,0152	0,0171	0,0130	0,0141	0,0086	0,0303	0,0178	0,0049	0,0197	0,0017	0,0180
Italy	-0,0079	0,0256	0,0200	0,0246	0,0305	0,0239	0,0307	0,0145	0,0329	0,0323	0,0234	0,0333	0,0087	0,0232
Japan	-0,0046	0,0164	0,0103	0,0112	0,0122	0,0099	0,0129	0,0086	0,0183	0,0176	0,0085	0,0160	0,0017	0,0166
Netherlands	-0,0064	0,0186	0,0137	0,0179	0,0166	0,0134	0,0173	0,0074	0,0218	0,0241	0,0114	0,0225	0,0058	0,0179
New Zealand	-0,0067	0,0207	0,0096	0,0169	0,0167	0,0137	0,0096	0,0071	0,0240	0,0223	0,0131	0,0243	0,0047	0,0154
Norway	-0,0091	0,0291	0,0163	0,0224	0,0280	0,0201	0,0282	0,0162	0,0325	0,0327	0,0206	0,0316	0,0081	0,0245
Portugal	-0,0038	0,0170	0,0155	0,0201	0,0181	0,0142	0,0189	0,0084	0,0202	0,0268	0,0202	0,0288	0,0097	0,0191
Singapore	-0,0066	0,0277	0,0180	0,0202	0,0238	0,0178	0,0202	0,0108	0,0301	0,0264	0,0167	0,0261	0,0041	0,0223
Spain	-0,0122	0,0296	0,0184	0,0224	0,0310	0,0230	0,0300	0,0124	0,0308	0,0315	0,0233	0,0335	0,0081	0,0223
Sweden	-0,0071	0,0183	0,0138	0,0188	0,0190	0,0155	0,0177	0,0097	0,0259	0,0227	0,0129	0,0233	0,0056	0,0171
Switzerland	-0,0033	0,0126	0,0086	0,0119	0,0104	0,0079	0,0106	0,0067	0,0156	0,0190	0,0095	0,0171	0,0036	0,0121
United Kingdom	-0,0050	0,0197	0,0144	0,0164	0,0227	0,0139	0,0189	0,0098	0,0282	0,0244	0,0149	0,0239	0,0044	0,0187
United States	-0,0041	0,0206	0,0124	0,0159	0,0155	0,0120	0,0131	0,0089	0,0236	0,0210	0,0100	0,0178	0,0035	0,0181
Bahrain	0,0026	0,0228	0,0139	0,0179	0,0219	0,0169	0,0169	0,0128	0,0278	0,0253	0,0207	0,0196	0,0087	0,0200
Bangladesh	0,0054	0,0159	0,0074	0,0043	0,0117	0,0099	0,0143	0,0073	0,0104	0,0092	0,0060	0,0050	0,0031	0,0120
Croatia	-0,0012	0,0213	0,0086	0,0150	0,0203	0,0158	0,0153	0,0074	0,0197	0,0229	0,0183	0,0225	0,0077	0,0138
Estonia	-0,0041	0,0246	0,0189	0,0253	0,0346	0,0240	0,0222	0,0114	0,0274	0,0275	0,0216	0,0304	0,0084	0,0248
Jordan	0,0495	0,0054	0,0015	0,0012	-0,0048	0,0008	0,0060	-0,0054	0,0069	-0,0046	0,0031	-0,0077	-0,0020	0,0028
Kazakhstan	0,0054	0,0837	0,0170	0,0240	0,0280	0,0218	0,0371	0,0109	0,0227	0,0330	0,0287	0,0285	0,0088	0,0237
Kenya	0,0015	0,0170	0,0409	0,0182	0,0142	0,0112	0,0237	0,0062	0,0218	0,0177	0,0093	0,0172	0,0091	0,0164
Lithuania	0,0012	0,0240	0,0182	0,0301	0,0167	0,0180	0,0194	0,0078	0,0281	0,0232	0,0174	0,0246	0,0098	0,0184
Mauritius	-0,0048	0,0280	0,0142	0,0167	0,0585	0,0278	0,0261	0,0176	0,0315	0,0265	0,0262	0,0295	0,0087	0,0237
Morocco	0,0008	0,0218	0,0112	0,0180	0,0278	0,0285	0,0194	0,0119	0,0267	0,0176	0,0218	0,0214	0,0092	0,0195
Nigeria	0,0060	0,0371	0,0237	0,0194	0,0261	0,0194	0,0945	0,0091	0,0244	0,0291	0,0297	0,0288	0,0056	0,0239
Oman	-0,0054	0,0109	0,0062	0,0078	0,0176	0,0119	0,0091	0,0234	0,0171	0,0160	0,0123	0,0125	0,0007	0,0148
Pakistan	0,0069	0,0227	0,0218	0,0281	0,0315	0,0267	0,0244	0,0171	0,0783	0,0299	0,0203	0,0286	0,0042	0,0322
Romania	-0,0046	0,0330	0,0177	0,0232	0,0265	0,0176	0,0291	0,0160	0,0299	0,0570	0,0302	0,0325	0,0118	0,0276
Serbia	0,0031	0,0287	0,0093	0,0174	0,0262	0,0218	0,0297	0,0123	0,0203	0,0302	0,0508	0,0286	0,0109	0,0182
Slovenia	-0,0077	0,0285	0,0172	0,0246	0,0295	0,0214	0,0288	0,0125	0,0286	0,0325	0,0286	0,0439	0,0126	0,0210
Tunisia	-0,0020	0,0088	0,0091	0,0098	0,0087	0,0092	0,0056	0,0007	0,0042	0,0118	0,0109	0,0126	0,0239	0,0087
Vietnam	0,0028	0,0237	0,0164	0,0184	0,0237	0,0195	0,0239	0,0148	0,0322	0,0276	0,0182	0,0210	0,0087	0,0481

Semi-Covariance-Variance Matrix

	Australia	Austria	Belgium	Canada	Denmark	Finland	France	Germany	Hong Kong	Ireland	Israel	Italy	Japan	Netherlands
Australia	0,0217	0,0220	0,0125	0,0150	0,0048	0,0102	0,0125	0,0126	0,0115	0,0102	0,0118	0,0139	0,0077	0,0090
Austria	0,0220	0,0351	0,0168	0,0190	0,0062	0,0120	0,0179	0,0181	0,0128	0,0158	0,0150	0,0224	0,0096	0,0137
Belgium	0,0125	0,0168	0,0166	0,0107	0,0070	0,0102	0,0114	0,0119	0,0074	0,0093	0,0107	0,0127	0,0078	0,0103
Canada	0,0150	0,0190	0,0107	0,0153	0,0035	0,0076	0,0111	0,0109	0,0080	0,0090	0,0098	0,0120	0,0061	0,0083
Denmark	0,0048	0,0062	0,0070	0,0035	0,0076	0,0052	0,0055	0,0063	0,0041	0,0050	0,0050	0,0065	0,0048	0,0056
Finland	0,0102	0,0120	0,0102	0,0076	0,0052	0,0096	0,0085	0,0091	0,0071	0,0067	0,0073	0,0097	0,0054	0,0066
France	0,0125	0,0179	0,0114	0,0111	0,0055	0,0085	0,0121	0,0121	0,0082	0,0098	0,0085	0,0138	0,0068	0,0095
Germany	0,0126	0,0181	0,0119	0,0109	0,0063	0,0091	0,0121	0,0137	0,0088	0,0101	0,0092	0,0144	0,0077	0,0098
Hong Kong	0,0115	0,0128	0,0074	0,0080	0,0041	0,0071	0,0082	0,0088	0,0136	0,0061	0,0074	0,0094	0,0059	0,0063
Ireland	0,0102	0,0158	0,0093	0,0090	0,0050	0,0067	0,0098	0,0101	0,0061	0,0127	0,0080	0,0113	0,0059	0,0081
Israel	0,0118	0,0150	0,0107	0,0098	0,0050	0,0073	0,0085	0,0092	0,0074	0,0080	0,0171	0,0099	0,0072	0,0078
Italy	0,0139	0,0224	0,0127	0,0120	0,0065	0,0097	0,0138	0,0144	0,0094	0,0113	0,0099	0,0208	0,0080	0,0107
Japan	0,0077	0,0096	0,0078	0,0061	0,0048	0,0054	0,0068	0,0077	0,0059	0,0059	0,0072	0,0080	0,0071	0,0063
Netherlands	0,0090	0,0137	0,0103	0,0083	0,0056	0,0066	0,0095	0,0098	0,0063	0,0081	0,0078	0,0107	0,0063	0,0091
New Zealand	0,0108	0,0110	0,0071	0,0082	0,0037	0,0058	0,0069	0,0072	0,0077	0,0062	0,0078	0,0074	0,0045	0,0054
Norway	0,0154	0,0184	0,0114	0,0127	0,0044	0,0091	0,0123	0,0120	0,0088	0,0089	0,0088	0,0128	0,0069	0,0090
Portugal	0,0094	0,0133	0,0086	0,0073	0,0052	0,0069	0,0093	0,0090	0,0061	0,0064	0,0048	0,0098	0,0046	0,0076
Singapore	0,0143	0,0175	0,0104	0,0115	0,0047	0,0087	0,0106	0,0109	0,0117	0,0084	0,0100	0,0129	0,0071	0,0080
Spain	0,0141	0,0219	0,0120	0,0132	0,0061	0,0087	0,0132	0,0137	0,0090	0,0109	0,0104	0,0176	0,0075	0,0106
Sweden	0,0105	0,0148	0,0099	0,0091	0,0055	0,0077	0,0099	0,0106	0,0082	0,0092	0,0082	0,0116	0,0064	0,0081
Switzerland	0,0066	0,0079	0,0070	0,0051	0,0047	0,0052	0,0061	0,0072	0,0045	0,0047	0,0056	0,0070	0,0049	0,0057
United Kingdom	0,0126	0,0160	0,0114	0,0108	0,0045	0,0082	0,0106	0,0108	0,0081	0,0092	0,0087	0,0117	0,0064	0,0085
United States	0,0111	0,0145	0,0102	0,0101	0,0045	0,0070	0,0089	0,0094	0,0070	0,0077	0,0096	0,0096	0,0065	0,0076
Bahrain	0,0128	0,0159	0,0075	0,0106	0,0025	0,0063	0,0091	0,0075	0,0074	0,0069	0,0064	0,0105	0,0038	0,0063
Bangladesh	0,0054	0,0057	0,0036	0,0037	0,0003	0,0018	0,0031	0,0029	0,0004	0,0019	0,0020	0,0039	0,0009	0,0021
Croatia	0,0080	0,0103	0,0062	0,0070	0,0026	0,0046	0,0065	0,0064	0,0038	0,0057	0,0048	0,0077	0,0039	0,0049
Estonia	0,0159	0,0208	0,0108	0,0124	0,0047	0,0078	0,0111	0,0107	0,0064	0,0089	0,0093	0,0130	0,0049	0,0076
Jordan	0,0045	0,0047	0,0009	0,0025	0,0001	0,0015	0,0024	0,0023	0,0013	0,0028	0,0004	0,0034	0,0016	0,0011
Kazakhstan	0,0164	0,0207	0,0080	0,0161	0,0032	0,0067	0,0104	0,0111	0,0071	0,0099	0,0105	0,0133	0,0067	0,0078
Kenya	0,0090	0,0110	0,0079	0,0080	0,0040	0,0049	0,0066	0,0067	0,0051	0,0051	0,0059	0,0084	0,0044	0,0054
Lithuania	0,0112	0,0153	0,0094	0,0100	0,0044	0,0069	0,0090	0,0085	0,0058	0,0076	0,0071	0,0111	0,0049	0,0065
Mauritius	0,0213	0,0267	0,0118	0,0180	0,0024	0,0101	0,0143	0,0137	0,0091	0,0124	0,0122	0,0177	0,0056	0,0091
Morocco	0,0131	0,0174	0,0082	0,0109	0,0027	0,0068	0,0093	0,0084	0,0059	0,0077	0,0069	0,0123	0,0039	0,0062
Nigeria	0,0140	0,0203	0,0086	0,0113	0,0044	0,0061	0,0112	0,0108	0,0053	0,0132	0,0085	0,0171	0,0061	0,0081
Oman	0,0086	0,0095	0,0045	0,0061	0,0022	0,0048	0,0059	0,0055	0,0055	0,0041	0,0055	0,0081	0,0033	0,0040
Pakistan	0,0199	0,0273	0,0157	0,0167	0,0036	0,0116	0,0149	0,0142	0,0111	0,0129	0,0184	0,0187	0,0090	0,0108
Romania	0,0142	0,0183	0,0127	0,0117	0,0042	0,0079	0,0109	0,0113	0,0065	0,0083	0,0103	0,0129	0,0070	0,0088
Serbia	0,0126	0,0167	0,0061	0,0106	0,0019	0,0053	0,0076	0,0071	0,0058	0,0076	0,0051	0,0109	0,0037	0,0049
Slovenia	0,0121	0,0155	0,0099	0,0101	0,0057	0,0082	0,0096	0,0099	0,0063	0,0071	0,0079	0,0124	0,0048	0,0076
Tunisia	0,0032	0,0035	0,0021	0,0030	0,0014	0,0024	0,0025	0,0027	0,0012	0,0033	0,0013	0,0027	0,0010	0,0021
Vietnam	0,0156	0,0180	0,0120	0,0116	0,0046	0,0090	0,0112	0,0100	0,0096	0,0085	0,0087	0,0133	0,0069	0,0088

	New Zealand	Norway	Portugal	Singapore	Spain	Sweden	Switzerland	United Kingdom	United States	Bahrain	Bangladesh	Croatia	Estonia
Australia	0,0108	0,0154	0,0094	0,0143	0,0141	0,0105	0,0066	0,0126	0,0111	0,0128	0,0054	0,0080	0,0159
Austria	0,0110	0,0184	0,0133	0,0175	0,0219	0,0148	0,0079	0,0160	0,0145	0,0159	0,0057	0,0103	0,0208
Belgium	0,0071	0,0114	0,0086	0,0104	0,0120	0,0099	0,0070	0,0114	0,0102	0,0075	0,0036	0,0062	0,0108
Canada	0,0082	0,0127	0,0073	0,0115	0,0132	0,0091	0,0051	0,0108	0,0101	0,0106	0,0037	0,0070	0,0124
Denmark	0,0037	0,0044	0,0052	0,0047	0,0061	0,0055	0,0047	0,0045	0,0045	0,0025	0,0003	0,0026	0,0047
Finland	0,0058	0,0091	0,0069	0,0087	0,0087	0,0077	0,0052	0,0082	0,0070	0,0063	0,0018	0,0046	0,0078
France	0,0069	0,0123	0,0093	0,0106	0,0132	0,0099	0,0061	0,0106	0,0089	0,0091	0,0031	0,0065	0,0111
Germany	0,0072	0,0120	0,0090	0,0109	0,0137	0,0106	0,0072	0,0108	0,0094	0,0075	0,0029	0,0064	0,0107
Hong Kong	0,0077	0,0088	0,0061	0,0117	0,0090	0,0082	0,0045	0,0081	0,0070	0,0074	0,0004	0,0038	0,0064
Ireland	0,0062	0,0089	0,0064	0,0084	0,0109	0,0092	0,0047	0,0092	0,0077	0,0069	0,0019	0,0057	0,0089
Israel	0,0078	0,0088	0,0048	0,0100	0,0104	0,0082	0,0056	0,0087	0,0096	0,0064	0,0020	0,0048	0,0093
Italy	0,0074	0,0128	0,0098	0,0129	0,0176	0,0116	0,0070	0,0117	0,0096	0,0105	0,0039	0,0077	0,0130
Japan	0,0045	0,0069	0,0046	0,0071	0,0075	0,0064	0,0049	0,0064	0,0065	0,0038	0,0009	0,0039	0,0049
Netherlands	0,0054	0,0090	0,0076	0,0080	0,0106	0,0081	0,0057	0,0085	0,0076	0,0063	0,0021	0,0049	0,0076
New Zealand	0,0114	0,0074	0,0060	0,0083	0,0077	0,0067	0,0050	0,0075	0,0068	0,0053	0,0013	0,0055	0,0073
Norway	0,0074	0,0176	0,0100	0,0113	0,0126	0,0099	0,0059	0,0114	0,0094	0,0113	0,0051	0,0075	0,0116
Portugal	0,0060	0,0100	0,0144	0,0063	0,0094	0,0066	0,0057	0,0069	0,0057	0,0085	0,0015	0,0055	0,0085
Singapore	0,0083	0,0113	0,0063	0,0155	0,0123	0,0098	0,0052	0,0102	0,0089	0,0098	0,0031	0,0068	0,0104
Spain	0,0077	0,0126	0,0094	0,0123	0,0184	0,0108	0,0065	0,0114	0,0094	0,0105	0,0044	0,0081	0,0131
Sweden	0,0067	0,0099	0,0066	0,0098	0,0108	0,0104	0,0056	0,0095	0,0082	0,0066	0,0018	0,0052	0,0082
Switzerland	0,0050	0,0059	0,0057	0,0052	0,0065	0,0056	0,0064	0,0059	0,0055	0,0036	0,0011	0,0037	0,0044
United Kingdom	0,0075	0,0114	0,0069	0,0102	0,0114	0,0095	0,0059	0,0112	0,0089	0,0080	0,0026	0,0062	0,0095
United States	0,0068	0,0094	0,0057	0,0089	0,0094	0,0082	0,0055	0,0089	0,0092	0,0067	0,0022	0,0048	0,0085
Bahrain	0,0053	0,0113	0,0085	0,0098	0,0105	0,0066	0,0036	0,0080	0,0067	0,0184	0,0041	0,0076	0,0122
Bangladesh	0,0013	0,0051	0,0015	0,0031	0,0044	0,0018	0,0011	0,0026	0,0022	0,0041	0,0127	0,0027	0,0043
Croatia	0,0055	0,0075	0,0055	0,0068	0,0081	0,0052	0,0037	0,0062	0,0048	0,0076	0,0027	0,0085	0,0082
Estonia	0,0073	0,0116	0,0085	0,0104	0,0131	0,0082	0,0044	0,0095	0,0085	0,0122	0,0043	0,0082	0,0210
Jordan	0,0008	0,0022	0,0007	0,0031	0,0030	0,0018	0,0008	0,0023	0,0015	0,0024	0,0024	0,0022	0,0031
Kazakhstan	0,0069	0,0146	0,0065	0,0125	0,0163	0,0078	0,0049	0,0099	0,0097	0,0135	0,0069	0,0097	0,0128
Kenya	0,0043	0,0073	0,0032	0,0089	0,0073	0,0053	0,0027	0,0058	0,0058	0,0064	0,0025	0,0039	0,0093
Lithuania	0,0062	0,0088	0,0070	0,0088	0,0108	0,0068	0,0039	0,0076	0,0069	0,0094	0,0033	0,0062	0,0119
Mauritius	0,0093	0,0159	0,0071	0,0149	0,0182	0,0109	0,0046	0,0134	0,0112	0,0176	0,0074	0,0109	0,0215
Morocco	0,0056	0,0104	0,0058	0,0102	0,0123	0,0070	0,0025	0,0077	0,0064	0,0119	0,0058	0,0079	0,0138
Nigeria	0,0054	0,0125	0,0058	0,0115	0,0177	0,0090	0,0042	0,0100	0,0071	0,0117	0,0077	0,0083	0,0121
Oman	0,0042	0,0075	0,0050	0,0071	0,0068	0,0046	0,0034	0,0053	0,0046	0,0092	0,0016	0,0046	0,0068
Pakistan	0,0107	0,0156	0,0073	0,0184	0,0176	0,0122	0,0057	0,0146	0,0128	0,0178	0,0057	0,0115	0,0196
Romania	0,0076	0,0127	0,0081	0,0113	0,0120	0,0081	0,0077	0,0106	0,0101	0,0120	0,0045	0,0077	0,0131
Serbia	0,0061	0,0085	0,0058	0,0093	0,0113	0,0058	0,0034	0,0077	0,0065	0,0127	0,0039	0,0082	0,0121
Slovenia	0,0075	0,0103	0,0094	0,0097	0,0133	0,0071	0,0057	0,0085	0,0068	0,0095	0,0038	0,0082	0,0126
Tunisia	0,0015	0,0023	0,0023	0,0021	0,0027	0,0020	0,0009	0,0019	0,0014	0,0032	0,0018	0,0023	0,0037
Vietnam	0,0074	0,0124	0,0081	0,0129	0,0126	0,0089	0,0054	0,0103	0,0088	0,0138	0,0054	0,0076	0,0132

	Jordan	Kazakhstan	Kenya	Lithuania	Mauritius	Morocco	Nigeria	Oman	Pakistan	Romania	Serbia	Slovenia	Tunisia	Vietnam
Australia	0,0045	0,0164	0,0090	0,0112	0,0213	0,0131	0,0140	0,0086	0,0199	0,0142	0,0126	0,0121	0,0032	0,0156
Austria	0,0047	0,0207	0,0110	0,0153	0,0267	0,0174	0,0203	0,0095	0,0273	0,0183	0,0167	0,0155	0,0035	0,0180
Belgium	0,0009	0,0080	0,0079	0,0094	0,0118	0,0082	0,0086	0,0045	0,0157	0,0127	0,0061	0,0099	0,0021	0,0120
Canada	0,0025	0,0161	0,0080	0,0100	0,0180	0,0109	0,0113	0,0061	0,0167	0,0117	0,0106	0,0101	0,0030	0,0116
Denmark	0,0001	0,0032	0,0040	0,0044	0,0024	0,0027	0,0044	0,0022	0,0036	0,0042	0,0019	0,0057	0,0014	0,0046
Finland	0,0015	0,0067	0,0049	0,0069	0,0101	0,0068	0,0061	0,0048	0,0116	0,0079	0,0053	0,0082	0,0024	0,0090
France	0,0024	0,0104	0,0066	0,0090	0,0143	0,0093	0,0112	0,0059	0,0149	0,0109	0,0076	0,0096	0,0025	0,0112
Germany	0,0023	0,0111	0,0067	0,0085	0,0137	0,0084	0,0108	0,0055	0,0142	0,0113	0,0071	0,0099	0,0027	0,0100
Hong Kong	0,0013	0,0071	0,0051	0,0058	0,0091	0,0059	0,0053	0,0055	0,0111	0,0065	0,0058	0,0063	0,0012	0,0096
Ireland	0,0028	0,0099	0,0051	0,0076	0,0124	0,0077	0,0132	0,0041	0,0129	0,0083	0,0076	0,0071	0,0033	0,0085
Israel	0,0004	0,0105	0,0059	0,0071	0,0122	0,0069	0,0085	0,0055	0,0184	0,0103	0,0051	0,0079	0,0013	0,0087
Italy	0,0034	0,0133	0,0084	0,0111	0,0177	0,0123	0,0171	0,0081	0,0187	0,0129	0,0109	0,0124	0,0027	0,0133
Japan	0,0016	0,0067	0,0044	0,0049	0,0056	0,0039	0,0061	0,0033	0,0090	0,0070	0,0037	0,0048	0,0010	0,0069
Netherlands	0,0011	0,0078	0,0054	0,0065	0,0091	0,0062	0,0081	0,0040	0,0108	0,0088	0,0049	0,0076	0,0021	0,0088
New Zealand	0,0008	0,0069	0,0043	0,0062	0,0093	0,0056	0,0054	0,0042	0,0107	0,0076	0,0061	0,0075	0,0015	0,0074
Norway	0,0022	0,0146	0,0073	0,0088	0,0159	0,0104	0,0125	0,0075	0,0156	0,0127	0,0085	0,0103	0,0023	0,0124
Portugal	0,0007	0,0065	0,0032	0,0070	0,0071	0,0058	0,0058	0,0050	0,0073	0,0081	0,0058	0,0094	0,0023	0,0081
Singapore	0,0031	0,0125	0,0089	0,0088	0,0149	0,0102	0,0115	0,0071	0,0184	0,0113	0,0093	0,0097	0,0021	0,0129
Spain	0,0030	0,0163	0,0073	0,0108	0,0182	0,0123	0,0177	0,0068	0,0176	0,0120	0,0113	0,0133	0,0027	0,0126
Sweden	0,0018	0,0078	0,0053	0,0068	0,0109	0,0070	0,0090	0,0046	0,0122	0,0081	0,0058	0,0071	0,0020	0,0089
Switzerland	0,0008	0,0049	0,0027	0,0039	0,0046	0,0025	0,0042	0,0034	0,0057	0,0077	0,0034	0,0057	0,0009	0,0054
United Kingdom	0,0023	0,0099	0,0058	0,0076	0,0134	0,0077	0,0100	0,0053	0,0146	0,0106	0,0077	0,0085	0,0019	0,0103
United States	0,0015	0,0097	0,0058	0,0069	0,0112	0,0064	0,0071	0,0046	0,0128	0,0101	0,0065	0,0068	0,0014	0,0088
Bahrain	0,0024	0,0135	0,0064	0,0094	0,0176	0,0119	0,0117	0,0092	0,0178	0,0120	0,0127	0,0095	0,0032	0,0138
Bangladesh	0,0024	0,0069	0,0025	0,0033	0,0074	0,0058	0,0077	0,0016	0,0057	0,0045	0,0039	0,0038	0,0018	0,0054
Croatia	0,0022	0,0097	0,0039	0,0062	0,0109	0,0079	0,0083	0,0046	0,0115	0,0077	0,0082	0,0082	0,0023	0,0076
Estonia	0,0031	0,0128	0,0093	0,0119	0,0215	0,0138	0,0121	0,0068	0,0196	0,0131	0,0121	0,0126	0,0037	0,0132
Jordan	0,0383	0,0061	0,0008	0,0024	0,0045	0,0040	0,0036	0,0003	0,0060	0,0019	0,0035	0,0023	0,0014	0,0037
Kazakhstan	0,0061	0,0395	0,0091	0,0111	0,0187	0,0132	0,0221	0,0081	0,0177	0,0139	0,0165	0,0132	0,0034	0,0113
Kenya	0,0008	0,0091	0,0157	0,0068	0,0097	0,0070	0,0106	0,0037	0,0110	0,0080	0,0044	0,0062	0,0029	0,0082
Lithuania	0,0024	0,0111	0,0068	0,0120	0,0137	0,0102	0,0105	0,0053	0,0143	0,0105	0,0084	0,0099	0,0035	0,0101
Mauritius	0,0045	0,0187	0,0097	0,0137	0,0404	0,0205	0,0181	0,0110	0,0291	0,0151	0,0177	0,0161	0,0048	0,0183
Morocco	0,0040	0,0132	0,0070	0,0102	0,0205	0,0157	0,0125	0,0068	0,0187	0,0093	0,0116	0,0108	0,0039	0,0130
Nigeria	0,0036	0,0221	0,0106	0,0105	0,0181	0,0125	0,0507	0,0073	0,0169	0,0133	0,0172	0,0140	0,0036	0,0107
Oman	0,0003	0,0081	0,0037	0,0053	0,0110	0,0068	0,0073	0,0097	0,0113	0,0086	0,0077	0,0067	0,0008	0,0079
Pakistan	0,0060	0,0177	0,0110	0,0143	0,0291	0,0187	0,0169	0,0113	0,0446	0,0200	0,0155	0,0135	0,0032	0,0213
Romania	0,0019	0,0139	0,0080	0,0105	0,0151	0,0093	0,0133	0,0086	0,0200	0,0240	0,0141	0,0116	0,0026	0,0148
Serbia	0,0035	0,0165	0,0044	0,0084	0,0177	0,0116	0,0172	0,0077	0,0155	0,0141	0,0251	0,0110	0,0025	0,0117
Slovenia	0,0023	0,0132	0,0062	0,0099	0,0161	0,0108	0,0140	0,0067	0,0135	0,0116	0,0110	0,0170	0,0042	0,0110
Tunisia	0,0014	0,0034	0,0029	0,0035	0,0048	0,0039	0,0036	0,0008	0,0032	0,0026	0,0025	0,0042	0,0077	0,0037
Vietnam	0,0037	0,0113	0,0082	0,0101	0,0183	0,0130	0,0107	0,0079	0,0213	0,0148	0,0117	0,0110	0,0037	0,0212

Efficient Frontier Composition

Efficient Frontier (P1-A)

	Standard deviation	Return	Sharpe
Portfolio 1	21,68%	5,81%	0,2208
Portfolio 2	13,64%	6,03%	0,3668
Portfolio 3	11,88%	6,24%	0,4394
Portfolio 4	**11,93%**	**6,33%**	**0,4449**
Portfolio 5	12,41%	6,46%	0,4382
Portfolio 6	13,48%	6,68%	0,4196
Portfolio 7	14,75%	6,90%	0,3982
Portfolio 8	16,19%	7,11%	0,3762
Portfolio 9	17,86%	7,33%	0,3531
Portfolio 10	19,85%	7,55%	0,3287
Portfolio 11	22,65%	7,77%	0,2976

Efficient Frontier (P1-B)

	Standard deviation	Return	Sharpe
Portfolio 1	20,67%	5,9%	0,2339
Portfolio 2	11,46%	6,4%	0,4709
Portfolio 3	9,91%	7,0%	0,6062
Portfolio 4	**9,76%**	**7,3%**	**0,6384**
Portfolio 5	11,16%	7,64%	0,5923
Portfolio 6	15,30%	8,2%	0,4720
Portfolio 7	17,97%	8,5%	0,4145

Efficient Frontier (P1-C)

	Standard deviation	Return	Sharpe
Portfolio 1	21,73%	5,9%	0,2259
Portfolio 2	13,05%	6,4%	0,4134
Portfolio 3	10,63%	7,0%	0,5650
Portfolio 4	9,14%	7,6%	0,7235
Portfolio 5	**8,23%**	**8,31%**	**0,8851**
Portfolio 6	10,11%	8,9%	0,7748
Portfolio 7	13,95%	9,5%	0,6052

Efficient Frontier (P2-A)

	Downside risk	Return	Sortino
Portfolio 1	13,26%	5,81%	0,3609
Portfolio 2	7,84%	6,03%	0,6379
Portfolio 3	**7,39%**	**6,30%**	**0,7140**
Portfolio 4	7,74%	6,46%	0,7024
Portfolio 5	8,41%	6,68%	0,6723
Portfolio 6	9,27%	6,90%	0,6332
Portfolio 7	10,30%	7,11%	0,5915
Portfolio 8	11,47%	7,33%	0,5499
Portfolio 9	12,77%	7,55%	0,5108
Portfolio 10	14,42%	7,77%	0,4673

Efficient Frontier (P2-B)

	Downside risk	Return	Sortino
Portfolio 1	13,23%	5,9%	0,3655
Portfolio 2	6,94%	6,4%	0,7779
Portfolio 3	**6,34%**	**7,0%**	**0,9488**
Portfolio 4	8,07%	7,6%	0,8194
Portfolio 5	11,02%	8,25%	0,6552
Portfolio 6	12,63%	8,5%	0,5896

Efficient Frontier (P2-C)

	Downside risk	Return	Sortino
Portfolio 1	14,68%	5,9%	0,3343
Portfolio 2	7,92%	6,4%	0,6810
Portfolio 3	6,43%	7,0%	0,9337
Portfolio 4	5,72%	7,6%	1,1556
Portfolio 5	**5,55%**	**8,08%**	**1,2706**
Portfolio 6	5,82%	8,2%	1,2411
Portfolio 7	8,00%	8,9%	0,9790
Portfolio 8	11,84%	9,5%	0,7127

Efficient Frontier (P3-A)

	Standard deviation	Return	Sharpe
Portfolio 1	12,42%	8,2%	0,5791
Portfolio 2	12,04%	9,3%	0,6840
Portfolio 3	12,74%	10,3%	0,7280
Portfolio 4	13,72%	11,3%	0,7514
Portfolio 5	**14,33%**	**11,86%**	**0,7558**
Portfolio 6	15,15%	12,4%	0,7492
Portfolio 7	17,08%	13,4%	0,7252
Portfolio 8	19,43%	14,4%	0,6909

Efficient Frontier (P3-B)

	Standard deviation	Return	Sharpe
Portfolio 1	11,95%	8,51%	0,6264
Portfolio 2	9,71%	9,40%	0,8624
Portfolio 3	10,17%	10,58%	0,9395
Portfolio 4	11,11%	11,75%	0,9662
Portfolio 5	**11,33%**	**11,98%**	**0,9669**
Portfolio 6	12,60%	12,93%	0,9451
Portfolio 7	14,88%	14,11%	0,8795
Portfolio 8	17,75%	15,29%	0,8037
Portfolio 9	21,15%	16,47%	0,7302

Efficient Frontier (P3-C)

	Standard deviation	Return	Sharpe
Portfolio 1	12,22%	9,0%	0,6485
Portfolio 2	8,86%	9,4%	0,9456
Portfolio 3	8,30%	10,6%	1,1506
Portfolio 4	**9,05%**	**11,7%**	**1,1819**
Portfolio 5	10,32%	12,93%	1,1537
Portfolio 6	12,22%	14,1%	1,0713
Portfolio 7	14,46%	15,3%	0,9863
Portfolio 8	17,45%	16,5%	0,8849